Barriers to Reallocation and Economic Growth: the Effects of Firing Costs*

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Abstract

Recent empirical studies have underlined the existence of large reallocation flows across firms. In this paper, we study how factors that hinder this reallocation process influence aggregate productivity growth. We extend Hopenhayn and Rogerson's (1993) general equilibrium firm dynamics model to allow for endogenous innovation. We calibrate the model using U.S. data, and then evaluate the effects of firing taxes on reallocation, innovation, and aggregate productivity growth. We find that firing taxes can have opposite effects on the entrants' and the incumbents' innovation.

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1 Introduction

Recent empirical studies have underlined the existence of large flows of productive resources across firms and their important role for aggregate productivity. Production inputs are constantly being reallocated as firms adjust to changing market environments and new products and techniques are developed. As documented recently by Micco and Pagés (2007) and Haltiwanger et al. (2014), labor market regulations may dampen this reallocation of resources. Using cross-country industry-level data, these studies show that restrictions on hiring and firing reduce the pace of job creation and job destruction. In a similar vein, Davis and Haltiwanger (2014) find that the introduction of common-law exceptions that limit firms' ability to fire their employees at will has a negative impact on job reallocation in the U.S. The objective of the paper is to study the implications of this reduced job reallocation for aggregate productivity growth.

We investigate the consequences of employment protection on job reallocation and productivity growth using a model of innovation-based economic growth. We extend Hopenhayn and Rogerson's (1993) model of firm dynamics by introducing an innovation decision. Firms can invest in research and development (R&D) to improve the quality of products. Hence, in contrast to Hopenhayn and Rogerson's (1993) model (and the Hopenhayn (1992) model that it is based on) where the productivity process is exogenous, job creation and job destruction in our model are the result of both idiosyncratic productivity shocks and *endogenous* innovation.

Following the seminal work of Grossman and Helpman (1991) and Aghion and Howitt (1992), we model innovation as a process of *creative destruction*: by innovating on existing products, entrants displace the incumbent producers. In addition to this Schumpeterian feature, we also incorporate the innovations developed by incumbent firms. We allow incumbent firms to invest in R&D to improve the quality of their own product.[1] In the model, produc-

[1] The importance of incumbents' innovation is emphasized in recent papers, such as Acemoglu and Cao (2015), Akcigit and Kerr (2015), and Garcia-Macia et al. (2015). Earlier papers that analyze incumbents' innovations in the quality-ladder framework include Segerstrom and Zolnierek (1999), Aghion et al. (2001), and Mukoyama (2003).

tivity growth thus results from the R&D of both entering and incumbent firms. The model highlights the crucial role of reallocation for economic growth. As products of higher quality are introduced into the market, labor is reallocated towards the firms producing higher quality products.[2] By limiting the reallocation of labor across firms, employment protection changes the firms' incentives to innovate.

We model employment protection as a firing tax and study its effect on innovation and growth. We find that the effects of the firing tax on aggregate productivity growth depend on the interaction between the innovation of entrants and incumbents. In fact, the firing tax can have opposite effects on the entrants' and the incumbents' innovation: while the firing tax tends to reduce the entrants' innovation, it may raise the incentives for incumbent firms to innovate. The firing tax reduces the entrants' innovation because the tax itself represents an additional cost that reduces expected future profits (direct effect). In addition, the misallocation of labor further reduces expected future profits (misallocation effect). For incumbents, the consequences of the firing tax are less clear-cut. In particular, the misallocation of labor has an ambiguous impact on the incumbents' incentive to innovate. Firms which are larger than their optimal size have stronger incentives to invest in R&D in the presence of firing costs. For those firms, innovation has the added benefit of allowing them to avoid paying the firing tax as they no longer need to reduce their employment (tax-escaping effect). By contrast, for firms that are smaller than their optimal size, the direct effect and the misallocation effect discourage innovation. In addition, the incumbents' incentive to innovate is affected by the rate at which entrants innovate. By reducing the entry rate, firing costs lower the probability for incumbents of being taken over by an entrant. This decline in the rate of creative destruction tends to raise the incumbents' innovation. In our baseline calibration, the entrants' innovation rate falls and the incumbents' innovation rate increases as a result of the firing tax. Overall, the negative effect on entrants dominates, and the firing tax leads to a fall in the rate of growth of aggregate productivity.[3]

[2] Aghion and Howitt (1994) is an earlier study that highlight this aspect of the Schumpeterian growth models in their analysis of unemployment.

[3] Saint-Paul (2002) makes a related argument that countries with a rigid labor market tend to produce relatively secure goods at a late stage of their product life cycle, so that these countries tend to specialize in

Past theoretical studies have shown that firing costs can have adverse consequences on aggregate productivity. The existing literature, however, has mainly focused on the effects of employment protection on the *level* of aggregate productivity. Using a general equilibrium model of firm dynamics, Hopenhayn and Rogerson (1993) have shown that employment protection hinders job reallocation and reduces allocative efficiency and aggregate productivity. They find that a firing cost that amounts to one year of wages reduces aggregate total factor productivity by 2%. Moscoso Boedo and Mukoyama (2012) consider a wider range of countries, and show that firing costs calibrated to match the level observed in low income countries can reduce aggregate total factor productivity by 7%. A recent paper by Da-Rocha et al. (2016) analyzes a continuous time model with two possible levels of employment at each firm, and also find that the firing cost reduces aggregate productivity. In line with these papers, we find that the *level* of employment and labor productivity falls.[4] We show that in addition to the level effect, employment protection also affects the *growth rate* of aggregate productivity.

In focusing on the consequences of barriers to labor reallocation on aggregate productivity growth, our analysis also goes one step beyond the recent literature on misallocation that follows the seminal work of Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). We highlight the fact that barriers to reallocation affect not only the allocation of resources across firms with different productivity levels, but also the productivity *process* itself as it modifies the firms' incentives to innovate. Empirical studies that evaluate the contribution of reallocation on productivity, such as Foster et al. (2001) and Osotimehin (2013), are designed to analyze the sources of productivity growth, rather than the level; in that sense, our analysis is more comparable to that literature. Poschke (2009) is one of the few exceptions that studies the effects of firing costs on aggregate productivity growth.[5] In Poschke (2009), firing costs

'secondary' innovations. A country with more flexible labor market tends to specialize in 'primary' innovations. Thus increasing firing costs may encourage 'secondary' innovation, and the effect on aggregate growth depends on which type of innovation is more important.

[4]This result is consistent with recent empirical evidence by Autor et al. (2007) who show that a common-law restriction that limits the firms' ability to fire (the "good faith exception") had a detrimental effect on state total factor productivity in manufacturing.

[5]Samaniego (2006*b*) analyzes how employment protection affects employment and profit for sectors that

act as an exit tax which lowers the exit rate of low productivity firms. We focus on a different channel and show that firing costs may also affect aggregate productivity growth through their effects on R&D and innovation.

Our paper is also related to firm-dynamics models with endogenous innovation, such as Klette and Kortum (2004), Lentz and Mortensen (2008), and Acemoglu et al. (2013). Our focus, however, is different. While our objective is to study the effects of employment protection, Lentz and Mortensen (2008) mainly focus on the structural estimation of the model, and Acemoglu et al. (2013) study the consequences of subsidies to R&D spending and the allocation of R&D workers across firms. Models by Akcigit and Kerr (2015) and Acemoglu and Cao (2015) extend the Klette-Kortum model and allow incumbents to innovate on their own products. Our model also exhibits that feature. Compared to these models, one important difference of our approach is that we use labor market data to discipline the model parameters, consistently with our focus on labor market reallocation and labor market policy.[6] Methodologically, these models typically are written in continuous time, while we use a discrete-time framework. This modeling strategy allows us to solve the model with firing taxes using a similar method to those used for standard heterogeneous-agent models (such as Huggett (1993) and Aiyagari (1994)) and standard firm-dynamics models (such as Hopenhayn and Rogerson (1993) and Lee and Mukoyama (2008)).

The paper is organized as follows. The next section sets up the model. Section 3 outlines our computational method and details of calibration. Section 4 describes the results. Section 5 concludes.

2 Model

We build a model of firm dynamics in the spirit of Hopenhayn and Rogerson (1993). We extend their framework to allow for endogenous productivity at the firm level. The innovation process of our model is built on the classic quality-ladder models of Grossman and Helpman

differ in the rates of technological progress. He treats technological progress as an exogenous process.

[6]Garcia-Macia et al. (2015) also utilizes labor market data to quantify their model, innovation is however exogenous in their model.

(1991) and Aghion and Howitt (1992), and also on the recent models of Acemoglu and Cao (2015) and Akcigit and Kerr (2015).

There is a continuum of differentiated intermediate goods on the unit interval $[0, 1]$ and firms innovate by improving the quality of these intermediate goods. Final goods are produced from the intermediate goods in a competitive final good sector. We first describe the optimal aggregate consumption choice. We then turn to the firms, first describing the final goods sector and the demand for each intermediate good, and then the decisions of the intermediate goods firms. Finally, we present the potential entrants' decision in the intermediate goods sector and the aggregate innovation in general equilibrium.

2.1 Consumers

The utility function of the representative consumer has the following form:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t [\log(C_t) - \xi L_t],$$

where C_t is consumption at time t, L_t is labor supply at time t, $\beta \in (0, 1)$ is the discount factor, and $\xi > 0$ is the parameter of the disutility of labor. Similarly to Hopenhayn and Rogerson (1993), we adopt the indivisible-labor formulation of Rogerson (1988).

The consumer's budget constraint is

$$A_{t+1} + C_t = (1 + r_t)A_t + w_t L_t + T_t,$$

where

$$A_t = \int_{\mathcal{N}_t} V_t^j dj$$

is the asset holding. The asset in this economy is the ownership of firms.[7] Here, V_t^j indicates the value of a firm that produces product j at time t, and $\mathcal{N}_t$ is the set of products that are actively produced at time t. In the budget constraint, r_t is the net return of the asset, w_t is the wage rate, and T_t is the lump-sum transfer of the firing tax to the consumer.

[7] We do not distinguish firms and establishments in this paper. Later we use establishment-level data in our calibration. Using firm-level data yields similar results.

The consumer's optimization results in two first-order conditions. The first is the Euler equation:

$$\frac{1}{C_t} = \beta(1 + r_{t+1})\frac{1}{C_{t+1}}, \tag{1}$$

and the second is the optimal labor-leisure choice:

$$\frac{w_t}{C_t} = \xi. \tag{2}$$

2.2 Final-good firms

The final good Y_t is produced by the technology

$$Y_t = \left(\int_{\mathcal{N}_t} \mathbf{q}_{jt}{}^{\psi} y_{jt}{}^{1-\psi} dj \right)^{\frac{1}{1-\psi}}.$$

The price of Y_t is normalized to one, y_{jt} is the amount of intermediate product j used at time t, $\mathbf{q}_{jt}$ is the *realized quality* of intermediate product j.[8] The realized quality is the combination of the *potential quality* q_t and the transitory shock α_{jt}:

$$\mathbf{q}_{jt} = \alpha_{jt} q_{jt}.$$

We assume that α_{jt} is i.i.d. across time and products. We also assume that the transitory shock realizes at the product level, rather than at the firm level, so that the value of α_{jt} does not alter the ranking of the realized quality compared to the potential quality.[9]

Let the *average potential quality* of intermediate goods be

$$\bar{q}_t \equiv \frac{1}{N_t} \left(\int_{\mathcal{N}_t} q_{jt} dj \right)$$

and the *quality index* Q_t be

$$Q_t \equiv \bar{q}_t^{\frac{\psi}{1-\psi}}.$$

Note that the quality index grows at the same rate as the aggregate output Y_t along the balanced-growth path.

[8] Similar formulations are used by Luttmer (2007), Acemoglu and Cao (2015), and Akcigit and Kerr (2015), among others.

[9] If the shock is at the firm level, it is possible that the incumbent firm i's realized quality $\alpha_{it} q_{it}$ is larger than the new firm j's realized quality $\alpha_{jt} q_{jt}$ even if $q_{jt} > q_{it}$.

The final goods sector is perfectly competitive, and the problem for the representative final good firm is

$$\max_{y_{jt}} \quad \left(\int_{\mathcal{N}_t} \mathbf{q}_{jt}{}^{\psi} y_{jt}{}^{1-\psi} dj\right)^{\frac{1}{1-\psi}} - \int_{\mathcal{N}_t} p_{jt} y_{jt} dj.$$

The first-order condition leads to the inverse demand function for y_{jt}:

$$p_{jt} = \mathbf{q}_{jt}{}^{\psi} y_{jt}{}^{-\psi} Y_t{}^{\psi}. \tag{3}$$

Final-good firms are introduced for ease of exposition; as in the standard R&D-based growth models, one can easily transform this formulation into a model without final goods, assuming that the consumers (and firms engaging in R&D activities) combine the intermediate goods on their own.[10] In this sense, the final-good sector is a veil in the model, and we ignore final-good firms when we map the model to the firm dynamics data.

2.3 Intermediate-good firms

Each intermediate-good firm produces one differentiated product and is the monopolist producer of that product. The core of the model is the dynamics of the heterogeneous intermediate-good firms. Intermediate-good firms enter the market, hire workers, and produce. Depending on variations in the quality of their products, they expand or contract over time, and they may be forced to exit. Compared to standard firm-dynamics models, the novelty of our model is that these dynamics are largely driven by endogenous innovations.

The intermediate firms conduct R&D activities to innovate. We consider two sources of innovations. One is the *innovation by incumbents*: an incumbent can invest in R&D in order to improve the potential quality of its own product. The other is the *innovation by entrants*: an entrant can invest in R&D to innovate a product that is either (i) not currently produced, or (ii) currently produced by another firm. If the product in not currently produced, the entrant becomes the monopolist for that product. If the product is currently produced by another firm, the entrant displaces the incumbent monopolist. The previous producer is, as a result, forced to exit.[11]

[10]See, for example, Barro and Sala-i-Martin (2004).

[11]Instead of assuming that the lower-quality producer automatically exits, we can resort to a market participation game with price competition as in Akcigit and Kerr (2015).

Our main policy experiment is imposing a firing tax on intermediate-good firms. We assume that the firm has to pay the tax τw_t for each worker fired,[12] including when it exits.[13]

2.3.1 Production of intermediate goods

Each product j is produced by the leading-edge monopolist who produces the highest quality for that particular product. The firm's production follows a linear technology

$$y_{jt} = \ell_{jt},$$

where ℓ_{jt} is the labor input of production workers. The monopolist decides on the production quantity given the inverse demand function, given by Equation (3).

2.3.2 Innovation by incumbents

An incumbent producer can innovate on its own product. The probability that an incumbent innovates on its product at time t is denoted x_{Ijt}. A successful innovation increases the quality of the product from q_{jt} to $(1 + \lambda_I)q_{jt}$, where $\lambda_I > 0$, in the following period. The cost of innovation, $\mathbf{r}_{Ijt}$, is assumed to be

$$\mathbf{r}_{Ijt} = \theta_I Q_t \frac{q_{jt}}{\bar{q}_t} {x_{Ijt}}^{\gamma},$$

where $\gamma > 1$ and θ_I are parameters.[14]

2.3.3 Innovation by entrants

A new firm can enter after having successfully innovated on either an intermediate good currently produced by an incumbent or on a good that is not currently produced. In order

[12] Following the literature (e.g. Hopenhayn and Rogerson (1993)), we assume that the firing costs are incurred only when the firm contracts or exits (that is, only when job destruction occurs). As is well documented (see, for example, Burgess et al. (2000)), worker flows are typically larger than job flows. The implicit assumption here is that all worker separations that are not counted as job destruction are voluntary quits that are not subject to the firing tax.

[13] An alternative specification is to assume that the firm does not need to incur firing costs when it exits. See Samaniego (2006*a*) and Moscoso Boedo and Mukoyama (2012) for discussions.

[14] The assumption that the innovation cost increases with productivity is frequently used in endogenous growth literature. See, for example, Segerstrom (1998), Howitt (2000), and Akcigit and Kerr (2015). Kortum (1997) provides empirical support for this assumption in a time-series context.

to innovate, a potential entrant has to spend a fixed cost ϕQ_t and a variable cost

$$\mathbf{r}_{Ejt} = \theta_E Q_t {x_{Ejt}}^{\gamma}$$

to innovate with probability x_{Ejt}.[15] Here, ψ and θ_E are parameters. As with the incumbents' innovation, a successful innovation increases the quality of product j from q_{jt} to $(1+\lambda_E)q_{jt}$ in the following period. Here, we are allowing the innovation step for the entrants, λ_E, to be different from the incumbents' innovation step λ_I. We assume that the entrants' innovation is not targeted: each entrant innovates on a product that is randomly selected. The entrants choose their innovation probability before learning the quality of the product they will innovate upon.

We assume free entry, that is, anyone can become a potential entrant by spending these costs. The free entry condition for potential entrants is

$$\max_{x_{E_t}} \left\{ -\theta_E Q_t {x_{Ejt}}^{\gamma} - \phi Q_t + \frac{1}{1+r_t} x_{Ejt} \bar{V}_{E,t+1} \right\} = 0, \tag{4}$$

where $\bar{V}_{E,t+1}$ is the expected value of an entrant at time $t+1$. Because the entrant decides on its innovation probability before learning its quality draw, the expected value $\bar{V}_{E,t+1}$ is constant across potential entrants and so is the innovation probability. The optimal value of the innovation probability, x^*_{Et}, is determined by

$$\frac{1}{1+r_t} \bar{V}_{E,t+1} - \gamma \theta_E Q_t {x_{Et}}^{\gamma-1} = 0 \tag{5}$$

and the value of aggregate innovation by entrants is $X_{Et} = m_t x^*_{Et}$, where m_t is the mass of potential entrants at time t. From (4) and (5), x^*_{Et} satisfies

$$-\theta_E {x^*_{Et}}^{\gamma} - \phi + \gamma \theta_E {x^*_{Et}}^{\gamma} = 0$$

and thus x^*_{Et} is a constant number x^*_E that can easily be solved as a function of parameters. The solution is

$$x^*_E = \left(\frac{\phi}{\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}}. \tag{6}$$

Note that x^*_E is not affected by the firing tax. The response of the entry rate to changes in firing tax occurs through variation in the mass of potential entrants m_t.

[15] Bollard et al. (2016) provide empirical support for the assumption that entry costs increase with productivity.

2.3.4 Exit

We assume that the firm can exit for two reasons: (i) the product line is taken over by en entrant with a better quality; (ii) the firm is hit by an exogenous, one-hoss-shay depreciation shock. While exit is an exogenous shock from the viewpoint of the incumbent firm in both cases, the first type of exit is endogenously determined in equilibrium.

The probability that an incumbent is taken over by an entrant is denoted μ; as we will see, this probability depends on the mass of potential entrants and on the innovation intensity of each entrant. The probability of the depreciation shock, assumed to be constant across firms, is denoted by $\delta > 0$. After this shock, the product becomes inactive until a new entrant picks up that product. From a technical viewpoint, the depreciation shock enables the economy to have a stationary distribution of (relative) firm productivity.[16]

2.4 Timing of events and value functions

The timing of events in the model is the following. Below, we omit the firm subscript j when there is no risk of confusion.

1. At the beginning of period t, all innovations from last period's R&D spending realize. Incumbent firms have to exit from the product lines on which entrants have innovated, including when both the incumbent and the entrant innovate at the same time.

2. The transitory productivity shock realizes.

3. The firms (including the newly-entered firms) receive the depreciation shock with probability δ.

4. Exiting firms pay the firing cost.

5. Let us express the distribution of the firm size at this point by the stationary measure over the individual state $(q_t, \alpha_t, \ell_{t-1})$, where q_t is the potential quality, α_t is the transitory shock, and ℓ_{t-1} is the size in the previous period.

[16] See, for example, Gabaix (2009).

6. Firms decide on hiring, firing, and innovation (this include the potential entrants' innovation), then the labor market clears and the production takes place. The consumer decides on consumption and saving.

We now express the firm's optimization problem as a dynamic programming problem. The expected value for the firm at the beginning of the period (after stage 2 of the timing) is

$$Z_t(q_t, \alpha_t, \ell_{t-1}) = (1-\delta)V_t^s(q_t, \alpha_t, \ell_{t-1}) + \delta V_t^o(\ell_{t-1}).$$

The first term in the right-hand side is the value from surviving and the second term is the value from exiting due to the exogenous exit shock. When exiting, the firm has to pay a firing tax on all the workers fired. The value of exiting is then

$$V_t^o(\ell_{t-1}) = -\tau w_t \ell_{t-1}.$$

The value of survival is

$$\begin{aligned} &V_t^s(q_t, \alpha_t, \ell_{t-1}) \\ &\quad = \max_{\ell_t, x_{It}} \left\{ \Pi_t(q_t, \alpha_t, \ell_{t-1}, \ell_t, x_{It}) + \frac{1}{1+r_{t+1}} \left((1-\mu_t) S_{t+1}(x_{It}, q_t, \ell_t) - \mu_t \tau w_{t+1} \ell_t \right) \right\}. \end{aligned}$$

Here, $S_{t+1}(x_{It}, q_t, \ell_t)$ is the value of not being displaced by an entrant and μ_t is the probability of being displaced by an entrant. The value of not being displaced by an entrant is

$$S_{t+1}(x_{It}, q_t, \ell_t) = (1-x_{It})E_{\alpha_{t+1}}[Z_{t+1}(q_t, \alpha_{t+1}, \ell_t)] + x_{It}E_{\alpha_{t+1}}[Z_{t+1}((1+\lambda_I)q_t, \alpha_{t+1}, \ell_t)],$$

where the period profit is

$$\Pi_t(q_t, \alpha_t, \ell_{t-1}, \ell_t, x_{It}) = ([\alpha_t q_t]^{\psi} {\ell_t}^{-\psi} {Y_t}^{\psi} - w_t)\ell_t - \theta_I Q_t \frac{q_t}{\bar{q}_t} {x_{It}}^{\gamma} - \tau w_t \max\langle 0, \ell_{t-1} - \ell_t \rangle.$$

Because the economy exhibits perpetual growth, we first need to transform the problem into a stationary one before applying the usual dynamic programming techniques.

3 Balanced-growth equilibrium

From this section, we focus on the balanced-growth path of the economy, where w_t, C_t, Y_t, Q_t grow at a common rate g. Our specification implies that $\bar{q}_t$ grows at a rate $g_q =$

$(1+g)^{\frac{1-\psi}{\psi}}-1$ along this path. Let us normalize all variables except q_t by dividing by Q_t. For q_t, we normalize with $\bar{q}_t$. All normalized variables are denoted with a hat (ˆ): for example, $\hat{Y}_t = Y_t/Q_t$, $\hat{C}_t = C_t/Q_t$, $\hat{q}_t = q_t/\bar{q}_t$, and so on.

3.1 Normalized Bellman equations

From the consumer's Euler equation (1),

$$\beta(1+r_{t+1}) = \frac{C_{t+1}}{C_t} = 1+g$$

holds. Therefore $(1+g)/(1+r) = \beta$ holds along the stationary growth path. This can be used to rewrite the firm's value functions as the following. (We use the hat notation for the value functions, in order to distinguish from the previous section.) The value at the beginning of the period is (given the stationarity, time subscripts are dropped)

$$\hat{Z}(\hat{q},\alpha,\ell) = (1-\delta)\hat{V}^s(\hat{q},\alpha,\ell) + \delta\hat{V}^o(\ell), \tag{7}$$

where

$$\hat{V}^o(\ell) = -\tau\hat{w}\ell.$$

The value of survival is

$$\hat{V}^s(\hat{q},\alpha,\ell) = \max_{\ell'\geq 0, x_I}\left\{\hat{\Pi}(\hat{q},\alpha,\ell,\ell',x_I) + \beta\left((1-\mu)\hat{S}\left(x_I, \frac{\hat{q}}{1+g_q},\ell'\right) - \mu\tau\hat{w}\ell'\right)\right\}, \tag{8}$$

where

$$\hat{S}\left(x_I,\frac{\hat{q}}{1+g_q},\ell'\right) = (1-x_I)E_{\alpha'}\left[\hat{Z}\left(\frac{\hat{q}}{1+g_q},\alpha',\ell'\right)\right] + x_I E_{\alpha'}\left[\hat{Z}\left(\frac{(1+\lambda_I)\hat{q}}{1+g_q},\alpha',\ell'\right)\right].$$

Note that in (8), ℓ is the previous period employment and ℓ' is the current period employment. The period profit can be rewritten as

$$\hat{\Pi}(q,\alpha,\ell,\ell',x_I) = ([\alpha\hat{q}]^{\psi}\ell'^{-\psi}\hat{Y}^{\psi} - \hat{w})\ell' - \theta_I\hat{q}{x_I}^{\gamma} - \tau\hat{w}\max\langle 0,\ell-\ell'\rangle. \tag{9}$$

Note that the Bellman equation, Equation (8), can be solved for given $\hat{Y}$, $\hat{w}$, g, and μ.

For the entrants, the free entry condition can be rewritten as:

$$\max_{x_E}\left\{-\theta_E {x_E}^{\gamma} - \phi + \beta x_E\hat{\hat{V}}_E\right\} = 0,$$

where x_E satisfies the optimality condition

$$\beta \hat{\hat{V}}_E = \gamma \theta_E {x_E}^{\gamma - 1}.$$

3.2 General equilibrium under balanced growth

Let the decision rule for x_I be $\mathcal{X}_I(\hat{q}, \alpha, \ell)$, and the decision rule for ℓ' be $\mathcal{L}'(\hat{q}, \alpha, \ell)$. Denote the stationary measure of the (normalized) individual state variables as $f(\hat{q}, \alpha, \ell)$ at the point of decision for innovation and hiring. Assume that innovating over a vacant line improves the quality of the product over a quality drawn from a given distribution $h(\hat{q})$. Let Ω denote the cumulative distribution function of α and ω denote the corresponding density function.

The stationary measure is the fixed point of the mapping $f \to \mathbf{T}f$, where $\mathbf{T}$ is given in Appendix A. The total mass of active product lines is

$$N \equiv \int \int \int f(\hat{q}, \alpha, \ell) d\hat{q} d\alpha d\ell.$$

From the steady-state condition (inflow equals outflow)

$$\delta N = \mu(1 - \delta)(1 - N),$$

the mass of active product lines can be computed easily as

$$N = \frac{\mu(1 - \delta)}{\delta + \mu(1 - \delta)}. \tag{10}$$

The aggregate innovation by incumbents is

$$X_I = \int \int \int \mathcal{X}_I(\hat{q}, \alpha, \ell) f(\hat{q}, \alpha, \ell) d\hat{q} d\alpha d\ell,$$

and the aggregate innovation by entrants is

$$X_E = m x_E^*.$$

The probability that an incumbent loses a product, μ, is equal to the aggregate innovation by entrants:

$$\mu = X_E.$$

The growth rate of $\bar{q}$ is given by

$$1 + g_q = \frac{\bar{q}'}{\bar{q}}.$$

Let

$$\bar{f}(\hat{q}) \equiv \int\int f(\hat{q}, \alpha, \ell) d\alpha d\ell.$$

Then the normalized value of entry in the stationary equilibrium can be calculated as:

$$\hat{\hat{V}}_E = \int \left[\int \hat{Z}\left(\frac{(1+\lambda_E)\hat{q}}{1+g_q}, \alpha, 0 \right) (\bar{f}(\hat{q}) + (1-N)h(\hat{q}))d\hat{q} \right] \omega(\alpha) d\alpha.$$

In the goods market, the final goods are used for consumption and R&D, and therefore

$$\hat{Y} = \hat{C} + \hat{R},$$

holds, where $\hat{R}$ is the normalized R&D spending which includes the potential entrants' fixed cost. In the labor market, the consumer's first-order condition with respect to the labor-leisure decision (2) is

$$\frac{\hat{w}}{\hat{C}} = \xi,$$

and thus

$$\frac{\hat{w}}{\hat{Y} - \hat{R}} = \xi$$

holds. Since $\hat{Y}$ is a function of intermediate-good production which utilizes labor, this equation (implicitly) clears the labor market.

4 Characterization of the model

The case without the firing tax can be characterized analytically. It provides a useful benchmark and gives some intuition for the determinants of innovation and growth in the model. Also, the economy without firing costs is later used for calibration in the quantitative analysis. The case with the firing tax is less straightforward to characterize. We provide a partial characterization of the model that facilitates the numerical computation of the equilibrium.

4.1 Analytical characterization of the frictionless economy

The solution to the economy without the firing tax boils down to a system of nonlinear equations. The full characterization is in Appendix C. Here, we present several key results.

In the first result, we characterize the value function and the innovation probability of incumbents.

Proposition 1 *Given $\hat{Y}$, μ, and g_q, the value function for the incumbents is of the form*

$$\hat{Z}(\hat{q}, \alpha) = \mathcal{A}\alpha\hat{q} + \mathcal{B}\hat{q},$$

and the optimal decision for x_I is

$$x_I = \left(\frac{\beta(1-\mu)\lambda_I(\mathcal{A}+\mathcal{B})}{(1+g_q)\gamma\theta_I} \right)^{\frac{1}{\gamma-1}}$$

where

$$\mathcal{A} = (1-\delta)\psi\frac{\hat{Y}}{N}$$

and $\mathcal{B}$ solves

$$\mathcal{B} = (1-\delta)\beta(1-\mu)\left(1 + \frac{\gamma-1}{\gamma}\lambda_I x_I\right)\frac{\mathcal{A}+\mathcal{B}}{1+g_q}.$$

Proof. See Appendix C. Note that N solves (10) for a given μ. ■

This result shows that x_I is constant across firms regardless of the values of α and $\hat{q}$. This is consistent with *Gibrat's law*: the expected growth of a firm is independent of its size.[17] This property implies that the process for firm productivity is a *stochastic multiplicative process with reset events*.[18] This process allows us to characterize the right tail of the firm productivity distribution as follows.

Proposition 2 *Suppose that the distribution of the quality for a innovation on vacant line, $h(\hat{q})$, is bounded. Then the right tail of the relative firm productivity $\hat{q}$ follows a Pareto*

[17]Various studies have found that Gibrat's law holds for large firms, while many document important deviations for young and small firms. See Sutton (1997) for a survey.

[18]See, for example, Manrubia and Zanette (1999).

distribution with the shape parameter κ (that is, the density has a form of $F\hat{q}^{\kappa+1}$) which solves

$$1 = (1-\delta)\left[(1-\mu)x_I\gamma_i^\kappa + \mu\gamma_e^\kappa + (1-\mu-(1-\mu)x_I)\gamma_n^\kappa\right].$$

where $\gamma_i \equiv (1+\lambda_I)/(1+g_q)$, $\gamma_e \equiv (1+\lambda_E)/(1+g_q)$, and $\gamma_n \equiv 1/(1+g_q)$.

Proof. See Appendix C. ■

Because the firm size (in terms of employment) is log-linear in $\hat{q}$ for a given α, the right-tail of the firm size also follows the Pareto distribution with the same shape parameter κ.

Finally, the growth rate of aggregate productivity is given by the following expression.

Proposition 3 *The growth rate of aggregate productivity is given by*

$$g_q = (1-\delta)[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu] + \delta(1+\lambda_E)\bar{q}^h - 1,$$

where $\bar{q}_h$ is the average value of the distribution of inactive product lines $h(\hat{q})$.

Proof. This can be shown by a simple accounting relation. Let the measure of q_t (without normalization) for active products be $z(q_t)$. Innovation by incumbents occurs on a fraction $(1-\mu)x_I(1-\delta)$ of active product lines, no innovation occurs on a fraction $(1-\mu-(1-\mu)x_I)(1-\delta)$ of active lines. There is innovation by entrants on a fraction $\mu(1-\delta)$ of active products. Among the inactive products, the fraction $\mu(1-\delta)$ becomes active from the innovation by entrants, but it is an upgrade from the distribution $h(q_t/\bar{q}_t)$ rather than $z(q_t)/N$. Thus g_q can be calculated from

$$1 + g_q = (1-\delta)\left[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu + (1+\lambda_E)\mu\frac{1-N}{N}\frac{\bar{q}^h}{\bar{q}^z}\right].$$

Here, $\bar{q}^h$ and $\bar{q}^z$ are averages of q_t with respect to the distributions h and z. Thus $\bar{q}^h/\bar{q}^z = \int q_t h(q_t/\bar{q}_t)dq_t / \int q_t[z(q_t)/N]dq_t = \int \hat{q}h(\hat{q})d\hat{q} / \int \hat{q}[\hat{z}(\hat{q})/N]d\hat{q}$. Combining this with the expression for N in (10) and the fact that $\bar{q}^z = 1$ yields the above result. ■

Once the firing tax is introduced, x_I is no longer constant across firms, and therefore this formula is not valid. However, it is still useful to think of the effect of the policy on growth through these three components: the incumbents' innovation, the entrants' innovation on active products, and the entrants' innovation on inactive products.

4.2 Some characterization of the economy with the firing tax

With the firing tax, the employment decision of the firm is no longer static, and therefore the characterization is not as easy as in the case without the firing tax. However, we can make a partial characterization that helps ease the computational burden of the numerical solution method. The main idea is to formulate the model in terms of the deviations from the frictionless outcome.

First, define the *frictionless level of employment without temporary shock* as

$$\ell^*(\hat{q};\hat{w},\hat{Y}) \equiv \arg\max_{\ell'}([\alpha\hat{q}]^{\psi}\ell'^{-\psi}\hat{Y}^{\psi} - \hat{w})\ell'$$

with $\alpha = 1$; that is,

$$\ell^*(\hat{q};\hat{w},\hat{Y}) = \left(\frac{1-\psi}{\hat{w}}\right)^{\frac{1}{\psi}} \hat{q}\hat{Y}.$$

Also define $\Omega(\hat{w},\hat{Y})$ by

$$\Omega(\hat{w},\hat{Y}) \equiv \frac{\ell^*(\hat{q};\hat{w},\hat{Y})}{\hat{q}}.$$

In addition, define the *deviation of employment from the frictionless level* by

$$\tilde{\ell} \equiv \frac{\ell}{\ell^*(\hat{q};\hat{w},\hat{Y})}.$$

Similarly, let

$$\tilde{\ell}' \equiv \frac{\ell'}{\ell^*(\hat{q};\hat{w},\hat{Y})}.$$

Then, the period profit (9) can be rewritten as

$$\hat{\Pi}(\hat{q},\alpha,\ell,\ell',x_I) = \\ \left(\left[\frac{\alpha}{\Omega(\hat{w},\hat{Y})}\right]^{\psi}\tilde{\ell}'^{-\psi}\hat{Y}^{\psi} - \hat{w}\right)\hat{q}\Omega(\hat{w},\hat{Y})\tilde{\ell}' - \theta_I\hat{q}x_I{}^{\gamma} - \tau\hat{w}\max\langle 0, \hat{q}\Omega(\hat{w},\hat{Y})\tilde{\ell} - \hat{q}\Omega(\hat{w},\hat{Y})\tilde{\ell}'\rangle.$$

Thus this is linear in $\hat{q}$, and can be rewritten as $\hat{q}\tilde{\Pi}(\alpha,\tilde{\ell},\tilde{\ell}',x_I)$, where

$$\tilde{\Pi}(\alpha,\tilde{\ell},\tilde{\ell}',x_I) \equiv \left(\left[\frac{\alpha}{\Omega(\hat{w},\hat{Y})}\right]^{\psi}\tilde{\ell}'^{-\psi}\hat{Y}^{\psi} - \hat{w}\right)\Omega(\hat{w},\hat{Y})\tilde{\ell}' - \theta_I x_I{}^{\gamma} - \tau\Omega(\hat{w},\hat{Y})\hat{w}\max\langle 0, \tilde{\ell} - \tilde{\ell}'\rangle.$$

Because the period return function is linear in $\hat{q}$, it is straightforward to show that all value functions are linear in $\hat{q}$. Defining $\tilde{Z}(\alpha, \tilde{\ell})$ from $\hat{Z}(\hat{q}, \alpha, \ell) = \hat{q}\tilde{Z}(\alpha, \tilde{\ell})$, (7) can be rewritten as

$$\tilde{Z}(\alpha, \tilde{\ell}) = (1-\delta)\tilde{V}^s(\alpha, \tilde{\ell}) + \delta\tilde{V}^o(\tilde{\ell}),$$

where $\tilde{V}^o(\tilde{\ell})$ is from $\hat{V}^o(\ell) = \hat{q}\tilde{V}^o(\tilde{\ell})$ and thus

$$\tilde{V}^o(\tilde{\ell}) = -\tau\hat{w}\Omega(\hat{w}, \hat{Y})\tilde{\ell}$$

and $\tilde{V}^s(\alpha, \tilde{\ell})$ is from $\hat{V}^s(\hat{q}, \alpha, \ell) = \hat{q}\tilde{V}^s(\alpha, \tilde{\ell})$ with

$$\tilde{V}^s(\alpha, \tilde{\ell}) = \max_{\tilde{\ell}' \geq 0, x_I} \left\{ \tilde{\Pi}(\alpha, \tilde{\ell}, \tilde{\ell}', x_I) + \beta \left((1-\mu)\frac{\tilde{S}(x_I, \tilde{\ell}')}{1+g_q} - \mu\tau\hat{w}\Omega(\hat{w}, \hat{Y})\tilde{\ell}' \right) \right\} \tag{11}$$

Here, the expression $\tilde{S}(x_I, \tilde{\ell}')/(1+g_q)$ comes from $\hat{S}(x_I, \hat{q}/(1+g_q), \ell') = \hat{q}\tilde{S}(x_I, \tilde{\ell}')/(1+g_q)$. The linearity of the value functions implies that

$$\frac{\tilde{S}(x_I, \tilde{\ell}')}{1+g_q} = (1-x_I)E_{\alpha'}\left[\tilde{Z}\left(\alpha', (1+g_q)\tilde{\ell}'\right)\right]\frac{1}{1+g_q} + x_I E_{\alpha'}\left[\tilde{Z}\left(\alpha', \frac{(1+g_q)\tilde{\ell}'}{1+\lambda_I}\right)\right]\frac{1+\lambda_I}{1+g_q}$$

also holds. Here we used that

$$\hat{Z}(\hat{q}', \alpha', \ell') = \hat{q}'\tilde{Z}\left(\alpha', \frac{\ell'}{\ell^*(\hat{q}'; \hat{w}', \hat{Y}')}\right) = \hat{q}'\tilde{Z}\left(\alpha', \frac{\ell^*(\hat{q}; \hat{w}, \hat{Y})}{\ell^*(\hat{q}'; \hat{w}', \hat{Y}')}\frac{\ell'}{\ell^*(\hat{q}; \hat{w}, \hat{Y})}\right)$$

with $\hat{w}' = \hat{w}$, $\hat{Y}' = \hat{Y}$; and that $\ell^*(\hat{q}; \hat{w}, \hat{Y})/\ell^*(\hat{q}'; \hat{w}', \hat{Y}') = \hat{q}/\hat{q}'$ yields

$$\hat{Z}(\hat{q}', \alpha', \ell') = \hat{q}'\tilde{Z}\left(\alpha', \frac{\hat{q}}{\hat{q}'}\tilde{\ell}'\right)$$

for $\hat{q}' = \hat{q}/(1+g_q)$ and $\hat{q}' = (1+\lambda_I)\hat{q}/(1+g_q)$.

The optimization problem in (11) has two choice variables, $\tilde{\ell}'$ and x_I. The first-order condition for x_I is

$$\gamma\theta_I x_I^{\gamma-1} = \Gamma_I$$

and thus x_I can be computed from

$$x_I = \left(\frac{\Gamma_I}{\gamma\theta_I}\right)^{1/(\gamma-1)},$$

where $\Gamma_I \equiv \beta(1-\mu)E_{\alpha'}\left[\tilde{Z}(\alpha', (1+g_q)\tilde{\ell}'/(1+\lambda_I))(1+\lambda_I) - \tilde{Z}(\alpha', (1+g_q)\tilde{\ell}')\right]/(1+g_q)$. From here, it is easy to see that x_I is uniquely determined once we know $\tilde{\ell}'$. Let the decision rule for $\tilde{\ell}'$ in the right-hand side of (11) be $\mathcal{L}'(\alpha, \tilde{\ell})$. Then the optimal x_I can be expressed as $x_I = \mathcal{X}_I(\alpha, \tilde{\ell})$. This implies that x_I is independent of $\hat{q}$.

5 Computation and calibration

The details of the computational methods are described in Appendix B. Our method involves similar steps to solving the standard general-equilibrium firm dynamics model. As in Hopenhayn and Rogerson (1993) and Lee and Mukoyama (2008), we first make a guess on relevant aggregate variables (in our case $\hat{w}$, μ, g, and $\hat{Y}$), solve the optimization problems given these variables, and then update the guess using the equilibrium conditions. This procedure is also similar to how the Bewley-Huggett-Aiyagari models of heterogeneous consumers are typically computed (see, for example, Huggett (1993) and Aiyagari (1994)). This separates our work from recent models of innovation and growth, such as Klette and Kortum (2004), Acemoglu et al. (2013), and Akcigit and Kerr (2015), as these models heavily rely on analytical characterization in a continuous-time setting. Being able to use a standardized numerical method to compute the equilibrium is particularly useful in our experiment, as the firing tax introduces a kink in the firm's maximization problem, which makes it difficult to obtain analytical solutions.

Following a strategy similar to Hopenhayn and Rogerson (1993), we calibrate the parameters of the model under the assumption that firing costs are equal to zero and use U.S. data to compute our targets. In addition to the standard targets that are widely used in the macroeconomic literature, we use establishment-level labor market data to pin down the parameters that relate to the establishment dynamics.[19]

The first set of targets are relatively standard. The model period is one year. The discount factor β is set to 0.947 in line with Cooley and Prescott (1995). Similarly to Hopenhayn and Rogerson (1993), we set the value of the disutility of labor ξ so that the employment to population ratio is equal to 0.6. The value of ψ is set to 0.2 which implies an elasticity of substitution across goods of 5. This value is in the range of Broda and Weinstein's (2006) estimates. Our value of 0.2 implies a markup of 25% which is in line with the estimates of De Loecker and Warzynski (2012). We set the curvature of the innovation cost γ to 2. As noted

[19] Our model does not distinguish between firms and establishments. As 95 percent of U.S. firms are single-establishment firms, the results would be similar if we had instead calibrated the model on firm-level labor market data.

by Acemoglu et al. (2013), $1/\gamma$ can be related to the elasticity of patents to R&D spending, which has been found to be between 0.3 and 0.6.[20] These estimates imply that γ is between 1.66 and 3.33.

Next, we turn to the size of innovation by entrants and incumbents, λ_E and λ_I. As underlined by Acemoglu and Cao (2015), various studies suggest that the innovations developed by entrants are more radical than those developed by incumbents; therefore, it is reasonable to assume that $\lambda_E > \lambda_I$. We set $\lambda_E = 1.5$ and $\lambda_I = 0.25$, based on the recent estimates of Bena et al. (2015). These numbers are also similar to the ones used by Acemoglu and Cao (2015). To set the innovation costs parameters, we first assume that the cost of innovation is proportional to its size, that is $\theta_E/\theta_I = \lambda_E/\lambda_I$, and thus radical innovations are more costly than incremental innovations. Second, we set the level of θ_I to match the output growth rate of 2.0%. When θ_I is smaller, the probability to innovate is higher, and thus the output growth rate is higher. Third, we set ϕ so that the job creation rate by entrants matches the value in the data. When ϕ is small, there is more entry, and therefore the job creation rate by entrants is larger. We assume that the transitory shock α is uniformly distributed, and can take three values $\{1-\varepsilon, 1, 1+\varepsilon\}$, with probability 1/3 for each value. The value of ε is set to replicate the aggregate job creation rate. The model job flows are larger when ε is larger. The overall job creation rate and the job creation rate by entrants, used as targets for ϕ and ε, are computed from the Business Dynamics Statistics published by the Census Bureau.[21]

When an entrant innovates on an inactive product line, the entrant draws the productivity upon which it innovates from a uniform distribution over $[0, 2\bar{q}^h]$. We set the mean $\bar{q}^h = 1$, so that the inclusion of new product lines does not alter the value of average $\hat{q}$.[22] The exogenous exit (depreciation) probability δ is set so that the tail index κ of the productivity distribution

[20] See for example Griliches (1990).

[21] The job creation rates data are publicly available at http://www.census.gov/ces/dataproducts/bds/. We use the average values computed over 1977-2012.

[22] Note that approximation over discrete states creates slight deviation from the target value of 1.

Table 1: Calibration

	Parameter	Calibrated values
Discount rate	β	0.947
Disutility of labor	ξ	1.515
Demand elasticity	ψ	0.2
Innovation step: entrants	λ_E	1.50
Innovation step: incumbents	λ_I	0.25
Innovation cost curvature	γ	2.0
Innovation cost level: entrants	θ_E	3.504
Innovation cost level: incumbents	θ_I	0.584
Entry cost	ϕ	0.302
Exogenous exit (depreciation) rate	δ	0.00112
Transitory shock	ε	0.258
Avg productivity from inactive lines	h mean	0.976
Firing tax	τ	0.0

Table 2: Comparison between the U.S. data and the model outcome

	Data	Model
growth rate of output g (%)	(2.00)	2.00
Employment L	(0.60)	0.60
Tail index κ	1.06	1.06
Job creation rate (%)	17.0	17.0
Job creation rate from entry (%)	6.4	6.4
Job destruction rate (%)	15.0	17.0
Job destruction rate from exit (%)	5.3	2.8
R&D spending ratio (R/Y)		0.12

matches the data value of 1.06.[23] A large δ implies a larger tail index (a thinner tail).[24] The parameter values are summarized in Table 1.

Table 2 compares the baseline outcome and the targets. We also report the R&D expenditures as a share of aggregate output though we do not use it as a target in the calibration. The R&D ratio, at about 12%, is larger than what we typically see from conventional mea-

[23]This is based on Axtell's (2001) estimate from the U.S. Census data (1.059). Axtell (2001) also reports the values ranging from 0.994 to 1.098 depending on the dataset used. Luttmer (2011) reports the value of 1.05 for U.S. firms. Ramsden and Kiss-Haypál (2000) reports the U.S. estimate of 1.25, along with estimates from other countries.

[24]See Section 4.1 for the expression of the tail index.

Table 3: The effects of firing costs

	Baseline $\tau = 0.0$	Experiment $\tau = 0.3$	Fixed entry $\tau = 0.3$
Growth rate of output g (%)	2.00	1.92	2.01
Average innovation probability by incumbents x_I	0.172	0.188	0.176
Innovation probability by entrants x_E	0.294	0.294	0.294
Creative destruction rate μ (%)	2.70	2.21	2.70
Employment L	100	98.9	99.8
Normalized output $\hat{Y}$	100	98.1	99.3
Normalized average productivity $\hat{Y}/L$	100	99.2	99.4
Number of active products N	0.96	0.95	0.96
Job creation rate (%)	17.0	4.6	5.4
Job creation rate from entry (%)	6.4	4.1	4.9
Job destruction rate (%)	17.0	4.6	5.4
Job destruction rate from exit (%)	2.8	2.3	2.8
R&D ratio R/Y	0.12	0.11	0.12

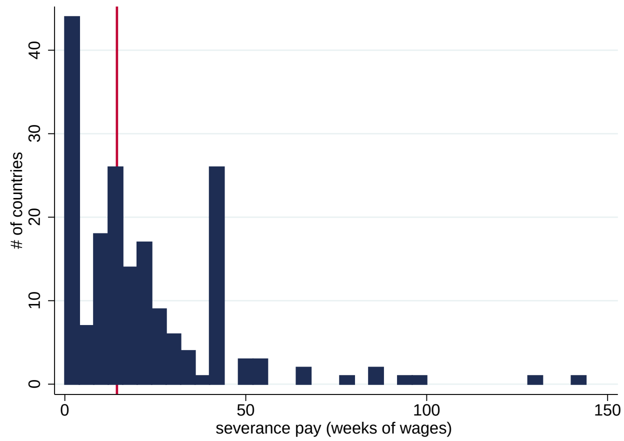
Note: L, $\hat{Y}$, and $\hat{Y}/L$ are set at 100 in the baseline outcome.

sures of R&D spending. However, because our model intends to capture innovation in a broad sense, which includes productivity improvements that come from non-R&D activities such as improvements in production floor, finding a better retail location, and learning by doing, it is more appropriate the compare the model R&D spending to a broader statistic than the conventional measure of R&D. Here, the output share of R&D spending is in line with Corrado et al.'s (2009) estimate of the U.S. intangible investments in the 1990s. We also report the innovation probabilities x_E and x_I, and the creative destruction rate μ.

6 Quantitative results

We now turn to our main experiment in which we evaluate the effects of employment protection. The first two columns of Table 3 compare the baseline result with the firing tax of $\tau = 0.3$; that is, the cost of dismissal per worker amounts to 3.6 months of wages. The choice of this level of tax is motivated by data from the World Bank Doing Business Dataset. The Doing Business dataset reports the severance payments due by firms upon firing a worker. To ensure comparability across countries, precise assumptions are made about the firm and

Figure 1: Severance payments across the world



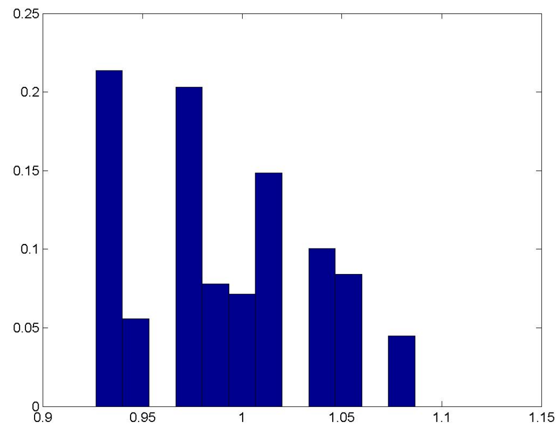
Notes: This figure shows the distribution of severance payments for a worker with ten years of tenure in the retail industry. Source: Doing Business dataset (2015), World Bank.

the worker. Among others, the worker is assumed to be a cashier in a supermarket and the firm is assumed to have 60 workers. Figure 1 displays the distribution of severance payments across countries for this typical firm and typical worker with ten years of tenure. We choose to set the firing tax to 0.3 which corresponds to the median severance payments indicated by the vertical line in Figure 1.[25]

In Table 3, the level variables L, $\hat{Y}$, and $\hat{Y}/L$ are normalized to 100 in the baseline case, to facilitate the comparison. Similarly to Hopenhayn and Rogerson (1993), employment L declines when the firing tax is imposed. The firing tax has two effects on employment. On the one hand, it reduces the firm's incentive to contract when a bad shock arrives. On the other hand, knowing this, the firm also becomes more reluctant to hire when there is a good shock. Here, as in Hopenhayn and Rogerson (1993) and Moscoso Boedo and Mukoyama

[25]This is also close to the level of firing costs in France, estimated by Kramarz and Michaud (2010) to be 25 percent of a worker's annual wages. This a somewhat milder level of firing tax compared to what has been examined in the literature. Hopenhayn and Rogerson (1993) consider $\tau = 0.5$ and $\tau = 1.0$ (their model is calibrated to five years, and thus 10% of five-year wages is 50% of annual wage). Moscoso Boedo and Mukoyama (2012) consider numbers ranging between $\tau = 0.7$ (average of high income countries) and $\tau = 1.2$ (average of low income countries). Moscoso Boedo and Mukoyama (2012) also use the Doing Business Data, but they consider a broader concept of firing tax than us.

Figure 2: Misallocation of labor

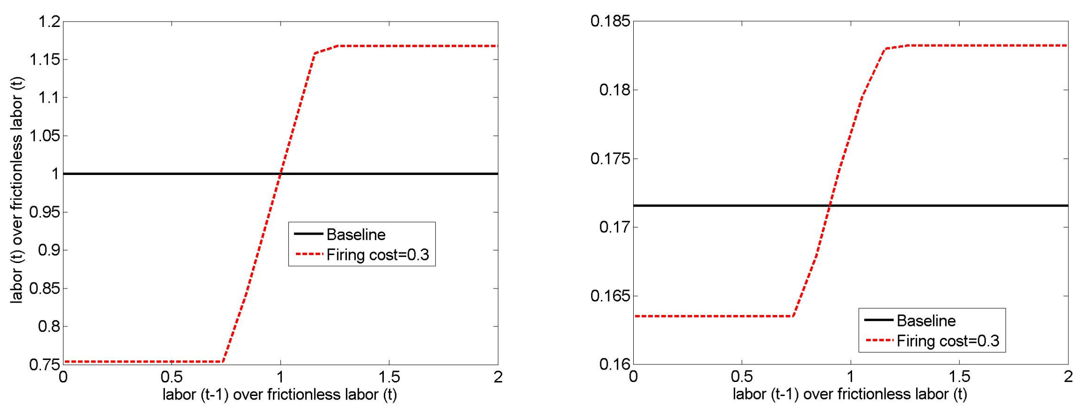


Notes: This figure shows the distribution of the marginal productivity of labor in the model for the baseline experiment where the firing tax is equal to 0.3. The marginal productivity is normalized by the wage rate $\hat{w}$. Without the firing tax, the marginal productivity of labor would be equalized across establishments and the normalized marginal productivity would be equal to 1.

Figure 3: Labor and innovation decision function, constant μ

(a) Labor decision

(b) Innovation probability



Notes: This figure displays the firm's labor decision $\tilde{l}'$ as a function of the previous labor level $\tilde{l}$. The labor variables $\tilde{l}'$ and $\tilde{l}$ are expressed in deviation from the current frictionless level. The transitory shock is set to one.

(2012), the latter effect dominates.[26]

The output level $\hat{Y}$ declines more than employment does. This is mainly because of *misallocation*: the allocation of labor input is not aligned with the productivity across firms when firms face firing costs. This can most vividly be seen by the large decline in job flows. The reduction in labor reallocation is consistent with the recent empirical evidence by Micco and Pagés (2007) and Haltiwanger et al. (2014). While the marginal product of labor is equalized across firms in the frictionless equilibrium, there is, by contrast, a notable dispersion in the economy with a firing tax as shown in Figure 2. In fact, the marginal product of labor deviates by more than 5 percent from the equilibrium wage for about 35 percent of firms. As shown in the Table, the amount of entry also decreases with the firing tax. This reduces the number of active intermediate products N, which further reduces the aggregate productivity level.

In addition to these *level effects* that have already been studied in the literature, our model features *growth effects*. First, firing costs reduce the entrants' incentives to innovate. The total innovation rate by entrants, represented by μ, falls by about 0.5 percentage points.[27] The entrants' incentive to innovate is reduced because of two factors. First, the firing tax has a direct effect on expected profits as it raises the cost of operating a firm. Second, firing costs prevent firms from reaching their optimal scale and this misallocation reduces the entrants' expected profits.

By contrast, the incumbents' innovation probability *increases* by about 2 percentage points as a result of the firing tax. The consequences of the firing tax on the incumbents' incentive to innovate are theoretically ambiguous. On the one hand, the misallocation of labor is costly because the firm will not operate at its optimal size after innovating (*misallocation effect*). But the negative misallocation effect only holds for firms that are below their optimal size. On the contrary, firms that are larger than their optimal size, either because of a

[26]In a recent empirical study, Autor et al. (2006) document that, during the 1970s and 1980s, many U.S. states have adopted common-law restrictions (wrongful-discharge laws) that limits firms' ability to fire. They show that these restrictions resulted in a reduction in state employment.

[27]Note that the equilibrium value of x_E is not affected by the tax (see equation (6)), and thus the change in μ is all due to the change in the number of potential entrants, m.

negative transitory shock or because they have been unsuccessful at innovating, now have stronger incentives to invest in R&D, because firms avoid paying the firing costs when they are successful at innovating as they no longer have to reduce their employment (*tax-escaping effect*).

In addition, the incumbents' incentives to innovate further depend on the entrants' innovation (creative destruction). A lower entry rate reduces the risk for incumbents of being taken over by an entrant, which raises the return of the firm's R&D investment (*creative-destruction effect*). In effect, a lack of creative destruction increases the planning horizon of incumbents.

To assess the importance of the creative-destruction effect, we conduct an additional experiment. There, we hold the value of μ fixed to the value in the baseline economy by not imposing the free-entry condition, Equation (5). The results are reported in the third column of Table 3. The experiment also allows us to illustrate the ambiguous effect of the firing tax on the incumbents' innovation. As shown in Figure 3, the firing tax leads firms that are below their optimal size to reduce their innovation probability. As explained above, this negative effect comes from the misallocation effect as firms: after a successful innovation, firms do not expand as much as they would without the firing tax. For firms that are larger than their optimal size, on the contrary, the tax-escaping effect leads to a higher innovation probability since innovating provides the added benefit of avoiding paying the firing tax. Overall, the results displayed in Table 3 indicate that those two effects largely offset each other. In fact, when the entry rate is held constant, the incumbents' innovation changes only about 30% of the total change. This result suggests that the decline in the entry rate is the key to understanding the increase in the incumbents' innovation. Without the decline in entry, the effects of the firing costs would have limited consequences on the incumbents' innovation.

Our results illustrate the importance of including the incumbents' innovation in the analysis. We find that, with our calibration, firing costs can affect the innovation of entrants and incumbents in opposite directions. The aggregate growth rate can, in principle, increase or decrease as a result of these two effects. In our baseline experiment, the negative effect on entrants dominates, and result in the negative overall growth effect of firing tax.

7 Extensions

[To be completed.]

8 Conclusion

In this paper, we constructed a general equilibrium model of firm dynamics with endogenous innovation. The firms not only decide on production and employment, but also entry and expansion through innovation. Therefore, the productivity shocks that firms face are *endogenous* in our framework, in contrast to the existing firm dynamics literature that evaluates the effect of reallocation on aggregate consequences. In our model, a policy that affects reallocation of productive inputs across firms not only has *level effects*, but also *growth effects*.

Our framework allows us to examine how barriers to reallocation influence firm dynamics and aggregate economic growth. This paper examined a particular type of barriers: a firing tax. We found that a firing tax can have opposite effects on entrants' innovation and incumbents' innovation. A firing tax reduces job reallocation and entrants' innovation, while it may enhance incumbents' innovation.

Because the process of innovation inherently involves randomness, the incentive to innovate affects the risks that each firm faces from their own innovation. It also affects the risks that firms face from other firms' innovation, in the form of creative destruction. When there are barriers to reallocating productive resources, there is a natural feedback process between the misallocation of resources and innovation: misallocation affects the incentive to innovate, and this in turn changes the process of shocks that affects misallocation. It is a promising future research topic to further investigate this interaction both theoretically and empirically.

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Appendix

A Stationary distribution

The stationary measure is the fixed point of the mapping $f \to \mathbf{T}f$, where $\mathbf{T}$ is defined by

$$\begin{aligned}\int_0^{\alpha'}\int_0^{\ell'}\int_0^{\hat{q}'}\mathbf{T}f(\hat{q},\alpha,\ell)d\hat{q}d\alpha d\ell = \quad & \Omega(\alpha')\Bigg[(1-\mu)M_s(\hat{q}',\ell') \\ & +\mu(1-N)\int_{(1+\lambda_E)\hat{q}/(1+g_q)\leq\hat{q}'}(1-\delta)h(\hat{q})d\hat{q} \\ & +\mu\int_{(1+\lambda_E)\hat{q}/(1+g_q)\leq\hat{q}'}\int\int(1-\delta)f(\hat{q},\alpha,\ell)d\hat{q}d\alpha d\ell\Bigg]\end{aligned}$$

The first term of the right hand side is the mass of surviving firms (defined below). The second line is the entry into inactive products. The last line refers to the products whose ownership changes because of entry. The mass of surviving firms is

$$\begin{aligned}M_s(\hat{q}',\ell') = \quad & \int_{\hat{q}/(1+g_q)\leq\hat{q}'}\int_{\mathcal{L}'(\hat{q},\alpha,\ell)\leq\ell'}(1-\mathcal{X}_I(\hat{q},\alpha,\ell))(1-\delta)f(\hat{q},\alpha,\ell)d\hat{q}d\alpha d\ell \\ & +\int_{(1+\lambda_I)\hat{q}/(1+g_q)}\int_{\mathcal{L}'(\hat{q},\alpha,\ell)\leq\ell'}\mathcal{X}_I(\hat{q},\alpha,\ell)(1-\delta)f(\hat{q},\alpha,\ell)d\hat{q}d\alpha d\ell.\end{aligned}$$

B Details of computation

Computation of the model is done by first guessing values for the relevant aggregate variables, performing optimization and deriving the value function trough iteration, and then updating the guess.

The procedure is as follows.

1. First, several variables can be computed from parameters. First, calculate x_E^* from
$$x_E^* = \left(\frac{\phi}{\theta_E(\gamma-1)}\right)^{\frac{1}{\gamma}}.$$
2. Then $\hat{\hat{V}}_E$ can be computed from
$$\hat{\hat{V}}_E = \frac{\gamma\theta_E}{\beta}{x_E}^{\gamma-1}.$$

3. Start the iteration. Guess $\hat{Y}$, $\hat{w}$, m, and g.

   Given m, we can calculate the value of μ by $\mu = X_E = mx_E^*$. Now we are ready to solve the Bellman equation for the incumbents.

   We have two choice variables, $\tilde{\ell}'$ and x_I. The first-order condition for x_I is

$$\gamma\theta_I x_I^{\gamma-1} = \Gamma_I$$

   and thus x_I can be computed from

$$x_I = \left(\frac{\Gamma_I}{\gamma\theta_I}\right)^{1/(\gamma-1)},$$

   where $\Gamma_I \equiv \beta(1-\mu)E_{\alpha'}\left[\tilde{Z}(\alpha', (1+g_q)\tilde{\ell}'/(1+\lambda_I))(1+\lambda_I) - \tilde{Z}(\alpha', (1+g_q)\tilde{\ell}')\right]/(1+g_q)$. We can see that x_I is uniquely determined once we know $\tilde{\ell}'$. Let the decision rule for $\tilde{\ell}'$ be $\mathcal{L}'(\alpha, \tilde{\ell})$. Then $x_I = \mathcal{X}_I(\alpha, \tilde{\ell})$.

4. Once all decision rules are computed, with iterative procedure we can find $f(\hat{q}, \alpha, \tilde{\ell})$ by iterating over the density.

5. Now, we check if the first guesses are consistent with the solution from the optimization. The values of $\hat{w}$ and

$$\begin{aligned}\hat{Y} &= \left(\int\int\int [\alpha\hat{q}]^{\psi}[\ell^*(\hat{q}; \hat{w}, \hat{Y})\mathcal{L}'(\alpha, \tilde{\ell})]^{1-\psi} f(\hat{q}, \alpha, \tilde{\ell})d\hat{q}d\alpha d\tilde{\ell}\right)^{\frac{1}{1-\psi}} \\ &= \left(\int\int \alpha^{\psi}[\Omega(\hat{w}, \hat{Y})\mathcal{L}'(\alpha, \tilde{\ell})]^{1-\psi} \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell})d\hat{q}d\alpha d\tilde{\ell}\right)^{\frac{1}{1-\psi}}\end{aligned}$$

   and

$$\frac{\hat{w}}{\hat{Y} - \hat{R}} = \xi,$$

   where

$$\begin{aligned}\hat{R} &= \int\int\int \theta_I \hat{q}\mathcal{X}_I(\alpha, \tilde{\ell})^{\gamma} f(\hat{q}, \alpha, \tilde{\ell})d\hat{q}d\alpha d\tilde{\ell} + m(\phi + \theta_E x_E^{\gamma}) \\ &= \theta_I \int\int \mathcal{X}_I(\alpha, \tilde{\ell})^{\gamma} \int \hat{q} f(\hat{q}, \alpha, \tilde{\ell})d\hat{q}d\alpha d\tilde{\ell} + m(\phi + \theta_E x_E^{\gamma})\end{aligned}$$

   In order to check the value of g_q, the condition $\frac{1}{N}\int\int\int \hat{q} f(\hat{q}, \alpha, \tilde{\ell})d\alpha d\ell dq = 1$ is used. Intuitively, when g_q is too small, the stationary density $f(\hat{q}, \alpha, \tilde{\ell})$ implies the values of $\hat{q}$ that are too large.

In order to set m, we look at the free-entry condition. Because a large m implies a large μ, which in turn lowers $\tilde{Z}$. Thus the value of m affects the computed value of $\hat{\hat{V}}_E$, through $\tilde{Z}$. Recall that

$$\hat{\hat{V}}_E = \frac{\gamma\theta_E}{\beta} {x_E}^{\gamma-1}$$

has to be satisfied, and this has to be equal to

$$\begin{aligned}
\hat{\hat{V}}_E &= \int \left[\int \hat{Z}((1+\lambda_E)\hat{q}/(1+g_q), \alpha, 0)(\bar{f}(\hat{q}) + (1-N)h(\hat{q}))d\hat{q} \right] \omega(\alpha)d\alpha \\
&= \int \left[\int \hat{q}\tilde{Z}(\alpha, 0)(1+\lambda_E)/(1+g_q)(\bar{f}(\hat{q}) + (1-N)h(\hat{q}))d\hat{q} \right] \omega(\alpha)d\alpha \\
&= \int \tilde{Z}(\alpha, 0)\omega(\alpha)d\alpha \left[N + (1-N) \int h(\hat{q})d\hat{q} \right] (1+\lambda_E)/(1+g_q)
\end{aligned}$$

because $\int \hat{q}\bar{f}(\hat{q})d\hat{q} = N$.

6. Go back to Step 3, until convergence.

C Analytical characterization of the case without firing tax

This section characterizes the model without the firing tax and boils it down to a system of nonlinear equations. The derivations also serve as proofs for Propositions 1 and 2.

C.1 Model solution

Note first that for a given μ, the number of actively produced product, N, is calculated by (10). Recall that μ is an endogenous variable, and it is determined by the entrants' innovation:

$$\mu = m{x_E}^*.$$

As we have seen, ${x_E}^*$ is a function of parameters

$${x_E}^* = \left(\frac{\phi}{\theta_E(\gamma-1)} \right)^{\frac{1}{\gamma}},$$

and thus μ (and also N) is a function of m. In particular, note that N is an increasing function of m.

Because there are no firing taxes, the previous period employment, ℓ, is not a state variable anymore. The measure of individual states can be written as $f(\hat{q}, \alpha)$, and because $\hat{q}$ and α

are independent, we can write $f(\hat{q}, \alpha) = \hat{z}(\hat{q})\omega(\alpha)$. In particular, note that $\int \hat{q}\hat{z}(\hat{q})d\hat{q} = N$, because $\hat{q}$ is the value of q_t normalized by its average. We also assume that $\omega(\alpha)$ is such that $\int \alpha\omega(\alpha)d\alpha = 1$.

Without firing costs, the labor can be adjusted freely. Thus the intermediate-good firm's decision for ℓ' is essentially static:

$$\max_{\ell'} \quad \hat{\pi} \equiv ([\alpha\hat{q}]^{\psi}\ell'^{-\psi}\hat{Y}^{\psi} - \hat{w})\ell'. \tag{12}$$

From the first-order condition,

$$\ell' = \left(\frac{1-\psi}{\hat{w}}\right)^{\frac{1}{\psi}} \alpha\hat{q}\hat{Y} \tag{13}$$

holds. Because $y = \ell'$, we can plug this into the definition of $\hat{Y}$:

$$\hat{Y} = \left(\int\int [\alpha\hat{q}]^{\psi} y^{1-\psi}\hat{z}(\hat{q})\omega(\alpha)d\hat{q}d\alpha\right)^{\frac{1}{1-\psi}}.$$

This yields

$$\hat{Y} = \hat{Y}\left(\frac{1-\psi}{\hat{w}}\right)^{\frac{1}{\psi}} N^{\frac{1}{1-\psi}}$$

and therefore

$$\hat{w} = (1-\psi)N^{\frac{\psi}{1-\psi}}. \tag{14}$$

Recall that N is a function of the endogenous variable m. Thus $\hat{w}$ is also a function of m.

The equations (13) and (14) can also be combined to

$$\ell' = \alpha\hat{q}\hat{Y}N^{-\frac{1}{1-\psi}}. \tag{15}$$

Integrating this across all active firms yield

$$\int\int \ell'\hat{z}(\hat{q})\omega(\alpha)d\hat{q}d\alpha = N^{-\frac{1}{1-\psi}}\hat{Y}\int\int \alpha\hat{q}\hat{z}(\hat{q})\omega(\alpha)d\hat{q}d\alpha = N^{-\frac{\psi}{1-\psi}}\hat{Y}.$$

The left-hand side is the aggregate employment L. Thus,

$$L = N^{-\frac{\psi}{1-\psi}}\hat{Y}.$$

One way of looking at this equation is that $\hat{Y}$ can be pinned down once we know L and N (and thus L and m). Plugging (14) and (15) into (12) yields

$$\hat{\pi} = \psi\alpha\hat{q}\frac{\hat{Y}}{N}.$$

Now, let us characterize the innovation decision of a intermediate-good firm. Recall that the value functions are (with our simplifications)

$$\hat{Z}(\hat{q},\alpha) = (1-\delta)\hat{V}^s(\hat{q},\alpha),$$

where

$$\hat{V}^s(\hat{q},\alpha) = \max_{x_I} \quad \psi\alpha\hat{q}\frac{\hat{Y}}{N} - \theta_I\hat{q}{x_I}^{\gamma} + \beta(1-\mu)\hat{S}(x_I,\hat{q}/(1+g_q)) \tag{16}$$

and

$$\hat{S}(x_I,\hat{q}/(1+g_q)) = (1-x_I)\int \hat{Z}(\hat{q}/(1+g_q),\alpha')\omega(\alpha')d\alpha' + x_I\int \hat{Z}((1+\lambda_I)\hat{q}/(1+g_q),\alpha')\omega(\alpha')d\alpha'.$$

We start from making a guess that $\hat{Z}(\hat{q},\alpha)$ takes the form

$$\hat{Z}(\hat{q},\alpha) = \mathcal{A}\alpha\hat{q} + \mathcal{B}\hat{q},$$

where $\mathcal{A}$ and $\mathcal{B}$ are constants. With this guess, the first-order condition in (16) for x_I is

$$\gamma\theta_I\hat{q}{x_I}^{\gamma-1} = \frac{\beta(1-\mu)\lambda_I(\mathcal{A}+\mathcal{B})\hat{q}}{1+g_q}.$$

Thus

$$x_I = \left(\frac{\beta(1-\mu)\lambda_I(\mathcal{A}+\mathcal{B})}{(1+g_q)\gamma\theta_I}\right)^{\frac{1}{\gamma-1}} \tag{17}$$

and x_I is constant across $\hat{q}$ and α. Using this x_I,

$$\hat{Z}(\hat{q},\alpha) = (1-\delta)\left(\psi\alpha\hat{q}\frac{\hat{Y}}{N} - \theta_I\hat{q}{x_I}^{\gamma} + \beta(1-\mu)\frac{1+x_I\lambda_I}{1+g_q}(\mathcal{A}+\mathcal{B})\hat{q}\right).$$

Thus, the guess is verified with

$$\mathcal{A} = (1-\delta)\psi\frac{\hat{Y}}{N}$$

and $\mathcal{B}$ is a value that solves

$$\mathcal{B} = (1-\delta)\left(-\theta_I{x_I}^{\gamma} + \beta(1-\mu)\frac{1+x_I\lambda_I}{1+g_q}(\mathcal{A}+\mathcal{B})\right) = (1-\delta)\beta(1-\mu)\left(1+\frac{\gamma-1}{\gamma}\lambda_I x_I\right)\frac{\mathcal{A}+\mathcal{B}}{1+g_q},$$

where x_I is given by (17). Therefore, we found that x_I (and the coefficients of $\hat{Z}(\hat{q},\alpha)$ function) is a function of endogenous aggregate variables μ, g_q, $\hat{Y}$, and N. We have already seen that we can pin down μ and N if we know m, and $\hat{Y}$ can be pinned down if we know m and L. How about g_q?

To calculate g_q, let us start from the measure of q_t (without normalization) for active products, $z(q_t)$. As we have seen above, the transitory shock α does not affect the innovation decision, and thus can be ignored when calculating the transition of q_t. The fraction $(1-\mu)x_I(1-\delta)$ of active lines experiences innovation by incumbents, and the fraction $(1-\mu-(1-\mu)x_I)(1-\delta)$ do not experience any innovation (but stay in the market). The fraction $\mu(1-\delta)$ of active products experiences innovation by entrants. Among the inactive products, the fraction $\mu(1-\delta)$ experiences innovation by entrants, but it is an upgrade from distribution $h(q_t/\bar{q}_t)$ rather than $z(q_t)/N$. Thus g_q can be calculated from

$$1+g_q = (1-\delta)\left[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu + (1+\lambda_E)\mu\frac{1-N}{N}\frac{\bar{q}^h}{\bar{q}^z}\right].$$

The first term is the productivity increase of the surviving incumbents, the second term is the entry into active products, and the last is the entry into inactive products. Here, $\bar{q}^h$ and $\bar{q}^z$ are averages of q_t with respect to distributions h and z. Thus $\bar{q}^h/\bar{q}^z = \int q_t h(q_t/\bar{q}_t)dq_t/\int q_t[z(q_t)/N]dq_t = \int \hat{q}h(\hat{q})d\hat{q}/\int \hat{q}[\hat{z}(\hat{q})/N]d\hat{q}$. Using the expression for N in (10) and the fact that $\bar{q}^z = 1$,

$$g_q = (1-\delta)[(1+\lambda_I x_I)(1-\mu) + (1+\lambda_E)\mu] + \delta(1+\lambda_E)\bar{q}^h - 1.$$

Thus, g_q can be written as a function of μ and x_I, and therefore m and L.

From above procedure, we found that once we pin down m and L, we can determine all endogenous variables in the economy. The values of m and L can be pinned down by two additional conditions: the labor-market equilibrium condition and the free-entry condition. To see this, let us first be explicit about each variable's (and each coefficient's) dependence on m and L: $\hat{w}(m)$, $N(m)$, $\hat{Y}(m,L)$, $x_I(m,L)$, $g_q(m,L)$, $\mathcal{A}(m,L)$, and $\mathcal{B}(m,L)$. Also note that the total R&D, $\hat{R}$, can be written as

$$\hat{R} = \int \theta_I \hat{q} x_I(m,L)^\gamma \hat{z}(\hat{q})d\hat{q} + m(\phi + \theta_E {x_E}^\gamma) = \theta_I N(m) x_I(m,L)^\gamma + m(\phi + \theta_E {x_E}^\gamma)$$

and therefore we can write $\hat{R}(m,L)$.

The labor-market equilibrium condition is

$$\frac{\hat{w}(m)}{\hat{Y}(m,L) - \hat{R}(m,L)} = \xi$$

and the free-entry condition is

$$\begin{aligned}\frac{\gamma\theta_E {x_E}^{\gamma-1}}{\beta} = \hat{\hat{V}}_E &= \int\left[\int \hat{Z}((1+\lambda_E)\hat{q}/(1+g_q),\alpha)(\hat{z}(\hat{q})+(1-N)h(\hat{q}))d\hat{q}\right]\omega(\alpha)d\alpha \\ &= \int \frac{\mathcal{A}(m,L)+\mathcal{B}(m,L)}{1+g_q(m,L)}(1+\lambda_E)\hat{q}(\hat{z}(\hat{q})+(1-N)h(\hat{q}))d\hat{q} \\ &= \frac{\mathcal{A}(m,L)+\mathcal{B}(m,L)}{1+g_q(m,L)}(1+\lambda_E)[N(m)+(1-N(m))\bar{q}^h].\end{aligned}$$

These two equations pin down the values of m and L.

C.2 Productivity distribution

The invariant distribution $\hat{z}(\hat{q})$ can easily be computed. The next-period mass at relative quality $\hat{q}$ is (i) upgrade by incumbents' innovation: $(1-\delta)(1-\mu)x_I\hat{z}((1+g_q)\hat{q}/(1+\lambda_I))d\hat{q}$, (ii) upgrade by entrants' innovation: $(1-\delta)\mu\hat{z}((1+g_q)\hat{q}/(1+\lambda_E))d\hat{q}$, (ii) natural downgrade from non-innovating products: $((1-\delta)(1-\mu-(1-\mu)x_I)\hat{z}((1+g_q)\hat{q})d\hat{q}$, and (iii) entry from inactive products, $(1-\delta)\mu(1-N)h(\hat{q}/(1+\lambda_E))dq$. The sum of these has to be equal to $\hat{z}(\hat{q})d\hat{q}$ along the stationary growth path.

In fact, it is possible to characterize the right tail of the distribution analytically, when the distribution $h(\hat{q})$ is bounded. Let the density function of the stationary distribution be $s(\hat{q}) \equiv \hat{z}(\hat{q})/N$. Because $h(\hat{q})$ is bounded, there is no direct inflow from the inactive product lines at the right tail.

Take the point $\hat{q}$ and interval Δ around that point. The outflow from that interval is $s(\hat{q})\Delta$, as entire firms there will either move up, move down, or exit.

The inflow is from two sources. First is the mass of firms who innovated. Innovation is either done by incumbents or entrants. Let $\gamma_i \equiv (1+\lambda_I)/(1+g_q) > 1$ be the (adjusted) improvement of $\hat{q}$ upon innovation by incumbents. The probability of innovation by either incumbents is $(1-\delta)(1-\mu)x_I$. Thus the mass of inflow into the above interval is $(1-\delta)(1-\mu)x_I s(\hat{q}/\gamma_i)\Delta/\gamma_i$. Similarly, letting $\gamma_e \equiv (1+\lambda_E)/(1+g_q) > 1$ be the improvement of $\hat{q}$ upon innovation by entrants, the mass of inflow due to entrants' innovation is $(1-\delta)\mu s(\hat{q}/\gamma_e)\Delta/\gamma_e$.

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The second inflow is the firms that didn't innovate or exit. With probability $(1-\delta)(1-\mu-(1-\mu)x_I)$, there are no innovations (nor exit). Let $\gamma_n \equiv 1/(1+g_q) < 1$ be the (adjusted) improvement (in this case the "negative improvement") when there is no innovation. Then the mass of inflow into above interval is $(1-\delta)(1-\mu-(1-\mu)x_I)s(\hat{q}/\gamma_n)\Delta/\gamma_n$.

In the stationary distribution, inflow equals outflow, and therefore

$$s(\hat{q})\Delta = (1-\delta)\left[(1-\mu)x_I s\left(\frac{\hat{q}}{\gamma_i}\right)\frac{\Delta}{\gamma_i} + \mu s\left(\frac{\hat{q}}{\gamma_e}\right)\frac{\Delta}{\gamma_e} + (1-\mu-(1-\mu)x_I)s\left(\frac{\hat{q}}{\gamma_n}\right)\frac{\Delta}{\gamma_n}\right],$$

or

$$s(\hat{q}) = (1-\delta)\left[(1-\mu)x_I s\left(\frac{\hat{q}}{\gamma_i}\right)\frac{1}{\gamma_i} + \mu s\left(\frac{\hat{q}}{\gamma_e}\right)\frac{1}{\gamma_e} + (1-\mu-(1-\mu)x_I)s\left(\frac{\hat{q}}{\gamma_n}\right)\frac{1}{\gamma_n}\right],$$

Guess that the right-tail of the density function is Pareto and has the form $s(x) = Fx^{-(\kappa+1)}$. $\kappa >$ is the shape parameter and the expected value of x exists only if $\kappa > 1$. Plugging this guess into the above yields

$$\begin{aligned} F\hat{q}^{-(\kappa+1)} = \\ & (1-\delta)\Bigg[(1-\mu)x_I F\left(\frac{\hat{q}}{\gamma_i}\right)^{-(\kappa+1)}\frac{1}{\gamma_i} + \mu F\left(\frac{\hat{q}}{\gamma_e}\right)^{-(\kappa+1)}\frac{1}{\gamma_e} \\ & \qquad + (1-\mu-(1-\mu)x_I)F\left(\frac{\hat{q}}{\gamma_n}\right)^{-(\kappa+1)}\frac{1}{\gamma_n}\Bigg], \end{aligned}$$

or

$$1 = (1-\delta)\left[(1-\mu)x_I\gamma_i^{\kappa} + \mu\gamma_e^{\kappa} + (1-\mu-(1-\mu)x_I)\gamma_n^{\kappa}\right].$$

Thus, κ is the solution of this equation.

D Robustness checks

[To be completed]