See-saws, Swings, and Roundabouts in Media Markets[1]

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Abstract

We analyze the positive and normative effects of policy interventions in oligopolistic advertising-financed media markets. We uncover a general see-saw effect: Changes in the market structure that make the consumer side better off tend to make the advertiser side worse off. In our framework the impacts of market structure changes on the three groups—advertisers, platforms and consumers—are tied together by a simple summary statistic. The prohibition of a profitable merger is consumer-surplus increasing, but tends to be advertiser-surplus decreasing if viewers have unlimited attention for ads. This result can be turned around if consumers have limited attention.

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1 Introduction

In advertising-financed media, changes in advertising prices and levels tend to affect consumers and advertisers in opposite directions. In this paper we establish this as a general see-saw principle. Advertisers and viewers tend to have opposed interests with respect to policy interventions and other exogenous changes in the market environment: when consumer surplus increases, advertiser surplus tends to decrease. To establish this principle and to investigate strategic interaction in media market more generally, we develop a framework with asymmetric oligopoly media platforms which can be formulated as an aggregative game. This allows us to obtain clear-cut results on the distribution of prices, market shares, and profits in equilibrium and on the competitive effects of mergers, entry, and advertising levels. Furthermore, we show that in the standard media economics setting consumer surplus can be tracked as functions depending only on the aggregator and not the individual actions.

The standard model of two-sided markets as applied to media economics (Anderson and Coate, 2005) builds in a "competitive bottleneck" feature (Armstrong, 2006) which implies there is no direct competition for advertisers. The first model proposed in this paper -a model with consumers allocating their scarce leisure time on the consumption of programming on various platforms - exhibits the same features. At a given point in time, a particular viewer can only be reached through the single channel she is watching. This gives platforms monopoly power over advertisers. The monopoly bottleneck means that competition among platforms is effectively competition for viewers, and so an increase in the number of platforms is predicted to decrease equilibrium ad levels, much like product prices decrease with the number of firms in standard oligopoly models of product competition. Similarly, according to the model a merger between media platforms results in higher advertising levels and higher ad prices. This model serves as a useful starting point of media economics, even though, as discussed by Anderson, Foros, Kind, and Peitz (2012), empirical support for these predictions are scarce.

We then introduce interaction on the advertiser side. Such interaction can arise for a number of reasons. The simplest approach would be to assume that advertisers single-home. Then, an advertiser attracted by one media platform is an advertiser lost for another platform. While this tends to reverse the above mentioned result, introducing single-homing on the advertiser side is unsatisfactory in many media markets since advertisers often use the option to advertise on multiple media platforms.[1] Alternative avenues can be pursued. First, multi-homing viewers introduce interaction on the advertiser side because any given viewer can possibly be reached through multiple channels.[2] Second, a limited attention for ads (or a limited capacity to recall ads) together with viewers spending time on multiple platforms introduces interaction on the advertiser side. In this paper we explore multi-homing viewers with a limited attention in detail.

Advertisers try hard to reach viewers. However, as is widely documented in the marketing literature, viewers pay attention to and recall only a fraction of ads and this fraction decreases in the total volume of ads. Another feature of mass media consumption is that viewers tend to spend only a fraction of their time on one particular media platform. Thus, any given media platform is not the sole access provider to the viewers attention and the viewer's attention can be seen as a common property resource to which multiple media platforms have access. One purpose of this paper is to develop an alternative model of the media market which takes these two features

[1] Kaiser and Wright () empirically investigate a two-sided single-homing model in selected duopoly markets. Advertisers may not be indifferent as to which viewer they reach raising the possibility of tailoring ads to consumers (see, e.g., Athey and Gans, and Bergemann and Bonnati); considering the effects of mergers in the presence of tailored ads is an interesting topic.

[2] Whether or not, multi-homing generates interaction on the advertiser side, depends on the advertising technology, as disussed below. Anderson, Foros, and Kind (2012, 2013) and Ambrus, Calvano, and Reisinger (2013) present models with multi-homing.

into account and delivers testable cross-section and comparative statics predictions. As we show, the predictions of this alternative model stand in contrast to the standard model of a media market as a two-sided platform market. An intervention which swings the equilibrium to a state which benefits consumers but harms advertisers under unlimited attention for ads swings in the opposite direction benefitting consumers and harming advertisers under limited attention.

The mechanism which reverses our initial findings is related to negative externalities due to congestion arising from limited attention. Suppose that a platform cannot deliver the viewer with certainty to advertisers. Then, through the congestion function, one firm's choice of ad level will affect the willingness to pay for advertising on other platforms when viewers mix their media consumption. Large platforms internalize congestion to a larger extent than small platforms implying that the former have fewer ads and charge more for them. Nevertheless, viewers are exposed to more ads on large than on small platforms. Entry of a media platform in this setting will lead media platforms to less internalization of the negative congestion effect. Thus, more competition among media platforms will increase ad levels (which is in line with some observed market facts, such as the entry of Fox television). A merger, by contrast, makes the merged firm reduce its advertising level leading to a lower advertising prices, and, in equilibrium, all firms reduce their advertising levels. A reduction in congestion tends to be welfare-increasing, while more congestions is welfare-decreasing.

This paper brings in information congestion into platform pricing using the general approach followed in the various papers of Anderson and de Palma on information congestion. Specifically, it is assumed here that the viewer only has a limited attention span for ads, and is therefore only able to process a fixed number, ϕ, of all the ads to which she is exposed. Notice that this analysis renders endogenous the platform prices in the presence of congestion, as well as dealing with multiple platforms competing for attention. By setting up the model so as to induce an aggregative game, it therefore allows for access to the comparative static and characterization properties of those games, allowing for asymmetries between any finite number of media platforms.

Our model applies to a number of media markets. A key property of the model is that although advertisers and viewers multi-home, an ad by a particular advertiser is seen at most once by a viewer. Television markets and internet markets may endogenously lead to this property. In the television context, this properties means that advertisers gain most from the first contact because a second contact is only valuable if the first contact did not reach the viewer due to congestion. Hence, if she were able to choose whether to contact a viewer a second time or to newly contact a different viewer, an advertiser would always go for the latter option. This implies that advertisers prefer to place theirs ads exactly at the same time slot when advertising on multiple platforms. Such a strategy can be applied in television markets. A different motivation can be given in the context of internet advertising. Here, the advertising level refers to the frequency e.g. a new ad pops up. Consumers randomly click through various websites and spend some time on them. Different from television, there are no time slots from advertising. Our model applies if the information on ad exposure is shared among platforms. Then, since consumer behavior is identified through cookies it is possible to track consumer behavior. Hence, a platforms knows whether a consumer has already been exposed to a given ad and for each pop-up randomly selects an ad among the winning bidders which has not been seen by the consumer before. In this case advertisers make bids for advertising space conditional on viewers not being exposed to the ad on some other platform. We call this type of advertising "synchronized advertising." Synchronized advertising avoids partly wasteful multiple exposures. It will emerge endogenously if participating advertisers are not too heterogeneous in their willingness to pay.

2 Preliminaries on aggregative games

Characterization and comparative static properties are derived from the aggregative game approach. The media market models in this paper will have an aggregative game structure: most of their properties are not derived here, but follow in quite a straightforward manner from the aggregative game analysis. For background references to such topics as the IIA property, logit and CES formulations, and differentiated product models of oligopoly, see Anderson, de Palma, and Thisse (1992). For aggregative game tools, see Anderson, Erkal, and Picinnin (2013).

Assume that each platform's profit can be written as $\Pi^i(\psi_i, \Psi)$ where ψ_i is firm i's action variable and $\Psi = \Sigma_{j=1}^{n} \psi_j$ is the aggregator (see Acemoglu and Jensen, 2011, for example). Prominent examples include oligopolies with price-setting firms and logit or CES demand.

Assume further that the game is competitive in the sense that a higher Ψ reduces profits (the simplest example is the homogeneous products Cournot model for which the aggregator is simply aggregate output). The first-order condition is

$$\Pi_1^i(\psi_i, \Psi) + \Pi_2^i(\psi_i, \Psi) = 0$$

where the subscripts denote partial derivatives with respect to the corresponding arguments. If Π_i is strictly quasi-concave, for interior solutions, this equation implicitly defines the cumulative best reply function, $r_i(\Psi)$, as i's action that brings the total to Ψ.[3] This differs from the standard way to write best replies as functions of the actions of all other players. However, for aggregative games, the two concepts are quite related. In particular, under some mild regularity conditions, r_i is an increasing function if actions are strategic complements.[4]

[3]This concept has been introduced by Selten (1970).

[4]See Anderson, Erkal and Picinnin (2013) for details.

Equilibrium constitutes a fixed point, namely the equilibrium value of the aggregator is given by $\Psi^* = \Sigma r_i(\Psi^*)$ which is depicted simply graphically as the point where the sum of the cumulative reaction functions crosses the 45-degree line. The equilibrium is unique if $\Sigma r_i'(\Psi) < 1$. Hence, for strategic complements, we need an upper bound of the slope of the cumulative best reply.

Assumption $r_i'(\Psi) < r_i(\Psi)/\Psi$ for all $i = 1, ..., n$.

Summing over all i, assumption 2 implies that $\Sigma r_i'(\Psi) < 1$.

Both the difference across agents in equilibrium and comparative statics can be depicted and derived simply with this device. In particular, "weaker" agents (in the sense of those with lower cumulative reaction functions) have lower equilibrium actions, and a change rendering an agent's behavior more aggressive (i.e., shifting up its reaction function) will increase its own equilibrium action, increase the aggregator and increase or decrease other players' actions according to whether actions are strategic complements or substitutes respectively.

3 Basic media economics

The model

We consider a market in which media deliver viewers to advertisers. Viewer and advertisers are atomless; in particular, each individual ad is atomless. We rank advertisers in terms of decreasing willingness to pay, p, to contact viewers and so $p(a_i)$ is the per-viewer willingness to pay of the marginal (a_ith) advertiser. This determines the demand price for ads on platform i, P_i, as the the probability of reaching a given consumer λ_i, which is equal to the fraction of time spent on platform i times the conditional willingness to pay, namely $P_i = \lambda_i p(a_i)$. [5]

[5] In our framework in which viewer obtain zero surplus within an advertiser-viewer interaction this assumption appears to be natural. More generally, one may want to allow for some correlation between product characteristics and the likelihood to recall an ad.

Advertisers multi-home: Advertisers with a high willingness to pay advertise on all platforms; advertisers with a rather low willingness to pay advertise on few platforms, if at all. However, advertisers do not waste impressions since ad placements are perfectly synchronized. In the context of television and radio advertising, a way to think about this assumption is to consider advertising within a given time window and to allow advertisers who post ads on multiple platforms to choose the time slot of their ads. Choosing the same time slot for all ads of a given advertiser is the most efficient use of an advertiser's advertising budget. It implies that an advertiser's ad can be viewed at most once by any given viewer even though viewers multi-home.[6] In the context of internet advertising we can think about perfect synchronization to arise when web sites place cookies and share this information with each other. To extract most from advertisers, they may fill an ad space only with those advertisers which did not yet have contacted with the same ad on a different platform. If visits to websites occur in random order, this is equivalent to perfect synchronized advertising in our model.

We assume that p is log-concave and twice continuously differentiable in a_i. This implies that the revenue per viewer, $R(a_i) = a_i p(a_i)$ is also log-concave in a_i. Platform i's profit is

$$\Pi_i = a_i \lambda_i(\mathbf{a}) p\left(a_i\right) = R(a_i)\lambda_i(\mathbf{a})$$

Suppose that viewer demand can be written as ψ_i/Ψ where ψ_i is a monotone transformation of a_i and $\Psi = \sum_i \psi_i$, in other words, demand of platform i depends only on its own transformed action ψ_i and the aggregator. Writing $a_i = a_i(\psi_i)$ viewer demand of the form $\lambda_i(a_1(\psi_1), ..., a_n(\psi_n)) = \psi_i/\Psi$ includes oligopoly models with logit and the Luce (1959)-form of demand and duopoly models such as the one

[6]The coordination of advertising across platforms makes the model very similar to a model in which viewers single-home. Thus, the mixing of programs by itself does not affect the results of the standard model; see also Peitz and Valletti (2008).

presented in Anderson and Coate (2005). We then have

$$\begin{aligned}\Pi_i &= \lambda_i a_i p(a_i) \\ &= R(a_i(\psi_i))\frac{\psi_i}{\Psi}.\end{aligned}$$

The first-order condition of platform i is thus

$$\frac{\partial \Pi_i}{\partial \psi_i} = R'(a_i(\psi_i))a_i'(\psi_i)\frac{\psi_i}{\Psi} + R(a_i(\psi_i))\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) = 0.$$

Rewriting this expression we obtain

$$-a_i'(\psi_i)\psi_i \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))} = 1 - \frac{\psi_i}{\Psi}. \tag{1}$$

To derive an explicit expression for viewer demand with the property that all viewers are identical and mix between media, we introduce a time-use consumer model. Suppose that the viewer time demand function is derived from the utility function

$$\max \sum_{i=1}^{m} [s_i(1-a_i)\lambda_i]^{\alpha} \qquad \text{s.t. } \sum_{i=1}^{n} \lambda_i = 1 \tag{2}$$

with $\alpha \in (0,1)$ so that viewers like to mix between different platforms. Here, λ_i is the fraction of time spent on platform i and s_i stands for the content quality offered by platform i. Only $(1-a_i)\lambda_i$ is actual program content, due to the ads interjecting, so $s_i(1-a_i)\lambda_i$ captures the "quality time" spent on platform i. The idea here is that the viewer only values the content part of a program and advertising sections gives a benefit normalized to zero.

Demand is

$$\lambda_i(\mathbf{a}) = \frac{(s_i(1-a_i))^{\alpha/(1-\alpha)}}{\Sigma(s_j(1-a_j))^{\alpha/(1-\alpha)}} = \frac{\psi_i}{\Psi}, \tag{3}$$

where $\psi_i = (s_i(1-a_i))^{\alpha/(1-\alpha)}$. Thus we have shown that the proposed time-use model has the property that demand for platform i depends only on the transformed action ψ_i and its aggregator Ψ.

Equilibrium properties

In the time-use model, the first-order conditions (1) can be rewritten as

$$\frac{\psi_i}{\Psi} = 1 - \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))} \frac{1 - a_i(\psi_i)}{\widetilde{\alpha}}, \tag{4}$$

where $\widetilde{\alpha} = \alpha/(1-\alpha)$. First-order conditions define cumulative best replies r_i which only depend on the aggregator Ψ. In an interior equilibrium, the right-hand side of equation (4) is strictly positive. It is denoted by $J(\psi_i)$ and, thus, $\psi_i/J(\psi_i) = \Psi$.

Lemma 1 *(Strategic complementarity) cumulative best replies are increasing in the aggregator, $r_i'(\Psi) > 0$.*

Proof. Differentiating the inverse of the best reply, $\Psi = \psi_i/J(\psi_i)$ we obtain

$$\frac{d\Psi}{d\psi_i} = \frac{1}{J(\psi_i)} - \frac{\psi_i J'(\psi_i)}{J^2(\psi_i)}.$$

This is positive if we can show that J' is negative.

Using (4),

$$\begin{aligned} sign(J') &= sign\left(-\frac{d}{d\psi_i}\left(\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}(1 - a_i(\psi_i))\right)\right) \\ &= sign\left(\frac{d}{da_i}\left(\frac{R'(a_i)}{R(a_i)}(1 - a_i)\right)\right) \\ &= sign\left(\frac{d}{da_i}\left(\frac{R'(a_i)}{R(a_i)}\right)(1 - a_i) - \frac{R'(a_i)}{R(a_i)}\right) \end{aligned}$$

Since R is log-concave and a_i is chosen in the increasing part of R both terms in the above expression are negative. ■

As mentioned in the previous section, equilibrium uniqueness with strategic complements follows from Assumption 2—i.e., $r_i'(\Psi) < r_i(\Psi)/\Psi$. In the following remark, we provide sufficient conditions that imply that Assumption 2 holds (the proof is relegated to Appendix 1).

Remark 1 *If R is concave and $\alpha < 1/2$, then $r_i' < r_i/\Psi$.*

We also note that $r_i' < r_i/\Psi$ (which becomes inequality (12)) implies that the second-order condition $d^2\Pi_i/d\psi_i^2 < 0$ holds.

To evaluate the effect of policy interventions, we have to understand how market shares $\lambda_i = \psi_i/\Psi$ depend on the aggregator. Suppose that we compare two situations with aggregators such that $\Psi'' > \Psi'$. We call all those platforms outsiders whose cumulative best reply function are the same in both situations—i.e., the policy intervention has no direct effect on the outsiders' behavior. For outsiders we can sign the change in market share, as the following result establishes.

Proposition 1 $\Psi' > \Psi''$ *implies that* $\lambda_i(\Psi') < \lambda_i(\Psi'')$.

Proof. Inequality $\lambda_i(\Psi') < \lambda_i(\Psi'')$ is equivalent to

$$\frac{r_i(\Psi')}{\Psi'} < \frac{r_i(\Psi'')}{\Psi''}.$$

Defining ψ_i^M as the monopoly action of platform i, we have that $r_i(\psi_i^M) = \psi_i^M$ and $r_i'(\psi_i^M) < 1$. We must have $\Psi'' > \psi_i^M$. Assumption 1 requires $r_i'(\Psi) < r_i(\Psi)/\Psi$. This implies that r_i/Ψ is decreasing in Ψ. Hence, $\lambda_i(\Psi') = r_i(\Psi')/\Psi' < r_i(\Psi'')/\Psi'' = \lambda_i(\Psi'')$. ■

In words, the market share of outsiders decreases if the aggregator is increased. This result will be useful to evaluate the changes on those platforms which are not affected by a policy intervention or an exogenous change of the market structure such as platform entry or a merger.

Summing the first-order conditions over all media platforms (see equation (4)), we obtain

$$\sum_i \frac{R'(a_i)}{R(a_i)}(1 - a_i) = (n-1)\widetilde{\alpha}$$

In the symmetric case ($s_1 = s_2 = ... = s_n$), $a_i = a$ for all i and the above equation boils down to

$$\frac{R'(a)}{R(a)}(1-a) = \frac{n-1}{n}\widetilde{\alpha}.$$

This implicitly defines the equilibrium advertising level as a function of the number of firms.

In asymmetric markets, not only the number of platforms, but the distribution over platform characteristics s_i matter. For a give number of platforms, we investigate how the relative position of platforms with respect to their characteristic s_i translates into their relative position with respect to market share λ_i and advertising level a_i. Ranking firms according to content quality, $s_1 \geq s_2 \geq ... \geq s_n$, it must hold that, in equilibrium, $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ and $a_1 \geq a_2 \geq ... \geq a_n$ if the only difference between media platforms is their content quality (in particular, no joint ownership or cross share-holdings). Whenever $s_i > s_j$, it must be that $\lambda_i > \lambda_j$ and $a_i > a_j$.

Proposition 2 *Consider any two platforms i and j. In equilibrium, $s_i > s_j$ implies that $\lambda_i > \lambda_j$ and $a_i > a_j$.*

The proof is relegated to Appendix 1. The proposition establishes that high-quality platforms carry more ads than lower-quality platforms, but are still more attractive such that consumers spend more time on them than on lower-quality platforms despite the higher nuisance. This finding is analogous to price competition models with horizontal product differentiation and quality differences between firms: In those models, a high-quality firm set a higher price and obtains a larger markets share than lower-quality firms.

Platform profits are $\Pi_i = R(a_i)\lambda_i$. As shown above, a platform with higher-quality content than another platform, $s_i > s_j$, has a larger market share $\lambda_i > \lambda_j$ and more advertising $a_i > a_j$. Platforms choose advertising in the increasing part of R and, therefore, $R(a_i) > R(a_j)$. Therefore, $s_i > s_j$ implies that, in equilibrium, $\Pi_i > \Pi_j$.

Properties of advertiser and consumer surplus

We start by considering consumer surplus CS, which is given by inserting the expression for demand given in (3) into the utility function provided in (2). This gives CS as utils per time unit. To express it in monetary units, we have to solve a larger problem in which consumers decide how much time $\sum_{i=1}^{n} \lambda_i$ to dedicate to media consumption (which can be interpreted as leisure time) and how much time λ_0 to work. In Appendix 2 we show that a Cobb-Douglas specification of the labor-leisure trade-off yields demand $\lambda_i = (1-\lambda_0)\psi_i/\Psi$ where λ_0 is independent of $(s_1, ..., s_n)$ and $(a_1, ..., a_n)$.

Proposition 3 *Consumer surplus is an increasing function of the aggregator.*

Note that we can track viewer welfare solely from changes in the aggregator. Under symmetry, the viewer welfare in the maximized utility function is $n^{1-\alpha} (s(1-a))^{\alpha}$.

We now turn to net advertiser surplus $AS^N = \sum_{i=1}^{n} AS_i^N$ where the advertiser surplus on platform i is

$$\begin{aligned} AS_i^N &= \lambda_i \int_0^{a_i} (p(x) - p(a_i))dx \\ &= \frac{\psi_i}{\Psi} \int_0^{a_i(\psi_i)} (p(x) - p(a_i))dx. \end{aligned} \tag{5}$$

Since media platforms choose actions $\psi_i = r_i(\Psi)$ as functions of the aggregator, net advertiser surplus is a function of the aggregator. For given r_i, we know that a larger value of the aggregator leads to a larger action ψ_i and, thus, a lower advertising level. Ignoring the composition effect of Ψ which affects market shares, an increase in Ψ is thus net advertiser surplus decreasing. This arguments suggests that a see-saw effect is at play, namely a larger value of the aggregator increases consumer surplus, but decreases advertiser surplus. We have to qualify this see-saw effect by the need to take into account the direct effect which triggers the change of Ψ leading to composition effects in the form of changing market shares λ_i.

In a comparative statics analysis the direct effect leads to a change of r_i for some i (e.g., for a merger)—these are the insiders. By contrast, the outsiders' cumulative best reply functions are unaffected. Hence, the overall effect on advertiser surplus is given by a change in r_i for the insiders and a consecutive change in the aggregator. We first consider advertiser surplus generated on outsider platforms.

Proposition 4 *On the set of outsider media platforms O, a decrease in the aggregator leads to higher advertisers' net surplus and higher media platform profits and, hence, higher advertisers' gross surplus, in particular, $dAS_i^N/d\Psi < 0$ and $d\Pi_i/d\Psi < 0$ for all $i \in O$.*

Proof.

$$\begin{aligned}
\frac{dAS_i^N}{d\Psi} &= -\frac{1}{\Psi^2}\psi_i \int_0^{a_i}(p(x)-p(a_i))dx + \frac{1}{\Psi}\left(r_i'(\Psi)\int_0^{a_i}(p(x)-p(a_i))dx + \psi_i(-a_i p'(a_i))a_i' r_i'(\Psi)\right) \\
&= -\frac{1}{\Psi}\left[-\left(\frac{\psi_i}{\Psi} - r_i'(\Psi)\right)\int_0^{a_i}(p(x)-p(a_i))dx - (\psi_i(-a_i p'(a_i))a_i' r_i'(\Psi))\right]
\end{aligned}$$

Since $r_i'(\Psi) < r_i(\Psi)/\Psi$ this expression must be negative. Hence, $\sum_{i\in O}\frac{dAS_i^O}{d\Psi} < 0$, where O is the set of outsider media platforms.

Recalling that $\Pi_i = \frac{\psi_i}{\Psi}R(a_i(\psi_i))$, we have

$$\begin{aligned}
\frac{d\Pi_i^*}{d\Psi} &= -\frac{\psi_i}{\Psi^2}R + \frac{\psi_i'}{\Psi}(R + \psi_i R' a_i') \\
&= -\frac{\psi_i}{\Psi^2}R + \frac{\psi_i'}{\Psi}R(1 + \psi_i \frac{R'}{R}a_i') \\
&= -\frac{\psi_i}{\Psi^2}R + \frac{\psi_i'}{\Psi}R\frac{\psi_i}{\Psi} \\
&= \frac{\psi_i}{\Psi^2}R(1-\psi_i') < 0
\end{aligned}$$

where we used equation (1) and the observation that $r_i'(\Psi) < r_i(\Psi)/\Psi$ implies $r_i'(\Psi) < 1$. ■

The effects on insiders depend on the particular exogenous variation. In what follows, we consider three such types, media mergers, entry, and advertising regulation.

Media mergers

Media mergers have received quite some attention in the policy debate. Here, we explore the implications of our model on the allocative effects of a media merger and its welfare implications.

Lemma 2 *A merger of two media platforms leads to a decrease of the aggregator.*

Proof. This is seen as follows. The merged entity (or indeed, a cooperative venture or other coordination between agents) maximizes joint profits $\Pi_i(\psi_i, \Psi) + \Pi_j(\psi_j, \Psi)$. The first-order condition then becomes

$$\frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \psi_i} + \frac{\partial \Pi_i(\psi_i, \Psi)}{\partial \Psi} + \frac{\partial \Pi_j(\psi_j, \Psi)}{\partial \Psi} = 0.$$

The two first-order conditions can be solved simultaneously to find ψ_i and ψ_j as functions of the aggregator, giving $\widetilde{r}_i(\Psi)$ and $\widetilde{r}_j(\Psi)$ as the individual cumulative best reply functions under merger. The last term on the left-hand side of both first-order conditions is negative. This implies that the cumulative reply function $\widetilde{r}_i$ must take a lower value than the corresponding function before the merger, i.e., $\widetilde{r}_i(\Psi) < r_i(\Psi)$ for all Ψ with $r_i(\Psi) > 0$. The same feature applies to the other platform j which is part of the merger. Hence, the total contribution of the merged entity $\widetilde{r}_i(\Psi) + \widetilde{r}_j(\Psi)$ is lower for a given Ψ. Therefore, the merger results very straightforwardly in a lower equilibrium value of the aggregator. ■

The above lemma is rather general and applies to various settings including the amended models presented in the following sections. Non-merging platforms are better off as a consequence of the lower equilibrium value of the aggregator, as $\Pi_i(\psi_i, \Psi)$ is decreasing in Ψ, which has been established in Proposition 4. (Note that this is independent of whether or not they increase or decrease their equilibrium actions.)

The merged entity is better off. To see this, we first observe that, for given Ψ, internalizing the "pricing" externality among the two platforms engaged in the merger necessarily leads to higher profits. Since the equilibrium value of the aggregator is lower after the merger, profits $\Pi_i(\psi_i, \Psi) + \Pi_j(\psi_j, \Psi)$ also increase due to the reaction of competitors (strategic complements). Thus, similar to mergers in price competition models with differentiated products, a merger is always profitable.

Concerning surplus on the two sides of the market, we have to evaluate consumer surplus and advertiser surplus. Since CS is increasing in Ψ (as established in Proposition 3), the merger is consumer surplus-decreasing.

Consider now advertiser surplus. As we have shown in Proposition 4, the advertiser surplus generated on each outsider media platform increases. What is happening is that, after the merger, each outsider's platform features a larger advertising level and, thus, $\int_0^{a_i}(p(x)-p(a_i))dx$ increases. Also the market share of each outsider media platform increases, as shown in Proposition 1. Hence, advertiser surplus associated to each outsider media platform must increase.

Consider now the insiders. Also here $\int_0^{a_i}(p(x)-p(a_i))dx$ is increased by the merger. However, the overall effect on the advertiser surplus associated with the merged entity is unclear because $\widetilde{\lambda}_i + \widetilde{\lambda}_j < \lambda_i + \lambda_j$ after the merger of media platforms i and j. We obtain an unambiguous result on net advertiser surplus AS^N if we consider a merger between the two lowest-quality advertising platforms when they are not too asymmetric.[7] We summarize our findings as follows:

Proposition 5 *Consider a two-firm merger among platforms. Consumer surplus decreases, while net advertiser surplus of outsiders increases. Net advertiser surplus of insiders necessarily increases if the merger involves the two lowest-quality media platforms and if $s_n - s_{n-1}$ small.*

Proof. The results on consumer surplus and net advertiser surplus of outsiders have

[7] We restrict attention to the case in which consumers spend a fixed amount of time on media consumption. The corresponding utility model endogenizing the labor-leisure trade-off is presented in the Appendix.

been established above. It remains to consider net advertiser surplus of insiders. We recall that platforms n and $n-1$ also feature more advertising after the merger than before. Furthermore, from Proposition 2 we know that a lower-quality-platform i has a smaller advertising level prior to the merger than platform i. Hence,

$$\int_0^{a_i} (p(x) - p(a_i))dx < \int_0^{a_{i-1}} (p(x) - p(a_i))dx < ....$$

Thus, market share needs to be shifted away from low-quality firms towards high-quality firms. It remains to be shown that λ_n and $\lambda_{n-1} + \lambda_n$ decrease after the merger. The latter is a direct implication of Proposition 1 since the aggregator is down and, thus, all outsiders have a larger market share. The former necessarily holds if $s_n = s_{n-1}$ and, thus, by continuity, for $s_n - s_{n-1}$ sufficiently small. ■

Propositions 3 and 5 together with Lemma 2 establish that the merger between the two lowest-quality firms decreases consumer surplus but increases net advertiser surplus. This is an instance of the see-saw effect: The change in market structure resulting from the merger makes the consumer side worse offer, while it makes the advertiser side better off. While we do need additional conditions to show this implications, in many instances the see-saw effects still holds if these conditions are violated. We also note that, when the time spent on media is fixed, the see-saw effect is always present in symmetric markets.[8]

Another instance of a clear cut result is in the special case of a uniform distribution of advertisers such that $p(a)$ is uniformly distributed on $[0, \overline{p}]$. We have established above, that any two-firm merger involving platforms i and j is profitable—i.e., we must have that $\lambda_i a_i p(a_i) + \lambda_j a_j p(a_j) < \widetilde{\lambda}_i \widetilde{a}_i p(\widetilde{a}_i) + \widetilde{\lambda}_j \widetilde{a}_j p(\widetilde{a}_j)$, where $\widetilde{a}_j$ denotes the advertising level after the merger. Suppose that $s_i = s_j$. If we can show that $\widetilde{\lambda}_i \widetilde{a}_i p(\widetilde{a}_i)/(\lambda_i a_i p(a_i)) \leq \widetilde{AS}_i / AS_i$, we have established that the merger is net adver-

[8] An outside option would divert some time spent on advertising-finances media on other activities. After a merger, advertising nuisance increases and, therefore, advertiser surplus can possibly be reduced.

tiser surplus increasing also on the insider platforms. The ratio of the net advertiser surplus can be written as

$$\frac{\widetilde{AS}_i}{AS_i} = \frac{\frac{\widetilde{\lambda}_i}{2}(\overline{p} - p(\widetilde{a}_i))\widetilde{a}_i}{\frac{\lambda_i}{2}(\overline{p} - p(a_i))a_i} = \frac{\widetilde{\lambda}_i(\overline{p} - p(\widetilde{a}_i))\widetilde{a}_i}{\lambda_i(\overline{p} - p(a_i))a_i}$$

We thus have to show that $\frac{\overline{p}-p(\widetilde{a}_i)}{\overline{p}-p(a_i)} \geq \frac{p(\widetilde{a}_i)}{p(a_i)}$ which necessarily holds as $a_i < \widetilde{a}_i$ and, thus, $p(a_i) > p(\widetilde{a}_i)$. Hence, in the uniform specification any merger between two platforms of equal quality is advertiser surplus increasing. By continuity, this also holds a merger including two firms of similar quality.

- Next we allow for synergies in the form of higher s_i of the merged parties. Consider synergies such that ψ_ι of the merged parties remain the same before and after the merger. This implies that the merged parties increase their advertising a_i so as to leave their action ψ_i unchanged. Consequently, Ψ is the same before and after the merger. This in turn implies that the merger is consumer-surplus neutral. However, since a_i has increased on the merged platform, advertiser net surplus has gone up.

- Considering only mergers which do not change the ψ_i among the merged platform appears to be restrictive. Consider now a consumer surplus-neutral merger after which the content quality of the parties involved in the merger weakly increase. If, after the merger, $\widetilde{s}_i > \widetilde{s}_j$ of the merged platforms i and j, then $\widetilde{a}_i > \widetilde{a}_j$ and $\widetilde{\lambda}_i > \widetilde{\lambda}_j$. Since the merger is welfare neutral, $\psi_i + \psi_j = \widetilde{\psi}_i + \widetilde{\psi}_j$. This implies that outside the special case considered above, after the merger either one of the platforms has a larger market share than before the merger. A sufficient condition for the merger to be advertiser surplus increasing is (i) each of the platform chooses a larger advertising level than the maximal advertising level among those two platforms prior to the merger—i.e., $\min\{\widetilde{a}_i, \widetilde{a}_j\} > \max\{\widetilde{a}_i, \widetilde{a}_j\}$—or (ii) the higher-quality platform prior to the merger gains market share as a result of the merger—i.e., $\widetilde{\psi}_i - \psi_i$ is positive whenever $s_i > s_j$ prior to the merger.

For mergers with lower synergies than those required for a consumer surplus neutral merger, the see-saw effect applies: Consumers are worse off after the merger, while advertisers and platforms are better off.[9]

Entry of media platforms

We now consider exogenous entry into a media platform—such exogenous entry may be the outcome of regulatory measures.

Proposition 6 *With symmetric platforms the equilibrium advertising level decreases with entry of media platforms.*

Proof. Writing profits as a function of the revised action variable, $\psi_i = (s_i(1-a_i))^{\widetilde{\alpha}}$, gives $\Pi_i = \frac{\psi_i}{\Psi} a_i(\psi_i) p(a_i(\psi_i))$. Substituting $\frac{d\psi_i}{da_i} = -\widetilde{\alpha} \frac{\psi_i}{(1-a_i)}$ into the first-order condition for a symmetric equilibrium, $-\frac{\psi n'}{n}(1+\varepsilon) = \left(1 - \frac{1}{m}\right)$ yields

$$\frac{1}{\widetilde{\alpha}} \frac{(1-n)}{n} (1+\varepsilon) = \left(1 - \frac{1}{m}\right).$$

Here $(1-n)/n$ is positive and decreasing in n and also, by assumption, $1+\varepsilon$ is positive and decreasing in n. Thus, the LHS is decreasing in n. Since an increase in m leads to an increase of the RHS, the equilibrium advertising level must decrease with entry.

■

Advertising regulation and platform-specific advertising

Suppose that some platforms are ad-capped as a result of advertising regulation. These platforms would be the large ones, as they choose the largest advertising level. The ad-cap renders the individual cumulative reaction function horizontal and, thus, increases the cumulative reaction function. Consequently, if it is binding for at least

[9] Clearly, the merged firm is better off after the merger. Since the aggregator is less than before the merger, also profits of outsider platforms are higher after the merger.

one platform it brings down the aggregator. Due to strategic complementarity, ψ_i of the non-constrained platforms must decrease. In equilibrium, we may then have a mix of large, ad capped stations and smaller, non-constrained firms, while the reverse cannot happen.

Since all platforms reduce advertising levels (leading to a higher aggregator Ψ) consumers are necessarily better off whenever advertising caps are introduced which are binding for at least one firm. Advertisers, by contrast, tend to suffer from such regulation. Advertisers have to accommodate lower advertising levels for all platforms. However, an effect in the opposite direction is that the ad-capped firms gain market share, which benefits advertisers since the ad-capped firms offer more advertising space then firms for which the cap is not binding. Hence, also advertising regulation tends to exhibit a see-saw effect.

- welfare analysis: positive direct effect of advertising regulation if advertising is socially excessive absent regulation; demand is shifted towards ad-capped platforms; this is also a positive effect; however, advertising goes down for non-capped platforms and advertising may be socially insufficient for small media platforms. The overall effect determines whether total surplus goes up or down.

An important extension is to extend the viewer model to allow for varying the degree of multi-homing viewership. To do so will enable the study of how competition among platforms varies as a function of the ability to pick up viewers elsewhere. In the viewer model (**??**), this can be done by having s_i depend on a type, x, and have x distributed in the population. This can be simply seen for "Hotelling" preferences (in duopoly) where types are distributed over an interval. Different configurations can be generated by varying the distribution over the interval: for example, a point-mass at each end can generate no multi-homing, while a point-mass in the middle generates symmetric full multi-homing. Alternatively, values of s could be drawn from a distribution: as its support collapses to a point, then full multi-homing symmetry is retained, and a bigger support gives more idiosyncratic preferences and a lower degree of multi-homing.

4 Advertising congestion

4.1 Advertising congestion with exogenous viewer behavior

The model

We amend the model of the previous section by introducing limited attention on the consumer side. In this section, we abstract from the indirect external effect that advertisers exert on viewers; thus we use the same model except that advertising does not enter as a nuisance.[10] Consequently consumers spend a fixed fraction λ_i of their time on each platform. We consider a market in which consumers can at most absorb ϕ ads. We can interpret ϕ as the attention span of consumers. Platform i contains a_i ads, so the expected number of ads seen on platform i is $\lambda_i a_i$. The expected total number of ads seen by a viewer is $A = \Sigma_{i=1}^{n} \lambda_i a_i$ so that the probability of retaining an ad from platform i is $\max\{1, \frac{\phi}{A}\}$.

We focus on situations in which, in oligopoly equilibrium, the expected total number of ads A is larger than the viewer attention span ϕ so that there is congestion in equilibrium.

Congestion can only be an equilibrium in oligopoly because a monopolist would never choose $a > \phi$. To see the latter, note that with $a > \phi$, the monopolist would be able to sell an ad at price $(\phi/a)p(a)$ yielding profit $\phi p(a)$. This choice is dominated by the choice $a = \phi$ yielding profit $\phi p(\phi)$ because p is decreasing in a.

[10]Specifically suppose that the viewer time demand function is derived from maximizing the utility function $\max_{\lambda_1,\ldots,\lambda_m} \sum_{i=1}^{m} (s_i \lambda_i)^{\alpha}$ subject to $\sum_{i=1}^{1} \lambda_i = 1$. Hence, viewer demand is $\lambda_i(\mathbf{s}) = s_i^{\tilde{\alpha}} / \sum_{j=1}^{m} s_j^{\tilde{\alpha}}$ which maps exogenous program qualitities into market shares. A program of higher quality thus enjoys a larger market share.

As before, we rank advertisers in terms of decreasing willingness to pay, p, to contact viewers and so $p(a_i)$ is the willingness to pay of the marginal (a_ith) advertiser, which determines the demand price for ads on platform i as the product of the probability that the viewer retains the ad and is on the platform and the conditional willingness to pay, namely $\frac{\lambda_i \phi}{A} p(a_i)$. Here, we implicitly assume that the likelihood of remembering an ad is independent of the particular product that is advertised. In particular, it is independent of the advertiser's willingness to pay.[11]

We continue to assume that p is log-concave and twice continuously differentiable. Platform i's profit is

$$\Pi_i = a_i \frac{\lambda_i \phi}{A} p(a_i).$$

Platform interdependence comes from the joint assumption that the A ads are seen across multiple channels (so viewers are mixing) and that there is advertising congestion. If there are a_i ads on platform i, the ad price per viewer is the per-viewer willingness to pay of the marginal advertiser - i.e., $p(a_i)\phi/A$.

We define the action variable $\psi_i = \lambda_i a_i$ which represent the advertising exposure on platform i. Then, the aggregator is $\Psi = A$. We can then replace a_i by ψ_i/λ_i and write platform i's profit as a function of ψ_i. Platform i's problem is

$$\max_{\psi_i} \ \phi \frac{\psi_i}{\Psi} p\left(\frac{\psi_i}{\lambda_i}\right), \quad i = 1, ..., n.$$

Notice the aggregative game structure here appears through the congestion function. Platform competition arises through the advertising side of the market because competition for viewers has been closed down for now.

(One way to see the decomposition of different effects is to write the platform's profits in logs:

[11] In our framework this assumption appears to be natural since viewers obtain zero surplus within an advertiser-viewer interaction and, thus, are ex post indifferent as to which ads they remembered. More generally, one may want to allow for some correlation between product characteristics and the likelihood to recall an ad.

$$\ln \Pi^i = \ln \lambda_i + \ln(a_i p_i(a_i)) + \ln[\phi/(\sum \lambda_j a_j)]. \tag{6}$$

An increase in a_i induces three changes: it reduces the number of viewers (first term), it affects the revenues per viewer $R(a_i) = a_i p_i(a_i)$, and it increases congestion.)

The first-order condition implicitly defines a relationship between ψ_i and Ψ. We note that the objective function has the same structure $\frac{\psi_i}{\Psi} f(\psi_i)$ as in the previous section. However, in the previous model, $a_i'(\psi_i) < 0$, while $a_i'(\psi_i) > 0$ in the present model.

The profit derivative is

$$\begin{aligned} \frac{d\Pi_i}{d\psi_i} &= \left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) p\left(\frac{\psi_i}{\lambda_i}\right) + \frac{\psi_i}{\lambda_i \Psi} p'\left(\frac{\psi_i}{\lambda_i}\right) \\ &= \frac{p\left(\frac{\psi_i}{\lambda_i}\right)}{\Psi} \left\{ \left(1 - \frac{\psi_i}{\Psi}\right) + \frac{\psi_i}{\lambda_i} \frac{p'\left(\frac{\psi_i}{\lambda_i}\right)}{p\left(\frac{\psi_i}{\lambda_i}\right)} \right\}. \end{aligned}$$

The pre-multiplying factor is positive, so to prove strict quasiconcavity of the objective function we need to show that the term in parentheses is strictly decreasing and positive for small values of ψ_i and negative for large values. The term $(1 - \frac{\psi_i}{\Psi})$ is linearly decreasing in ψ_i and takes values between 1 and zero. The elasticity on the advertiser side $\varepsilon(a_i) \equiv a_i \frac{p'(a_i)}{p(a_i)}$ is negative. Also, it is decreasing, as p is log-concave.

Thus, there is a unique solution to the first-order conditions characterized as

$$1 + \frac{\psi_i}{\lambda_i} \frac{p'\left(\frac{\psi_i}{\lambda_i}\right)}{p\left(\frac{\psi_i}{\lambda_i}\right)} = \frac{\psi_i}{\Psi}.$$

Since p is continuously differentiable, the implicitly defined cumulative best reply $\psi_i = r_i(\Psi)$ is continuous in Ψ.

First-order conditions can be written as

$$1 + \varepsilon\left(\frac{\psi_i}{\lambda_i}\right) = \frac{\psi_i}{\Psi}. \tag{7}$$

Since $\frac{\psi_i}{\Psi} \in (0,1)$, we have that $\varepsilon \in (-1,0)$ along the cumulative best reply.

Equilibrium properties

Lemma 3 *Actions ψ_i are strategic complements.*

Proof. Using equation (7), the slope of the cumulative best reply, writing its inverse as $\Psi = \frac{\psi_i}{1+\varepsilon}$, is given from $\Psi' = \frac{1}{1+\varepsilon} - \frac{\varepsilon' \psi_i}{(1+\varepsilon)^2}$, which is positive since ε is decreasing. ■

We now explore various cross-section variations.

Proposition 7 *In equilibrium, a larger platform shows fewer ads - i.e.,$\lambda_i > \lambda_j$ if and only if $a_i > a_j$. Larger platforms are more profitable.*

Proof. Using $1 + \varepsilon - \frac{\psi_i}{\Psi} = 0$ implies that

$$\frac{d\psi_i}{d\lambda_i} = -\frac{\varepsilon'\left(-\frac{\psi_i}{\lambda_i^2}\right)}{\varepsilon'\left(\frac{1}{\lambda_i}\right) - \frac{1}{\Psi}} = \frac{\varepsilon'\left(\frac{\psi_i}{\lambda_i^2}\right)}{\varepsilon'\left(\frac{1}{\lambda_i}\right) - \frac{1}{\Psi}}$$

so as ε is decreasing this is positive. While the action variable, the advertising exposure on channel i, ψ_i, is increasing in the size of the media channel, we are more interesting in knowing how the advertising level of a channel is related to its size.

$$\begin{aligned}
\frac{da_i}{d\lambda_i} &= \frac{d\left(\frac{\psi_i}{\lambda_i}\right)}{d\lambda_i} = \frac{\psi_i}{\lambda_i^2}\left(\frac{d\psi_i}{d\lambda_i}\frac{\lambda_i}{\psi_i} - 1\right) \\
&= \frac{\psi_i}{\lambda_i^2}\left(\frac{\varepsilon'\left(\frac{1}{\lambda_i}\right)}{\varepsilon'\left(\frac{1}{\lambda_i}\right) - \frac{1}{\Psi}} - 1\right) \\
&= \frac{\psi_i}{\lambda_i^2}\left(\frac{\frac{1}{\Psi}}{\varepsilon'\left(\frac{1}{\lambda_i}\right) - \frac{1}{\Psi}}\right)
\end{aligned}$$

The numerator of the term in round bracket is positive, while the denominator is negative. Thus a larger network shows fewer ads.

To show that a larger platform is more profitable than a smaller one, recall first that $\Pi_i = \phi \frac{\psi_i}{\Psi} p(a_i)$. We just showed above that a larger λ_i entails a smaller a_i, and hence a larger $p(a_i)$. Thus we just need to show that a larger λ_i entails a larger action variable, ψ_i. To show this property, recall that (see (7))

$$1 + a_i \frac{p'(a_i)}{p(a_i)} - \frac{\psi_i}{\Psi o} = 0,$$

with $a_i = \frac{\psi_i}{\lambda_i}$. Hence

$$\frac{d\psi_i}{d\lambda_i} = \frac{\frac{\psi_i}{\lambda_i^2}\varepsilon'(a_i)}{\frac{\varepsilon'(a_i)}{\lambda_i} - \frac{1}{\Psi}}$$

which is positive as desired because both the numerator and denominator are negative. ■

The negative relationship between market share λ_i and advertising level a_i is due to the negative externality a platform exerts on other platforms by increasing its advertising. Considering the limit cases, if $\lambda_i = 1$ platform i fully internalizes congestion and, thus, will decide not to have advertising beyond the viewer attention span. By contrast, a platform i with $\lambda_i = 0$ does not suffer from increasing the amount of advertising in terms of a reduced likelihood to reach viewers.

The result that larger platforms have fewer ads stands in stark contrast in our analysis in the previous section.

Properties of advertiser and consumer surplus

Consumer surplus in this setting is not affected by the decisions platforms make and completely exogenous. However, we still may want to track consumer surplus when considering policy interventions or exogenous changes in market structure as these may affect market shares λ_i. Absent indirect externalities (in the previous model they arose from ad nuisance), the present model does not feature a see-saw effect since consumer surplus is not linked to the outcome in the advertiser market.

Advertisers' net surplus is the sum of advertiser net surplus on each platform AS_i^N. Thus, $AS^N = \sum_{i=1}^n AS_i^N$ where

$$
\begin{aligned}
AS_i^N &= \int_0^{a_i} (p(x) - p(a_i)) \frac{\lambda_i \phi}{\Psi} dx \\
&= \frac{\phi}{\Psi} \lambda_i \int_0^{a_i} (p(x) - p(a_i)) dx. \qquad (8)
\end{aligned}
$$

where $a_i = \psi_i / \lambda_i$. We note that advertiser surplus depends on the aggregator and the cumulative best replies. Clearly, AS_i^N is increasing in a_i.

Media mergers

- cumulative best-reply of the merged firm down
- Hence, in equilibrium after the merger aggregator down; less congestion
- Due to strategic complementarity, also outsiders choose lower ψ_i than before the merger—i.e., they advertise less. Merged firm advertises less on each of its platforms.
- market shares λ_i not affected by the merger. Per-viewer willingness to pay of advertisers shifted up due to higher probability that the ad is remembered. However, fewer advertisers are served.
- Advertiser surplus tends to go up.

Entry of media platforms

Here, we consider the exogenous entry of a platform. The viewing time consumers spend on this new platform may previously have been spent on outside consumption or work. Alternatively, it may have been spent previously on one of initial platforms.

Amending the consumer model in Appendix 2 and postulating that advertising is not seen as a nuisance, viewers would reallocate the time spent on platforms

$1, ..., n$ and spend the available time now on $n+1$ platforms. The relative viewing times among existing platforms would not be affected by entry—i.e., λ_i/λ_j for $i, j \in \{1, ..., n\}$ is not affected by entry. First, we cover situations in the neighborhood of the symmetric case $\lambda_i = \lambda$ for all platforms in the market.

Proposition 8 *In a market in which the asymmetry between media platforms with respect to market share is sufficiently small, the entry of a media platform leads to lower advertising levels by all firms and a higher price per ad as well as a higher price per ad per viewer.*

Proof. Suppose that the market is fully covered and equally split, so that $\lambda_i = \frac{1}{n}$ for all i. Consider a symmetric equilibrium so that $\psi_i = \frac{\Psi}{n}$. Then the first-order condition becomes $1 + \varepsilon = \frac{1}{n}$, so that a higher n induces a lower ε and so the amount of advertising per channel goes up in this case too. By continuity, this holds also for asymmetric cases in which $\lambda_i - \lambda_j$ is sufficiently small for all i, j. ■

This shows how the model delivers advertising-increasing entry through more congestion. The intuition for this result is that entry here leads to a more pronounced negative externality via the congestion function. Under symmetry this implies that advertiser surplus decreases after entry. The reason is that a fraction of high-value advertisers do not reach viewers being replaced by lower-value advertisers. For exogenous reasons, consumer surplus increases: By increasing variety, consumers are necessarily better off in this model. Thus, consumer surplus and advertiser surplus move in opposite directions as a result of entry. However, as pointed out above, since consumer surplus is not linked to the outcome on the advertiser side, this model obviously lacks the see-saw effect.

Advertising regulation and platform-specific advertising

To evaluate the effect of advertising caps, we investigate the effect of a marginal change of a platform's advertising on gross advertiser surplus. Intuitively, if one firm increases its advertising level, then, everything else given, we would expect that advertiser surplus is decreasing. This is clearly the case if platforms are symmetric. However, if platforms are asymmetric, it is not necessarily the case since the largest platform with the smallest advertising level shifts some of the attention to other channels with higher advertising levels when it reduces its own advertising level. Thus, it is not necessarily the case that less advertising increases welfare. However, what may not hold for the largest platform, necessarily holds for the smallest platform. Since this is the platform with the largest advertising level, advertising regulation which starts to become binding is necessarily welfare-increasing.

Proposition 9 *If the level of advertising of the smallest platform is decreased, gross advertiser surplus increases. There is a platform $j \in \{1, 2, ...n\}$ such that a lower advertising level a_j is necessarily welfare-increasing, and also a lower advertising level a_k with $k > j$ is also necessarily welfare-increasing.*

Proof. We first have to show that advertiser gross surplus satisfies $-dAS^G/da_n > 0$.

$$\begin{aligned}
\frac{\partial AS}{\partial a_n} &= p(a_n)\frac{\lambda_n \phi}{\Psi} - \frac{\lambda_n \phi}{\Psi^2}\sum_{i=1}^{n}\lambda_i \int_0^{a_i} p(x)dx \\
&= \frac{\lambda_n \phi}{\Psi^2}\left[\sum_{i=1}^{n}\lambda_i a_i p(a_n) - \sum_{i=1}^{n}\lambda_i \int_0^{a_i} p(x)dx\right] \\
&= \frac{\lambda_n \phi}{\Psi^2}\left[\sum_{i=1}^{n}\lambda_i \left(a_i p(a_n) - \int_0^{a_i} p(x)dx\right)\right] \\
&< \frac{\lambda_n \phi}{\Psi^2}\left[\sum_{i=1}^{n}\lambda_i \left(a_i p(a_i) - \int_0^{a_i} p(x)dx\right)\right] \\
&< 0
\end{aligned}$$

Hence, for platform n the claim is satisfied. Suppose that $\frac{\partial W}{\partial a_j} < 0$ holds for some platform j. This implies that $\sum_{i=1}^{n}\lambda_i\left(a_i p(a_j) - \int_0^{a_i} p(x)dx\right) < 0$ must hold. Since $p(a_k) < p(a_j)$ for $k > j$, we must have $\frac{\partial W}{\partial a_k} < 0$ for all $k > j$. ■

It is important to note that the above proposition only considers the partial effect—i.e., a_{-i} is kept fixed. We would also like to know what happens to welfare, if for instance advertising regulation in the form of an upper bound $\overline{a}$ is introduced. In terms of congestion, this is clearly desirable as long as $(\sum_{i=1}^{n} \lambda_i)\overline{a} \geq \phi$ since it limits the advertising of those platforms constrained by the cap and reduces the advertising level chosen by the other platforms due to strategic complementarity. In effect, there are instances in which $\overline{a} < a_i$ absent regulation while $\overline{a} > a_i$ in the presence of regulation. However, a general welfare analysis is involved as it leads to reshuffling of ad exposure across platforms. Using expression (8), we obtain

$$\begin{aligned} dAS^G &= \sum_{i=1}^{n} \lambda_i \left[p(a_i) - \frac{1}{\sum_{j=1}^{n} a_j \lambda_j} \sum_{j=1}^{n} \lambda_j \int_0^{a_j} p(x)dx \right] da_i \\ &= \sum_{i=1}^{n} \lambda_i \left[p(a_i) - \frac{\sum_{j=1}^{n} a_j \lambda_j \overline{p}(a_j)}{\sum_{j=1}^{n} a_j \lambda_j} \right] da_i \end{aligned}$$

where $\overline{p}(a_j) = \int_0^{a_j} p(x)dx/a_j$ is the average surplus among a_j active intermediaries on platform j. Clearly, $p(a_i) < \overline{p}(a_i)$. However, here we are taking averages implying that some of the terms in square bracket may be positive. Even in the case of an advertising cap which, as the direct effect, reduces advertising on the platform which congests most and, as an equilibrium effect, also reduces advertising of all other platforms, it is not obvious that total surplus increases. For this one has to evaluate how the other platforms adjust in response to the introduction or the tightening of the regulation and we cannot show that advertising regulation necessarily is welfare-increasing. Of course, in the symmetric model, advertising regulation is binding for all platforms and total surplus is decreasing in $\overline{a}$ with $\overline{a} > \phi / \sum_i \lambda_i$. In this case,

$$\frac{dAS^G}{da} = \frac{\phi}{a^2}(ap(a) - \int_0^a p(x)dx) < 0.$$

By continuity, this holds in the neighborhood of a symmetric market. Even when market shares are very heterogeneous, a tightening of regulation is welfare increasing in the vicinity of $\phi/\sum_i \lambda_i$ as the regulation is binding for all platforms provided that the largest firm 1, as a monopolist, would not choose an interior solution—i.e., $a^m > \phi/\lambda_1$ where $a^m = \arg\max_a ap(a)$. The argument then follows from the fact that actions are strategic complements giving rise to an upward sloping cumulative best reply and, thus, also firm 1 chooses a socially excessive advertising level.

Note that in the model with congestion and multi-homing viewers the opposite may hold compared to the model with unlimited attention: The smaller stations tend to be ad-capped, while the larger channels are not constrained.

Let us now evaluate the effects of exogenous mergers and exogenous entry.

Public service broadcast. It is straightforward to introduce public broadcasters into the model. Suppose that the firm $i = 1, ..., m$ are public broadcasters, while private broadcasters are firms $i = m+1, ..., n$. Suppose furthermore that the advertising level of broadcasters is set by the regulator; this includes the possibility not to have any advertising on the public channels.

Program quality and viewer demand. To close the model, we have to endogenize viewer demand. Consider a market in which advertiser do not exert a negative externality on viewers, i.e. advertising is not seen as a nuisance but it provides utility in the same way as the surrounding program. In equilibrium, a high-quality platform chooses a lower advertising level than a platform which offers lower quality. Thus, our model delivers the cross-section prediction that quality and advertising levels are negatively correlated; put differently, higher content quality go hand in hand with more time for content. The causal chain in our model is that higher quality leads to larger market shares. This in turn implies that the media platform will choose a lower advertising level than a smaller alternative platform so as to keep the congestion low.

We note that viewer utility here depends on exogenous content quality and is independent of the advertising level. We may want to understand, how firms set content quality. More importantly, the property that viewer utility is independent of the advertising level is a bit odd, at least in the limit as a media platform fills all its air time with ads. In such a case, content quality would become irrelevant, which is not captured by our utility function. We address these issues in the following section.

5 Advertising congestion and advertising nuisance

The model

This extension allows for the full two-sided nature of the market. The externality from advertisers to viewers arises when viewers are annoyed by ads. We therefore analyze the full platform balance problem of delivering reluctant viewers to advertisers. The profit function depending on whether there is congestion takes the form

$$\Pi^i = \begin{cases} \frac{\lambda_i \phi a_i p(a_i)}{\sum \lambda_j a_j} & \text{for } \phi < \sum_{j=1}^m \lambda_j a_j \\ \lambda_i a_i p(a_i) & \text{for } \phi \geq \sum_{j=1}^m \lambda_j a_j \end{cases}$$

As before suppose that the viewer time demand function is given by (3), which we recall is

$$\lambda_i = \frac{[s_i(1-a_i)]^{\widetilde{\alpha}}}{\sum_{j=1}^m [s_j(1-a_j)]^{\widetilde{\alpha}}},$$

where $\widetilde{\alpha} = \alpha/(1-\alpha)$.

Finding an action variable to yield an aggregator is somewhat challenging, but we can take a convex combination of the action variables in the single-homing and fixed-λ congestion cases, and we now use the action variable $\psi_i = a_i[s_i(1-a_i)]^{\widetilde{\alpha}}$ and $\Psi = \sum_{j=1}^m \psi_j$. The profit of channel i is then

$$\begin{aligned} \Pi_i &= \frac{a_i[s_i(1-a_i)]^{\widetilde{\alpha}}\phi}{\sum_{j=1}^m a_j[s_j(1-a_j)]^{\widetilde{\alpha}}} p(a_i) \\ &= \phi \frac{\psi_i}{\Psi} p(a_i(\psi_i)) \end{aligned}$$

The objective function looks very similar to the one in the previous section. However, the relationship between ψ_i and a_i is different. Note that $\psi_i(a_i)$ is hump-shaped.

Nonetheless, the formulation still yields a viable aggregative game because p is decreasing, and so we can restrict attention to the increasing part of $\psi_i(a_i)$ along the cumulative best reply. Thus, $\psi_i(a_i)$ can be inverted: since $\psi_i = a_i[s_i(1-a_i)]^{\widetilde{\alpha}}$ then

$$\frac{d\psi_i}{da_i} = \frac{\psi_i(1-a_i-\widetilde{\alpha}a_i)}{a_i(1-a_i)} > 0. \tag{9}$$

In contrast to the decreasing relation $a_i(\psi_i)$ in the model with unlimited attention, under limited attention the relation is now increasing.

The first-order condition defining the cumulative best reply is now

$$p\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) + \frac{\psi_i}{\Psi}p'\frac{da_i}{d\psi_i} = 0, \tag{10}$$

where $\frac{da_i}{d\psi_i}$ is given as the reciprocal of (9).[12]

Equilibrium properties

First, we analyze the link between action ψ_i and the advertising level a_i.

Lemma 4 *In equilibrium in the congestion model with endogenous advertising nuisance, a larger action ψ_i is associated with a lower advertising level.*

Proof. Inserting the expression for $da_i/d\psi_i$ from (9) into the cumulative best reply (10) yields

$$\left(\frac{1}{\Psi} - \frac{\psi_i}{\Psi^2}\right) + \frac{\psi_i}{\Psi}\frac{p'}{p}\frac{a_i(1-a_i)}{\psi_i(1-a_i-\widetilde{\alpha}a_i)} = 0.$$

This simplifies to

$$\left(1 - \frac{\psi_i}{\Psi}\right) = -\frac{p'}{p}\frac{a_i(1-a_i)}{(1-a_i-\widetilde{\alpha}a_i)}. \tag{11}$$

The left-hand side is decreasing in ψ_i. It remains to be shown that the right-hand side is increasing in a_i. The slope of the right-hand side is

$$\begin{aligned} &-\left(\frac{p'}{p}\right)'\frac{a_i(1-a_i)}{1-a_i-\widetilde{\alpha}a_i} \\ &-\frac{p'}{p}\frac{(1-2a_i)(1-a_i-\widetilde{\alpha}a_i) + a_i(1-a_i)(1+\widetilde{\alpha})}{(1-a_i-\widetilde{\alpha}a_i)^2}. \end{aligned}$$

[12] Alternatively, this can be written in elasticity form as $\Psi - \psi_i = -\varepsilon(a_i)\,\eta(\psi_i)$ where $\eta(\psi_i) = \frac{\psi_i}{a_i}\frac{da_i}{d\psi_i}$.

Since

$$\begin{aligned} & (1-2a_i)(1-a_i-\widetilde{\alpha}a_i)+a_i(1-a_i)(1+\widetilde{\alpha}) \\ = \; & (1-a_i)(1-a_i-\widetilde{\alpha}a_i)+\widetilde{\alpha}a_i(1-a_i)+\widetilde{\alpha}a_i^2, \end{aligned}$$

the second term is positive. Log-concavity of p implies that $-\left(\frac{p'}{p}\right)'$ is positive. Hence, also the first term is positive and the right-hand side of (11) is increasing in a_i. ∎

Furthermore, it is readily shown that ψ_i is increasing in s_i and thus the equilibrium a_i is decreasing in s_i.

Properties of advertiser and consumer surplus

Advertiser net surplus is

$$\begin{aligned} AS^G &= \sum_i \lambda_i \int_0^{a_i(\psi_i)} (p(x)-p(a_i))dx \frac{\phi}{\sum_j a_j\lambda_j} \\ &= \sum_i \frac{\psi_i}{\Psi}\phi\frac{1}{a_i(\psi_i)}\int_0^{a_i(\psi_i)}(p(x)-p(a_i))dx. \end{aligned}$$

Since, in equilibrium $\psi_i = r_i(\Psi)$ for all i this surplus function depends only on the aggregator.

Consumer surplus for media consumption is

$$CS = \left(\sum_i \frac{\psi_i}{a_i(\psi_i)}\right)^{1-\alpha}$$

We note that

$$\begin{aligned} \frac{dCS}{d\Psi} &\propto \sum_i \frac{\psi_i'}{a_i(\psi_i)} - \frac{\psi_i}{a_i^2(\psi_i)}a_i'(\psi_i)\psi_i' \\ &= \sum_i \frac{\psi_i'}{a_i(\psi_i)}\left(1-\frac{\psi_i}{a_i(\psi_i)}a_i'(\psi_i)\right). \end{aligned}$$

The sign is negative if the elasticity $\frac{\psi_i}{a_i(\psi_i)}a_i'(\psi_i)$ is greater than one for all i and $\psi_i' > 0$. In our specification

$$a_i'(\psi_i) = \frac{1}{\frac{\psi_i}{a_i(\psi_i)} - a_i s_i^{\widetilde{\alpha}}(1-a_i)^{\widetilde{\alpha}-1}\widetilde{\alpha}}.$$

Thus,

$$\frac{dS_C}{d\Psi} \propto \sum_i \frac{\psi_i'}{a_i(\psi_i)} \left(1 - \frac{\psi_i}{\psi_i - a_i^2 s_i^{\widetilde{\alpha}}(1-a_i)^{\widetilde{\alpha}-1}\widetilde{\alpha}}\right) < 0$$

for given upward sloping cumulative best replies.

Thus, when AS^G is increasing in the aggregator we have the naive form of the see-saw effect that the two surpluses are diametrically opposed.

Mergers of media platforms

Entry of media platforms

To determine the effects of platform entry, consider a symmetric equilibrium at which

$$\frac{n-1}{\Psi}\frac{1}{a'} = -\frac{p'}{p}$$

Substituting from (9):

$$-\frac{p'}{p} = \frac{a-1}{\Psi}\frac{s^{\widetilde{\alpha}}(1-a-\widetilde{\alpha}a)}{(1-a)^{1-\widetilde{\alpha}}}$$

The LHS is increasing in a, while the RHS is decreasing in a (it can be shown that $\frac{(1-(1-\widetilde{\alpha})a)}{(1-a)^{1-\widetilde{\alpha}}}$ is decreasing for $\widetilde{\alpha} < 1$). Since a larger n shifts the RHS upward, the equilibrium advertising per channel increases with more platforms.

Proposition 10 *Entry of platforms implies higher advertising levels of all channels.*

This is an unambiguous result (and is still true with a non-viewing option) even though platforms compete for viewers and a larger a puts them at a disadvantage. Thus the externality effect through the congestion function dominates the competition effect and more competition leads to wasteful advertising in the sense that this advertising does not enter the attention span of viewers and is thus purely wasteful from a welfare perspective.

6 Appendix

6.1 Relegated proofs

Proof of Remark 1. Implicitly differentiating the first-order condition (1) we obtain

$$r_i' = \frac{R\psi_i/\Psi^2}{-a_i''\psi_i R' - a_i'R' - (a_i')^2\psi R'' - a_i'R'(1-\psi_i/\Psi) + R/\Psi}.$$

Thus, $r_i' < r_i/\Psi$ is equivalent to

$$R/\Psi < -a_i''\psi_i R' - a_i'R' - (a_i')^2\psi R'' - a_i'R'(1-\psi_i/\Psi) + R/\Psi$$

which reduces to

$$0 < -a_i''\psi_i R' - a_i'R' - (a_i')^2\psi R'' - a_i'R'(1-\psi_i/\Psi). \tag{12}$$

The second and the fourth term are positive since $a_i' > 0$ and R' positive when evaluated at the best response. If R concave, the third term is positive. Concerning the first term, in the time-use model, at $\alpha = 1/2$, we have that $a_i'' = 0$ as $s_i(1-a_i) = \psi_i$. Since $a_i(\psi_i) = 1 - \psi_i^{\frac{1-\alpha}{\alpha}}/s_i$, we have that $a_i'' < 0$ if $\alpha < 1/2$. In this case also the first term is positive. ■

Proof of Proposition 2. We first show that $\lambda_i > \lambda_j$ if and only if $a_i > a_j$. The proof is by contradiction. Suppose that $\lambda_i > \lambda_j$. Due to the first-order condition (4) we must have

$$1 - \frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\frac{1-a_i(\psi_i)}{\widetilde{\alpha}} > 1 - \frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))}\frac{1-a_j(\psi_j)}{\widetilde{\alpha}}$$

which is equivalent to

$$\frac{R'(a_i(\psi_i))}{R(a_i(\psi_i))}\Big/\frac{R'(a_j(\psi_j))}{R(a_j(\psi_j))} < \frac{1-a_j(\psi_j)}{1-a_i(\psi_i)}.$$

The log-concavity of R implies that R'/R is decreasing. Hence, if $a_i \leq a_j$ the left-hand side is greater or equal to one. However, the right-hand side is less or equal to one. This is a contradiction and, therefore, we must have $a_i > a_j$.

Since $\lambda_i = \psi_i/\Psi$, we have that $\psi_i > \psi_j$ if and only if $a_i > a_j$. We recall that

$$\psi_i = (s_i\,(1-a_i))^{\alpha/(1-\alpha)}.$$

Hence,

$$(s_i\,(1-a_i))^{\alpha/(1-\alpha)} > (s_j\,(1-a_j))^{\alpha/(1-\alpha)}$$

and $a_i > a_j$. The former inequality can only be satisfied if $s_i > s_j$. ■

Proof of Proposition 3. The consumers' maximized utility function is

$$\begin{aligned}
CS &= \sum_{i=1}^{n}\left[s_i(1-a_i)\frac{[s_i(1-a_i)]^{\alpha/(1-\alpha)}}{\sum_{j=1}^{m}[s_j(1-a_j)]^{\alpha/(1-\alpha)}}\right]^{\alpha} \\
&= \sum_{i=1}^{n}\left[\frac{[s_i(1-a_i)]^{1/(1-\alpha)}}{\sum_{j=1}^{m}[s_j(1-a_j)]^{\alpha/(1-\alpha)}}\right]^{\alpha} \\
&= \sum_{i=1}^{n}\frac{[s_i(1-a_i)]^{\alpha/(1-\alpha)}}{\left(\sum_{j=1}^{m}[s_j(1-a_j)]^{\alpha/(1-\alpha)}\right)^{\alpha}} \\
&= \sum_{i=1}^{n}\frac{\psi_i}{\Psi^{\alpha}} = \Psi^{1-\alpha}
\end{aligned}$$

CS is increasing in Ψ since $\alpha < 1$. In the monetized version, consumer surplus is an increasing function of $\Psi^{1-\alpha}$ and, therefore, inherits the property (see Appendix 2). ■

6.2 The labor-leisure decision and consumer surplus in dollars

Utility with labor-leisure decision

$$\max_{\boldsymbol{\lambda}} \beta \ln(\sum_{i=1}^{n}\lambda_i^{\alpha}u_i) + (1-\beta)\ln(\frac{\lambda_0 w + y}{p_0})$$

$$\text{s.t. } \lambda_0 + \sum_{i=1}^{n}\lambda_i = 1$$

where $u_i = (s_i(1-a_i))^\alpha$, w is the wage income if all time is spent on work, p_0 the price for the outside good and y is non-labor income.

The first-order conditions with respect to λ_i, $i \in \{1, ..., n\}$, are

$$\beta \frac{\alpha \lambda_i^\alpha u_i}{\sum_{i=1}^n \lambda_i^\alpha u_i} = \mu \lambda_i \tag{13}$$

where μ is the Lagrange multiplier. Summed over all $i = 1, ..., n$ we obtain

$$\alpha\beta = \mu(\sum_{i=1}^n \lambda_i) = \mu(1-\lambda_0)$$

or $\mu = \alpha\beta/(1-\lambda_0)$.

The first-order conditions with respect to λ_0

$$\begin{aligned} \mu &= \frac{(1-\beta)\frac{w}{p_0}}{\frac{\lambda_0 w + y}{p_0}} \\ &= \frac{1-\beta}{\lambda_0(1+y/(\lambda_0 w))} \end{aligned}$$

Thus, we obtain an explicit expression of λ_0 which only depends on α and β.

$$\frac{\alpha\beta}{1-\lambda_0} = \frac{1-\beta}{\lambda_0(1+y/(\lambda_0 w))}$$

or, equivalently,

$$\lambda_0 = \frac{1-\beta-\alpha\beta(y/w)}{1-\beta+\alpha\beta}$$

We can rewrite (13) as

$$(1-\lambda_0)\frac{\lambda_i^\alpha u_i}{\sum_{j=1}^n \lambda_j^\alpha u_j} = \lambda_i$$

Dividing this equation of platform offer i by the foc for platform offer j we obtain

$$\frac{\lambda_i^\alpha u_i}{\lambda_j^\alpha u_j} = \frac{\lambda_i}{\lambda_j}$$

which is equivalent to

$$\left(\frac{u_i}{u_j}\right)^{\frac{1}{1-\alpha}} = \frac{\lambda_i}{\lambda_j}$$

Solving for λ_j we can write the foc for platform offer i as

$$\begin{aligned}
\lambda_i &= (1-\lambda_0)\frac{\lambda_i^\alpha u_i}{\sum_{j=1}^n \lambda_i^\alpha \left(\frac{u_j}{u_i}\right)^{\frac{\alpha}{1-\alpha}} u_j} \\
&= (1-\lambda_0)\frac{u_i}{\sum_{j=1}^n \left(\frac{u_j}{u_i}\right)^{\frac{\alpha}{1-\alpha}} u_j} \\
&= (1-\lambda_0)\frac{u_i^{1+\frac{\alpha}{1-\alpha}}}{\sum_{j=1}^n u_j^{\frac{\alpha}{1-\alpha}} u_j} \\
&= (1-\lambda_0)\frac{u_i^{\frac{1}{1-\alpha}}}{\sum_{j=1}^n u_j^{\frac{1}{1-\alpha}}} \\
&= (1-\lambda_0)\frac{(s_i\,(1-a_i))^{\alpha/(1-\alpha)}}{\Sigma(s_j\,(1-a_j))^{\alpha/(1-\alpha)}}
\end{aligned}$$

We can then express consumer surplus in dollars.

$$\begin{aligned}
CS &= \beta \ln\left[\sum_{i=1}^n (\lambda_i s_i (1-a_i))^\alpha\right] + (1-\beta)\ln\left(\frac{\lambda_0 w + y}{p_0}\right) \\
&= \beta \ln\left[(1-\lambda_0)^\alpha \sum_{i=1}^n \left(\frac{(s_i\,(1-a_i))^{\alpha/(1-\alpha)}}{\Sigma(s_j\,(1-a_j))^{\alpha/(1-\alpha)}} s_i(1-a_i)\right)^\alpha\right] + \text{constant} \\
&= \beta \ln\left[(1-\lambda_0)^\alpha \sum_{i=1}^n \left(\frac{[s_i(1-a_i)]^{1/(1-\alpha)}}{\sum_{j=1}^m [s_j(1-a_j)]^{\alpha/(1-\alpha)}}\right)^\alpha\right] + \text{constant} \\
&= \beta \ln\left[(1-\lambda_0)^\alpha \sum_{i=1}^n \frac{[s_i(1-a_i)]^{\alpha/(1-\alpha)}}{\left(\sum_{j=1}^m [s_j(1-a_j)]^{\alpha/(1-\alpha)}\right)^\alpha}\right] + \text{constant} \\
&= \beta \ln\left[(1-\lambda_0)^\alpha \sum_{i=1}^n \frac{\psi_i}{\Psi^\alpha}\right] + \text{constant} \\
&= (1-\alpha)\beta \ln \Psi + \alpha\beta \ln(1-\lambda_0) + (1-\beta)\ln\left(\frac{\lambda_0 w + y}{p_0}\right)
\end{aligned}$$

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