

Shopping and Pricing on Online Marketplaces*

Michael Choi Yifan Dai Kyungmin Kim[†]

University of Iowa

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Abstract

We present an oligopoly model that captures some salient features of online marketplaces. Consumers engage in non-stationary sequential search based on partial product information and advertised prices. We characterize consumers' optimal shopping behavior and study its implications for price competition among the sellers. Under some regularity assumptions, we establish the existence and uniqueness of market equilibrium. We then study how equilibrium prices are influenced by the market environment. Among others, we show that a reduction in search costs increases market prices, whereas providing better product information before consumer search may or may not increase market prices.

JEL Classification Numbers: D43, D83, L13.

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1 Introduction

Shopping on online marketplaces, which is already in our everyday lives, typically takes place in the following sequence. A consumer either searches with a specific query or chooses

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[†]Choi: yufai-choi@uiowa.edu, Dai: yifan-dai-1@uiowa.edu, Kim: teddy.kyungmin.kim@gmail.com

an appropriate category. The website displays a list of suitable products, with a brief description of each product, such as its image, manufacturer, model name, and price. The consumer clicks an item and obtains more detailed information about the product. There are more images and a more complete product description. Customer reviews are also often available. The consumer either purchases the product or checks other products. This process continues until the consumer either purchases a particular product or leaves the website. Most consumers check multiple products but rarely examine all offered products,¹ which suggests that *search costs* are significant even on online marketplaces.

We present a market model that captures these salient features of online shopping. In other words, we develop a market structure which particularly well represents market interactions on online platforms. There are a finite number of sellers and a large number of consumers. Each seller posts his price.² Consumers have different tastes for the products (horizontal product differentiation) but do not possess full information about their values for the products. A consumer needs to visit a seller (i.e., clicks an item and reads its full description) in order to fully gauge her value for the product. Each consumer purchases the best product among the ones she has examined or leaves the market at any point in the process. Note that, although the model describes consumer experiences on online marketplaces particularly well, it is also applicable to traditional markets with active price advertisements.

Our model is closely related to two strands of the literature on oligopoly market structures. The first strand studies Bertrand competition under product differentiation. In particular, our model adopts a taste-based framework by Perloff and Salop (1985) and extends it to accommodate search aspects of online shopping. Indeed, our model reduces to that of Perloff and Salop (1985) if consumers face either no search costs (in which case they visit all the sellers) or prohibitively high search costs (in which case they visit at most one seller).³

¹For some concrete evidence, see, e.g., Kim, Albuquerque and Bronnenberg (2010) and Dinerstein, Einav, Levin and Sundaresan (2014).

²We restrict attention to the posted price selling mechanism. This is not only for analytical tractability, but also because it is the dominant selling mechanism on online marketplaces. Even on eBay, which is a leading platform for retail auctions in the U.S., most sales occur through posted prices (see, e.g., Einav, Farronato, Levin and Sundaresan, 2013).

³We contribute to the literature itself by answering some open questions in Perloff and Salop (1985). In particular, we incorporate consumers' outside options (which were assumed away in Perloff and Salop (1985) for the sake of tractability), provide a technique to establish the existence and uniqueness of equilibrium (which have been partially completed in Perloff and Salop (1985)), and show that dispersive order is an appropriate measure for preference diversity (Perloff and Salop (1985) only showed that mean-preserving spreads have

The second strand introduces consumer search problems into oligopoly settings. Our main departure from this large literature is our assumption that prices are observable to consumers before search. In other words, in most existing studies, consumers have imperfect information about prices and, therefore, mainly search for a better price, while in our model, they search only to collect more precise information about product values, knowing all offered prices.⁴ As demonstrated in what follows, this difference not only requires a distinct equilibrium analysis, but also leads to quite different comparative statics results.

We first solve for consumers' optimal shopping strategies. A consumer's shopping problem can be interpreted as a non-stationary sequential search problem. Based on her prior information about the products and advertised prices, she decides in which order to visit the sellers. In addition, after each visit, she decides whether to continue to search or stop and purchase from any visited seller. We employ an elegant solution by Weitzman (1979), who considered a more general class of non-stationary search problems, and provide a complete description of consumers' optimal shopping behavior.⁵

We then consider the pricing problem of the sellers. Prices affect demands through their influence on consumers' search behavior. The dynamic and non-stationary nature of consumers' search behavior complicates the derivation of demand functions. Nevertheless, we show that the structure of our model allows us to precisely predict each consumer's eventual purchase decision and, therefore, summarize consumers' shopping outcomes in a simple fashion. This enables us to interpret the pricing game as a familiar discrete choice model and, therefore, apply canonical techniques to establish the existence and uniqueness of equilibrium and characterize equilibrium market prices.

We also study how equilibrium market prices are influenced by various market factors. We begin by examining some familiar ideas in the literature. We demonstrate that the conventional wisdom that prices are lower in more competitive environments holds in our en-

ambiguous effects on market prices).

⁴We provide a detailed discussion on one exception regarding the price-observability assumption, Armstrong and Zhou (2011), at the end of the introduction. A few papers consider the case in which consumers search for both prices and match values (that is, if a consumer visits a seller, then she observes both the seller's price and her idiosyncratic value for the product). See, e.g., Wolinsky (1986), Anderson and Renault (1999), Armstrong, Vickers and Zhou (2009), and Chen and He (2011).

⁵Choi and Smith (2015) considered a more restricted class of optimal search problems, which still encompasses the optimal shopping problem in our model. Both Weitzman (1979) and Choi and Smith (2015) focused on optimal search behavior, which our main focus is its implications for price competition among the sellers. In other words, our model can be interpreted as an equilibrium model of those two studies.

vironment. Specifically, we show that market prices decrease as the number of sellers increases (inside competition) or consumers' outside option improves (outside competition).⁶ In addition, we show that market prices increase as consumers' preferences become more dispersive. Importantly, our analysis reveals that *dispersive order* is an appropriate measure in the Perloff-Salop framework, thereby unifying several dispersed studies in the literature.⁷

We provide two particularly intriguing results. It was recognized early on that the Internet can dramatically reduce market frictions and, therefore, should deliver more efficient market outcomes, by transforming traditional businesses as well as creating many new markets. This promise has been fulfilled in various ways by now, but several phenomena that are at odds with it still persist. In particular, it has been repeatedly reported that the Internet has not significantly lowered markups and reduced price dispersion (see, e.g., Ellison and Ellison, 2005; Baye, Morgan and Scholten, 2006) These suggest that search frictions are significant even in online markets and cast doubt on the conventional wisdom that a reduction in search frictions is necessarily beneficial to consumers. The following two results of ours provide new insights on these fundamental issues.

We find that market prices increase as search costs decrease in our model. This is exactly opposite to the standard result in the literature. As search costs increase, a consumer is less likely to leave for another seller and, therefore, more likely to purchase from the first seller. The sellers then have an incentive to extract more from visiting consumers and, therefore, charge higher prices. This is the main mechanism behind the opposite result in the literature. However, it crucially depends on the assumption of no price advertisement, which implies that the sellers cannot influence consumers' search strategies (i.e., consumer search is effectively random). In our model, the sellers compete in prices to attract consumers (i.e.,

⁶We note that our regularity assumptions about the distributions are crucial for the competition results. It is well-known that in the Perloff-Salop framework, market prices may increase as the number of sellers increases (see, e.g., Perloff and Salop, 1985; Chen and Riordan, 2008). It occurs when each firm has significant mass of loyal customers (who value the firm's product a lot more than the other products). In such a case, when the environment becomes more competitive, each firm attempts to extract more from its loyal customers, rather than trying to steal away others' customers. Although our regularity assumptions exclude this possibility, it is easy to show that the same phenomenon can arise in our model. See Section 5 for a more detailed discussion.

⁷Perloff and Salop (1985) found that a natural stochastic ordering, mean-preserving spread, has ambiguous effects on market prices, that is, market prices may increase or decrease as consumers' value distributions for the products become more spread. Based on the observation, subsequent studies have restricted attention to rather simple measures of preference diversity (product differentiation), such as constant scaling and variances in Gaussian environments.

consumer search is price-directed). When search costs increase, this competition becomes more important, given that more consumers would purchase from the first seller. This induces the sellers to lower their price.

In contrast, providing better product information for consumers before search may or may not increase market prices. Specifically, we show that market prices tend to increase under severe (inside or outside) competition, while the opposite is true under mild competition. From a consumer's perspective, obtaining better product information is similar to a reduction in search costs, in that both mitigate her search burden and, therefore, make her less price-sensitive (more value-sensitive). However, their effects on consumer search behavior are opposite to each other: the former induces consumers to search less, while the latter increases their incentive to search more. For the same logic as in the previous paragraph, this intensifies price competition among the sellers and, therefore, has an effect of lowering market prices. The ambiguous result of better product information is driven by the presence of these two opposing forces.

These results allow us to reinterpret various empirical findings in the literature, which, conversely, justifies the empirical relevance of our model. For instance, Lynch and Ariely (2000) ran a field experiment with online wine sales and found that providing more product information lowers consumers' price sensitivity. This suggests that sellers have an incentive to raise prices as consumer preferences become more diverse or the value of search increases and, therefore, is consistent with our results. Bailey (1998) and Ellison and Ellison (2014) reported that online prices are often higher than off-line prices.⁸ This naturally arises in our model, given that search costs are significantly lower in online markets than in off-line markets. Ellison and Ellison (2009) reported that markups are relatively higher for high-quality products than for low-quality products. Within our model, this can be understood as consumer preferences being more diverse, or the relative cost of search being lower, for high-quality products.

Our paper belongs to the fast-growing literature on electronic commerce. It is particularly related to two subsets of the literature. First, there are several theoretical studies that develop an equilibrium online shopping model. For example, Baye and Morgan (2001) analyzed a model in which both the sellers and consumers decide whether to participate

⁸We note that this is not a universal finding in the literature. For example, Brynjolfsson and Smith (2000), Brown and Goolsbee (2002) and Baye, Morgan and Scholten (2004) reported the opposite pattern.

in an online marketplace, while Chen and He (2011) and Athey and Ellison (2011) presented an equilibrium model that combines position auctions with consumer search. Our paper is unique in that the focus is on consumer search within an online marketplace. Second, a growing number of papers bring search theory to study online markets. For example, Kim, Albuquerque and Bronnenberg (2010) developed a non-stationary search model to study the online market for camcoders. De los Santos, Hortacısu and Wildenbeest (2012) tested some classical search theories with online book sale data and argued that fixed sample size (i.e., simultaneous) search theory explains the data better than sequential search theory. Dinerstein, Einav, Levin and Sundaresan (2014) estimated online search costs and retail margins with a consumer search model based on the “consideration set” approach,⁹ and apply them to evaluate the effect of search redesign by eBay in 2011. Although empirical analysis is beyond the scope of this paper, we think that our equilibrium model is tractable and structured enough to be taken to data.

In terms of modelling, our paper is particularly close to Armstrong and Zhou (2011).¹⁰ They presented three models in which firms can influence consumers’ search orders. Their second model is based on observable prices and, therefore, particularly close to our model. The specific model is very different from ours. In particular, they adopted a spatial duopoly (i.e., Hotelling) model and assumed perfect negative correlation for consumers’ values for the products. Nevertheless, they also obtained one of our key results, namely that a reduction in search costs leads to higher market prices. This suggests that the result is likely to hold in an even more general environment.

The rest of the paper is organized as follows. We introduce the formal model in Section 2. We analyze consumers’ optimal shopping problems in Section 3 and characterize the

⁹This approach is similar to simultaneous search, in that a consumer is assumed to consider only and all the options presented to her. It is simpler and, therefore, more suitable for empirical analysis than simultaneous search, because the set of options for each consumer is exogenously determined, for example, by firm advertisements, rather than optimally selected by herself. See, for example, Goeree (2008) for an application and an empirical implementation strategy of this approach.

¹⁰A precursor to this paper is Bakos (1997). He analyzed several versions of a (circular) location model. The main model is similar to Wolinsky (1986) and Anderson and Renault (1999): a consumer randomly selects a seller and observes both his price and her value for the product. One of his extensions considers the case where quality (value) information is significantly costlier than price information. The limit version where price information can be obtained at zero cost is equivalent to the case where prices are publicly observable and, therefore, correspond to Armstrong and Zhou (2011) and our paper. He did not provide a full characterization for the limit model.

market equilibrium in Section 4. In Sections 5, 6, 7, and 8, we study the effects of competition, preference diversity, search costs, and information quality, respectively, on equilibrium market prices. We conclude in Section 9. All omitted proofs are in the appendix.

2 The Model

The market consists of n sellers and a unit mass of consumers. The sellers supply differentiated products, each with no fixed cost and constant marginal cost $c \geq 0$. At the beginning of the market, each seller announces a price. We denote by p_i seller i 's price.¹¹ In addition, we let \mathbf{p} denote the price vector for all sellers (i.e., $\mathbf{p} = (p_1, \dots, p_n)$) and \mathbf{p}_{-i} denote the price vector except for seller i 's price (i.e., $\mathbf{p}_{-i} = (p_1, \dots, p_{i-1}, p_{i+1}, \dots, p_n)$). Denote by $D_i(\mathbf{p})$ the measure of consumers who eventually purchase from seller i . Seller i 's profit is then defined as $\pi_i(\mathbf{p}) \equiv D_i(\mathbf{p})(p_i - c)$. Each seller maximizes his profit $\pi_i(\mathbf{p})$.

Each consumer has unit demand. A consumer's value for seller i 's product is given by $v_i = x_i + y_i$, where x_i is known to the consumer before search, while y_i is revealed through her visit to seller i . The known component x_i represents a consumer's prior estimate on the value of the product, based on easily observable characteristics, such as its brand and basic design. The hidden component y_i captures more precise information about the product, which is available once a consumer inspects the product more carefully. As above, we let $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ to denote a (representative) consumer's value profile for each component. We assume that for each consumer, x_i 's are independently and identical drawn according to the distribution function F and y_i 's are independently and identical drawn according to the distribution function G . In addition, x_i and y_j are independent of each other for any i and j . Finally, both F and G have full support over the real line and continuously differentiable density f and g , respectively.

Search is costly and with perfect recall. Specifically, each consumer needs to incur constant cost $s(> 0)$ to visit each seller. This mainly captures the opportunity cost of time spent for each visit but may also come from "obfuscation" (see, e.g., Ellison and Ellison, 2009;

¹¹We assume that the sellers do not have the capacity, or are not allowed, to do any sort of price discrimination. In the current search context, a particularly intriguing possibility is to discriminate consumers based on whether they are first visitors or returning ones. See Zhu (2012) and Armstrong and Zhou (2014) for some developments along this line.

Ellison and Wolitzky, 2012). A consumer can purchase from any visited seller without additional costs.

A consumer's ex post utility depends on her value for the purchased product v_i , its price p_i , and the number of visits she has made before purchase. Specifically, if a consumer has visited k sellers and eventually purchases product i , then her ex post utility is equal to

$$U(x_i, y_i, p_i, k) = x_i + y_i - p_i - sk.$$

Each consumer can leave the market, without buying from any seller, at any point. A leaving consumer takes an outside option and receives utility u . This outside option summarizes the opportunity cost of shopping on the current marketplace (i.e., the value of shopping on another marketplace or visiting a local store). Each consumer is risk neutral and maximizes her expected utility.

We maintain the following regularity assumption about the distribution functions F and G through the paper.

Assumption 1 *Both density functions f and g are log-concave.*

It is well-known that log-concavity is satisfied with various well-behaved distributions (see, e.g., Bagnoli and Bergstrom, 2005) and an appropriate distributional assumption in various contexts. For example, it plays a crucial role in ensuring the existence of equilibrium in certain models (see, e.g., Caplin and Nalebuff, 1991; Burdett and Coles, 1997), and yields intuitive comparative statics results in various situations (see, e.g., Burdett, 1996). This also holds in our model. We fully utilize Assumption 1 to ensure the existence and uniqueness of equilibrium and obtain several unambiguous comparative statics results.

The market proceeds as follows. First, the sellers simultaneously announce prices \mathbf{p} . Then, each consumer shops (searches) based on available information (\mathbf{p}, \mathbf{x}) . We study subgame Nash equilibrium of this market game.

3 Consumer Behavior

We first analyze consumers' optimal shopping (search) problems, given prices $\mathbf{p} \in \mathcal{R}_+^n$. The characterization is used in the next section to study the sellers' optimal pricing problems and

characterize market equilibrium.

3.1 Optimal Shopping

Given prices \mathbf{p} , each consumer faces an optimal search problem. Specifically, each consumer decides in which order to visit the sellers. In addition, after completion of each visit, she decides whether to stop, in which case she chooses from which seller, if any, to purchase among those she has visited so far, or visit another seller. We implement an elegant solution by Weitzman (1979), who considered a broader class of optimal search problems. The linear and symmetric structure of our model permits a sharper characterization, as reported in the following proposition.¹²

Proposition 1 *Given the sellers' price announcements $\mathbf{p} = (p_1, \dots, p_n)$, the optimal shopping strategy for a consumer with $\mathbf{x} = (x_1, \dots, x_n)$ is given as follows:*

- (i) *Optimal search order: The consumer visits the sellers in the decreasing order of $x_i - p_i$ (i.e., she visits seller i before seller j if $x_i - p_i > x_j - p_j$).*
- (ii) *Optimal stopping: The consumer visits seller i if and only if $x_i - p_i + y^*$ exceeds the best available option by the point, that is,*

$$x_i + y^* - p_i > \max\{\underline{u}, x_j + y_j - p_j\}, \text{ for all } j \text{ such that } x_j - p_j > x_i - p_i,$$

where y^* is the value that satisfies

$$s = \int_{y^*}^{\infty} (1 - G(y)) dy. \quad (1)$$

The general solution in Weitzman (1979) is based on a single (Gittins) index for each option (seller). Specifically, let r_i be the value such that a consumer is indifferent between obtaining utility r_i immediately (which saves additional search costs s) and visiting seller i

¹²The event that a consumer is indifferent between two choices occurs with probability 0. For notational convenience, we ignore such independence (equality) cases through the paper.

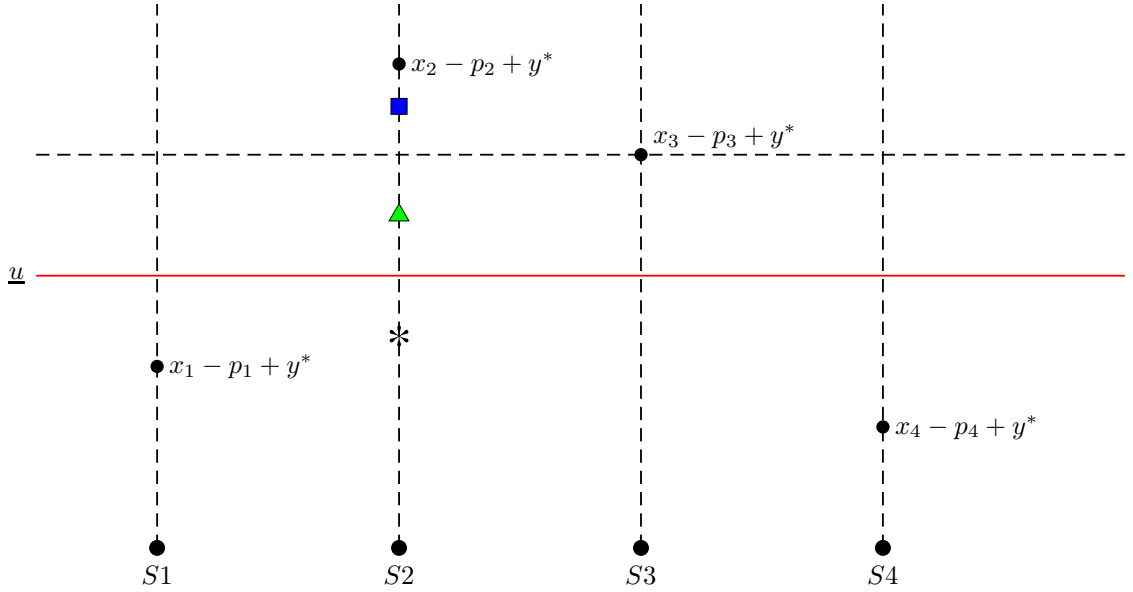


Figure 1: A non-stationary shopping problem when there are 4 sellers. It is optimal for the consumer to visit seller 2 first and decide whether to visit seller 3 or not, depending on the realization of y_2 (specifically, she visits seller 3 if and only if $x_2 - p_2 + y_2 < x_3 - p_3 + y^*$). She never visits sellers 1 and 4.

(which gives her an option to choose between r_i and $x_i - p_i + y_i$): formally,

$$r_i = -s + \int \max\{r_i, x_i - p_i + y_i\} dG(y_i).$$

Weitzman (1979) showed that the optimal search strategy is to visit the sellers in the decreasing order of r_i and stop as soon as a realized value $x_i - p_i + y_i$ is greater than all remaining r_i 's. The solution is simpler in our model, because, due to the linear utility specification and symmetry among the sellers, each r_i reduces to $x_i - p_i + y^*$.

To see consumer shopping behavior more concretely, consider an example depicted in Figure 1. Given the consumer's information before search (\mathbf{x}, \mathbf{p}) , the four sellers are ranked in the following order: S_2, S_3, S_1, S_4 . If the consumer would visit all the sellers, she would follow the same order. However, a consumer visits seller i only when $x_i - p_i + y^*$ exceeds the outside option \underline{u} . Therefore, in Figure 1, she visits at most two sellers, sellers 2 and 3. She first visits seller 2. If the realized value of y_2 is sufficiently high (square dot), she immediately purchases from seller 2. If not (triangle or asterisk dot), she visits seller 3 and

decides whether to purchase from seller 2 (if $x_2 + y_2 - p_2 > \max\{x_3 + y_3 - p_3, \underline{u}\}$), purchase from seller 3 (if $\max\{x_3 + y_3 - p_3 > x_2 + y_2 - p_2, \underline{u}\}$), or take the outside option.

Despite complexity, consumers' optimal shopping strategies exhibit various intuitive properties. In particular, given prices \mathbf{p} , consumers tend to visit more sellers as the unit search cost s decreases: if s decreases, then y^* increases, and thus a consumer is less likely to stop. In the limit as s tends to 0 (∞), almost all consumers never stop (never continue), because y^* tends to ∞ ($-\infty$).

3.2 Shopping Outcomes

In order to characterize market equilibrium, it is necessary to derive demand functions. This task is rather straightforward if consumers' purchase decisions depend only on prices and their observable preferences (e.g., Perloff and Salop, 1985) or consumers do random search (e.g., Wolinsky, 1986; Anderson and Renault, 1999). In our model, consumers engage in non-stationary sequential search, which significantly complicates the derivation of demand functions.

Consider the simplest case where there are two sellers and no outside option (so that each consumer must purchase from one of the sellers). Even in this case, there are three different paths through which a consumer eventually purchases from seller i . First, a consumer may visit seller i first and purchases immediately (the solid region in the left panel of Figure 2). Second, a consumer may visit seller j first but eventually purchases from seller i (the solid region in the right panel of Figure 2). Third, a consumer may visit seller i first, tries seller j as well, but comes back and purchases from seller i (the shaded region in the left panel of Figure 2). Total demand for seller i is the sum of all these demands. Therefore, in order to evaluate price effects on total demand, it is necessary to aggregate the effects on all possible paths. Notice that the number of paths grows exponentially fast as the number of sellers n increases, and the outside option introduces additional complication.

We overcome this difficulty by focusing on eventual purchase decisions, not on different purchase paths. To see this concretely, consider the same duopoly case as above (without the outside option). The precise conditions for the three purchase paths are given as follows:

- $x_i + y^* - p_i > x_j + y^* - p_j$ and $x_i + y_i - p_i > x_j + y^* - p_j$: visit seller i first (first inequality) and purchase product i without visiting seller i (second inequality).

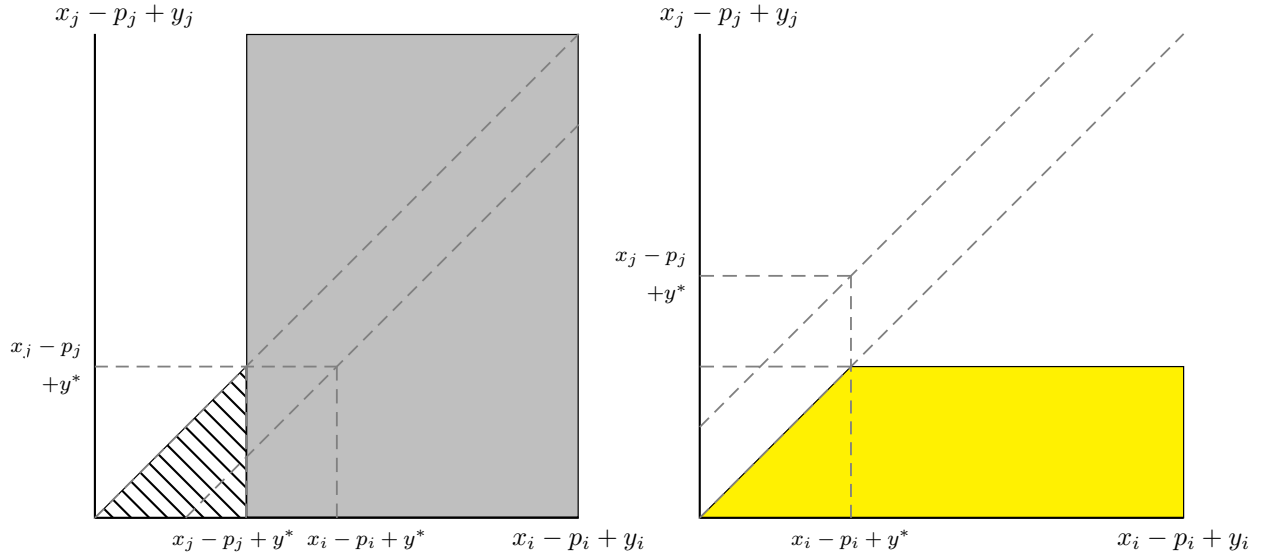


Figure 2: The condition for a consumer to eventually choose seller i over seller j . The left panel depicts the case when the consumer visits seller i before seller j ($x_i - p_i > x_j - p_j$), while the right panel is for the opposite case ($x_i - p_i < x_j - p_j$).

- $x_i + y^* - p_i > x_j + y^* - p_j$, $x_i + y_i - p_i < x_j + y^* - p_j$, and $x_i + y_i - p_i > x_j + y_j - p_j$: visit seller i first (first inequality), also visit seller j (second inequality), but recall product i (third inequality).
- $x_i + y^* - p_i < x_j + y^* - p_j$, $x_i + y^* - p_i < x_j + y_j - p_j$, and $x_i + y_i - p_i > x_j + y_j - p_j$: visit seller j first (first inequality), but come to seller i (second inequality) and purchase product i (third inequality).

Notice that the first condition can be simplified to $x_i + \min\{y_i, y^*\} - p_i > x_j + y^* - p_j$, and the second and the third conditions together can be reduced to $x_i + \min\{y_i, y^*\} - p_i > x_j + y_j - p_j$. Intuitively, a consumer purchases product i if she either does not visit seller i or finds a sufficiently low realized value of y_j . Combining the last two inequalities, we arrive at the following simple condition:

$$x_i + \min\{y_i, y^*\} - p_i > x_j + \min\{y_j, y^*\} - p_j.$$

This is a necessary and sufficient condition for a consumer to eventually purchase from seller

i. This significantly simplifies the subsequent analysis, because it suffices to evaluate how this inequality responds as each seller's price changes, instead of calculating marginal effects on each purchase path.

Furthermore, the condition can be readily extended into the general case. It is easy to see that with more than two sellers, the condition applies to any pair of sellers and, therefore, a consumer purchases product *i* if and only if the inequality holds for any $j \neq i$ (provided that she purchases at all). For the outside option, it suffices to add $x_i + \min\{y_i, y^*\} - p_i > \underline{u}$, because it implies that the consumer will visit at least one seller ($x_i + y^* - p_i > \underline{u}$) and not leave without a purchase ($x_i + y_i - p_i > \underline{u}$). We summarize the results so far in the following lemma.

Lemma 1 (Eventual Purchase) *Let $z_i \equiv x_i + \min\{y_i, y^*\}$ for each i . Given \mathbf{p} , \mathbf{x} , and \mathbf{y} , the consumer eventually purchases from seller i if and only if $z_i - p_i > \max\{\underline{u}, z_j - p_j\}$ for all $j \neq i$.*

Lemma 1 suggests that the random variable z_i summarizes all necessary consumer value information regarding eventual purchase decisions: in what follows, we often refer to z_i as effective consumer value. The hidden component y_i affects a consumer's purchase decision only partially. In particular, conditional on $y_i > y^*$, a consumer's purchase decision is independent of y_i . This is a consequence of Proposition 1: if a consumer visits seller i and draws y_i above y^* , then she purchases from seller i with probability 1, independent of the exact value of y_i . Even if $y_i < y^*$, the consumer may eventually purchase from seller i , but with probability less than 1, because she may visit other sellers or simply prefer a previous seller to seller i .

Two remarks are in order. First, it is straightforward to accommodate seller heterogeneity into this lemma. It suffices to apply equation (1) and identify individual-specific y_i^* . Then, a consumer purchases from seller i if and only if $x_i - p_i + \min\{y_i, y_i^*\} > \max\{\underline{u}, x_j - p_j + \min\{y_j, y_j^*\}\}$ for each $j \neq i$. Second, it still depends on the specifics of our model. For example, this result does not hold if prices are not observable before search. In that case, a consumer's search decision is based on her expectations about the sellers' prices, while her final purchase decision depends on the actual prices charged. Therefore, if a seller deviates, then a consumer's eventual purchase decision cannot be summarized as in Lemma 1. In addition, if a consumer can discriminate consumers based on whether they are new visitors

or returning ones, then the result obviously fails.

In order to utilize Lemma 1, we let H denote the distribution function for a random variable $z = x + \min\{y, y^*\}$, that is,

$$H(z) \equiv \int_{-\infty}^{y^*} F(z - y)dG(y) + \int_{y^*}^{\infty} F(z - y^*)dG(y). \quad (2)$$

The distribution function H effectively summarizes all relevant preference information regarding eventual purchase decisions. To see this more concretely, consider the case where s is sufficiently large that consumers visit at most one seller. This means that only the known component x affects consumers' eventual purchase decisions. Observe that, indeed, $H(z)$ becomes independent of y . Now consider the case where s is sufficiently close to 0. In this case, most consumers visit all sellers (y^* is arbitrarily large in this case) and make final purchase decisions based on full information (x, y) . This implies that both the known component x and the hidden component y equally affect consumers' purchase decisions. Observe that $H(z)$ tends to $\Pr\{x + y \leq z\} = \int F(z - y)dG(y)$ as s tends to 0. In general, the known component x is fully reflected in consumers' purchase decisions, while the hidden component y affects consumers' decisions only partially. The distribution function H incorporates this difference between two value components in a simple fashion.

Due to its particular form, the distribution function H inherits certain properties from the two underlying distributions F and G , but not all. For example, its density, which we denote by h in what follows, may not be single-peaked when both f and g are single-peaked. For our purpose, more important is the log-concavity of the induced distribution, as it plays an important role in establishing the existence and uniqueness of equilibrium. In general, the density function h does not inherit log-concavity, because of the mass point on y^* . The following result shows that, nevertheless, log-concavity is inherited into the distribution function H . As shown later, this suffices for our purpose.

Lemma 2 *The distribution function H is log-concave.*

4 Market Equilibrium

This section analyzes the pricing game among the sellers, building upon the characterization of consumer shopping behavior in Section 3. We first derive demand functions and derive their basic properties. We then establish the existence and uniqueness of market equilibrium.

4.1 Demand Functions

Recall that $D_i(\mathbf{p})$ denotes total demand for seller i . In a slight abuse of notation, we use $D_i(p_i, p^*)$ to denote the demand for seller i when he posts p_i , while all other sellers post an identical price p^* . We further abuse notation and denote by $D_i(p^*)$ the demand for seller i when all sellers, including seller i , announce an identical price p^* .

Lemma 1 implies that the demand function for seller i is given as follows:

$$D_i(\mathbf{p}) = \int_{\underline{u}+p_i}^{\infty} \left(\prod_{j \neq i} H(z_i - p_i + p_j) \right) dH(z_i). \quad (3)$$

Notice that this formulation is a familiar one in discrete choice models (except that the distribution function H is not exogenously given here). As such, the demand function $D_i(\mathbf{p})$ exhibits various standard properties. Among others, $D_i(\mathbf{p})$ is decreasing in p_i and increasing in p_j for any $j \neq i$, which means that the products are imperfect substitutes one another.

The following lemma reports two crucial properties of the demand function $D_i(\mathbf{p})$.

Lemma 3 *The demand function $D_i(\mathbf{p})$ is log-concave and log-supermodular in \mathbf{p} .*

Proof. Rewriting equation (3),

$$D_i(\mathbf{p}) = \int_{\underline{u}-y^*+p_i}^{\infty} \int_{\underline{u}-x+p_i}^{\infty} \left(\prod_{j \neq i} H(x + \min\{y, y^*\} - p_i + p_j) \right) g(y) f(x) dy dx.$$

Changing the variables with $k = x - p_i$ and $l = x + y - p_i$ yields

$$D_i(\mathbf{p}) = \int_{\underline{u}-y^*}^{\infty} \int_{\underline{u}}^{\infty} \left(\prod_{j \neq i} H(\min\{l, k + y^*\} + p_j) \right) g(l - k) f(k + p_i) dl dk.$$

All the ingredients in the integrand are log-concave (because of Lemma 2 and the log-concavity of f and g), and thus the integrand as a whole is log-concave in (l, k, \mathbf{p}) . Then, by Prékopa's theorem,¹³ $D_i(\mathbf{p})$ is log-concave in \mathbf{p} .

For the log-supermodularity, first observe that the log-concavity of g implies that $g(l - k)$ is log-supermodular in $(-l, -k)$:

$$\frac{\partial \log(g(l - k))}{\partial(-l)} = -\frac{g'(l - k)}{g(l - k)}.$$

Since $g'(y)/g(y)$ is decreasing in y , $-g'(l - k)/g(l - k)$ is increasing in $-k$. Similarly, $f(k + p_i)$ is log-supermodular in $(p_i, -k)$ (by the log-concavity of f), and $H(\min\{l, k + y^*\} + p_j)$ is log-supermodular in $(-l, -k, p_j)$ (by Lemma 2). Since the log-supermodularity is preserved under multiplication as well as under partial integration (see Karlin and Rinott, 1980), $D_i(\mathbf{p})$ is log-supermodular in \mathbf{p} . ■

The two properties in Lemma 3 are prevalent in various oligopoly models. The difference is that they are driven by exogenous restrictions on the demand function $D_i(\mathbf{p})$ (or, indirectly, on the effective distribution function H) in most existing models, while $D_i(\mathbf{p})$ and H are endogenously determined in our model.

4.2 Market Equilibrium

We now state and prove our main characterization result.

Theorem 1 *There exists a unique equilibrium, in which all sellers announce p^* such that*

$$\frac{1}{p^* - c} = \frac{\int h(\max\{\underline{u} + p^*, z\}) dH(z)^{n-1}}{\frac{1}{n}(1 - H(\underline{u} + p^*))^n}. \quad (4)$$

Proof. We establish this result with an elegant theory of supermodular games (see, e.g., Vives, 2005). Notice that the log-supermodularity of the demand function $D_i(\mathbf{p})$ implies the same property for the profit function $\pi_i(\mathbf{p}) = D_i(\mathbf{p})(p_i - c)$, because $\log \pi_i(\mathbf{p}) = \log D_i(\mathbf{p}) + \log(p_i - c)$. This implies that the pricing game among the sellers is a supermodular game, from which the existence of equilibrium immediately follows.

¹³The theorem effectively suggests that log-concavity is preserved under partial integration. See, for example, Caplin and Nalebuff (1991) and Choi and Smith (2015) for a formal statement of the theorem.

For equilibrium uniqueness, let p^* denote a symmetric equilibrium price. It is necessary and sufficient (due to the log-concavity of $\pi_i(\mathbf{p}) = D_i(\mathbf{p})(p_i - c)$) that the equilibrium price p^* satisfies an individual seller's first-order condition, and thus

$$\frac{1}{p^* - c} = -\frac{\partial D_i(p^*, p^*)/\partial p_i}{D_i(p^*)}. \quad (5)$$

The left-hand side is strictly decreasing in p^* , while the right-hand side is increasing in p^* (see the proof of Proposition 3 in the appendix for a formal proof). Therefore, there exists a unique symmetric equilibrium. A standard result in supermodular games then implies that there cannot exist any asymmetric equilibrium. ■

4.3 Alternative Equilibrium Pricing Formula

The following transformation of the equilibrium pricing function (4), which can be obtained by applying the definition of H and changing the variables, is useful for ensuing comparative statics exercises.

Proposition 2 *The unique equilibrium price p^* satisfies the following equation:*

$$\frac{1}{p^* - c} = -\int \frac{f'(x_i)}{f(x_i)} dJ(x_i), \quad (6)$$

where J is a distribution function over $[\underline{u} + p^* - y^*, \infty)$ such that

$$J(x_i) \equiv \frac{\int_{\underline{u}+p^*-y^*}^{x_i} \left(\int_{\underline{u}+p^*-t}^{\infty} H(t + \min\{y_i, y^*\})^{n-1} dG(y_i) \right) dF(t)}{\int_{\underline{u}+p^*-y^*}^{\infty} \left(\int_{\underline{u}+p^*-t}^{\infty} H(t + \min\{y_i, y^*\})^{n-1} dG(y_i) \right) dF(t)}. \quad (7)$$

To understand this result, first let $\Gamma(x_i, p_i)$ denote the probability that a consumer with x_i eventually purchases product i , conditional on the event that seller i posts p_i (while all other sellers post the equilibrium price p^*). Applying Lemma 1, $\Gamma(x_i, p_i)$ is given as follows:

$$\Gamma(x_i, p_i) = \begin{cases} 0 & \text{if } x_i + y^* - p_i \leq \underline{u}, \\ \int_{\underline{u}+p_i-x_i}^{\infty} H(x_i + \min\{y_i, y^*\} - p_i + p^*)^{n-1} dG(y_i), & \text{if } x_i + y^* - p_i > \underline{u}. \end{cases}$$

Intuitively, a consumer purchases product i if and only if she has an incentive to visit the seller (thus, $x_i + y^* - p_i > \underline{u}$), the seller's product is acceptable even after the hidden component is revealed (thus, $x_i + y_i - p_i > \underline{u}$), and she either does not visit other sellers or does not find others' products more desirable than seller i 's (thus, $H(x_i + \min\{y_i, y^*\}) - p_i + p^*)^{n-1}$). Naturally, $\Gamma(x_i, p_i)$ is increasing in x_i and decreasing in p_i . Importantly, $\Gamma(x_i, p_i)$ depends only on $x_i - p_i$, which implies that the purchase probability is invariant if x_i and p_i change proportionally (that is, $\Gamma(x_i + \Delta, p_i + \Delta) = \Gamma(x_i, p_i)$).

Integrating $\Gamma(x_i, p_i)$ over x_i produces total demand for seller i (equivalently, the unconditional probability that a consumer purchases product i). It follows that $J(x_i)$ gives the equilibrium proportion of consumers whose observable components are below x_i among those who eventually purchase product i (equivalently, the probability that a consumer's observable component is less than x_i , conditional on her purchasing product i).

Suppose seller i decreases his price from p^* to $p^* - \varepsilon$. This increases the purchase probability from $\Gamma(x_i, p^*)$ to $\Gamma(x_i, p^* - \varepsilon)$ at each $x_i \in X$. The invariance property mentioned above, however, implies that $\Gamma(x_i, p^*) = \Gamma(x_i - \varepsilon, p^* - \varepsilon)$. This means that the effect of the price change on the aggregate demand can be written as

$$D_i(p^*, p^*) - D_i(p^* - \varepsilon, p^*) = \int \Gamma(x_i, p^*)(f(x_i) - f(x_i - \varepsilon))dx_i = \int \Gamma(x_i, p^*)f'(x_i)\varepsilon dx_i.$$

Proposition 2 follows once this condition is combined with the familiar inverse elasticity pricing rule.

5 Inside and Outside Competition

In this section, we address a classic question in industrial organization, namely the effects of competition on market prices. Specifically, we examine how the equilibrium price responds to an increase in the number of sellers and to an increase in the outside option.

The following result reports that more intense competition, whether inside or outside, lowers market prices in our model.

Proposition 3 *The equilibrium price p^* decreases in the number of sellers n and the outside option \underline{u} .*

In order to understand this intuitive result more deeply, first notice that consumers tend to visit more sellers as n increases: a consumer is less likely to stop because the next best seller becomes more attractive on average. This implies that consumers become more selective and are more likely to have higher values of x_i , conditional on eventually purchasing from seller i . Similarly, when the outside option \underline{u} improves, consumers tend to stop earlier. Since consumers visit in the decreasing order of $x_i - p^*$, this also means that they are more likely to purchase from sellers with relatively higher values of x_i . In both cases, the distribution function J in Proposition 2 increases in the sense of first-order stochastic dominance. Therefore, for equation (6) to be preserved, p^* must decrease.

The effects of inside competition on market prices have been widely investigated in the Perloff-Salop framework. Our result complements existing findings by incorporating consumer search and proving the robustness of existing insights in such an environment. There are two particularly intriguing results in the literature, one that equilibrium markups ($p^* - c$) do not necessarily vanish as n tends to infinity and the other that market prices increase in n under some distributions. Both results continue to hold in our model. Inspecting equation (6), it follows that the right-hand side converges to $\lim_{x_i \rightarrow \infty} -f'(x_i)/f(x_i)$ as n tends to infinity. Therefore, equilibrium markups do not vanish if and only if $f'(x_i)/f(x_i)$ is bounded.¹⁴ The increasing-price result also depends on the behavior of $f'(x_i)/f(x_i)$. We obtain Proposition 3 under the assumption that f is log-concave. If f is log-convex (i.e., $f'(x)/f(x)$ is increasing), instead, then the opposite result holds. Assuming the existence and uniqueness of equilibrium (which is not guaranteed without Assumption 1), by the same reasoning as above, market prices increase in n .

The outside competition result is, to our knowledge, new to the literature. The outside option is known to significantly complicate the analysis, but also be crucial for some results (see, e.g., Perloff and Salop, 1985; Chen and Riordan, 2008). Due to the emergence of various online marketplaces, it has become a lot more relevant issue now. Contemporary sellers, whether on- or off-line, compete not only within a market(place), but also across different markets (platforms). Our simple reduced-form approach obviously has various limitations, but it is clear that the question is an integral one in the current Internet age.

¹⁴For example, if f is a normal distribution, then p^* converges to 0, while if f is a logistic distribution with scale parameter σ , then $p^* - c$ converges to σ as n tends to infinity.

6 Preference Diversity

Product differentiation is a classic solution to the Bertrand paradox (see Tirole, 1988): a seller can retain the consumers who particularly value his product even if he charges a higher price than the other sellers. It is plausible that the more differentiated consumers' preferences are, the higher prices the sellers charge. This conjecture has been one of central questions in oligopoly models of product differentiation (see, e.g., Perloff and Salop, 1985; Anderson et al., 1992; Anderson and Renault, 1999). In this section, we address this question within our framework. In so doing, we also generalize some known results in the literature.

The following measure of stochastic orders, so called *dispersive order*, plays a crucial role in what follows.

Definition 1 *The distribution function H_2 is more dispersed than the distribution function H_1 if $H_2^{-1}(b) - H_2^{-1}(a) \geq H_1^{-1}(b) - H_1^{-1}(a)$ for any $0 < a \leq b < 1$.*

Intuitively, a more dispersed distribution function increases more slowly, as its density is more spread. Note that this order is location-free (in the sense that it is independent of the absolute values of the distribution function) and, therefore, neither is implied by nor implies first-order or second-order stochastic dominance. Mean-preserving dispersive order, however, implies mean-preserving spread: if H_2 is more dispersed than H_1 with the same mean, then H_2 is a mean-preserving spread of H_1 .¹⁵

We first provide a result concerning the relationship between the equilibrium price p^* and the (endogenous) distribution function H .¹⁶ Our result complements the findings in Perloff and Salop (1985). They studied the same problem for the case where there is no outside option (i.e., $\underline{u} = -\infty$). They found that constant scaling of consumers' preferences necessarily increases the equilibrium price, but failed to further generalize the result. In particular, they showed that the effect of mean-preserving spreads on the equilibrium price is ambiguous in general. Our result reveals that there is a good sense in which dispersive order is an appropriate measure of product differentiation (preference diversity).

¹⁵See Shaked and Shanthikumar (2007) for further details.

¹⁶A very close result was independently discovered by Zhou (2015), who studied the effects of bundling in the Perloff-Salop framework. Precisely, his Lemma 2 is equivalent to our Proposition 4, provided that there is no outside option (i.e., $\underline{u} = -\infty$). Our result is more general than his, in that we allow for the outside option.

Proposition 4 *The equilibrium price p^* increases as the distribution function H becomes more dispersive and $H(\underline{u} + c)$ weakly decreases.*

Proof. Equation (4) in Theorem 1 can be rewritten as

$$\frac{1}{p^* - c} = \frac{\int h(\max\{\underline{u} + p^*, z\})dH(z)^{n-1}}{\frac{1}{n}(1 - H(\underline{u} + p^*))^n} = \frac{h(H^{-1}(\phi))\phi^n + \int_{\phi}^1 h(H^{-1}(a))da^{n-1}}{\frac{1}{n}(1 - \phi^n)},$$

where $\phi \equiv H(\underline{u} + p^*)$ and the second term in the numerator is obtained through changing the variable with $a = H(z)$. Observe that if H becomes more dispersive, $dH^{-1}(a)/da = 1/h(H^{-1}(a))$ increases (i.e., $h(H^{-1}(a))$ decreases) for each a . If, in addition, $H(\underline{u} + c)$ decreases, then $\phi = H(\underline{u} + p^*)$ also decreases for any $p^* \geq c$, because a distribution function crosses a less dispersive one only once from above. Notice that both of these lower the right-hand side. Now recall that the left-hand side is strictly decreasing in p^* , while the right-hand side is strictly increasing in p^* (equivalently, ϕ). Therefore, for the equilibrium equation to be restored, p^* must increase. ■

Proposition 4 becomes more transparent if there is no outside option. In this case, the second condition about $H(\underline{u} + c)$ is vacuous, and thus dispersive order alone dictates how market prices vary. To see the necessity of the second condition, recall that dispersive order and first-order stochastic dominance can go in the opposite direction, that is, a distribution function H_1 can first-order stochastically dominate a more dispersed distribution function H_2 . This means that an increase of H in the sense of first-order stochastic dominance may decrease p^* if the change reduces the dispersion of H . This counter-intuitive result arises only when there is no outside option. With an outside option, other distributional changes (in particular, first-order stochastic dominance) also influence market prices. The second condition captures the effect. Notice that it is significantly weaker than, and is implied by, first-order stochastic dominance.

We now return to our model and analyze the effects of varying our primitive distribution functions F and G . Proposition 6 suggests that it is most crucial to study their effects on the distribution function H . The following proposition provides three ways to increase the dispersion of H , each of which then can be combined with Proposition 4.

Proposition 5 (i) *Scaling: Suppose x_i and y_i are drawn according to the distribution func-*

tions $F^\theta(x_i)$ and $G^\theta(y_i)$, respectively, where $F^\theta(x_i) = F(x_i/\theta)$ and $G^\theta(y_i) = G(y_i/\theta)$. The induced distribution function H becomes more dispersive as θ increases.

(ii) *Hidden component: The distribution function H becomes more dispersive as the distribution function G becomes more dispersed.*

(iii) *Known component: If the density function f is decreasing over its support, then H becomes more dispersive as F becomes more dispersed.*

The results are fairly intuitive. The underlying random variable for the distribution function H is $z = x + \min\{y, y^*\}$. Therefore, simultaneous scaling of both x and y scales up z proportionally, which naturally increases the dispersion of H . Intuitively, making each component more dispersive should also increase the dispersion of the overall distribution. The argument works in general for the hidden component y , but not for the known component x . This is, of course, because of the asymmetry between the two. In particular, the upper truncation structure of y generates a probability mass for each x . This does not interfere in the dispersion of y being transferred to that of z , but may between x and z . If the density function f is decreasing, it is still possible to establish the desired result. However, it does not hold in general. Indeed, we have a counterexample in which market prices strictly decrease when x becomes more dispersed.

7 Search Costs

In this section, we study the relationship between the equilibrium price p^* and search costs s . Although this is a classic question in search models, it has become an even more relevant question, due to fast developments of communication technologies and online marketplaces, which are believed to have dramatically reduced search costs. Intuitively, a reduction in search costs induces more consumer search, intensifies price competition and, therefore, lowers market prices. This intuition has been confirmed in existing search models (see, e.g., Wolinsky, 1986; Anderson and Renault, 1999). In stark contrast, we find that the opposite result holds in our model.

Proposition 6 *The equilibrium price p^* decreases as the unit search cost s increases.*

Interestingly, we prove this result by applying Proposition 4: we show that the distribution function H increases in the sense of first-order stochastic dominance and becomes more

dispersive as s decreases. Intuitively, consumers visit more sellers as search costs decrease. This makes consumers' effective values increase (as they find better match values) as well as more dispersed (as y_i 's get better reflected). Then, for the reasons given in the previous section (i.e., more dispersed consumer preferences), the sellers charge a higher price.

In most existing search models, prices are not observable to consumers before search. In this case, an increase in search costs decreases the value of additional search and, therefore, increases the probability that a consumer purchases from a given seller. This induces the sellers to charge a higher price as search costs increase. In our model, however, prices are observable before search and influence consumers' search behavior (see Proposition 3): the lower price a seller offers, the more consumers visit him first. As search costs increase, consumers search less and are more likely to purchase from their first visit. This intensifies price competition among the sellers and leads to lower market prices.¹⁷

In models without price advertisements, the effect of increasing search costs on the sellers' profits is ambiguous. As s increases, the sellers charge a higher price and extract more from remaining consumers. However, less consumers shop in the first place, and thus the sellers face overall lower demand. In our model, the effect is clearly negative, as shown in the following proposition.

Proposition 7 *Each seller's profit $\pi_i(p^*)$ decreases in search costs s .*

This result is particularly easy to see where there is no outside option. In that case, total demand is constant and, in equilibrium, each seller serves $1/n$ of consumers. Therefore, equilibrium prices and profits always move in the same direction. The sellers' profits increase in s without price advertisements (in existing models) but decrease in s with price advertisements (in our model). When there is an outside option, total demand depends both on search costs s and market prices p^* . In our model, p^* decreases in s , which offsets a direct reduction in total demand due to an increase in s . However, in equilibrium, total demand never increases sufficiently fast, and thus the seller's profits always decrease as s increases.

In contrast, Proposition 6 raises an interesting possibility regarding consumer welfare,

¹⁷In a comparable monopoly setting, the same result holds, because the monopolist must compensate consumers for their search costs. Intuitively, if search costs are sufficiently large, then consumers would not even bother to search, unless the price is sufficiently low. Although this effect is present in our model, it is not the driving force for Proposition 6. This is best reflected in the fact that Proposition 6 holds even if there is no outside option, and thus all consumers must visit at least one seller.

namely that it may increase when search costs increase. An increase in search costs has a negative direct effect on consumer welfare. However, if the sellers lower their prices dramatically in response, overall consumer welfare may rise. Indeed, we have an example in which an increase in search costs is beneficial to consumers. It arises when consumers' outside option is sufficiently low and there are sufficiently few sellers. In this case, the sellers possess strong market power and, therefore, charge a high price. An increase in search costs induces them to drop their prices quickly, up to the point where the indirect effect outweighs the direct effect and, therefore, consumer welfare increases.

8 Information Quality

In this section, we analyze the effects of improving information quality on online marketplaces. Specifically, we study how consumers and sellers respond when consumers receive more accurate information about their values before search.

For tractability, we specialize our model into the Gaussian learning environment through this section.¹⁸ Specifically, we assume that both F and G are given by normal distributions with mean 0. In addition, F has variance α^2 , while G has variance $1 - \alpha^2$, for some $\alpha \in (0, 1)$. The variances are deliberately chosen so that a change in α does not affect the distribution for underlying ex post values $x + y$: notice that $x \sim \mathcal{N}(0, \alpha^2)$ and $y \sim \mathcal{N}(0, 1 - \alpha^2)$, and thus $x + y \sim \mathcal{N}(0, 1)$, independent of α .

We interpret α as the parameter that measures information quality. To see this clearly, first consider the case when α is close to 1. In this case, consumers' values for the products are fully known before search. Then, the model shrinks to that in Perloff and Salop (1985), and consumers visit at most one seller. Now consider the case where α is close to 0. In this case, consumers have little information about their values and, therefore, are likely to visit multiple sellers. More generally, as α increases, consumers possess more prior information about their values and, therefore, tend to search less.

In general, the effect of increasing α on the equilibrium market price p^* is ambiguous and may take a complex structure, such as multiple peaks. This is because of the presence of two opposing effects. On the one hand, effective consumer values (z) become more diverse: pre-

¹⁸See Choi and Smith (2015) for a thorough discussion on the advantages and foundation of this specification.

cisely, an increase of α incurs a mean-preserving spread of the distribution function H .¹⁹ As shown in Section 6, this tends to make consumers less price-sensitive (more value-sensitive) and, therefore, increase market prices. On the other hand, it decreases the value of visiting an additional seller and obtaining further information and, therefore, lowers consumers' search incentives. Similarly to an increase in search costs s in Section 7, this intensifies price competition and, therefore, depresses market prices. Whether p^* increases or decreases in α depends on the relative strength of these two effects.

We provide two results that help us understand what goes when. The first result highlights the role of inside competition (n), while the second result illustrates the role of outside competition (\underline{u}). We provide an intuitive explanation for each result, while relegating formal proofs to the appendix.

Proposition 8 *Suppose there is no outside option (i.e., $\underline{u} = -\infty$). There exists an integer $n^*(\alpha)$ such that a marginal increase in α increases the equilibrium market price p^* if and only if the number of sellers n exceeds $n^*(\alpha)$.*

To understand this result, first notice that consumer demand for each seller becomes more elastic as the number of sellers n increases, because it becomes more likely that there are close substitutes for each seller's product. This implies that the first preference-diversity effect has a larger marginal impact on the market price when there are more sellers. In contrast, the second search-incentive effect is less sensitive to the number of sellers. Therefore, market prices tend to decrease in α when there are few sellers and increase in α when there are many sellers.

Proposition 9 *If the outside option exceeds y^* (given $\alpha \in (0, 1)$), then a marginal increase in α increases the equilibrium market price p^* .*

If the outside option \underline{u} is sufficiently lucrative, then consumers explore a product only when it looks sufficiently promising (i.e., x_i is sufficiently large). In other words, consumers have little search incentives. This implies that the second search-incentive effect has only a small marginal impact. Since the first preference-diversity effect dominates, market prices increase in α .

¹⁹See the proof of Proposition 8 in the appendix.

9 Conclusion

We have developed a new market structure that captures some prominent features of on-line shopping and examined many basic properties of the model. Consumers undergo non-stationary sequential search based on partial product information and advertised prices. We have explained how to accommodate such non-stationary search behavior in an equilibrium framework and how to apply existing techniques to characterize the market equilibrium of the model. In addition, we have studied various implications for price competition among the sellers. Among others, our model predicts that market prices increase as search costs decrease, which is opposite to a common result in existing consumer search models. In contrast, providing better product information before search, which also eases off consumer search, may or may not lead to higher market prices. These results are consistent with some existing empirical findings and provide concrete guidance for future work.

Price dispersion is a pervasive phenomenon even in online markets.²⁰ We have abstracted away from it, in order to glean main insights from our model more efficiently. There are two straightforward ways to generate price dispersion within our framework. First, if there is no heterogeneity among consumers about the known component x (i.e., the distribution function F is degenerate), then the sellers' demand functions are not continuous (as all consumers use the same search order), and thus equilibrium necessarily involves price mixing. Although this alternative specification generates price dispersion, it is well-known that it is analytically intractable and, therefore, not suitable for further analyses. Second, more directly, it suffices to introduce heterogeneity among the sellers. Various specifications are possible: the sellers may have different search costs (s_i), marginal cost (c_i), or different consumer values (F_i, G_i). For either specification, our techniques in Sections 3 and 4 can be used to establish the existence of equilibrium. Equilibrium uniqueness is much harder to establish, but some comparative statics results would be feasible, due to the supermodular structure of the game. We leave these and related extensions for future research.

Our framework can be used to evaluate the effects of various policies by platform providers. To begin with, it is clear that ex ante price information is beneficial to consumers: notice that if search costs are negligible (i.e., s is close to 0), then it does not matter whether prices

²⁰See, e.g., Brynjolfsson, Hu and Smith (2003), Baye, Morgan and Scholten (2004), and Ellison and Ellison (2005).

are observable before search or not. However, market prices increase in s if prices are not observable, while they decrease in s if prices are observable. Therefore, price advertisements always lower market prices. It also follows that a platform provider would be willing to restrict the use of hidden fees. We did not specify a platform provider's preferences, but her profit maximization problem is obviously interesting. Our model provides a micro-foundation of interactions among market participants and, therefore, might help enrich our understanding of optimal platform pricing.

Appendix

Proof of Lemma 1. Suppose $z_i - p_i > \underline{u}$ and $z_i - p_i > z_j - p_j$ for any $j \neq i$. The former implies that the consumer visits at least one seller ($x_i + y^* - p_i > \underline{u}$) and makes a purchase ($x_i + y_i - p_i > \underline{u}$). The latter implies that, by the same reasoning as in the duopoly case, she does not purchase any other product. Combining the two results, it follows that the consumer purchases product i .

Now suppose either $z_i - p_i < \underline{u}$ or there exists j such that $z_i - p_i < z_j - p_j$. In the former case, the consumer does not visit seller i ($x_i + y^* - p_i < \underline{u}$) or does not purchase product i even if she visits ($x_i + y_i - p_i < \underline{u}$). In the latter case, she either does not purchase at all or purchase a different product, whether she visits seller i or not. ■

Proof of Lemma 2. Integrating equation (2) by parts and changing the variable with $y = z - x$ leads to

$$H(z) = F(z - y^*) + \int_{z-y^*}^{\infty} G(z - x) dF(x) = \int_{-\infty}^{\infty} \tilde{G}(y) f(z - y) dy, \quad (8)$$

where $\tilde{G}(y) \equiv 1$ for $y > y^*$ and $\tilde{G}(y) \equiv G(y)$ for $y \leq y^*$. Then

$$\frac{h(z)}{H(z)} = \frac{\int_{-\infty}^{\infty} \tilde{G}(y) f'(z - y) dy}{\int_{-\infty}^{\infty} \tilde{G}(y) f(z - y) dy} = \frac{\int_{-\infty}^{\infty} \tilde{G}(z - s) f(s) \frac{f'(s)}{f(s)} ds}{\int_{-\infty}^{\infty} \tilde{G}(z - s) f(s) ds}.$$

The second equality is through a change of variable with $s = z - y$. To show that $H(z)$ is log-concave in z , it suffices to show $h(z)/H(z)$ falls in z . To this end, note that the ratio

$f'(s)/f(s)$ falls in s by the log-concavity of f . Therefore, the RHS falls in z if the random variable induced by the probability density $\tilde{G}(z-s)f(s)$ rises in z in the first order stochastic dominance sense. We argue this is true by showing that $\tilde{G}(z-s_2)f(s_2)/[\tilde{G}(z-s_1)f(s_1)]$ rises in z for all $s_2 > s_1$, or equivalently, $\tilde{G}(z-s_2)/\tilde{G}(z-s_1)$ rises in z . If $z-s_1 > z-s_2 \geq y^*$, then $\tilde{G}(z-s_2)/\tilde{G}(z-s_1) = 1$ and it remains constant as z rises. If $z-s_1 \geq y^* > z-s_2$, then $\tilde{G}(z-s_2)/\tilde{G}(z-s_1) = G(z-s_2)$ rises in z . If $y^* > z-s_1 > z-s_2$, then $\tilde{G}(z-s_2)/\tilde{G}(z-s_1) = G(z-s_2)/G(z-s_1)$ rises in z by the log-concavity of G . ■

Proof of Proposition 3. Recall that the equilibrium price p^* solves equation (6) and the ratio $f'(x)/f(x)$ falls in x by the log-concavity of f . Hence it suffices to show that the distribution function J in Proposition 2 increases in p^* , \underline{u} , and n in the sense of first-order stochastic dominance.

Let $\Omega(u, t, n) = \int_{u-t}^{\infty} H(t + \min\{y_i, y^*\})^{n-1} dG(y_i)$ so that (??) becomes

$$J(x_i) = \frac{\int_{\underline{u}+p^*-y^*}^{x_i} \Omega(\underline{u} + p^*, t, n) dF(t)}{\int_{\underline{u}+p^*-y^*}^{\infty} \Omega(\underline{u} + p^*, t, n) dF(t)} = \frac{\int_{\underline{u}+p^*-y^*}^{\infty} \mathbb{1}\{t \leq x_i\} \Omega(\underline{u} + p^*, t, n) dF(t)}{\int_{\underline{u}+p^*-y^*}^{\infty} \Omega(\underline{u} + p^*, t, n) dF(t)}$$

where $\mathbb{1}\{t \leq x_i\}$ is an indicator function and thus it falls in t . This can be interpreted as an expectation $E[\mathbb{1}\{T \leq x_i\}]$ where the random variable T has density $\Omega(\underline{u} + p^*, t, n)f(t)$ and support $[\underline{u} + p^* - y^*, \infty)$. The random variable T rises in the first order stochastic dominance sense in p^* or \underline{u} through two channels. First, $\Omega(\underline{u} + p^*, t)$ is log-supermodular in (p^*, t) or (\underline{u}, t) by Lemma 4 below. Second, the lower support $\underline{u} + p^* - y^*$ rises in p^* or \underline{u} . Altogether, $J(x_i)$ falls in p^* or \underline{u} . Similarly, $J(x_i)$ falls in n because $\Omega(\underline{u} + p^*, t)$ is log-supermodular in (t, n) by Lemma 4 below. ■

Lemma 4 Assume $\Omega(u, t, n) = \int_{u-t}^{\infty} H(t + \min\{y, y^*\})^{n-1} dG(y)$. Then $\Omega(u, t, n)$ is log-supermodular in (u, t) and (t, n) .

Proof. For (u, t) , differentiate wrt u and change variable $s = y - u + t$:

$$\frac{\partial \Omega(u, t, n) / \partial u}{\Omega(u, t, n)} = \frac{-H(\min\{u, t + y^*\})^{n-1} g(u-t)}{\int_0^{\infty} H(\min\{s+u, t+y^*\})^{n-1} g(s+u-t) ds}.$$

The ratio $H(\min\{u, t + y^*\})/H(\min\{s + u, t + y^*\})$ falls in t because H is an increasing function. The ratio $g(u - t)/g(s + u - t)$ falls in t by the log-concavity of g . Therefore, the RHS rises in t . This proves the log-supermodularity in (u, t) .

For (t, n) , change variable $s = y - u + t$. Then

$$\Omega(u, t, n) = \int_0^\infty H(\min\{s + u, t + y^*\})^{n-1} g(s + u - t) ds.$$

The function $H(\min\{s + u, t + y^*\})$ and $g(s + u - t)$ are log-supermodular in (s, t) by the log-concavity of H and g respectively. Therefore the integrand $H(\min\{s + u, t + y^*\})^{n-1} g(s + u - t)$ is log-supermodular in (s, t, n) . Since log-supermodularity is preserved under partial integration by Karlin and Rinott (1980), $\Omega(u, t, n)$ is log-supermodular in (t, n) . ■

The following lemma is used in the proofs of Propositions 5 and 6.

Lemma 5 *The effective consumer value $z = x + \min\{y, y^*\}$ becomes more dispersive (i) as the hidden component y grows more dispersive or (ii) as search costs s decrease.*

Proof. By Theorem 3.B.8 in Shaked and Shanthikumar (2007) (SS, hereafter), z grows more dispersive if $\min\{y, y^*\}$ grows more dispersive and x has log-concave density. Since we assume f is log-concave, it suffices to show that $\min\{y, y^*\}$ becomes more dispersive as y grows more dispersive or as s falls. To this end, let \tilde{G} be the cdf of $\min\{y, y^*\}$. The slope of the quantile function is $\partial\tilde{G}^{-1}(a)/\partial a = \partial G^{-1}(a)/\partial a$ for $a < G(y^*)$ and 0 for $a > G(y^*)$.²¹

Proof of (i): Assume $y_2 \geq_{disp} y_1$, namely y_2 is more dispersive than y_1 . For $i = 1, 2$, let G_i be the cdf of y_i and let y_i^* be the solution for equation (1) when the hidden component is y_i . Also, let \tilde{G}_i be the cdf of $\min\{y_i, y_i^*\}$. Since $\partial G_2^{-1}(a)/\partial a \geq \partial G_1^{-1}(a)/\partial a$ by the definition of the dispersive order and $G_2(y_2^*) \geq G_1(y_1^*)$ (see Choi and Smith (2015) for a proof), $\tilde{G}_2^{-1}(a)$ is weakly steeper than $\tilde{G}_1^{-1}(a)$ for all $a \in (0, 1)$, or equivalently $\min\{y_2, y_2^*\}$ is more dispersive than $\min\{y_1, y_1^*\}$.

Proof of (ii): Assume $s_2 > s_1$ and let y_i^* be the solution for equation (1) when the search cost is s_i for $i = 1, 2$. Let \tilde{G}_i be the cdf of $\min\{y_i, y_i^*\}$. As s falls, y^* increases by equation (1). Thus $y_2^* \leq y_1^*$ and $G(y_2^*) \leq G(y_1^*)$. Since $\partial\tilde{G}_i^{-1}(a)/\partial a = \partial G^{-1}(a)/\partial a$ for

²¹The derivative at $a = G(y^*)$ does not exist but this does not affect the dispersion of \tilde{G} .

$a < G(y_i^*)$ and is 0 otherwise, $\tilde{G}_1^{-1}(a)$ is weakly steeper than $\tilde{G}_2^{-1}(a)$ and thus $\min\{y, y_1^*\}$ is more dispersive than $\min\{y, y_2^*\}$. \blacksquare

Proof of Proposition 5. (i) Assume the cdf of x and y are $F(x/\theta)$ and $G(y/\theta)$ respectively. Define $\tilde{x} = x/\theta$, $\tilde{y} = y/\theta$ and $\tilde{y}^* = y^*/\theta$, and thus $z = \theta(\tilde{x} + \min\{\tilde{y}, \tilde{y}^*\})$. The random variable z grows more dispersive in θ through two channels. First, an increase in θ scales up z and thus increases the dispersion of z . Second, \tilde{y}^* rises in θ and z grows more dispersive as \tilde{y}^* rises by Lemma 5 (ii). To see why \tilde{y}^* rises in θ , recall that when the cdf of y is $G(y/\theta)$, we have $s = \int_{y^*}^{\infty} [1 - G(y/\theta)] dy$ by equation (1). Substitute \tilde{y} and \tilde{y}^* into this equation to derive $s = \theta \int_{\tilde{y}^*}^{\infty} [1 - G(\tilde{y})] d\tilde{y}$. Then it is easy to see \tilde{y}^* rises in θ .

(ii) See (i) in Lemma 5.

(iii) Consider two random variables x_i for $i = 1, 2$. Assume they have lower support $\underline{x} > -\infty$ and decreasing density $f_i(x)$. Also assume $x_2 \geq_{disp} x_1$. Then x_2 is higher than x_1 in the first order stochastic dominance sense by Theorem 3.B.13 in SS. We will prove $x_2 + \min\{y, y^*\} \geq_{disp} x_1 + \min\{y, y^*\}$.

Define a random variable x_t with cdf $F(x, t)$ and pdf $f(x, t)$ for $t \in (0, 1)$. Assume the quantile function of x_t is $F^{-1}(a, t) = (1 - t)F_1^{-1}(a) + tF_2^{-1}(a)$ for $a, t \in (0, 1)$. It is easy to check that (i) x_t rises in the first order stochastic dominances sense in t , namely that $F(x, t)$ falls in t , (ii) $f(x, t)$ falls in x and (iii) x_t grows more dispersed in t , namely that $\partial F^{-1}(a, t)/\partial a$ rises in t .²²

To prove the proposition, it is sufficient to show $z_t \equiv x_t + \min\{y, y^*\}$ grows more dispersive in t . This occurs if $\partial H^{-1}(a, t)/\partial a$ rises in t for all $a \in (0, 1)$, or equivalently $\partial^2 H^{-1}(a, t)/\partial a \partial t = -\partial[H_t(z, t)/h(z, t)]/\partial z \geq 0$ for all z . Therefore, z_t grows more dispersive if $H_t(z, t)/h(z, t)$ falls in z . By (8),

$$\begin{aligned} \frac{H_t(z, t)}{h(z, t)} &= \frac{\int_{-\infty}^{\infty} F_t(z - \min(y, y^*), t) g(y) dy}{\int_{-\infty}^{\infty} f(z - \min(y, y^*), t) g(y) dy} \\ &= \frac{\int_{-\underline{x}}^{-\underline{x}} \frac{F_t(\max(-r, z - y^*), t)}{f(\max(-r, z - y^*), t)} f(\max(-r, z - y^*), t) g(r + z) dr}{\int_{-\infty}^{-\underline{x}} f(\max(-r, z - y^*), t) g(r + z) dr}. \end{aligned}$$

The last line applies a change of variable $r = y - z$. The last line can be interpreted

²²To see (iii), note that $\partial^2 F^{-1}(a, t)/\partial a \partial t = \partial F_2^{-1}(a)/\partial a - \partial F_1^{-1}(a)/\partial a \geq 0$. The last inequality is true because x_2 is more dispersed than x_1 , and thus F_2^{-1} is steeper than F_1^{-1} .

as $E[F_t(\max(-R, z - y^*), t)/f(\max(-R, z - y^*), t)]$ where the random variable R has density $f(\max(-r, z - y^*), t)g(r + z)$. The function $F_t(x, t)/f(x, t)$ falls in x because $\partial[F_t(x, t)/f(x, t)]/\partial x = -\partial^2 F^{-1}(a, t)/\partial a \partial t \leq 0$. The last inequality is true because $\partial F^{-1}(a, t)/\partial a$ rises in t by the dispersive order. The expectation falls in z for two reasons. First, $F_t(\max(-R, z - y^*), t)/f(\max(-R, z - y^*), t)$ falls in z for any given R . Second, the random variable R falls stochastically in z if by the log-concavity of g and $f' \leq 0$. ■

Proof of Proposition 6. As mentioned in the main text, we prove that the distribution function H increases in the sense of first-order stochastic dominance and becomes more dispersive as s decreases. The result then follows from Proposition 5. For first-order stochastic dominance, recall that

$$H(z) = \int F(\max\{x, z - y^*\})g(z - x)dx.$$

As s decreases, y^* increases, and thus $F(\max\{x, z - y^*\})$ weakly decreases. Since this holds for any x , $H(z)$ weakly decreases, which establishes the desired result. See Lemma 5 (ii) for the claim about the dispersive order. ■

Proof of Proposition 7. An increase in s affects each firm's profit $\pi_i(\mathbf{p}) = D_i(p_i, p_{-i})(p_i - c)$ through the following three channels:

$$\frac{\partial \pi_i(\mathbf{p})}{\partial s} = \frac{\partial p_i}{\partial s} \frac{\partial \pi_i(\mathbf{p})}{\partial p_i} + \frac{\partial p_{-i}}{\partial s} \frac{\partial \pi_i(\mathbf{p})}{\partial p_{-i}} + \frac{\partial y^*}{\partial s} \frac{\partial \pi_i(\mathbf{p})}{\partial y^*}.$$

Each term represents the marginal effect of own price, that of the other sellers' prices, and that of consumer shopping behavior, respectively. In equilibrium, the first effect vanishes (the envelope theorem), while the other two effects are negative. The result for the second follows from $\partial p_{-i}/\partial s \leq 0$ (Proposition 6) and $\partial \pi_i(\mathbf{p})/\partial p_{-i} \geq 0$ (as the products are imperfect substitutes one another). The result for the last term is due to the fact that $\partial y^*/\partial s < 0$ (see Proposition 1), while $\partial \pi_i(\mathbf{p})/\partial y^* \geq 0$. To see the last inequality, recall that $z = x + \min\{y, y^*\}$, and thus an increase in y^* increases the distribution function H in the sense of first-order stochastic dominance. This implies that consumers are less likely to exercise the outside option and, therefore, $D_i(p^*, p^*)$ increases in y^* . ■

The following lemma is used in the proofs of Propositions 8 and 9.

Lemma 6 (i) *The mean of H is equal to $-s$, independent of α .*

(ii) *$H(z)$ decreases in α (i.e., $\partial H(z)/\partial\alpha < 0$) if and only if $z > y^*$.*

(iii) *There exists $a'(\leq H(y^*))$ such that $h(H^{-1}(a))$ decreases in α (i.e., $\partial h(H^{-1}(a))/\partial\alpha < 0$) if and only if $a > a'$.*

Proof. (i) The mean of H is equal to

$$E[z] = E[x + \min\{y, y^*\}] = E[x] + E[\min\{y, y^*\}] = \int_{-\infty}^{y^*} y dG(y) + (1 - G(y^*))y^*.$$

Since $E[y] = 0$,

$$E[z] = - \int_{y^*}^{\infty} y dG(y) + (1 - G(y^*))y^*.$$

Combining this with the fact that

$$s = \int_{y^*}^{\infty} (1 - G(y))dy = -(1 - G(y^*))y^* + \int_{y^*}^{\infty} y dG(y),$$

it follows that $E[z] = -s$, independent of α .

(ii) Let Φ denote the distribution function for the standard normal distribution and ϕ denote its density function. Since $x \sim \mathcal{N}(0, \alpha^2)$ and $y \sim \mathcal{N}(0, 1 - \alpha^2)$, $F(x) = \Phi(x/\alpha)$ and $G(y) = \Phi(y/\sqrt{1 - \alpha^2})$. Differentiating equation (8) with respect to α yields

$$H_\alpha(z) \equiv \frac{\partial H(z)}{\partial\alpha} = - \left[1 - \Phi \left(\frac{y^*}{\sqrt{1 - \alpha^2}} \right) \right] \left(\frac{z - y^*}{\alpha^2} \right) \phi \left(\frac{z - y^*}{\alpha} \right),$$

where $\partial y^*/\partial\alpha$ can be obtained from equation (1) by applying the implicit function theorem. The desired result is immediate from this equation.

(iii) From the equation above (or by differentiating the density of H with respect to z), we get

$$h_\alpha(z) \equiv \frac{\partial h(z)}{\partial\alpha} = - \left[1 - \Phi \left(\frac{y^*}{\sqrt{1 - \alpha^2}} \right) \right] \left[1 - \left(\frac{z - y^*}{\alpha} \right)^2 \right] \frac{1}{\alpha^2} \phi \left(\frac{z - y^*}{\alpha} \right).$$

Now observe that

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = h_\alpha(H^{-1}(a)) - H_\alpha(H^{-1}(a)) \frac{h'(H^{-1}(a))}{h(H^{-1}(a))}.$$

Let $z = H^{-1}(a)$ and apply $H_\alpha(z)$ and $h_\alpha(z)$ to the equation. Then,

$$\frac{\partial h(H^{-1}(a))}{\partial \alpha} = \frac{-1}{\alpha^2} \left[1 - \Phi \left(\frac{y^*}{\sqrt{1-\alpha^2}} \right) \right] \phi \left(\frac{z-y^*}{\alpha} \right) \left[1 - \frac{(z-y^*)^2}{\alpha^2} - (z-y^*) \frac{h'(z)}{h(z)} \right].$$

Since

$$h(z) = \frac{1}{\sqrt{1-\alpha^2}} \int_{-\infty}^{\infty} \phi \left(\frac{z-y^*}{\alpha} + \max\{r, 0\} \right) \phi \left(\frac{y^* - \alpha r}{\sqrt{1-\alpha^2}} \right) dr,$$

we have

$$\frac{h'(z)}{h(z)} = -\frac{z-y^*}{\alpha^2} - \frac{\int_{-\infty}^{\infty} \max\{r, 0\} \phi \left(\frac{z-y^*}{\alpha} + \max\{r, 0\} \right) \phi \left(\frac{y^* - \alpha r}{\sqrt{1-\alpha^2}} \right) dr}{\alpha \int_{-\infty}^{\infty} \phi \left(\frac{z-y^*}{\alpha} + \max\{r, 0\} \right) \phi \left(\frac{y^* - \alpha r}{\sqrt{1-\alpha^2}} \right) dr}.$$

Applying this to the above equation leads to

$$\begin{aligned} \frac{\partial h(H^{-1}(a))}{\partial \alpha} &\propto -1 + \left(\frac{z-y^*}{\alpha} \right)^2 + (z-y^*) \frac{h'(z)}{h(z)} \\ &= -1 - \frac{(z-y^*) \int_0^{\infty} r \phi \left(\frac{z-y^*}{\alpha} + \max\{r, 0\} \right) \phi \left(\frac{y^* - \alpha r}{\sqrt{1-\alpha^2}} \right) dr}{\alpha \int_{-\infty}^{\infty} \phi \left(\frac{z-y^*}{\alpha} + \max\{r, 0\} \right) \phi \left(\frac{y^* - \alpha r}{\sqrt{1-\alpha^2}} \right) dr}. \end{aligned}$$

The last expression is clearly negative if $z > y^*$. In addition, it converges to ∞ as z tends to $-\infty$ and decreases in z whenever $z \leq y^*$. Therefore, there exists $z' (< y^*)$ such that the expression is positive if and only if $z < z'$. The desired result follows from the fact that $z = H^{-1}(a)$ is strictly increasing in a . ■

Proof of Proposition 8. If $\underline{u} = -\infty$, then equation (4) shrinks to

$$\frac{1}{p^* - c} = n \int h(z) dH(z)^{n-1} = n \int_0^1 h(H^{-1}(a)) da^{n-1}.$$

Differentiating this equation with respect to α leads to

$$\frac{\partial p^*}{\partial \alpha} = -(p^* - c)^2 n \int_0^1 \frac{\partial h(H^{-1}(a))}{\partial \alpha} da^{n-1}.$$

The desired result follows from (iii) in Lemma 6 and the fact that for any real value function $\gamma : \mathcal{R} \rightarrow \mathcal{R}$, if $\int_0^1 \gamma(a) da^n = 0$ and there exists a' such that $\gamma(a) < 0$ if and only if $a > a'$, then

$$\int_0^1 \gamma(a) da^{n+1} = \frac{n+1}{n} \int_0^1 \gamma(a) a da^n \leq 0.$$

The last inequality is due to the fact that a is positive and strictly increasing and, therefore, assigns more weight to the negative portion of $\gamma(a)$ in the integral. The result follows by letting $\gamma(a) = \partial h(H^{-1}(a))/\partial \alpha$. ■

Proof of Proposition 9. A necessary condition for a consumer to purchase from seller i is $z_i - p_i \geq \underline{u}$. If $\underline{u} \geq y^*$, then only the right tail of $H(z)$ (above y^*) affects the equilibrium price. For this region, $H(z)$ grows more dispersive (by (iii) in Lemma 6) and increases in α in the sense of first-order stochastic dominance (by (ii) in Lemma 6). The result then follows from Proposition 5. ■

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