Cohabitation versus marriage: Marriage matching with peer effects

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Abstract

The Cobb Douglas marriage matching function (MMF) is an easy to estimate MMF. It encompasses the Choo Siow (CS) MMF, CS with peer effects, Dagsvik Menzel MMF and Chiappori, Salanié and Weiss MMF. Given population supplies, the Cobb Douglas MMF exists and is unique. This MMF is estimated on US marriage and cohabitation data by states from 1990 to 2010. There are scale effects in US marriage markets. CS with peer effect, which admits both peer and scale effects, is not rejected. Positive assortative matching in marriage and cohabitation by educational attainment are stable from 1990 to 2010.

Since the seventies, marital behavior in the United States have changed significantly.[1] First, for most adult groups, marriage rates have fallen. Second, starting from a very low initial rate, cohabitation rates have risen significantly.

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[1]Lundberg and Pollak (2013) has a longer and broader description of marital changes in the US.

Because the initial cohabitation rates were so low, the rise in cohabitations did not compensate for the fall in marriages. So third, the fraction of adults who are unmatched, i.e. not married or cohabitating, have risen significantly. Evidence for these trends for women and men between ages 26-30 and 28-32 respectively are shown in Figure 1 in Appendix A.

Researchers have investigated different causes for these changes including changes in reproductive technologies as well as access to them, changes in family laws, changes in household technologies, changes in earnings inequality and changes in welfare regimes.[2] Most of this research ignored changes in population supplies over time. Often, they also ignore peer effects in marital behavior.

There were significant changes in population supplies over the time period. The sex ratio (ratio of male to female) of new college graduates have decreased from above one in the seventies to below one currently. See Figure 2 in Appendix A for women and men between ages 26-30 and 28-32 respectively. This change in the sex ratio may have exacerbated the decline in the marriage rate and also potentially changed marriage matching patterns.

Researchers estimate marriage matching functions (MMF) to analyze how changes in causes and population supplies affect marital behavior. Consider a static marriage market. There are I, $i = 1, .., I$, types of men and J, $j = 1, .., J$, types of women. Let M be the population vector of men where a typical element is m_i, the supply of type i men. F is the population vector of women where a typical element is f_j, the supply of type j women. Each individual can choose to enter a relationship, marriage or cohabitation, $r = [\mathcal{M}, \mathcal{C}]$, and a partner (by type) of the opposite sex for the relationship or not. An unmatched individual chooses a partner of type 0.

Let θ be a vector of parameters. A marriage matching function (MMF)

[2] E.g. Burtless (1999); Choo Siow (2006a); Fernandez, Guner and Knowles (2005); Fernandez-Villaverde, et. al. (2014); Goldin and Katz (2002); Greenwood, et. al. (2012, 2014); Lundberg and Pollak (2013); Moffitt, et. al. (1998); Stevenson and Wolderers (2007); Waite and Bachrach (2004).

is a R_+^{2IJ} vector valued function $\mu(M, F, \theta)$ whose typical element is μ_{ij}^r, the number of (r, i, j) relationships. μ_{0j} and μ_{i0} are the numbers of unmatched women and men respectively. μ_{ij}^r have to satisfy the following $I+J$ accounting identities:

$$\sum_{j=1}^{J} \mu_{ij}^{\mathcal{M}} + \sum_{j=1}^{J} \mu_{ij}^{\mathcal{C}} + \mu_{i0} = m_i, \quad 1 \leq i \leq I \tag{1}$$

$$\sum_{i=1}^{I} \mu_{ij}^{\mathcal{M}} + \sum_{i=1}^{I} \mu_{ij}^{\mathcal{C}} + \mu_{0j} = f_j, \quad 1 \leq j \leq J \tag{2}$$

$$\mu_{0j}, \mu_{i0} \geq 0, \quad 1 \leq j \leq J, 1 \leq i \leq I.$$

There are two main difficulties with constructing MMFs. First, due to multicollinearity and the proliferation of parameters, without apriori restrictions, it is usually intractable to estimate the dependence of μ on the population vectors, M and F. Most empirical researchers impose a behaviorally implausible no spillover rule which says that μ_{ij}^r depends only on the sex ratio, m_i/f_j, and not other population supplies (E.g. Qian and Preston 1993; Schoen 1981). This no spillover rule excludes general equilibrium effects. Second, it is difficult to construct MMFs which satisfy the accounting identities above.

Recently, Choo and Siow (2006a, 2006b; hereafter CS) used McFadden's (1973) random utility model to model spousal demand in a transferable utility model of the marriage market, in order to obtain an empirically tractable MMF. General equilibrium and population supplies effects on μ_{ij}^r are fully absorbed by the numbers of unmatched men and women of each type, μ_{0j} and μ_{i0}. The CS marriage matching function is:

$$\ln \frac{\mu_{ij}^r}{\sqrt{\mu_{i0}\mu_{0j}}} = \gamma_{ij}^r \ \forall \ (r, i, j)$$

CS interprets γ_{ij}^r as the expected gain in utility to a randomly chosen (i, j) pair in relationship r relative to the alternative of them remaining unmatched. The left hand side of the above equation can be motivated as follows. As the gain increases, the number of such pairs, μ_{ij}^r, will increase relative the numbers of i and j individuals, μ_{i0} and μ_{0j}, remaining unmatched. Given population supplies and parameters, Decker, et. al. (2013) showed that the marriage distribution exists and is unique. The CS MMF satisfies constant returns to scale in population supplies (CRS), meaning that, holding the type distributions of men and women fixed, increasing market size has no effect on the probability of forming a match (i,j) in a relationship r, and the effects of μ_{i0} and μ_{0j} in the CS MMF is symmetric.

Ignoring cohabitation, retaining CRS, Chiappori, Salanié and Weiss (2012; hereafter CSW) relaxed the symmetric effect of the unmatched in CS to obtain:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{i0}^{\alpha}\mu_{0j}^{1-\alpha}} = \gamma_{ij}^{\mathcal{M}} \; \forall \, (r, i, j)$$

Also ignoring cohabitation, Dagsvik (2000), Dagsvik et al. (2001), and Menzel (2015) study non-transferable utility models of the marriage market to obtain the DM MMF:

$$\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{i0}\mu_{0j}} = \gamma_{ij}^{\mathcal{M}} \; \forall \, (r, i, j)$$

Simulations show that DM has increasing returns to scale in population supplies. The symmetric effect of the unmatched in the MMF is retained.

Building on the above, this paper proposes the Cobb Douglas MMF:

$$\ln \mu_{ij}^r = \gamma_{ij}^r + \alpha_{ij}^r \ln \mu_{i0} + \beta_{ij}^r \ln \mu_{0j}; \;\; \alpha_{ij}^r, \beta_{ij}^r > 0 \; \forall \, (r, i, j) \tag{3}$$

The Cobb Douglas MMF has some useful properties:

1. It nests a large class of behavioral MMFs.[3]

2. Scale effects show up in the parameters α_{ij}^r and β_{ij}^r.

3. The effects of μ_{i0} and μ_{0j} on μ_{ij}^r do not have to be gender neutral.

[3]Other related MMFs which are not in the Cobb Douglas class include Galichon and Salanié (2013); Dupuy and Galichon (2012). Chiappori and Salanié (2015) has a survey.

4. Following the above CS interpretation of γ_{ij}^r one can parametrize γ_{ij}^r to study how a particular behavioral mechanism affects marital matching.[4]

5. Given population supplies and parameters, the equilibrium marriage matching distribution $\mu(M, F, \theta)$ exists and is unique. It is easy to simulate for policy evaluations. See section 1.

6. Without restrictions on γ_{ij}^r, the MMF fits any observed marital behavior in a single marriage market. In fact, the model must be restricted to obtain identification even with multimarket data. Luckily, identification is transparent. Due to the log linear estimating equations (3), we do not need to add any identifying restriction over and above what the empirical literature, which uses state and time variation to estimate different aspects of US marriage market behavior, imposes.[5]

7. Estimation is easy. The parameters of the MMF can be estimated using multi-market data by difference in differences and using population supplies as instruments for the unmatched.

While the equations (3) are in the Cobb Douglas form, they are not standard production functions.[6] Rather, they form a set of equilibrium relationships which defines the Cobb Douglas MMF.

Compared with the other behavioral MMFs above, the Cobb Douglas MMF relaxes CRS and symmetry of the unmatched on the MMF. But is there a behavioral MMF which has these properties? Building on Brock and Durlauf

[4]CS used it to study marital effects of the legalization of abortion. Brandt, Siow and Vogel (2008) used CS to study the effects of the famine in China due to the Great Leap Forward on the marriage market of the famine affected birth cohorts. Cornelson and Siow (2015) used it to study the effect of increased earnings inequality on marital behavior.

[5]E.g. Bitler, et. al. (2004); Chiappori, Fortin and Lacroix (2002); Dahl (2010); Mechoulan (2011), Stevenson and Wolfers (2006); Wolfers (2006).

[6]The standard Cobb Douglas model, $\ln \mu_{ij}^{rst} = \alpha_{ij}^r \ln m_i^{st} + \beta_{ij}^r \ln f_j^{st} + \gamma_{ij}^{rst}$, is not a well behaved MMF. In general, it will not satisfy the accounting relationships (1) and (2). Nor does it have spillover effects.

(2001), Section 3 develops a CS MMF with peer effects (CSPE) with these properties. Peer effects, as well as changes in cultural norms, affect cohabitation and other marital behavior (E.g. Adamopoulou (2012); Waite, et. al. 2000; Fernandez-Villaverde, et. al. (2014)). Marriage and cohabitation are costly individual investments and commitments. Individual who never married or cohabitated are not likely to be very confident of their payoffs from these relationships. Thus it is reasonable to expect that individuals will be affected by the relationship choices of their peers. Moreover, cohabitation is a relatively new form of socially accepted relationship in the US. The US census first asked about cohabitating relationships in 1990. So peer effects may be more salient for cohabitation compared with marriage (E.g. Thornton and Young-DeMarco (2001)). Our peer effects model also incorporates scale effects. We cannot separately identify the importance of direct peer effects versus indirect (scale) effects.

CSPE is a special testable case of the Cobb Douglas MMF. When we extend the CS, and DM MMF to additional types of relationships, the log odds of the numbers of different types of relationships, $\ln(\mu_{ij}^{\mathcal{M}}/\mu_{ij}^{\mathcal{C}})$, is independent of the sex ratio, $\ln(m_i/f_j)$. Independence is a very strong assumption. Arciadiacono, et. al. (2010) shows that independence does not hold for sexual versus non-sexual boy girl relationships in high schools. This paper shows that independence also does not hold for cohabitation versus marriage. A suitable extension of CSW to additional types of relationships also does not impose independence but still imposes CRS. However, CSPE simultaneously relaxes CRS and independence albeit in a restricted manner, which allows us to discriminate CSPE from other behavioral models. The Cobb Douglas MMF relaxes independence more flexibly.

The Cobb Douglas MMF nests CS, CSW, DM and CSPE as special cases. Since the special cases include frictionless transferable utility models and non-transferable utility models as well as a CS model with frictional transfers (Mourifié and Siow: in process), we should be modest in our ability to determine the importance of transfers in equilibrating the marriage market. Although we are partial to CSPE and it is not rejected empirically, it should be clear that we propose the Cobb Douglas MMF precisely because we do not want to insist on a particular behavioral model of the marriage market.

Galichon, et. al. (2014) studied a model with an imperfect transfer technology and without peer effects. They proposed a MMF which is qualitatively motivated by their behavioral model. Their MMF is related to our Cobb Douglas MMF. They used a different proof from that in this paper to show uniqueness and existence of their MMF. They did not focus on identification of their MMF nor provide an empirical application. Our paper focuses on deriving an empirically tractable MMF with peer effects and we provide an empirical application. Thus our two papers are complementary.

Section 4 estimates the Cobb Douglas MMF with marriage and cohabitation data across states for women and men between ages 26-30 and 28-32 respectively from the US Censuses in 1990 and 2000, and the American Community Surveys around 2010. Men and women are differentiated by their educational attainment. This empirical analysis builds on Siow (2015) and CSW. Our empirical results show that:

1. From a descriptive (goodness of fit) point of view, a simplified Cobb Douglas MMF with relationship match (r, i, j), state and year fixed effects, provides a reasonably complete and parsimonious description of the US marriage market by state from 1990 to 2010.

2. There are scale effects in US marriage markets.

3. CS, CSW and DM are rejected by the data.

4. CSPE is not rejected by the data. Homogenous peer effects $\grave{a}$ la Manski (2003) is rejected.

5. The value of cohabitation is less sensitive to peer effects than the value for marriage.

6. The value of remaining unmatched is less sensitive to peer effects for women than for men.

7. Consistent with CSW and Siow (2015), to a first order, there is no general increase in positive assortative matching (PAM) by educational attainment from 1990 to 2010.

8. Consistent with CSW and many other observers, gains to marriage declined from 1990 to 2010. We further show that gains to cohabitation increased. Both findings are consistent with the observation that the average age of first marriage has increased over this period.

The remainder of the paper is organized as follows. Section 1 presents the Cobb Douglas MMF and discusses existence uniqueness of the equilibrium. Section 2 presents our identification and estimation strategy. Section 3 introduces a behavorial matching model with peer effects. Section 4 discussed the empirical application. The last section concludes. Proofs of the main results are collected in the appendix.

1 The Cobb Douglas MMF

Consider the Cobb Douglas MMF defined by:

$$\ln \frac{\mu_{ij}^r}{(\mu_{i0})^{\alpha_{ij}^r}(\mu_{0j})^{\beta_{ij}^r}} = \gamma_{ij}^r \ \forall \ (r,i,j) \tag{4}$$
$$\alpha_{ij}^r, \beta_{ij}^r \geq 0$$

Consistent with the behavioral models, and the fact that γ_{ij}^r can be negative, we interpret γ_{ij}^r as proportional to the mean gross gains to relationship r minus the sum of the mean gains to them remaining unmatched for two randomly chosen (i,j) individuals.

The matching equilibrium in this model is characterized by the Cobb Douglas MMF (4) and the population constraint equations. Since the equations are not derived from a behavioral model of the marriage market, to the best of our knowledge, nothing was known about the existence and the uniqueness of the equilibrium for the Cobb Douglas MMF[7]. We propose an approach that proves the existence and uniqueness of this model. The details of the complete development of our approach are derived in Appendix B.

Following CS, an important simplification in the proof is to first reduce the $2r \times I \times J$ system of non-linear equations to an $I + J$ system of the numbers of unmatched individuals by substituting the Cobb Douglas MMF in equation (4) into the population constraints, (1) and (2), to get:

Lemma 1

$$m_i = \mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{\mathcal{M}}} \mu_{0j}^{\beta_{ij}^{\mathcal{M}}} e^{\gamma_{ij}^{\mathcal{M}}} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^{\mathcal{C}}} \mu_{0j}^{\beta_{ij}^{\mathcal{C}}} e^{\gamma_{ij}^{\mathcal{C}}}, \; \textit{for } 1 \leq i \leq I, \tag{5}$$

$$f_j = \mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{\mathcal{M}}} \mu_{0j}^{\beta_{ij}^{\mathcal{M}}} e^{\gamma_{ij}^{\mathcal{M}}} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^{\mathcal{C}}} \mu_{0j}^{\beta_{ij}^{\mathcal{C}}} e^{\gamma_{ij}^{\mathcal{C}}}, \; \textit{for } 1 \leq j \leq J. \tag{6}$$

Although there are $2 \times I \times J$ elements in μ, the analyst only has to first solve a sub-system of $I + J$ non-linear equations whose solution is unique (see Theorem 1 below). The rest of the system is linear. Using this two steps approach, the MMF is easy to simulate for policy evaluations.[8]

The following theorem summarizes our results:

Theorem 1 *[Existence and Uniqueness of the Equilibrium matching] For every fixed matrix of relationship gains and coefficients $\beta_{ij}^r; \alpha_{ij}^r \geq 0$, the equilibrium matching of the Cobb Douglas MMF model exists and is unique.*

[7]Separately, Galichon and al (2014) provide an existence and uniqueness proof of a general related MMF. Although different, either proof can be easily adapted to show the existence and uniqueness of both MMFs. We simultaneously became aware about our two results during a conference on matching at the Fields Institute on September 15, 2014.

[8]Feedback from users of the CS MMF (a special case) suggest that a one step numerical solution is difficult to achieve.

Notice that using (4),

$$\ln \frac{\mu_{ij}^r}{\mu_{ij}^{r'}} = (\alpha_{ij}^r - \alpha_{ij}^{r'}) \ln(\mu_{i0}) + (\beta_{ij}^r - \beta_{ij}^{r'}) \ln(\mu_{0j}) + \gamma_{ij}^r - \gamma_{ij}^{r'} \; \forall \; (r, i, j)$$

So:

Lemma 2 *When $(\alpha_{ij}^r - \alpha_{ij}^{r'}) = (\beta_{ij}^r - \beta_{ij}^{r'}) = 0$ as in CS, CSW and DM, the log odd of μ_{ij}^r to $\mu_{ij}^{r'}$ is independent of the sex ratio m_i/f_j. Otherwise the log odd is not independent of the sex ratio.*

Arciadiacono, et. al. (2010) shows that independence does not hold for sexual versus non-sexual boy girl relationships in high schools. We show here that it does not hold for cohabitation versus marriage. CSPE provides a behavioral model which relaxes independence. We can also relax independence under CSW by letting α and $(1 - \alpha)$ be dependent on r.

With multimarket data, β_{ij}^r and α_{ij}^r are identified under some usual restrictions, as shall be clearer soon. However β_{ij}^r and α_{ij}^r cannot be estimated precisely with the data which we have[9]. So often, we will assume that the exponents on the Cobb Douglas MMF are gender and relationship specific but independent of the types of couples, (i, j): $\beta_{ij}^r = \beta^r$ and $\alpha_{ij}^r = \alpha^r$. With multimarket data, type independent exponents, $\beta_{ij}^r = \beta^r$ and $\alpha_{ij}^r = \alpha^r$, is in principle a testable relationship. From a practical point of view, the most flexible model that we estimate in this paper imposes type independent exponents.[10]

When i and j are unidimensional and ordered, and we impose type independent exponents, the local log odds, $l(r, i, j)$, of the Cobb Douglas MMF become:

[9]In our application, we do not observe enough different markets to have precise estimate of β_{ij}^r and α_{ij}^r, however this could be obtained using richer data set.

[10]We estimated the fully flexible Cobb Douglas MMF but the point estimates on the unmatched interacted with the type of the match were too imprecisely estimated to be useful.

$$l(r,i,j) = \ln \frac{\mu^r_{ij}\mu^r_{i+1,j+1}}{\mu^r_{i+1,j}\mu^r_{i,j+1}} = \gamma^r_{ij} + \gamma^r_{i+1,j+1} - \gamma^r_{i+1,j} - \gamma^r_{i,j+1}$$

Following all the other behavioral MMFs considered in this paper, we interpret $l(r,i,j)$ as proportional to the degree of local complementarity of the marital output function of the couple at (r,i,j).

Since different cases of the Cobb Douglas MMF imply the presence of scale effects or otherwise, we provide a restriction for scale effects. Then:

Proposition 1 (Constant return to scale) *The equilibrium matching distribution of the Cobb Douglas MMF model satisfies the Constant return to scale property if $\beta^r + \alpha^r = 1$ i.e.,*

$$\beta^r + \alpha^r = 1 \text{ for } r \in \{\mathcal{M}, \mathcal{C}\} \Rightarrow \sum_{i=1}^{I} \frac{\partial \mu}{\partial m_i} m_i + \sum_{j=1}^{J} \frac{\partial \mu}{\partial f_j} f_j = \mu.$$

The result claims that the Cobb Douglas MMF model exhibits constant results to scale if $\beta^r + \alpha^r = 1$. The proposition generalizes to $\beta^r_{ij} + \alpha^r_{ij} = 1$ for all (r,i,j) implies constant returns to scale.

Comparative statics

Building on Graham (2013), we derive in Theorem 2 (relegated in Appendix C.2 for sake of exposition) some comparative statics results for the Cobb Douglas MMF model. We show that:

1. For any admissible k and l, the unmatched rate for type l individual is increasing in the supply of type k individual of the same gender.

2. For any admissible k and l, the unmatched rate for type l of individual is decreasing in the supply of type k of individual of the opposite gender.

3. Variation of the log ratio $\ln \frac{\mu^{\mathcal{M}}_{ij}}{\mu^{\mathcal{C}}_{ij}}$:
   If $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ and $\beta^{\mathcal{C}} > \beta^{\mathcal{M}}$ we have

$$
\text{(a)}\ \ \frac{1}{\partial m_i}[\ln \frac{\mu^{\mathcal{M}}_{kj}}{\mu^{\mathcal{C}}_{kj}}] \geq \begin{cases} > 0 & \text{if } k \neq i \\ > \alpha^{\mathcal{M}} - \alpha^{\mathcal{C}} & \text{if } k = i, \end{cases} \quad 1 \leq k \leq I
$$

$$
\text{(b)}\ \ \frac{1}{\partial f_j}[\ln \frac{\mu^{\mathcal{M}}_{ik}}{\mu^{\mathcal{C}}_{ik}}] \leq \begin{cases} < 0 & \text{if } k \neq j \\ < -(\alpha^{\mathcal{M}} - \alpha^{\mathcal{C}}) & \text{if } k = j, \end{cases} \quad 1 \leq k \leq J
$$

The above results generalize Theorem 2 of Decker et. al. (2012), and the case i) of Theorem 1 of Graham (2013).

2 Identification and estimation

Consider the general Cobb Douglas MMF in presence of independent multi-market data where an isolated marriage market is defined by the state s and time t:

$$
\ln \mu^{rst}_{ij} = \alpha^r_{ij} \ln \mu^{st}_{i0} + \beta^r_{ij} \ln \mu^{st}_{0j} + \gamma^{rst}_{ij}. \tag{7}
$$

This section provides flexible specifications which are identified and can be estimated using a difference in differences instrumental variables methodology. Many studies use variations across state and time in marriage markets to estimate models of marital behavior.[11] A maintained assumption in these studies is that the variation in population supplies is orthogonal to variation in the payoffs to marital behavior. Otherwise most of the estimates of marital behavior using state time variation will be inconsistent. We and the empirical research which relies on this assumption recognizes that there is

[11] First, there were often significant changes in the payoffs to marriage and cohabitation across state and time. This variation has been exploited in previous research to study how changes in divorce laws (E.g. Wolfers (2006)), changes in laws affecting reproductive choice (E.g. CS; Galichon Salanie), changes in rules governing welfare receipts (E.g. Bitler, et. al. (2004)), and minimum age of marriage laws (Dahl (2010)) affect marital outcomes. Second, variations in sex ratio across state and time have also been used to study its effects on marital behavior as well as intrahousehold allocations (E.g. Kerwin and Luoh (2010); Mechoulan (2011); Chiappori, Fortin and Lacroix (2002)).

migration across states. The large number of studies, on different marital outcomes, which have obtained behaviorally plausible estimates, suggest that the orthogonality assumption is empirically reasonable.

Even with multimarket data, the most general Cobb Douglas MMF is not identified. There are $2 \times I \times J \times S \times T$ elements in the observed matching distribution (i.e. μ_{ij}^{rst}) and there are $2 \times I \times J \times S \times T + 4 \times I \times J$ parameters i.e. (γ_{ij}^{rst}, α_{ij}^{r}, and β_{ij}^{r}). Therefore, to obtain identification of the general Cobb Douglas MMF we will impose additional standard restrictions on the structure of the gains i.e. γ_{ij}^{rst}.

Assumption 1 1. *(Additive separability of the gain).* $\gamma_{ij}^{rst} = \pi_{ij}^{r} + \eta_{ij}^{rs} + \zeta_{ij}^{rt} + \epsilon_{ij}^{rst}$ *where* π_{ij}^{r} *represents the type fixed effect,* η_{ij}^{rs} *the state fixed effect,* ζ_{ij}^{rt} *the time fixed effect, and* ϵ_{ij}^{rst} *the residual terms.*

2. *(Instrumental Variable (IV)).* $\mathbb{E}[\epsilon_{ij}^{rst}|z_{ij}^{11}, ..., z_{ij}^{ST}] = 0$, *where* $z_{ij}^{st} = (m_i^{st}, f_j^{st})'$.

Assumption 1 (1) decomposes γ_{ij}^{rst} into match type fixed effect, state fixed effect and time fixed effect, and the error term of the regression, ϵ_{ij}^{rst}. π_{ij}^{r}, η_{ij}^{rs} and ζ_{ij}^{rt} are identified. ϵ_{ij}^{rst} is not identified. Assumption 1 (1) allows us to reduce the number of parameters. When ϵ_{ij}^{rst} increases, the gain to the match increases which will increase μ_{ij}^{rst} and therefore likely reduces μ_{i0}^{st} and μ_{0j}^{st}. Thus μ_{i0}^{st} and μ_{0j}^{st} and the error term ϵ_{ij}^{rst} are likely negatively correlated. So in general, using ordinary least square (OLS) to estimate equation (7) is inconsistent. Assumption 1 (2) allows us to use the population supplies, m_i^{st} and f_j^{st}, as instruments for μ_{i0}^{st} and μ_{0j}^{st}. The assumption says that the population supplies must be orthogonal to ϵ_{ij}^{rst}. As discussed in the introduction to this section, Assumption 1 does not impose any additional restriction over and above what is standard in the empirical literature on US marriage markets which uses state and time variation for estimation. And just like that literature, we cannot identify parameters which vary by s and t without additional restrictions.[12] Under Assumption 1 (1), equation (7) becomes:

$$\ln \mu_{ij}^{rst} = \alpha_{ij}^{r} \ln \mu_{i0}^{st} + \beta_{ij}^{r} \ln \mu_{0j}^{st} + \pi_{ij}^{r} + \eta_{ij}^{rs} + \zeta_{ij}^{rt} + \epsilon_{ij}^{rst}. \tag{8}$$

Notice that for a fixed (i, j) type we have now $3 + S + T$ parameters and ST observations. Therefore, the parameter of interests would be identified whenever $2 + S + T < ST$. The OLS estimator of $\widetilde{\lambda_{ij}^{r}} \equiv (\alpha_{ij}^{r}, \beta_{ij}^{r})'$ in equation (7) is equivalent to the regression of $\widetilde{y_{ij}^{rst}} \equiv \ln \mu_{ij}^{rst} - \overline{\ln}\mu_{ij}^{rs} - \overline{\ln}\mu_{ij}^{rt} + \overline{\overline{\ln}}\mu_{ij}^{r}$ on $\widetilde{x_{ij}^{st}} \equiv (\ln \mu_{i0}^{st} - \overline{\ln}\mu_{i0}^{s} - \overline{\ln}\mu_{i0}^{t} + \overline{\overline{\ln}}\mu_{i0}, \ln \mu_{0j}^{st} - \overline{\ln}\mu_{0j}^{s} - \overline{\ln}\mu_{0j}^{t} + \overline{\overline{\ln}}\mu_{0j})'$ where $\overline{\ln}\mu_{ij}^{rs} = T^{-1} \sum_{t=1}^{T} \ln \mu_{ij}^{rst}$, $\overline{\ln}\mu_{ij}^{rt} = S^{-1} \sum_{S=1}^{S} \ln \mu_{ij}^{rst}$, and $\overline{\overline{\ln}}\mu_{ij}^{r} = (ST)^{-1} \sum_{s=1}^{S} \sum_{t=1}^{T} \ln \mu_{ij}^{rst}$.

Since μ_{i0}^{st} and μ_{0j}^{st} are potentially correlated with the residual terms ϵ_{ij}^{rst} OLS will not be able to identify $\widetilde{\lambda_{ij}^{r}}$. Therefore, we will instrument μ_{i0}^{st} and μ_{0j}^{st} respectively with m_{i}^{st} and f_{j}^{st}. Notice that to be a valid instrument μ_{i0}^{st} and μ_{0j}^{st} should be respectively correlated with μ_{i0}^{st} and μ_{0j}^{st} and respect the exogeneity condition summarizes in Assumption 1 (2).

As can be seen in Theorem 2, the comparative statics show the correlation between m_{i}^{st} and f_{j}^{st} and the unmatched. Therefore, $\widetilde{\lambda_{ij}^{r}}$ can be identify using the IV estimand if $\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']$ is of full column rank. The identification result is summarized in the following proposition.

Proposition 2 *Under Assumption 1, the general Cobb Douglas MMF is identified if $\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']$ is of full column rank. The identification equation is given by $\widetilde{\lambda_{ij}^{r}} = \{\mathbb{E}[z_{ij}^{st}\widetilde{x_{ij}^{st}}']\}^{-1}\mathbb{E}[z_{ij}^{st}\widetilde{y_{ij}^{rst}}]$.*

We have a few comments. First, whenever $\widetilde{\lambda_{ij}^{r}}$ is identified, we can identify the gain matrix γ_{ij}^{rst} using equation (7). Second, this model can also be estimated using the generalized method of moments (GMM). Third, whenever the numbers of state S and period T are not high, we do not need to do the double differentiation. We can use a sequence of state and time dummies fixed-effects.

[12] Cornelson and Siow (2015) provides an example in which the effect of covariates which vary by (i, j, s, t) on γ_{ij}^{rst} can be estimated.

3 Marriage matching with peer effects

In this model every individual can decide to cohabit, marry or remain unmatched. For a type i man to match with a type j woman in relationship r, he must transfer to her a part of his utility that he values as τ_{ij}^r. The woman values the transfer as τ_{ij}^r. τ_{ij}^r may be positive or negative.

There are $2 \times I \times J$ matching sub-markets for every combination of relationship, and types of men and women. A matching market clears when, given equilibrium transfers τ_{ij}^r, the demand by men of type i for type j women in the relationship r is equal to the supply of type j women for type i men in the relationship r for all (r, i, j). To implement the above framework empirically, we adopt the extreme value random utility model of McFadden (1973) to generate market demands for matching partners. Each individual considers matching with a member of the opposite gender. Let the utility of male g of type i who matches a female of type j in a relationship r be:

$$U_{ijg}^r = \tilde{u}_{ij}^r + \phi_i^r \ln \mu_{ij}^r - \tau_{ij}^r + \varsigma_{ijg}^r, \text{ where} \tag{9}$$

$\tilde{u}_{ij}^r + \phi_i^r \ln \mu_{ij}^r$: Systematic gross return to a male of type i matching to a female of type j in relationship r.

ϕ_i^r: Coefficient of peer effect for relationship r. $1 \geq \phi_i^r \geq 0$.

μ_{ij}^r: Equilibrium number of (r, i, j) relationships.

τ_{ij}^r: Equilibrium transfer made by a male of type i to a female of type j in relationship r.

ς_{ijg}^r: denotes the errors terms (idiosyncratic payoffs) which are assumed to be i.i.d. random variables distributed according to the extreme value Type-I (Gumbel) distribution. It is worth noting that the errors are assumed to be also independent across genders.

Due to the peer effects, the net systematic return is increased when more type i men are in the same relationships. It is reduced when the equilibrium transfer τ_{ij}^r is increased.

And $\widetilde{u}_{i0} + \phi_i^0 \ln \mu_{i0}^0$ is the systematic payoff that type i men get from remaining unmatched. We allow the peer effect to differ by relationship. For example, unmarried individuals spend more time with their unmarried friends than married individuals with their married friends. On the other hand, due to their higher shadow cost of time, married individuals may not value interacting with their peers as much.

Also, we estimate our MMFs with market level data. Each peer effect coefficient consists of a direct effect and an indirect effect. The direct peer effect is already discussed in the previous paragraph, i.e. how individual g's utility is affected when he *observes* how many others like him choose the same action. The indirect effect is a market level effect. As there are more (i, j, r) relationships in a community, local firms will provide services to them (E.g. Compton and Pollak (2007); Costa and Kahn (2000)). This community response will make it cheaper for g to choose (i, j, r) relationships. Marriage market participants do not necessarily recognize the impact of their aggregate actions on the prices of goods and services which they face. Thus the indirect peer effect is a scale effect. The peer coefficients, ϕ_i^0 and ϕ_i^r, capture both the direct and indirect effects. Either effect is not individually identified.

Our peer effects specification is chosen for analytic and empirical convenience. We want CSPE to be nested in the Cobb Douglas MMF. We also want CSPE to be testable. Proposition 3 below shows that Manski's (1993) justifiably famous peer effect specification is a special case of CSPE. Since this paper is the first attempt of introducing peer effects in two sided matching models, other specifications are left for future investigation.

Individual g will choose according to:

$$U_{ig} = \max_{j,r}\{U_{i0g}, U_{i1g}^{\mathcal{M}}, ..., U_{ijg}^{\mathcal{M}}, ..., U_{iJg}^{\mathcal{M}}, U_{i0g}^{\mathcal{C}}, ..., U_{ijg}^{\mathcal{C}}, ..., U_{iJg}^{\mathcal{C}}\}$$

Let $(\mu_{ij}^r)^d$ be the number of (r, i, j) matches demanded by i type men and $(\mu_{i0})^d$ be the number of unmatched i type men. Following the well known

McFadden result, we have:

$$
\begin{aligned}
\frac{(\mu_{ij}^r)^d}{m_i} &= \mathbb{P}(U_{ijg}^r - U_{ikg}^{r'} \geq 0, k = 1, ..., J; r' = (\mathcal{M}, \mathcal{C})) \\
&= \frac{e^{\tilde{u}_{ij}^r + \phi_i^r \ln \mu_{ij}^r - \tau_{ij}^r}}{e^{\tilde{u}_{i0} + \phi_i^0 \ln \mu_{i0}} + \sum_{r' \in \{\mathcal{M}, \mathcal{C}\}} \sum_{k=1}^J e^{\tilde{u}_{ik}^{r'} + \phi_i^{r'} \ln \mu_{ij}^{r'} - \tau_{ik}^{r'}}},
\end{aligned} \tag{10}
$$

where m_i denotes the number of men of type i. Using (10) we can easily derive the following relationship:

$$
ln \frac{(\mu_{ij}^r)^d}{(\mu_{i0})^d} = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \phi_i^r \ln \mu_{ij}^r - \phi_i^0 \mu_{i0} - \tau_{ij}^r, \tag{11}
$$

The above equation is a quasi-demand equation by type i men for (r, i, j) relationships.

The random utility function for women is similar to that for men except that in matching with a type i men in an (r, i, j) relationship, a type j women receives the transfer, τ_{ij}^r. Let $\tilde{v}_{ij}^r + \Phi_j^r \ln \mu_{ij}^r$ denotes the systematic gross gain that type j women get from matching type i men in the relationship r. Φ_j^r, $1 \geq \Phi_j^r \geq 0$, is her peer effect coefficient in relationship (r, i, j). And $\tilde{v}_{0j} + \Phi_j^0 \ln \mu_{0j}^0$ is the systematic payoff that type j women get from remaining unmatched. Let $(\mu_{ij}^r)^s$ be the number of i, j matches offered by j type women for the relationship r and $(\mu_{0j})^s$ the number of type j women who want to remain unmatched. The quasi-supply equation of type j women for (r, i, j) relationships is given by:

$$
\ln \frac{(\mu_{ij}^r)^s}{(\mu_{0j})^s} = \tilde{v}_{ij}^r - \tilde{v}_{0j} + \Phi_j^r \ln \mu_{ij}^r - \Phi_j^0 \ln \mu_{0j} + \tau_{ij}^r. \tag{12}
$$

The matching market clears when, given equilibrium transfers τ_{ij}^r, the demand of type i men for (r, i, j) relationships is equal to the supply of type j women for (r, i, j) relationships for all (r, i, j):

$$
(\mu_{ij}^r)^d = (\mu_{ij}^r)^s = \mu_{ij}^r. \tag{13}
$$

Substituting (13) into equations (11) and (12) we get:

$$\ln \mu_{ij}^r = \frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{i0} + \frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r}\ln \mu_{0j} + \frac{\pi_{ij}^r}{2-\phi_i^r-\Phi_j^r} \tag{14}$$

$$\pi_{ij}^r = \tilde{u}_{ij}^r - \tilde{u}_{i0} + \tilde{v}_{ij}^r - \tilde{v}_{0j} \tag{15}$$

The above is the CS model with peer effects, the CSPE MMF.

Now, we will show how previous MMF can be recovered using the CSPE MMF.

When there is no peer effect or all the peer effect coefficients are the same (homogeneous peer effects),

$$\phi_i^0 = \Phi_j^0 = \phi_i^r = \Phi_j^r$$

we recover the CS MMF. That is,

Proposition 3 *No peer effect, or homogenous peer effects, generates observationally equivalent MMFs.*

Put another way, the above proposition says if we cannot reject CS using marriage matching data alone, we also cannot reject homogenous peer effects. This result is related to Manski (1993).

By imposing homogenous peer effects, we can rewrite the individual's spousal choice utilities, U_{ijg}^r and V_{ijk}^r using a specification which is in the spirit of Manski and Brock Durlauf:

$$U_{ijg}^r = \tilde{u}_{ij}^r + \phi \ln \frac{\mu_{ij}^r}{m_i} - \tau_{ij}^r + \varsigma_{ijg}^r,\ j = 0, 1.., J$$
$$V_{ijg}^r = \tilde{v}_{ij}^r + \phi \ln \frac{\mu_{ij}^r}{f_j} + \tau_{ij}^r + \varrho_{ijk}^r,\ i = 0, 1.., I$$

Interestingly, non-homogenous peer effects are generically detectable:

Corollary 1 *When* $\frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r} \neq \frac{1}{2}$ *and/or* $\frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r} \neq \frac{1}{2}$*, non-homogenous peer effects are present.*

This corollary is related to identification of linear models with non-homogenous peer effects.[13] As pointed out in the introduction, relaxing homogenous peer effects in the marriage matching context is behaviorally sound.

When

$$\frac{1-\phi_i^0}{2-\phi_i^r-\Phi_j^r} = \frac{1-\Phi_j^0}{2-\phi_i^r-\Phi_j^r} = 1$$

we recover the DM MMF. Intuitively in this case, we want the peer effect on relationships to be significantly more powerful than that for remaining unmatched. E.g. $\phi_i^0 = \Phi_j^0 = 0$ and $\phi_i^r = \Phi_j^r = \frac{1}{2}$.

Also, when

$$\phi_i^0 + \Phi_j^0 = \phi_i^r + \Phi_j^r = \phi_i^{r'} + \Phi_j^{r'},$$

CSW MMF is obtained.

From (14), you cannot distinguish ϕ_i^r from Φ_j^r. However, behaviorally interesting peer effect responses can be learned. Please see Lemmata 3 and 4 below.

Again using Eq (14) we have:

$$\ln\frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{\mathcal{C}}} = \frac{(\phi_i^{\mathcal{M}}+\Phi_j^{\mathcal{M}}-\phi_i^{\mathcal{C}}-\Phi_j^{\mathcal{C}})}{(2-\phi_i^{\mathcal{M}}-\Phi_j^{\mathcal{M}})(2-\phi_i^{\mathcal{C}}-\Phi_j^{\mathcal{C}})}\left[(1-\phi_i^0)\ln\mu_{i0}+(1-\Phi_j^0)\ln\mu_{0j}\right] + \frac{\pi_{ij}^{\mathcal{M}}}{2-\phi_i^{\mathcal{M}}-\Phi_j^{\mathcal{M}}} - \frac{\pi_{ij}^{\mathcal{C}}}{2-\phi_i^{\mathcal{C}}-\Phi_j^{\mathcal{C}}} \tag{16}$$

Since μ_{i0} and μ_{0j} appears on the right hand side of (16), the log odds of the number of r to r' relationships will not be independent of the sex ratio.

It is easy to check that under CS and DM, the log odds of the number of r to r' relationships is independent of the sex ratio. Independence is a very strong assumption and unlikely to hold every two types of relationships. CSPE relaxes the independence assumption. However because the coefficients on unmatched men and women have the same sign, this independence is restricted.

[13]Blume, et. al. (forthcoming) has a state of the art survey. Also see Djebbari, et. al. (2009).

We will now study PAM patterns. Let the heterogeneity across males (females) be one dimensional and ordered. Without loss of generality, let male (female) ability be increasing in i (j).

Also let:

$$\phi_i^0 = \phi^0; \Phi_j^0 = \Phi^0; \phi_i^r = \phi^r; \Phi_j^r = \Phi^r \tag{17}$$

that is, the peer effects depend on their gender and the relationship they pick but not their type or their partner's type. We call equation (17) type independent peer effects. Type independence peer effects leads to type independent exponents in the Cobb Douglas MMF, a testable restriction.

Then using (14), the local log odds for (r, i, j) is:

$$l(r,i,j) = \ln \frac{\mu_{ij}^r \mu_{i+1,j+1}^r}{\mu_{i+1,j}^r \mu_{i,j+1}^r} = \frac{\pi_{ij}^r + \pi_{i+1,j+1}^r - \pi_{i+1,j}^r - \pi_{i,j+1}^r}{2 - \phi^r - \Phi^r} \tag{18}$$

$$= \frac{\tilde{u}_{ij}^r + \tilde{v}_{ij}^r + \tilde{u}_{i+1,j+1}^r + \tilde{v}_{i+1,j+1}^r - (\tilde{u}_{i+1,j}^r + \tilde{v}_{i+1,j}^r) - (\tilde{u}_{i,j+1}^r + \tilde{v}_{i,j+1}^r)}{2 - \phi^r - \Phi^r} \tag{19}$$

According to (18), if the marital output function, $\tilde{u}_{ij}^r + \tilde{v}_{ij}^r$, is supermodular in i and j, then the local log odds, $l(r, i, j)$, are positive for all (i, j), or totally positive of order 2 ($TP2$). Statisticians use $TP2$ as a measure of stochastic positive assortative matching. Thus even when peer effects are present, we can test for supermodularity of the marital output function, a cornerstone of Becker's theory of positive assortative matching in marriage. This result generalizes Siow (2015), CSW and Graham (2011).

Now, it will be convenient to summarize different MMFs existing in the literature and clarify their relations to the Cobb Douglas MMF.[14]

[14] Other behavioral MMFs can also be nested in the Cobb Douglas MMF. Dagsvik (2000, Page 43) provides another example of MMF which allows correlation between idiosyncratic payoffs. However, this extension still does not relax the independence assumption, and imposes $1 < \alpha + \beta \leq 2$.

Models and restrictions on α^r and β^r of Cobb Douglas MMF				
Model	α^r	β^r	γ^r_{ij}	Restrictions
Cobb Douglas MMF	α^r	β^r	γ^r_{ij}	$\alpha^r \geq 0, \beta^r \geq 0$
CS	$\frac{1}{2}$	$\frac{1}{2}$	π^r_{ij}	$\alpha^r = \beta^r = \frac{1}{2}$
DM	1	1	π^r_{ij}	$\alpha^r = \beta^r = 1$
CSW	$\frac{\sigma^r}{\sigma^r+\Sigma^r}$	$\frac{\Sigma^r}{\sigma^r+\Sigma^r}$	$\frac{\pi^r_{ij}}{\sigma^r+\Sigma^r}$	$\alpha^r, \beta^r > 0; \alpha^r + \beta^r = 1$
CSPE	$\frac{1-\phi^0}{2-\phi^r-\Phi^r}$	$\frac{1-\Phi^0}{2-\phi^r-\Phi^r}$	$\frac{\pi^r_{ij}}{2-\phi^r-\Phi^r}$	$\alpha^r, \beta^r \geq 0, \frac{\alpha^{\mathcal{M}}}{\alpha^{\mathcal{C}}} = \frac{\beta^{\mathcal{M}}}{\beta^{\mathcal{C}}}$

The CSPE MMF imposes the following restriction on the Cobb Douglas MMF parameters:

$$\frac{\alpha^r}{\alpha^{r'}} = \frac{\beta^r}{\beta^{r'}}$$

which is a testable restriction. In other words, scale effects in the Cobb Douglas are restricted under CSPE.

Although individual peer effect coefficients, i.e., Φ^0, ϕ^0, ϕ^r, and Φ^r are not point identified, economically meaningful information can be learned through the reduced form parameters α^r, β^r.

Lemma 3 *Under CSPE, i.e.,* $\frac{\alpha^r}{\beta^r} = \frac{\alpha^{r'}}{\beta^{r'}}$, $\frac{\alpha^r}{\beta^r} \begin{cases} = 1 \Leftrightarrow \Phi^0 = \phi^0 \\ > 1 \Leftrightarrow \Phi^0 > \phi^0 \\ < 1 \Leftrightarrow \Phi^0 < \phi^0. \end{cases}$

With this result, we can know which gender's value of being unmatched is more sensitive to peer effects. For instance, if the coefficient on unmatched males (α^r) is smaller than that for unmatched females (β^r) for both relationships, then the value that women derive from being unmatched will be more sensitive to peer effects than for men.

Lemma 4 *Under CSPE, i.e.,* $\frac{\alpha^r}{\alpha^{r'}} = \frac{\beta^r}{\beta^{r'}}$, $\frac{\alpha^r}{\alpha^{r'}} \begin{cases} = 1 \Leftrightarrow \phi^{r'} + \Phi^{r'} = \phi^r + \Phi^r \\ > 1 \Leftrightarrow \phi^r + \Phi^r > \phi^{r'} + \Phi^{r'} \\ < 1 \Leftrightarrow \phi^r + \Phi^r < \phi^{r'} + \Phi^{r'}. \end{cases}$

This latter lemma says which type of relationship is more affected by the peer effects. For instance, if the ratio of the coefficient of unmatched men (women) in marriage is larger than the coefficient of unmatched men (women) in cohabitation, then the value that a couple derives from cohabitation will be more affected by peer effects than for marriage.

4 Empirical results

We study the marriage matching behavior of 26-30 years old women and 28-32 years old men with each other in the US for 1990, 2000 and 2010.

The 1990 and 2000 data is from the 5% US census. The 2010 data is from aggregating three years of the 1% American Community Survey from 2008-2010. A state year is considered as an isolated marriage market. There were 51 states which includes DC. Individuals are distinguished by their schooling level: less than high school (L), high school graduate (M) and university graduate (H).

A cohabitating couple is one where a respondent answered that they are the "unmarried partner" of the head of the household.

An observation in the dataset is the number of (r, i, j) relationships in a state year. Since there are three types of men and three types of women, there are potentially 9 types of matches for each type of relationship, marriage versus cohabitation.

Table 1 in Appendix A provides some summary statistics.

There are 1113 and 1283 non-zero number of cohabitations and marriages respectively. There are close to an average of 50,000 males and females of each type. The number of unmatched individuals exclude individuals whose partners are not in the (r, i, j) matches considered here. For example, if a woman has a husband older than 32, she will be counted in the number of females with her educational level and excluded in the count of the unmatched. There are close to an average of 20,000 unmatched individuals of each type.

The educational distributions by types and year are in Figure 2.

Due the small numbers of cohabitations for some observations, we reduce the effect of sampling error on our estimates by doing weighted regressions.[15]

Table 2 presents estimates of equation (3) by OLS. Although the OLS estimates are inconsistent, the estimates anticipate what we will find by IV. The smallest model, model 1, is in columns (1a) and (1b) where $\gamma_{ij}^{srt} = \gamma_{ij}^{r}$. The CS model, where $\alpha^r = \beta^r = \frac{1}{2}$, cannot be rejected in column 1a at the 5% significance level. But there is already evidence against the CS model in column 1b for the estimated coefficient on the unmatched females.[16] We can also reject the hypothesis of constant return to scale (CRS), $\alpha + \beta = 1$, in column 1b.

Model 2, in columns (2a) and (2b) add unrestricted year and match effects. The estimated year effects show that compared with 1990, the gains to cohabitation increased in 2000 and again in 2010, whereas the gains to marriage fell in 2000 and again in 2010.

Since the estimates of the match effects are difficult to interpret, we present instead the local log odds, equation (18). With three types of individuals by gender, there are four local log odds. In columns (2a) and (2b), all the local log odds are significantly positive. Thus there is strong evidence for PAM by educational attainment in both cohabitation and marriage. In fact PAM is present in both cohabitation and marriage in all our empirical models. There is mild evidence against CRS in columns (2a) and (2b).

Model 3, in columns (3a) and (3b) add state effects to the covariates. In model 1, the $R^2 s$ are in the 0.5 range. The $R^2 s$ increase to 0.9 by adding match and year effects in model 2. The $R^2 s$ increase to 0.92 and 0.97 in columns (3a) and (3b) respectively with the addition of state effects. As a descriptive model of marital behavior by state and year, the Cobb Douglas MMF is a very good

[15]Each observation is weighted by the average of m_i^{st} and f_j^{st}.

[16]In their empirical work which uses only time variations and aggregate data, CSW did not reject $\alpha^{\mathcal{M}} = \beta^{\mathcal{M}} = \frac{1}{2}$. We use both across state and time variations and so have a lot more power to test the same hypothesis.

summary of the data for these individuals.

The estimated coefficients on the unmatched increased significantly in model 3 compared with the estimates in model 1 consistent with our hypothesis that the error terms in model 1 are primarily gains to relationships effects. Since the gain to a relationship is negatively correlated with the unmatched, the estimates in model 1 are biased down relative to model 3. In model 3, CRS is easily rejected.

The test of CSPE, that $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}} = 1$, is in the second last row of the table. CSPE cannot be rejected in all three models. And in model 3, the point estimate of $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$ is essentially 1. As will also be the cases later, there will be no evidence against CSPE in the model with match, state and year effects by either OLS or IV.

Except for column (1a), the estimate of $\frac{\alpha}{\beta}$ is smaller than one which means that the peer effect coefficient for unmatched females, Φ^0, is smaller than the peer coefficient for unmatched males, ϕ^0. Put another way, compared with men, women's utilities as unmatched are less affected by their peers choosing to remain unmatched.

Finally in the last row of the Table 2, using lemma 2, we present the p-value for testing independence of the log odds of cohabitation versus marriage with respect to the sex ratio. Independence is not rejected at the 5% significance level in models 1 and 3. It is rejected in model 2. As will be shown later, these conclusions are not robust to IV estimation.

Table 3 presents IV estimates where the instruments for the unmatched, μ_{i0}^{st} and μ_{0j}^{st}, are the population supplies m_i^{st} and f_j^{st}. Since the error terms in the regressions represent the gains to a relationship and thus are negatively correlated with the number of unmatched, we expect the IV estimates to be larger than their OLS counterparts. Compared with their OLS counterparts, the IV estimates of the coefficients of the unmatched are marginally larger. They are significantly larger for model 3.

The IV estimates of the local log odds and year effects are similar to their OLS counterparts. This should not be surprising. The R^2s for the first stage regressions of the unmatched on population supplies exceed 0.95. Because the IV estimates for the unmatched are larger than the OLS estimates, the IV estimates of the constant term is smaller than their OLS counterparts.

Similar to the OLS estimates, the IV estimates for model 3 fits well. Interestingly, the estimates for $\alpha + \beta$ is close to 2 in column (3b), presenting the most suggestive evidence for the DM MMF although we can reject $\alpha = 1$ or $\beta = 1$. The estimates for cohabitation in column (3a) is less compelling for DM. Still, there is significant evidence for both peer and scale effects in marriage matching.

There is no evidence against CSPE. In particular in model 3, the point estimate for $\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$ is again essentially 1 and the standard error is small.

The one big difference in the IV estimates compared with the OLS estimates is the test of independence of the log odds of cohabitation versus marriage with respect to the sex ratio. For all three models in Table 3, independence is rejected at lower than the 1% significance level.

Table 4 presents IV estimates where we allow the marriage matching patterns to change over time. Model 1, in columns 1a and 1b, includes time varying match effects and year effects. Model 2, in columns 2a and 2b, add state effects. Unsurprisingly, based on estimates of the local log odds in 1990, PAM remains strong and significant. There is little evidence for systematic changes in the local log odds in 2000, 10 years later. There is more evidence for an increase in PAM along the main diagonal in cohabitation in 2010, 20 years later. There is also evidence of an increase in PAM in marriage between high school graduates and less than high school graduates. In general, except for a mild increase in PAM among cohabitants, there is little change in the degree of complementarity of the relationship output functions between 1990 and 2010. The stability in marriage matching pattern was anticipated in CSW and Siow (2015). This is in strong contrast to a loss of the gains to marriage and an increase in the gains to cohabitation over the same period. Again, there is little evidence against CSPE. Finally, independence is rejected around the 5% significance level for model 2 and less than 1% for model 3.

In summary, model 3 in Table 3, where gains to a relationship is captured by match, state and year effects, provides a reasonable summary of marital behavior for the individuals under study. Moreover, to a first order the match effects have not changed significantly over the period of study. There is a second order increase in PAM for cohabitants. Thus analysts can focus on studying mechanisms which affected state and year effects to the gains in relationships relative to remaining unmatched. Contrary to Botticini and Siow (2008) but anticipated by Fernandez-Villaverde, et.al. (2014), Adamopoulou (2012) and Drewianka (2003), peer and scale effects in the marriage market are empirically important.[17] Finally, independence of the log odds of the number of marriages to cohabitation with respect to the sex ratio is rejected.

4.1 Behavioral interpretation of estimated peer effects

Here, we use CSPE to interpret our estimated model 3 in Table 3. First, the condition for lemma 3 is satisfied and $\Phi^0 < \phi^0$. As discussed above, having more unmatched women do not increase the utilities of unmatched women as much as the same exercise for men. This result is apparently contrary to Adamopolou. But Adamopolou's estimates of peer effects only consist the direct peer effects because she uses small peer groups to estimate her effects. We use market level data which also include indirect (or scale) effects. Thus our estimates and hers are not directly comparable.

We cannot identify whether women or men are more responsive to peer effects in marriage or cohabitation due to our frictionless transferable utility model where only the joint gain to marriage or cohabitation is identified.

Second, the condition for lemma 4 is satisfied and $\Phi^{\mathcal{C}} + \phi^{\mathcal{C}} < \Phi^{\mathcal{M}} + \phi^{\mathcal{M}}$. So the mean value which an (i, j) couple derives from cohabitation is less sensitive

[17] Using across cities variation in the US, pre-reform China and medieval Tuscany, Botticini and Siow's (2008) could not reject constant returns to scale with an aggregate marriage rate. This finding may be related to our inability to show that $\beta^r + \alpha^r > 1$ implies increasing returns with an aggregate unmatched rate.

to peer effects than from marriage. Because cohabitation is a significantly newer form of publicly accepted relationship, we expect the direct peer effect of cohabitation to be stronger than marriage. Our estimates suggests that the indirect (or scale) effects for marriage is stronger than that for cohabitation.

The above two conclusions on how peer effects affect marital behavior is dependent on our estimates as well as our behavioral model, CSPE. We are not aware of other estimates of these peer effects using aggregate data. Thus we do not view our results as conclusive. Rather, we hope they will stimulate more research on these effects.

5 Conclusion

This paper presented an easy to estimate and simulate MMF, the Cobb Douglas MMF. Several behavioral MMFs are special cases including CSPE. Our empirical results show that the Cobb Douglas MMF provides a reasonably complete and parsimonous characterization of the recent evolution the US marriage market. Scale effects are quantitatively important. Independence of the log odds of the number of marriages to cohabitation with respect to the sex ratio is rejected. We also show that changes in marital matching behavior over this period are best explained by mechanisms which explain year and state effects in the gains to relationships relative to remaining unmatched.

And as we discussed in the introduction, although we are partial to CSPE and it is not rejected empirically, it should be clear that we propose the Cobb Douglas MMF precisely because we do not want to insist on a particular behavioral model of the marriage market. Rather, we view the evidence here in support of CSPE as a proof of concept that it is a useful empirical model.

In order to keep the paper within a reasonable length, our empirical study focused on a small subset of the marriageable population. Estimating the model on a larger subset of the population is an important agenda for future research. Also using the model to study particular mechanisms for marital change is another important topic for future research. For e.g., Cornelson and Siow (2015) used a special case of the above framework to show that increased earnings inequality cannot explain the decline the marriage rate of young Americans from 1970 to 2010.

From an analytic perspective, we are working on sufficient conditions for characterizing increasing and/or decreasing returns to scale. It will also be useful to study other behavioral models which allow for more varied relationships between the log odds of the number of marriages to cohabitation and changes in the sex ratio. There is also room to investigate more general peer effects specifications.

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A Figures and Tables

Figure 1: Marital Status by Gender and Year.

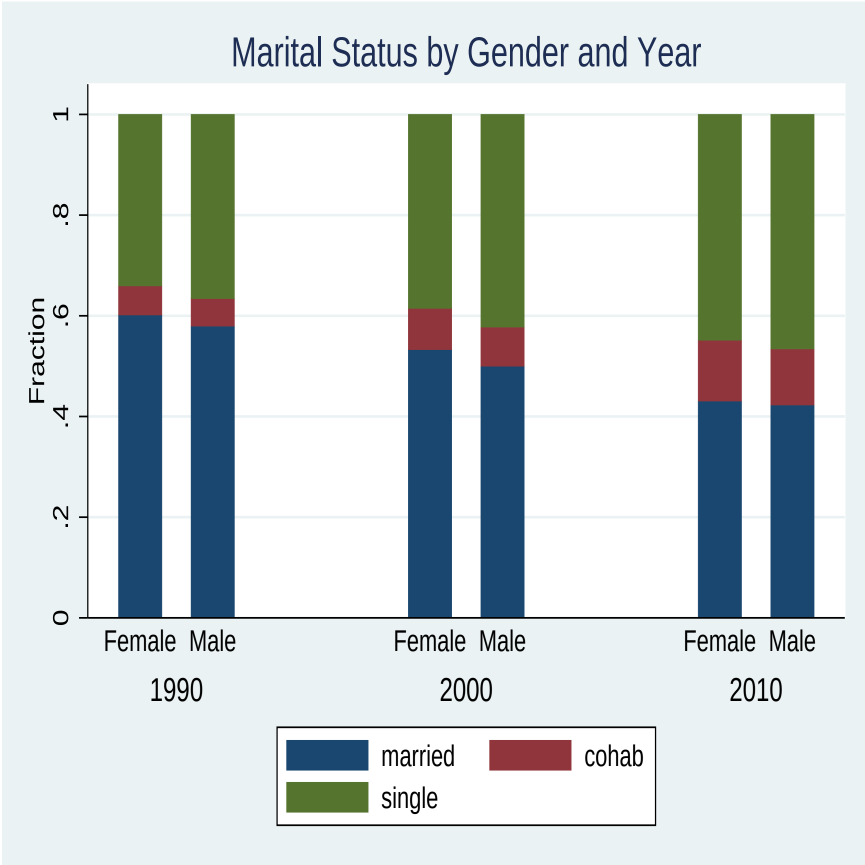

Figure 2: Fraction of individual by gender, education and year.

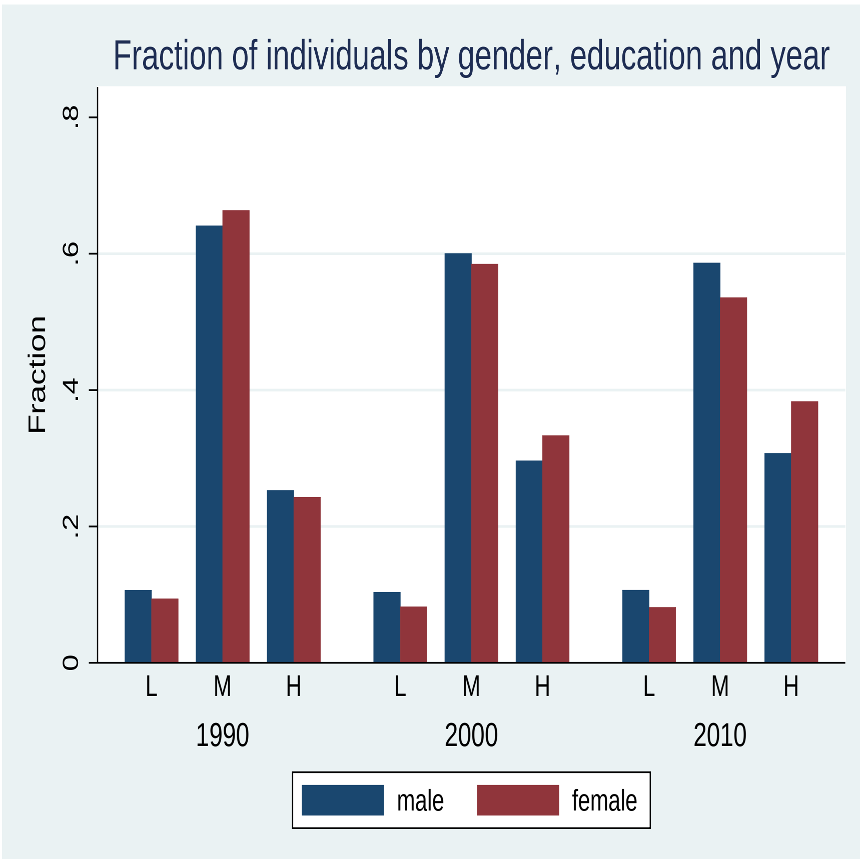

Table 1: Summary Statics*.

Variable	Obs	Mean	Std. Dev.	Min	Max
N cohabitations	1113	710.7491	1367.211	3	15362
N marriages	1283	5023.924	9981.822	6	118867
N males	1377	49097.82	67662.33	174	568449
N females	1377	48319.55	67411	143	580493
N unmatched males	1377	20361.38	30701.33	165	262267
N umatched females	1377	18757.32	28271.38	76	236391
Year	1377	2000	8.167932	1990	2010

*An observation is a state/year. There are 51 states which includes DC. Observations with 0 cohabitation or marriages are excluded.

Table 2: Ordinary Least Square (OLS).

	1a	1b	2a	2b	3a	3b
Dep. Var.	LCOH	LMAR	LCOH	LMAR	LCOH	LMAR
LU_M (α)	0.562	0.536	0.320	0.244	0.557	0.626
	(0.041)**	(0.048)**	(0.075)**	(0.061)**	(0.084)**	(0.056)**
LU_F (β)	0.531	0.665	0.630	0.609	0.827	0.939
	(0.040)**	(0.047)**	(0.076)**	(0.058)**	(0.078)**	(0.051)**
$L\left[\frac{HH*MM}{HM*MH}\right]$			2.31	2.44	2.290	2.412
			(0.078)**	(0.069)**	(0.071)**	(0.048)**
$L\left[\frac{MM*LL}{LM*ML}\right]$			1.78	2.52	1.781	2.464
			(0.087)**	(0.082)**	(0.080)**	(0.063)**
$L\left[\frac{HM*ML}{MM*HL}\right]$			0.784	1.43	0.796	1.426
			(0.147)**	(0.092)**	(0.145)**	(0.084)**
$L\left[\frac{MH*LM}{MM*LH}\right]$			1.33	1.37	1.344	1.379
			(0.141)**	(0.101)**	(0.135)**	(0.083)**
Y2000			0.289	-0.287	0.313	-0.277
			(0.042)**	(0.032)**	(0.037)**	(0.022)**
Y2010			0.627	-0.604	0.615	-0.667
			(0.041)**	(0.037)**	(0.040)**	(0.030)**
STATE					Y	Y
_cons	-4.788	-3.981	-2.842	1.172	-7.359	-5.015
	(0.383)**	(0.441)**	(0.196)**	(0.158)**	(0.772)**	(0.543)**
R^2	0.51	0.45	0.90	0.95	0.92	0.97
N	1,113	1,283	1,113	1,283	1,113	1,283
$\frac{\alpha}{\beta}$	1.058	0.806	0.508	0.400	0.673	0.667
	(0.137)	(0.115)	(0.178)	(0.138)	(0.146)	(0.082)
$\alpha+\beta$	1.093	1.202	0.950	0.853	1.384	1.57
	(0.039)	(0.045)	(0.018)	(0.016)	(0.079)	(0.057)
$\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$		0.762		0.788		0.991
		(0.147)		(0.387)		(0.246)
$prob\begin{bmatrix}\alpha^{\mathcal{M}}=\alpha^{\mathcal{C}}\\ \beta^{\mathcal{M}}=\beta^{\mathcal{C}}\end{bmatrix}$		0.091		0.001		0.117

* $p<0.05$; ** $p<0.01$

Table 3: Instrumental Variable (IV).

	1a	1b	2a	2b	3a	3b
Dep. Var.	LCOH	LMAR	LCOH	LMAR	LCOH	LMAR
LU_M (α)	0.574	0.599	0.267	0.164	0.556	0.728
	(0.044)**	(0.053)**	(0.086)**	(0.074)*	(0.095)**	(0.070)**
LU_F (β)	0.527	0.717	0.682	0.731	0.915	1.186
	(0.042)**	(0.052)**	(0.089)**	(0.070)**	(0.090)**	(0.058)**
$L\left[\frac{HH*MM}{HM*MH}\right]$			2.315	2.436	2.282	2.393
			(0.079)**	(0.073)**	(0.071)**	(0.051)**
$L\left[\frac{MM*LL}{LM*ML}\right]$			1.783	2.514	1.773	2.440
			(0.087)**	(0.081)**	(0.080)**	(0.051)**
$L\left[\frac{HM*ML}{MM*HL}\right]$			0.784	1.430	0.802	2.440
			(0.149)	(0.092)**	(0.147)**	(0.064)**
$L\left[\frac{MH*LM}{MM*LH}\right]$			1.340	1.377	1.350	1.428
			(0.141)	(0.104)**	(0.135)**	(0.086)**
Y2000			0.293	-0.277	0.320	-0.258
			(0.042)**	(0.035)**	(0.038)**	(0.025)**
Y2010			0.624	-0.612	0.607	-0.694
			(0.041)**	(0.038)**	(0.039)**	(0.030)**
STATE					Y	Y
_cons	-4.806	-5.215	-2.806	0.775	-8.045	-8.216
	(0.384)**	(0.452)**	(0.196)**	(0.180)**	(0.798)**	(0.663)**
R^2	0.51	0.45	0.90	0.95	0.92	0.97
N	1,113	1,283	1,113	1,283	1,113	1,283
$\frac{\alpha}{\beta}$	1.045	0.808	0.346	0.201	0.605	0.606
	(0.132)	(0.107)	(0.165)	(0.103)	(0.135)	(0.065)
$\alpha+\beta$	1.102	1.316	0.950	0.892	1.501	1.908
	(0.038)	(0.045)	(0.018)	(0.016)	(0.082)	(0.064)
$\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$		0.771		0.521		1.00
		(0.140)		(0.347)		(0.249)
$prob\begin{bmatrix}\alpha^{\mathcal{M}}=\alpha^{\mathcal{C}}\\ \beta^{\mathcal{M}}=\beta^{\mathcal{C}}\end{bmatrix}$		0.000		0.090		0.000

* $p<0.05$; ** $p<0.01$

Table 4: IV with time varying match effects.

	1a	1b	2a	2b
Dependent variable	LCOH	LMAR	LCOH	LMAR
LU_M (α)	0.415	0.357	0.576	0.754
	(0.071)**	(0.064)**	(0.077)**	(0.057)**
LU_F (β)	0.528	0.524	0.688	0.885
	(0.071)**	(0.063)**	(0.073)**	(0.050)**
$L\left[\frac{HH*MM}{HM*MH}\right]$	2.288	2.458	2.278	2.440
	(0.148)**	(0.087)**	(0.145)**	(0.045)**
$L\left[\frac{MM*LL}{LM*ML}\right]$	1.504	2.255	1.514	2.198
	(0.116)**	(0. .121)**	(0.103)**	(0.067)**
$L\left[\frac{HM*ML}{MM*HL}\right]$	1.169	1.698	0.787	1.702
	(0.283)**	(0.136)**	(0.211)**	(0.110)**
$L\left[\frac{MH*LM}{MM*LH}\right]$	0.693	1.305	1.177	1.313
	(0.260)**	(0.134)**	(0.266)**	(0.119)
$L\left[\frac{HH*MM}{HM*MH}\right]*Y2000$	0.259	-0.011	0.663	-0.097
	(0.181)	(0.191)	(0.230)**	(0.119)
$L\left[\frac{MM*LL}{LM*ML}\right]*Y2000$	0.346	0.346	0.257	0.337
	(0.323)	(0.186)	(0.153)	(0.124)**
$L\left[\frac{HM*ML}{MM*HL}\right]*Y2000$	0.032	-0.279	0.332	-0.274
	(0.330)	(0.187)	(0.317)	(0.151)
$L\left[\frac{MH*LM}{MM*LH}\right]*Y2000$	1.133	-0.042	0.040	-0.052
	(0.252)	(0.193)	(0.311)	(0.152)
$L\left[\frac{HH*MM}{HM*MH}\right]*Y2010$	1.133	-0.208	1.062	-0.396
	(0.252)**	(0.199)	(0.232)**	(0.128)
$L\left[\frac{MM*LL}{LM*ML}\right]*Y2010$	0.692	0.534	0.679	0.540
	(0.190)**	(0.200)**	(0.181)**	(0.162)**
$L\left[\frac{HM*ML}{MM*HL}\right]*Y2010$	-0.384	-0.609	-0.323	-0.655
	(0.261)	(0.268)**	(0.261)	(0.246)**
$L\left[\frac{MH*LM}{MM*LH}\right]*Y2010$	0.394	0.264	0.391	0.237
	(0.392)	(0.234)	(0.376)	(0.203)
Y2000	0.693	0.014	0.669	-0.071
	(0.104)**	(0.075)	(0.091)**	(0.054)
Y2010	1.122	-0.097	1.061	-0.274
	(0.102)**	(0.079)	(0.092)**	(0.057)**
STATE			Y	Y
_cons	-3.075	0.618	-6.512	-5.900
	(0.185)**	(0.161)**	(0.735)**	(0.506)**
R^2	0.91	0.96	0.93	0.98
N	1,113	1,283	1,113	1,283
$\frac{\alpha}{\beta}$	0.786	0.681	0.836	0.852
	(0.239)	(0.203)	(0.174)	(0.096)
$\alpha+\beta$	0.943	0.881	1.264	1.640
	(0.017)	(0.016)	(0.076)	(0.055)
$\frac{\alpha^{\mathcal{M}}}{\beta^{\mathcal{M}}}\frac{\beta^{\mathcal{C}}}{\alpha^{\mathcal{C}}}$		0.866		1.019
		(0.369)		(0.241)
$prob\left[\begin{matrix}\alpha^{\mathcal{M}}=\alpha^{\mathcal{C}}\\ \beta^{\mathcal{M}}=\beta^{\mathcal{C}}\end{matrix}\right]$		0.051		0.000

APPENDIX: FOR ONLINE PUBLICATION

B Existence and Uniqueness of the Matching Equilibrium

To ease the notation, denote $\mathcal{M} \equiv a$ and $\mathcal{C} \equiv b$ in the rest of the paper. The matching equilibrium in this model is characterized by the Cobb Douglas MMF (4) and the population constraint equations

$$\sum_{j=1}^{J} \mu_{ij}^{a} + \sum_{j=1}^{J} \mu_{ij}^{b} + \mu_{i0} = m_i, \quad 1 \leq i \leq I \tag{20}$$

$$\sum_{i=1}^{I} \mu_{ij}^{a} + \sum_{i=1}^{I} \mu_{ij}^{b} + \mu_{0j} = f_j, \quad 1 \leq j \leq J \tag{21}$$

$$\mu_{0j}, \mu_{i0} \geq 0, \quad 1 \leq j \leq J, 1 \leq i \leq I.$$

Let $m \equiv (m_1, ..., m_I)'$, $f \equiv (f_1, ..., f_J)'$, $\mu \equiv (\mu_{10}, ..., \mu_{I0}, \mu_{01}, ..., \mu_{0J})'$, $\gamma^r \equiv (\gamma_{11}^r, ..., \gamma_{1I}^r, ..., \gamma_{I1}^r, ..., \gamma_{IJ}^r)'$ for $r \in \{a, b\}$, $\beta^r \equiv (\beta_{11}^r, ..., \beta_{1I}^r, ..., \beta_{I1}^r, ..., \beta_{IJ}^r)'$, $\alpha^r \equiv (\alpha_{11}^r, ..., \alpha_{1I}^r, ..., \alpha_{I1}^r, ..., \alpha_{IJ}^r)'$ $\beta \equiv ((\beta^a)', (\beta^b)')'$, $\alpha \equiv ((\alpha^a)', (\alpha^b)')'$ and $\theta \equiv ((\gamma^a)', (\gamma^b)', \alpha', \beta')'$. Let Γ be a closed and bounded subset of $\mathbb{R}^{2IJ}$ such that $\theta \in \Gamma \times (0, \infty)^2$. Equation (4) can be written as follows:

$$\mu_{ij}^r = \mu_{i0}^{\alpha_{ij}^r} \mu_{0j}^{\beta_{ij}^r} e^{\gamma_{ij}^r} \qquad \text{for } r \in \{a, b\}. \tag{22}$$

Now, let consider the following mapping $g : (\mathbb{R}_+^*)^{I+J} \to (\mathbb{R}_+^*)^{I+J}$

$$g_i(\mu; \theta) = \mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b}, \text{ for } 1 \leq i \leq I, \tag{23}$$

$$g_{j+I}(\mu; \theta) = \mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b}, \text{ for } 1 \leq j \leq J. \tag{24}$$

This mapping is obtained by just rewriting the left side of the population constraints with the Cobb Douglas MMF. We show later in Section B.1 that for every $\theta \in \Gamma \times (0,\infty)^2$ g is a proper mapping[18] and that the Jacobian of $g(\mu;\theta)$ (i.e. $J_g(\mu;\theta)$) does not vanish for all μ in $(\mathbb{R}_+^*)^{I+J}$. Thus, we can invoke Hadamard's theorem [19] (see Krantz and Park (2003, Theorem 6.2.8 p 126)), which tells us that g is an homeomorphism i.e. (one-to-one mapping) whenever the latter two properties of g hold. Then, for every $\theta \in \Gamma \times (0,\infty)^2$ and for all $m > 0$ and $f > 0$ the system of equations

$$\mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b} = m_i, \text{ for } 1 \leq i \leq I \tag{25}$$

$$\mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b} = f_j, \text{ for } 1 \leq j \leq J. \tag{26}$$

admit a unique solution $0 < \mu < (m', f')'$. Therefore the equilibrium matching of the Cobb Douglas MMF model exists and is unique. The following theorem summarizes our discussion:

B.1 Proof of Theorem 1

Proof. Consider the following continuously differentiable function $g : (\mathbb{R}_+^*)^{I+J} \rightarrow (\mathbb{R}_+^*)^{I+J}$

$$g_i(\mu) = \mu_{i0} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{j=1}^{J} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b}, \tag{27}$$

$$g_{j+I}(\mu) = \mu_{0j} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^a} \mu_{0j}^{\beta_{ij}^a} e^{\gamma_{ij}^a} + \sum_{i=1}^{I} \mu_{i0}^{\alpha_{ij}^b} \mu_{0j}^{\beta_{ij}^b} e^{\gamma_{ij}^b}. \tag{28}$$

Notice that,

$$\begin{aligned} \mu_{i0}^{\alpha_{ij}^r} \mu_{0j}^{\beta_{ij}^r} e^{\gamma_{ij}^r} &= e^{\alpha_{ij}^r ln\mu_{i0} + \beta_{ij}^r ln\mu_{0j} + \gamma_{ij}^r}, \\ &\equiv e^{\delta_{ij}^r}. \end{aligned}$$

[18]A continuous function between topological spaces is called proper if the inverse images of compact subsets are compact.

[19]We thank Marcin Peski for pointing to us out this Hadamard's result.

Therefore, the mapping $g(\mu)$ can be written equivalently as follows:

$$g_i(\mu) = \mu_{i0} + \sum_{j=1}^{J} e^{\delta^a_{ij}} + \sum_{j=1}^{J} e^{\delta^b_{ij}}, \tag{29}$$

$$g_{j+I}(\mu) = \mu_{0j} + \sum_{i=1}^{I} e^{\delta^a_{ij}} + \sum_{i=1}^{I} e^{\delta^b_{ij}}. \tag{30}$$

Let $J_g(\mu)$ be the Jacobian of g. After a simple derivation we can show that $J_g(\mu)$ takes the following form:

$$J_g(\mu) = \begin{pmatrix} (J_g)_{11}(\mu) & (J_g)_{12}(\mu) \\ (J_g)_{21}(\mu) & (J_g)_{22}(\mu) \end{pmatrix}$$

with

$$(J_g)_{11}(\mu) = \begin{pmatrix} 1 + \sum_{j=1}^{J}\left[\frac{\alpha^a_{1j}}{\mu_{10}} e^{\delta^a_{1j}} + \frac{\alpha^b_{1j}}{\mu_{10}} e^{\delta^b_{1j}}\right] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 + \sum_{j=1}^{J}\left[\frac{\alpha^a_{Ij}}{\mu_{I0}} e^{\delta^a_{Ij}} + \frac{\alpha^b_{Ij}}{\mu_{I0}} e^{\delta^b_{Ij}}\right] \end{pmatrix},$$

$$(J_g)_{12}(\mu) = \begin{pmatrix} \frac{\beta^a_{11}}{\mu_{01}} e^{\delta^a_{11}} + \frac{\beta^b_{11}}{\mu_{01}} e^{\delta^b_{11}} & \cdots & \frac{\beta^a_{1J}}{\mu_{0J}} e^{\delta^a_{1J}} + \frac{\beta^b_{1J}}{\mu_{0J}} e^{\delta^b_{1J}} \\ \vdots & \ddots & \vdots \\ \frac{\beta^a_{I1}}{\mu_{01}} e^{\delta^a_{I1}} + \frac{\beta^b_{I1}}{\mu_{01}} e^{\delta^b_{I1}} & \cdots & \frac{\beta^a_{IJ}}{\mu_{0J}} e^{\delta^a_{IJ}} + \frac{\beta^b_{IJ}}{\mu_{0J}} e^{\delta^b_{IJ}} \end{pmatrix},$$

$$(J_g)_{21}(\mu) = \begin{pmatrix} \frac{\alpha^a_{11}}{\mu_{10}} e^{\delta^a_{11}} + \frac{\alpha^b_{11}}{\mu_{10}} e^{\delta^b_{11}} & \cdots & \frac{\alpha^a_{I1}}{\mu_{I0}} e^{\delta^a_{I1}} + \frac{\alpha^b_{I1}}{\mu_{I0}} e^{\delta^b_{I1}} \\ \vdots & \ddots & \vdots \\ \frac{\alpha^a_{1J}}{\mu_{10}} e^{\delta^a_{1J}} + \frac{\alpha^b_{1J}}{\mu_{10}} e^{\delta^b_{1J}} & \cdots & \frac{\alpha^a_{IJ}}{\mu_{I0}} e^{\delta^a_{IJ}} + \frac{\alpha^b_{IJ}}{\mu_{I0}} e^{\delta^b_{IJ}} \end{pmatrix},$$

$$(J_g)_{22}(\mu) = \begin{pmatrix} 1 + \sum_{i=1}^{I}\left[\frac{\beta^a_{i1}}{\mu_{01}} e^{\delta^a_{i1}} + \frac{\beta^b_{i1}}{\mu_{01}} e^{\delta^b_{i1}}\right] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 + \sum_{i=1}^{I}\left[\frac{\beta^a_{iJ}}{\mu_{0J}} e^{\delta^a_{iJ}} + \frac{\beta^b_{iJ}}{\mu_{0J}} e^{\delta^b_{iJ}}\right]. \end{pmatrix}.$$

Claim 1 *The Jacobian $J_g(\mu)$ does not vanish for all μ in $(\mathbb{R}^*_+)^{I+J}$.*

Proof. $J_g(\mu)$ is a column diagonally dominant matrix or diagonally dominant in the sense of McKenzie (1960) and therefore $J_g(\mu)$, for all $\mu > 0$, is a non-singular matrix. See McKenzie (1960, p 47-50) for more details on this result. Indeed, let us denote every element of $J_g(\mu)$, $b_{k,l}$ with $1 \leq k, l \leq I + J$. $J_g(\mu)$

is diagonally dominant in the sense of McKenzie (1960) if there exist numbers $d_l > 0$ such that $d_l|b_{ll}| > \sum_{k \neq l}^{I+J} d_k|b_{kl}|$ for $l = 1, ..., I + J$. Here, it is sufficient to take $d_l = 1$ for $1 \leq l \leq I + J$. Indeed, if you take one element in the diagonal of the matrix $(J_g)_{11}(\mu)$, it can be seen that this element is greater than the summation of all elements in the same column of the matrix $(J_g)_{21}(\mu)$. ■

Claim 2 *$(\mathbb{R}_+^*)^{I+J}$ is a smooth manifold and simply connected.*

Claim 3 *g is proper.*

Proof.

Definition 1 *Let $\mathbb{X}$, $\mathbb{Y}$ be topological spaces and g: $\mathbb{X} \to \mathbb{Y}$ be a mapping. g is said to be proper if whenever $K \subseteq \mathbb{Y}$ is compact then $g^{-1}(K) \subseteq \mathbb{X}$ is compact.*

Krantz and Park (2003, p 125) pointed out the following lemma that gives more operational criteria for checking if a mapping is proper.

Lemma 5 *Let U and V be connected open sets in $\mathbb{R}^{I+J}$, g: $U \to V$ is a proper mapping if and only if whenever $\{x_j\} \subseteq U$ satisfies $x_j \to \partial U$ then $g(x_j) \to \partial V$.*

Notice that ∂V is used for the boundary of set V. Therefore, the following result completes the proof; define $\underline{\mu} = (0, ..., 0, 0, ..., 0)'$ and $\overline{\mu} = (+\infty, ..., +\infty, +\infty, ..., +\infty)'$. We can easily show that $lim_{\mu \to \underline{\mu}} g(\mu) = (0, ..., 0, 0, ..., 0)'$ and $lim_{\mu \to \overline{\mu}} g(\mu) = (+\infty, ..., +\infty, +\infty, ..., +\infty)'$. Notice that there are many different ways to show that our mapping g(.) is proper as soon as we remark that $g_i(\mu) > \mu_{i0}$. ■ After presenting the previous claims, we can now invoke Hadamard's theorem as stated in Krantz and Park (2003, Theroem 6.2.8 p 126). This theorem ensures that g is an homeomorphism. Moreover, it is easy to see that the solution μ^{eq} of system of equations (25) satisfies the restriction $0 < \mu^{eq} < (m', f')'$. Notice that the existence of at least an equilibrium can be shown by invoking Brouwer's fixed point theorem. It is also worth noting that the same proof can be used for a more general MMF i.e., $\mu_{ij} = g(\mu_{i0}, \mu_{0j})$ as used in Galichon and al (2014) and the latter can be extended to multiple relationship as entertained here. This completes our proof. ■

C Comparative Statistics

C.1 Fixed point representation of the equilibrium of the Cobb Douglas MMF

After rearranging equation (22) we have four equalities that holds for all (i, j) pairs:

$$\frac{\mu^r_{ij}}{\mu_{i0}} = exp\Big[\gamma^r_{ij} + (\alpha^r - 1)\ln\mu_{i0} + \beta^r \ln\mu_{0j}\Big] \equiv \eta^r_{ij} \qquad \text{for } r \in \{a, b\}, \tag{31}$$

$$\frac{\mu^r_{ij}}{\mu_{0j}} = exp\Big[\gamma^r_{ij} + \alpha^r \ln\mu_{i0} + (\beta^r - 1)\ln\mu_{0j}\Big] \equiv \zeta^r_{ij} \qquad \text{for } r \in \{a, b\}. \tag{32}$$

Using equations (31) and (32) we have:

$$\sum_{j=1}^{J}\mu^a_{ij} + \sum_{j=1}^{J}\mu^b_{ij} = \mu_{i0}\sum_{j=1}^{J}\Big[\eta^a_{ij} + \eta^b_{ij}\Big], \quad 1 \le i \le I,$$
$$\sum_{i=1}^{I}\mu^a_{ij} + \sum_{i=1}^{I}\mu^b_{ij} = \mu_{0j}\sum_{i=1}^{I}\Big[\zeta^a_{ij} + \zeta^b_{ij}\Big], \quad 1 \le j \le J.$$

Manipulating the population constraints (20), (21) we have the following:

$$\mu_{i0} = \frac{m_i}{1 + \sum_{j=1}^{J}\Big[\eta^a_{ij} + \eta^b_{ij}\Big]} \equiv B_{i0}, \quad 1 \le i \le I \tag{33}$$

$$\mu_{0j} = \frac{f_j}{1 + \sum_{i=1}^{I}\Big[\zeta^a_{ij} + \zeta^b_{ij}\Big]} \equiv B_{0j}, \quad 1 \le j \le J. \tag{34}$$

Let $B(\mu; m, f, \theta) \equiv (B_{10}(.), ..., B_{I0}(.), B_{01}(.), ..., B_{0J}(.))'$. For a fixed θ we have shown that the $(I + J)$ vector μ of the number of agents of each type who choose not to match is a solution to $(I + J)$ vector of implicit functions

$$\mu - B(\mu; m, f, \theta) = 0. \tag{35}$$

Let $\mathbb{T}_\epsilon = \{\epsilon \le \mu_{10} \le m_1, ..., \epsilon \le \mu_{I0} \le m_I, \epsilon \le \mu_{01} \le f_1, ..., \epsilon \le \mu_{0J} \le f_J\}$ be a closed and bounded rectangular region in $\mathbb{R}^{I+J}$ with ϵ some arbitrarily small positive constant. We know from Theorem 1 that the fixed point representation has a unique solution $\mu^{eq} > 0$. We can verify that $\mu^{eq} \in \mathbb{T}_\epsilon$. Now, let $J(\mu) = I_{I+J} - \bigtriangledown_\mu B(\mu; m, f, \theta)$ with $\bigtriangledown_\mu B(\mu; m, f, \theta) = \frac{\partial B(\mu; m, f, \theta)}{\partial \mu'}$ be the $(I+J) \times (I+J)$ Jacobian matrix associated with (36). For a fixed θ we have shown that the $(I+J)$ vector μ of the number of agents of each type who choose not to match is a solution to $(I+J)$ vector of implicit functions

$$\mu - B(\mu; m, f, \theta) = 0. \tag{36}$$

C.2 Comparative Statistics

Theorem 2 *Let μ be the equilibrium matching distribution of the Cobb Douglas MMF model. If the coefficients β^r and α^r respect the restrictions*

1. $0 < \beta^r; \alpha^r \leq 1$ *for* $r \in \{\mathcal{M}, \mathcal{C}\}$;
2. $\max(\beta^{\mathcal{C}} - \alpha^{\mathcal{C}}, \beta^{\mathcal{M}} - \alpha^{\mathcal{M}}) < \min_{i \in I} \left(\frac{1-\rho_i^m}{\rho_i^m} \right)$;
3. $\min(\beta^{\mathcal{C}} - \alpha^{\mathcal{C}}, \beta^{\mathcal{M}} - \alpha^{\mathcal{M}}) > -\max_{j \in J} \left(\frac{1-\rho_j^f}{\rho_j^f} \right)$;

where ρ_i^m is the rate of matched men of type i and ρ_j^f is the rate of matched women of type j, then the following inequalities hold in the neighbourhood of μ^{eq}:

1. *Type-specific elasticities of unmatched.*

(a) $$\frac{m_i}{\mu_{k0}} \frac{\partial \mu_{k0}}{\partial m_i} \geq \begin{cases} \frac{1}{m_i^*} \frac{m_k}{m_k^*} \sum_{j=1}^J \frac{[\alpha^{\mathcal{M}} \mu_{kj}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{kj}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{kj}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{kj}^{\mathcal{C}}]}{f_j^*} > 0 & \text{if } k \neq i \\ \frac{m_i}{m_i^*} [1 + \frac{1}{m_i^*} \sum_{j=1}^J \frac{[\alpha^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}]}{f_j^*}] > 1 & \text{if } k = i, \end{cases}$$
$1 \leq k \leq I$.

(b) $$\frac{f_j}{\mu_{0k}} \frac{\partial \mu_{0k}}{\partial f_j} \geq \begin{cases} \frac{1}{f_j^*} \frac{f_k}{f_k^*} \sum_{i=1}^I \frac{[\alpha^{\mathcal{M}} \mu_{ik}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{ik}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{ik}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{ik}^{\mathcal{C}}]}{m_i^*} > 0 & \text{if } k \neq j \\ \frac{f_j}{f_j^*} [1 + \frac{1}{f_j^*} \sum_{i=1}^I \frac{[\alpha^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \alpha^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}} \mu_{ij}^{\mathcal{M}} + \beta^{\mathcal{C}} \mu_{ij}^{\mathcal{C}}]}{m_i^*}] > 1 & \text{if } k = j, \end{cases}$$
$1 \leq k \leq J$,

(c)

$$\frac{m_i}{\mu_{0j}}\frac{\partial \mu_{0j}}{\partial m_i} \leq -\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}} + \alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} m_i < 0, \text{ for } 1 \leq i \leq I \text{ and } 1 \leq j \leq J,$$

(d)

$$\frac{f_j}{\mu_{i0}}\frac{\partial \mu_{i0}}{\partial f_j} \leq -\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}} + \beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} f_j < 0, \text{ for } 1 \leq i \leq I \text{ and } 1 \leq j \leq J,$$

2. *Variation of the log ratio* $\ln \frac{\mu_{ij}^{\mathcal{M}}}{\mu_{ij}^{\mathcal{C}}}$:
If $\alpha^{\mathcal{M}} > \alpha^{\mathcal{C}}$ *and* $\beta^{\mathcal{C}} > \beta^{\mathcal{M}}$ *we have*

(a) $$\frac{1}{\partial m_i}[\ln \frac{\mu_{kj}^{\mathcal{M}}}{\mu_{kj}^{\mathcal{C}}}] \geq \begin{cases} \frac{\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}}{m_i^* m_i}\frac{m_k}{m_k^*}\sum_{j=1}^{J}\frac{[\alpha^{\mathcal{M}}\mu_{kj}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{kj}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{kj}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{kj}^{\mathcal{C}}]}{f_j^*} \\ \qquad +(\beta^{\mathcal{M}}-\beta^{\mathcal{C}})\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} > 0 & \text{if } k \neq i \\ \frac{\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}}{m_i^*}\left[1+\frac{1}{m_i^*}\sum_{j=1}^{J}\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{f_j^*}\right] \\ \qquad +(\beta^{\mathcal{M}}-\beta^{\mathcal{C}})\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} > \alpha^{\mathcal{M}}-\alpha^{\mathcal{C}} & \text{if } k = i, \end{cases}$$

$1 \leq k \leq I$

(b) $$\frac{1}{\partial f_j}[\ln \frac{\mu_{ik}^{\mathcal{M}}}{\mu_{ik}^{\mathcal{C}}}] \leq \begin{cases} \frac{\beta^{\mathcal{M}}-\beta^{\mathcal{C}}}{f_j^* f_j}\frac{f_k}{f_k^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ik}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ik}^{\mathcal{C}}]}{m_i^*} \\ \qquad -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}})\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} < 0 & \text{if } k \neq j \\ \frac{\beta^{\mathcal{M}}-\beta^{\mathcal{C}}}{f_j^*}[1+\frac{1}{f_j^*}\sum_{i=1}^{I}\frac{[\alpha^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\alpha^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}][\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^*}] \\ \qquad -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}})\frac{[\beta^{\mathcal{M}}\mu_{ij}^{\mathcal{M}}+\beta^{\mathcal{C}}\mu_{ij}^{\mathcal{C}}]}{m_i^* f_j^*} f_j < -(\alpha^{\mathcal{M}}-\alpha^{\mathcal{C}}) & \text{if } k = j, \end{cases}$$

$1 \leq k \leq J$

where

$$m_i^* \equiv m_i - \sum_{j=1}^{J}[(1-\alpha^{\mathcal{M}})\mu_{ij}^{\mathcal{M}} + (1-\alpha^{\mathcal{C}})\mu_{ij}^{\mathcal{C}}], \text{ for } 1 \leq i \leq I,$$

$$f_j^* \equiv f_j - \sum_{i=1}^{I}[(1-\beta^{\mathcal{M}})\mu_{ij}^{\mathcal{M}} + (1-\beta^{\mathcal{C}})\mu_{ij}^{\mathcal{C}}], \text{ for } 1 \leq j \leq J.$$

It is worth noting that the restriction imposed on β^r and α^r are only necessary and would be very mild depending on the model. For instance, those restrictions directly holds for the CS and DM model; Graham (2013) shows that those restrictions are not necessary to derive the comparative statistics in the CSW model.

C.3 Proof of Theorem 2

All derivation in this section will be done at the matching equilibrium μ^{eq}. However, to ease notation we will use the notation μ.

Proof.

Step 0: Derivation of the $J(\mu)$ matrix.

To ease the notation, in the following we will use $B(\mu)$ to denote $B(\mu; m, f, \theta)$ whenever no confusion is possible.

$J(\mu) = I_{I+J} - \bigtriangledown_\mu B(\mu)$. After tedious but simple manipulations we can show that

$$\bigtriangledown_\mu B(\mu) = \begin{pmatrix} E_{11}(\mu) & E_{12}(\mu) \\ E_{21}(\mu) & E_{22}(\mu) \end{pmatrix}$$

with

$$E_{11}(\mu) = diag\Big\{ \textstyle\sum_{j=1}^{J} e_{j|1}(\mu), ..., \sum_{j=1}^{J} e_{j|I}(\mu) \Big\},$$

$$E_{22}(\mu) = diag\Big\{ \textstyle\sum_{i=1}^{I} g_{i|1}(\mu), ..., \sum_{i=1}^{I} g_{i|J}(\mu) \Big\} \text{ where}$$

$$e_{j|i} = \frac{m_i}{\mu_{i0}} \left[\frac{(1-\alpha^a)\eta^a_{ij} + (1-\alpha^b)\eta^b_{ij}}{\Big(1+\sum_{j=1}^{J}[\eta^a_{ij}+\eta^b_{ij}]\Big)^2} \right],\; g_{i|j} = \frac{f_j}{\mu_{0j}} \left[\frac{(1-\beta^a)\zeta^a_{ij} + (1-\beta^b)\zeta^b_{ij}}{\Big(1+\sum_{i=1}^{I}[\zeta^a_{ij}+\zeta^b_{ij}]\Big)^2} \right].$$

$$E_{12}(\mu) = -\begin{pmatrix} \frac{\mu_{10}}{\mu_{01}}\hat{e}_{1|1} & \cdots & \frac{\mu_{10}}{\mu_{0J}}\hat{e}_{J|1} \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}}\hat{e}_{1|I} & \cdots & \frac{\mu_{I0}}{\mu_{0J}}\hat{e}_{J|I} \end{pmatrix},\quad E_{21}(\mu) = -\begin{pmatrix} \frac{\mu_{01}}{\mu_{10}}\hat{g}_{1|1} & \cdots & \frac{\mu_{01}}{\mu_{I0}}\hat{g}_{I|1} \\ \vdots & \ddots & \vdots \\ \frac{\mu_{0J}}{\mu_{10}}\hat{g}_{1|J} & \cdots & \frac{\mu_{0J}}{\mu_{I0}}\hat{g}_{I|J} \end{pmatrix}$$

where

$$\hat{e}_{j|i} = \frac{m_i}{\mu_{0j}} \left[\frac{\beta^a\eta^a_{ij} + \beta^b\eta^b_{ij}}{\Big(1+\sum_{j=1}^{J}[\eta^a_{ij}+\eta^a_{ij}]\Big)^2} \right],\; \hat{g}_{i|j} = \frac{f_j}{\mu_{i0}} \left[\frac{\alpha^a\zeta^a_{ij} + \alpha^b\zeta^b_{ij}}{\Big(1+\sum_{i=1}^{I}[\zeta^a_{ij}+\zeta^b_{ij}]\Big)^2} \right].$$

Now, it is important to remark that at the **equilibrium** when (36) holds, we get simplified versions of $e_{j|i}$, $g_{i|j}$, $\hat{e}_{j|i}$, and $\hat{g}_{i|j}$ which are the following:

$$e_{j|i} = \frac{(1-\alpha^a)\eta^a_{ij} + (1-\alpha^b)\eta^b_{ij}}{1+\sum_{j=1}^{J}[\eta^a_{ij}+\eta^b_{ij}]} = \frac{1}{m_i}[(1-\alpha^a)\mu^a_{ij} + (1-\alpha^b)\mu^b_{ij}];$$

$g_{j|i} = \frac{(1-\beta^a)\zeta_{ij}^a + (1-\beta^b)\zeta_{ij}^b}{1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]} = \frac{1}{f_j}[(1-\beta^a)\mu_{ij}^a + (1-\beta^b)\mu_{ij}^b]$;
$\hat{e}_{j|i} = \frac{\beta^a\eta_{ij}^a + \beta^b\eta_{ij}^b}{1+\sum_{j=1}^J [\eta_{ij}^a + \eta_{ij}^b]} = \frac{1}{m_i}[\beta^a\mu_{ij}^a + \beta^b\mu_{ij}^b]$;
$\hat{g}_{j|i} = \frac{\alpha^a\zeta_{ij}^a + \alpha^b\zeta_{ij}^b}{1+\sum_{i=1}^I [\zeta_{ij}^a + \zeta_{ij}^b]} = \frac{1}{f_j}[\alpha^a\mu_{ij}^a + \alpha^b\mu_{ij}^b]$;
An appropriate adaptation of the supplement calculation of Graham (2013) (not published) would help the reader to understand some details of the calculations, that we have done here. Note that $0 < \sum_{j=1}^J e_{j|i}(\mu) < 1$, for all $1 \le i \le I$, and $0 < \sum_{i=1}^I g_{i|j}(\mu) < 1$ for all $1 \le j \le J$ whenever $0 < \beta^r < 1$ and $0 < \alpha^r < 1$ for $r \in \{a, b\}$. Now, we can write $J(\mu)$ at the equilibrium. We have the following: $J(\mu) = \begin{pmatrix} J_{11}(\mu) & J_{12}(\mu) \\ J_{21}(\mu) & J_{22}(\mu) \end{pmatrix}$
where $J_{11}(\mu) = I\{I\} - E_{11}(\mu)$, $J_{22}(\mu) = I\{J\} - E_{22}(\mu)$, $J_{12}(\mu) = -E_{12}(\mu)$, $J_{21}(\mu) = -E_{21}(\mu)$

Step 1: Factorization of the $J(\mu)$ matrix

Recall $J(\mu) = \begin{pmatrix} J_{11}(\mu) & J_{12}(\mu) \\ J_{21}(\mu) & J_{22}(\mu) \end{pmatrix}$, where

$$J_{12}(\mu) = diag(m)^{-1}\left\{\beta^a \begin{pmatrix} \frac{\mu_{10}}{\mu_{01}}\mu_{11}^a & \cdots & \frac{\mu_{10}}{\mu_{0J}}\mu_{1J}^a \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}}\mu_{I1}^a & \cdots & \frac{\mu_{I0}}{\mu_{0J}}\mu_{IJ}^a \end{pmatrix} + \beta^b \begin{pmatrix} \frac{\mu_{10}}{\mu_{01}}\mu_{11}^b & \cdots & \frac{\mu_{10}}{\mu_{0J}}\mu_{1J}^b \\ \vdots & \ddots & \vdots \\ \frac{\mu_{I0}}{\mu_{01}}\mu_{I1}^b & \cdots & \frac{\mu_{I0}}{\mu_{0J}}\mu_{IJ}^b \end{pmatrix}\right\}$$

Define $diag(\mu_{.0}) = diag(\mu_{10}, ..., \mu_{I0})$, $diag(\mu_{0.}) = diag(\mu_{01}, ..., \mu_{0J})$ and $R^r = \begin{pmatrix} \mu_{11}^r & \cdots & \mu_{1J}^r \\ \vdots & \ddots & \vdots \\ \mu_{I1}^r & \cdots & \mu_{IJ}^r \end{pmatrix}$ Therefore,

$$J_{12}(\mu) = diag(\mu_{.0})diag(m)^{-1}[\beta^a R^a + \beta^b R^b]diag(\mu_{0.})^{-1}$$

Similarly, we can show that $J_{21}(\mu)$ can be factored as follows:

$$J_{21}(\mu) = diag(\mu_{0.})diag(f)^{-1}[\alpha^a(R^a)' + \alpha^b(R^b)']diag(\mu_{.0})^{-1}$$

We also factor also $J_{11}(\mu)$ and $J_{22}(\mu)$ as follows:

$$J_{11}(\mu) = I_I - diag(m)^{-1}[(1-\alpha^a)R_{\iota J}^a + (1-\alpha^a)R_{\iota J}^b],$$

$$J_{22}(\mu) = I_J - diag(f)^{-1}[(1-\beta^a)(R^a)'_{\iota I} + (1-\beta^b)(R^b)'_{\iota I}].$$

where $R^r_{\iota J} = (\sum_{j=1}^{J} \mu^r_{1j}, ..., \sum_{j=1}^{J} \mu^r_{Ij})'$ and $(R^r)_{\iota I} = (\sum_{i=1}^{I} \mu^r_{i1}, ..., \sum_{i=1}^{I} \mu^r_{iJ})$.
After rearranging we can show that:

$$J(\mu) = C(\mu)^{-1}[A(\mu) + U(\mu)B_0(\mu)U(\mu)^{-1}]$$

where
$C(\mu) = \begin{pmatrix} diag(m) & 0 \\ 0 & diag(f) \end{pmatrix}$
$A(\mu) = \begin{pmatrix} diag(m - (1-\alpha^a)R^a_{\iota J} - (1-\alpha^b)R^b_{\iota J}) & 0 \\ 0 & diag(f - (1-\beta^a)(R^a)'_{\iota I} - (1-\beta^b)(R^b)'_{\iota I}) \end{pmatrix}$
$U(\mu) = \begin{pmatrix} diag(\mu_{.0}) & 0 \\ 0 & diag(\mu_{0.}) \end{pmatrix}$
$B_0(\mu) = \begin{pmatrix} 0 & \beta^a R^a + \beta^b R^b \\ \alpha^a (R^a)' + \alpha^b (R^b)' & 0) \end{pmatrix}.$
Therefore, $J(\mu)$ can be equivalently rewritten as:

$$\begin{aligned} J(\mu) &= U(\mu)C(\mu)^{-1}[A(\mu) + B_0(\mu)]U(\mu)^{-1} \\ &= U(\mu)H(\mu)U(\mu)^{-1} \end{aligned}$$

where

$$H(\mu) = C(\mu)^{-1}[A(\mu) + B_0(\mu)].$$

■ Let us write $H(\mu)$ in detail:
$H(\mu) = \begin{pmatrix} H_{11}(\mu) & H_{12}(\mu) \\ H_{21}(\mu) & H_{22}(\mu) \end{pmatrix}$
with
$H_{11}(\mu) = \begin{pmatrix} 1 - \frac{\sum_{j=1}^{J}[(1-\alpha^a)\mu^a_{1j} + (1-\alpha^b)\mu^b_{1j}]}{m_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - \frac{\sum_{j=1}^{J}[(1-\alpha^a)\mu^a_{Ij} + (1-\alpha^b)\mu^b_{Ij}]}{m_I} \end{pmatrix}$, $H_{12}(\mu) =$
$\begin{pmatrix} \frac{\beta^a\mu^a_{11} + \beta^b\mu^b_{11}}{m_1} & \cdots & \frac{\beta^a\mu^a_{1J} + \beta^b\mu^b_{1J}}{m_1} \\ \vdots & \ddots & \vdots \\ \frac{\beta^a\mu^a_{I1} + \beta^b\mu^b_{I1}}{m_I} & \cdots & \frac{\beta^a\mu^a_{IJ} + \beta^b\mu^b_{IJ}}{m_I} \end{pmatrix}$, $H_{21}(\mu) = \begin{pmatrix} \frac{\alpha^a\mu^a_{11} + \alpha^b\mu^b_{11}}{f_1} & \cdots & \frac{\alpha^a\mu^a_{I1} + \alpha^b\mu^b_{I1}}{f_1} \\ \vdots & \ddots & \vdots \\ \frac{\alpha^a\mu^a_{1J} + \alpha^b\mu^b_{1J}}{f_J} & \cdots & \frac{\alpha^a\mu^a_{IJ} + \alpha^b\mu^b_{IJ}}{f_J} \end{pmatrix}$,

$$H_{22}(\mu) = \begin{pmatrix} 1 - \frac{\sum_{i=1}^{I}[(1-\beta^a)\mu_{i1}^a + (1-\beta^b)\mu_{i1}^b]}{f_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 - \frac{\sum_{i=1}^{I}[(1-\beta^a)\mu_{iJ}^a + (1-\beta^b)\mu\alpha^b b_{iJ}]}{f_J} \end{pmatrix}.$$

Similar to Graham (2013, p 16), we observe that all elements of $H(\mu)$ are non-negative whenever $0 < \beta^r; \alpha^r \leq 1$.

Step 2: Derivation of M-matrix property

The main goal of this step is to show that the Schur complements of $H(\mu)$ the upper $I \times I$ (H_{11}) and lower $J \times J$ (H_{22}) diagonal blocks, (i.e. $SH_{11} = H_{22} - H_{21}H_{11}^{-1}H_{12}$ and $SH_{22} = H_{11} - H_{12}H_{22}^{-1}H_{21}$) are M-matrices which implies $SH_{11}^{-1} \geqq 0$ and $SH_{22}^{-1} \geqq 0$. To show that, we first need to show that $H(\mu)$ is row diagonally dominant. In other terms, if we denote the element of $H(\mu)$, h_{ij} with $1 \leq i, j \leq I + J$ we need to show that there exist $d_i > 0$ such that $d_i|h_{ii}| > \sum_{j \neq i}^{I+J} d_j|h_{ij}|$. This will be difficult to show without further restrictions on β^r and α^r. Graham (2013, p15) showed this result in the particular case where the two following restrictions hold simultaneously: $\beta^r + \alpha^r$=1 and $\beta^a = \beta^b$. Here, we will impose some conditions on the coefficients β^r and α^r that ensure $H(\mu)$ to be row diagonally dominant. Let first assume that $0 < \beta^r; \alpha^r < 1$, then $h_{ij} \geq 0$ for $1 \leq i, j \leq I + J$.

Case 1: $1 \leq i \leq I$

$$|h_{ii}| > \sum_{j \neq i}^{I+J} |h_{ij}| \Leftrightarrow \sum_{j=1}^{J} \Big((1 - \alpha^a + \beta^a)\mu_{ij}^a + (1 - \alpha^b + \beta^b)\mu_{ij}^b\Big) < m_i. \quad (37)$$

Notice that

$$\max\Big((1 - \alpha^a + \beta^a), (1 - \alpha^b + \beta^b)\Big) \sum_{j=1}^{J} \Big(\mu_{ij}^a + \mu_{ij}^b\Big) < m_i \Rightarrow$$

$$\sum_{j=1}^{J} \Big((1 - \alpha^a + \beta^a)\mu_{ij}^a + (1 - \alpha^b + \beta^b)\mu_{ij}^b\Big) < m_i,$$

and

$$\max\Big((1-\alpha^a+\beta^a),(1-\alpha^b+\beta^b)\Big)\sum_{j=1}^{J}\Big(\mu_{ij}^a+\mu_{ij}^b\Big)<m_i \quad\Leftrightarrow$$
$$\max\Big((1-\alpha^a+\beta^a),(1-\alpha^b+\beta^b)\Big)\rho_i^m<1,$$

where $\rho_i^m \equiv \frac{m_i-\mu_{i0}}{m_i}$ is the rate of matched men of type i. The latter inequality is equivalent to $\max(\beta^b-\alpha^b,\beta^a-\alpha^a)<\frac{1-\rho_i^m}{\rho_i^m}$. Therefore, if $\max(\beta^b-\alpha^b,\beta^a-\alpha^a)<\frac{1-\rho_i^m}{\rho_i^m}$ for all i then $|h_{ii}|>\sum_{j\neq i}^{I+J}|h_{ij}|$.

Case 2: $I+1\leq i\leq I+J$.

Similarly, we can show that if $\min(\beta^b-\alpha^b,\beta^a-\alpha^a)>-\frac{1-\rho_j^f}{\rho_j^f}$ for all j where $\rho_j^f\equiv\frac{f_j-\mu_{0j}}{f_j}$ is the rate of matched women of type j, then we have $|h_{ii}|>\sum_{j\neq i}^{I+J}|h_{ij}|$.

Assume that the two latter restrictions on β^r and α^r hold in the rest of the proof. The Schur complements of the $H(\mu)$ upper $I\times I$ and lower $J\times J$ diagonal blocks are $SH_{11}=H_{22}-H_{21}(H_{11})^{-1}H_{12}$ and $SH_{22}=H_{11}-H_{12}(H_{22})^{-1}H_{21}$. Since H has been showed to be diagonally dominant, Theorem 1 of Carlson and Markham (1979 p 249) implies that the two schur complements are also diagonally dominant. Therefore, SH_{11} and SH_{22} are also row diagonally dominant. We can easily see that SH_{11} and SH_{22} are also Z-matrices (i.e., members of the class of real matrices with nonpositive off-diagonal elements). By applying Theorem 4.3 of Fiedler and Ptak (1962) it follows that they are M-matrices and then $SH_{11}^{-1}\geqq 0$ and $SH_{22}^{-1}\geqq 0$. These results are sufficient to establish the sign structure of $H^{-1}(\mu)$. $H^{-1}(\mu)=\begin{pmatrix}W_{11}&W_{12}\\W_{21}&W_{22}\end{pmatrix}=\begin{pmatrix}+&\vdots&-\\\dots&&\dots\\-&\vdots&+\end{pmatrix}$ where W_{ij} are exactly defined as defined in Graham (2013. p 16).

Step 3: Derivation of $H^{-1}(\mu)$

Following Graham we can show the following inequalities:

$$\begin{aligned} W_{11} &\geq H_{11}^{-1} + H_{11}^{-1} H_{12} H_{22}^{-1} H_{21} H_{11}^{-1} = LW_{11} \\ W_{22} &\geq H_{22}^{-1} + H_{22}^{-1} H_{21} H_{11}^{-1} H_{12} H_{22}^{-1} = LW_{22} \\ W_{12} &\leq -H_{11}^{-1} H_{12} H_{22}^{-1} = UW_{12} \\ W_{21} &\leq -H_{22}^{-1} H_{21} H_{11}^{-1} = UW_{21}. \end{aligned}$$

Using the expression of the matrix $H(\mu)$ and after some tedious calculations we can show the following: $LW_{11} = H_{11}^{-1} +$

$$\begin{pmatrix} \frac{1}{m_1^*}\frac{m_1}{m_1^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{1j}^a+\alpha^b\mu_{1j}^b][\beta^a\mu_{1j}^a+\beta^b\mu_{1j}^b]}{f_j^*} & \cdots & \frac{1}{m_1^*}\frac{m_I}{m_I^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{Ij}^a+\alpha^b\mu_{Ij}^b][\beta^a\mu_{1j}^a+\beta^b\mu_{1j}^b]}{f_j^*} \\ \vdots & \ddots & \vdots \\ \frac{1}{m_I^*}\frac{m_1}{m_1^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{1j}^a+\alpha^b\mu_{1j}^b][\beta^a\mu_{Ij}^a+\beta^b\mu_{Ij}^b]}{f_j^*} & \cdots & \frac{1}{m_I^*}\frac{m_I}{m_I^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{Ij}^a+\alpha^b\mu_{Ij}^b][\beta^a\mu_{Ij}^a+\beta^b\mu_{Ij}^b]}{f_j^*} \end{pmatrix}$$

where

$$m_i^* \equiv m_i - \sum_{j=1}^J [(1-\alpha^a)\mu_{ij}^a + (1-\alpha^b)\mu_{ij}^b], \text{ for all } 1 \leq i \leq I$$

and

$$f_j^* \equiv f_j - \sum_{i=1}^I [(1-\beta^a)\mu_{ij}^a + (1-\beta^b)\mu_{ij}^b], \text{ for all } 1 \leq j \leq J.$$

Moreover, we can show that:

$$\begin{aligned} (LW_{11})_{ii} &= \frac{m_i}{m_i^*}\Big[1 + \frac{1}{m_i^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{ij}^a + \alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a + \beta^b\mu_{ij}^b]}{f_j^*}\Big] \\ &> 1, \end{aligned}$$

for all $1 \leq i \leq I$. Therefore we have $LW_{11} > I_I$. Similarly, we have also the following:

$LW_{22} = H_{22}^{-1} +$

$$\begin{pmatrix} \frac{1}{f_1^*}\frac{f_1}{f_1^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{i1}^a+\alpha^b\mu_{i1}^b][\beta^a\mu_{i1}^a+\beta^b\mu_{i1}^b]}{m_i^*} & \cdots & \frac{1}{f_1^*}\frac{f_J}{f_J^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{i1}^a+\alpha^b\mu_{i1}^b][\beta^a\mu_{iJ}^a+\beta^b\mu_{iJ}^b]}{m_i^*} \\ \vdots & \ddots & \vdots \\ \frac{1}{f_J^*}\frac{f_1}{f_1^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{iJ}^a+\alpha^b\mu_{iJ}^b][\beta^a\mu_{i1}^a+\beta^b\mu_{i1}^b]}{m_i^*} & \cdots & \frac{1}{f_J^*}\frac{f_J}{f_J^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{iJ}^a+\alpha^b\mu_{iJ}^b][\beta^a\mu_{iJ}^a+\beta^b\mu_{iJ}^b]}{m_i^*} \end{pmatrix}$$

Moreover, we can show that:

$$
\begin{aligned}
(LW_{22})_{jj} &= \frac{f_j}{f_j^*}\Big[1+\frac{1}{f_j^*}\sum_{i=1}^{I}\frac{[\alpha^a\mu_{ij}^a+\alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a+\beta^b\mu_{ij}^b]}{m_i^*}\Big] \\
&> 1,
\end{aligned}
$$

for all $1 \leq j \leq J$. Therefore, we have $LW_{11} > I_J$. Now, let us look at the off-diagonal blocks of $H(\mu)^{-1}$.

$$
UW_{12} = -\begin{pmatrix} \frac{[\beta^a\mu_{11}^a+\beta^b\mu_{11}^b]}{m_1^*f_1^*}f_1 & \cdots & \frac{[\beta^a\mu_{1J}^a+\beta^b\mu_{1J}^b]}{m_1^*f_J^*}f_J \\ \vdots & \ddots & \vdots \\ \frac{[\beta^a\mu_{I1}^a+\beta^b\mu_{I1}^b]}{m_I^*f_1^*}f_1 & \cdots & \frac{[\beta^a\mu_{IJ}^a+\beta^b\mu_{IJ}^b]}{m_I^*f_J^*}f_J \end{pmatrix}
$$

and $UW_{21} = -\begin{pmatrix} \frac{[\alpha^a\mu_{11}^a+\alpha^b\mu_{11}^b]}{m_1^*f_1^*}m_1 & \cdots & \frac{[\alpha^a\mu_{I1}^a+\alpha^b\mu_{I1}^b]}{m_1^*f_J^*}m_I \\ \vdots & \ddots & \vdots \\ \frac{[\alpha^a\mu_{1J}^a+\alpha^b\mu_{1J}^b]}{m_I^*f_1^*}m_1 & \cdots & \frac{[\alpha^a\mu_{IJ}^a+\alpha^b\mu_{IJ}^b]}{m_I^*f_J^*}m_I \end{pmatrix}$

Step 4: Main results

Case 1: Type specific elasticities of single hood

By applying the implicit function theorem to the equation (36) we have: $\frac{\partial\mu}{\partial m_i} = J(\mu)^{-1}\frac{\partial B}{\partial m_i}$ for $1 \leq i \leq I$ and $\frac{\partial\mu}{\partial f_j} = J(\mu)^{-1}\frac{\partial B}{\partial f_j}$ for all $1 \leq j \leq J$, where $\frac{\partial B}{\partial m_i} = (0,...,0,\frac{\mu_{i0}}{m_i},0,...,0)'$ and $\frac{\partial B}{\partial f_j} = (0,...,0,\frac{\mu_{0j}}{f_j},0,...,0)'$ are $(I+J)$ vectors such that the non-zero entries are respectively at the i^{th} row and the $(I+j)^{th}$ row. Let $h_k = (0,...,0,1,0,...,0)'$ be a $(I+J)$ vector such that the non-zero entry is at the k^{th} row. We have the following:

$$
\begin{aligned}
U(\mu)^{-1}\frac{\partial\mu}{\partial m_i}m_i &= U(\mu)^{-1}J(\mu)^{-1}\frac{\partial B}{\partial m_i}m_i \\
&= H(\mu)^{-1}U(\mu)^{-1}h_i\mu_{i0} \\
&= H(\mu)^{-1}h_i \\
&= [H(\mu)^{-1}]_{\cdot i} \qquad (38)
\end{aligned}
$$

for $1 \leq i \leq I$, where $[H(\mu)^{-1}]_{\cdot i}$ represents the i^{th} column of the matrix $H(\mu)^{-1}$. Similarly, we can show that $U(\mu)^{-1}\frac{\partial\mu}{\partial f_j}f_j = [H(\mu)^{-1}]_{\cdot(I+j)}$ for $1 \leq j \leq J$. Putting these results together, we get the following inequalities:

$$\frac{m_i}{\mu_{k0}}\frac{\partial\mu_{k0}}{\partial m_i} \geq \begin{cases} \frac{1}{m_i^*}\frac{m_k}{m_k^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{kj}^a+\alpha^b\mu_{kj}^b][\beta^a\mu_{kj}^a+\beta^b\mu_{kj}^b]}{f_j^*} > 0 & \text{if } k \neq i \\ \frac{m_i}{m_i^*}[1+\frac{1}{m_i^*}\sum_{j=1}^J \frac{[\alpha^a\mu_{ij}^a+\alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a+\beta^b\mu_{ij}^b]}{f_j^*}] > 1 & \text{if } k = i, \end{cases} \text{for } 1 \leq k \leq I.$$

$$\frac{f_j}{\mu_{0k}}\frac{\partial\mu_{0k}}{\partial f_j} \geq \begin{cases} \frac{1}{f_j^*}\frac{f_k}{f_k^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{ik}^a+\alpha^b\mu_{ik}^b][\beta^a\mu_{ik}^a+\beta^b\mu_{ik}^b]}{m_i^*} > 0 & \text{if } k \neq j \\ \frac{f_j}{f_j^*}[1+\frac{1}{f_j^*}\sum_{i=1}^I \frac{[\alpha^a\mu_{ij}^a+\alpha^b\mu_{ij}^b][\beta^a\mu_{ij}^a+\beta^b\mu_{ij}^b]}{m_i^*}] > 1 & \text{if } k = j, \end{cases} \text{for } 1 \leq k \leq J.$$

$$\frac{m_i}{\mu_{0j}}\frac{\partial\mu_{0j}}{\partial m_i} \leq -\frac{[\alpha^a\mu_{ij}^a+\alpha^b\mu_{ij}^b]}{m_i^* f_j^*}m_i < 0$$

and

$$\frac{f_j}{\mu_{i0}}\frac{\partial\mu_{i0}}{\partial f_j} \leq -\frac{[\beta^a\mu_{ij}^a+\beta^b\mu_{ij}^b]}{m_i^* f_j^*}f_j < 0$$

for $1 \leq i \leq I$ and $1 \leq j \leq J$.

C.4 Proof of Proposition 1

Recall, from the result of Theorem 1 we know that the fixed point representation (36) admits a unique solution. Therefore, $\mu - B(\mu; m, f, \theta)$ must be at least locally invertible at the equilibrium. This ensures that it jacobian matrix $J(\mu)$ does not vanish at the equilibrium. Then, $det(J(\mu)) \neq 0$ for all $\beta^r, \alpha^r > 0$. Since we shown within Step 1 of proof of Theorem 2 that $J(\mu) = U(\mu)H(\mu)U(\mu)^{-1}$ for all $\beta^r, \alpha^r > 0$, we have then $det(H(\mu)) \neq 0$. Moreover, we have shown that

$$\sum_{i=1}^I U(\mu)^{-1}\frac{\partial\mu}{\partial m_i}m_i + \sum_{j=1}^J U(\mu)^{-1}\frac{\partial\mu}{\partial f_j}f_j \quad = \quad \sum_{i=1}^I [H(\mu)^{-1}]_{\cdot i} + \sum_{j=1}^J [H(\mu)^{-1}]_{\cdot (I+j)}.$$

If $\beta^r + \alpha^r = 1$, we observe that all elements of $H(\mu)$ are non-negative and the rows sum to one. Therefore, $H(\mu)$ is a row stochastic matrix, see Horn and Johnson (2013, p.547), with an inverse whose rows also sum to one. Then,

$$[H(\mu)^{-1}]_{\cdot i} + \sum_{j=1}^J [H(\mu)^{-1}]_{\cdot (I+j)} \quad = \quad \iota_{I+J}.$$

where $\iota_{I+J} = (1, ..., 1)'$. The last equality holds since the rows of $[H(\mu)^{-1}]$ sum to one.