On the Number and Size of Banks: Efficiency and Equilibrium

Angela Kui Huang[*†]
National University of Singapore

September 18, 2016

Abstract

I develop a model where banking arises endogenously from economies of scale in monitoring. Only a fraction of agents are designated bankers, to reduce monitoring costs, but that implies more deposits per bank and therefore greater incentives to divert profits opportunistically. Hence, with fewer bankers, they need higher rewards. The optimal number of banks decreases with monitoring costs, impatience and the temptation to default, and increases with investment returns. To implement efficient allocations, there is a tension between free entry and the positive bank profits required for incentives. Therefore, equilibrium is optimal only if we limit entry by taxation or a quota on bank charters.

Keywords: Banking; Limited Commitment; Imperfect Monitoring

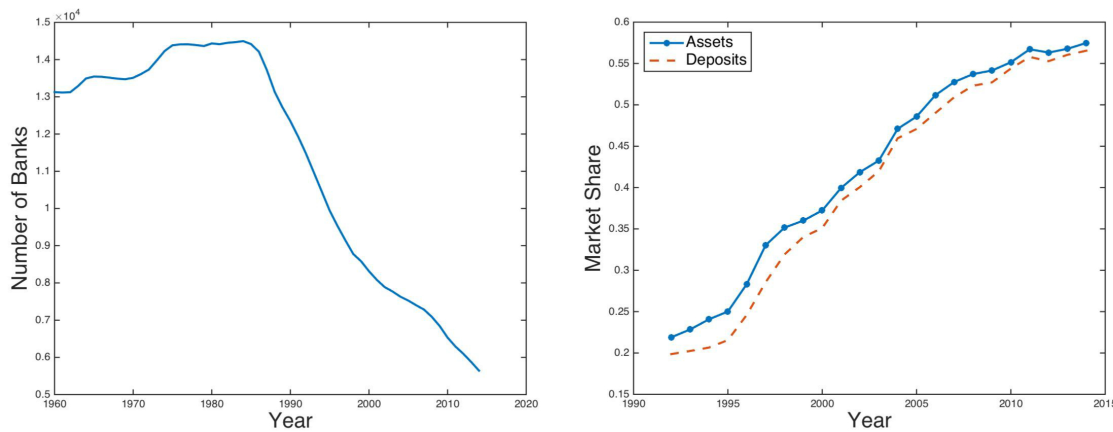
JEL: D02, E02, E44, E62, G21

[*]I am grateful to Randall Wright, Dean Corbae, Erwan Quintin, and Enghin Atalay for their guidance and feedback. I thank Chao Gu, Cyril Monnet, Alberto Trejos, Rasmus Lentz, Kenneth West, Briana Chang, Aloysius Siow, Han Han, Chao He, Yu Zhu and Michael Choi for comments and suggestions. Any remaining errors are my own.

[†]Department of Economics, National University of Singapore, 1 Arts Link, Singapore 117570. Email:ecskha@nus.edu.sg

1 Motivation

In the United States, between 1960 and 2014, the number of banks fell by more than half from about 13,000 to around 5,500. Between 1992 and 2014, the market share of the 10 largest banks grew dramatically from 21% to 57%. A great many of these changes started during the deregulation in the 1980s and 1990s. In 1960, banks could not branch across states and some states even forbade branching within a state. These legal and regulatory limits on bank size were subsequently removed. Figure 1 reports the time paths for the number of banks and the market share of the 10 largest banks. I use two measures of bank size, commercial bank assets, and commercial bank deposits. I use fourth-quarter data on all commercial banks in the United States.[1]



(a) Drop in the number of banks

(b) Market share of the 10 largest banks

Figure 1: Structural Change in the Banking Industry

My goal is to develop a theoretical model to address the following questions: Why did this structural change occur in the banking industry and is it desirable? Under what conditions is it socially optimal to have few large banks versus many small

[1]Following Berger et al. (1995), I treat all banks and bank holding companies under a higher-level holding company as a single independent banking enterprise. For convenience, I will typically refer to each of these entities as a bank. Data on banks are taken from the Federal Deposit Insurance Corporation dataset.

banks? Why don't we want too few or too many banks? Is "unfettered competition" in banking optimal?

I proceed with minimal assumptions about who bankers are or what they do. The agents that become bankers are ex-ante the same as the depositors. Obviously, some frictions are needed because models such as Arrow-Debreu have no roles for banks. There are two frictions in my model, arising from limited commitment: The agents that become bankers have a temptation to abscond with the proceeds (as in the cash-diversion models of Demarzo and Fishman 2007, or Biais et al. 2007), and there is imperfect monitoring. Related to a classic challenge in monetary economics—what makes money essential—I want to first ask what makes banking arrangements essential.[2]

My background environment shares features with Gu et al. (2013b), although there are also important differences as discussed below. The formal model incorporates the following ingredients. Time is discrete and continues forever. Each period is divided into two subperiods. There are two types of infinitely lived agents: type 1 and type 2. Type 1 agents consume in the first subperiod while type 2 agents consume in the second. Both types produce the other type's consumption goods in the first subperiod. In a first-best world, it would be efficient to have type 2 deliver his production good to type 1 in the first subperiod, enabling type 1 to consume first, then invest and deliver his production good to type 2 in the second subperiod. In the second subperiod, however, type 1 is tempted to abscond with the proceeds. If type 1 defaults, type 2 knows it and needs to pay a monitoring cost to verify the default and communicate it to the mechanism (or the court/legal system). With probability π, the mechanism receives and records the information, and the deviating type 1 is punished to future autarky. In general, we need to impose an incentive constraint guaranteeing type 1 does not default.

[2] I want to know which frictions lead to banking. As in Townsend (1988): "the theory should explain why markets sometimes exist and sometimes do not, so that economic organization falls out in the solution to the mechanism design problem" . Relatedly, I stick to a generalization of Wallace's (1998) dictum: "money should not be a primitive in monetary theory—in the same way that a firm should not be a primitive in industrial organization theory or a bond a primitive in finance." By extension, banks should not be a primitive in banking theory; they should arise endogenously.

An efficient mechanism is to designate a fraction of the ex-ante homogenous type 1 to be bankers and concentrate monitoring efforts on them. A banker in my model is an agent that has three features: he takes deposits, makes investments on behalf of depositors, and his liabilities (claims on deposits) facilitate third-party transactions. Of course, banks may do more, such as providing liquidity insurance or information processing. I downplay these functions, which have been studied extensively elsewhere, and focus instead on banking arising endogenously as a response to commitment problems and economies of scale.

Consider the cost-benefit trade-off of decreasing the number of bankers from the planner's perspective: Having fewer bankers reduces total monitoring costs, but this means more deposits per banker. Having more deposits, however, increases the bankers' incentives to divert deposits for their own profit, so that they may need to be monitored more rigorously. The result is that the planner needs to give the bankers reward to dissuade opportunistic behavior.

To implement efficient allocations in decentralized competitive markets, there is a tension between equilibrium with free entry and having positive bank profit for incentive reasons. Since bankers have higher payoffs than the non-bankers, all the agents would want to be bankers, which will lead to excess entry. To improve efficiency, the government needs to limit entry of banks, either by charging a tax or rationing bank charters. If the tax on banks is not too high, there exist stationary equilibriums with banks; if the tax on banks is higher, there exists an equilibrium with no banks. For a given tax, we can have too much or too little entry, compared with the efficient outcome. If the tax is almost zero, nearly everyone wants to be a banker, and, thus, there is too much entry. If the tax is almost at the cut-off value, there is too little entry. When the government charges an optimal tax on the banker and gives an optimal transfer to the non-banker, the competitive equilibrium is efficient.

In the decentralization, inside money also helps to implement efficient outcomes. Specifically, type 1 non-banker deposits his production with the banker, who invests on his behalf. The bankers issue receipts for deposits to type 1 non-bankers, which are then transferred to type 2 in the first subperiod and redeemed in the second. The receipts, like bank notes through history, and later checks and debit cards, constitute a transactions medium—inside money.[3]

The model can answer under what conditions it is socially optimal to have few large banks versus many small banks. The optimal number of banks is negatively related to the fixed and marginal monitoring costs, impatience, and the temptation to default, but positively related to the return on real investments. It can explain why the number of banks dropped in the United States. Because the world is more complex than before and it's easier to cheat, the temptation to default increases. With the rise of the temptation to default, the efficient number of banks decreases. The model can explain why we do not want too few banks. The recent literature has stressed "financial fragility" or "too big to fail", but I propose a different explanation. If we have too few banks, each would have too many deposits and this increases their incentive to misbehave. It can also explain why we cannot have too many banks, because of the monitoring cost. The theory provides an explanation for why "executive compensation" is so high in the financial sector: optimally we have to offer these agents big rewards to dissuade them from opportunistic behavior. Finally, it can explain whether "unfettered competition" in banking is optimal. Free competition will lead to too much entry compared with the efficient outcome.

The model is related to several papers about credit with limited commitment, such as Kehoe and Levine (1993), Alvarez and Jermann (2000), and Gu et al. (2013a), but the application and emphasis concern banking. In terms of the mainstream banking literature, Gorton and Winton (2002) and Freixas and Rochet (2008) provide surveys. One approach, originated by Leland and Pyle (1977) and developed by Boyd and Prescott (1986), interprets banks as information-sharing coalitions. Another strand, pioneered by Diamond and Dybvig (1983), interprets banks as coalitions providing liquidity insurance. A related approach, following Diamond (1984) and Williamson (1986, 1987), interprets banks as delegated monitors taking advantage

[3]This is a commonly understood role of banking. Consider Selgin (2007): "Genuine banks are distinguished from other kinds of financial intermediaries by the readily transferable or spendable nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money. Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic debit cards."

of returns to scale. I abstract from liquidity provision and information sharing, and instead highlight banking arising endogenously as a response to commitment problems and economies of scale. Compared with Diamond (1984) and Williamson (1986, 1987), my paper is an infinite-horizon model.[4] It allows banker's reputation to have a role ("reputation" in the sense of Kehoe-Levine). Also, bankers have the incentive to honor their notes that circulate; this would not happen in a finite-horizon world because they would choose not to redeem the notes in the last period and by induction, would renege in any period. Another major difference from most banking literature is that who is a banker, plus how many plus how big, are all endogenous variables.

I also highlight literature where bank liabilities are payment instruments, such as Gu et al. (2013b), Cavalcanti and Wallace (1999a, 1999b), and He et al. (2005, 2008). My model is based on but different from Gu et al. (2013b). In their paper, banking arises endogenously because of heterogeneity, some people are more trustworthy to be bankers.[5] More trustworthy agents accept deposits by less trustworthy agents and invest them. Then these less trustworthy agents use their claims on deposits to facilitate trade with third parties. While in this model, even if the bankers and depositors are ex-ante homogenous, banking can still arise because of economies of scale. Another difference is that the monitoring probability is exogenous in their paper, whereas I endogenize it. Compared with Cavalcanti and Wallace (1999a, 1999b), and He et al. (2005, 2008), where inside money also facilitates trade, a major difference is that they do not have deposits, delegated investments or endogenous monitoring.[6]

With regard to literature on bank number and bank size, there are some empirical

[4]Diamond (1984) and Williamson (1986) are finite-horizon models, and Williamson (1987) is an overlapping generations model where each agent lives for two periods.

[5]In Gu et al. (2013b), agents are better suited to banking when they have a good combination of the following characteristics that make them more trustworthy: they are relatively patient; they are more visible, by which they mean more easily monitored; they have a greater connection to the economic system; they have access to better investment opportunities; and they derive lower payoffs from opportunistically diverting resources.

[6]In addition, see Wallace (2005), Koeppl et al. (2008), Andolfatto and Nosal (2009), Huangfu and Sun (2011), Mills (2008), Sanches andWilliamson (2010), and Monnet and Sanches (2012).

papers. Janicki and Prescott (2006) document the changes in the size distribution of U.S. banks between 1960 and 2005, but they don't provide a theory. Corbae and D'Erasmo (2013) is one of the few papers where both the number and size of banks are endogenously determined. However, their work focuses on the industrial organization approach to banking. They analyze a Stackelberg game between banks and the endogenous bank size distribution arises out of entry and exit in response to shocks to borrowers' production technologies. They focus on mechanisms such as "too big to fail", while I look at something else. Also, a main goal here is a tractable if somewhat stylized framework, so that it is possible to derive analytic and not only numerical results.

The other related literature is that of monitoring. Monitoring has a broad sense of meanings. In Diamond (1984), and Townsend (1979), it means punishing or auditing a borrower who fails to meet contractual obligations in the context of costly state verification. In Broecker (1990), it means screening projects a priori in the context of adverse selection. In Holmstrom and Tirole (1997), and Diamond and Rajan (2001), it means preventing opportunistic behavior of a borrower during the realization of a project (moral hazard). The monitoring here is similar to Diamond (1984), in which the deviation is costly to verify. If there is a default, the banker is detected by the mechanism with probability π.

The rest of the paper is organized as follows. Section 2 describes the basic environment without banking, which provides a simple model of credit with limited commitment and imperfect monitoring. Section 3 describes the environment with banking. Section 4 solves the planner's problem. All of the analysis here focuses on stationary allocations. Section 5 describes the decentralization, which shows how to implement efficient allocations using inside money (bank notes). Section 6 is Conclusion.

2 Environment without Banking

Time is discrete and continues forever. Each period is divided into two subperiods. There are two types of agents: measure 1 of type 1 agents, and measure 1 of type 2 agents. Type 1 agents consume good x and produce good y; type 2 agents consume good y and produce good x. Both goods are produced in the first subperiod; good x is consumed in the first subperiod, while good y is consumed in the second. There is a role for credit since type 1 consumes before type 2, and there is a notion of collateral since good y is produced in the first subperiod. Type 1 agents store and invest good y across subperiods, with fixed gross return ρ in terms of second-subperiod goods. There is no investment across periods, only across subperiods. This may be as simple as pure storage, perhaps for safekeeping, or any other investment; merely for ease of presentation do we impose a fixed return. To generate gains from trade in a simple way, type 2 agents cannot invest for themselves; more generally, we could let them invest, just not as efficiently. We can interpret type 1 agents as borrowers and type 2 agents as lenders.

Utility of type 1 is $U^1(x, y)$, and utility of type 2 is $U^2(\rho y, x)$. Both utility functions are strictly increasing in consumption and decreasing in production, strictly concave, twice differentiable, and $U^j(0, 0) = 0$, $j = 1, 2$.

The timeline of the environment with credit is shown in Figure 2

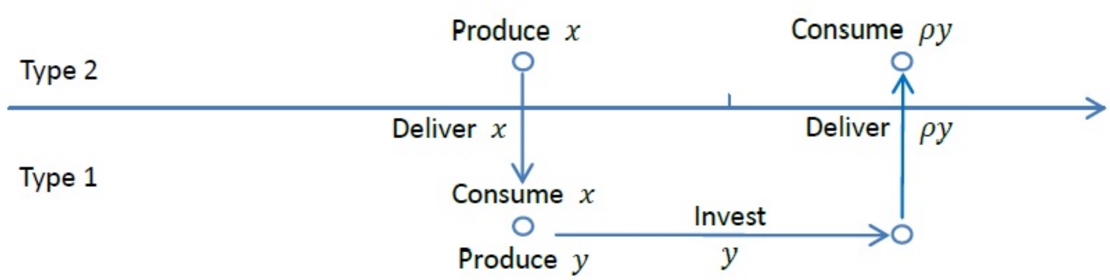


Figure 2: Timeline of the environment with no banking

There are two important frictions:

- Limited Commitment.

  When type 1 agents are supposed to deliver the goods, in the second subperiod, they can renege to obtain a payoff $\lambda\rho y$, over and above $U^1(x, y)$. This is the key incentive issue in the model. If $\lambda = 0$, investment constitutes perfect collateral, since type 1 agent has no gains from reneging when the production cost is sunk. However, if $\lambda > 0$, there is an opportunity cost to deliver the goods. Formally, diversion can be interpreted as type 1 agent consuming the investment returns, but it stands in for the more general idea that investors can divert resources opportunistically.

- Imperfect monitoring.

  Any deviation from the suggested outcome is detected by the mechanism with probability π, punished with future autarky with payoff 0,[7] and is not detected by the mechanism with probability $1 - \pi$. Here, π is endogenous, which means the mechanism can choose monitoring intensity.

  We have many ways to rationalize this monitoring probability; a straightforward one is to assume imperfect record keeping: information concerning deviations "gets lost" with probability $1 - \pi$ across periods. More specifically, if a type 1 agent defaults, the type 2 agent who got defaulted on knows it and needs to verify the default (communicate with and report it to the mechanism, or court/legal system). One example of such costly communication is a lawsuit. With probability π, the mechanism (court/legal system) knows it and records it, and the deviator is punished to future autarky. There are various elements required to punish a deviation: (1) it must be observed by someone; (2) it must be communicated with the mechanism; and (3) it must be recorded/remembered. Failure on one of these dimensions—which is called imperfect memory by Kocherlakota (1998)—is enough to hinder punishments based on reputation.

Assume monitoring each type 1 with probability π implies a utility cost $c(\pi)$, where

$$c(\pi) = \begin{cases} k_0 + \pi k & \text{if } \pi > 0 \\ 0 & \text{if } \pi = 0 \end{cases} \tag{1}$$

[7] We can consider weaker punishments but this is obviously the most effective.

The cost is paid by type 2 agent. Here, k_0 is a fixed cost, and k is a marginal cost, and the cost function implies increasing returns to scale (economies of scale).

The incentive feasible set with no commitment entails two participation constraints for type 1 and type 2 agents and one repayment constraint for type 1 agent. All of the analysis here focuses on stationary allocations.

$$U^1(x, y) \geq 0, \tag{2}$$

$$U^2(\rho y, x) - c(\pi) \geq 0, \tag{3}$$

$$U^1(x, y) + \beta V^1(x, y) \geq U^1(x, y) + \lambda \rho y + (1 - \pi)\beta V^1(x, y), \tag{4}$$

where $V^1(x, y) = U^1(x, y)/(1 - \beta)$ is the continuation value for the type 1 agent. When type 1 agent invests y, he promises to deliver ρy in the second subperiod, but he can always renege for a short-term gain $\lambda \rho y$, and so he delivers the goods only if the repayment constraint satisfies. The LHS is the payoff of not deviating, and the RHS is the payoff to behave opportunistically, again caught with probability π, and punished to future autarky with payoff 0. Note that $U^1(x, y)$ is sunk at the time of decision. The repayment constraint reduces to $U^1(x, y) \geq \frac{(1-\beta)\lambda\rho y}{\beta\pi} = \frac{r\lambda\rho y}{\pi}$ where $r = (1 - \beta)/\beta$. A high r or high λ both increase the temptation to default. We say an agent is more trustworthy when he has smaller $r\lambda$, which means he can credibly promise more (or has better credit).[8]

3 Environment with Banking

The planner designates measure μ of type 1 agents to be bankers and concentrates monitoring efforts on them. The other measure $1 - \mu$ of type 1 agents are non-bankers. (I will explain why those measure μ of agents resemble bankers and why their activity resembles banking later.) The type 1 bankers and non-bankers are

[8] In Gu et al. (2013b), they have one more parameter γ, which is the probability that an agent will want to participate in the "market" each period. This "attachment to the market" parameter provides one more way to make an agent more or less trustworthy, since agents more attached to the market can be more trustworthy. Because it operates very much like r or λ, I omit it.

ex-ante homogeneous. Each type 1 non-banker produces part y_n of good y, deposits his production with type 1 banker, and consumes part x_n of good x. Each type 1 banker produces part y_b of good y, accepts deposits from type 1 non-banker, and consumes part x_b of good x. The bankers can store and invest the combined good y, from their own production and the deposits from the non-bankers, across subperiods, with fixed gross return ρ in terms of second-subperiod goods. The size (assets) of each bank after investment is $\rho y/\mu$.

The cost-benefit trade-off is that having fewer bankers reduces total monitoring costs, but this means more deposits per bank. Having more deposits, however, increases the bankers' incentives to divert deposits for their own profit, and thereby reduces the benefit to the economy.

There are two feasibility constraints for good x and good y. If the type 2 agent produces good x, each type 1 banker consumes part x_b of good x, and each type 1 non-banker consumes part x_n of good x, then $x = \mu x_b + (1-\mu)\, x_n$. Similarly, if each type 1 banker produces part y_b of good y and each type 1 non-banker produces part y_n of good y, we can define $y \equiv \mu y_b + (1-\mu)\, y_n$. Type 1 bankers store and invest y in total across subperiods, get ρy after investment, and deliver the goods to type 2. Each type 2 agent consumes good ρy.

The timeline of the environment with banking is shown in figure 3.

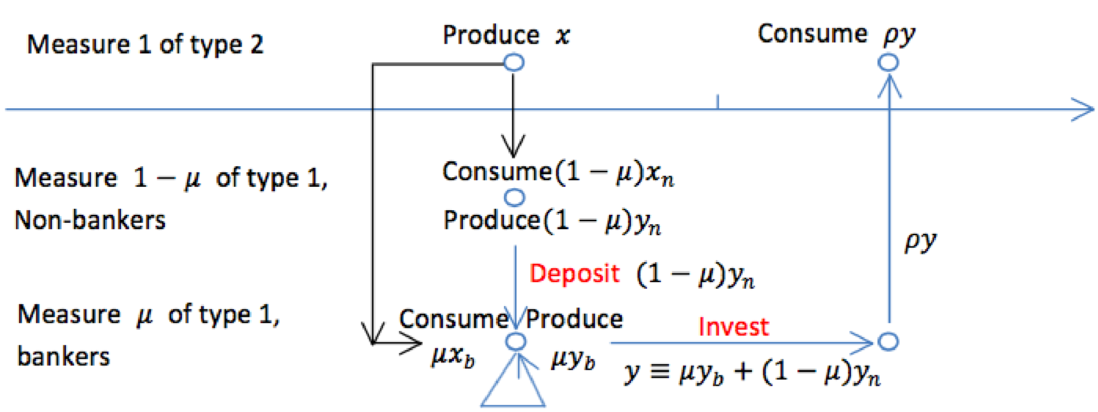


Figure 3: Timeline of the environment with banking

Utility of type 1 banker is $U^1(x_b, y_b)$, utility of type 1 non-banker is $U^1(x_n, y_n)$, and utility of type 2 is $U^2(\rho y, x)$. Both utility functions are strictly increasing in consumption and decreasing in production, strictly concave, twice differentiable, and $U^j(0,0) = 0$, $j = 1, 2$. We assume a discount factor across periods $\beta \in (0,1)$, there is no discount across subperiods with no loss in generality.

The banker in the model is an agent that has three features: he takes deposits, and makes investments on behalf of depositors; and his liabilities (claims on deposits) facilitate third-party transactions. The non-bankers here are depositors. I downplay other functions of banks, such as providing liquidity insurance or information processing, but these can be added using standard methods. Notice that a special case is $\mu = 1$; then we are back to the previous model with pure credit, where there are no depositors and, thus, no banking, all the type 1 agents can invest their own production goods and are tempted to divert the resources for their own profit. However, I am going to show that it is better for the planner to choose $\mu < 1$. Since we have a fixed monitoring cost k_0, it is better to monitor some of the people more intensely, and economize the number of bankers. Why would the planner not choose μ to be a tiny ϵ? In this case, each banker would have so many deposits, and they are more likely to default. Thus, the optimal number of banks is an interior solution. I define the case of no trade to be $\mu = 0$.

We will discuss both the planner's problem and the decentralization in the following two sections. In the planner's problem, the mechanism designer recommends the optimal number and size of bankers as well as efficient consumption and production to different agents. In the decentralization, inside money is needed to implement the optimal outcomes. Specifically, bankers issue receipts for deposits which are then transferred to type 2 in the first subperiod and redeemed in the second. The receipts constitute a transactions medium-inside money.[9]

[9] Here we can compare the theory with some facts from banking history. Institutions that accepted commodity deposits were operating long before the invention of coinage, let alone fiat currency. As Davies (2002) describes the situation, in ancient Mesopotamia and Egypt, goods were often deposited in temple and palace based banks, and, later, private banking houses. "Receipts testifying to these deposits gradually led to transfers to the order not only of depositors but also to a third party." In ancient Babylon, also, as Ferguson (2008) says: "Debts were transferable, hence pay to the bearer rather than a named creditor. Clay receipts or drafts were issued to those who

4 Efficiency

Now what a planner or mechanism can do is to recommend an incentive feasible allocation in the group, as long as no one wants to deviate. All of the analysis here focuses on stationary allocations. The monitoring cost is paid by type 2, and the utility of type 2 is $U^2(\rho y, x) - \mu(k_0 + \pi k)$.[10]

We can define the ex post (conditional on type) welfare as

$$\begin{aligned} W(x_b, y_b, x_n, y_n, x, y) = \ & \theta\left[\mu U^1(x_b, y_b) + (1-\mu) U^1(x_n, y_n)\right] \\ & + (1-\theta)\left[U^2(\rho y, x) - \mu(k_0 + \pi k)\right] \end{aligned} \tag{5}$$

where we put the same weight θ on type 1 banker and non-banker, since they are ex-ante homogeneous, and weight $1-\theta$ on type 2 agent.[11]

The incentive feasible set with no commitment should satisfy the following participation constraints and incentive constraints:

Participation constraints for type 1 banker, non-banker and type 2 agent

$$U^1(x_b, y_b) \geq 0 \tag{6}$$

$$U^1(x_n, y_n) \geq 0 \tag{7}$$

$$U^2(\rho y, x) - \mu(k_0 + \pi k) \geq 0 \tag{8}$$

Repayment constraint for type 1 banker

$$U^1(x_b, y_b) + \beta V^1(x_b, y_b) \geq U^1(x_b, y_b) + \lambda \rho y/\mu + (1-\pi)\beta V^1(x_b, y_b) \tag{9}$$

where $V^1(x_b, y_b) = \frac{U^1(x_b, y_b)}{1-\beta}$ is the continuation value for type 1 banker. The LHS

deposited grain or other commodities at royal palaces or temples." And, also as in the model, "the foundation on which all of this rested was the underlying credibility of a borrower's promise to repay."

[10]There are measure μ of bankers and measure 1 of type 2. The cost of monitoring the bankers are paid by type 2 evenly.

[11]When we put the same weight on type 1 banker and non-banker, it's like there is a lottery where the planner randomly puts μ of type 1 agents as bankers and $1-\mu$ of them as non-bankers, and the summation of utility implies an ex-ante expected utility of a representative type 1 agent.

is the payoff from following the recommendation, while the RHS is the deviation payoff. This reduces to

$$U^1(x_b, y_b) \geq r\lambda\rho y/\pi\mu \tag{10}$$

where $r = (1-\beta)/\beta$. From expression (10), as we decrease the number of banks, the repayment constraint is tighter. If the number of banks is too small, this repayment constraint could be violated. On the other hand, from the welfare function, if the number of banks is too large, the monitoring cost would be too high. Thus, the optimal number of banks is interior.

To sum up, the incentive feasible set with no commitment should satisfy (7), (8) and (10) above. A planner can recommend an incentive feasible solution (x_b, y_b, x_n, y_n, x, y, π, μ) in the group.

$$\begin{aligned}
&\max_{(x_b,y_b,x_n,y_n,x,y,\pi,\mu)} \left\{\theta\left[\mu U^1(x_b,y_b) + (1-\mu)\,U^1(x_n,y_n)\right]\right. \\
&\qquad\left. + (1-\theta)\left[U^2(\rho y, x) - \mu\,(k_0 + \pi k)\right]\right\} \\
\text{s.t.}\quad &\mu x_b + (1-\mu)\,x_n = x \\
&\mu y_b + (1-\mu)\,y_n = y \\
&U^1(x_n,y_n) \geq 0 \\
&U^2(\rho y, x) - \mu\,(k_0+\pi k) \geq 0 \\
&U^1(x_b,y_b) \geq \tfrac{r\lambda\rho y}{\pi\mu}
\end{aligned}$$

Lemma 1. *The repayment constraint must bind,* $U^1(x_b, y_b) = \frac{r\lambda\rho y}{\pi\mu}$.

Proof: If not, we could reduce π to increase the objective function. □

With a bit more structure on preferences, by using quasi-linearity as is usual in these models, we can get even more predictions, especially clean comparative statics results. Suppose

$$\begin{aligned}
U^1(x_b,y_b) &= u(x_b) - y_b \\
U^1(x_n,y_n) &= u(x_n) - y_n \\
U^2(\rho y,x) &= \rho y - v(x)
\end{aligned}$$

where u is strictly increasing and concave, satisfies Inada conditions: $\lim_{x\to 0} u'(x) = +\infty$, $\lim_{x\to+\infty} u'(x) = 0$, and v is strictly increasing and convex.

From the binding repayment constraint for type 1 banker, $u(x_b) - y_b = \frac{r\lambda\rho y}{\pi\mu}$, we know $\pi = \frac{r\lambda\rho y}{[u(x_b)-y_b]\mu}$. Substituting $\pi = \frac{r\lambda\rho y}{[u(x_b)-y_b]\mu}$, $x = \mu x_b + (1-\mu)\, x_n$, and $y_n = \frac{y-\mu y_b}{1-\mu}$ into the planner's problem, it becomes

$$\max_{(x_b,y_b,x_n,y,\mu)} \Big\{\theta\left[\mu u(x_b) + (1-\mu)\, u(x_n) - y\right] + (1-\theta)\left[\rho y - v\left(\mu x_b + (1-\mu)\, x_n\right) - \mu k_0 - \frac{r\lambda\rho y k}{u(x_b)-y_b}\right]\Big\}$$

FOCs

$$u'(x_b) - \frac{1-\theta}{\theta} v'(x) + \frac{1-\theta}{\theta}\frac{r\lambda\rho y k u'(x_b)}{\mu[u(x_b)-y_b]^2} = 0$$

$$y_b^* = 0$$

$$u'(x_n) - \frac{1-\theta}{\theta} v'(x) = 0$$

$$-1 + \frac{1-\theta}{\theta}\left[\rho - \frac{r\lambda\rho k}{[u(x_b)-y_b]}\right] = 0 \Rightarrow x_b^* = \overline{x_b}\,(k, r, \lambda, \rho)$$

$$u(x_b) - u(x_n) - \frac{1-\theta}{\theta} v'(x)\,(x_b - x_n) - \frac{1-\theta}{\theta} k_0 = 0$$

Proposition 1. *$x_b^* > x_n^*$, and $y_b^* = 0$. That is to say, the bankers can consume more than the non-bankers and do not need to produce.*[12]

Proof: See the Appendix.

The intuition is that the planner needs to give the bankers some reward to dissuade opportunistic behavior (satisfies the repayment constraint).

Proposition 2. *$\frac{\partial x_b^*}{\partial k} > 0$, $\frac{\partial x_b^*}{\partial r} > 0$, $\frac{\partial x_b^*}{\partial \lambda} > 0$, and $\frac{\partial x_b^*}{\partial \rho} < 0$.*

Proof: See the Appendix.

As the marginal monitoring cost increases, the first order effect is to reduce monitoring probability, and, thus, the bankers are more likely to renege, we have to compensate them more such that they don't deviate. Similarly, if the interest rate (impatience) increases, or if there is more temptation to behave badly, we need to

[12] If we use the general additively separable utility, $U^1(x, y) = u(x) - v(y)$, we can get $x_b^* > x_n^*$ and $y_b^* < y_n^*$, the bankers can consume more than the non-bankers and produce less. With quasilinear utility, however, bankers specialize to just invest and not produce.

give the bankers more compensation. If the rate of return increases, we can give the bankers less compensation.

Proposition 3. *Suppose* $u(x) = \frac{x^{1-\alpha}-1}{1-\alpha}$, *where* $\alpha > 0$, we have $\frac{\partial\left(x_b^*-x_n^*\right)}{\partial k} > 0$, $\frac{\partial\left(x_b^*-x_n^*\right)}{\partial r} > 0$, $\frac{\partial\left(x_b^*-x_n^*\right)}{\partial \lambda} > 0$, $\frac{\partial\left(x_b^*-x_n^*\right)}{\partial \rho} < 0$.

Proof: See the Appendix.

Note that $x_b^*-x_n^*$ is the premium that the bankers take because of the commitment problems. It's sort of the rent extracted by the banker. The premium that the bankers take is positively related to the marginal monitoring cost, the interest rate (impatience), and the temptation to default, but negatively related to the rate of return.

Proposition 4. $\frac{\partial \mu^*}{\partial k_0} < 0$, $\frac{\partial \mu^*}{\partial k} < 0$, $\frac{\partial \mu^*}{\partial r} < 0$, $\frac{\partial \mu^*}{\partial \lambda} < 0$, *and* $\frac{\partial \mu^*}{\partial \rho} > 0$.

Proof: See the Appendix.

As the fixed monitoring cost increases, we definitely should have fewer bankers. We can interpret the other comparative statics results of the optimal number of bankers through the premium that the bankers take. As the marginal monitoring cost (or impatience, or the temptation to default) increases, we should have less bankers because it's more expensive to use them. As the rate of return increases, we should have more bankers because it's cheaper to use them.

The proposition can explain why the number of banks dropped in the United States. Because the world is more complex than before and it's easier for people to cheat, the temptation to default λ increases. According to the effects of parameter changes, with the rise of the temptation to default, the number of banks decreases.

5 Equilibrium

From the planner's problem, we can get the second-best solution with frictions (limited commitment). Then I want to find a decentralized pricing mechanism such that the second-best allocations can be realized. Here, I am using the Walrasian pricing mechanism, where everyone takes prices as given. Because the agents who are selected to be bankers have a higher payoff than the non-bankers in the planner's problem, all the agents would want to be bankers, which is not efficient. Thus, the government needs to limit entry of banks. one natural way is to charge a tax τ on bankers and give a transfer t to non-bankers; another way is to simply impose a quota by limiting the number of bank charters.

5.1 Charging a Tax

To implement the efficient outcomes, we also need inside money (bank notes). When a type 1 non-banker wants to consume in the first subperiod, he produces and deposits output y_n with a type 1 banker in exchange for a receipt. Think of the receipt as a bearer note for goods y. He then gives this note to a type 2 agent in exchange for his consumption good x_n. Naturally, the type 2 agent accepts it, and carries this note to the second subperiod. Each type 1 banker borrows $\hat{y}$ from the non-banker, produces y_b by himself, and gives some notes to a type 2 agent in exchange for his consumption good x_b. When the type 2 agent wants to consume in the second subperiod, he redeems all the notes for his consumption good. Type 1 banker pays type 2 agent out of deposits—principal plus return on investments, ρy—to clear, or settle, the obligation. In this way the bank liabilities serve as inside money, like banknotes, checkbooks and debit cards.

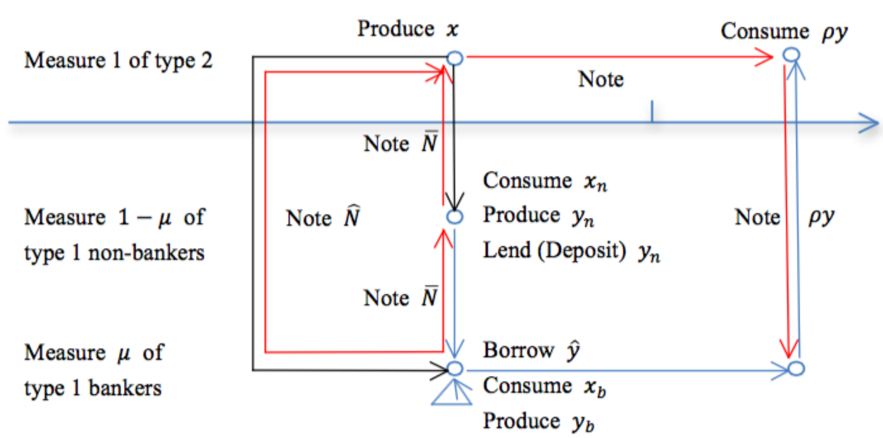


Figure 4: Timeline of decentralization with banking

In sum, there are three types of trades. In the first subperiod, agents trade good x and bank notes issued by the banker; type 1 non-banker and banker trade good y and banknotes. In the second subperiod, A banknote entitles type 2 one unit of good y from the banker. The timeline is shown in Figure 4.

Let V^1_{bt} be the banker's value function at time t given an allocation (x_{bt}, y_{bt}), which specifies that the banker consumes x_{bt} and produces y_{bt}, then the Bellman equation for the banker is

$$V^1_{bt} = U^1(x_{bt}, y_{bt}) + \beta V^1_{bt+1}. \tag{11}$$

Similarly, the bellman equations for type 1 non-banker and type 2 agent are, respectively,

$$V^1_{nt} = U^1(x_{nt}, y_{nt}) + \beta V^1_{nt+1}, \tag{12}$$

$$V^2_t = U^2(\rho y_t, x_t) + \beta V^2_{t+1}. \tag{13}$$

The repayment constraint for the banker is

$$\lambda\rho\left(\hat{y}_t + y_{bt}\right) + (1 - \Pi)\,\beta V^1_{bt+1} \leq \beta V^1_{bt+1}. \tag{14}$$

The RHS is the payoff from following the recommendation while the LHS is the deviation payoff. It reduces to

$$\hat{y}_t + y_{bt} \leq \frac{\beta\Pi}{\lambda\rho} V^1_{bt+1}. \tag{15}$$

By difining $\phi_t \equiv \frac{\beta\Pi}{\lambda\rho} V^1_{bt+1}$ as the debt limit, it is convenient to rewrite the repayment constraint as

$$\hat{y}_t + y_{bt} \leq \phi_t. \tag{16}$$

Using the bellman equation (11), we can express this recursively to make it clear that the debt limit in one period depends on the debt limit in the next period:

$$\phi_{t-1} = \frac{\beta\Pi}{\lambda\rho} U^1(x_{bt}, y_{bt}) + \beta\phi_t \tag{17}$$

There are a large number of spatially distinct Walrasian markets, and the agents trade short-term (across subperiod) credit contracts taking prices as given. Let goods y in the second subperiod be numeraire, the price of goods x in the first subperiod is p_{xt}, and the price of goods y in the first subperiod is p_{yt}. The banker maximizes utility given his budget constraint and repayment constraint. We drop the participation constraint because autarky is always feasible, and use the same preference functions as in the efficiency part.

$$\begin{aligned} & \max_{(x_{bt}, y_{bt}, \hat{y}_t)} u(x_{bt}) - y_{bt} - \tau \\ \text{s.t.} \quad & p_{xt} x_{bt} + p_{yt} \hat{y}_t = \rho(\hat{y}_t + y_{b_t}) \\ & \hat{y}_t + y_{bt} \leq \phi_t \end{aligned} \tag{18}$$

where $\rho > 1$.

Type 1 non-banker maximizes utility given his budget constraint.

$$\max_{(x_{nt}, y_{nt})} u(x_{nt}) - y_{nt} + t \quad \text{s.t.} \quad p_{xt} x_{nt} = p_{yt} y_{nt} \tag{19}$$

Type 2 agent maximizes utility given his budget constraint.

$$\max_{(x_t, y_t)} \rho y_t - v(x_t) - \mu_t(k_0 + \Pi k) \quad \text{s.t.} \quad \rho y_t = p_{xt} x_t \tag{20}$$

Notice that the monitoring probability Π is exogenous in the decentralization, because otherwise, there will be a free-rider problem here. The cost $\mu_t(k_0 + \Pi k)$ is kind of a tax on type 2 to be used by the "government" to pay monitoring.

We have the following goods market clearing conditions: For goods y in the first subperiod, we have

$$\mu_t \hat{y}_t = (1 - \mu_t)\, y_{nt.} \tag{21}$$

For goods y in the second subperiod, we have

$$\rho y_t = \mu_t \rho \hat{y}_t + \mu_t \rho y_{bt}. \tag{22}$$

For goods x in the first subperiod, we have

$$\mu_t x_{bt} + (1 - \mu_t)\, x_{nt} = x_t. \tag{23}$$

Combining the first two conditions, we have

$$y_t = \mu_t y_{bt} + (1 - \mu_t)\, y_{nt.} \tag{24}$$

The free entry conditions are

$$\begin{aligned} \mu_t = 0 \quad & \text{if} \quad u(x_{bt}) - y_{bt} - \tau < u(x_{nt}) - y_{nt} + t \\ \mu_t \in (0,1) \quad & \text{if} \quad u(x_{bt}) - y_{bt} - \tau = u(x_{nt}) - y_{nt} + t \\ \mu_t = 1 \quad & \text{if} \quad u(x_{bt}) - y_{bt} - \tau > u(x_{nt}) - y_{nt} + t \end{aligned} \tag{25}$$

Following Alvarez and Jermann (2000), for all t, the equilibrium debt limit ϕ_t is defined as follows: the banker is indifferent between repaying ϕ_t and defaulting. In any feasible allocation, payoffs, and hence ϕ_t , must be bounded (so, as in many other models, we rule out explosive bubbles). We can also bound (x_{bt}, y_{bt}, $\hat{y}_t$, x_{nt}, y_{nt}, x_t, y_t) without loss in generality. Hence we have the following definition:

Definition 1. An equilibrium is a specification of nonnegative and bounded sequences of quantities $\{x^e_{bt},\, y^e_{bt},\, \hat{y}^e_t,\, x^e_{nt},\, y^e_{nt},\, x^e_t,\, y^e_t\}_{t=1}^{\infty}$, prices $\{p^e_{xt},\, p^e_{yt}\}_{t=1}^{\infty}$, measure of bankers $\{\mu^e_t\}_{t=1}^{\infty}$ and credit limits $\{\phi^e_t\}_{t=1}^{\infty}$ such that for all t

1. $(x^e_{bt}, y^e_{bt}, \hat{y}^e_t)$ solves the banker's problem given ϕ^e_t.
2. (x^e_{nt}, y^e_{nt}) solves the type 1 non-banker's problem.
3. (x^e_t, y^e_t) solves the type 2 agent's problem.
4. Markets clear.
5. Free entry.
6. ϕ^e_t solves the difference equation (17) given (x^e_{bt}, y^e_{bt}).

Solve the type 1 non-banker's problem, we have

$$u'(x^e_{nt}) = p_{xt}/p_{yt} \Rightarrow x^e_{nt} = u'^{-1}(p_{xt}/p_{yt}) \tag{26}$$

The demand of goods x for type 1 non-banker x^e_{nt} is decreasing in p_{xt}.

Solve the type 2 agent's problem, we have

$$p_{xt} = v'(x^e_t) \Rightarrow x^e_t = v'^{-1}(p_{xt}) \tag{27}$$

The supply of goods x for type 2 agent x^e_t is increasing in p_{xt}.

Lemma 2. *There is an equilibrium only if $p_{yt} \leq \rho$.*

Proof: See the Appendix.

This lemma says that for the banker, the return of borrowing is always larger than or equal to the cost in equilibrium.

Lemma 3. *When $p_{yt} = \rho$, there is an equilibrium with no banks (trade).*

Proof: See the Appendix.

This lemma says when the return of borrowing is equal to the cost, there is an equilibrium with no banks (trade) if we charge a tax on bankers and give a transfer to non-bankers.

Lemma 4. *When $p_{yt} < \rho$, the repayment constraint must bind, $\hat{y}^e_t + y^e_{bt} = \phi_t$.*

Proof: If not, the banker could increase $\hat{y}_t$ to increase the objective function. □

From the budget constraint and the binding repayment constraint for type 1 banker, we have

$$y_{bt} = \frac{p_{xt}x_{bt} - (\rho - p_{yt})\,\phi_t}{p_{yt}} \tag{28}$$

Lemma 5. *When $p_{yt} < \rho$, $y^e_{bt} = 0$.*

Proof: Since the return of borrowing is larger than the cost, the banker would like to borrow as much as possible and produce nothing. □

From $y^e_{bt} = \frac{p_{xt}x^e_{bt} - (\rho - p_{yt})\phi_t}{p_{yt}} = 0$, we have $x^e_{bt} = (\rho - p_{yt})\,\phi_t/p_{xt}$. The demand of goods x for the banker x^e_{bt} is decreasing in p_{xt}.

The bellman equation (17) can be rewritten as

$$\phi_{t-1} = f(\phi_t) \equiv \begin{cases} \frac{\beta\Pi}{\lambda\rho}\left[u((\rho - p_{yt})\,\phi_t/p_{xt}) - \tau\right] + \beta\phi_t & \text{if } 0 < \phi_t < y_b^{**} + \hat{y}^{**} \\ \frac{\beta\Pi}{\lambda\rho}\left[u((x_b^{**}) - y_b^{**} - \tau\right] + \beta\phi_t & \text{if } \phi_t \geq y_b^{**} + \hat{y}^{**} \\ 0 & \text{if } \phi_t = 0 \end{cases} \tag{29}$$

where x_b^{**}, y_b^{**} and $\hat{y}^{**}$ denote equilibrium sollutions ignoring the repayment constraint. The dynamical system describes the evolution of the debt limit in terms of itself. The three cases represent the evolution when the repayment constraint is binding, not binding, and when the debt limit is zero respectively.[13] This system is forward looking, naturally, in the sense that the debt limit in one period depends on the debt limit in the next period.

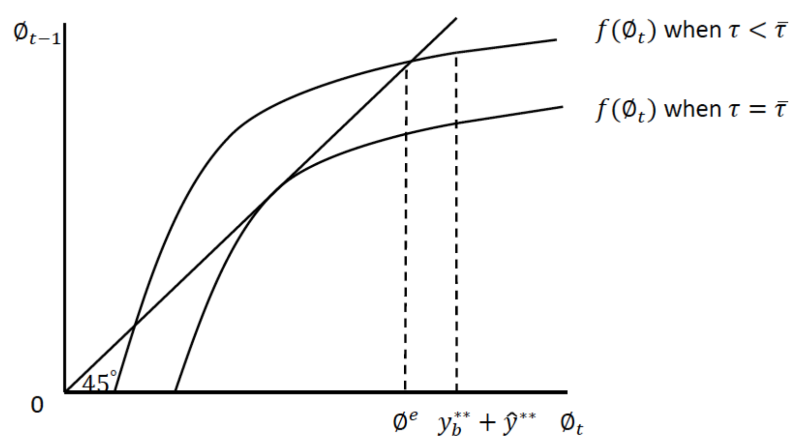


Figure 5: Steady state in terms of ϕ when $p_y < \rho$

A stationary equilibrium, or steady state, is a fixed point such that $f(\phi) = \phi$. Obviously $\phi = 0$ is one such point. A non-degenerate steady state is a solution to

[13]When the debt limit is zero, there is to be no credit in the future, you have nothing to lose by reneging, so no one will extend you credit today. Note that in this case there is no banker (trades), and no one needs to pay the tax.

$f(\phi) = \phi > 0$. The graph of the steady state in terms of ϕ when $p_y < \rho$ is shown in Figure 5. The assumption that u satisfies Inada conditions $\lim_{x \to 0} u'(x) = +\infty$ and $\lim_{x \to +\infty} u'(x) = 0$ guaranties:

Proposition 5. *When $p_y < \rho$, if $0 < \tau < \bar{\tau}$, there are two stationary equilibriums with banks (trade), one is stable and the other is unstable; if $\tau = \bar{\tau}$, there is a unique stationary equilibrium with banks; if $\tau > \bar{\tau}$, there is an equilibrium with no banks.*

Proof: When $p_y < \rho$, the repayment constraint is binding. If $\tau = \bar{\tau}$, $\exists\,!\ \phi^e > 0$; if $\tau < \bar{\tau}$, $\exists$ two positive solutions. However, the one with larger ϕ^e such that $f'(\phi) < 1$ is stable, while the one with smaller ϕ^e such that $f'(\phi) > 1$ is unstable. The larger credit limit ϕ^e corresponds to a higher x_b^e and a higher payoff with $u'(x_b) < \frac{r\lambda\rho p_x}{\Pi(\rho - p_y)}$, and the smaller credit limit ϕ^e corresponds to a lower x_b^e and a lower payoff with $u'(x_b) > \frac{r\lambda\rho p_x}{\Pi(\rho - p_y)}$. □

When $\phi_{t-1} = \phi_b = \phi$, $x_{bt-1} = x_{bt} = x_b$, $p_{xt-1} = p_{xt} = p_x$ and $p_{yt-1} = p_{yt} = p_y$, The steady state condition regarding ϕ is $\phi = \frac{\beta\Pi}{\lambda\rho}\left[u((\rho - p_y)\,\phi/p_x) - \tau\right] + \beta\phi$, which reduces to

$$u(\frac{(\rho - p_y)\,\phi}{p_x}) = \frac{r\lambda\rho\phi}{\Pi} + \tau. \tag{30}$$

From $y_b = 0$, we have $\phi = x_b p_x / (\rho - p_y)$, thus the steady state condition regarding x_b is

$$u(x_b) = \frac{r\lambda\rho p_x x_b}{\Pi\,(\rho - p_y)} + \tau. \tag{31}$$

Use the market clearing conditions: For goods x

$$\mu x_b + (1 - \mu)\, x_n = x \tag{32}$$

For goods y, $y = \mu y_b + (1 - \mu)\, y_n$. Using the budget constraints, where $y = p_x x/\rho$ and $y_n = p_x x_n / p_y$, and $y_b = 0$, we have

$$x = (1 - \mu)\,\rho x_n / p_y \tag{33}$$

Substituting (33) into (32), we have

$$\mu^e = \frac{(\rho/p_y - 1)\, x_n}{(\rho/p_y - 1)\, x_n + x_b} \in (0,1) \tag{34}$$

When $\tau \leq \bar{\tau}$, the equilibrium $(x_b^e, x_n^e, x^e, p_x^e, p_y^e, \mu^e)$ solves

$$\begin{aligned}
u(x_b) &= \frac{r\lambda\rho p_x x_b}{\Pi\,(\rho - p_y)} + \tau \\
u'(x_n) &= p_x/p_y \\
p_x &= v'(x) \\
\mu x_b + (1-\mu)\, x_n &= x \\
x &= (1-\mu)\,\rho x_n/p_y \\
u(x_b) - \tau &= u(x_n) - p_x x_n + t
\end{aligned}$$

where the first three equations are from the maximization problems of the type 1 banker, type 1 non-banker and type 2 agent; the fourth and fifth one are the market-clearing conditions for good x and good y, and the last one is the free-entry condition.

Compare it with the planner's problem, where $(x_b^*, x_n^*, x^*, \mu^*)$ solves

$$\begin{aligned}
u(x_b) &= \frac{r\lambda\rho k}{\rho - \frac{\theta}{1-\theta}} \\
u'(x_n) &= \frac{1-\theta}{\theta} v'(x) \\
\mu x_b + (1-\mu)\, x_n &= x \\
u(x_b) - u(x_n) + \frac{1-\theta}{\theta}\frac{u'(x_n)(x - x_b)}{1-\mu} &= \frac{1-\theta}{\theta} k_0
\end{aligned}$$

Proposition 6. *The competitive equilibrium implements the efficient allocations if the following conditions are satisfied: (i)* $p_y^e = \frac{\theta}{1-\theta} < \rho$*, and (ii)* $\tau = \tau^*$*, where* τ^* *solves* $\frac{p_x^e(\tau) x_b^e(\tau)}{\Pi} + \frac{\tau(\rho - \frac{\theta}{1-\theta})}{r\lambda\rho} = k$*, and (iii)* $t = t^*$*, where* t^* *solves* $u\left[x_b^e(t)\right] - u\left[x_n^e(t)\right] + \frac{1-\theta}{\theta}\frac{u'[x_n^e(t)]\left[x^e(t) - x_b^e(t)\right]}{1-\mu^e(t)} = \frac{1-\theta}{\theta} k_0$*.*

Proof: See the Appendix.

For a given tax, we can have too much or too little entry, compared with the efficient outcome. If the tax is almost zero, nearly everyone wants to be a banker, and, thus, there is too much entry. If the tax is almost at the cut-off value, there is too little entry. When the government charges an optimal tax on the banker and gives an optimal transfer to the non-banker, the competitive equilibrium implementes the efficient allocations.

5.2 Rationing Bank Charters

The government can also impose a quota by limiting the number of bank charters at the efficiency level μ^*. In this way, we don't have the free entry condition and there is excess demand. A lottery is the easiest way to do the rationing scheme.

The equilibrium $(x_b^e, x_n^e, x^e, p_x^e, p_y^e)$ solves

$$\begin{aligned} u(x_b) &= \frac{r\lambda\rho p_x x_b}{\Pi\left(\rho - p_y\right)} \\ u'(x_n) &= p_x/p_y \\ p_x &= v'(x) \\ \mu^* x_b + \left(1-\mu^*\right) x_n &= x \\ x &= \left(1-\mu^*\right)\rho x_n/p_y \end{aligned}$$

Proposition 7. *The competitive equilibrium implements the efficient allocations if the following conditions are satisfied: (i) $p_y^e = \frac{\theta}{1-\theta} < \rho$, and (ii) $\mu = \mu^*$, where μ^* is the efficient number of bankers.*

6 Conclusion

I develop a theoretical model with limited commitment and endogenous monitoring to study the optimal number and size of bankers from the planner's point of view. I begin by specifying preferences, technologies, and frictions, then illustrate how it can be desirable to designate some part of the ex-ante homogeneous agents to perform certain functions resembling banking: they accept deposits, they make investment, and their liabilities facilitate third party transactions. The mechanism is that if we have a utility cost to monitor the bankers, we can consider the cost-benefit trade-off of decreasing the number of bankers. Having fewer bankers reduces total monitoring cost, but for a given amount of total deposits, this means more deposits per banker. Having more deposits, however, increases the bankers' incentives to divert deposits for their own profit and thereby, reduces the benefit to the economy. The result is that the planner needs to give the bankers some reward to dissuade such opportunistic behavior.

To implement efficient allocations, there is a tension between equilibrium with free entry and having positive bank profit for incentive reasons. In the competitive equilibrium, when the tax on banks is not too high, there exist non-degenerate stationary equilibriums. The allocation is optimal only if the government limits entry of banks. One natural way is to charge a tax on bankers and give a transfer to non-bankers; another way is to simply impose a quota by limiting the number of bank charters.

The model can answer under what conditions it is socially optimal to have few large banks versus many small banks. The optimal number of banks is negatively related to the fixed and marginal monitoring costs, impatience, and the temptation to default, but positively related to the return on real investments. It can explain why the number of banks dropped in the United States through the rise of the temptation to default. The model can explain why we do not want too few banks. If we have too few banks, each would have too many deposits, and this increases their incentive to misbehave. It can also explain why we cannot have too many banks, because of the monitoring cost. The theory provides an explanation for why "executive compensation" is so high in the financial sector: because we have to offer these agents big rewards to dissuade them from opportunistic behavior. Finally, it can explain whether "unfettered competition" in banking is optimal. Free competition will lead to excess entry compared with the efficient outcome.

Appendix

Proof of Proposition 1:

From the first and third FOCs, we have $u'(x_b) = u'(x_n) - \frac{1-\theta}{\theta}\frac{r\lambda\rho yku'(x_b)}{\mu[u(x_b)-y_b]^2}$, thus, $u'(x_b^*) < u'(x_n^*)$. Because $u''(.) < 0$, we have $x_b^* > x_n^*$. From the second FOC, we have $y_b^* = 0$. □

Proof of Proposition 2:

The maximization problem for choosing y is $\max\limits_{y} - \theta y + (1-\theta)\left[\rho y - \frac{r\lambda\rho ky}{u(x_b)}\right]$

$y =$ anything if $-1 + \frac{1-\theta}{\theta}\left[\rho - \frac{r\lambda\rho k}{u(x_b)}\right] = 0 \Rightarrow u(x_b) = \frac{r\lambda k}{1-\frac{\theta}{1-\theta}\frac{1}{\rho}}$. Thus, $\frac{\partial x_b^*}{\partial k} = \frac{\partial \overline{x_b}}{\partial k} > 0$, $\frac{\partial x_b^*}{\partial r} = \frac{\partial \overline{x_b}}{\partial r} > 0$, $\frac{\partial x_b^*}{\partial \lambda} = \frac{\partial \overline{x_b}}{\partial \lambda} > 0$, and $\frac{\partial x_b^*}{\partial \rho} = \frac{\partial \overline{x_b}}{\partial \rho} < 0$. Then, from the other FOCs, we can solve y^*.

$y = +\infty$ if $-1 + \frac{1-\theta}{\theta}\left[\rho - \frac{r\lambda\rho k}{u(x_b)}\right] > 0$, it is not the solution.

$y = 0$ if $-1 + \frac{1-\theta}{\theta}\left[\rho - \frac{r\lambda\rho k}{u(x_b)}\right] < 0$, it is not the solution. □

Proof of Proposition 3:

Differentiation of the FOCs yields

$$\begin{bmatrix} -\frac{1-\theta}{\theta}v''(x)\,(1-\mu) & \frac{A}{y} & -\frac{1-\theta}{\theta}v''(x)(\overline{x_b}-x_n) - \frac{A}{\mu^2} \\ u''(x_n) - \frac{1-\theta}{\theta}v''(x)\,(1-\mu) & 0 & -\frac{1-\theta}{\theta}v''(x)(\overline{x_b}-x_n) \\ -\frac{1-\theta}{\theta}v''(x)(\overline{x_b}-x_n)\,(1-\mu) & 0 & -\frac{1-\theta}{\theta}v''(x)(\overline{x_b}-x_n)^2 \end{bmatrix} \begin{bmatrix} dx_n \\ dy \\ d\mu \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & \frac{A}{k} + B\frac{\partial \overline{x_b}}{\partial k} & \frac{A}{r} + B\frac{\partial \overline{x_b}}{\partial r} & \frac{A}{\lambda} + B\frac{\partial \overline{x_b}}{\partial \lambda} & \frac{A}{\rho} + B\frac{\partial \overline{x_b}}{\partial \rho} \\ 0 & C\frac{\partial \overline{x_b}}{\partial k} & C\frac{\partial \overline{x_b}}{\partial r} & C\frac{\partial \overline{x_b}}{\partial \lambda} & C\frac{\partial \overline{x_b}}{\partial \rho} \\ -1 & \Sigma\frac{\partial \overline{x_b}}{\partial k} & \Sigma\frac{\partial \overline{x_b}}{\partial r} & \Sigma\frac{\partial \overline{x_b}}{\partial \lambda} & \Sigma\frac{\partial \overline{x_b}}{\partial \rho} \end{bmatrix} \begin{bmatrix} dk_0 \\ dk \\ dr \\ d\lambda \\ d\rho \end{bmatrix}$$

$=0$,

where $A = \frac{1-\theta}{\theta} \frac{r\lambda\rho yku'(\overline{x_b})}{\mu[u(\overline{x_b})]^2}$, $B = u''(\overline{x_b}) - \frac{1-\theta}{\theta} v''(x)\mu + \frac{1-\theta}{\theta} \frac{r\lambda\rho yk}{\mu} \frac{u''(\overline{x_b})[u(\overline{x_b})]^2 - 2u(\overline{x_b})[u'(\overline{x_b})]^2}{[u(\overline{x_b})]^4}$, $C = -\frac{1-\theta}{\theta} v''(x)\mu$, $\Sigma = u'(\overline{x_b}) - u'(x_n) - \frac{1-\theta}{\theta}\mu v''(x)(\overline{x_b} - x_n)$. The determinant of the square matrix is

$$D = \frac{\left(\frac{1-\theta}{\theta}\right)^2 r\lambda\rho k u'(\overline{x_b}) u''(x_n) v''(x) (\overline{x_b} - x_n)^2}{\mu \left[u(\overline{x_b})\right]^2} < 0.$$

The partial derivatives of x_n^* with respect to each of its arguments are, respectively,

$$\begin{aligned}
\frac{\partial x_n^*}{\partial k} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} \left\{ \left[u'(\overline{x_b}) - u'(x_n)\right] v''(x)(\overline{x_b} - x_n) \right\} \frac{\partial \overline{x_b}}{\partial k}}{D} > 0, \\
\frac{\partial x_n^*}{\partial r} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} \left\{ \left[u'(\overline{x_b}) - u'(x_n)\right] v''(x)(\overline{x_b} - x_n) \right\} \frac{\partial \overline{x_b}}{\partial r}}{D} > 0, \\
\frac{\partial x_n^*}{\partial \lambda} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} \left\{ \left[u'(\overline{x_b}) - u'(x_n)\right] v''(x)(\overline{x_b} - x_n) \right\} \frac{\partial \overline{x_b}}{\partial \lambda}}{D} > 0, \\
\frac{\partial x_n^*}{\partial \rho} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} \left\{ \left[u'(\overline{x_b}) - u'(x_n)\right] v''(x)(\overline{x_b} - x_n) \right\} \frac{\partial \overline{x_b}}{\partial \rho}}{D} < 0.
\end{aligned}$$

The partial derivatives of $x_b^* - x_n^*$ with respect to each of its arguments are, respectively,

$$\begin{aligned}
\frac{\partial \left(x_b^* - x_n^*\right)}{\partial k} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} v''(x)(\overline{x_b} - x_n)\Phi \frac{\partial \overline{x_b}}{\partial k}}{D} > 0, \\
\frac{\partial \left(x_b^* - x_n^*\right)}{\partial r} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} v''(x)(\overline{x_b} - x_n)\Phi \frac{\partial \overline{x_b}}{\partial r}}{D} > 0, \\
\frac{\partial \left(x_b^* - x_n^*\right)}{\partial \lambda} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} v''(x)(\overline{x_b} - x_n)\Phi \frac{\partial \overline{x_b}}{\partial \lambda}}{D} > 0, \\
\frac{\partial \left(x_b^* - x_n^*\right)}{\partial \rho} &= \frac{\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2} v''(x)(\overline{x_b} - x_n)\Phi \frac{\partial \overline{x_b}}{\partial \rho}}{D} < 0.
\end{aligned}$$

where $\Phi = u''(x_n)(\overline{x_b} - x_n) - \left[u'(\overline{x_b}) - u'(x_n)\right]$. According to the mean value theorem, there exists a point ξ in $(x_n, \overline{x_b})$ such that $u''(\xi) = \frac{u'(\overline{x_b}) - u'(x_n)}{\overline{x_b} - x_n}$, thus, for $u(x) = \frac{x^{1-\alpha} - 1}{1-\alpha}$, where $\alpha > 0$, we have $\Phi = \left[u''(x_n) - u''(\xi)\right](\overline{x_b} - x_n) < 0$. □

Proof of Proposition 4:

The partial derivatives of μ^* with respect to each of its arguments are, respectively,

$$\frac{\partial \mu^*}{\partial k_0} = \frac{-\left(\frac{1-\theta}{\theta}\right)^2 \frac{r\lambda\rho k u'(\overline{x_b})}{\mu[u(\overline{x_b})]^2}\left[u''(x_n) - \frac{1-\theta}{\theta}(1-\mu)\, v''(x)\right]}{D} < 0,$$

$$\frac{\partial \mu^*}{\partial k} = \frac{-\frac{1-\theta}{\theta}\frac{r\lambda\rho k u'(\overline{x_b})\Omega}{\mu[u(\overline{x_b})]^2}\frac{\partial \overline{x_b}}{\partial k}}{D} < 0,$$

$$\frac{\partial \mu^*}{\partial r} = \frac{-\frac{1-\theta}{\theta}\frac{r\lambda\rho k u'(\overline{x_b})\Omega}{\mu[u(\overline{x_b})]^2}\frac{\partial \overline{x_b}}{\partial r}}{D} < 0,$$

$$\frac{\partial \mu^*}{\partial \lambda} = \frac{-\frac{1-\theta}{\theta}\frac{r\lambda\rho k u'(\overline{x_b})\Omega}{\mu[u(\overline{x_b})]^2}\frac{\partial \overline{x_b}}{\partial \lambda}}{D} < 0,$$

$$\frac{\partial \mu^*}{\partial \rho} = \frac{-\frac{1-\theta}{\theta}\frac{r\lambda\rho k u'(\overline{x_b})\Omega}{\mu[u(\overline{x_b})]^2}\frac{\partial \overline{x_b}}{\partial \rho}}{D} > 0,$$

where $\Omega = \frac{1-\theta}{\theta}(1-\mu)\, v''(x)\left[u'(\overline{x_b}) - u'(x_n)\right] - u''(x_n)\left[u'(\overline{x_b}) - u'(x_n) - \frac{1-\theta}{\theta}\mu v''(x)(\overline{x_b} - x_n)\right].$

□

Proof of Lemma 2:

The Lagrangean function for the banker is:

$$\mathcal{L} = u(x_b) - y_b - \tau + \lambda_1\left[\rho(\hat{y} + y_b) - p_x x_b - p_y \hat{y}\right] + \lambda_2\left(\phi - \hat{y} - y_b\right) + \lambda_3 y_b.$$

The critical points of the Lagrangean are the solutions $(x_b,\ y_b,\ \hat{y},\ \lambda_1,\ \lambda_2,\ \lambda_3)$ to the following system of equations:

1. $u'(x_b) - \lambda_1 p_x = 0,$

2. $-1 + \lambda_1\rho - \lambda_2 + \lambda_3 = 0,$

3. $\lambda_1\rho - \lambda_1 p_y - \lambda_2 = 0$,

4. $\lambda_2 \geq 0,\ \phi - \hat{y} - y_b \geq 0,\ \lambda_2\left(\phi - \hat{y} - y_b\right) = 0,$

5. $\lambda_3 \geq 0$, $y_b \geq 0$, $\lambda_3 y_b = 0$,

where the first three equations are the first order conditions for x_b, y_b, and $\hat{y}$, and the last two equations are the complementary slackness conditions. Because $\lambda_1 (\rho - p_y) = \lambda_2 \geq 0$, we have $p_y \leq \rho$. $\square$

Proof of Lemma 3:

When $p_y = \rho$, we have $\lambda_2 = 0$.

If $y_b > 0$, we have $\lambda_3 = 0 \Rightarrow \lambda_1 = 1/\rho$, $u'(x_b) = p_x/\rho = p_x/p_y = u'(x_n) \Rightarrow x_b^e = x_n^e$, $y_b^e = y_n^e$. If we charge a tax on bankers and give a transfer to non-bankers, the bankers have a lower payoff than the non-bankers, there is no banks (trade).

If $y_b = 0$, from $p_y = \rho$, the budget constraint becomes $p_x x_b = \rho y_b$, thus $x_b = 0$, there is no banks (trade). $\square$

Proof of Proposition 6:

Compare the results in the efficiency part and the equilibrium part, we have three different equations, the binding repayment constraint, the free-entry condition, and the optimal consumption relationship between the non-banker and type 2. We can prove the three conditions step by step:

(i) Prove $p_y^e = \frac{\theta}{1-\theta} < \rho$.

From the efficiency part, we have $u'(x_n) = \frac{1-\theta}{\theta} v'(x)$, while from the equilibrium part, we have $u'(x_n) = v'(x)/p_y$. We need to have $p_y = \frac{\theta}{1-\theta}$ such that the efficient allocations can be implemented.

(ii) Prove $\tau = \tau^*$, where τ^* solves $\frac{p_x^e(\tau) x_b^e(\tau)}{\Pi} + \frac{\tau(\rho - \frac{\theta}{1-\theta})}{r\lambda\rho} = k$.

The first equation in the efficiency part is

$$u(x_b) = \frac{r\lambda\rho k}{\rho - \frac{\theta}{1-\theta}},$$

while the first equation in the equilibrium can be rewritten as

$$u(x_b) = \frac{r\lambda\rho}{\rho - p_y}\left[\frac{p_x x_b}{\Pi} + \frac{\tau(\rho - p_y)}{r\lambda\rho}\right].$$

Using $p_y^e = \frac{\theta}{1-\theta}$, to let the banker's consumption in the equilibrium reach the optimal outcome, we need

$$\frac{p_x^e(\tau)\, x_b^e(\tau)}{\Pi} + \frac{\tau(\rho - \frac{\theta}{1-\theta})}{r\lambda\rho} = k.$$

(iii) Prove $t = t^*$, where t^* solves $u\left[x_b^e(t)\right] - u\left[x_n^e(t)\right] + \frac{1-\theta}{\theta}\frac{u'[x_n^e(t)]\left[x^e(t) - x_b^e(t)\right]}{1-\mu^e(t)} = \frac{1-\theta}{\theta}k_0$.

We need to set t to the optimal level such that the equilibrium allocations satisfy the last equation that is different in the efficiency part. $\square$

References

[1] Alvarez, F., and Jermann, U. (2000), "Efficiency, Equilibrium, and Asset Pricing with Risk of Default", *Econometrica*, 68, 775–798.

[2] Andolfatto, D. and Nosal, E. (2009), "Money, Intermediation and Banking", *Journal of Monetary Economics*, 56, 289–294.

[3] Berger, A. N., Kashyap, A. K., and Scalise, J. M. (1995), "The Transformation of the U.S. Banking Industry: What a Long, Strange Trip it's been", *Brookings Papers on Economic Activity,* 2, 55–201.

[4] Biais, B., Mariotti, T., Plantin, G., and Rochet, J-C. ((2007), "Dynamic Security Design: Convergence to Continuous Time and Asset Pricing Implications", *Review of Economic Studies*, 74, 345–390.

[5] Boyd, J. and Prescott, E. (1986), "Financial Intermediary Coalitions", *Journal of Economic Theory*, 38, 211–232.

[6] Broecker, T. (1990), "Credit Worthiness Tests and Interbank Competition", *Econometrica*, 58, 429-452.

[7] Cavalcanti, R. and Wallace, N. (1999a), "A Model of Private Bank Note Issue", *Review of Economic Dynamics*, 2, 104–136.

[8] Cavalcanti, R. and Wallace, N. (1999b), "Inside and Outside Money as Alternative Media of Exchange", *Journal of Money, Credit, and Banking*, 31, 443–457.

[9] Corbae, D. and D'Erasmo, P. (2013) "A Quantitative Model of Banking Industry Dynamics", *mimeo.*

[10] Davies, G. (2002), *A History of Money From Ancient Times to the Present Day*, 3rd edn (Cardiff: University of Wales Press).

[11] Demarzo, P. and Fishman, M. (2007), "Agency and Optimal Investment Dynamics", *Review of Financial Studies*, 20, 151–188.

[12] Diamond, D. and Dybvig, P. (1983), "Bank Runs, Deposit Insurance, and Liquidity", *Journal of Political Economy*, 91, 401–419.

[13] Diamond, D. (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies,* 51, 393–414.

[14] Diamond, D. and Rajan, R. (2001), "Liquidity Risk, Liquidity Creation and Financial Fragility: A Theory of Banking," *Journal of Political Economy*, 109, 287–327.

[15] Ferguson, N. (2008), *The Ascent of Money* (New York: The Penguin Press).

[16] Freixas, X. and Rochet, J. C. (2008), *Microeconomics of Banking* (Cambridge: MIT Press).

[17] Gorton, G. and Winton, A. (2002), "Financial Intermediation", in Constantinides, G., Harris, M. and Stulz, R. (eds) *Handbook of the Economics of Finance* (Amsterdam/Boston: Elsevier/North Holland).

[18] Gu, C., Mattesini, F., Monnet, C. and Wright, R. (2013a), "Endogenous Credit Cycles", *Journal of Political Economy*, 121, 940–965.

[19] Gu, C., Mattesini, F., Monnet, C. and Wright, R. (2013b), "Banking: A New Monetarist Approach", *Review of Economic Studies,* 80(2): 636–662.

[20] He, P., Huang, L. and Wright, R. (2005), "Money and Banking in Search Equilibrium", *International Economic Review,* 46, 637–670.

[21] He, P., Huang, L. and Wright, R. (2008), "Money, Banking and Monetary Policy", *Journal of Monetary Economics,* 55, 1013–1024.

[22] Holmstrom, B., and Tirole, J. (1997), "Financial Intermediation, Loanable Funds, and the Real Sector", *Quarterly Journal of Economics,* 112(3), 663–691.

[23] Huangfu, X. and Sun, H. (2011), "Private Money and Bank Runs", *Canadian Journal of Economics*, 44, 859–879.

[24] Janicki, H. and Prescott, E. (2006) "Changes in the Size Distribution of U.S. Banks: 1960-2005", *Federal Reserve Bank of Richmond Quarterly Review*, 92, 291–316.

[25] Kehoe, T. J., and Levine, D. K. (1993), "Debt-Constrained Asset Markets", *Review of Economic Studies,* 60, 865–888.

[26] Kocherlakota, N. R. (1998), "Money Is Memory", *Journal of Economic Theory*, 81, 232–251.

[27] Koeppl, T., Monnet, C. and Temzelides, T. (2008), "A Dynamic Model of Settlement", *Journal of Economic Theory*, 142, 233–246.

[28] Leland, H. E. and Pyle, D. H. (1977), "Informational Asymmetries, Financial Structure and Financial Intermediation", *Journal of Finance*, 32, 371–387.

[29] Mills, D. (2008), "Imperfect Monitoring and the Discounting of Inside Money", International Economic Review, 49, 737–754.

[30] Selgin, G. (2007), "Banking", written for *Encyclopedia Britannica.*

[31] Townsend, R. (1979), "Optimal contracts and competitive markets with costly state verification", *Journal of Economic Theory*, 22, 265–293.

[32] Townsend, R. (1988), "Models as Economies", Economic Journal, 98, 1–24.

[33] Wallace, N. (1998), "A Dictum for Monetary Theory", *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter, 20–26.

[34] Wallace, N. (2005), "From Private Banking to Central Banking: Ingredients of a Welfare Analysis", *International Economic Review*, 46, 619–636.

[35] Williamson, S. (1986), "Costly Monitoring, Financial Intermediation and Equilibrium Credit Rationing", *Journal of Monetary Economics*, 18, 159–179.

[36] Williamson, S. (1987), "Financial Intermediation, Business Failures, and Real Business Cycles", *Journal of Political Economy*, 95, 1196–1216.