TEAM PRODUCTION, ENDOGENOUS LEARNING ABOUT ABILITIES AND CAREER CONCERNS

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August 2015

COWLES FOUNDATION DISCUSSION PAPER NO. 2020

COWLES FOUNDATION FOR RESEARCH IN ECONOMICS
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Team production, endogenous learning about abilities and career concerns*

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Abstract

This paper studies career concerns in teams where the support a worker receives depends on fellow team members' effort *and* ability. In this setting, by exerting effort and providing support, a worker can influence her own and her teammates' performances in order to bias the learning process in her favor. To manipulate the market's assessments, we argue that in equilibrium, a worker has incentives to help or even sabotage her colleagues in order to signal that she is of higher ability. In a multiperiod stationary framework, we show that the stationary level of work effort is above and help effort is below their efficient levels. We also examine career concerns with explicit contracts.

Keywords: career concerns, team incentives, incentives to help, incentives to sabotage, relative performance

Jel codes: D83, J24, M54

*I am indebted to Lambros Pechlivanos for the very useful discussions on this topic. I am also grateful to Dan Bernhardt, Marco Celentani, George Deltas, Johannes Hörner, Patrick Rey and Spyros Vassilakis for their comments and suggestions. I give special thanks to Costas Arkolakis, Roberto Burguet, Mehmet Ekmekci, Simona Fabrizi, Eiichiro Kazumori, Ting Liu, Chrisovalantou Milliou, Michael Powell, Larry Samuelson, Konstantinos Serfes, Spyridon Skouras, Juuso Välimäki, Anastasios Xepapadeas and the participants at various conferences and meetings. Financial support from the State Scholarships Foundation in Greece is also acknowledged.

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1 Introduction

Modern corporations launch innovative employment practices in the workplace, including teamwork, job rotation and problem-solving groups, to raise productivity and profits.[1] However, providing team incentives creates challenges. Workers, who may be subject to explicit incentives that arise from compensation contracts, may also be involved in productive activities for free. A prominent example is the development of open source software.[2] Top programmers contribute freely to this process because there are delayed rewards (Lerner & Tirole (2002)). They have implicit incentives that arise from career concerns; i.e., concerns about the effect of reputation on external labor markets and thus on future remuneration.[3] In the open source mode, the market can see outcomes and whether the problem was addressed in a clever way (Von Hippel & Von Krogh (2003)).[4] In turn, a programmer is able to signal her talent to peers and prospective employers, thereby increasing future monetary payments. However, due to the collaborative nature of this activity (Weber (2004)), the individual outcome also depends on the contribution and thus the qualities of fellow team members.

This paper studies career concerns in teams where the support a worker receives depends on fellow team members' effort *and* ability. We address the questions of how the learning process about a worker's ability is shaped by teamwork interactions and how career concerns arise in this setting. A worker's effort and ability are inputs in her teammates' production functions. Thus, by exerting effort and providing support, a worker can influence her own *and* her teammates' performances in order to manipulate the market's assessments about her own ability. We argue that in equilibrium, a worker has incentives either to help or even to sabotage her colleagues, in order to bias the learning process in her favor. The existing literature on career concerns in teams, based on Auriol, Friebel & Pechlivanos (2002), assumes that a teammate's support depends exclusively on her teammates' effort (not ability). The learning process about a worker's ability is therefore independent of the quality of fellow team members and her career concerns depend exclusively on her own performance.

We employ Holmström's (1982, 1999) career concerns framework, in which neither the workers nor the market know workers' innate abilities and both learn from past performances. We consider

[1] The 5th European Working Conditions Survey (2012) reports that the pace of respondents' work depends on direct control of their boss (43% in all workplaces), production or performance targets (47%), and work done by colleagues (45%). The 2011 Workplace Employment Relations Study about British workplaces finds that the incidence of methods for knowledge transmission and teamwork interactions are considerable; i.e., meetings involving all staff (in 80% of workplaces), team briefings (60%) and problem solving groups (14%).

[2] Apache, Linux, Perl and Sendmail, among others, are developed as open source software. The national value of Europe's investment in free/libre/open-source (FLOSS) software in 2006 is 22 billion euros representing 20.5% of total software investment. In USA, this value is 36 billion (in euros) (Ghosh (2007)).

[3] Explicit incentives to perform a job or a task are provided through explicit contractual commitments by a principal. However, implicit incentives arise when principals competing in a labor market have some *ex post discretion* how to respond to an agent's performance. This agent has implicit incentives to change her current effort in order to influence the learning process about her ability and thus increase her future payments.

[4] The Apache project makes a point of recognizing all contributors on its website, http://httpd.apache.org/contributors/#colm. Kogut & Metiu (2000) state that many programmers reportedly believe that being a member of the LINUX community "commands a $10,000 premium on annual wages". Hann, Roberts, Slaughter & Fielding (2004) argue that star programmers are an order of magnitude more productive than their peers, so there is much to signal.

a simple setting with two agents who work and interact for two periods. Agents consider work and help as two separate tasks, and have task-specific cost functions. A worker's "project" output is observable and linear in her own innate ability and "work" effort, her teammate's support, and a transitory shock. The support a teammate provides also depends on her own "help" effort *and* ability; i.e., the teammate's ability matters for an agent's performance. Agents' abilities and the transitory shocks are independently and normally distributed. Additionally, we consider different degrees of *initiated* and *received* teamwork interactions; i.e., the fraction of a teammate's support that is appropriated by an agent may differ from the fraction of an agent's help that increases a teammate's production.

The dependence of future rewards on past performances plays a key role in agents' labor supply. The market draws inferences about the levels of agents' abilities via current project outputs. Since labor is a substitute for ability, an agent can influence the learning process in her favor by distorting both her efforts upwards.[5] What complicates inferences is that because both teammates' abilities are inputs in the production function, an agent's project output as a signal of her own ability is noisier. However, her colleague's output also conveys information.

By exerting work and help effort, an agent can influence both performance measures and manipulate market perceptions. If the initiated interactions are strong enough relative to the received interactions so that an agent's support has a great impact on her colleague's production, the market attributes high performance by a teammate to the agent's ability and revises its assessment about the ability upwards. In this case, we argue that an agent has incentives to work *and* help her colleague in order to build up her reputation. The opposite occurs if received interactions are strong enough relative to initiated interactions. High performance by a teammate is attributed to the teammate's ability. This causes the market to put a negative weight on that performance when forecasting an agent's ability. In this case, an agent's help will increase the teammate's performance further, which biases the learning process against her. Thus, an agent now has incentives to sabotage her colleague. She can induce an upward revision of her own ability only by destroying some part of her teammate's production.

This analysis shows that what matters for career concerns is how many components of the production and learning process an agent can affect in order to shape the market's assessments. An agent cashes in a reputational bonus that increases with effort exertion and support provision or sabotage. Holmström (1982) studies career concerns when there are no interactions, while Auriol et al. (2002) assume that the support an agent receives depends exclusively on her colleague's effort. In their model, by looking at a teammate's performance, the market cannot draw any additional information about an agent's ability. The process of inference about each teammate's ability is independent and

[5] Empirical studies find evidence for the existence of career concerns for professionals (Gibbons & Murphy (1992)), and for economists (Coupe, Smeets & Warzynski (2006)); i.e., past performance and the probability of promotion are positively related, and the sensitivity of promotion to performance declines with experience, indicating the presence of a learning process. Borland (1992) provides a survey.

the quality of fellow team members has no effect on an agent's reputation incentives.

This paper also investigates Fama's (1980) conjecture that career concerns induce agents to behave efficiently. Holmström (1999) formalizes this idea by considering a "stationary" single-agent model where ability is not fixed but fluctuates over time, thereby preventing the market from fully learning its level. He states that if there is no discounting, Fama's result is correct: agents exert the efficient level of work effort. We argue that in a multi-agent model where there are teamwork interactions and the quality of fellow team members matters for an agent's decisions, this result does not necessarily hold. In particular, if initiated interactions occur, even though there is no discounting, the stationary work effort is higher and help effort lower than their efficient levels. Because we add noise to the learning process, both performance measures become more vague. An agent can more effectively shape the market's assessments by increasing her own project output, and thus the work effort is distorted upwards, while the help effort is distorted downwards. The balance between the reputation incentives in a stationary model indicates that an agent is oriented to focus on tasks that increase her own project output, dragging her attention from helping or sabotaging her teammate. In a stationary equilibrium, career concerns induce an agent to over-provide work effort.

An agent's stationary effort levels are efficient only in two cases, provided that there is no discounting. On the one hand, this happens as long as an agent's ability is not an input in her teammate's production function as in the settings of Holmström (1999) and Auriol et al. (2002), although received interactions may occur. A teammate's output as a performance measure should not convey any information about an agent's ability and hence has no effect on reputation incentives. In this case, the supplied work effort is efficient regardless of the intensity of received interactions, and thus of how noisy the signal of an agent's performance is about her own ability. Exerting zero help effort is also efficient. On the other hand, efficient effort levels are obtained in a stationary setting as long as both initiated and received interactions are perfect, implying that agent's work and help efforts need to be equally productive.

We extend the analysis to examine team incentives when explicit contracts are provided. We consider a risk-neutral principal who appoints two risk-averse agents whose individual outputs are observable and contractible, allowing the principal to treat agents separately through individual-based schemes (Itoh (1992), Auriol et al. (2002)). The incentive packages are derived in a linear principal-agent model (Holmström & Milgrom (1987), Gibbons & Murphy (1992)) and are based on explicit comparisons of team members' outputs.[6] The existing literature uses such contracts when the market shocks that hit agents' production are correlated. In our setting, market shocks are independent. However, as in Chalioti (2015), individual outputs are correlated due to teamwork

[6] Some principal-agent models allow both parties to hold some bargaining power (e.g., Pitchford (1998)) while other models assume that either party can make a 'take-it-or-leave-it' offer (e.g., Mookherjee & Ray (2002)). Bernhardt (1995) studies how the composition of an agent's skills and the non-observability of her ability affect wage and promotion paths. Ferrer (2010) studies the effects of lawyers' career concerns on litigation when the outcome of a trial depends on the opposing lawyers' effort and abilities. Bilanakos (2013) argues that the provision of general training increases the worker's bargaining power vis-à-vis the employer.

interactions. Team-incentive contracts are the consequence of the efficient use of information conveyed by both performance measures about an agent's effort and ability.

The optimal contractual parameter based on an agent's own project output is always positive, indicating that higher agent performance is rewarded with a higher payment. However, the sign of the contractual parameter based on her teammate's output is less clear cut. If initiated interactions are large enough, the principal rewards the agent for the support she provides to her colleague. In contrast, if the market and the principal anticipate that initiated interactions are small and an agent's contribution in her teammate's production is insignificant, the principal penalizes the agent when the teammate performs better by setting this contractual parameter negative. The principal now filters out the effect of teamwork interactions from agents' compensation. Implicit sabotage incentives now arise due to an agent's increasing willingness to persuade the principal that she is teamed with a less productive teammate. An agent wants to signal that she is the more productive team member in absolute and relative terms. In fact, an agent has implicit incentives to induce a downward readjustment of the market's assessment of her colleague's ability. This happens because a colleague's reputation cannot benefit the agent. She is unable to capitalize on the increase in her colleague's bonus; it hurts her instead. In particular, if the teammate is perceived as being highly productive, the principal expects to pay a large part of the compensation through the contractual incentive components.[7] Given that individual remuneration is pinned down by the outside option, the fixed part of the salary will decrease, making an agent worse off.

This paper contributes to the existing literature on career concerns in teams where the ratchet effect or sabotage incentives arise. Lazear (1989) considers sabotage incentives in tournaments that arise because explicit payments condition the reward of an agent negatively on her colleagues' performances. In Auriol et al. (2002), explicit contracts are provided and the source of sabotage incentives is a lack of commitment by the principal. In a two-agent model, Meyer & Vickers (1997) use Holmström's (1999) production function where an agent's effort and ability only matters for her own outcome. Thus, an agent cannot influence another's production. However, the learning process depends on whether agents' abilities are correlated. They argue that because there is a positive externality, each agent free-rides on the effort of the other to enhance reputation. Due to free-riding, reputation incentives are weakened and the ratchet effect arises. Agents have a decreasing willingness to work. In our setting, the teammates' innate characteristics are independent, but due to teamwork interactions, an agent has incentives to take action in order to affect her teammates' performance. Even if no explicit contracts are provided, an agent exerts effort either to help her teammate or sabotage her by destroying some part of her production. Incentives to sabotage arise when the market puts a negative weight on a teammate's performance when predicting an agent's ability.

The literature on moral hazard problems remains narrow in its focus on whether market forces

[7] Dewatripont, Jewitt & Tirole (1999) argue that the implicit and explicit incentives are substitutes in a production function where the inputs are additive, while they may become complements if the agent's ability is multiplicative to her effort. Andersson (2002) provides a discussion on unobservable contracts.

alone can remove them. Fama (1980) states that there will be no need for explicit contracts in order to solve the principal-agent conflicts. The market already provides efficient implicit contracts, inducing the "right" level of labor supply. Holmström (1999) shows that risk-aversion and discounting place limitations on the market's ability to urge adequate incentives. However, if these limitations are lifted in a stationary model, agents exert efficient effort levels. Bar-Isaac & Hörner (2014) consider an agent who has different abilities - specialized and generalized abilities - to perform two tasks. They compare the value of specializing with acting as a generalist in an infinite-horizon model and find that, if there is no discounting, the stationary level of effort is also efficient. Bonatti & Hörner (2014) consider a dynamic framework with exponential learning. We show that in our model where teammates' abilities affect their reputation incentives, the stationary levels of efforts on both tasks are inefficient. The stationary work effort is higher and the help effort is lower than their efficient levels. Thus, the work effort is distorted upwards and the help effort is distorted downwards.

This paper is also tied to the literature on team incentives when the degree of visibility of an agent's characteristics is an issue. In team production models, the market only observes the team output and uses this (single) measure to infer the level of workers' abilities. Ortega (2003) examines the effect of the allocation of power within the firm on workers' career concerns, where power confers visibility: as an agent becomes more visible, the visibility of her colleague must decline. He argues that uneven allocation of authority is optimal. Jeon (1996) shows the optimality of equal sharing of the team output among workers as well as the advantage of mixing young and old workers in a team. Bar-Isaac (2007) analyzes workers' incentives to work for their own reputations when young but for their firms' reputation when old. Arya & Mittendorf (2011) examine the desirability of aggregate performance measures in models with reputation incentives. They assume that an agent can impact multiple dimensions of a firm's operation and the output of each operation depends on her own effort and ability. Effort can influence all signals to varying degrees. There are no teamwork interactions and the process of inference of an agent's ability depends only on her own efforts. They argue that an aggregate signal of the outputs of these operations can improve efficiency. In a single agent model, Dewatripont, Jewitt & Tirole (2000) consider multitasking and claim that increasing the number of tasks reduces the total effort because performance becomes noisier. Dewatripont et al. (1999) use a production function where an agent's effort and ability are multiplicative and argue that what matters is market expectations about focus on a task and not observability of tasks as in an additive case. They also examine incentives under a "fuzzy mission" where the market is ignorant about the allocation of an agent's effort across tasks. In a different setting, Effinger & Polborn (2001) assume that an agent is most valuable if she is the only smart agent. If this value is sufficiently large, the other expert opposes her predecessor's report. 'Antiherding' may result.

In our two-agent model, individual project outputs are observable (separate signals) and subject to market shocks that are independent of each other. Teammates' abilities are also uncorrelated. The degree of visibility of agents' abilities changes with teamwork interactions that also make individual production noisier. This happens because a teammate's ability affects an agent's project output. However, a teammate's performance also conveys information about an agent's ability and it is likely that the signals will be jointly more informative. In our model, what drives the optimal reputation incentives is not the amount of available information about teammates' abilities per se, but how agents' performances are related. An agent's attempts to shape the market's assessments may induce her to exert inefficiently high work effort, and help or sabotage her teammate, even in the absence of explicit motivation.[8,9] This model can be used to analyze reputation incentives of team workers when their individual performance depends on the quality of fellow members. This is likely to happen in research collaborations or even in sports teams.

The paper is organized as follows. Section 2 presents the model. It discusses the process of learning about abilities and the effect of teamwork interactions on the amount of available information. Section 3 solves the game and derives teammates' reputation incentives in a setting where there is no explicit motivation. The optimal incentives to help or sabotage are analyzed. We also discuss the reputation incentives when market shocks are correlated. In section 4, we consider a multiperiod model and focus on the stationary level of labor supply. In section 5, we derive the optimal incentives when explicit contracts are also provided. Section 6 concludes.

2 The model

This section describes the model where no contingent contracts can be made and thus only reputation (implicit) incentives arise. We assume that there are two effort-averse agents 1 and 2, indexed by i and j where $i \neq j$. Agents are also rational and forward-looking. Employment lasts for two periods indexed by $t = \{1, 2\}$, and at each period, each agent carries out her own project.

2.1 Production technology

Agents are engaged in a stochastic production process. At each period t, agent i's "project" output, z_t^i, depends on her own innate ability, θ^i, her "work" effort, e_t^i, and a transitory shock, ε_t^i. In addition, z_t^i depends on the teammate's support, $\theta^j + a_t^j$, weighted by a parameter h_j, where $0 \leq h_j \leq 1$:

$$z_t^i = \theta^i + e_t^i + h_j \left(\theta^j + a_t^j\right) + \varepsilon_t^i. \tag{1}$$

The teammate's innate ability, θ^j, and her "help" effort, a_t^j, increase agent i's project output in an additive way. Thus, each agent exerts *work* effort to accomplish her own project as well as *help* effort

[8] Heterogeneous teams in terms of seniority or learning by doing are beyond the scope of this analysis.

[9] Milgrom & Oster (1987) study the role of a worker's visibility in the job market: the abilities of visible workers are known to all parties while those of invisibles are concealed by an employer from other potential employers. Mukherjee (2008) examines a firm's decision to disclose information about its workers' productivity.

to improve her colleague's performance.[10]

When agent i enters the labor market, her ability is not known with certainty. However, all parties share the common prior that abilities are independently and identically distributed, where θ^i is drawn from a normal distribution with mean m_1^i and variance σ_i^2. Prendergast & Topel (1996) consider θ^i as the fit between the agent and her job that is contingent on some systemic variation, (symmetrically) unknown to all parties at each stage.[11] The parameter h_j measures the degree of *received* teamwork interactions - the fraction of agent j's support that is appropriated by agent i - and h_i indicates the degree of *initiated* interactions - the fraction of agent i's support that contributes to agent j's production. These parameters may differ. They are also exogenous and lie in $[0,1]$. Teamwork interactions are value-creating and their intensity depends on the characteristics of the technology used by each agent or, for instance, the degree of tacit knowledge required in production. The fact that h_i and h_j are less than one reflects the imperfect nature of teamwork interactions: providing help to a fellow member of the team is (somewhat) less productive than putting effort into one's own task. The random terms ε_t^i, ε_t^j are also independently and normally distributed, across agents and periods, with zero mean and variance σ_ε^2.

2.2 Learning process

In multi-agent career concerns models with uncorrelated shocks, the market updates from an agent's past performance in order to infer the level of her ability. In our model, teamwork interactions occur and support also depends on the ability of the fellow member. Since the unknown θ^j enters agent i's production function, agent i's project output, z_t^i, as a signal of her ability θ^i becomes noisier. Teamwork interactions weaken the link between an agent's performance and her ability, implying that this relationship becomes less autonomous and accountable. However, θ^i is also an input in the teammate's production function. Therefore, z_1^j also conveys information about agent i's ability. The market has two performance measures from which to draw inference about an agent's ability.

In Holmström's (1999) model where $z_t^i = \theta^i + e_t^i + \varepsilon_t^i$, there are no interactions, while in a two agent version of Auriol et al. (2002) model, agent i's production function is $z_t^i = \theta^i + e_t^i + h_j a_t^j + \varepsilon_t^i$, so the support an agent receives depends exclusively on her colleague's effort. There is no link between θ^j and z_t^i, $corr\left(\theta^i, z_1^j\right) = 0$. Thus, the processes of inference of θ^i and θ^j are completely independent.

Following DeGroot (1970), Lemma 1 specifies the mean and variance of the conditional distribution of abilities after the realizations of z_1^i and z_1^j. All parties (the market and the two teammates) observe the outputs of both projects that are realized in the end of the first period. We denote by $\widehat{e}_1^i$ and $\widehat{a}_1^i$ the market conjectures about agent i's first period efforts.

[10] The price of the outputs is normalized to one and the scale of production is identical in all t periods.

[11] Laffont & Tirole (1988), among others, analyze the optimal incentives when an agent has private information about her own ability before she goes to the market.

Lemma 1 (Conditional distribution of abilities) *Given the realizations of the first-period project outputs, z_1^i and z_1^j, the mean and variance of the conditional distribution of θ^i in period 2 are*

$$\begin{aligned} m_2^i &\equiv E\left\{\theta^i \mid z_1^i, z_1^j\right\} = \mu_1^i m_1^i + \rho_1^{ii}\left(z_1^i - \widehat{e}_1^i - h_j \widehat{a}_1^j - h_j m_1^j\right) + \rho_1^{ij}\left(z_1^j - \widehat{e}_1^j - m_1^j - h_i \widehat{a}_1^i\right), \\ \sigma_{i,2}^2 &\equiv var\left\{\theta^i \mid z_1^i, z_1^j\right\} = \sigma_i^2\left(1 - \rho_1^{ii} - h_i \rho_1^{ij}\right), \end{aligned}$$

where $\mu_1^i \equiv 1 - \rho_1^{ii} - h_i \rho_1^{ij}$. The conditional correlation coefficients of z_1^i and z_1^j are, respectively,

$$\begin{aligned} \rho_1^{ii} &\equiv corr\left(\theta^i, z_1^i \mid z_1^j\right) = \frac{\sigma_i^2}{\lambda_1}\left[\sigma_\varepsilon^2 + \left(1 - h_i h_j\right)\sigma_j^2\right], \\ \rho_1^{ij} &\equiv corr\left(\theta^i, z_1^j \mid z_1^i\right) = \frac{\sigma_i^2}{\lambda_1}\left[h_i \sigma_\varepsilon^2 - \left(1 - h_i h_j\right) h_j \sigma_j^2\right], \end{aligned}$$

where $\lambda_1 \equiv \sigma_\varepsilon^4 + \left(1 - h_i h_j\right)^2 \sigma_i^2 \sigma_j^2 + \sigma_\varepsilon^2\left[\left(1 + h_i^2\right)\sigma_i^2 + \left(1 + h_j^2\right)\sigma_j^2\right]$ for all h_i and h_j.

Proof. In appendix $(A.1)$. ■

Provided that all parties have rational expectations, the equilibrium conjectures must be correct: $\widehat{e}_1^i = e_1^{i*}$ and $\widehat{a}_1^i = a_1^{i*}$. There are no off-equilibrium realizations of observables because of the presence of noise. Each agent is compelled to exert the equilibrium effort levels that are expected of her, since working less will bias the learning process against her. Remark 1 highlights the informativeness of the signals about an agent's ability.[12]

Remark 1 (Informativeness of signals) *(a) Given z_1^j, the conditional correlation between agent i's ability, θ^i, and her own project output, z_1^i, is always positive: $\rho_1^{ii} > 0$ for all h_i and h_j.*

(b) Given z_1^i, the conditional correlation between agent i's ability, θ^i, and her teammate's project output, z_1^j, is positive as long as initiated interactions are substantial:

$$\rho_1^{ij} > 0 \text{ if and only if } h_i > \frac{h_j \sigma_j^2}{\sigma_\varepsilon^2 + h_j^2 \sigma_j^2}.$$

The coefficient ρ_1^{ii} represents the correlation of agent i's ability and her own project output, given a teammate's performance; i.e., the linear dependence between θ^i and z_1^i, given z_1^j.[13] This correlation coefficient is always positive, $\rho_1^{ii} > 0$, because $\frac{cov\left(\theta^i, z_1^i\right)}{cov\left(\theta^i, z_1^j\right)} > \frac{cov\left(z_1^i, z_1^j\right)}{var\left(z_1^j\right)} \Leftrightarrow \frac{1}{h_i} > \frac{h_j \sigma_j^2}{\sigma_\varepsilon^2 + \sigma_j^2}$ for all h_i and h_j. Thus, given the realization of her colleague's project output, an agent's high "own" performance

[12] If the estimate of θ^i is based only on z_1^i, we have $E\left\{\theta^i \mid z_1^i\right\} = (1 - \xi) m_1^i + \xi\left(z_1^i - \widehat{e}_1^i - h_j \widehat{a}_1^j - h_j m_1^j\right)$ and $Var\left\{\theta^i \mid z_1^i\right\} = \sigma_i^2(1 - \xi)$ where $\xi \equiv \sigma_i^2\left[\sigma_i^2 + h_j^2 \sigma_j^2 + \sigma_\varepsilon^2\right]^{-1}$. ξ exceeds ρ_1^{ii}, $\xi \geq \rho_1^{ii}$, implying that the market puts a lower weight on z_1^i to perceive the level of θ^i if another signal is also available. However, the two signals are jointly more informative, allowing for a better estimate: $\rho_1^{ii} + h_i \rho_1^{ij} \geq \xi$ for all h_i and h_j.

[13] The correlation coefficients of the unconditional distribution of θ^i are $corr\left(\theta^i, z_t^i\right) = \sigma_i\left[\sigma_i^2 + h_j^2 \sigma_j^2 + \sigma_\varepsilon^2\right]^{-\frac{1}{2}}$ and $corr\left(\theta^i, z_t^j\right) = h_i corr\left(\theta^i, z_t^i\right)$. Both are positive.

signals high "own" ability and vice versa. If there are no teamwork interactions as in Holmström (1999), or if support depends only on teammate's effort as in Auriol et al. (2002), the variance of agent i's ability after the observation of z_1^i, $var\left(\theta^i \mid z_1^i\right)$, is independent of σ_j^2 and equal to $\frac{\sigma_\varepsilon^2 \sigma_i^2}{\sigma_\varepsilon^2+\sigma_i^2}$. The correlation of θ^i and z_1^j is also zero.

In our model, z_1^j conveys information about θ^i, but the sign of the (conditional) correlation coefficient ρ_1^{ij} is less straightforward. The sign of ρ_1^{ij} depends on the *relative* intensity of the degrees of teamwork interactions (rather than on their absolute values) as well as on the variance of θ^j and ε_1^i; these two inputs are apart from agent i's characteristics and beyond her control. A positive ρ_1^{ij} requires $\frac{cov\left(\theta^i, z_1^j\right)}{cov\left(\theta^i, z_1^i\right)} > \frac{cov\left(z_1^i, z_1^j\right)}{var\left(z_1^i\right)} \Leftrightarrow h_i > \frac{h_j \sigma_j^2}{\sigma_\varepsilon^2 + h_j^2 \sigma_j^2}$. Initiated interactions must be strong enough so that agent j's performance is sensitive to θ^i, while received interactions, h_j, must be weak (z_1^i must not be sensitive to θ^j). If this is the case, both signals are more likely to reflect the level of θ^i. Thus, given z_1^i, higher z_1^j is "good news" for agent i's ability. The market perceives that a high z_1^j is due to a high agent i's ability and updates its assessments upwards. In the polar case where $h_j = 0$, ρ_1^{ij} is positive for all h_i.

The opposite occurs if received interactions, h_j, are large enough while initiated interactions, h_i, are small. In this case, as z_1^j increases, $E\left[\theta^j \mid z_1^i, z_1^j\right]$ will increase for a given fixed z_1^i. Hence, a larger proportion of this z_1^i will also be attributed to θ^j rather than θ^i, so that $E\left[\theta^i \mid z_1^i, z_1^j\right]$ will decrease. In particular, if h_i is small and the variance of θ^j is large enough, it is more likely that both performance measures indicate the level of θ^j. Thus, if both agents perform well, the market attributes these outcomes to high θ^j, causing the estimate of θ^i to be updated downwards. Agent j is now perceived as the high-quality member of the team. In the polar case where $h_i = 0$, θ^i does not contribute to agent j's project output at all. However, the market still uses this performance measure to draw valuable information about θ^j (and indirectly about θ^i). Under these conditions, given z_1^i, the market always puts a negative weight on z_1^j to assess θ^i: if $h_i = 0$, $\rho_1^{ij} < 0$ for all h_j.

To obtain better insight, we also examine how the variances of θ^i and θ^j affect the weights the market puts on outputs in estimating teammates' abilities. In particular, we have: (i) $\frac{\partial \rho_1^{ii}}{\partial \sigma_i^2} > 0$ for all h_i and h_j; (ii) $\frac{\partial \rho_1^{ij}}{\partial \sigma_i^2} > 0$ if and only if $\rho_1^{ij} > 0$. As long as initiated interactions are strong enough relative to the degree of received interactions so that a higher z_1^i or z_1^j is attributed to a higher θ^i, an increase in the variance of agent i's ability, σ_i^2, will trigger the market to rely more on both signals. The market will be willing and able to learn more about θ^i. On the other hand, we have: (i) $\frac{\partial \rho_1^{ii}}{\partial \sigma_j^2} < 0$ if and only if $\rho_1^{ji} \equiv corr\left(\theta^j, z_1^i \mid z_1^j\right) > 0$ (see Lemma 1); (ii) $\frac{\partial \rho_1^{ij}}{\partial \sigma_j^2} < 0$ for all h_i and h_j. For strong received now interactions (large h_j), a teammate's ability is key for an agent's performance and $\rho_1^{ji} > 0$. In this case, as σ_j^2 increases, z_1^i is more likely to reflect the level of θ^j, while as a signal of θ^i, it becomes more vague, $\frac{\partial \rho_1^{ii}}{\partial \sigma_j^2} < 0$. The opposite occurs when received interactions are weak (small h_j). To interpret this case, let us assume $h_j = 0$, implying that agent i's output is now independent of θ^j and $\rho_1^{ji} < 0$ for all h_i. The negative sign of ρ_1^{ji} indicates that given z_1^j, a higher z_1^i is "bad news" for agent j. The market attributes a higher z_1^i to a higher θ^i. An increase in σ_j^2 now works in favor of agent i and induces the market to rely more on an agent's project output, z_1^i, to perceive the level of her ability, θ^i. We have $\frac{\partial \rho_1^{ii}}{\partial \sigma_j^2} > 0$.

The variance of agents' abilities and the degrees of teamwork interactions also affect the "total" amount of available information in the market. Learning about abilities is captured by a decrease in the variance of the posterior estimate of the θs, and thus by an increase in

$$\rho_1^{ii} + h_i \rho_1^{ij} = \frac{\sigma_i^2}{\lambda_1} \left[\left(1 + h_i^2\right) \sigma_\varepsilon^2 + \left(1 - h_i h_j\right)^2 \sigma_j^2 \right],$$

where λ_1 is given in Lemma 1. The market can obtain a better estimate of agent i's ability as σ_i^2 increases and σ_j^2, σ_ε^2 decrease. Remark 2 shows that h_i also increases learning as long as ρ_1^{ij} is positive.[14]

Remark 2 (Information extraction & teamwork interactions) *Given z_1^i and z_1^j, the conditional variance of θ^i: (i) decreases with initiated interactions h_i, $\frac{\partial\left(\rho_1^{ii} + h_i \rho_1^{ij}\right)}{\partial h_i} > 0$, if and only if such interactions are strong enough so that $\rho_1^{ij} > 0$; (ii) increases with received interactions h_j, $\frac{\partial\left(\rho_1^{ii} + h_i \rho_1^{ij}\right)}{\partial h_j} < 0$, for all h_i.*

[Figures 1 are about here.]

As h_j increases, the joint signal $\left(z_1^i, z_1^j\right)$ about θ^i becomes more vague. The market finds it harder to disentangle the contribution of agent i's ability to both teammates' project outputs and the information conveyed by z_1^i and z_1^j about θ^i is less pronounced. The market relies less on the performance measures to assess agent i's ability as the impact of θ^j to z_1^i increases. Similarly, as long as $\rho_1^{ij} < 0$, a small increase in h_i prevents the market from learning, since it makes the joint signal $\left(z_1^i, z_1^j\right)$ about θ^i to reveal less information. Nevertheless, if h_i exceeds a threshold such that ρ_1^{ij} becomes positive, the conditional variance of θ^i decreases. In this regime, ρ_1^{ii} decreases with h_i. However, as agent i's help matters more for agent j's performance, z_1^j becomes more informative. The effect of h_i on z_1^j exceeds that on z_1^i, making both signals jointly "speak" more about ability. Higher h_i helps the market to learn, resulting in better estimates of θ^i.

2.3 Agents' preferences and objectives

In carrying out her own task and providing support to her teammate, agent i incurs disutility that is task specific. The cost functions of work effort and help effort are $\psi\left(e_t^i\right)$ and $\psi\left(a_t^i\right)$, respectively. The function $\psi\left(.\right)$ is twice continuously differentiable and convex, implying that there are diminishing returns to scale in the production process. We also assume that $\psi'\left(0\right) = 0$, $\lim_{e_t^i \to \infty} \psi'\left(e_t^i\right) = \infty$

[14] We have $\frac{\partial\left(\rho_1^{ii} + h_i \rho_1^{ij}\right)}{\partial h_j} = -\frac{2\sigma_i^2 \sigma_j^2 \sigma_\varepsilon^2}{\lambda_1^2} \left(h_i + h_j\right) \left[\sigma_\varepsilon^2 + \left(1 - h_i h_j\right) \sigma_j^2\right] < 0$ for all h_i, σ_i^2, σ_j^2 and σ_ε^2. The derivative with respect to h_i gives $\frac{\partial\left(\rho_1^{ii} + h_i \rho_1^{ij}\right)}{\partial h_i} = \frac{2\sigma_i^2 \sigma_\varepsilon^2}{\lambda_1^2} \left[\sigma_\varepsilon^2 + \left(1 + h_j^2\right) \sigma_j^2\right] \left[h_i \sigma_\varepsilon^2 - \left(1 - h_i h_j\right) h_j \sigma_j^2\right]$. Note that $sign\left\{h_i \sigma_\varepsilon^2 - \left(1 - h_i h_j\right) h_j \sigma_j^2\right\} = sign\left\{\rho_1^{ij}\right\}$.

and $\lim_{a_t^i \to \infty} \psi'(a_t^i) = \infty$. Task-specific cost functions are used in multi-agent models as in Auriol et al. (2002), and Itoh (1992). However, they are in stark contrast to other multitask models based on Holmström & Milgrom (1991) that assume $\psi(e_t^i + a_t^i)$. In the latter models, the cross-partial derivatives with respect to two efforts are positive. That is, tasks are (perfect) substitutes in an agent's cost function. These total-effort-cost functions introduce negative externalities between a given agent's tasks. As an agent increases the effort devoted to one task, the marginal cost of effort to the other task will grow larger. Thus, providing support to a teammate would be costly to an agent and it crowds out effort directed to her own task, decreasing her own project output. Agents care for the sum of effort exerted and the allocation of effort between the tasks depends on the relative benefits an agent derives by these two tasks. In fact, the agent must equate the marginal return to effort in both tasks. These models focus on the allocation of an agent's "attention" between the tasks.

In our model with task-specific-cost functions, disaggregated information, and separation of tasks - work effort and help effort are inputs in different production functions - benefits of providing help or sabotage emerge. Allocating a given total effort to both tasks entails lower disutility. The cross-partial of the cost function is zero, hence the cost of exerting effort to perform a given task is independent of the other task. An agent can focus on eliciting effort to affect her teammate's project output without having to consider simultaneously technologically founded externalities. Putting effort in a task does not require effort away from the other task. There are benefits from task-specific costs that can emerge exactly when there is separation of tasks and each teammate's project output is observable. This cost function allows us to compare agents' effort decisions for the same tasks and capture the results of influencing another agent's project output. Multitasking in the absence of crowding out effects between the tasks keep a worker highly motivated to exert effort in environments where career concerns are an issue.

Agent i receives the reward w_t^i and has constant absolute risk-averse (CARA) preferences. She derives utility

$$U^i = -\exp\left(-r\left[\sum_{t=1}^{2}\left[w_t^i - \psi\left(e_t^i\right) - \psi\left(a_t^i\right)\right]\right]\right), \tag{2}$$

where r is the Arrow-Pratt measure of risk aversion, $r > 0$.[15] This function is additively separable across periods, implying that agents behave as if they have access to perfect capital markets. They also do not discount the future.

Agent i's reward is determined in equilibrium and depends on the available information conveyed

[15] Risk-aversion on the part of the agents is essential when explicit contracts are provided in order for the incentives parameters to be less than one (in absolute terms). Otherwise, the optimal contract will impose substantial human capital risk on the agents. If only implicit incentives arise, our results are also obtained with risk-neutrality; i.e., agent i's utility could be $U^i = \sum_{t=1}^{2}\left[w_t^i - \psi\left(e_t^i\right) - \psi\left(a_t^i\right)\right]$. We consider risk-aversion in order to be consistent in both settings.

by both agents' past performance measures.[16,17] A competitive market will set

$$w_t^i = (1 + h_i) E \left\{ \theta^i \mid z_{t-1}^i, z_{t-1}^j \right\} + \widehat{e}_t^i + h_i \widehat{a}_t^i \equiv \widetilde{\theta}_t^i. \tag{3}$$

Each agent receives a fixed payment equal to the reputational bonus she can claim for her contribution to both teammates' project outputs. This bonus is the total rent an agent can get by exerting effort *and* providing support. Given the available information, her payment increases with an upward revision of the market's estimate of her own ability.

3 Reputation incentives

We now solve the two-period game and derive the teammates' optimal efforts. The conventional wisdom in career concerns models is that an agent works harder at the beginning of her career in order to improve her own performance and thus manipulate market assessments about her ability. We show that in our multi-agent model where an agent's ability inserts a fellow member's production function, additional reputation incentives arise. To influence the learning process, under certain conditions, an agent has incentives either to help or even to sabotage her colleague. Then, we perform this analysis when the output shocks are correlated.

3.1 Work and help effort

In period 2, agent i receives $w_2^i = (1 + h_i) E \left\{ \theta^i \mid z_1^i, z_1^j \right\} + \widehat{e}_2^i + h_i \widehat{a}_2^i$. However, this reward does not depend on her current actions. There are no career concerns and thus she exerts zero effort: $e_2^{i*} = 0$ and $a_2^{i*} = 0$. In period 1, agent i maximizes her current and future utility:

$$-\exp\left(-r\left[E\left\{w_1^i\right\} - \psi\left(e_1^i\right) - \psi\left(a_1^i\right) + E\left\{w_2^i \mid z_1^i, z_1^j\right\} - \psi\left(e_2^{i*}\right) - \psi\left(a_2^{i*}\right)\right]\right).$$

The reward w_1^i is independent of e_1^i and a_1^i because $z_0^i = \emptyset$ and $E\left\{\theta^i\right\} = m_1^i$. Given also that $\psi\left(e_2^{i*}\right)$ and $\psi\left(a_2^{i*}\right)$ are zero, agent i's problem reduces to maximizing

$$-\exp\left(-r\left[-\psi\left(e_1^i\right) - \psi\left(a_1^i\right) + (1 + h_i) E\left\{\theta^i \mid z_1^i, z_1^j\right\}\right]\right).$$

Career concerns arise because the levels of current project outputs, z_1^i and z_1^j, affect the reputational bonus (wage) in the second period. As long as ability is unknown, there are returns to supplying labor, since past performances will influence the markets' perceptions about θ^i. Labor is a substitute

[16] The principals maximizes the sum of outputs minus the agents' payments. However, the competition among them will drive their profits down to zero and each agent will receive her reputational bonus.

[17] Recall that $t = \{1, 2\}$. If employment lasts for T periods where $T > 2$, the market's perceptions of abilities will depend on all past performances. The reputational bonus will be $w_t^i = (1 + h_i) E\left\{\theta^i \mid z_1^i, z_1^j, ..., z_{t-1}^i, z_{t-1}^j\right\} + \widehat{e}_t^i + h_i \widehat{a}_t^i$.

for ability. Thus, by increasing labor supply, an agent can potentially bias the process of inference in her favor. Proposition 1 presents the optimal efforts.[18]

Proposition 1 (Career concerns) *In equilibrium, agent i has reputation (implicit) incentives to work, increasing her own project output, as well as to help or sabotage her teammate's production:*

$$\psi'\left(e_1^{i*}\right) = \left(1+h_i\right)\rho_1^{ii} \text{ and } \psi'\left(a_1^{i*}\right) = \underbrace{\left(1+h_i\right)h_i\rho_1^{ij}}_{\text{help or sabotage}}$$

where ρ_1^{ii} and ρ_1^{ij} are given in Lemma 1.

The optimal efforts are contingent on the measures that the market uses to draw inferences about ability. In line with the literature, career concerns depend on the weight the market puts on outputs in estimating ability. However, we argue that what also matters for career concerns is how many components of the production process and the learning process an agent can affect in order to manipulate the market's perceptions in her favor and how many "pieces" of future remuneration depend on an agent's current actions. By exerting work effort in the current period and providing support, an agent affects both teammates' performance measures, z_1^i and z_1^j, in order to induce an upward revision of the market's estimate of her own ability. Thus, an agent has two tools available to use to shape the market's assessments. In Auriol et al. (2002) where the support an agent receives depends only on her teammate's effort (not on her ability) and the market shocks are not correlated, market assessments about agent i's ability only depend on her own performance. Thus, providing support has no effect on an agent's future remuneration. Agent i's utility-maximizing help effort is zero. Her work effort is equal to $\frac{\sigma_i^2}{\sigma_i^2+\sigma_\varepsilon^2}$ and independent of the degrees of teamwork interactions.

In our model, additional reputation incentives arise. Agent i exerts effort to increase her future remuneration by $M_1^{ii} \equiv \left(1+h_i\right)\rho_1^{ii}$ through her work and by $M_1^{ij} \equiv \left(1+h_i\right)h_i\rho_1^{ij}$ through help or sabotage. In particular, if initiated interactions are strong enough (large h_i) relative to the degree of received interactions h_j so that $\rho_1^{ij} > 0$, agent i anticipates that good teammate performance (high z_1^j) will entail an upward revision of the market's estimate of her own ability, θ^i. Therefore, she has additional incentives to help her colleague, $M_1^{ij} > 0$. However, for a small h_i so that $\rho_1^{ij} < 0$, such reputation incentives are reversed, $M_1^{ij} < 0$. If initiated interactions are weak, a higher z_1^j is attributed to θ^j and the market updates its assessments about θ^i downwards. Thus, by helping a teammate to further increase her project output, agent i will induce market inferences to be revised against her. Instead, a bad performance by her teammate will be a good signal about her own ability.

[18] One can consider the normalization $z_t^i = \left(1-h_i\right)\left(\theta^i + e_t^i\right) + h_j\left(\theta^j + a_t^j\right) + \varepsilon_t^i$ for any i and j. The reputational bonus now is $E\left\{\theta^i \mid z_{t-1}^i, z_{t-1}^j\right\} + \left(1-h_i\right)\widehat{e}_t^i + h_i\widehat{a}_t^i$. This normalization serves to guarantee that agents tend to put effort in both tasks exactly in order to manipulate market's perceptions rather than because the "pie" gets larger by helping a teammate. Qualitatively, *all* our results also hold in this setting. The optimal work effort will satisfy $\psi'\left(e_1^{i*}\right) = \left(1-h_i\right) corr\left(\theta^i, z_1^i \mid z_1^j\right)$ and the optimal help effort is given by $\psi'\left(a_1^{i*}\right) = h_i corr\left(\theta^i, z_1^j \mid z_1^i\right)$, which can be either positive or negative.

A decrease in z_1^j will increase agent i's reputation so that she now has incentives to sabotage her colleague. We can interpret negative effort as hiding, stealing or even destroying some part of a teammate's project output. In the polar case where "one-way" teamwork interactions occur - $h_i > 0$ while $h_j = 0$ - agent i always has incentives to help.

This analysis boils down to the following: agent i has stronger reputation incentives as more pieces of information during the learning process depend on current actions and as the impact of the estimate of θ^i on future remuneration increases. An agent always has incentives to exert work effort in order to increase her own project output. As long as a teammate's performance is sensitive to agent's own ability so that $\rho_1^{ij} > 0$, we argue that this agent has additional incentives to help her colleague in order to build up her reputation. In contrast, if the impact of an agent's support to her teammate's performance is insignificant so that the market puts a negative weight on her teammate's output to estimate her ability, $\rho_1^{ij} < 0$, an increase in z_1^j will bias the learning process against her. Thus, incentives to sabotage her teammate arise.

[Figures 2 are about here.]

We can also compare the teammates' effort decisions, given the differences in the variance of their abilities.[19] In particular, if received and initiated interactions are identical, $h_i = h_j$, the agent with the higher variance of ability, say $\sigma_i^2 > \sigma_j^2$, exerts more work effort, $\psi'(e_1^{i*}) > \psi'\left(e_1^{j*}\right)$, and help effort, $\psi'(a_1^{i*}) > \psi'\left(a_1^{j*}\right)$. Due to higher σ_i^2, the market is able to draw additional information about θ^i, and agent i's attempts to manipulate market perceptions are more effective. More generally, as long as the interactions initiated by the agent with the higher variance, $\sigma_i^2 > \sigma_j^2$, are large enough relative to the intensity of received interactions, this agent exerts more work and help effort than her colleague. The market anticipates that this agent's efforts are key determinants of both project outputs and relies more on both signals that likely reflect the level of her ability.[20]

3.2 Correlated output shocks

We analyze the reputation incentives when the transitory shocks, ε_t^i and ε_t^j, are correlated. Suppose that $\phi \equiv \frac{cov\left(\varepsilon_t^i, \varepsilon_t^j\right)}{\sigma_\varepsilon^2}$ denotes the correlation coefficient, where $|\phi| \leq 1$. The type of correlation (positive or negative) may depend on whether the team members use similar or different technologies

[19] We have $\psi'(e_1^{i*}) > \psi'\left(e_1^{j*}\right)$ if and only if $\sigma_i^2 > \frac{(1+h_j)\sigma_j^2\sigma_\varepsilon^2}{(1+h_i)\sigma_\varepsilon^2 + (h_i - h_j)(1 - h_i h_j)\sigma_j^2}$. Additionally, $\psi'(a_1^{i*}) > \psi'\left(a_1^{j*}\right)$ if and only if $\sigma_i^2 > \frac{h_j^2}{h_i}\frac{(1+h_j)\sigma_j^2\sigma_\varepsilon^2}{h_i(1+h_i)\sigma_\varepsilon^2 + h_j(h_j - h_i)(1 - h_i h_j)\sigma_j^2}$ for any h_i, h_j and σ_ε^2.

[20] One can also consider the degrees of teamwork interactions to be decision variables; i.e., agents decide how much they will appropriate from a teammate's support. Agent i's "appropriation" effort, (say) b_t^i, and θ^j are multiplicative, $z_t^i = \theta^i + e_t^i + b_t^i\left(\theta^j + a_t^j\right) + \varepsilon_t^i$. There are now multiple equilibria. The optimal efforts satisfy $\psi'(e_1^{i*}) = \left(1 + b_1^{j*}\right)\rho_1^{ii}$, $\psi'(a_t^{i*}) = \left(1 + b_1^{j*}\right) b_1^{j*}\rho_1^{ij}$ and $\psi'(b_1^{i*}) = \left(1 + b_1^{j*}\right)\rho_1^{ii}\left(m_j + e_1^{j*}\right)$ for any i and j. Dewatripont et al. (1999) assume that agent i's (work) effort is multiplied with her 'own' ability, and thus career concerns depend only on the mean of θ^i, and not on θ^j as in our setting.

in production.[21] Now, there are two "forms" of correlation between the team members' project outputs: one due to teamwork interactions and one due to the correlation of the random terms. Given the realized performances z_1^i and z_1^j, the correlation coefficients of the (conditional) distribution of θ^i are

$$\widetilde{\rho}_1^{ii} = \frac{\sigma_i^2}{\widetilde{\lambda}_1}\left[(1-h_i\phi)\,\sigma_\varepsilon^2 + (1-h_ih_j)\,\sigma_j^2\right] \text{ and } \widetilde{\rho}_1^{ij} = \frac{\sigma_i^2}{\widetilde{\lambda}_1}\left[(h_i-\phi)\,\sigma_\varepsilon^2 - (1-h_ih_j)\,h_j\sigma_j^2\right],$$

where $\widetilde{\lambda}_1 = \sigma_\varepsilon^4\left(1-\phi^2\right) + (1-h_ih_j)^2\,\sigma_i^2\sigma_j^2 + \sigma_\varepsilon^2\left[\left(1+h_i^2\right)\sigma_i^2 + \left(1+h_j^2\right)\sigma_j^2 - 2\phi\left(h_i\sigma_i^2 + h_j\sigma_j^2\right)\right]$.

Teammates' project outputs are informative about an agent's ability as long as at least one form of correlation is imperfect. If $h_i = h_j = \phi = 1$, both performance measures are identical and the market cannot draw any information about an agent's ability: $\widetilde{\rho}_1^{ii} = \widetilde{\rho}_1^{ij} = 0$ for any i and j. Thus, teammates give up on influencing the market's perceptions. There are no career concerns.

In settings where there is some degree of asymmetry in performance measures - i.e., h_i, h_j or ϕ are less than one - the correlation coefficient $\widetilde{\rho}_1^{ii}$ is always positive, $\widetilde{\rho}_1^{ii} > 0$. Given the teammate's performance, an agent's higher project output is attributed to her own higher ability and vice versa. However, the effect of an increase in ϕ on $\widetilde{\rho}_1^{ii}$ and thus on the intensity of an agent's (utility-maximizing) work effort, e_1^{i*}, is not straightforward. For example, let $\sigma_\varepsilon^2 = \sigma_i^2 = \sigma_j^2 = 1$ and $h_j = 0$ in order to isolate the effects of h_i and ϕ on agent i' reputation incentives. If $\phi = 0.9$ while $h_i = 0.1$, we have $\frac{\partial \widetilde{\rho}_1^{ii}}{\partial \phi} > 0$: agent i's contribution in z_1^j is negligible, but the observation of this additional signal effectively reduces the variance of the "noise" of her own project, ε_1^i, allowing the market to put a higher weight on z_1^i in estimating θ^i. Thus, an increase in the correlation between the output shocks leads an agent to exert higher work effort in order to build up her reputation.[22]

The relationship between e_1^{i*} and ϕ becomes negative, $\frac{\partial \widetilde{\rho}_1^{ii}}{\partial \phi} < 0$, when $\phi = 0.1$ while $h_i = 0.9$. Assuming that agent i does not receive any help while her support is critical to her teammate's performance, high project outputs are mainly attributed to her own ability. The market perceives that both signals indicate the level of θ^i and thus, given z_1^j, z_1^i is a good estimate of its level. However, as ϕ increases and the market accumulates more information about the market conditions, a lower weight is put on z_1^i in estimating θ^i. As the 'prior' variance of the noise terms decreases and the market factors affect teammates' project outputs the same way (recall $h_j = 0$), the market anticipates that both teammates' good performances are influenced by market factors, revising the estimate of θ^i downwards. Thus, higher correlation between the shocks will decrease agent i's optimal work effort. However, if a teammate's support in agent i's project output is significant (say $h_j = 1$), the derivative $\frac{\partial \widetilde{\rho}_1^{ii}}{\partial \phi}$ becomes positive, because now additional information about the market environment will be nothing else but useful. If cross-agent teamwork interactions are intensive, the market finds it harder to perceive the levels of the θs. Thus, as ϕ increases, the market can better identify whether

[21] For instance, one can consider a team that produces hard disks but the team members use different technologies; i.e., magnetic and holographic. A market shock may hit the output of the projects that are based on these two technologies in a different way.

[22] Under the assumption that $\sigma_\varepsilon^2 = \sigma_i^2 = \sigma_j^2 = 1$ and $h_j = 0$, for $\phi < 0$, we have $\frac{\partial \widetilde{\rho}_1^{ii}}{\partial \phi} < 0$ for any h_i.

the outputs signal the level of teammates' abilities or are influenced by marketwide factors.

This analysis highlights that given the available information, a larger ϕ will discourage agent i to exert work effort if this increase leads to a worse market estimate of θ^i. More precisely, an increase in a small $\phi > 0$ will decrease agent i's optimal work effort when initiated interactions, h_i, are strong enough while received interactions, h_j, are weak: $\frac{\partial \psi'\left(e_1^{i*}\right)}{\partial \phi} < 0$ if and only if

$$\phi < \frac{\sigma_\varepsilon^2 + (1 - h_i h_j)\,\sigma_j^2 - \left(\sigma_\varepsilon^2 + h_i^2 \sigma_i^2 + \sigma_j^2\right)^{\frac{1}{2}} \left[(1 - h_i^2)\,\sigma_\varepsilon^2 + \left(1 - h_i^2 h_j^2\right)\sigma_j^2\right]^{\frac{1}{2}}}{h_i \sigma_\varepsilon^2}.$$

Meyer & Vickers (1997) also examine the relationship between reputation incentives and the correlation of the output shocks. They consider a two-agent setting in which each agent's output depends only on her own effort and ability, as in Holmström (1999). They find that when agents' output shocks are correlated (while their abilities are independent), a larger correlation ϕ, where $\phi > 0$, leads an agent to exert higher effort, $\widetilde{e}_1^{i*}$, in order to increase her reputation. There is a negative externality and some rivalry between agents. The observation of another agent's outcome exactly reduces the variance of the "noise" and allows the market to rely more on an agent's performance to infer the level of her own ability.[23] This effect is also present in our setting where teamwork interactions occur. However, we argue that this relationship can turn out to be negative when h_i is large enough while h_j is small, where an increase in ϕ induces the market to decrease the weight it puts on agent i's project output to perceive the level of her ability.

The sign of $\widetilde{\rho}_1^{ij}$ is also not clear cut. It depends on the *relative* intensity of the two forms of correlation between the project outputs. For $\widetilde{\rho}_1^{ij}$ to be positive, initiated interactions, h_i, must be sufficiently large in order for θ^i to be a key determinant of z_1^j. For instance, if the market shocks vary substantially (high σ_ε^2) and are negatively correlated, $\phi < 0$, $\widetilde{\rho}_1^{ij}$ is more "likely" to be positive. A high realization of z_1^i should be associated with a low z_1^j. However, if agent j's project output is also high, this is attributed to high θ^i, especially for relatively intensive initiated interactions h_i. In turn, agent i cashes in an increase in her reputational bonus due to a higher z_1^j and thus, she has incentives to help her fellow member.

Remark 3 (Different forms of correlation of performance measures & help effort) *An agent will have reputation incentives to help a teammate when initiated interactions are substantially larger than the correlation of the output shocks:* $\psi'\left(a_1^{i*}\right) > 0$ *if and only if* $h_i - \frac{\sigma_j^2}{\sigma_\varepsilon^2} h_j\,(1 - h_i h_j) > \phi$.

[Figures 3 are about here.]

In a setting where the random shocks are positively correlated, $\phi > 0$, but ϕ exceeds h_i, $\phi > h_i$, then $\widetilde{\rho}_1^{ij}$ is negative. This happens because the contribution of θ^i in z_1^j is relatively small and high

[23] Meyer & Vickers (1997) argue that this relationship between reputation incentives and the correlation of the output shocks is the counterpart of the insurance effect in a static principal-agent model where "comparative performance information" compensation schemes are provided. The observation of another agent's output increases the precision with which an agent's effort is estimated, leading the principal to provide additional motivation.

teammates' project outputs are mainly attributed to market factors. The market believes that the teammates act in a favorable environment and updates its assessments about θ^i downwards. Therefore, there is some rivalry between the agents and incentives to sabotage arise.

4 Multiperiod models

We now focus on career concerns when employment extends to many periods and the output shocks are uncorrelated. We also use a stationary model as in Holmström (1999) to examine whether the equilibrium efforts are efficient under the assumption that the quality of a fellow member of a team matters for an agent's reputation.

4.1 The T-period case

At each period t, the market's assessments of abilities now depend on the history of agent i's and j's project outputs $z_1^i, z_1^j, ..., z_{t-1}^i, z_{t-1}^j$. The optimal efforts satisfy the equations $\psi'(e_t^{i*}) = (1+h_i)\sum_{\tau=t}^{\infty}\rho_\tau^{ii}$ and $\psi'(a_t^{i*}) = (1+h_i)\, h_i \sum_{\tau=t}^{\infty}\rho_\tau^{ij}$. The signals are

$$\rho_\tau^{ii} = \frac{\sigma_i^2}{\lambda_\tau}\left[\sigma_\varepsilon^2 + (\tau-1)(1-h_ih_j)\,\sigma_j^2\right] \text{ and } \rho_\tau^{ij} = \frac{\sigma_i^2}{\lambda_\tau}\left[h_i\sigma_\varepsilon^2 - (\tau-1)(1-h_ih_j)\,h_j\sigma_j^2\right],$$

where $\lambda_\tau \equiv \sigma_\varepsilon^4 + (\tau-1)^2(1-h_ih_j)^2\sigma_i^2\sigma_j^2 + (\tau-1)\,\sigma_\varepsilon^2\left[(1+h_i^2)\,\sigma_i^2 + \left(1+h_j^2\right)\sigma_j^2\right]$.

In line with the literature, the signal ρ_τ^{ii} is always positive but decreasing in τ. The returns to an agent's work effort are bigger the more uncertainty there is about her ability. Thus, early in the process when there is less available information, the market puts more weight on the most recent output observation when updating its assessments about θ^i. Eventually, θ^i will be revealed almost completely and new output observations will have little impact on market perceptions. For small h_i, the presence of teamwork interactions slows down the learning process about θ^i. However, agent i's attempts to influence output are only temporarily effective (only early in career). In this multi-period setting, the signal ρ_τ^{ij} deserves special attention.

Remark 4 (Signals over the periods) *For strong initiated interactions (large h_i), ρ_τ^{ij} is positive only in the early stages of agent i's career: $\rho_\tau^{ij} > 0$ if and only if $1 + \frac{h_i\sigma_\varepsilon^2}{(1-h_ih_j)h_j\sigma_j^2} > \tau$.*

The signal ρ_τ^{ij} may even switch signs, from positive to negative, as τ increases. Note that although the variance of a performance measure depends on the variance of its transitory shock - i.e., $var(z_t^i) = \sigma_i^2 + h_j^2\sigma_j^2 + \sigma_\varepsilon^2$ - the covariance of project outputs realized in the same or different periods depends only on the variance of teammates' abilities: $cov\left(z_t^i, z_{t+1}^i\right) = \sigma_i^2 + h_j^2\sigma_j^2$ and $cov\left(z_t^i, z_{t+1}^j\right) = h_i\sigma_i^2 + h_j\sigma_j^2$. Thus, over the periods, the noise in the performance measures driven by the output shocks becomes (relatively) less significant in the process of estimating abilities. To put it differently, the signals incorporate information about the covariances of all project outputs that have been realized in the past. Under the assumption of independently distributed random terms, such convariances depend solely on σ_i^2 and σ_j^2 (not σ_ε^2), implying that over the periods, the noise introduced by teammates' abilities matters more in the learning process and for reputation incentives. Thus, even when teamwork interactions are such that the market puts a positive weight on agent j's project outputs to infer the level of θ^i early in the process, as performance observations accumulate, ρ_T^{ij} diminishes. At later stages of an agent's career, as θ^j becomes key in predicting θ^i, this signal can turn out to be negative. As the market learns more about θ^j by observing z_t^j, agent i's reputation incentives reverse and, in fact, she has incentives to sabotage. Even if early in the process an agent has incentives to help her colleague, sabotage incentives can arise for those agents who are about to retire.

4.2 The stationary case

We now investigate the relationship between the intensity of reputation incentives over time and the efficient level of efforts in a stationary setting where teammates' abilities remain unknown to the parties. In this setting, we can examine whether agents' desire to shape market perceptions in order to increase future remuneration can induce them to exert the "right" level of efforts. We also need to assume that the agents discount the future by some factor ζ. For higher ζ, agents put a lower weight on the future and thus value the "delayed" payments less. Provided that career concerns arise exactly because of agents' attempts to increase their reputation, seeking higher future monetary payments, such incentives will be stronger in a setting with no discounting. However, the presence of discounting in this analysis will allow for additional insights on whether the market forces alone can remove the moral hazard problems and provide adequate incentives for workers to perform.

In line with the literature based on Holmström (1999), we assume that the ability fluctuates over the agents' working life, according to the process

$$\theta_{t+1}^i = \theta_t^i + \eta_t^i,$$

where η_t^i is independently and normally distributed with zero mean and variance σ_η^2. Thus, at period $t+1$, agent i's project output is $z_{t+1}^i = \theta_t^i + \eta_t^i + e_{t+1}^i + h_j\left(\theta_t^j + \eta_t^j + a_{t+1}^j\right) + \varepsilon_{t+1}^i$. The shocks η_t^i and η_t^j add uncertainty that prevents agents' abilities from becoming fully known. Lemma 2 derives the variance of θ_{t+1}^i in this stationary setting.

Lemma 2 (Stationary variance) *Let* $\widehat{\sigma}_{i,t}^2 \equiv \sigma_{i,t}^2\left(1-\rho_t^{ii}-h_i\rho_t^{ij}\right)$ *be the variance of* θ_{t+1}^i *before observing the realizations of* z_{t+1}^i *and* z_{t+1}^j*. After observing* z_{t+1}^i *and* z_{t+1}^j*, the stationary variance of agent* i*'s ability at* $t+1$ *is*

$$\overline{\sigma}_{i,t+1}^2 = \left(\widehat{\sigma}_{i,t}^2 + \sigma_\eta^2\right)\left(1-\widehat{\rho}_t^{ii} - h_i\widehat{\rho}_t^{ij}\right),$$

where $\widehat{\rho}_t^{ii} \equiv \dfrac{\widehat{\sigma}_{i,t}^2+\sigma_\eta^2}{\widehat{\lambda}_t^i}\left[\sigma_\varepsilon^2 + (1-h_ih_j)\left(\widehat{\sigma}_{j,t}^2+\sigma_\eta^2\right)\right]$, $\widehat{\rho}_t^{ij} \equiv \dfrac{\widehat{\sigma}_{i,t}^2+\sigma_\eta^2}{\widehat{\lambda}_t^i}\left[h_i\sigma_\varepsilon^2 - (1-h_ih_j)\,h_j\left(\widehat{\sigma}_{j,t}^2+\sigma_\eta^2\right)\right]$, *and* $\widehat{\lambda}_t^i \equiv \sigma_\varepsilon^4 + (1 - h_i h_j)^2 \left(\widehat{\sigma}_{i,t}^2 + \sigma_\eta^2\right)\left(\widehat{\sigma}_{j,t}^2 + \sigma_\eta^2\right) + \sigma_\varepsilon^2 \left[\left(1 + h_i^2\right)\left(\widehat{\sigma}_{i,t}^2 + \sigma_\eta^2\right) + \left(1 + h_j^2\right)\left(\widehat{\sigma}_{j,t}^2 + \sigma_\eta^2\right)\right].$

The learning process becomes

$$m_{t+1}^i = \widehat{\mu}_t^i m_t^i + \widehat{\rho}_t^{ii}\left(z_t^i - \widehat{e}_t^i - h_j \widehat{a}_t^j - h_j m_t^j\right) + \widehat{\rho}_t^{ij}\left(z_t^j - \widehat{e}_t^j - m_t^j - h_i \widehat{a}_t^i\right),$$

where $\widehat{\mu}_t^i = 1 - \widehat{\rho}_t^{ii} - h_i \widehat{\rho}_t^{ij}$. The shocks η_t^i and η_t^j prevent the market from learning, and thus the variance of abilities declines deterministically with t but does not go to zero. The optimal efforts satisfy

$$\psi'\left(e_t^{i*}\right) = (1 + h_i)\,\widehat{\rho}_t^{ii} \sum_{s=t+1}^{\infty} \zeta^{s-t} \Pi_{\kappa=t+1}^{s} \mu_\kappa^i \equiv T_{e_t^i}, \tag{4}$$

$$\psi'\left(a_t^{i*}\right) = (1 + h_i)\, h_i \widehat{\rho}_t^{ij} \sum_{s=t+1}^{\infty} \zeta^{s-t} \Pi_{\kappa=t+1}^{s} \mu_\kappa^i \equiv T_{a_t^i}. \tag{5}$$

In period 1, we have $\psi'\left(e_1^{i*}\right)$ to be given by the sum of the terms $\zeta(1 + h_i)\,\widehat{\rho}_1^{ii}$, $\zeta^2(1 + h_i)\,\widehat{\rho}_1^{ii}\widehat{\mu}_2^i$, $\zeta^3(1 + h_i)\,\widehat{\rho}_1^{ii}\widehat{\mu}_2^i\widehat{\mu}_3^i$, etc. In the stationary case where $\widehat{\rho}_{t+1}^{ii} = \widehat{\rho}_t^{ii} = \widehat{\rho}^{ii*}$, $\widehat{\rho}_{t+1}^{ij} = \widehat{\rho}_t^{ij} = \widehat{\rho}^{ij*}$ and $\widehat{\mu}^{i*} = 1 - \widehat{\rho}^{ii*} - h_i \widehat{\rho}^{ij*}$, equation (4) becomes

$$\psi'\left(e_1^{i*}\right) = \zeta(1 + h_i)\,\widehat{\rho}^{ii*}\left[1 + \zeta\widehat{\mu}^{i*} + \zeta^2\left(\widehat{\mu}^{i*}\right)^2 + \zeta^3\left(\widehat{\mu}^{i*}\right)^3 + ...\right],$$

where the sum in the brackets is equal to $\frac{1}{1-\zeta\mu^{i*}}$. This analysis gives the stationary work and help effort levels:

$$\psi'\left(e^{i*}\right) = \frac{\zeta(1 + h_i)\,\widehat{\rho}^{ii*}}{1 - \zeta\left(1 - \widehat{\rho}^{ii*} - h_i\widehat{\rho}^{ij*}\right)} \text{ and } \psi'\left(a^{i*}\right) = \frac{\zeta(1 + h_i)\, h_i\widehat{\rho}^{ij*}}{1 - \zeta\left(1 - \widehat{\rho}^{ii*} - h_i\widehat{\rho}^{ij*}\right)}. \tag{6}$$

Holmström (1999) formalizes Fama's (1980) major conclusion that the market induces the agents to exert the efficient effort levels. In a single-agent model, he shows that this happens if there is no discounting. We argue that in our model where the team members interact and an agent's individual performance depends on the quality of her team, even if there is no discounting, for $h_i > 0$, Fama's conclusion generically fails. The stationary levels of efforts are above or below their efficient levels. We perform this analysis by discussing first the only two cases where teammates' stationary effort levels are also efficient in our model.

Under full information (first-best), agent i's remuneration is a fixed payment equal to the sum of the disutilities of work and help efforts, $\psi\left(e_t^i\right) + \psi\left(a_t^i\right)$, and the efficient effort levels $e_t^{i,fb}$ and $a_t^{i,fb}$ satisfy the conditions $\psi'\left(e_t^{i,fb}\right) = 1$ and $\psi'\left(a_t^{i,fb}\right) = h_i$, respectively. The first-best reward at each period t is the reward that is optimal in a one-shot game.

Agent i's stationary work and help efforts are efficient as long as there is no discounting, $\zeta = 1$, and an agent's ability *does not* affect her teammate's project output, $h_i = 0$, as in Holmström (1999) and Auriol et al. (2002), although received interactions may occur, $h_j > 0$ (agent j's stationary efforts will be inefficient). Therefore, $\widehat{\rho}^{ij*}$ as a signal should play no role in agent i's reputation decisions. In turn, the stationary work effort is efficient and equal to one: $\psi'\left(e^{i*}\right)=\psi'\left(e_t^{i,fb}\right)=1$. Since an agent's help effort does not affect the process of inference about her own ability, any incentive to influence a teammate's performance disappears. Exerting zero help effort in a stationary model is also efficient: $\psi'\left(a^{i*}\right)=\psi'\left(a_t^{i,fb}\right)=0$.

It is rather striking that in our model where teammates' abilities matter for reputation concerns, the stationary effort levels can also be efficient as long as the initiated *and* received interactions are *perfect*, $h_i=h_j=1$. Now, providing help to a colleague is as productive as putting effort into one's own task. An agent's help and work efforts weight equally to both performance measures. Under full information, the efficient effort levels are given by $\psi'\left(e_t^{i,fb}\right)=\psi'\left(a_t^{j,fb}\right)=1$. The stationary efforts also satisfy $\psi'\left(e^{i*}\right)=\psi'\left(a^{i*}\right)=\psi'\left(e^{j*}\right)=\psi'\left(a^{j*}\right)=1$. They become efficient as soon as we add any amount of noise in the learning process about agents' ability levels. There is a balance between the incentives to work and help in order to build up reputation. Proposition 2 establishes that in our model, under any other condition, Fama's conclusion does not hold.

Proposition 2 (Stationary labor supply) *In the stationary model where $\sigma_\eta^2>0$ and $\sigma_\varepsilon^2>0$, for all $h_j\in(0,1)$, if initiated teamwork interactions occur, $h_i>0$, agent i exerts*

(a) higher work effort than its efficient level: $\psi'\left(e^{i*}\right)>\psi'\left(e_t^{i,fb}\right)$;

(b) lower help effort than its efficient level: $\psi'\left(a^{i*}\right)<\psi'\left(a_t^{i,fb}\right)$.

In the stationary case where $h_j<1$, for any h_i, an agent's work effort is higher and help effort is lower than its efficient level. Agent i has incentives to distort her work effort upwards in order to signal that she is of higher ability and induce the market to revise its beliefs in her favor. The stationary reputation incentives indicate that an agent is oriented to exert more work effort in order to improve her own performance, rather than to focus on helping or sabotaging her colleague. The optimal a^{i*} is distorted downwards. Thus, agents will overprovide work effort and underprovide help effort. For small initiated interactions such that $\widehat{\rho}^{ij*}<0$, the stationary level of help effort can even be negative, while its efficient level is always positive.

To complete this analysis, we also need to examine the convergence to the stationary state. We need to explore reputation incentives before a stationary state is reached. In Holmström (1999), learning follows the process $m_{t+1}^i=v_t^i m_t^i+\left(1-v_t^i\right)z_t^i$, where $z_t^i=\theta^i+e_t^i+\varepsilon_t^i$ and $v_t^i>0$, implying that the convergence of an agent's effort to the stationary state is directly related to the dynamics of v_t^i. In our model, $\widehat{\mu}_t^i$ incorporates both signals $\widehat{\rho}_t^{ii}$ and $\widehat{\rho}_t^{ii}$. Thus, the sequence of $\widehat{\mu}_t^i$ will depend on the amount of information extracted by both signals, $\widehat{\rho}_t^{ii}+h_i\widehat{\rho}_t^{ii}$, in each period. However, the convergence of an agent's work effort will depend on the dynamics of $\widehat{\rho}_t^{ii}$, while the convergence of help effort will depend on the dynamics of $\widehat{\rho}_t^{ij}$.

We show in Appendix $(A.2)$ that the sequence of agent i's optimal work effort $\left\{e_t^{i*}\right\}$ converges monotonically to the stationary state level e^{i*}, given by equation (6). If $\left\{\widehat{\rho}_t^{ii}\right\}$ is an increasing sequence, then so is $\{T_{e_t^i}\}$, and the convergence of $\{e_t^{i*}\}$ is from below. Similarly, if $\widehat{\rho}_t^{ii}$ is a decreasing sequence, the convergence of $\{e_t^{i*}\}$ is from above. The same dynamics govern the convergence of $\{a_t^{i*}\}$ to its stationary level. If (positive or negative) $\{\widehat{\rho}_t^{ij}\}$ is an increasing sequence, the convergence of $\{a_t^{i*}\}$ is from below, while if $\{\widehat{\rho}_t^{ij}\}$ is a decreasing sequence, $\{a_t^{i*}\}$ converges from above to the stationary state.

5 Explicit contracts

We now examine career concerns in the presence of teamwork interactions when explicit contracts are provided and employment lasts for two periods.[24] We assume that the agents are appointed by a risk-neutral and profit-seeking principal. Their project outputs are observable and contractible, allowing the principal to deal with each agent separately as in Itoh (1991, 1992). The principal and agents interact and play the two-period game described in Figure 4.

Period 1 *Contracting stage:*	$\longrightarrow$	*Production stage:*	$\longrightarrow$	Period 2 *Contracting stage:*	$\longrightarrow$	*Production stage:*
The principal simultaneously makes a contract offer to each teammate.		If the agent accepts the offer, she chooses effort levels, e_1^i and a_1^i. Events beyond the agents' control occur, the outputs are determined, and the rewards are received.		All parties observe 1st-period outputs and update their beliefs. A new contract offer is made to each teammate.		If the agent stays in the firm, she makes effort choices, e_2^i and a_2^i. The outputs are realized and the contracts are executed.

Figure 4. Timing of the game

5.1 Principal's problem

The contracts depend linearly on both agents' project outputs since the latter are correlated due to teamwork interactions. Holmström & Milgrom (1987) establish that in a model much like the single-period version of this model (but lacking the uncertainty about an agent's ability), the optimal contract is linear.[25] Gibbons & Murphy (1992), in a single-agent model, and Auriol et al. (2002), in their multi-agent framework, also consider contracts that are linear in outputs.

Holmström (1999) also argues that, for a risk-neutral agent whose payoff is linear in the posterior belief about her ability, there is no divergence between the agent's effort decision to build up reputation and the first best, provided that the market can observe her project output. Reputation incentives distort the agent's effort away from first best only if she is risk averse or the production technology is nonlinear in agent's ability so that the process of inference about her ability is also nonlinear. The intuition is as follows. Assuming linear contracts, the agent's expected payment is exactly based on the prior assessment of her ability. Thus, a risk-neutral agent is willing to exert the "right" level of effort from the principal's perspective. Reputation incentives matter only if the agent is risk averse or her wage is nonlinear in the posterior of her ability. Both these assumptions

[24] The model where employment lasts for T periods is solved in Appendix ($A.5$).

[25] Holmström & Milgrom (1987) show that in a static version of their dynamic model, the optimal compensation scheme that is offered to an agent with CARA preferences is a linear function of the performance measures.

serve to guarantee that there is no symmetry between the risk faced by the firm and the risk face by the agent. Gibbons & Murphy (1992) consider explicit payments and state that risk-aversion is necessary so that optimal contracts do not completely eliminate career concerns. In particular, a risk-averse agent wishes to be insured against low realizations of her project output, and thus weaker explicit incentives are provided. Given that explicit payments will decrease, reputation incentives will increase.

Relative performance evaluations provide a richer information base on which to write contracts and allow the principal to better assess agent i's efforts and ability. Teamwork interactions necessitate the use of two-piece rate contracts, which promote efficiency in designing incentives. In particular, we assume that, at each period t, the principal offers contracts of the form $C_t^i \equiv \left(\omega_t^i, \beta_t^i, \gamma_t^i\right)$ and agent i receives

$$w_t^i = \omega_t^i + \beta_t^i z_t^i + \gamma_t^i z_t^j, \tag{7}$$

where ω_t^i denotes the fixed salary component and β_t^i, γ_t^i are the incentive parameters. Such "team-incentive" schemes introduce either cooperation or competition between the teammates, depending on the sign of γ_t^i.

As in Gibbons & Murphy (1992) and Auriol et al. (2002), we also assume that side-contracts between agents are not possible and long-term (multiperiod) contracts are not feasible.[26] In particular, the second-period contracts depend implicitly on z_1^i and z_1^j rather than explicitly through commitment at the beginning of the first period. That is, at each period, agents choose the most attractive contract offers. This assumption also allows us to derive the substitutive relationship between implicit and explicit incentives: the contractual incentives should be strong when reputation incentives are weak.

In case that an agent rejects the contract offer, she receives an outside option that equals her reputational bonus $\widetilde{\theta}_t^i$, given by equation (3).[27] Competing employers cannot make a better offer than $\widetilde{\theta}_t^i$. The principal is equally well-off by hiring either a high reputation agent at a high wage or a low reputation agent at a low wage. This bargaining outcome can arise as the equilibrium of an extensive-form game. In this game, an agent is randomly assigned to a prospective principal and queues with the other job applicants. The principal makes a contract offer to the first agent in line. If the agent accepts the offer, she works for this principal. Otherwise, the agent queues for another job and the principal makes an offer to the next agent in line. Therefore, each agent receives only

[26] If a principal can commit herself to a second-period salary before the observation of the first-period outputs, the principal succeeds in insulating an agents' expected life-time compensation from the uncertainty she faces with respect to all team members' true abilities. An agent's problem is identical in each period. However, in our model, the principal cannot commit to such long-term schemes. A new contract is offered in every period, after observing the realizations of past performances. Such contracts will depend on these observations and reputation incentives will arise.

[27] Gibbons & Murphy (1992), Holmström (1999), among others, assume that the agent has all the bargaining power, and thus the principal maximizes subject to a zero-profit condition. In a multi-agent setting, this assumption would be problematic. Following Auriol et al. (2002), we consider a bargaining process that effectively makes each teammate the residual claimant only to her reputational bonus. One can also verify that if we set $\beta_2^{i*} = \gamma_2^{i*} = 0$, the equilibrium implicit incentives in our setting (derived in subsection 5.2) are as in Proposition 1.

her reputational bonus that arises due to work and support provision.

The principal is the residual claimant on firm's net profits, which equal the sum of the project outputs net of agents' compensations. In a two-period model, given the available information, the principal's problem becomes

$$\max_{C_t^i, e_t^i, a_t^i, C_t^j, e_t^j, a_t^j} E\left\{U^P\right\} = E\left\{\sum_{t=1}^{2}\sum_{i=1}^{2}\left(z_t^i - w_t^i\right) \mid z_{t-1}^i, z_{t-1}^j\right\}$$

$$\begin{aligned} \text{subject to} \quad & e_t^{i*} = \arg\max_{e_t^i} E\left\{U^i\left(w_t^i\right) \mid z_{t-1}^i, z_{t-1}^j\right\}, \ \forall i,t && \left(IC_{e,t}^i\right) \\ & a_t^{i*} = \arg\max_{a_t^i} E\left\{U^i\left(w_t^i\right) \mid z_{t-1}^i, z_{t-1}^j\right\}, \ \forall i,t && \left(IC_{a,t}^i\right) \\ & E\left\{U^i\left(w_t^i\right) \mid z_{t-1}^i, z_{t-1}^j\right\} \geq \widetilde{\theta}_t^i, \ \forall i,t && \left(IR_t^i\right) \end{aligned}$$

The incentive compatibility constraints, $\left(IC_{e,t}^i\right)$ and $\left(IC_{a,t}^i\right)$, guarantee that, in each period t, an agent chooses the (expected) utility maximizing efforts. The individual rationality constraint $\left(IR_t^i\right)$ shows that the agent will participate in the production process only if her expected utility of doing so exceeds her outside option. We recursively solve this game.[28]

5.2 Equilibrium explicit incentives

In period 2, agent i maximizes the certainty equivalent of her utility that takes the mean-variance form $CE_2^i \equiv E\left\{w_2^i \mid z_1^i, z_1^j\right\} - \psi\left(e_2^i\right) - \psi\left(a_2^i\right) - \frac{r}{2}Var\left\{w_2^i \mid z_1^i, z_1^j\right\}$. For any value of β_2^{i*}, the optimal γ_2^{i*} is choosing to minimize the variance of the wage w_2^i. Thus, the optimal work effort and help effort satisfy, respectively,

$$\beta_2^i = \psi'\left(e_2^{i*}\right) \text{ and } h_i\gamma_2^i = \psi'\left(a_2^{i*}\right). \tag{8}$$

Each agent also accepts the contract that allows her to earn (at least) her reputational bonus: in equilibrium, the IR_2^i constraint is binding. The principal signs the most appealing contracts. Thus, agent i's base payment is

$$\omega_2^{i*} = \widetilde{\theta}_2^i - E\left\{\beta_2^i z_2^i + \gamma_2^i z_2^j \mid z_1^i, z_1^j\right\} + \psi\left(e_2^{i*}\right) + \psi\left(a_2^{i*}\right) + \frac{r}{2}Var\left\{w_2^i \mid z_1^i, z_1^j\right\}, \tag{9}$$

where the conditional variance of the wage is

$$Var\left\{w_2^i \mid z_1^i, z_1^j\right\} = \left[\left(\beta_2^i + h_i\gamma_2^i\right)^2\sigma_{i,2}^2 + \left(h_j\beta_2^i + \gamma_2^i\right)^2\sigma_{j,2}^2\right] + \left[\left(\beta_2^i\right)^2 + \left(\gamma_2^i\right)^2\right]\sigma_\varepsilon^2. \tag{10}$$

[28] In this multi-agent framework, the monotone likelihood ratio property (MLRP) and the convexity of the distribution function condition (CDFC) are not sufficient for the first-order approach to be valid as in a single-agent setting. Itoh (1991) argues that, in a model with cross-agent interactions, a generalized CDFC for the joint probability distribution of the outputs is needed and the wage schemes must be nondecreasing. The coefficient of absolute risk aversion must also not decline too fast. Our model with agents' CARA preferences, linear contracts and production technologies satisfies all these assumptions and thus the first-order approach applies.

The second-period principal's problem is solved in Appendix $(A.3)$. Let $\Delta_t^i \equiv \frac{h_i^2\left[1+r\Sigma_t^{jj}\psi''\left(e_t^i\right)\right]-r\Sigma_t^{ij}\psi''\left(a_t^i\right)}{r\Sigma_t^{ii}\psi''\left(a_t^i\right)+h_i^2\left[1-r\Sigma_t^{ij}\psi''\left(e_t^i\right)\right]}$ and $\Omega_t^i \equiv \left[\sigma_\varepsilon^2 + \left(1+h_i\Delta_t^i\right)\sigma_{i,t}^2 + \left(h_j+\Delta_t^i\right)h_j\sigma_{j,t}^2\right]$, where $\Sigma_t^{ii} \equiv \sigma_\varepsilon^2 + h_i^2\sigma_{i,t}^2 + \sigma_{j,t}^2$ and $\Sigma_t^{ij} \equiv h_i\sigma_{i,t}^2 + h_j\sigma_{j,t}^2$ for any period t. Using equations (8), (9) and (10), we derive the second period explicit incentives:

$$\beta_2^{i*} = \frac{1}{1+r\Omega_2^i\psi''\left(e_2^i\right)} \text{ and } \gamma_2^{i*} = \Delta_2^i\beta_2^{i*}. \tag{11}$$

Proposition 3 (Relative performance evaluation) *In the second period, the optimal*

(a) pay-for-own-performance incentive parameter is positive, $\beta_2^{i*} > 0$, for all h_i and h_j;

(b) pay-for-teammate-performance incentive parameter is negative, $\gamma_2^{i*} < 0$, and thus, agent i has incentives to sabotage if and only if the degree of risk aversion is small enough so that

$$\frac{h_i^2}{\Sigma_2^{ij}\psi''\left(a_2^i\right) - h_i^2\Sigma_2^{jj}\psi''\left(e_2^i\right)} > r.$$

The positive sign of β_2^{i*} indicates that an agent's higher own project output is compensated with a higher wage. The sign of γ_2^{i*} is less straightforward. The principal sets γ_2^{i*} positive for low degrees of risk aversion and strong initiated interactions (high h_i) so that $\Delta_2^i > 0$, giving the agent a long position in her teammate's performance. The principal anticipates the support an agent provides to her colleague and rewards her when the teammate does better. The "compensation ratio" $\left|\frac{\gamma_2^{i*}}{\beta_2^{i*}}\right|$ is also higher in compensation packages that are rewritten to accommodate an increasing h_i. The higher h_i is, the more valuable is the information conveyed by the colleague's project output, and thus the use of relative performance evaluations becomes more essential. Such evaluation schemes can effectively be used as means of internalizing the positive effects of providing support. In contrast, for a high risk averse agent, if the initiated interactions are weak (low h_i), Δ_2^i becomes negative: due to risk-sharing and the fact that agent i's contribution in her teammate's project output is insignificant, the principal infers that agent j is the high productivity worker in the team and penalizes agent i as z_2^j increases. By setting γ_2^{i*} negative, the principal filters out the teamwork interactions from agent i's compensation. Recall also that the optimal γ_2^{i*} is such that it minimizes the variance of agent i's wage. In fact, for substantially high risk averse agents, it is likely the variance of her wage w_2^i to be minimized when γ_2^{i*} is negative.

To shed additional insight, let us assume that the cost-of-effort functions are $\psi\left(e_2^i\right) = \frac{1}{2}\left(e_2^i\right)^2$ and $\psi\left(a_2^i\right) = \frac{1}{2}\left(a_2^i\right)^2$. Suppose also that agent j does not provide any support to agent i, $h_j = 0$, while the contribution of agent i to her teammate performance is significant, provided that $h_i = 1$. For any degree of risk-aversion, the principal encourages agent i to help her teammate and compensates her by setting γ_2^{i*} positive, $\gamma_2^{i*} = \frac{1+r\sigma_\varepsilon^2}{1+r\left(\sigma_\varepsilon^2+\sigma_{j,2}^2\right)}\beta_2^{i*}$, while the principal decreases her teammate's (agent j) payment for an increase in z_2^i by setting γ_2^{j*} negative, $\gamma_2^{j*} = -\frac{\sigma_{i,2}^2}{\sigma_\varepsilon^2+\sigma_{i,2}^2}\beta_2^{j*}$. The principal provides opposing incentives to team members.

In period 1, agent i anticipates the implicit dependence of the second-period wage on the first-period project outputs and maximizes

$$CE_1^i \equiv E\left\{w_1^i\right\} - \psi\left(e_1^i\right) - \psi\left(a_1^i\right) + E\left\{w_2^{i*}\right\} - \psi\left(e_2^{i*}\right) - \psi\left(a_2^{i*}\right) - \frac{r}{2}Var\left\{\widetilde{w}_1^i + w_2^i\right\}, \tag{12}$$

where $Var\left\{\widetilde{w}_1^i\right\} = Var\left\{w_1^i\right\} + Var\left\{\omega_2^{i*}\right\} + 2Cov\left\{w_1^i, \omega_2^{i*}\right\}$ (see Appendix ($A.3$)). The optimal work effort e_1^{i*} and help effort a_1^{i*} solve, respectively,

$$\psi'\left(e_1^{i*}\right) = \beta_1^i + M_1^{ii} \text{ and } \psi'\left(a_1^{i*}\right) = h_i\left(\gamma_1^i + M_1^{ij}\right)$$

where $\frac{\partial \omega_2^{i*}}{\partial e_1^i} = M_1^{ii}$ and $\frac{\partial \omega_2^{i*}}{\partial a_1^i} = h_i M_1^{ij}$. By equation (9), we have

$$\begin{aligned} M_1^{ii} &\equiv \left(1+h_i\right)\rho_1^{ii} - \beta_2^{i*}\left(\rho_1^{ii} + h_j\rho_1^{ji}\right) - \gamma_2^{i*}\left(h_i\rho_1^{ii} + \rho_1^{ji}\right), \\ M_1^{ij} &\equiv \left(1+h_i\right)\rho_1^{ij} - \beta_2^{i*}\left(\rho_1^{ij} + h_j\rho_1^{jj}\right) - \gamma_2^{i*}\left(h_i\rho_1^{ij} + \rho_1^{jj}\right). \end{aligned} \tag{13}$$

Agents are motivated by the total explicit incentives from the first-period contract and the implicit incentives from career concerns. In particular, current effort only affects the intercept of future wage ω_2^{i*}. This is because there are no wealth effects in agents' utility and the production functions are additive. Both agents have the same marginal product of effort regardless of their true ability. In turn, the second-period explicit incentives β_2^{i*}, γ_2^{i*} are independent of z_1^i, z_1^j and thus of agent i's reputation.

By exerting more work effort in the first period, e_1^i, an agent gains from the subsequent increase in her reputational bonus by $\left(1+h_i\right)\rho_1^{ii}$. However, because of the "explicit incentive component" of the second-period reward, this bonus is diminished by $\beta_2^{i*}\left(\rho_1^{ii} + h_j\rho_1^{ji}\right) + \gamma_2^{i*}\left(h_i\rho_1^{ii} + \rho_1^{ji}\right)$. If initiated interactions are strong so that $\gamma_2^{i*} > 0$, the principal anticipates that agent i will be assessed as being of higher ability and the explicit incentive component of her future remuneration will be large. Thus, the principal offers a contract whose base payment increases by less than the increase in the agent's reputational bonus. However, if h_i is small so that $\gamma_2^{i*} < 0$, the principal now perceives that the explicit incentive component in the second period will also be small and does not lower the fixed part of the salary as much.

Implicit incentives also arise due to help effort provision, captured by M_1^{ij}. As long as agent i's contribution in a teammate's project output is significant (high h_i) so that $\rho_1^{ij} > 0$, by undertaking more help effort in the first period, a_1^i, agent i gains from an improvement in her teammate's performance and thus from the subsequent increase in her own reputational bonus $\widetilde{\theta_2^i}$. In particular, an increase in z_1^j will make agent i better off, since the principal will infer that agent i is a high productivity agent. Thus, agent i has positive (reputation) implicit incentives to help. Such incentives decrease by $\beta_2^{i*}\left(\rho_1^{ij} + h_j\rho_1^{jj}\right) + \gamma_2^{i*}\left(h_i\rho_1^{ij} + \rho_1^{jj}\right)$, where $\rho_1^{ij} + h_j\rho_1^{jj} > 0$ and $h_i\rho_1^{ij} + \rho_1^{jj} > 0$. If initiated interactions are weak (low h_i) so that $\rho_1^{ij} < 0$, M_1^{ij} becomes negative. It is in agent i's interest to convince the principal that she is teamed with a lower productivity agent and thus she has implicit incentives to sabotage. However, if $\gamma_2^{i*} < 0$, the second period explicit incentives now diminish agent i's appetite to sabotage. By setting γ_2^{i*} negative, the principal now makes agent i less interested in destroying part of her teammate's output in order to build up her own reputation. Thus, a negative γ_2^{i*} encourages agent i to focus more on her own project in the first-period rather than sabotaging her teammate in her attempt to shape market assessments about her ability. From the principal's perspective, career motives can be either beneficial or detrimental.

Sabotage incentives also arise in Auriol et al. (2002). However, the context and intuition differ. In their model, agent i's higher work effort increases only the conditional priors of her own ability, while her help effort decreases the expectations about her colleague's ability. Thus, an agent's reputation incentives are straightforward: she wants to induce an upward revision of the market's estimate of θ^i and a downward revision of the estimate of θ^j. Provided that agent i does not internalize any benefits of an increase in the estimate of θ^j and her remuneration is determined by her outside option, the optimal help effort is always negative due to explicit motivation. In particular, the optimal first-period work effort, e_1^{i*}, depends exclusively on β_2^{i*}, while the optimal a_1^{i*} depends exclusively on γ_2^{i*}; i.e., using the same notation, $\frac{\partial \omega_2^{i*}}{\partial e_1^i} = \left(1 - \beta_2^{i,AFP}\right) \frac{\sigma_i^2\left(\sigma_\varepsilon^2+\sigma_i^2\right)}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^2+2\sigma_i^2\right)}$ and $\frac{\partial \omega_2^{i*}}{\partial a_1^i} = -\gamma_2^{i,AFP} \frac{\sigma_i^2\left(\sigma_\varepsilon^2+\sigma_i^2\right)}{\sigma_\varepsilon^2\left(\sigma_\varepsilon^2+2\sigma_i^2\right)}$, where $\beta_2^{i,AFP} > 0$ and $\gamma_2^{i,AFP} > 0$. However, in our model, additional incentives arise, either through agents' attempts to increase their reputation or though explicit motivation. The reputation incentives that emerge through changes in a teammate's performance and are captured by $\frac{\partial \omega_2^{i*}}{\partial a_1^i} \left(= h_i M_1^{ij}\right)$ can even be positive.

Agents' incentives also differ from those in Lazear (1989) where sabotage incentives arise in tournaments because an agent's compensation is conditioned negatively on her colleagues' performances. Using such schemes, agents may want to destroy other workers' output rather than to work hard on their own project. In our model, the optimal γ_2^{i*} can be positive. However, even if $\gamma_2^{i*} > 0$, as long as ρ_1^{ij} is negative, agent i has *implicit* incentives to sabotage. This result is also different from Meyer & Vickers (1997) where the agents' abilities are correlated and implicit incentives are weakened due to the ratchet effect.

The decomposition of the equilibrium incentive parameters in the first period gives[29]

$$\beta_1^{i*} = \underbrace{\frac{1}{1 + r\Omega_1^i \psi''\left(e_1^{i*}\right)}}_{\substack{\text{noise reduction} \\ \text{effect}}} - \underbrace{M_1^{ii}}_{\substack{\text{implicit work} \\ \text{incentives}}} - \underbrace{\frac{r\left(\sigma_i^2 + h_j\sigma_j^2 - r\Sigma_1^{ij}\delta_1^{i\beta}\right)\beta_2^{i*}\psi''\left(e_1^{i*}\right)}{1 + r\Omega_1^i \psi''\left(e_1^{i*}\right)}}_{\substack{\text{own-performance} \\ \text{insurance effect}}} - \underbrace{\frac{r\Sigma_1^{ij}\left(1 - r\delta_1^{i\gamma}\right)\gamma_2^{i*}\psi''\left(e_1^{i*}\right)}{1 + r\Omega_1^i \psi''\left(e_1^{i*}\right)}}_{\substack{\text{teammate-performance} \\ \text{insurance effect}}} \tag{14}$$

$$\text{and } \gamma_1^{i*} = \underbrace{\Delta_1^i\left(\beta_1^{i*} + M_1^{ii}\right)}_{\substack{\text{compensation} \\ \text{ratio effect}}} - \underbrace{M_1^{ij}}_{\substack{\text{implicit help} \\ \text{incentives}}} - \underbrace{r\delta_1^{i\beta}\beta_2^{i*}}_{\substack{\text{own-performance} \\ \text{insurance effect}}} - \underbrace{r\delta_1^{i\gamma}\gamma_2^{i*}}_{\substack{\text{teammate-performance} \\ \text{insurance effect}}} \tag{15}$$

[29] The principal's problem in the first period is solved in Appendix ($A.4$).

where $\delta_1^{i\beta} \equiv \frac{(h_i\sigma_i^2+h_j\sigma_j^2)\psi''(a_1^{i*})-h_i^2(\sigma_i^2+h_j\sigma_j^2)\psi''(e_1^{i*})}{h_i^2[1-r\Sigma_1^{ij}\psi''(a_1^{i*})]+r\Sigma_1^{ii}\psi''(a_1^{i*})}$ and $\delta_1^{i\gamma} \equiv \frac{(h_i^2\sigma_i^2+\sigma_j^2)\psi''(a_1^{i*})-h_i^2(h_i\sigma_i^2+h_j^2\sigma_j^2)\psi''(e_1^{i*})}{h_i^2[1-r\Sigma_1^{ij}\psi''(a_1^{i*})]+r\Sigma_1^{ii}\psi''(a_1^{i*})}$ (see Appendix $(A.4)$). The coefficients $\delta_1^{i\beta}$ and $\delta_1^{i\beta}$ are positive and less than one.

We use this decomposition of the optimal explicit incentives in order to examine the underlying effects (this analysis is generalized in a T-period model, analyzed in Appendix $(A.5)$). There is the *noise reduction effect* that arises due to changes in the "amount" of available information about ability. In the next period, as the market learns more about abilities and their conditional variance decreases, we have $\Omega_1^i > \Omega_2^i$. Therefore, the optimal trade-off between incentive provision and insurance becomes better for the principal (over time), in the sense that lower risks are incurred and a higher-power explicit incentive, β_2^i, can be provided: $\beta_2^{i*} > \frac{1}{1+r\Omega_1^i\psi''(e_1^{i*})}$. Higher h_i shifts the incentive-insurance trade-off towards the former even more.

The principal also adjusts the optimal explicit incentives to account for agents' reputation incentives. Given that $M_1^{ii} > 0$, the principal imposes a lower pay-for-own performance relation when the optimal implicit incentives to work are stronger. Similarly, for high h_i so that $M_1^{ij} > 0$, γ_1^{i*} is adjusted downwards. The opposite occurs and thus the optimal γ_1^i increases when initiated interactions are weak so that $M_1^{ij} < 0$. The negative reputation incentives need to be undone by a higher γ_1^{i*}.

Risk-aversion and uncertainty about abilities also induce each agent to require insurance against low realizations of both θ^i *and* θ^j: the *own-performance* and *teammate-performance (human capital) insurance* effects arise. In particular, the principal offers a contract that insures the agents against the intertemporal risk they face. For large initiated interactions so that both β_2^{i*} and γ_2^{i*} are positive, agents incur higher risk in the second period. Thus, the principal reduces the power of the first-period incentive scheme in order to share part of this risk. Both human capital insurance effects are negative, implying that lower β_1^{i*} and γ_1^{i*} can provide insurance against low realizations of both agents' abilities. In our model, the principal provides additional insurance to agent i due to θ^j taking the form of a further reduction in β_1^{i*}.[30] However, a negative γ_2^{i*} increases γ_1^{i*}. In the second period, relative performance evaluation schemes filter out the common uncertainty, allowing for a higher γ_1^{i*} in the first period. In the absence of teamwork interactions as in Gibbons & Murphy (1992), the teammate-performance human capital insurance effect does not hold.

The compensation ratio effect reflects the relationship between the "effective" incentives - the sum of the explicit and implicit incentives - agent i is influenced by. We have $\frac{\gamma_1^{i*}+M_1^{ij}}{\beta_1^{i*}+M_1^{ii}} = \Delta_1^i$, which is positive when the initiated interactions are strong enough so that the principal finds it optimal to induce cooperation between the teammates. For low h_i so that Δ_1^i becomes negative, the principal induces competition between them.[31]

[30] The own-performance and teammate-performance human capital insurance effects can be separated because agents' project outputs are observable and relative performance evaluation schemes are used. If learning is based on an aggregate measure and wages are contingent on the team production, these two effects are merged.

[31] Assuming that long-term contracts are not feasible is equivalent to assuming that long-term contracts exist but must be Pareto efficient at each period. Gibbons & Murphy (1992) show that a sequence of (optimal) short-term contracts provides exactly the same incentives as the optimal renegotiation-proof long-term contract. This result also holds in our model where the optimal pay-for-own and pay-for-teammate incentive parameters of the renegotiation-

6 Conclusion

We examine career concerns in teams in a setting where there are interactions between the fellow members of a team and the help an agent receives depends on both her colleagues' effort and innate ability. Teamwork interactions affect the learning process and are at the heart of this analysis. By exerting effort and providing support, an agent can affect both her own and her teammate's project output. Thus, she can use both performance measures to induce the market to revise its assessment about her own ability upwards.

We show that career concerns depend on how many signals the agent can affect in order to manipulate the market's inference. In particular, we argue that if initiated interactions are substantial so that an agent's support is a key determinant of her teammate's production, the agent has incentives to work and help her colleague. By providing support, an agent can signal that she is a high-productivity agent. In contrast, if initiated interactions are weak and received interactions are intensive so that the market updates its beliefs about an agent's ability downwards when the colleague performs well, sabotage incentives arise. This happens because an agent's higher help effort increases a teammate's performance, which biases the process of inference against her. Thus, the agent has incentives to sabotage her teammate in order to signal that she is of higher ability and increase her reputation. In a stationary model where we add uncertainty into the performance measures in order for abilities to remain unknown, initiated interactions induce the agents to supply work effort above its efficient level and help effort below its efficient level. The optimal implicit incentives are distorted as long as teamwork interactions are imperfect and there is no discounting.

This model can be used to analyze reputation incentives of team workers when their individual performance is observable and depends on the quality of fellow members. This is likely to happen in research collaborations, sports or finance reams. There are also extensions and directions for future work that are of special interest, using the present model as a reference point. For instance, one can consider differences in the mean of the distribution of teammates' ability and address the question of whether a researcher has incentives to be teamed with senior or junior colleagues. We can also assume that a worker contributes to multiple projects and is teamed with different workers in each of them. We can then examine if she has incentives to work in projects where teammates' ability is more visible or in projects where teammates are of lower productivity. The size of the team with heterogeneous workers is another key determinant of career concerns. For instance, biotechnology requires large teams and may lack the ability to break up large projects into small independent modules, as is possible in the software industry.

Market conditions may also alter team workers' incentives to signal their abilities. For instance, the existence of a dominant competitor tends to align the goals of the team members, and thus sabotage incentives may be weakened or even reversed. Competition may necessitate cooperation within a heterogeneous group and reputation incentives may encourage support provision in order

proof contract are equal to $\beta_1^{i*} + M_1^{ii}$ and $\gamma_1^{i*} + M_1^{ij}$, respectively.

to ensure the success of the projects. If explicit contracts are provided, allowing for side payments between the agents as well as for different allocations of the bargaining power may boost this analysis further.

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A APPENDIX

A.1 Proof of Lemma 1: conditional distribution of abilities

The variance-covariance matrix of the multivariate normal distribution of θ^i, z_t^i and z_t^j is

$$\begin{pmatrix} \sigma_i^2 & \sigma_i^2 & h_i\sigma_i^2 \\ \sigma_i^2 & \sigma_i^2 + h_j^2\sigma_j^2 + \sigma_\varepsilon^2 & h_i\sigma_i^2 + h_j\sigma_j^2 \\ h_i\sigma_i^2 & h_i\sigma_i^2 + h_j\sigma_j^2 & \sigma_j^2 + h_i^2\sigma_i^2 + \sigma_\varepsilon^2 \end{pmatrix}.$$

Following DeGroot (1970), after the observation of z_1^i and z_1^j, the conditional mean of θ^i is

$$\begin{aligned} E\left\{\theta^i \mid z_1^i, z_1^j\right\} &= m_1^i + \sigma_{i1}'\Sigma_{i1}^{-1}\begin{pmatrix} z_1^i - \widehat{e}_1^i - m_1^i - h_j\left(\widehat{a}_1^j + m_1^j\right) \\ z_1^j - \widehat{e}_1^j - m_1^j - h_i\left(\widehat{a}_1^i + m_1^i\right) \end{pmatrix}, \\ \text{where } \sigma_{i1}' &= \begin{pmatrix} cov\left(\theta^i, z_1^i\right) & cov\left(\theta^i, z_1^j\right) \end{pmatrix} \text{ and } \Sigma_{i1} = \begin{pmatrix} var\left(z_1^i\right) & cov\left(z_1^i, z_1^j\right) \\ cov\left(z_1^i, z_1^j\right) & var\left(z_1^j\right) \end{pmatrix}. \end{aligned}$$

Thus, $\Sigma_{i1}^{-1} = \frac{1}{\lambda_1}\begin{pmatrix} var\left(z_1^j\right) & -cov\left(z_1^i, z_1^j\right) \\ -cov\left(z_1^i, z_1^j\right) & var\left(z_1^i\right) \end{pmatrix}$ where $\lambda_1 = var\left(z_1^i\right)var\left(z_1^j\right) - \left[cov\left(z_1^i, z_1^j\right)\right]^2$. The correlation coefficient matrix is

$$\sigma_{i1}'\Sigma_{i1}^{-1} = \begin{pmatrix} \rho_1^{ii} & \rho_1^{ij} \end{pmatrix} = \frac{1}{\lambda_1}\begin{pmatrix} \sigma_i^2 & h_i\sigma_i^2 \end{pmatrix}\begin{pmatrix} \sigma_j^2 + h_i^2\sigma_i^2 + \sigma_\varepsilon^2 & -\left(h_i\sigma_i^2 + h_j\sigma_j^2\right) \\ -\left(h_i\sigma_i^2 + h_j\sigma_j^2\right) & \sigma_i^2 + h_j^2\sigma_j^2 + \sigma_\varepsilon^2 \end{pmatrix},$$

where $\lambda_1 = \sigma_\varepsilon^4 + \left(1 - h_ih_j\right)^2\sigma_i^2\sigma_j^2 + \sigma_\varepsilon^2\left[\left(1 + h_i^2\right)\sigma_i^2 + \left(1 + h_j^2\right)\sigma_j^2\right]$. The conditional variance of θ^i has as

$$var\left\{\theta^i \mid z_1^i, z_1^j\right\} = var\left(\theta^i\right) - \sigma_{i1}'\Sigma_{i1}^{-1}\sigma_{i1} \text{ where } \sigma_{i1}'\Sigma_{i1}^{-1}\sigma_{i1} = \sigma_i^2\left(\rho_1^{ii} + h_i\rho_1^{ij}\right).$$

A.2 Convergence to the stationary effort levels

We first elaborate the dynamics of the learning process. The sequence of μ_t^is reveals how fast updating about agent i's ability occurs. In particular, the recursive relationship between the $\widehat{\mu}_t^i$s is $\widehat{\mu}_{t+1}^i = \frac{1}{2 + l^i\left(h^i, h^j\right) - \widehat{\mu}_t^i}$, where $l^i\left(h^i, h^j\right)$ is always positive.[32,33] Thus, $\widehat{\mu}_{t+1}^i$ is increasing in $\widehat{\mu}_t^i$. Stationarity requires $\widehat{\mu}_{t+1}^i = \widehat{\mu}_t^i = \widehat{\mu}^{i*}$, implying that $\widehat{\mu}^{i*} = 1 + \frac{l^i}{2} - \left(l^i\right)^{\frac{1}{2}}\left(1 + \frac{l^i}{4}\right)^{\frac{1}{2}}$, as in Holmström (1999). From the latter equation follows that, within the interval $(0, 1)$, there exists exactly one stationary state

[32] We have $l^i\left(h^i, h^j\right) = \frac{\left(h_i + h_j\right)^2\sigma_\varepsilon^4\sigma_j^4 v + \lambda_1^i\left(1 - h_ih_j\right)^2\sigma_\eta^4 + \sigma_\eta^2\sigma_\varepsilon^2\left[\left(1 + h_i^2\right)\left[\sigma_\varepsilon^4 + 2\left(1 - h_ih_j\right)^2\sigma_i^2\sigma_j^2\right] + \left(1 - h_j^2 - 2h_ih_j + h_i^2 + h_i^4\right)\sigma_i^2\sigma_\varepsilon^2 + \sigma_j^2\sigma_\varepsilon^2 k\right]}{\sigma_\varepsilon^4\left[\left(1 + h_j^2\right)\sigma_\eta^2 + \sigma_\varepsilon^2\right]\left[\left(1 + h_i^2\right)\sigma_i^2 + \sigma_\varepsilon^2\right] + \sigma_j^2\sigma_\varepsilon^2\left[\sigma_\varepsilon^2\left(1 + h_j^2\right)\left[\left(1 + h_j^2\right)\sigma_\eta^2 + 2\sigma_\varepsilon^2\right] + \sigma_i^2\left[\left(1 + h_j^2\right)\left(1 - h_ih_j\right)^2\sigma_\eta^2 + k\sigma_\varepsilon^2\right]\right]}$ where $v \equiv \frac{\sigma_i^2}{\lambda_1^i}\left[\left(1 + h_j^2\right)\sigma_\varepsilon^2 + \left(1 - h_ih_j\right)^2\sigma_i^2\right]$ and $k \equiv 2\left(1 - h_ih_j\right) + \left(1 + 2h_i^2\right)h_j^2 + \left(1 + 2h_j^2\right)h_i^2$.

[33] If $h_i = h_j = 0$, as in Holmström (1999), l^i is equal to $\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$. If σ_ε^2 is sufficiently larger than σ_η^2 so that l^i is close to zero, the stationary level μ^{i*} is close to 1, implying that learning occurs slowly. In contrast, if $\sigma_\varepsilon^2 = \sigma_\eta^2$, then $l^i = 1$ and updating about m_t^i occurs quickly. In our model, if $h_i = 0$ and $h_j = 1$, teamwork interactions slow down learning since $l^i < 1$, while they speed up learning if $h_i = 1$ and $h_j = 0$ since $l^i > 1$. l^i exceeds $\frac{\sigma_\eta^2}{\sigma_\varepsilon^2}$ and thus $\widehat{\mu}^{i*}$ is lower than the stationary level of this measure in Holmström's model. The updating of m_t occurs faster, diminishing the negative effects of discounting.

and thus the system is stable. If one starts with $\widehat{\mu}_1^i$ that exceeds the stationary level, $\widehat{\mu}_1^i > \widehat{\mu}^{i*}$, $\widehat{\mu}_t^i$ will converge from above to $\widehat{\mu}^{i*}$, while if one starts with $\widehat{\mu}_1^i$ that is lower than the stationary level, $\widehat{\mu}_1^i < \widehat{\mu}^{i*}$, $\widehat{\mu}_t^i$ will converge from below to $\widehat{\mu}^{i*}$.

The dynamics of agent i's work effort are revealed in equation (4). We have

$$\psi'\left(e_1^{i*}\right) = \zeta\left(1+h_i\right)\widehat{\rho}_1^{ii} + \zeta^2\left(1+h_i\right)\widehat{\rho}_1^{ii}\widehat{\mu}_2^i + \zeta^3\left(1+h_i\right)\widehat{\rho}_1^{ii}\widehat{\mu}_2^i\widehat{\mu}_3^i + \ldots \equiv T_{e_t^i}. \tag{16}$$

Each term in (16) is increasing in $\widehat{\rho}_1^{ii}$, hence so is $T_{e_t^i}$. We can show that this positive relationship holds by induction. Let $\xi_s\left(\widehat{\rho}_1^{ii}\right) \equiv \widehat{\rho}_1^{ii}\widehat{\mu}_2^i\widehat{\mu}_3^i \ldots \widehat{\mu}_s^i$, which is increasing in $\widehat{\rho}_1^{ii}$, and thus, $\xi_s\left(\widehat{\rho}_2^{ii}\right) = \widehat{\rho}_2^{ii}\widehat{\mu}_3^i\widehat{\mu}_4^i \ldots \widehat{\mu}_{s+1}^i$. We also have

$$\xi_{s+1}\left(\widehat{\rho}_1^{ii}\right) = \widehat{\rho}_1^{ii}\widehat{\mu}_2^i\widehat{\mu}_3^i \ldots \widehat{\mu}_{s+1}^i = \frac{\widehat{\rho}_1^{ii}}{\widehat{\rho}_2^{ii}}\widehat{\mu}_2^i\xi_s\left(\widehat{\rho}_2^{ii}\right).$$

The correlation coefficients $\widehat{\rho}_1^{ii}$ and $\widehat{\rho}_2^{ii}$ are both positive. By the inductive hypothesis, $\xi_s\left(.\right)$ is increasing. Note that $\widehat{\mu}_2^i = 1 - \widehat{\rho}_2^{ii} - h_i^{ii}\widehat{\rho}_2^{ij} = \frac{\sigma_\varepsilon^2}{\widehat{\lambda}_2^i}\left[\sigma_\varepsilon^2 + \left(1+h_j^2\right)\left(\widehat{\sigma}_{j,2}^2 + \sigma_\eta^2\right)\right]$. Thus, we have $\frac{1}{\widehat{\rho}_2^{ii}}\widehat{\mu}_2^i = \frac{\sigma_\varepsilon^2\left[\sigma_\varepsilon^2 + \left(1+h_j^2\right)\left(\widehat{\sigma}_{j,2}^2+\sigma_\eta^2\right)\right]}{\left(\widehat{\sigma}_{i,2}^2+\sigma_\eta^2\right)\left[\sigma_\varepsilon^2+\left(1-h_ih_j\right)\left(\widehat{\sigma}_{j,2}^2+\sigma_\eta^2\right)\right]}$, which is decreasing in μ_1^{ii} and thus increasing in $\widehat{\rho}_1^{ii}$. In turn, the positive coefficient $\frac{\widehat{\rho}_1^{ii}}{\widehat{\rho}_2^{ii}}\widehat{\mu}_2^i$ is also increasing in $\widehat{\rho}_1^{ii}$. It follows that $\xi_{s+1}\left(\widehat{\rho}_1^{ii}\right)$ and thus $\psi'\left(e_1^{i*}\right)$ are also increasing as functions of $\widehat{\rho}_1^{ii}$. It boils down to the following: if $\left\{\widehat{\rho}_t^{ii}\right\}$ is an increasing (decreasing) sequence, $\left\{T_{e_t^i}\right\}$ is also an increasing (decreasing) sequence. The sequence of agent i's optimal work effort $\{e_t^{i*}\}$ converges monotonically to the stationary state level e^{i*}. If $\left\{\widehat{\rho}_t^{ii}\right\}$ is an increasing sequence, the convergence of $\{e_t^{i*}\}$ is from below. Similarly, if $\left\{\widehat{\rho}_t^{ii}\right\}$ is a decreasing sequence, the convergence of $\{e_t^{i*}\}$ is from above.

The dynamics of agent i's help effort follow by studying equation (5). We have

$$\psi'\left(a_1^{i*}\right) = \zeta\left(1+h_i\right)\widehat{\rho}_1^{ij} + \zeta^2\left(1+h_i\right)\widehat{\rho}_1^{ij}\widehat{\mu}_2^i + \zeta^3\left(1+h_i\right)\widehat{\rho}_1^{ij}\widehat{\mu}_2^i\widehat{\mu}_3^i + \ldots \equiv T_{a_t^i}. \tag{17}$$

The sign of each term in equation (17) is the same as the sign of $\widehat{\rho}_1^{ij}$. First, we will examine the convergence to the stationary level of help effort when this is positive. If each term in (17) is increasing in (the positive) $\widehat{\rho}_1^{ij}$, the same is true for $T_{a_t^i}$. As above, we prove this statement by induction. Suppose $\xi_s\left(\widehat{\rho}_1^{ij}\right) = \widehat{\rho}_1^{ij}\widehat{\mu}_2^i\widehat{\mu}_3^i \ldots \widehat{\mu}_s^i$, which implies $\xi_s\left(\widehat{\rho}_2^{ij}\right) = \widehat{\rho}_2^{ij}\widehat{\mu}_3^i\widehat{\mu}_4^i \ldots \widehat{\mu}_{s+1}^i$. These sequences give

$$\xi_{s+1}\left(\widehat{\rho}_1^{ij}\right) = \widehat{\rho}_1^{ij}\widehat{\mu}_2^i\widehat{\mu}_3^i \ldots \widehat{\mu}_{s+1}^i = \frac{\widehat{\rho}_1^{ij}}{\widehat{\rho}_2^{ij}}\widehat{\mu}_2^i\xi_s\left(\widehat{\rho}_2^{ij}\right).$$

We have $\frac{1}{\widehat{\rho}_2^{ij}}\widehat{\mu}_2^i = \frac{\sigma_\varepsilon^2\left[\sigma_\varepsilon^2+\left(1+h_j^2\right)\left(\widehat{\sigma}_{j,2}^2+\sigma_\eta^2\right)\right]}{\left(\widehat{\sigma}_{i,2}^2+\sigma_\eta^2\right)\left[h_i\sigma_\varepsilon^2-\left(1-h_ih_j\right)h_j\left(\widehat{\sigma}_{j,2}^2+\sigma_\eta^2\right)\right]}$, which is increasing in $\widehat{\rho}_1^{ij}$. It follows that the coefficient $\frac{\widehat{\rho}_1^{ij}}{\widehat{\rho}_2^{ii}}\widehat{\mu}_2^i$, the product $\xi_{s+1}\left(\widehat{\rho}_1^{ij}\right)$ and thus the optimal help effort a_1^{i*} are also increasing in $\widehat{\rho}_1^{ij}$. Therefore, if $\left\{\widehat{\rho}_t^{ij}\right\}$ is a positive and increasing sequence, $\left\{T_{a_t^i}\right\}$ is also an increasing sequence. The convergence of agent i's optimal help effort $\{a_t^{i*}\}$ to the stationary state level a^{i*} will be monotonically from below. If $\left\{\widehat{\rho}_t^{ij}\right\}$ is a decreasing sequence, the convergence of $\{a_t^{i*}\}$ is from above.

We can perform the same analysis to examine the convergence to the stationary level of help effort when this is negative. Now, $\xi_{s+1}\left(\widehat{\rho}_1^{ij}\right)$ and $\xi_s\left(\widehat{\rho}_2^{ij}\right)$ are also negative. However, the coefficient $\frac{\widehat{\rho}_1^{ij}}{\widehat{\rho}_2^{ij}}\mu_2^i$ is positive and increasing in the negative $\widehat{\rho}_1^{ij}$. This coefficient reinforces the dynamics of the negative sequence of $\xi_s(.)$. If $\left\{\widehat{\rho}_t^{ij}\right\}$ is a negative but increasing sequence ($\frac{\widehat{\rho}_1^{ij}}{\widehat{\rho}_2^{ij}}\widehat{\mu}_2^i$ becomes smaller), hence so is $\left\{T_{a_t^i}\right\}$, and the convergence of $\{a_t^{i*}\}$ is from below. If $\left\{\widehat{\rho}_t^{ij}\right\}$ is a negative and decreasing sequence, $\{a_t^{i*}\}$ converges from above to the stationary state.

A.3 Second period explicit incentives

In the second period, given (8), the principal maximizes

$$\begin{aligned} L_2 &= \sum_{i=1}^{2} E\left\{z_2^i - \omega_2^i - \beta_2^i z_2^i - \gamma_2^i z_2^j \mid z_1^i, z_1^j\right\} + \sum_{i=1}^{2} \lambda_i \left\{\beta_2^i - \psi'\left(e_2^i\right)\right\} + \sum_{i=1}^{2} \mu_i \left\{h_i\gamma_2^i - \psi'\left(a_2^i\right)\right\} \\ &\quad + \sum_{i=1}^{2} \xi_i \left\{E\left\{\omega_2^i + \beta_2^i z_2^i + \gamma_2^i z_2^j \mid z_1^i, z_1^j\right\} - \psi\left(e_2^i\right) - \psi\left(a_2^i\right) - \frac{r}{2}Var\left\{w_2^i \mid z_1^i, z_1^j\right\} - \widetilde{\theta}_2^i\right\}. \end{aligned}$$

Omitting details, the Kuhn-Tucker condition with respect to α_2^i gives $-1+\omega_i = 0 \Leftrightarrow \omega_i = 1$, implying that the IR_2^i constraint binds at the optimum and the base payment is given by (9). Agent i accepts a contract that allows her to earn her reputational bonus. Thus, the Kuhn-Tucker conditions become

$$\begin{aligned} \frac{\partial L_2}{\partial \lambda_i} &= \beta_2^i - \psi'\left(e_2^i\right) \geq 0 \text{ or } \lambda_i \geq 0,\ \lambda_i\left[\beta_2^i - \psi'\left(e_2^i\right)\right] = 0, \forall i \\ \frac{\partial L_2}{\partial \mu_i} &= h_i\gamma_2^i - \psi'\left(a_2^i\right) \geq 0 \text{ or } \mu_i \geq 0,\ \mu_i\left[h_i\gamma_2^i - \psi'\left(e_2^i\right)\right] = 0, \forall i \\ \frac{\partial L_2}{\partial \beta_2^i} &= -r\left\{\left(\beta_2^i + h_i\gamma_2^i\right)\sigma_{i,2}^2 + \left(h_j\beta_2^i + \gamma_2^i\right)h_j\sigma_{j,2}^2 + \beta_2^i\sigma_\varepsilon^2\right\} + \lambda_i \leq 0 \text{ or } \beta_2^i \geq 0,\ \frac{\partial L_2}{\partial \beta_2^i}\beta_2^i = 0, \forall i \\ \frac{\partial L_2}{\partial \gamma_2^i} &= -r\left\{\left(\beta_2^i + h_i\gamma_2^i\right)h_j\sigma_{i,2}^2 + \left(h_j\beta_2^i + \gamma_2^i\right)\sigma_{j,2}^2 + \gamma_2^i\sigma_\varepsilon^2\right\} + h_i\mu_i \leq 0 \text{ or } \gamma_2^i \geq 0,\ \frac{\partial L_2}{\partial \gamma_2^i}\gamma_2^i = 0, \forall i \\ \frac{\partial L_2}{\partial e_2^i} &= 1 - \lambda_i\psi''\left(e_2^i\right) - \psi'\left(e_2^i\right) \leq 0 \text{ or } e_2^i \geq 0,\ \frac{\partial L_2}{\partial e_2^i}e_2^i = 0, \forall i \\ \frac{\partial L_2}{\partial a_2^i} &= h_i - \mu_i\psi''\left(a_2^i\right) - \psi'\left(a_2^i\right) \leq 0 \text{ or } a_2^i \geq 0,\ \frac{\partial L_2}{\partial a_2^i}a_2^i = 0, \forall i \end{aligned}$$

We have $\lambda_i = \frac{1-\psi'\left(e_2^i\right)}{\psi''\left(e_2^i\right)}$ and $\mu_i = \frac{h_i-\psi'\left(a_2^i\right)}{\psi''\left(a_2^i\right)}$. Given also the equations in (8), the conditions with respect to β_2^i and γ_2^i imply

$$\frac{\left(\beta_2^i + h_i\gamma_2^i\right)\sigma_{i,2}^2 + \left(h_j\beta_2^i + \gamma_2^i\right)h_j\sigma_{j,2}^2 + \beta_2^i\sigma_\varepsilon^2}{\left(\beta_2^i + h_i\gamma_2^i\right)h_i\sigma_{i,2}^2 + \left(h_j\beta_2^i + \gamma_2^i\right)\sigma_{j,2}^2 + \gamma_2^i\sigma_\varepsilon^2} = \frac{\left(1-\beta_2^i\right)\psi''\left(a_2^i\right)}{h_i^2\left(1-\gamma_2^i\right)\psi''\left(e_2^i\right)}.$$

Solving with respect to γ_2^{i*}, we obtain the optimal pay-for-teammate performance parameter γ_2^{i*} (equation (11)). Thus, by the condition with respect to β_2^i, we get

$$\lambda_i = r\left\{\left(1 + h_i\Delta_2^i\right)\sigma_{i,2}^2 + \left(h_j + \Delta_2^i\right)h_j\sigma_{j,2}^2 + \sigma_\varepsilon^2\right\}\beta_2^i. \tag{18}$$

The optimal β_2^i (equation (11)) is obtained by substituting λ_i (equation (18)) into the condition with respect to e_2^i.

A.4 First period explicit incentives

To find the optimal contractual parameters in period 1, we first need to derive the form of

$$Var\left\{\widetilde{w}_1^i + w_2^i\right\} = Var\left\{\widetilde{w}_1^i\right\} + Var\left\{w_2^i\right\} + 2Cov\left\{\widetilde{w}_1^i, w_2^i\right\}, \tag{19}$$

where

$$Var\left\{\widetilde{w}_1^i\right\} = Var\left\{w_1^i\right\} + Var\left\{\omega_2^{i*}\right\} + 2Cov\left\{w_1^i, \omega_2^{i*}\right\}. \tag{20}$$

The variance of the first-period wage is given by

$$Var\left\{w_1^i\right\} = \left(\beta_1^i + h_i\gamma_1^i\right)^2 \sigma_i^2 + \left(h_j\beta_1^i + \gamma_1^i\right)^2 \sigma_j^2 + \left[\left(\beta_1^i\right)^2 + \left(\gamma_1^i\right)^2\right]\sigma_\varepsilon^2. \tag{21}$$

Note that

$$\widetilde{\theta}_t^i - E\left\{\beta_2^i z_2^i + \gamma_2^i z_2^j \mid z_1^i, z_1^j\right\} =$$

$$\begin{aligned}
&= E\left\{\left(1 + h_i - \beta_2^i - h_i\gamma_2^i\right)\theta^i - \left(h_j\beta_2^i + \gamma_2^i\right)\theta^j + \left(1 - \beta_2^i\right)e_2^i + h_i\left(1 - \gamma_2^i\right)a_2^i - h_j\beta_2^i a_2^j - \gamma_2^i e_2^j \mid z_1^i, z_1^j\right\} \\
&= M_1^{ii}\left(z_1^i - \widehat{e}_1^i - h_j\widehat{a}_1^j\right) + M_1^{ij}\left(z_1^j - \widehat{e}_1^j - h_i\widehat{a}_1^i\right) + \left(1 - \beta_2^i\right)\widehat{e}_2^i + h_i\left(1 - \gamma_2^i\right)\widehat{a}_2^i - h_j\beta_2^i\widehat{a}_2^j - \gamma_2^i\widehat{e}_2^j,
\end{aligned}$$

where M_1^{ii} and M_1^{ij} are given by (13). Thus, the variance of $\omega_2^i\left(\beta_2^{i*}, \gamma_2^{i*}\right)$ is

$$Var\left\{\omega_2^{i*}\right\} = \left(M_1^{ii} + h_i M_1^{ij}\right)^2 \sigma_i^2 + \left(h_j M_1^{ii} + M_1^{ij}\right)^2 \sigma_j^2 + \left[\left(M_1^{ii}\right)^2 + \left(M_1^{ij}\right)^2\right]\sigma_\varepsilon^2. \tag{22}$$

We also have

$$\begin{aligned}
Cov\left\{w_1^i, \omega_2^{i*}\right\} &= \left(\beta_1^i + h_i\gamma_1^i\right)\left(M_1^{ii} + h_i M_1^{ij}\right)\sigma_i^2 + \left(h_j\beta_1^i + \gamma_1^i\right)\left(h_j M_1^{ii} + M_1^{ij}\right)\sigma_j^2 \\
&\quad + \left[\beta_1^i M_1^{ii} + \gamma_1^i M_1^{ij}\right]\sigma_\varepsilon^2.
\end{aligned} \tag{23}$$

To obtain the variance of $\widetilde{w}_1^i$, let $B_1^i \equiv \beta_1^i + M_1^{ii}$ and $\Gamma_1^i \equiv \gamma_1^i + M_1^{ij}$ be the effective pay-for-own and pay-for-teammate performance parameters. Then, by (21), (22) and (23), the variance in (20) becomes

$$Var\left\{\widetilde{w}_1^i\right\} = \left(B_1^i + h_i\Gamma_1^i\right)^2 \sigma_i^2 + \left(h_j B_1^i + \Gamma_1^i\right)^2 \sigma_j^2 + \left[\left(B_1^i\right)^2 + \left(\Gamma_1^i\right)^2\right]\sigma_\varepsilon^2,$$

and the covariance of $\widetilde{w}_1^i$ and w_2^i is

$$Cov\left\{\widetilde{w}_1^i, w_2^i\right\} = \left(\beta_2^i + h_i\gamma_2^i\right)\left(B_1^i + h_i\Gamma_1^i\right)\sigma_i^2 + \left(h_j\beta_2^i + \gamma_2^i\right)\left(h_j B_1^i + \Gamma_1^i\right)\sigma_j^2.$$

Therefore, equation (19) takes the form

$$\begin{aligned}
Var\left\{\widetilde{w}_1^i + w_2^i\right\} &= \left[B_1^i + \beta_2^i + h_i\left(\Gamma_1^i + \gamma_2^i\right)\right]^2 \sigma_i^2 + \left[h_j\left(B_1^i + \beta_2^i\right) + \Gamma_1^i + \gamma_2^i\right]^2 \sigma_j^2 \\
&\quad + \left[\left(B_1^i\right)^2 + \left(\Gamma_1^i\right)^2 + \left(\beta_2^i\right)^2 + \left(\gamma_2^i\right)^2\right]\sigma_\varepsilon^2.
\end{aligned}$$

In period 1, the certainty equivalent of agent i's utility is given by equation (12). Provided that the IR_i constraint binds and $\widetilde{\theta}_1^i$ is zero, the first-period base payment is

$$\omega_1^i = -E\left\{\beta_1^i z_1^i + \gamma_1^i z_1^j\right\} + \psi\left(e_1^i\right) + \psi\left(a_1^i\right) - E\left\{w_2^{i*}\right\} + \psi\left(e_2^{i*}\right) + \psi\left(a_2^{i*}\right) + \frac{r}{2} Var\left\{\widetilde{w}_1^i + w_2^i\right\}.$$

We take the Kuhn-Tucker conditions as in subsection $(A.3)$ and solve the equations:

$$\begin{aligned}
\left[B_1^i + \beta_2^i + h_i\left(\Gamma_1^i + \gamma_2^i\right)\right]\sigma_i^2 + \left[h_j\left(B_1^i + \beta_2^i\right) + \Gamma_1^i + \gamma_2^i\right]h_j\sigma_j^2 + B_1^i\sigma_\varepsilon^2 &= \frac{\lambda_i}{r} \\
\left[B_1^i + \beta_2^i + h_i\left(\Gamma_1^i + \gamma_2^i\right)\right]h_i\sigma_i^2 + \left[h_j\left(B_1^i + \beta_2^i\right) + \Gamma_1^i + \gamma_2^i\right]\sigma_j^2 + \Gamma_1^i\sigma_\varepsilon^2 &= \frac{h_i\mu_i}{r}
\end{aligned}$$

$$\begin{aligned}
1 - \lambda_i\psi''\left(e_2^i\right) - B_1^i &= 0 \\
h_i - \mu_i\psi''\left(a_2^i\right) - h_i\Gamma_1^i &= 0
\end{aligned}$$

These equations imply the optimal β_1^{i*} and γ_1^{i*}, given by equations (14) and (15).

A.5 The multi-period model

Given the history of realized outputs, $Z_{t-1}^i \equiv \left(z_1^i, ..., z_{t-1}^i\right)$ and $Z_{t-1}^j \equiv \left(z_1^j, ..., z_{t-1}^j\right)$, the conditional expectation of agent i's project output in period t is

$$E\left\{z_t^i \mid Z_{t-1}^i, Z_{t-1}^j\right\} = m_{t-1}^i + \widehat{e}_t^i + h_j\left(m_{t-1}^j + \widehat{a}_t^j\right),$$

where m_{t-1}^i is exactly the market's expectation of agent i's ability as of the beginning of period t. After the observation of z_{t-1}^i and z_{t-1}^j, the signals that are used by the market to form its beliefs about agent i's ability become

$$\rho_{t-1}^{ii} = \frac{\sigma_i^2}{\lambda_{t-1}}\left[\sigma_\varepsilon^2 + (t-1)\left(1 - h_ih_j\right)\sigma_j^2\right] \text{ and } \rho_{t-1}^{ij} = \frac{\sigma_i^2}{\lambda_{t-1}}\left[h_i\sigma_\varepsilon^2 - (t-1)\left(1 - h_ih_j\right)h_j\sigma_j^2\right],$$

where

$$\lambda_{t-1} = \sigma_\varepsilon^4 + (\tau-1)^2\left(1 - h_ih_j\right)^2\sigma_i^2\sigma_j^2 + (\tau-1)\sigma_\varepsilon^2\left[\left(1 + h_i^2\right)\sigma_i^2 + \left(1 + h_j^2\right)\sigma_j^2\right].$$

Given also a sequence of contracts with pay-for-performance parameters $\left(\beta_1^i, \gamma_1^i, ..., \beta_T^i, \gamma_T^i\right)$, agent i maximizes

$$CE^i = \sum_{t=1}^{T}\left[E\left\{w_t^i \mid Z_{t-1}^i, Z_{t-1}^j\right\} - \psi\left(e_t^i\right) - \psi\left(a_t^i\right)\right] - \frac{r}{2} var\left\{\sum_{t=1}^{T} w_t^i \mid Z_{t-1}^i, Z_{t-1}^j\right\}.$$

Thus, agent i's optimal work and help effort will satisfy, respectively,

$$\beta_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ii} = \psi'\left(e_t^{i*}\right) \text{ and } h_i\left(\gamma_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ij}\right) = \psi'\left(a_t^{i*}\right), \tag{24}$$

$$\text{where} \quad \begin{aligned}
M_\tau^{ii} &= \left(1 + h_i\right)\rho_{\tau-1}^{ii} - \beta_\tau^{i*}\left(\rho_{\tau-1}^{ii} + h_j\rho_{\tau-1}^{ji}\right) - \gamma_\tau^{i*}\left(h_i\rho_{\tau-1}^{ii} + \rho_{\tau-1}^{ji}\right), \\
M_\tau^{ij} &= \left(1 + h_i\right)\rho_{\tau-1}^{ij} - \beta_\tau^{i*}\left(\rho_{\tau-1}^{ij} + h_j\rho_{\tau-1}^{jj}\right) - \gamma_\tau^{i*}\left(h_i\rho_{\tau-1}^{ij} + \rho_{\tau-1}^{jj}\right).
\end{aligned} \tag{25}$$

Letting $B_t^i \equiv \beta_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ii}$ and $\Gamma_t^i \equiv \gamma_t^i + \sum_{\tau=t+1}^{T} M_\tau^{ij}$, the variance of the wage is given by

$$var\left\{\sum_{\tau=t}^{T} w_\tau^i\right\} = \left(\sum_{\tau=t}^{T}\left\{B_\tau^i + h_j\Gamma_\tau^i\right\}\right)^2 \sigma_i^2 + \left(\sum_{\tau=t}^{T}\left\{h_j B_\tau^i + \Gamma_\tau^i\right\}\right)^2 \sigma_j^2 + \sum_{\tau=t}^{T}\left\{\left(B_\tau^i\right)^2 + \left(\Gamma_\tau^i\right)^2\right\}\sigma_\varepsilon^2.$$

The Kuhn-Tucker conditions with respect to β_t^{i*} and γ_t^{i*} give

$$\begin{aligned} \sigma_{i,t}^2 \sum_{\tau=t}^{T}\left\{B_\tau^i + h_i\Gamma_\tau^i\right\} + h_j\sigma_{j,t}^2 \sum_{\tau=t}^{T}\left\{h_j B_\tau^i + \Gamma_\tau^i\right\} + B_t^i\sigma_\varepsilon^2 &= \frac{\lambda_i}{r}, \\ h_i\sigma_{i,t}^2 \sum_{\tau=t}^{T}\left\{B_\tau^i + h_i\Gamma_\tau^i\right\} + \sigma_{j,t}^2 \sum_{\tau=t}^{T}\left\{h_j B_\tau^i + \Gamma_\tau^i\right\} + \Gamma_t^i\sigma_\varepsilon^2 &= \frac{h_i\mu_i}{r}, \end{aligned}$$

where $\sigma_{i,t}^2 = \left[1-(t-1)\left(\rho_{\tau-1}^{ii} + h_i\rho_{\tau-1}^{ij}\right)\right]\sigma_i^2$. Let also $\Delta_t^i \equiv \frac{h_i^2\left[1+r\Sigma_t^{jj}\psi''\left(e_t^i\right)\right]-r\Sigma_t^{ij}\psi''\left(a_t^i\right)}{r\Sigma_t^{ii}\psi''\left(a_t^i\right)+h_i^2\left[1-r\Sigma_t^{ij}\psi''\left(e_t^i\right)\right]}$ and $\Omega_t^i \equiv \left[\sigma_\varepsilon^2 + \left(1+h_i\Delta_t^i\right)\sigma_{i,t}^2 + \left(h_j+\Delta_t^i\right)h_j\sigma_{j,t}^2\right]$ where $\Sigma_t^{ii} \equiv \sigma_\varepsilon^2 + h_i^2\sigma_{i,t}^2 + \sigma_{j,t}^2$ and $\Sigma_t^{ij} \equiv h_i\sigma_{i,t}^2 + h_j\sigma_{j,t}^2$. We solve with respect to γ_t^{i*} and obtain

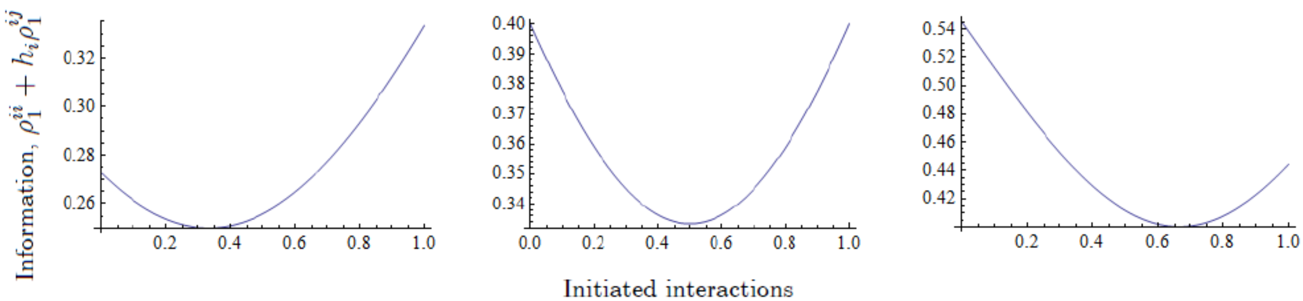
$$\gamma_t^{i*} = \Delta_t^i\left(\beta_t^{i*} + \sum_{\tau=t}^{T} M_\tau^{ii}\right) - \sum_{\tau=t}^{T} M_\tau^{ij} - r\sum_{\tau=t+1}^{T}\delta_\tau^{i\beta}B_\tau^i - r\sum_{\tau=t+1}^{T}\delta_\tau^{i\gamma}\Gamma_\tau^i$$

where $\delta_t^{i\beta} \equiv \frac{\left(h_i\sigma_{i,t}^2+h_j\sigma_{j,t}^2\right)\psi''\left(a_t^{i*}\right)-h_i^2\left(\sigma_{i,t}^2+h_j\sigma_{j,t}^2\right)\psi''\left(e_t^{i*}\right)}{h_i^2\left[1-r\Sigma_t^{ij}\psi''\left(a_t^{i*}\right)\right]+r\Sigma_t^{ii}\psi''\left(a_t^{i*}\right)}$ and $\delta_t^{i\gamma} \equiv \frac{\left(h_i^2\sigma_{i,t}^2+\sigma_{j,t}^2\right)\psi''\left(a_t^{i*}\right)-h_i^2\left(h_i\sigma_{i,t}^2+h_j^2\sigma_{j,t}^2\right)\psi''\left(e_t^{i*}\right)}{h_i^2\left[1-r\Sigma_1^{ij}\psi''\left(a_t^{i*}\right)\right]+r\Sigma_1^{ii}\psi''\left(a_t^{i*}\right)}$. We also derive

$$\begin{aligned} \beta_t^{i*} &= \frac{1}{1+r\Omega_t^i\psi''\left(e_t^{i*}\right)} - \sum_{\tau=t+1}^{T} M_\tau^{ii} - \frac{r\left(\sigma_{i,t}^2 + h_j\sigma_{j,t}^2 - r\Sigma_t^{ij}\delta_t^{i\beta}\right)\sum_{\tau=t+1}^{T}\beta_\tau^{i*}\psi''\left(e_t^{i*}\right)}{1+r\Omega_t^i\psi''\left(e_t^{i*}\right)} \\ &\quad - \frac{r\Sigma_t^{ij}\left(1-r\delta_t^{i\gamma}\right)\sum_{\tau=t+1}^{T}\gamma_\tau^{i*}\psi''\left(e_t^{i*}\right)}{1+r\Omega_t^i\psi''\left(e_t^{i*}\right)}. \end{aligned}$$

We can derive the contractual parameters of the contracts that are offered in each period. These optimal parameters change monotonically with t. They indicate that greater motivation is provided explicitly over the periods, while reputation incentives decrease for those about to retire.

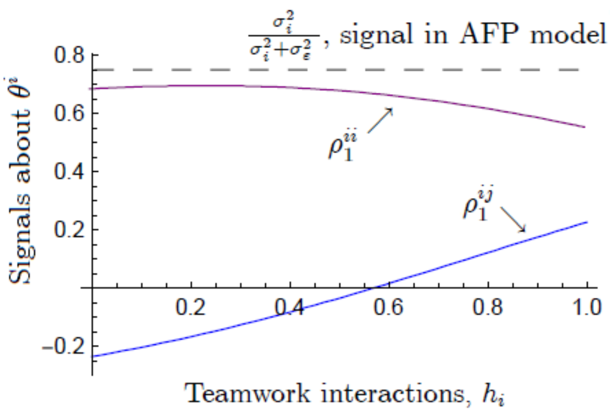
Figures 1. Effect of h_i on learning, captured by $\rho_1^{ii} + h_i\rho_1^{ij}$.



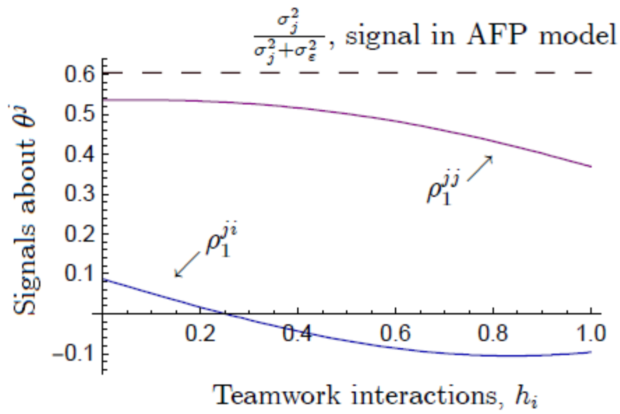
$\sigma_i^2 = \sigma_j^2 = 2,\ \sigma_\varepsilon^2 = 4$ $\quad$ $\sigma_i^2 = \sigma_j^2 = 4,\ \sigma_\varepsilon^2 = 4$ $\quad$ $\sigma_i^2 = \sigma_j^2 = 4,\ \sigma_\varepsilon^2 = 2$

Figures 1 show the changes in $\rho_1^{ii} + h_i\rho_1^{ij}$ as h_i increases under different assumptions about the level of σ_i^2, σ_j^2 and σ_ε^2. In all three figures, it is also assumed that $h_j = 1$.

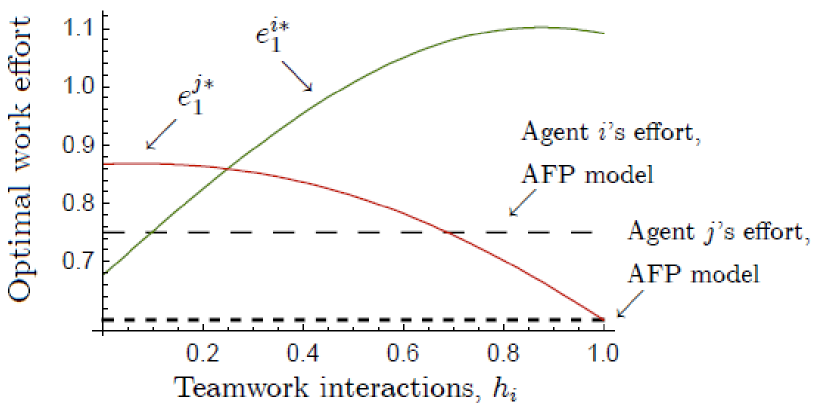
Figures 2. Effect of h_i on the signals about abilities and teammates' optimal efforts.



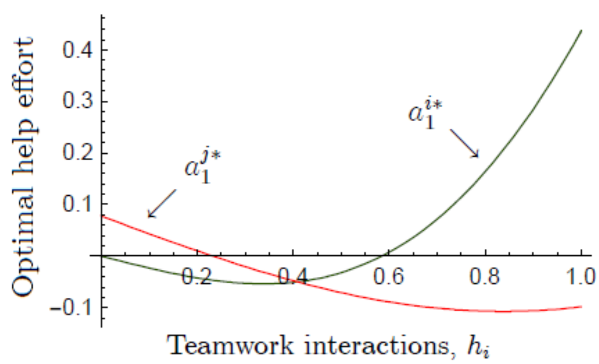
2.1



2.2



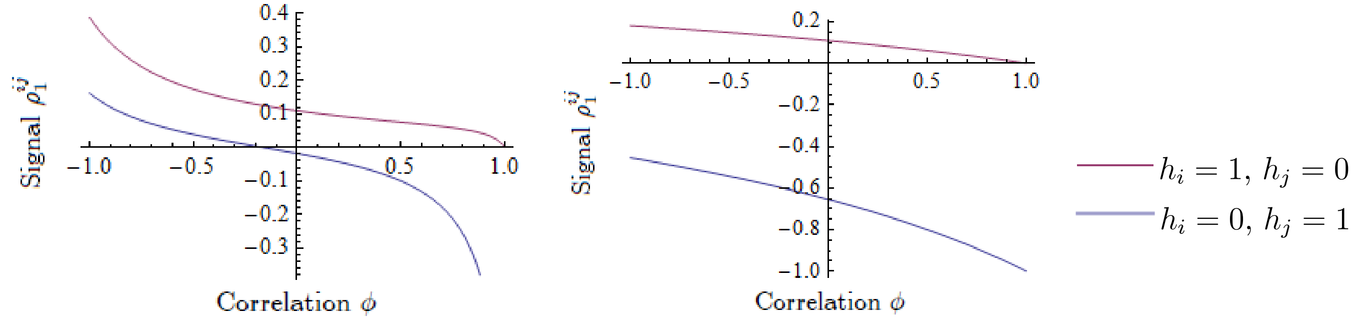
2.3



2.4

Figures 2.1 and 2.2 show the effect of teamwork interactions h_i on the signals about θ^i and θ^j, respectively, under the assumptions that $\sigma_i^2 = 6$, $\sigma_j^2 = 3$, $\sigma_\varepsilon^2 = 2$ and $h_j = 0.6$. In Auriol et al. (2002) and Holmström (1999), $corr\left(\theta^i \mid z_1^i\right) = \frac{\sigma_i^2}{\sigma_\varepsilon^2+\sigma_i^2}$ and $corr\left(\theta^j \mid z_1^j\right) = \frac{\sigma_j^2}{\sigma_\varepsilon^2+\sigma_j^2}$, while $corr\left(\theta^i \mid z_1^j\right) = corr\left(\theta^j \mid z_1^i\right) = 0$. Figures 2.3 and 2.4 show how the optimal work efforts and help efforts change with h_i. We assume that $\psi\left(e_t^i\right) = \frac{1}{2}\left(e_t^i\right)^2$and $\psi\left(a_t^i\right) = \frac{1}{2}\left(a_t^i\right)^2$.

Figures 3. Effect of the correlation of the random terms, ϕ, on the signal ρ_1^{ij}.



$\sigma_i^2 = \sigma_j^2 = 1$, $\sigma_\varepsilon^2 = 6$ $\qquad$ $\sigma_i^2 = \sigma_j^2 = 6$, $\sigma_\varepsilon^2 = 1$

Figures 3 show the changes in the sign of ρ_1^{ij} as ϕ increases, under certain assumptions about σ_i^2, σ_j^2, σ_ε^2, h_i and h_j.