Loss Aversion in Sequential Auctions: Endogenous Interdependence, Informational Externalities and the "Afternoon Effect"

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Abstract

Empirical evidence from sequential auctions shows that prices of identical goods tend to decline between rounds. In this paper, I show how expectations-based reference-dependent preferences and loss aversion can rationalize this phenomenon. I analyze two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand. Equilibrium bids in the second round are history-dependent and subject to a "discouragement effect": the higher the winning bid in the first auction is, the less aggressive the behavior of the remaining bidders in the second auction. When choosing his strategy in the first round, however, a bidder conditions his bid on being pivotal and hence underestimates the discouragement effect. Equilibrium behavior, therefore, leads the first-round winner to overestimate the next-round price so that in equilibrium prices decline. Moreover, I show that sequential and simultaneous auctions are not bidder-payoff equivalent nor revenue equivalent.

JEL classification: D03; D44; D81; D82.

Keywords: Reference-Dependent Preferences; Loss Aversion; Sequential Auctions; Afternoon Effect.

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1 Introduction

Sequential auctions are often used to sell multiple lots of identical or similar goods. How should one expect prices to vary from one round to the next? Weber (1983) and Milgrom and Weber (2000) showed that with symmetric, risk-neutral, unit-demand bidders having independent private values, the law of one price should hold and on average prices should be the same across different rounds. Intuitively, if they were not, then demand from the rounds with a higher expected price would shift towards those rounds with a lower expected price, due to arbitrage opportunities. The intuition for this result is very general and does not depend on the specific type of auction.[1]

However, this neat theoretical result does not seem to be supported by the data. Ashenfelter (1989), McAfee and Vincent (1993) and Ginsburgh (1998) document a puzzling *declining price anomaly* or *afternoon effect* (reflecting that later auctions often take place in the afternoon whereas earlier ones are in the morning) in sequential second-price and English auctions for identical bottles of wine. Declining prices have been also found by Beggs and Graddy (1997) in English auctions for artwork, Ashenfelter and Genesove (1992) in first-price auctions for identical condominium units, Van den Berg *et al.* (2001) in Dutch auctions for flowers and Lambson and Thurston (2006) in English auctions for fur. There is also experimental evidence of declining prices; see, for instance, Keser and Olson (1996), Février *et al.* (2007), and Neugebauer and Pezanis-Christou (2007). Moreover, while declining prices are more frequent, increasing prices have also been documented by Chanel *et al.* (1996), Gandal (1997), and Deltas and Kosmopoulou (2004).[2] Overall, declining prices have been documented in more instances than rising prices have (Ashenfelter and Graddy, 2003). Declining prices do not occur in every auction, but they seem to be an empirically robust feature of sequential auctions.

In this paper, I study two-round sealed-bid auctions with symmetric bidders having independent private values and unit demand and I argue that reference-dependent preferences and loss aversion provide an explanation for the afternoon effect. More generally, I show that reference-dependent preferences with expectations as the reference point induce an endogenous form of interdependence in the bidders' payoffs even though values are private and independent. Indeed, the derivation of the equilibrium strategies resembles the standard reference-free model with interdependent (common) values. The reason is that even though a bidder's valuation does not depend directly on his competitors' types, these affect his likelihood of winning the auction and hence his reference point. Moreover, I also show that if bidders have expectations-based reference-dependent preferences, sequential and simultaneous auctions are not revenue equivalent anymore.

[1] Technically, with independent private values, the price sequence of any standard auction is a martingale, so that the expected price in round $k+1$, conditional on p_k, the price in round k, is equal to p_k.

[2] Milgrom and Weber (2000) showed that if bidders' signals are affiliated and values are interdependent, then the equilibrium price sequence is a submartingale and the expected value of p_{k+1}, conditional on p_k, is higher than p_k. Mezzetti (2011) showed that affiliated signals are not necessary to explain increasing-price sequences: interdependent values with informational externalities — that is, when a bidder's value is increasing in all bidders' private signals — even with independent signals, push prices to increase between rounds.

Section 2 introduces the model of bidders' preferences and the solution concept. Following Köszegi and Rabin (2006), I assume that in addition to classical *consumption utility*, a bidder also derives *gain-loss utility* from the comparison of his consumption to a reference point equal to his lagged expectations regarding the same outcomes, with losses being more painful than equal-sized gains are pleasant. To account for the intrinsic dynamic nature of sequential auctions, I develop a dynamic version of the Choice Acclimating Personal Equilibrium (*CPE)* introduced in Köszegi and Rabin (2007) that I call Sequential Choice Acclimating Personal Equilibrium (*SCPE).* In a *SCPE*, a decision maker uses backward induction to correctly predict his (possibly stochastic) strategy at each point in the future and then applies the same (static) *CPE* as in Köszegi and Rabin (2007) at every stage of the game, with his reference point in a given stage being his beliefs over final outcomes conditional on the information available at that stage.[3]

Sections 3 analyzes sequential first-price and second-price auctions. First, I show that loss aversion creates an informational externality that renders equilibrium bids history-dependent, even if bidders have independent private values. Intuitively, learning the outcome of the previous auction modifies a bidder's expectations about how likely he is to win in the current one. Since expectations are the reference point, the optimal bid in the second auction depends also on what a bidder learns from the first one as this modifies his reference point. More precisely, I identify what I call the *discouragement effect*: the higher the winning bid in the first auction is, the less aggressive the bidding strategy of the remaining bidders in the second auction. The intuition is that, from the point of view of a bidder who lost the first auction, the higher the type of the winner is, the less likely he is to win in the second one; this in turn lowers the bidder's reference point who does not feel a strong attachment to the item and therefore reduces his equilibrium bid.

Notice that the history dependence arising in my model has the opposite effect of the one stemming from interdependent (common) values. With interdependent values, since in equilibrium he conditions his bid on himself having the highest signal, if a bidder loses the current round he learns that the winner had a higher signal than his; this in turn makes a losing bidder revise his estimate of the value of the good upward and therefore he will bid more aggressively in subsequent rounds. The discouragement effect instead goes in the opposite direction by pushing bidders to bid less aggressively in later rounds.

In equilibrium, bidders are indifferent between winning in the first round or in the second one. In order for a bidder to be indifferent between winning in the current round or the next one, in the current round he must bid the expectation of the next-round price as though he was tied with his closest competitor. By conditioning on being tied with his closest competitor, however, a bidder underestimates the discouragement effect which will lower his competitors' bids in the next round. Equilibrium behavior, therefore, leads the winner of the first round to overestimate the next-round

[3]The original notion of *CPE* in Köszegi and Rabin (2007) is related to the models of "disappointment aversion" of Bell (1985), Loomes and Sugden (1986), and Gul (1991), where outcomes are also evaluated relative to a reference lottery that is identical to the chosen lottery; likewise the notion of *SCPE* introduced in this paper is related to the notion of dynamic disappointment aversion proposed in Artstein-Avidan and Dillenberger (2015).

price so that equilibrium prices tend to decline.

With risk-neutral bidders having independent private values, sequential and simultaneous auctions are revenue equivalent for the seller and payoff-equivalent for the bidders. In Section 4 I show that these equivalences break down if bidders are expectations-based loss-averse. The key difference between sequential and simultaneous auctions is the timing of information. Sequential auctions provide bidders, in between rounds, with the opportunity to update their beliefs about the intensity of competition. Such feedback, however, is absent in simultaneous auctions. In the classical model this difference is irrelevant since bidding strategies in sequential auctions are history-independent. Loss-averse bidders, instead, update their reference point based on the outcome of the previous round. I show that bidders with high (resp. low) values prefer sequential (simultaneous) auctions and that sequential auctions generate a higher (resp. lower) revenue than simultaneous ones when the number of bidders is large (small). It is well known that sequential auctions are more vulnerable to collusion than simultaneous ones; see, for instance, Cramton and Schwartz (2000), Klemperer (2002), and Sherstyuk and Dulatre (2008). As collusion among bidders tends to reduce the seller's revenue, we would expect sellers to prefer simultaneous auctions over sequential ones. Yet, loss aversion provides a novel reason why in some situations sequential auctions are better for the seller.

For most of the paper, when studying sequential auctions, I assume that the winning bid in the first round is publicly announced by the seller prior to the second round. This assumption is inconsequential in the classical reference-free model when bidders have independent private values, but it is not if bidders have reference-dependent preferences. Therefore, in Section 5 I analyze sequential auctions under two alternative disclosure policies. First, I consider sequential auctions with no bid announcement and I show that the equilibrium strategies are radically different. If the winning bid from the first round is not publicly revealed, a losing bidder must use his own past bid to update his expectations about how likely he is to win in the second one. As auctions without bid announcement provide them with a noisier feedback mechanism, thus exposing them to greater risk, loss-averse bidders react by bidding less aggressively so that the seller's expected revenue decreases. Nevertheless, the afternoon effect still arises in equilibrium. Next, I discuss sequential second-price auctions with announcement of the first-round winning price and I show via an example that in this case existence of a symmetric equilibrium in increasing strategies is not guaranteed. The reason is that, just like in the classical model with interdependent values, revealing the winning price makes the game in the second round highly asymmetric as one of the remaining bidders would have his exact bid known to the others.

Section 6 presents two extensions of the main model. In Section 6.1 I consider sequential common-value auctions. It is well-known that with risk-neutral bidders and common values, the equilibrium price sequence is a submartingale so that prices drift upward. I show that if bidders are loss-averse, however, the equilibrium price sequence can be either increasing or decreasing depending on the magnitude of the loss aversion coefficient.

For tractability, as well as to make the contrast with the previous literature as stark as possible, for most of the paper I assume that bidders are loss-averse with respect to consumption but risk-neutral over money. In Section 6.2 I relax this assumption and analyze sequential first-price auctions with bidders who are loss-averse over both consumption and money. The effect of loss aversion over money on the equilibrium price path can either go in the same direction as that of loss aversion over consumption, or in the opposite one, depending on the number of bidders and the shape of the distribution of values. Yet, I show that if money loss aversion is not too strong, the afternoon effect still arises in equilibrium.

Many different explanations for the afternoon effect have been proposed. Ashenfelter (1989) hypothesized risk aversion as a plausible explanation. Yet, McAfee and Vincent (1993) argue that risk aversion is not a convincing explanation. They study two-round first-price and second-price auctions with independent private values, and show that equilibrium prices decline only if bidders display increasing absolute risk aversion. Under the more plausible assumption of decreasing absolute risk aversion, a monotone symmetric pure-strategy equilibrium fails to exist and prices need not decline. Black and De Meza (1992) and Février *et al.* (2005) argue that declining prices are no anomaly if the winner in one round is allowed to purchase subsequent lots at the same price.[4] Bernhardt and Scoones (1994), Engelbrecht-Wiggans (1994) and Gale and Hausch (1994) consider sequential auctions of "stochastically equivalent" objects — that is, when bidders' values are identically distributed across the objects, but are not perfectly correlated — and show that in this case equilibrium prices decline. Eyster (2002) models the behavior of an agent who has a taste for rationalizing past actions by taking current actions for which those past actions were optimal. He shows that this taste for consistency gives rise to an "unsunk-cost fallacy" that can rationalize declining prices in sequential auctions. Other studies have emphasized demand complementarity (Menezes and Monteiro, 2003), supply uncertainty (Jeitschko, 1999), asymmetric bidders (Gale and Stegeman, 2001), order-of-sale effects (Chakraborty *et al.*, 2006) and budget constraints (Pitchik and Schotter, 1988; Ghosh and Liu, 2016) in accounting for the declining price anomaly.

More recently, Mezzetti (2011) introduced a special case of risk aversion, called *aversion to price risk,* according to which a bidder prefers to win an object at a certain price rather than at a random price with the same expected value.[5] Under this different notion of risk aversion, in sequential auctions with independent private values a monotone equilibrium in pure strategies always exists and in equilibrium prices decline.[6] Although aversion to price risk and loss aversion are both able to explain the afternoon effect, the intuition behind the result is quite different. In Mezzetti (2011), the afternoon effect is due to the bidders' dislike of uncertainty over money; in my model, instead, the afternoon effect arises because bidders dislike uncertainty over consumption.

[4] However, Ashenfelter (1989) finds declining prices also for the case of bidders with unit demand.

[5] Similarly to the model of Köszegi and Rabin (2006), Mezzetti's notion also assumes separability of a bidder's payoff between the utility from winning the object and the disutility from paying the price.

[6] Hu and Zou (2015) generalize the analysis in Mezzetti (2011) by considering bidders who are heterogeneous in exposure to background risk.

The different intuition translates also into different testable predictions. When bidders are averse to price risk, the equilibrium bids do not depend on the history of the game and therefore the seller's information revelation policy does not affect revenue; with loss aversion, instead, bidders' strategies are history-dependent and the seller is always better off by committing to publicly reveal the history of the winning bids.

Section 7 concludes the paper by recapping the results of the model and pointing out some of its limitations as well as possible avenues for future research.

2 Model

2.1 Environment

Suppose 2 identical items are sold to $N > 2$ bidders via a series of sealed-bid auctions. More specifically, one item is sold using a sealed-bid auction and the winning bid is publicly announced; then, the remaining item is sold using another sealed-bid auction (of the same kind). Announcing the winning bid from former auctions prior to the current one is in accord with government procurement statutes and with actual practice in some auctions.

I assume bidders want at most one unit and have independent private values. Each bidder's valuation (type) θ_i, $i = 1, ..., n$, is drawn independently from the same continuous and strictly increasing distribution F which admits a continuous and positive density f everywhere on the support $\left[0, \overline{\theta}\right]$. I will consider two types of games. In the first one, the goods are sold sequentially via a series of first-price auctions. In the second one, the goods are sold sequentially via a series of second-price auctions. Both auctions have a zero reserve price.[7] Throughout the paper, I restrict attention to symmetric equilibria in pure and (strictly) monotone strategies. It is convenient to think of the auctions as being held in different periods of the day, the first one in the morning and the second one in the afternoon; however I assume the auctions are held over a short enough time so that bidders do not discount payoffs from the second auction.

2.2 Bidders' Preferences

Bidders have expectations-based reference-dependent preferences as formulated by Köszegi and Rabin (2006). In this formulation, a bidder's utility function has two components:

$$U\left[c^g, c^p | r^g, r^p, \theta\right] = \underbrace{\theta c^g + c^p}_{\text{consumption utility}} + \underbrace{\mu\left(\theta\left(c^g - r^g\right)\right) + \mu\left(c^p - r^p\right)}_{\text{gain-loss utility}} \tag{1}$$

where $c^g, r^g \in \{0, 1\}$ capture the good dimension and $c^p, r^p \in \mathbb{R}$ capture the money dimension. First, if he wins the auction at price p, a type-θ bidder experiences *consumption utility* $\theta - p$, which

[7] See Rosenkranz and Schmitz (2007) for an analysis of reserve prices in auctions as reference points.

represents the classical notion of outcome-based utility. Second, the bidder also derives *gain-loss utility* from the comparison of his actual consumption to a reference point given by his recent expectations (probabilistic beliefs).[8] I assume μ is two-piece linear with a slope of $\eta > 0$ for gains and a slope of $\eta\lambda > \eta$ for losses. The parameter η captures the relative weight a consumer attaches to gain-loss utility while λ is the coefficient of loss aversion.

Because in many situations expectations are stochastic, Köszegi and Rabin (2006) extend the utility function in (1) to allow for the reference point to be a pair of probability distributions $\mathbf{H} := (H^g, H^p)$. In this case a bidder's utility can be written as

$$U\left[\left(c^g, c^p\right)|H^g, H^p, \theta\right] = \theta c^g + c^p + \int_{r^g} \mu\left(\theta\left(c^g - r^g\right)\right) dH^g + \int_{r^p} \mu\left(c^p - r^p\right) dH^p \tag{2}$$

In words, a bidder compares the realized consumption outcome with each possible outcome in the reference lottery. For example, if he expected to win the auction with probability q, then winning the auction entails a gain of $\theta(1-q)$ while losing the auction results in a loss of θq. Thus, the weight on the loss (gain) in the overall experience is equal to the probability with which he was expecting to win (lose) the auction. Slightly departing from the original model of Köszegi and Rabin (2006), for most of the paper I will assume that bidders have reference-dependent preferences only with respect to their value for the item, but not with respect to the price they might pay; in other words, bidders are risk neutral over money. This assumption is reasonable if bidders' income is already subject to large background risk as argued by Köszegi and Rabin (2009). Relatedly, Novemsky and Kahneman (2005) propose that money given up in purchases is not generally subject to loss aversion.[9]

2.3 Solution Concept

Each bidder learns his type before submitting his bids and, therefore, maximizes his interim expected utility. If the distribution of the reference point is $\mathbf{H}$ and the distribution of consumption outcomes is $\mathbf{G} := (G^g, G^p)$, a type-θ bidder's interim expected utility is given by

$$EU\left[\mathbf{G}|\mathbf{H}, \theta\right] = \int_{\{c^g, c^p\}} \int_{\{r^g, r^p\}} U\left[\left(c^g, c^p\right)|H^g, H^p, \theta\right] d\mathbf{H} d\mathbf{G}.$$

For each auction in which he participates, after placing a bid, a bidder basically faces a lottery between winning or losing the auction and the probabilities and potential payoffs depend on his

[8]Recent experimental evidence lends support to Köszegi and Rabin's (2006) expectations-based model of reference-dependent preferences and loss aversion; see Abeler *et al.* (2011), Ericson and Fuster (2011), Gill and Prowse (2012), Banerji and Gupta (2014), Karle *et al.* (2015), Sprenger (2015) and Rosato and Tymula (2016).

[9]One advantage of assuming reference-dependence only over consumption is that it preserves revenue equivalence between first-price and second-price auctions. Moreover, by assuming that bidders are risk neutral over money I am shutting down risk in payments as a possible explanation for the afternoon effect. Indeed, differently from the works of Eyster (2002), Mezzetti (2011) and Hu and Zou (2015), all the results in my paper will be due to uncertainty and information updating over consumption.

own as well as other players' bids. The final outcome is then evaluated with respect to any possible outcome from this lottery as a reference point. As laid out in Köszegi and Rabin (2007), Choice Acclimating Personal Equilibrium (*CPE)* is the most appropriate solution concept for such decisions under risk when uncertainty is resolved after the decision is made so that the decision maker's strategy determines the distribution of the reference point as well as the distribution of final consumption outcomes; that is, $\mathbf{H} = \mathbf{G}$.

A strategy for bidder i is a pair of bidding functions $\boldsymbol{\beta}_i = (\beta_1, \beta_2)$, one for each auction. Fixing all other bidders' strategies, $\boldsymbol{\beta}_{-i}$, bidder i's strategy $\boldsymbol{\beta}_i$ induces a distribution over the set of final consumption outcomes, $\mathcal{A} := \{0, 1\} \times \mathbb{R}$. For $k = 1, 2$, let $\Gamma_k \left(\mathcal{A} | \boldsymbol{\beta}_i, \boldsymbol{\beta}_{-i}\right)$ denote the distribution over final (i.e., after the second auction) consumption outcomes from auction k point of view conditional on all available information.[10] Similarly, let EU_k denote a bidder expected utility from auction k point of view. To account for the intrinsic dynamic nature of sequential auctions, I introduce a slightly modified version of *CPE.*

Definition 1. *A strategy profile $\boldsymbol{\beta}^*$ constitutes a Sequential Choice Acclimating Personal Equilibrium (SCPE) if for all i, and for $k = 1, 2$:*

$$EU_k \left[\Gamma_k \left(\mathcal{A} | \boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^*\right) | \Gamma_k \left(\mathcal{A} | \boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^*\right), \theta\right] \geq EU_k \left[\Gamma_k \left(\mathcal{A} | \widetilde{\boldsymbol{\beta}}_i, \boldsymbol{\beta}_{-i}^*\right) | \Gamma_k \left(\mathcal{A} | \widetilde{\boldsymbol{\beta}}_i, \boldsymbol{\beta}_{-i}^*\right), \theta\right]$$

for any $\widetilde{\boldsymbol{\beta}}_i \neq \boldsymbol{\beta}_i^*$.

In words, in a *SCPE* a bidder correctly predicts his (possibly stochastic) strategy in the future using backward induction and applies the same (static) *CPE* as in Köszegi and Rabin (2007) at every stage of the game. The interpretation of *SCPE* is that each bidder understands that once consumption occurs, i.e. once all the auctions are over, he evaluates the realized outcome against the reference lottery. Notice that in round k, a bidder's reference point is given by his round-k expectations, $\Gamma_k \left(\mathcal{A} | \boldsymbol{\beta}_i^*, \boldsymbol{\beta}_{-i}^*\right)$, about his final consumption outcomes. Hence, in the second round bidders update their reference point based on the outcome of the first round.[11] The following assumption, maintained for the remainder of the paper, guarantees that all bidders participate in the auction for any realization of their own type, and that the equilibrium bidding functions derived in the next sections are strictly increasing:

Assumption 1 (*No dominance of gain-loss utility*) $\Lambda \equiv \eta (\lambda - 1) \leq 1$.

[10] Hence, Γ_2 is the distribution over final allocations conditional on the outcome of the first auction.

[11] The concept of *SCPE* differs from the dynamic version of Preferred Personal Equilibrium (*PPE*) introduced by Köszegi and Rabin (2009) on two main aspects. First, in their dynamic model, agents have a reference point for every period in which they expect to consume — so that consumption levels in different periods are treated like different dimensions. Second, agents are loss-averse over changes in beliefs about present as well future consumption and experience paper gains or paper losses by comparing old beliefs to new ones even when no actual consumption takes place. In my model, instead, an agent's reference point is given by his beliefs about final consumption in the last period, and while he updates his reference point, gains and losses are only felt in the last period by comparing his most recent beliefs with his actual consumption.

This assumption places, for a given η (λ), an upper bound on λ (η) and ensures that a bidder's expected utility is increasing in his type by imposing that the weight a bidder places on expected gain-loss utility does not (strictly) exceed the weight he puts on consumption utility.[12] Finally, notice that risk neutrality is embedded in the model as a special case (for either $\eta = 0$ or $\lambda = 1$).

3 Sequential Auctions

3.1 First-price Auctions

Suppose two identical items are sold sequentially via first-price auctions. In this case, a symmetric equilibrium consists of two bidding functions (β_1, β_2), one for each auction. I assume that both functions are strictly increasing and differentiable. The first-round bidding strategy is a function $\beta_1 : \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ that depends only on the bidder's type. The bid in the second auction, instead, might depend also on the price paid in the first auction. Since we are focusing on a symmetric equilibrium, it is useful to take the point of view of one of the bidders, say bidder i with type θ_i, and to consider the order statistics associated with the types of the other bidders. Let $Y_1^{(N-1)} \equiv Y_1$ be the highest of $N-1$ values, $Y_2^{(N-1)} \equiv Y_2$ be the second-highest and so on. Also, let F_1 and F_2 be the distributions of Y_1 and Y_2 respectively, with corresponding densities f_1 and f_2. As the first-round bidding function β_1 is assumed to be invertible, after the first auction is over, and its winning bid is revealed, the type of the winning bidder is commonly known to be $y_1 = \beta_1^{-1}(p_1)$. Thus, the second-period strategy can be described as a function $\beta_2 : \left[0, \overline{\theta}\right] \times \left[0, \overline{\theta}\right] \to \mathbb{R}_+$ so that a bidder with value θ bids $\beta_2(\theta, y_1)$ if $Y_1 = y_1$. To find an equilibrium that is sequentially rational, I start by solving the bidder's problem in the last auction.

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N-2$ remaining bidders follow the equilibrium strategy $\beta_2(\cdot, y_1)$.[13] His expected payoff is

$$\begin{aligned} EU_2\left(\widetilde{\theta}, \theta; y_1\right) = \; & F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right] + F_2\left(\widetilde{\theta}|y_1\right)\eta\left\{(\theta - 0)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\right\} \\ & + \left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\eta\lambda\left\{(0 - \theta)\, F_2\left(\widetilde{\theta}|y_1\right)\right\} \end{aligned} \tag{3}$$

where $F_2\left(\widetilde{\theta}|y_1\right)$ is the probability that Y_2, the second highest valuation among $N-1$, is less than $\widetilde{\theta}$ conditional on $Y_1 = y_1$ being the highest. The first term on the right-hand-side of (3), $F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right]$, is standard expected consumption utility. The other terms capture ex-

[12] Herweg *et al.* (2010) first introduced Assumption 1 and referred to it as "no dominance of gain-loss utility". This assumption, which has been used also by Lange and Ratan (2010) and Eisenhuth (2012) in the context of single-unit sealed-bid auctions, ensures that a loss-averse agent does not select first-order stochastically-dominated options. Using data from first-price and all-pay auctions, Eisenhuth and Ewers (2012) obtain an estimate for Λ of 0.42 (with a standard error of 0.16), which is statistically different from 0 and 1 at all conventional significance levels; similarly, using data from a BDM-like auction, Banerji and Gupta (2014) obtain an estimate for Λ of 0.283 (with a standard error of 0.08), also statistically different from 0 and 1 at all conventional significance levels.

[13] The analysis is virtually identical for the case $\widetilde{\theta} < \theta$.

pected gain-loss utility and are derived as follows. A bidder of type θ who bids as if his type were $\widetilde{\theta}$ expects to win the auction with probability $F_2\left(\widetilde{\theta}|y_1\right)$ and if he wins he gets consumption utility θ; thus, winning the auction feels like a gain of $\eta(\theta-0)$ compared to the outcome of losing the auction and getting 0, which the bidder expected to happen with probability $\left[1-F_2\left(\widetilde{\theta}|y_1\right)\right]$. Similarly, with probability $\left[1-F_2\left(\widetilde{\theta}|y_1\right)\right]$ the bidder loses the auction and gets 0; thus, losing the auction entails a loss of $\eta\lambda(0-\theta)$ compared to winning the auction and getting θ, which the bidder expected to happen with probability $F_2\left(\widetilde{\theta}|y_1\right)$. Collecting terms we can re-write (3) as

$$EU_2\left(\widetilde{\theta},\theta;y_1\right)=F_2\left(\widetilde{\theta}|y_1\right)\left[\theta-\beta_2\left(\widetilde{\theta},y_1\right)\right]-\Lambda\theta F_2\left(\widetilde{\theta}|y_1\right)\left[1-F_2\left(\widetilde{\theta}|y_1\right)\right]$$

where $\Lambda\equiv\eta(\lambda-1)$ is the weight on expected gain-loss utility. Notice that expected gain-loss utility is always negative as, since $\lambda>1$, losses are felt more heavily than equal-size gains.

Differentiating $EU_2\left(\widetilde{\theta},\theta;y_1\right)$ with respect to $\widetilde{\theta}$ yields the first-order condition:

$$\beta_2'\left(\widetilde{\theta},y_1\right)F_2\left(\widetilde{\theta}|y_1\right)=f_2\left(\widetilde{\theta}|y_1\right)\left[\theta-\beta_2\left(\widetilde{\theta},y_1\right)\right]-\Lambda\theta f_2\left(\widetilde{\theta}|y_1\right)\left[1-2F_2\left(\widetilde{\theta}|y_1\right)\right]\tag{4}$$

where β_2' is the derivative of β_2 with respect to its first argument. It is useful to compare condition (4) with its risk-neutral analog. The term on the left-hand-side of (4) captures the bidder's cost from raising his bid (i.e., paying a higher price when winning) which at the margin, without loss aversion ($\Lambda=0$), must equal the gain from raising his bid (i.e., winning more often) which is captured by the first term on the right-hand-side of (4). With loss aversion, however, there is an additional term on the right-hand-side of (4) which captures how raising his bid affects a bidder's reference point and expected gain-loss utility. In particular, the term $-\Lambda\theta f_2\left(\widetilde{\theta}|y_1\right)\left[1-2F_2\left(\widetilde{\theta}|y_1\right)\right]$ is positive if and only if $F_2\left(\widetilde{\theta}|y_1\right)>\frac{1}{2}$ which follows from the fact that expected gain-loss utility is proportional to the variance of the Bernoulli distributed outcome of winning or losing the auction.

Substituting $\theta=\widetilde{\theta}$ into (4) and re-arranging yields the following differential equation

$$\frac{\partial}{\partial\theta}\left\{\beta_2(\theta,y_1)F_2(\theta|y_1)\right\}=f_2(\theta|y_1)\,\theta\left\{1-\Lambda\left[1-2F_2(\theta|y_1)\right]\right\}\tag{5}$$

together with the boundary condition that $\beta_2(0,y_1)=0$. Because the different values are drawn independently, we have that

$$F_2(\theta|y_1)=\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}$$

and substituting into (5) yields

$$\beta_2^*(\theta,y_1)=\frac{\int_0^\theta x\left\{1-\Lambda\left[1-\frac{2F(x)^{N-2}}{F(y_1)^{N-2}}\right]\right\}dF(x)^{N-2}}{F(\theta)^{N-2}}.\tag{6}$$

In order to verify that $\beta_2^*(\theta, y_1)$ is indeed an equilibrium notice that since $\frac{dEU_2\left(\widetilde{\theta},\theta;y_1\right)}{d\widetilde{\theta}}|_{\widetilde{\theta}=\theta} = 0$ must hold for all types, it must also hold for type $\widetilde{\theta}$; that is

$$0 = \frac{dEU_2\left(\widetilde{\theta},\widetilde{\theta};y_1\right)}{d\widetilde{\theta}} = f_2\left(\widetilde{\theta}|y_1\right)\left[\widetilde{\theta} - \beta_2\left(\widetilde{\theta},y_1\right)\right] - \beta_2'\left(\widetilde{\theta},y_1\right)F_2\left(\widetilde{\theta}|y_1\right) - f_2\left(\widetilde{\theta}|y_1\right)\left[1 - 2F_2\left(\widetilde{\theta}|y_1\right)\right]\widetilde{\theta}\Lambda.$$

Hence, we have that

$$\begin{aligned}\frac{dEU_2\left(\widetilde{\theta},\theta;y_1\right)}{d\widetilde{\theta}} &= \frac{dEU_2\left(\widetilde{\theta},\theta;y_1\right)}{d\widetilde{\theta}} - \frac{dEU_2\left(\widetilde{\theta},\widetilde{\theta};y_1\right)}{d\widetilde{\theta}} \\ &= f_2\left(\widetilde{\theta}|y_1\right)\left(\theta - \widetilde{\theta}\right)\left\{1 - \Lambda\left[1 - 2F_2\left(\widetilde{\theta}|y_1\right)\right]\right\}.\end{aligned}$$

Since $\Lambda \leq 1$ it follows that $\frac{dEU_2\left(\widetilde{\theta},\theta;y_1\right)}{d\widetilde{\theta}} > 0$ when $\widetilde{\theta} < \theta$ and $\frac{dEU_2\left(\widetilde{\theta},\theta;y_1\right)}{d\widetilde{\theta}} < 0$ when $\widetilde{\theta} > \theta$ and therefore $EU_2\left(\widetilde{\theta},\theta;y_1\right)$ is maximized at $\widetilde{\theta} = \theta$.

The complete bidding strategy for a type-θ bidder is to bid $\beta_2^*(\theta, y_1)$ if $\theta < y_1$ and to bid $\beta_2^*(y_1, y_1)$ if $\theta \geq y_1$. The latter might occur if a bidder of type $\theta \geq y_1$ underbid in the first round causing a lower type to win. (Of course this is an off-equilibrium event.)

The bidding function in expression (6) can be re-written as:

$$\beta_2^*(\theta, y_1) = (1-\Lambda)\frac{\int_0^\theta x dF(x)^{N-2}}{F(\theta)^{N-2}} + \Lambda\frac{\int_0^\theta \frac{2xF(x)^{N-2}}{F(y_1)^{N-2}} dF(x)^{N-2}}{F(\theta)^{N-2}}.$$

Thus, the bid under loss aversion is a convex combination of the risk-neutral bid and a term that depends on the bidder's expectations about how likely he is to win the auction. It is worth noticing is that, even if bidders have independent private values, with expectations-based reference-dependent preferences the equilibrium bidding strategy in the second period is history-dependent, as it is a function of y_1; with risk-neutral preferences ($\Lambda = 0$), instead, this is not the case:

$$\beta_2^{RN}(\theta) = \frac{\int_0^\theta x dF(x)^{N-2}}{F(\theta)^{N-2}} = \mathbb{E}\left[Y_2|Y_2 \leq \theta\right].$$

Under risk neutrality, a bidder submits a bid equal to his estimation of the highest valuation of his opponents, conditional on his own valuation being the highest. Because of this conditioning, bids are independent of the prior history of the game. Similarly, previous winning bids have no influence on the remaining active bidders' strategies if bidders are risk-averse, as in the models by McAfee and Vincent (1993), Mezzetti (2011) and Hu and Zou (2015). With reference-dependent preferences, instead, the second-round equilibrium bid is decreasing in the first-round winning bid, as shown by the following lemma.[14]

[14] All proofs are relegated to Appendix A.

Lemma 1. *(Discouragement Effect) If $\Lambda > 0$, then $\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1} < 0$. Furthermore, $\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1}$ is decreasing in N.*

According to the result in Lemma 1, the higher is the type of the winner in the first round — and hence his bid — the less aggressively the remaining bidders will bid in the second round. The rationale for this negative effect, which I call the *discouragement* effect, is as follows. From the perspective of a bidder who lost the first auction, the higher is the type of the winner, the less likely he is to win in the second auction; with expectations-based reference-dependent preferences a bidder who thinks that most likely he is not going to win does not feel a strong attachment to the item and this pushes him to bid more conservatively. Thus, revealing the first-period winner's bid creates an informational externality. However, notice that the effect of this informational externality on the second-period bids is exactly the opposite of the one that arises with interdependent (or common) values. Indeed, with interdependent values the higher is the type of the first-round winner, the higher is the value of the object to all remaining bidders who in turn bid more aggressively in the second auction. Furthermore, Lemma 1 also says that the more bidders take part in the auction, the stronger the discouragement effect. Intuitively, fixing the type of the first-round winner, a bidder gets more pessimistic about his chances of winning the second auction the more competitors he faces. Hence, by analyzing the distribution of bids in the second auction, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical risk-neutral model with either independent private values (where there is no history dependence) or with common values (where the higher is the winning price in the first auction, the more aggressively the remaining bidders behave in the second auction).

Figure 1 displays the bidding strategy $\beta_2^*(\theta, y_1)$ for two different values of y_1 assuming $\theta \overset{U}{\sim} [0,1]$, $\Lambda = \frac{1}{2}$ and $N = 4$: the dashed curve is for the case $y_1 = \frac{1}{2}$ while the solid one is for the case $y_1 = \frac{3}{4}$. As we would expect from Lemma 1, for $\theta \leq \frac{1}{2}$ $\beta_2^*\left(\theta, \frac{3}{4}\right)$ is always below $\beta_2^*\left(\theta, \frac{1}{2}\right)$.

Next, I turn to analyzing the bidder's problem in the first round. Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} > \theta$ when all other $N-1$ bidders follow the strategy $\beta_1(\cdot)$. Furthermore, suppose that all bidders expect to follow the equilibrium strategy $\beta_2^*(\theta, y_1)$ in the second auction, regardless of what happens in the first one (sequential rationality). The bidder's expected total utility at the beginning of the first round is

$$
\begin{aligned}
EU_1\left(\widetilde{\theta}, \theta\right) = {} & F_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta | y_1)\left[\theta - \beta_2^*(\theta, y_1)\right] f_1(y_1)\, dy_1 \\
& -\Lambda\theta\left[F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta | y_1) f_1(y_1)\, dy_1\right]\left[1 - F_1\left(\widetilde{\theta}\right) - \int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta | y_1) f_1(y_1)\, dy_1\right]
\end{aligned} \tag{7}
$$

where $F_1\left(\widetilde{\theta}\right)$ is the probability that Y_1, the highest valuation among $N-1$, is less than $\widetilde{\theta}$, and $F_2(\theta | y_1)$ and Λ are defined as before. The first line on the right-hand-side of (7) is the sum of expected consumption utility in periods 1 and 2. The second line captures expected gain-loss

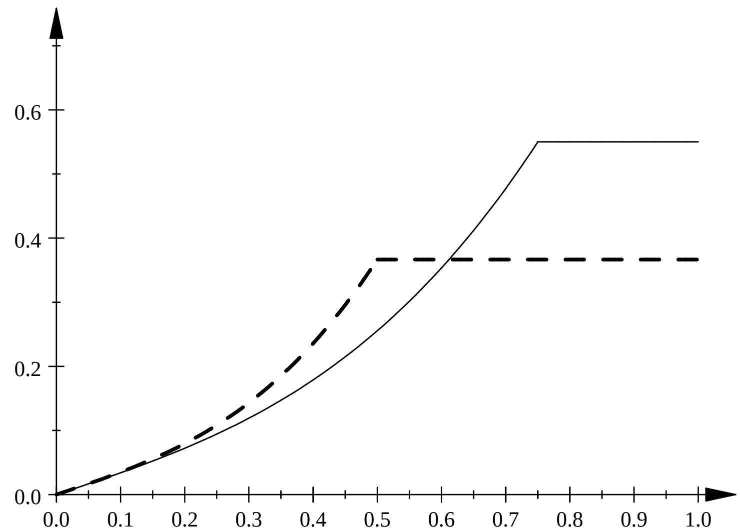

Figure 1: The effect of y_1 on $\beta_2^*(\theta, y_1)$ for $N = 4$ and $\Lambda = \frac{1}{2}$ with θ distributed uniformly on $[0, 1]$.

utility. Indeed, $F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1) \, dy_1$ is the sum of the probability with which a bidder of type θ expects to win the first auction if he pretends to be of type $\widetilde{\theta}$ and of his expectation, in the first round, of the probability of winning the second auction given that he pretends to be of type $\widetilde{\theta}$ in the first auction but expects to behave as his real type in the second one. Hence, in accordance with the definition of *SCPE* in Section 2, a bidder's reference point in the first round is given by his overall probability of consumption in both rounds.

Differentiating $EU_1\left(\widetilde{\theta}, \theta; y_1\right)$ with respect to $\widetilde{\theta}$ yields the following first-order condition:

$$\begin{aligned} \beta_1'\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right) &= f_1\left(\widetilde{\theta}\right)\left[\theta - \beta_1\left(\widetilde{\theta}\right)\right] - F_2\left(\theta|\widetilde{\theta}\right)\left[\theta - \beta_2^*\left(\theta, \widetilde{\theta}\right)\right] f_1\left(\widetilde{\theta}\right) \\ &\quad -\Lambda\theta\left[f_1\left(\widetilde{\theta}\right) - F_2\left(\theta|\widetilde{\theta}\right) f_1\left(\widetilde{\theta}\right)\right]\left\{1 - 2\left[F_1\left(\widetilde{\theta}\right) + \int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1) \, dy_1\right]\right\}. \end{aligned} \tag{8}$$

Substituting $\theta = \widetilde{\theta}$ into (8) results in the following differential equation

$$F_1(\theta) \beta_1'(\theta) = f_1(\theta) \left[\beta_2^*(\theta, \theta) - \beta_1(\theta)\right] \tag{9}$$

together with the boundary condition that $\beta_1(0) = 0$. In expression (9), the term on the left-hand-side represents the bidder's cost from raising his bid (i.e., paying a higher price when winning) while the term on the right-hand-side represents the bidder's gain from raising his bid which, at the margin, is simply the difference between winning in the current round or in the next one.[15]

[15] If a bidder deviates from the symmetric equilibrium strategy by slightly overbidding, there are only two possible consequences. First, if he was already going to win then he still wins, but pays a slightly higher price. Second, his deviation might make him win the current round when he was otherwise going to lose. In this case, however, it must be that the type of his closest competitor is so close that the bidder was almost certain to win the next round.

Moreover, notice that the last term on the right-hand-side of (8) vanishes when $\theta = \widetilde{\theta}$. Indeed, with a marginal increase in his first-round bid, because such a change only matters he is close to winning in either round, a bidder is not modifying his overall probability of consumption but simply reallocating it between the two rounds. Re-arranging condition (9) yields

$$\frac{d}{d\theta}\left\{\beta_1(\theta) F_1(\theta)\right\} = f_1(\theta)\,\beta_2^*(\theta,\theta)$$

whose solution is

$$\beta_1^*(\theta) = \frac{\int_0^\theta \beta_2^*(s,s)\, f_1(s)\, ds}{F_1(\theta)} = \mathbb{E}\left[\beta_2^*(Y_1,Y_1)\,|Y_1 \leq \theta\right].^{16} \tag{10}$$

The first thing worth noticing is that the bidding function in (10) resembles the one of the classical reference-free model with interdependent values. That is, in the first round bidder i with type θ submits a bid equal to the expectation of the second-round bid of his closest competitor conditional on winning and on the closest competitor assuming that he has the same type as bidder i. To see the intuition, suppose bidder i wins in the first round if he bids as type θ; that is, suppose $Y_1 \leq \theta$. Bidder i also has the option to bid as low as to lose in the first round and discover the value of Y_1; then, he can win for sure in the second round by bidding $\beta_2^*(Y_1, Y_1)$ (i.e., by bidding as if his type were Y_1). Hence, in order to be indifferent between winning in the current round or the next one, in the first round bidder i must bid the expectation of the second-round price as though he was tied with his closest competitor. Furthermore, notice that $\beta_1^*(\theta)$ depends on Λ only indirectly, through $\beta_2^*(s,s)$. Indeed, just like in the standard model, in the first round a bidder chooses his optimal bid by conditioning on himself having the highest type. This is because a small change in his bid only matters when the bidder wins or is close to winning. When conditioning on having the highest type, however, a bidder expects that if he were to lose the current auction, he would win the next one for sure and this is why Λ does not directly appear into the first-period bidding function. Finally, it is easy to see that for $\Lambda = 0$ we get back to the risk-neutral benchmark:

$$\beta_1^{RN}(\theta) = \frac{\int_0^\theta \beta_2^{RN}(s)\, f_1(s)\, ds}{F_1(\theta)} = \mathbb{E}\left[\beta_2^{RN}(Y_1)\,|Y_1 \leq \theta\right]$$

where $\beta_2^{RN}(s)$ does not depend of the type of the winner of the first auction.

[16]Notice that

$$\frac{d\beta_1^*(\theta)}{d\theta} = \frac{\left[F_1(\theta)\,\beta_2^*(\theta,\theta) - \int_0^\theta \beta_2^*(s,s)\, f_1(s)\, ds\right] f_1(\theta)}{\left[F_1(\theta)\right]^2} > 0$$

since

$$F_1(\theta)\,\beta_2^*(\theta,\theta) > \int_0^\theta \beta_2^*(s,s)\, f_1(s)\, ds = F_1(\theta)\,\beta_2^*(\theta,\theta) - \int_0^\theta F_1(s)\,\beta_2^*(s,s)'\, ds.$$

where the equality follows from integration by parts. Hence, $\beta_1^*(\theta)$ is increasing.

Let $y_1 = \beta_1^{-1}(p_1)$. Then, the expected equilibrium price in the second auction conditional on the price of the first auction is

$$\mathbb{E}[p_2|p_1] = \mathbb{E}[p_2|\beta_1(y_1)] = \mathbb{E}[\beta_2^*(Y_1, y_1)|Y_1 \leq y_1] = \frac{\int_0^{y_1} \beta_2^*(\theta, y_1) f_1(\theta) d\theta}{F_1(y_1)}.$$

The following proposition delivers the first main result of the paper.

Proposition 1. *(Afternoon Effect) If $\Lambda > 0$, then the price sequence in a two-round sequential first-price auction is a supermartingale and the afternoon effect arises in equilibrium. That is,*

$$p_1 = \beta_1^*(y_1) > \mathbb{E}[p_2|\beta_1^*(y_1)] = \mathbb{E}[p_2|p_1].$$

The intuition behind Proposition 1 is that, just like in the reference-independent case, in equilibrium bidders must be indifferent between winning in the first round or in the second one. Hence, in the first round a bidder bids the expectation of the second-round price conditional on himself having the highest type. Recall from (10) that bidder i's expectation of the second-round price is equal to the expectation of the second-round bid of his closest competitor computed as if he was tied with his closest competitor. Yet, by conditioning on being tied with his closest competitor, a bidder underestimates the discouragement effect which will lower his competitors' bids in the next round. In essence, optimal equilibrium behavior leads the current-round price setter to overestimate the next-round bid of his closest opponent and hence the next-round price. Example 1 illustrates the afternoon effect for the case of uniformly-distributed types.

Example 1. *Suppose that $\theta \overset{U}{\sim} [0,1]$. The first-round equilibrium bid and price are*

$$\beta_1^*(\theta) = (1-\Lambda)\left(\frac{N-2}{N}\right)\theta + \Lambda\left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)\theta$$

and

$$p_1 = \beta_1^*(y_1) = (1-\Lambda)\left(\frac{N-2}{N}\right)y_1 + \Lambda\left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)y_1.$$

The conditional second-round expected price is

$$\mathbb{E}[p_2|p_1] = \mathbb{E}[p_2|\beta_1^*(y_1)] = (1-\Lambda)\left(\frac{N-2}{N}\right)y_1 + \Lambda\left(\frac{N-2}{2N-3}\right)y_1.$$

Hence,

$$\mathbb{E}[p_2|p_1] - p_1 = \Lambda\left[\left(\frac{N-2}{2N-3}\right) - \left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right)\right]y_1$$

and

$$\mathbb{E}[p_2|p_1] < p_1 \Leftrightarrow \left(\frac{N-2}{2N-3}\right) < \left(\frac{2N-4}{2N-3}\right)\left(\frac{N-1}{N}\right) \Leftrightarrow N > 2.$$

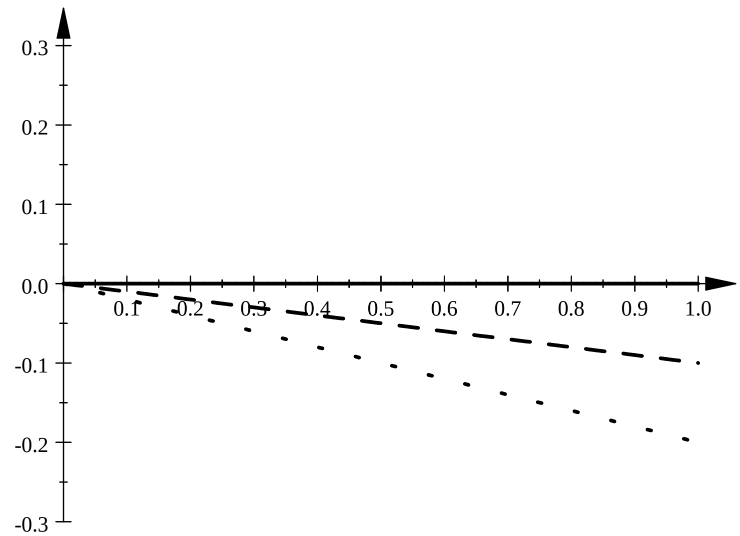

Figure 2: $\mathbb{E}[p_2|p_1] - p_1$ as a function of y_1 for three different values of Λ (0, $\frac{1}{2}$, 1) when $N = 4$ and θ is distributed uniformly on $[0, 1]$

In Figure 2, I plot the difference $\mathbb{E}[p_2|p_1] - p_1$ as a function of y_1 for three different values of Λ when $N = 4$ and θ is distributed uniformly on $[0, 1]$: $\Lambda = 0$ (solid), $\Lambda = \frac{1}{2}$ (dashed), and $\Lambda = 1$ (dotted). From the plot, we notice: (i) for a given y_1, the higher Λ the stronger the afternoon effect, and (ii) for a fixed strictly positive Λ, the higher y_1 the stronger the afternoon effect.

It is useful to compare the logic behind the afternoon effect in my model with the learning effect that arises in common-value auctions. In the symmetric equilibrium of a common-value auction, a bidder conditions his estimate of the value of the item (and hence his bid) on his strongest rival having a (weakly) lower signal than his. In this case, since a bidder revises his estimate of the value of the good upward after losing the first auction, the equilibrium price sequence drifts upward. Conversely, with the informational externalities that arise in a private-value auction with expectations-based loss aversion, after losing the first auction a bidder becomes more pessimistic about how likely he is to win the second one (compared to his first-round expectations); this creates a discouragement effect that pushes bidders to behave less aggressively and, in turn, generates a declining price path in equilibrium.

3.2 Second-price Auctions

In this section I assume that two identical items are sold using a sequence of second-price sealed-bid auctions. I continue to focus on symmetric strategies (b_1, b_2) that are strictly increasing and to assume that the winning bid of the first round is publicly disclosed by the seller prior to the second one.[17] As before, I begin by analyzing the bidder's problem in the second round.

[17] Notice that in a second-price auction the winning bid is not the price the winner actually ends up paying. While restrictive, this assumption is common in the literature for tractability reasons (McAfee and Vincent, 1993; Milgrom and Weber, 2000; Mezzetti, 2011; Hu and Zhou, 2015). See also the discussion in Section 5.

Fixing the strategies of the other bidders, let $\Phi(b_2|y_1)$ denote the probability with which a bidder of type θ expects to win with a bid equal to b_2 conditional on y_1 being the type of the first-round winner. The payment he has to make if he wins the auction is given by the second largest bid and follows the distribution $\Phi(\cdot|y_1)$. Then, the bidder's expected utility is

$$EU_2(b_2, \theta; y_1) = \int_0^{b_2} (\theta - p)\, d\Phi(p|y_1) - \theta\Lambda\Phi(b_2|y_1)\left[1 - \Phi(b_2|y_1)\right] \tag{11}$$

Differentiating (11) with respect to b_2 yields the first-order condition:

$$\theta - b_2 - \theta\Lambda\left[1 - 2\Phi(b_2|y_1)\right] = 0.$$

In a symmetric equilibrium, $\Phi(b_2|y_1) = F_2(\theta|y_1)$ and hence we obtain:

$$b_2^*(\theta, y_1) = \theta\left\{1 - \Lambda\left[1 - \frac{2F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\}. \tag{12}$$

While it is well known that without loss aversion ($\Lambda = 0$) in a symmetric equilibrium a bidder submits a bid equal to his own valuation, the above expression shows that this is not the case with reference-dependent preferences. Indeed, we have:

$$\frac{\partial b_2^*(\theta, y_1)}{\partial \Lambda} = \left[\frac{2F(\theta)^{N-2}}{F(y_1)^{N-2}} - 1\right]\theta > 0 \Leftrightarrow \frac{F(\theta)}{F(y_1)} > \left(\frac{1}{2}\right)^{\frac{1}{N-2}}.$$

Therefore, higher (lower) types bid higher (lower) than their valuation. Furthermore, it is easy to verify that the right-hand-side of (12) is decreasing in N so that when competition gets fiercer, bidders respond by decreasing their bids. This is another testable implication of my model: in the risk-neutral benchmark as well as in the risk-averse models of McAfee and Vincent (1993), Mezzetti (2011) and Hu and Zou (2015) equilibrium bids in the last round do not vary with the intensity of competition. Finally, as for first-price auctions, we have the following result about the effect of the first-round winning bid on second-round bids:

Lemma 2. *(Discouragement Effect II) If $\Lambda > 0$, then $\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} < 0\ \forall \theta$.*

The intuition is like in Lemma 1: the higher the type of the winner in the first auction, the less likely a remaining bidder is to win in the second one and, therefore, he bids less aggressively.

As shown by Lange and Ratan (2010) for the case of single-unit auctions, if bidders are not loss-averse over money, first-price and second-price auctions are revenue equivalent. The reason is that, when bidders are risk-neutral over money expected gain-loss utility depends only on the probability with which a bidder expects to win the auction and this is the same in both formats. It is easy to see that the same intuition applies also to multi-unit auctions. Hence, we can use the revenue equivalence theorem to derive the first-round equilibrium bidding function.

In the first auction a type-θ bidder wins with probability $F_1(\theta)$ and, if he wins, the price he pays is $b_1^*(y_1)$, the highest among his rivals' bids. Thus, his expected first-round payment is

$$F_1(\theta)\int_0^\theta b_1^*(y_1)\, f_1(y_1|\theta)\, dy_1.$$

In a first-price auction, instead, the winning bidder pays his own bid and therefore his expected payment in the first round is:

$$F_1(\theta)\,\beta_1^*(\theta) = F_1(\theta)\left[\frac{\int_0^\theta \beta_2^*(s,s)\, f_1(s)\, ds}{F_1(\theta)}\right]$$

where the equality follows from (10). From revenue equivalence it follows that

$$\int_0^\theta b_1^*(y_1)\, f_1(y_1|\theta)\, dy_1 = \frac{\int_0^\theta \beta_2^*(s,s)\, f_1(s)\, ds}{F_1(\theta)}$$

and differentiating both sides of the equality with respect to θ yields

$$b_1^*(\theta) = \beta_2^*(\theta,\theta)\,. \tag{13}$$

Therefore, the equilibrium bid in the first of two sequential second-price auctions is equal to the second round's bid of a sequential first-price auction where, in the latter, the bidder conditions his bid on himself having the highest type. The following proposition shows that the afternoon effect arises in equilibrium.

Proposition 2. *(Afternoon Effect II) If $\Lambda > 0$, then the price sequence in a two-round sequential second-price auction is a supermartingale and the afternoon effect arises in equilibrium.*

The intuition for the afternoon effect is essentially the same in a first-price and a second-price sequential auction, with just one (minor) difference. In both types of auction, in the first round a bidder bids the expectation of the second-round price conditional on having the highest type and being the price setter, and hence expects not to be discouraged in the second round. In a sequential first-price auction, if a bidder has the highest type and he bids according to the equilibrium bidding function, then he is automatically the price setter. In a sequential second-price auction, however, the winner is also the price setter only if his type is tied with the type of another bidder (cf. (13)).

4 Sequential vs. Simultaneous Auctions

In this section, I analyze simultaneous auctions; that is, auctions in which all the items are allocated after only one round of bidding. I derive the equilibrium bidding strategy in a two-unit discriminatory (pay-your-bid) auction. In a discriminatory auction, bidders submit sealed bids and the highest bidders each receive one object and each pays his own bid. This procedure generalizes the single-object first-price auction.[18] As before, I focus on symmetric monotone strategies.

Consider a bidder with type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N-1$ bidders follow the strategy $\beta(\cdot)$. His expected utility is

$$EU\left(\theta, \widetilde{\theta}\right) = F_2\left(\widetilde{\theta}\right)\left[\theta - \beta\left(\widetilde{\theta}\right)\right] - \Lambda\theta F_2\left(\widetilde{\theta}\right)\left[1 - F_2\left(\widetilde{\theta}\right)\right] \tag{14}$$

where $F_2\left(\widetilde{\theta}\right) \equiv F_1\left(\widetilde{\theta}\right) + (N-1)\left[1 - F\left(\widetilde{\theta}\right)\right] F\left(\widetilde{\theta}\right)^{N-2}$ is the probability that Y_2, the second highest valuation among $N-1$, is less than $\widetilde{\theta}$ and Λ is defined as before. Notice that it is not necessary for a bidder to outbid all his competitors in order to be awarded an object; it is enough to outbid $N-2$ of them. Differentiating (14) with respect to $\widetilde{\theta}$ yields the first-order condition:

$$\beta'\left(\widetilde{\theta}\right) F_2\left(\widetilde{\theta}\right) = f_2\left(\widetilde{\theta}\right)\left[\theta - \beta\left(\widetilde{\theta}\right)\right] - \Lambda\theta f_2\left(\widetilde{\theta}\right)\left[1 - 2F_2\left(\widetilde{\theta}\right)\right].$$

Substituting $\theta = \widetilde{\theta}$ and re-arranging results in the following differential equation

$$\frac{d}{d\theta}\left\{\beta(\theta) F_2(\theta)\right\} = f_2(\theta)\,\theta\left\{1 - \Lambda\left[1 - 2F_2(\theta)\right]\right\}$$

together with the boundary condition that $\beta(0) = 0$. Solving the differential equation yields

$$\begin{aligned}
\beta^*(\theta) &= \frac{\int_0^\theta s\left\{1 - \Lambda\left[1 - 2F_2(s)\right]\right\} f_2(s)\, ds}{F_2(\theta)} \\
&= (1-\Lambda)\frac{\int_0^\theta s f_2(s)\, ds}{F_2(\theta)} + \Lambda\frac{\int_0^\theta 2sF_2(s) f_2(s)\, ds}{F_2(\theta)}.
\end{aligned}$$

Again, the equilibrium bidding function can be re-written as a convex combination of the risk-neutral bid and a term that depends on the bidder's expectations (reference point). Similarly, it is easy to verify that the symmetric equilibrium in a uniform-price auction is

$$b^*(\theta) = (1-\Lambda)\theta + \Lambda 2\theta F_2(\theta).$$

Now I compare the bidders' equilibrium utility and the seller's expected revenue under simultaneous and sequential auctions.[19] Let $V^{sim}(\theta)$ and $V^{seq}(\theta)$ denote a bidder's equilibrium expected utility in a simultaneous and sequential auction, respectively. With independent private values and risk neutrality ($\Lambda = 0$), it is well known that a bidder's equilibrium expected utility in a

[18] An alternative procedure is the uniform-price auction, where bidders submit sealed bids and the winners all pay the same price, equal to the highest rejected bid. This procedure generalizes the single-object second-price auction.

[19] I do the comparison for first-price (sequential) auctions and discriminatory (simultaneous) auctions, but the same results apply for second-price (sequential) and uniform-price (simultaneous) auctions by revenue equivalence.

simultaneous auction is the same as in a sequential auction. Under loss aversion, instead, we have:

Proposition 3. *(Bidder-payoff Equivalence) If* $\Lambda > 0$, *then there exists a cutoff type* θ^* *such that* $V^{seq}(\theta) \geq V^{sim}(\theta)$ *if and only if* $\theta \geq \theta^*$.

According to Proposition 3, bidders with higher types prefer sequential auctions while bidders with lower types prefer simultaneous ones. This is a strategic consequence of the discouragement effect. To understand the intuition behind the result in Proposition 3 notice that in equilibrium a bidder's ex-ante probability of obtaining an item is the same under both formats and this implies, trivially, that a bidder's expected gain-loss utility is also the same under both formats. Hence, the difference between $V^{\text{seq}}(\theta)$ and $V^{sim}(\theta)$ is simply given by the difference in the expected payments. When bidders are risk-neutral, a bidder's expected payment in either a simultaneous or sequential auction is equal to $\mathbb{E}[Y_2|Y_2 \leq \theta]$; that is, the expected value of the valuation of his second-highest competitor. With reference-dependent preferences, however, expected payments in the two formats depend also on the bidders' beliefs about how likely they are to win since these determine their reference point. More precisely, a loss-averse bidder's expected payment equals the expectation of a convex combination of the valuation and the reference point of his second-highest competitor. In a sequential auction, losing the first round is bad news for a high-type bidder but it is even worse news for his competitors with lower types who are even more discouraged than he is. This is the strategic aspect of the discouragement effect. Hence, conditional on winning an item, high-type bidders expect to pay a lower price in a sequential auction than in a simultaneous one. Figure 3 shows how $V^{\text{seq}}(\theta) - V^{sim}(\theta)$ varies with θ for five different values of N when the bidders' types are uniformly distributed on $[0, 1]$ (a darker color corresponds to a higher value for N). For a given N there is a cutoff type θ^* who is indifferent between the two formats. Furthermore, the value of this cutoff is increasing in N.

Next, I compare the seller's expected revenue between the two formats. It is convenient to take the point of view of the seller and consider the order statistics of the values of all N bidders. Let $Z_1^{(N)} \equiv Z_1$ be the highest of N values, $Z_2^{(N)} \equiv Z_2$ be the second-highest and so on. Under risk neutrality ($\Lambda = 0$), the two auction formats are revenue-equivalent, both yielding an expected revenue equal to $2\mathbb{E}[Z_3]$ (Milgrom and Weber, 2000). Under loss aversion, instead, we have:

Proposition 4. *(Revenue non-Equivalence) If* $\Lambda > 0$, *then simultaneous and sequential auctions are not revenue-equivalent. Furthermore, there exists a* $N^* \geq 3$ *such that sequential auctions yield a higher expected revenue than simultaneous ones if and only if* $N \geq N^*$.

Therefore, which format yields a higher revenue depends on the number of bidders. The intuition for this result relies on how loss aversion with expectations as the reference point modifies the notion of a bidder's "type". In the classical, risk-neutral model a bidder's type coincides with his intrinsic value, θ; and the expected revenue for the seller depends on the expected value of the type of the marginal bidder, the bidder with the third-highest value. With reference-dependent

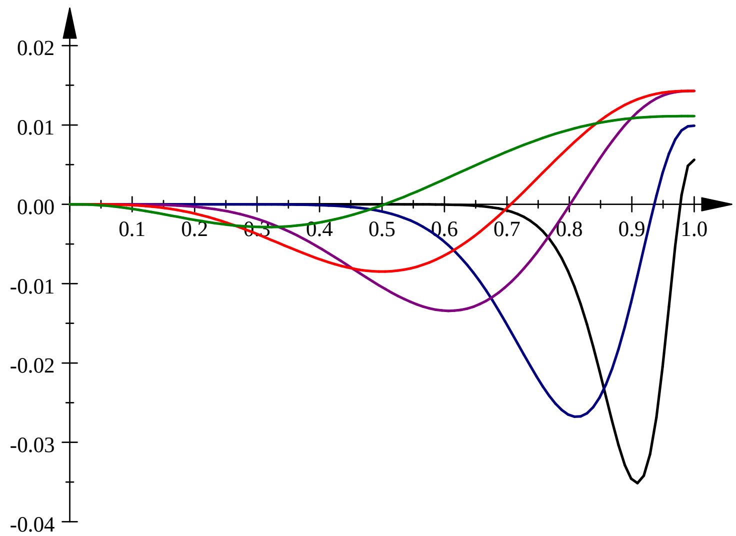

Figure 3: $V^{\text{seq}}(\theta) - V^{sim}(\theta)$ for $N = 3, 4, 5, 10$ and 20 with θ distributed uniformly on $[0, 1]$.

preferences, however, a bidder's "modified" type is a convex combination of his intrinsic value and his reference point which, in equilibrium, is given by his probability of obtaining an item. In a simultaneous auction the marginal bidder is the bidder with the third-highest $(1-\Lambda)\theta + \Lambda 2\theta F_2(\theta)$. In a sequential auction, instead, the marginal bidder is the one with the third-highest $(1-\Lambda)\theta + \Lambda 2\theta F_2(\theta|y_1)$. The difference is due to the different timing of information in the two auction formats. In both formats, ex ante a bidder's reference point is to get an item if he is one of the two highest types (and not getting an item otherwise). In a simultaneous auction, all uncertainty is resolved at once and so a bidder learns only whether he is among the two highest types. In a sequential auction, on the other hand, a bidder first learns in the first round whether he is the highest type; if not, he then updates his reference point which, entering the second round, becomes the probability of being the second-highest type given that he is not the highest. Hence, the "modified" type of the marginal bidder in a simultaneous auction differs from the "modified" type of the marginal bidder in a sequential auction as the two bidders have different reference points despite having the same intrinsic value. When the number of bidders is large, y_1 approaches $\overline{\theta}$, in which case $F_2(\theta|y_1)$ first-order stochastically dominates $F_2(\theta)$. For the case of a power distribution function, the threshold N^* can be computed explicitly, as the following example shows.

Example 2. *Let $\overline{\theta} = 1$ and $F(\theta) = \theta^\alpha$ with $\alpha > 0$. Then, it can be easily verified that*

$$N^* = \max\left\{3,\ \frac{3\alpha + \sqrt{8\alpha + \alpha^2 + 7} + 1}{2\alpha}\right\}.$$

When $\alpha = 1$, the distribution of types is uniform and $N^* = 4$ so that simultaneous auctions yield a higher expected revenue than sequential ones for $N = 3$ whereas for $N = 4$ the two formats yield exactly the same expected revenue. For $N \geq 5$ sequential auctions yield a higher expected *revenue than simultaneous ones. If* $\alpha > \frac{\sqrt{97}+7}{8}$*, so that the right-hand tail gets thicker, then* $N^* = 3$*; that is, sequential auctions always yield a higher expected revenue than simultaneous ones. On the other hand, when the right-hand tail gets thinner, then* $\lim_{\alpha \to 0^+}\left(\frac{3\alpha+\sqrt{8\alpha+\alpha^2+7}+1}{2\alpha}\right) = \infty$*; in this case, simultaneous auctions always yield a higher expected revenue than sequential ones.*

The result that sequential auctions yield a higher revenue when the number of bidders is large is reminiscent of a similar result in models analyzing information revelation in simultaneous auctions with risk-neutral bidders who are uncertain about their values as in Board (2009) and Ganuza (2004).[20] In such models, bidders have only an estimate of their values, and the seller can reveal additional information that would allow the bidders to refine their prior beliefs. Providing additional information entails a trade off for the seller. On one hand, more information improves the match between bidders' preferences and the object for sale and, by doing so, it increases the willingness to pay of the winning bidder and hence the seller's revenue. On the other hand, more information about the object increases the informational rent of the winning bidder, and this lowers the seller's revenue. When the number of bidders is large enough, the seller finds it optimal to reveal additional information to the bidders.[21] One important difference between these models and mine is the following. In models of information revelation in auctions where bidder have uncertain (private) values, the seller's decision of whether to reveal information or not (and how much) also has consequences for the final allocation and its efficiency. Indeed, as argued by Ganuza (2004) and Board (2009), without information revelation the bidder who is awarded the object might not be the one with the highest valuation ex post. Hence, there is a trade off between revenue and efficiency. In my model, instead, the choice between sequential and simultaneous auctions does not affect the final allocation, which is efficient in both formats. The reason is that in my model bidders know their values perfectly and the information that the seller can reveal them via a sequential auction is not about their own values but rather about the intensity of competition.

Gathering the results from this section and the previous one, we obtain the following corollary:

Corollary 1. *(Comparison of Different Auction Formats) If* $\Lambda > 0$*, revenue equivalence holds within formats but not between. That is, sequential first-price auctions are revenue-equivalent to sequential second-price auctions and discriminatory auctions are revenue-equivalent to uniform-price auctions. Yet, sequential first-price auctions are not revenue-equivalent to discriminatory auctions and sequential second-price auctions are not revenue-equivalent to uniform-price auctions.*

Recall that in equilibrium a bidder's expected probability of consumption and expected gain-loss utility are the same under all four types of auctions considered in this paper. Thus, the non-equivalence result in Corollary 1 is due to the effect that sequential (partial) information revelation has on the expected payments of a loss-averse bidder.

[20] See also Eső and Szentes (2007), Ganuza and Penalva (2010) and Schweizer and Szech (2015).

[21] This result holds as long as the additional information provided by the seller makes the posterior distribution (i.e., based on the additional information) of the bidders' values more dispersed than the prior distribution (i.e., without information revelation).

5 Alternative Information Revelation Policies

A delicate issue in sequential auctions is what information should the auctioneer reveal from one round to the next. Following most of the literature, in Section 3 I have assumed that the seller publicly reveals the first-round winning bid. Yet, other information revelation policies are possible. A natural candidate is to reveal the winning price in each round; that is, to reveal the highest bid in the case of a first-price auction and the second-highest bid in the case of a second-price auction. Yet, revealing the current round's winning price in second-price sequential auctions entails revealing the offer of a bidder who will take part in the next round. Hence, if the auctioneer is concerned with protecting bidders' privacy, she might decide against such disclosure policy. An alternative option for the seller is to reveal no information at all in between rounds. Milgrom and Weber (2000) showed that with risk-neutral bidders having independent private values, the seller's information revelation policy is inconsequential and equilibrium bids are the same no matter what information (if any) the seller discloses in between rounds. In this section I show that if bidders are expectations-based loss-averse, different information revelation policies result in different equilibrium bids.

5.1 Sequential Auctions without Announcement of the Winning Bid

In the classical reference-free model with independent private values, the optimal bidding strategy does not depend on the (public) history of the winning bids. However, this is no longer the case with expectations-based reference-dependent preferences. Hence, some questions naturally arise: Is equilibrium bidding different if the seller commits to not revealing the history of winning bids? Does the rationale for the afternoon effect with expectations-based reference-dependent preferences rely on the history of winning bids being publicly available? And, finally, would the seller be better off by not disclosing the history of winning bids? I answer these questions in the context of sequential first-price auctions but the same results also apply for the case of sequential second-price auctions via revenue equivalence. To find an equilibrium that is sequentially rational, I start by solving the bidder's problem in the last auction.

Consider a type-θ bidder who bids as type $\widetilde{\theta} > \theta$ when all other $N-2$ remaining bidders follow the strategy β_2. Let σ be the type that the bidder pretended to be in the first auction. If he lost the first auction, he knows that $y_1 > \sigma$. Then his expected second-round payoff is

$$EU_2\left(\widetilde{\theta},\theta;\sigma\right) = \varphi\left(\sigma\right)F\left(\widetilde{\theta}\right)^{N-2}\left[\theta-\beta_2\left(\widetilde{\theta},\sigma\right)\right] - \Lambda\theta\varphi\left(\sigma\right)F\left(\widetilde{\theta}\right)^{N-2}\left[1-\varphi\left(\sigma\right)F\left(\widetilde{\theta}\right)^{N-2}\right] \quad (15)$$

where $\varphi\left(\sigma\right) = \frac{(N-1)[1-F(\sigma)]}{1-F(\sigma)^{N-1}}$. Thus, $\varphi\left(\sigma\right)F\left(\widetilde{\theta}\right)^{N-2}$ denotes the probability that the second highest of $N-1$ draws is below $\widetilde{\theta}$ given that the highest is above σ; or, in other words, the probability that a bidder who pretends to be of type $\widetilde{\theta}$ in the second auction wins this auction given that he pretended to be of type σ in the first auction and lost it.[22] Notice that, crucially, the second-round bid might, in principle, depend also on σ.

As first conjectured by Milgrom and Weber (2000) and later shown by Mezzetti *et al.* (2008), with no bid announcement and interdependent values it is optimal for a bidder of type θ to behave according to his type in the second auction if and only if he behaved as type $\sigma \leq \theta$ in the first auction. By contrast, if a bidder of type θ behaved as if his type were higher than θ in the first auction, he might want to over-bid in the second auction as well.[23] Recall that, with interdependent values, a better estimate of the winning bid is also a better estimate of the value of the object for sale. In our case, however, values are private and independent; hence, it is optimal for a bidder in the second auction to bid according to his true type, no matter what he did in the first one.

Differentiating $EU_2\left(\widetilde{\theta}, \theta; \sigma\right)$ with respect to $\widetilde{\theta}$ yields the first-order condition:

$$\begin{aligned} 0 \quad = \quad & \left[\theta - \beta_2\left(\widetilde{\theta}, \sigma\right)\right](N-2)\, F\left(\widetilde{\theta}\right)^{N-3} f\left(\widetilde{\theta}\right) - \frac{\partial \beta_2\left(\widetilde{\theta}, \sigma\right)}{\partial \widetilde{\theta}} F\left(\widetilde{\theta}\right)^{N-2} \\ & -\theta\,(N-2)\, F\left(\widetilde{\theta}\right)^{N-3} f\left(\widetilde{\theta}\right) \Lambda \left[1 - 2\varphi(\sigma)\, F\left(\widetilde{\theta}\right)^{N-2}\right]. \end{aligned}$$

Hence, $\varphi(\sigma)$ enters the FOC only through the reference point, but it does not affect the "direct" part of a bidder's payoff and since $\Lambda \leq 1$ the "direct" part carries a higher weight than the reference-dependent part. Substituting $\theta = \widetilde{\theta}$ into the FOC and re-arranging results in the following differential equation

$$\frac{\partial}{\partial \theta}\left\{\beta_2(\theta, \sigma)\, F(\theta)^{N-2}\right\} = \theta \left\{1 - \Lambda\left[1 - 2\varphi(\sigma)\, F(\theta)^{N-2}\right]\right\}(N-2)\, F(\theta)^{N-3} f(\theta)$$

with the boundary condition that $\beta_2(0, \sigma) = 0$. Thus, the equilibrium bidding function is

$$\widehat{\beta}_2(\theta, \sigma) = \frac{\int_0^\theta x \left\{1 - \Lambda\left[1 - \frac{2(N-1)(1-F(\sigma))F(x)^{N-2}}{1-F(\sigma)^{N-1}}\right]\right\} dF(x)^{N-2}}{F(\theta)^{N-2}}.$$

The equilibrium bidding strategy is a function of the type that the bidder mimicked in the previous auction since, if the seller does not publicly reveal the first-round winning bid, a bidder who lost the first auction must use his own bid from the previous round to infer where he stands in the ranking of the remaining bidders' values. Hence, the equilibrium strategy depends on

[22] Technically, we have that

$$\frac{(N-1)\left[1 - F(\sigma)\right] F\left(\widetilde{\theta}\right)^{N-2}}{1 - F(\sigma)^{N-1}} = \frac{\int_\sigma^{\overline{\theta}} \int_0^{\widetilde{\theta}} h(y_1, y_2)\, dy_2 dy_1}{\int_\sigma^{\overline{\theta}} \int_0^{\widetilde{y}_1} h(\widetilde{y}_1, \widetilde{y}_2)\, d\widetilde{y}_2 d\widetilde{y}_1}$$

where $h(y_1, y_2) = (N-1)(N-2)\, f(y_1)\, f(y_2)\, F(y_2)^{N-3}$ is the joint density of Y_1 and Y_2.

[23] This happens because, as Milgrom and Weber (2000) pointed out, "a bidder might choose a bid a bit higher in the first round in order to have a better estimate of the winning bid, should he lose".

the (private) history of the game and, as the following lemma shows, a slightly different form of discouragement effect arises in equilibrium.

Lemma 3. *(Discouragement Effect III) If* $\Lambda > 0$*, then* $\frac{\partial \widehat{\beta}_2(\theta,\sigma)}{\partial \sigma} < 0 \; \forall \theta$.

The intuition for this result slightly differs from the one behind lemmas 1 and 2. When the winning bid from the first auction is not publicly revealed, a bidder can only use his own first-round bid to assess how likely he is to win in the second one. The higher the type he pretended to be in the first auction, the less likely he feels to win in the current one since not winning the first auction, given that he pretended to have a high type, is bad news about how fierce competition is. This, in turn, implies that the higher is the type a bidder pretended to be in the first auction, the less aggressive his bidding will be in the second auction. Comparing the second-round equilibrium strategies with and without bid announcement yields the following result.

Lemma 4. *(Effect of information I) Equilibrium bidding in the second auction is more aggressive when the seller does not reveal the winning bid of the first auction if and only if*

$$\frac{(N-1)\left[1-F(\sigma)\right]}{1-F(\sigma)^{N-1}} > \frac{1}{F(y_1)^{N-2}}. \tag{16}$$

First, notice that condition (16) can hold only if $y_1 > \sigma$. The term on the left-hand-side of (16) represents what a bidder learns from losing in the first round without revelation of the winning bid: he knows that the first-round winner's type is above σ. Similarly, the term on the right-hand-side of (16) represents what a bidder learns from losing in the first round when the winning bid is announced: he knows that all remaining bidders' types are below y_1. Hence, with no bid announcement, bidders are asymmetrically informed about the intensity of competition in the second round whereas with bid announcement they all have the same information. It is easy to see that the right-hand-side of condition (16) is decreasing in y_1 implying that, for a fixed σ, the higher is the type of the winner in the first auction, the more aggressive second-round bidding behavior is when the winning bid is not revealed. Similarly, the left-hand-side of condition (16) is decreasing in σ implying that, for a fixed y_1 a bidder who pretended to be a low type in the first auction behaves more aggressively in the second one when the winning bid is not revealed. Of course, as I am about to show next, in equilibrium a bidder will behave according to his type in both auctions so that $\sigma = \theta$.

Consider a bidder of type θ who plans to bid in the first round as if his type were $\widetilde{\theta} > \theta$ when all other $N-1$ bidders follow the strategy β_1. Suppose that all bidders expect to follow the equilibrium strategy $\widehat{\beta}_2$ in the second auction. Then, the bidder will solve the following problem:

$$
\begin{aligned}
EU_1\left(\widetilde{\theta},\theta\right) &= F_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\left[\theta-\widehat{\beta}_2\left(\theta,\widetilde{\theta}\right)\right] f_1\left(y_1\right) dy_1 \\
&\quad -\Lambda\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]
\end{aligned}
$$

Differentiating $EU_1\left(\widetilde{\theta},\theta\right)$ with respect to $\widetilde{\theta}$ yields the following first-order-condition:

$$
\begin{aligned}
0 &= f_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]-\beta_1^{'}\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right) \\
&\quad -F_2\left(\theta|\widetilde{\theta}\right) f_1\left(\widetilde{\theta}\right)\left[\theta-\beta_2\left(\theta,\widetilde{\theta}\right)\right]-\frac{\partial\beta_2\left(\theta,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}(N-1)\left[1-F\left(\widetilde{\theta}\right)\right] F\left(\theta\right)^{N-2} \\
&\quad -\Lambda\theta\left[f_1\left(\widetilde{\theta}\right)-F_2\left(\theta|\widetilde{\theta}\right) f_1\left(\widetilde{\theta}\right)\right]\left\{1-2\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]\right\}.
\end{aligned}
$$

Substituting $\theta=\widetilde{\theta}$ and re-arranging results in the following differential equation

$$
f_1\left(\theta\right)\beta_2\left(\theta,\theta\right)-(N-1)\left[1-F\left(\theta\right)\right]F\left(\theta\right)^{N-2}\left.\frac{\partial\beta_2\left(\theta,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}\right|_{\widetilde{\theta}=\theta}=\frac{d}{d\theta}\left\{\beta_1\left(\theta\right)F_1\left(\theta\right)\right\} \tag{17}
$$

together with the boundary condition that $\beta_1\left(0\right)=0$. Notice that, crucially, $\frac{\partial\beta_2\left(\theta,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}\neq 0$. That is, by mimicking another type in the first auction, a bidder is not just affecting the probability of getting to the second auction — like in the classical reference-free model — but he is also affecting his own future bid in the second auction. This occurs because, with no bid announcement between auctions, a player's bid in the current round affects his reference point in the next one. Solving the differential equation in (17) yields

$$
\widehat{\beta}_1\left(\theta\right)=\frac{\int_0^{\theta}\widehat{\beta}_2\left(s,s\right)f_1\left(s\right)ds}{F_1\left(\theta\right)}-\frac{\int_0^{\theta}\left\{\left.\frac{\partial\widehat{\beta}_2\left(s,\widetilde{\theta}\right)}{\partial\widetilde{\theta}}\right|_{\widetilde{\theta}=s}\frac{1-F(s)}{f(s)}\right\}f_1\left(s\right)ds}{F_1\left(\theta\right)}. \tag{18}
$$

The following lemma shows that, compared to the case analyzed in Section 3, bidders behave less aggressively in the first auction when the seller commits to not revealing the winning bid.

Lemma 5. *(Effect of information II) Equilibrium bidding in the first round is more aggressive when the seller commits to publicly reveal the winning bid prior to the second round; that is,* $\beta_1^*\left(\theta\right)-\widehat{\beta}_1\left(\theta\right)\geq 0\ \forall\theta$ *and the inequality is strict if* $\theta<\overline{\theta}$.

The intuition behind Lemma 5 is the following. When anticipating that the seller will not reveal the winning bid of the first auction prior to the second one, a bidder knows that his bid in the first auction — in case he does not win— will determine his reference point in the second one. A high bid in the first auction, hence, implies also a high reference point in the second auction.

Having a high reference point in the second auction, however, exposes the bidder to a greater disappointment in case he were to lose the second auction as well. Therefore, if the seller commits to not revealing the first-round winning price, bidders bid less aggressively in the first auction. Furthermore, the seller's total expected revenue is higher when she commits to revealing the first round's winning bid.

Proposition 5. *(Revenue) The seller's expected revenue is higher when she commits to disclose the winning bid from the first auction prior to the second one.*

From an ex-ante perspective, bidders going into the second round without knowing the type of the winner in the first round are exposed to much more uncertainty about competition compared to bidders who know the type of the first-round winner. Indeed, in the latter case every bidder going into the second auction knows that all of his competitors' types are below a certain cutoff type while in the former a bidder only knows that the winner has a higher type than his. An expectations-based loss-averse bidder dislikes uncertainty in his consumption outcomes because he dislikes the possibility of a resulting loss more than he likes the possibility of a resulting gain (so he is "first-order" risk averse; see Köszegi and Rabin, 2007). As auctions without bid announcements expose bidders to greater risk, they react by bidding less aggressively. Therefore, compared to the analysis in Section 3, if the seller does not reveal the winning bid of the first auction prior to the second one, her expected revenue decreases.[24]

Mezzetti *et al.* (2008) show that, under the assumption of affiliated private values, the seller's expected revenue in a sequential auction with winning-bid announcement is the same as in a sequential auction with no bid announcement, and is lower than in a simultaneous auction. By contrast, if bidders have independent private values and expectations-based reference-dependent preferences, sequential auctions with winning-bid announcement always yield a higher revenue than sequential auctions with no bid announcement (Proposition 5) and might yield a higher revenue than simultaneous auctions (Proposition 4). The following proposition shows that even with no bid announcement, however, the afternoon effect still arises in equilibrium.

Proposition 6. *(Afternoon Effect III) If $\Lambda > 0$, then the price sequence in a two-round first-price auction without bid announcement is a supermartingale and the afternoon effect arises in equilibrium.*

Like in Section 3 in equilibrium prices decline because of the discouragement effect, but the intuition is slightly different. When the seller commits to not disclosing the winning bid, in the first round bidders are willing to pay a positive premium — equal to the second term on the right-hand-side of (18) — in order to avoid having to go to the second round and being discouraged.

Summing up, with expectations-based reference-dependent preferences the equilibrium bidding strategy changes depending on whether the seller commits to publicly reveal the winning bids from

[24] This result is akin to the famous "Linkage Principle" of Milgrom and Weber (1982): auctioneers have an incentive to pre-commit to revealing all available information.

the previous rounds. With winning-bid announcement first-round bids are always higher whereas bids in the second round can be higher or lower than without bid announcement. Furthermore, the seller's expected revenue is higher when she commits to disclose the previous round's winning bid. In either case, however, equilibrium prices follow a declining path.

5.2 Revealing the Winning Price in Sequential Second-Price Auctions

Unlike in a first-price auction, announcing the winning price in a second-price auction amounts to revealing the type of the highest loser, a bidder who will be present in the next round. As a consequence, existence of a symmetric equilibrium with increasing bidding functions is problematic. Indeed, for the case of risk-neutral bidders De Frutos and Rosenthal (1998) and Mezzetti (2011) showed that with interdependent values a symmetric equilibrium in which in the first auction bidders use the same strictly increasing bidding function cannot exist.[25]

For the case of private values Mezzetti (2011) also showed that increasing equilibrium bidding functions exist and coincide with the ones under the policy of revealing the bid of the winner. The reason is that with private values in the last round it is a (weakly) dominant strategy for all remaining bidders to bid their valuations. The bids in earlier rounds are then determined recursively via the usual indifference condition and history does not matter because in each round a bidder bids as if he were pivotal. If bidders are expectations-based loss averse, however, bidding one's own value in the last round is not a dominant strategy anymore. Hence, existence of a symmetric equilibrium in monotone strategies is not warranted as the following example shows.

Example 3. *Let* $\theta \overset{U}{\sim} [0,1]$ *and* $N = 3$*. Let* p_1 *denote the first-round price and let bidder* i *be the price setter. Suppose bidders use symmetric increasing strategies,* $\widehat{b}_1(\cdot)$ *and* $\widehat{b}_2(\cdot)$*. Then, announcing the first-round winning price reveals the type of price setter:* $z_2 = \widehat{b}_1^{-1}(p_1)$*. Therefore, bidder* j*,* $j \neq i$*, participating in the second round does not face any risk as he knows that he will not win the auction. Hence, in the second round it is a weakly-dominant strategy for him to submit a bid equal to his valuation, i.e.* $\widehat{b}_2(\theta_j) = \theta_j$*. Now consider the behavior of bidder* i*. If* $\theta_i \geq z_2$*, that is if he pretended to be a (weakly) lower type in the first round, then he knows he will win for sure by bidding* $\widehat{b}_2(\theta_i) = \theta_i$ *in the second round and it is optimal for him to do so. However, if* $\theta_i < z_2$ *then the situation is more intricate. In this case, bidder* i *has two options: (i) he can win for sure by keeping pretending of being of type* z_2 *and bidding* z_2*; or, (ii) since he knows that bidder* j*'s type must be lower than* z_2*, he can choose to imitate another type* ρ_2 *by solving the following maximization program:*

$$\max_{\rho_2} \int_0^{\rho_2} (\theta_i - y_2)\, dF_2(y_2|z_2) - \Lambda\theta_i F_2(\rho_2|z_2)\left[1 - F_2(\rho_2|z_2)\right]. \tag{19}$$

[25] De Frutos and Rosenthal (1998) focused on two-round sequential auctions with pure common values. Mezzetti (2011) showed that the result extends to the more general case of sequential auctions with interdependent values and more than two rounds.

where $F_2(y_2|z_2)$ denotes the CDF of the second-highest order statistic (among $N-1$), conditional on it being lower than z_2. With F uniform and $N=3$, the solution to the program is

$$\rho_2^* = \frac{\theta_i(1-\Lambda)}{1-\frac{2\Lambda\theta_i}{z_2}}.^{26}$$

It is easy to verify that it is optimal for bidder i to bid z_2 if $\frac{z_2}{\theta_i} \leq 1+\Lambda$ and to bid ρ_2^* otherwise. The intuition is as follows. By bidding z_2 bidder i is guaranteed to win the second auction; yet, as z_2 is higher than θ_i, he risks paying a price higher than his value. By bidding $\rho_2^* < z_2$, instead, bidder i lowers his chances of winning which reduces his chances of paying a price higher than his value, but also increases his gain-loss disutility. Hence, when the ratio $\frac{z_2}{\theta_i}$ is relatively low, bidder i prefers to bid z_2 and win for sure, whereas when the ratio $\frac{z_2}{\theta_i}$ is relatively high, he prefers to lower his bid. Either way, however, if the price setter mimicked a higher type in the first auction, he will not bid truthfully in the second auction either. Hence, bidding one's own value is not a dominant strategy in the last round.

When would a bidder mimic a higher type in the first round? Again, suppose all bidders other than bidder i follow symmetric strategies. It is easy to verify that if bidder j, $j \neq i$, expects to play $\widehat{b}_2(\theta_j) = \theta_j$ in the second round, then in the first round he bids according to $\widehat{b}_1(\theta_j) = \frac{\int_0^{\theta_j} x dF(x)^{N-2}}{F(\theta_j)^{N-2}}$ which with F uniform and $N=3$ simplifies to $\widehat{b}_1(\theta_j) = \frac{\theta_j}{2}$. Suppose bidder i plans to bid as if his type were $\widetilde{\theta} > \theta_i$. He will choose the $\widetilde{\theta}$ that solves the following program:

$$\max_{\widetilde{\theta}} \int_0^{\widetilde{\theta}} \left(\theta_i - \frac{y}{2}\right)(3-1)y^{3-2}dy + \int_{\widetilde{\theta}}^1 \left(\int_0^{\widetilde{\theta}} (\theta_i - t)\frac{(3-2)t^{3-3}}{y^{3-2}}dt\right)(3-1)y^{3-2}dy$$
$$-\Lambda\theta_i\left[\widetilde{\theta}^{3-1} + (3-1)\left(1-\widetilde{\theta}\right)\widetilde{\theta}^{3-2}\right]\left[1-\widetilde{\theta}^{3-1} - (3-1)\left(1-\widetilde{\theta}\right)\widetilde{\theta}^{3-2}\right].$$

Taking FOC and re-arranging yields

$$\widetilde{\theta} = \frac{4\theta_i\Lambda - 1 + \sqrt{8\theta_i^2\Lambda^2 - 8\theta_i\Lambda + 8\theta_i^2\Lambda + 1}}{4\theta_i\Lambda}.$$

Notice that for bidder i to be willing to bid as type $\widetilde{\theta}$ in the second round if he is the first-round price setter, it must be that $\frac{z_2}{\theta_i} \leq 1+\Lambda$. Letting $\widetilde{\theta} = z_2$ and re-arranging yields

$$-16\theta_i^2\Lambda^2(\theta_i + \theta_i\Lambda - 1)^2 < 0$$

which is trivially satisfied. Finally, notice that $\widetilde{\theta} \geq \theta_i \Leftrightarrow \theta_i \geq 1 - \frac{\sqrt{2}}{2}$. Therefore, if his type is greater than $1-\frac{\sqrt{2}}{2}$, bidder i will deviate from the posited symmetric equilibrium and mimic type $\widetilde{\theta}$ instead. But why would a bid prefer to bid more aggressively in the first round by pretending to have a higher type? By increasing his bid, the bidder increases his chance of winning. This, in

[26] Notice that $\rho_2^* \geq \theta_i \Leftrightarrow \frac{z_2}{\theta_i} \leq 2$.

turn, has two effects on the bidder's expected utility. On one hand, he expects to pay a higher price which, with some probability, might even exceed his intrinsic value. On the other hand, a higher probability of winning lowers his expected gain-loss disutility as he now is less likely to lose the auction and be disappointed. If the bidder's type is relatively high, the second effect dominates.

Example 3 shows that there cannot be a symmetric equilibrium in monotone strategies in sequential second-price auctions with winning-price announcement. The reason is that the situation that the remaining bidders face in the second round is highly asymmetric, as one of the bidders would have his exact bid known to the others. It is worthwhile to highlight that the incentives to deviate for loss-averse bidders with private values are exactly opposite to the incentives of risk-neutral bidders with interdependent values. Indeed, when values are interdependent, those bidders with relatively low private signals have an incentive to deviate by decreasing their bids in the first round in order to pay a lower price in the second round in the unlikely (but germane) event that they were to be the price setter. Conversely, with private values and reference-dependent preferences it is the bidders with relatively high values who have incentive to deviate by bidding more aggressively in the first round in order to reduce their gain-loss disutility in the unlikely (but germane) event that they were to be the price setter.

6 Extensions and Robustness

In this section, I analyze two extensions and discuss further predictions of the model.[27]

6.1 Common Values

In this section, I consider common-value sequential auctions. I use the simplest possible model to make the point that the afternoon affect can still arise with loss-averse bidders even with common values. Suppose each bidder i observes a private signal t_i independently and identically distributed on the support $[0, \bar{t}]$ according to the distribution function $F(t_i)$, $i = 1, 2, ..., n$. Assume $F(\cdot)$ admits a continuous, positive density f everywhere on its support. The common value of the objects for sale is given by $V = \sum_{i=1}^{n} t_i$. This structure of value and signals is known as the "Wallet Game" (see Klemperer, 1998 and Bulow and Klemperer, 2002).[28] I focus on sequential first-price auctions with announcement of the winning bids, but the same analysis applies also to second-price auctions via revenue equivalence.

[27] Proofs of the results in this section are relegated to a Web Appendix which also gathers further technical details.

[28] One advantage of this formulation of the common value is that it preserves revenue equivalence. An alternative formulation for modeling common-value auctions is one where the common value has some known prior distribution and bidders' signals are draws conditional on the particular realization of V. It is important to point out that both formulations have the same qualitative features. First, the object for sale is worth the same to all bidders. Second, in both formulations bidders should realize that winning means that their signal is likely to be too optimistic; hence, in order not to fall prey to a "winner's curse", bidders must shade their bids accordingly.

Notice that at the time of the sale, buyers can only estimate the value of the good and they are well aware that the true value to them will be revealed only some time after the sale. Hence, as the value of the good for sale is subject to ex post risk, a loss-averse bidder can derive feelings of loss and/or gain from comparing the actual realized value of the good (if he gets it) with all the possible values the good could have taken with positive probability. In order to keep the analysis tractable, however, henceforth I shall assume that a bidder's reference point is given by the expected value of the good for sale rather than the full probabilistic distribution of its possible values.[29]

Let y_1 denote the signal of the first-round winner. Then, as shown in Web Appendix B, the equilibrium bidding strategy in the second auction is given by:

$$\beta_2^*(t,y_1) = \frac{(1-\Lambda)\int_0^t \left[2s+y_1+\frac{(N-3)\int_0^s xf(x)dx}{F(s)}\right] dF(s)^{N-2}}{F(t)^{N-2}} + \frac{\Lambda\int_0^t \left[3s+2y_1+\frac{(2N-5)\int_0^s xf(x)dx}{F(s)}\right] F_2(s|y_1)\, dF(s)^{N-2}}{F(t)^{N-2}}. \tag{20}$$

Notice that for $\Lambda = 0$, the above expression reduces to the well-known risk-neutral bid:

$$\beta_2^{RN}(t,y_1) = \frac{\int_0^t \left[2s+y_1+\frac{(N-3)\int_0^s xf(x)dx}{F(s)}\right] dF(s)^{N-2}}{F(t)^{N-2}} = \mathbb{E}\left[V(Y_2,Y_2,y_1)\,|Y_1=y_1,t,Y_2\le t<y_1\right]$$

where $V(Y_2,Y_2,y_1)$ denotes the expected value of bidder i's closest competitor conditional on the closest competitor assuming he has the same type as i and on y_1 being the type of first-round winner.[30] It is easy to verify that $\beta_2^{RN}(t,y_1)$ is increasing in y_1. Intuitively, the higher the signal of the first-round winner is the more valuable the good is to *all* bidders who, in turn, will bid more aggressively. Yet, when bidders are loss-averse, the affect of y_1 on second-round bidding is more subtle as the following lemma shows.

Lemma 6. *(Discouragement Effect IV) For $\Lambda > 0$, there exists a threshold $\widehat{N} \ge 3$ such that if $N \ge \widehat{N}$ then there exists a threshold $\widehat{\Lambda}$ such that $\frac{\partial\beta_2^*(t,y_1)}{\partial y_1} < 0$ if and only if $\Lambda > \widehat{\Lambda}$.*

While the statement of the Lemma is quite cumbersome, the intuition behind it is straightforward. By looking at the expression on the right-hand side of (20), it is easy to see that y_1 has three effects on the second-round bidding function. The first one, which is captured by the first term of

[29] If the reference point were given by the full distribution of the good's possible values, then a precautionary bidding effect similar to the one in the model of Esö and White (2004) would arise. Yet, while making the analysis more intricated, this additional effect would not modify this section's main result about the equilibrium price path.

[30] To understand the risk-neutral bidding function, recall that in a standard, single-item, first-price auction with risk-neutral bidders and private values, the bid of player i is equal to the expectation, conditional on winning, of the item's value to his closest competitor, the competitor with the next-highest signal. Yet, with interdependent values, the expected value of i's closest competitor j must be computed as if i's signal were equal to j's signal. Finally, for type t of bidder i conditioning on winning in the second round amounts to conditioning that $Y_2 \le t < Y_1$.

(20) and is always positive, is the standard effect that arises also in the risk-neutral benchmark: the higher y_1 is, the more valuable the good is. The other two effects, instead, appear in the second term on the right-hand side of (20) and pertain to how y_1 affects the bidders' reference point in the second round. On the one hand, a higher y_1 tends to raise a bidder's reference point *conditional on winning* by making the value of the item go up; on the other hand, a higher y_1 also tends to lower the bidder's reference point by reducing the bidder's likelihood of winning in the second round. When N is relatively large, the third effect dominates the second one, making the second term on the right-hand side of (20) decreasing in y_1. The reason is that when the number of bidders is large, the signal of just one bidder, the first-round winner, has a relatively small impact on the value of the good V; on the other hand, the larger is N, the less likely a bidder is to win in the current round when y_1 increases. Hence, when N is large the direct effect of y_1 on $\beta_2^*(t, y_1)$ is positive whereas the effect via the reference point is negative. As $\beta_2^*(t, y_1)$ is a convex combination between the risk-neutral bid and a term that captures the bidder's reference point, if Λ is large enough, the negative effect via the reference point outweighs the positive direct effect so that $\beta_2^*(t, y_1)$ declines with y_1.

As shown in Web Appendix B, the equilibrium bidding strategy in the first round is given by:

$$\beta_1^*(t) = \frac{\int_0^t \beta_2^*(s, s) f_1(s) \, ds}{F_1(t)}.$$

As the next proposition shows, the equilibrium price path of a sequential auction with loss-averse bidders having interdependent values can be either increasing or decreasing depending on the strength of loss aversion.

Proposition 7. *(Afternoon Effect IV) The price sequence in a two-round sequential first-price auction with common values is a supermartingale if* $\frac{\partial \beta_2^*(t,y_1)}{\partial y_1} < 0$ *and a submartingale otherwise.*

Intuitively, if $\frac{\partial \beta_2^*(t,y_1)}{\partial y_1} < 0$ the discouragement effect outweighs the standard positive informational externality of interdependent values so that equilibrium prices follow a declining path. Vice versa, if $\frac{\partial \beta_2^*(t,y_1)}{\partial y_1} > 0$, equilibrium prices follow an increasing path.

6.2 Loss Aversion over Money

In this section, I consider again sequential first-price auctions with private values and winning-bid announcement. The analysis mirrors the one carried out in Section 3; the only, crucial, difference is that now I allow for the bidders to be loss-averse over money as well as consumption. Let $\eta^m > 0$ and $\lambda^m > 1$ be the the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion for the money dimension, respectively. Similarly, let $\eta^g > 0$ and $\lambda^g > 1$ be the the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion over the consumption dimension, respectively. Finally, let $\Lambda^l = \eta^l \left(\lambda^l - 1\right)$ and assume $\Lambda^l \leq 1$, for $l \in \{g, m\}$.

As shown in Web Appendix C, when bidders are loss-averse over consumption as well as money the equilibrium bidding strategy in the second auction is given by:

$$\beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m) = \frac{\int_0^\theta x \left\{1 - \Lambda^g \left[1 - 2\frac{F(x)^{N-2}}{F(y_1)^{N-2}}\right]\right\} dF(x)^{N-2}}{F(\theta)^{N-2} \left\{1 + \Lambda^m \left[1 - \frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\}}. \tag{21}$$

It is easy to see that $\frac{\partial \beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial \Lambda^m} < 0$, implying that money loss aversion pushes all bidders to behave less aggressively compared to to the case in Section 3 as well as to the risk-neutral case. Notice, therefore, that while risk aversion and aversion to price risk as in Mezzetti (2011) and Hu and Zou (2015) induce bidders to behave more aggressively than their risk-neutral counterparts, loss aversion over money has the exact opposite effect. Moreover, as the following Lemma shows, loss aversion over money does not invalidate the (negative) effect that y_1 has on the second-round bidding function; that is, there still is a discouragement effect.

Lemma 7. *(Discouragement Effect V) If* $\Lambda^g \leq 1$*, then* $\frac{\partial \beta_2^*(\theta, y_1; \Lambda^g, \Lambda^m)}{\partial y_1} < 0$.

The intuition for Lemma 7 is the same as for Lemma 1 and the proof is only slightly different. Hence, money loss aversion does not introduce substantial modifications in the second auction.

Yet, money loss aversion makes the analysis of the first auction somewhat intricate for the following reasons. First, as bidders now dislike uncertainty over money, they experience gain-loss utility from comparing the price they would pay by winning in the first round with the one they would pay by winning in the second round. Moreover, as the strategy in (21) is history dependent, in the first round bidders face a distribution of possible second-round prices they could pay (conditional on winning) and should assess potential gains and/or losses by comparing the actual realized price with all the possible prices they could have paid. As shown in Web Appendix C, the equilibrium bidding strategy in the first auction is given by:

$$\beta_1^*(\theta; \Lambda^g, \Lambda^m) = \frac{\int_0^\theta \beta_2^*(s, s) \{1 + \Lambda^m [1 - 2F_1(s)]\} f_1(s) ds}{F_1(\theta) \{1 + \Lambda^m [1 - F_1(\theta)]\}}.$$

It is easy to see that $\Lambda^m \leq 1$ is a sufficient condition for β_1^* to be strictly increasing in θ. Moreover, notice that, like in Section 3, β_1^* depends on Λ^g only indirectly, through $\beta_2(s, s)$. On the other hand, β_1^* depends directly on Λ^m — but not indirectly as $\beta_2^*(s, s)$ is independent of Λ^m (cf. (21)). This renders the comparison between the first-round price and the (conditional) expected second-round price somewhat more involved. Indeed, as the term $\Lambda^m [1 - 2F_1(s)]$ becomes negative when N is relatively small and s is close to $\overline{\theta}$, the effect of money loss aversion on the price path can either go in the same direction as that of loss aversion over consumption, or in the opposite one. In particular, if $\Lambda^g = 0$ equilibrium prices can either increase or decline depending on the number of bidders and how high the first-round price is.[31] Yet, as the following Proposition shows,

[31]Figure 4 in Web Appendix C shows that with $\theta \overset{U}{\sim} [0, 1]$, $\Lambda^g = 0$ and $\Lambda^m = 1$ the afternoon effect arises for

for $\Lambda^g > 0$ loss aversion over money does not eliminate the afternoon effect.

Proposition 8. *(Afternoon Effect V) If $\Lambda^g > 0$, then there exists a threshold $\widetilde{\Lambda}^m$ such that if $\Lambda^m < \widetilde{\Lambda}^m$ the price sequence in a two-round sequential first-price auction is a supermartingale and the afternoon effect arises in equilibrium.*

Intuitively, we know from Section 3 that with $\Lambda^g > 0$ and $\Lambda^m = 0$ equilibrium prices decline; therefore, if Λ^m is not too large, the afternoon effect continues to arise.[32] Finally, notice that the results in this section do not immediately extend to sequential second-price auctions because with loss aversion over money first-price and second-price auctions are not revenue equivalent anymore.

7 Conclusions

Sequential auctions are often used to sell identical or similar goods such as bottles of wine, condominiums, flowers and stamps. Similarly, online auctions for identical goods that have different closing times can also be viewed as sequential auctions. In this paper I have proposed a novel, preference-based explanation for the afternoon effect commonly observed in sequential auctions by positing that bidders are expectations-based loss-averse. Reference-dependent preferences with expectations as the reference point create an informational externality, the discouragement effect, that renders the equilibrium strategy history-dependent: the higher is the type of the winner in the first auction, the less aggressively the remaining bidders will bid in the second one. Therefore, one can use the discouragement effect to empirically test the implications of loss aversion against the implications of the classical risk-neutral model with either private values or common values.

In equilibrium a bidder must be indifferent between winning in the first auction or in the second one. Hence, in the first auction he chooses the optimal bid conditional on having the highest type and being pivotal. By conditioning his bid in the first auction on being pivotal, however, a bidder underestimates the discouragement effect in the second auction. Thus, in equilibrium bidders bid more aggressively in the first auction and prices decline.

In addition to rationalizing the afternoon effect, loss aversion delivers new testable implications for the design of multi-unit auctions that are of independent interest for theorists and practitioners alike. For example, if bidders are expectations-based loss-averse simultaneous and sequential auctions are not revenue-equivalent nor bidder-payoff equivalent. Furthermore, in sequential auctions a seller always achieves a higher expected revenue by committing to reveal the previous round's winning bid.

There are several directions left for future research. For example, in some of the auctions discussed in the Introduction the goods up for sale are not sought after by the bidders for their

$N > 3$. When $N = 3$, however, whether prices increase or decline depends on how high the first-round price is.

[32] Notice that in Proposition 8 $\Lambda^m < \widetilde{\Lambda}^m$ is a sufficient condition. Hence, the afternoon effect can still arise for $\Lambda^m \geq \widetilde{\Lambda}^m$ as well. For further details, see Web Appendix C.

consumption value, but rather for commercial purposes (i.e., a production or a resale motive). In this case what bidders care about is the monetary value of the goods and a model of reference-dependent preferences where gains and losses are evaluated with respect to the overall gains from trade $(\theta - p)$ might be more appropriate.

Moreover, a model of sequential auctions with loss-averse bidders who demand more than one unit has the potential to yield new interesting results, as bidders' reference point would change during the game depending (also) on how many units they have already acquired. However, it is well known that, even in the risk-neutral case, relaxing the assumption of unit demand introduces several additional effects that complicate the analysis substantially. For instance, when bidders demand more than one unit, revenue equivalence between sequential and simultaneous auctions does not hold anymore even if bidders are risk neutral and have independent private values.

Finally, another interesting direction for future research would be to study sequential dynamic (open) auctions, like English and Dutch auctions. If bidders are risk-neutral and have independent private values, it is well-known that the English (resp. Dutch) auction is strategically equivalent to the second-price (resp. first-price) sealed-bid auction. This equivalence, however, is unlikely to hold if bidders are expectations-based loss-averse.[33]

[33] See Ehrhart and Ott (2014) for a first analysis of single-object dynamic auctions with expectations-based loss-averse bidders.

A Proofs

A.1 Proofs of Section 3

Proof of Lemma 1: We have

$$\begin{aligned}\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1} &= -\frac{2\Lambda (N-2)^2 F(y_1)^{N-3} f(y_1) \int_0^\theta x F(x)^{2N-5} f(x)\, dx}{[F(y_1) F(\theta)]^{2(N-2)}} \\ &= -f(y_1) \Lambda \left\{ \frac{(N-2) \int_0^\theta 1 - \left[\frac{F(x)}{F(\theta)}\right]^{2N-4} dx}{F(y_1)^{N-1}} \right\} < 0.\end{aligned}$$

where the second equality follows by integration by parts. Furthermore, notice that $F(y_1)^{N-1}$ is decreasing in N whereas $1 - \left[\frac{F(x)}{F(\theta)}\right]^{2N-4}$ is increasing in N. Hence, $\frac{\partial \beta_2^*(\theta, y_1)}{\partial y_1}$ is decreasing in N. ■

Proof of Proposition 1: We have

$$\begin{aligned}\beta_1^*(y_1) &= \frac{\int_0^{y_1} \beta_2^*(\theta, \theta) f_1(\theta)\, d\theta}{F_1(y_1)} \\ &> \frac{\int_0^{y_1} \beta_2^*(\theta, y_1) f_1(\theta)\, d\theta}{F_1(y_1)} = \mathbb{E}[p_2 | p_1]\end{aligned}$$

where the inequality follows from Lemma 1. ■

Proof of Lemma 2: We have

$$\frac{\partial b_2^*(\theta, y_1)}{\partial y_1} = -\frac{2\Lambda (N-2) \theta F(\theta)^{N-2}}{F(y_1)^{N-1}} < 0. \ ■$$

Proof of Proposition 2: Let y_1 denote the value of the first-round winner. As the winner pays the second-highest bid, let y_2 be the type of the first-round price setter; that is, $y_2 = b_1^{-1}(p_1)$. Hence, we have

$$\begin{aligned}p_1 &= b_1^*(y_2) \\ &= \frac{\int_0^{y_2} \theta \left\{1 - \Lambda \left[1 - 2\frac{F(\theta)^{N-2}}{F(y_2)^{N-2}}\right]\right\} dF(\theta)^{N-2}}{F(y_2)^{N-2}} \\ &> \int_0^{y_2} \theta \left\{1 - \Lambda \left[1 - 2\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\} dF_2(\theta | y_2) \\ &= \int_0^{y_2} b_2^*(\theta, y_1) f_2(\theta | y_2)\, d\theta = \mathbb{E}[p_2 | p_1]\end{aligned}$$

where $f_2(\theta | y_2)$ denotes the density of Y_2, the second-highest value among $N-1$, conditional on $Y_2 \le y_2$. The inequality follows since $y_1 > y_2$. ■

A.2 Proofs of Section 4

Proof of Proposition 3: We have that

$$V^{sim}(\theta) = F_2(\theta)[\theta - \beta^*(\theta)] - \Lambda\theta F_2(\theta)[1 - F_2(\theta)]$$

and

$$\begin{aligned} V^{seq}(\theta) &= F_1(\theta)[\theta - \beta_1^*(\theta)] + \int_\theta^{\overline{\theta}} F_2(\theta|y_1)[\theta - \beta_2^*(\theta, y_1)] f_1(y_1)\, dy_1 \\ &\quad -\Lambda\theta\left[F_1(\theta) + \int_\theta^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1)\, dy_1\right]\left[1 - F_1(\theta) - \int_\theta^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1)\, dy_1\right]. \end{aligned}$$

Hence,

$$\begin{aligned} V^{seq}(\theta) - V^{sim}(\theta) &= F_2(\theta)\beta^*(\theta) - F_1(\theta)\beta_1^*(\theta) - \int_\theta^{\overline{\theta}} F_2(\theta|y_1)\beta_2^*(\theta, y_1) f_1(y_1)\, dy_1 \\ &= (1-\Lambda)\int_0^\theta s f_2(s)\, ds + \Lambda\int_0^\theta 2F_2(s) s f_2(s)\, ds \\ &\quad -(1-\Lambda)\int_0^\theta \int_0^x s f_2(s|x)\, ds f_1(x)\, dx - \Lambda\int_0^\theta\int_0^x 2F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx \\ &\quad -(1-\Lambda)\int_\theta^{\overline{\theta}} \int_0^\theta s f_2(s|x)\, ds f_1(x)\, dx - \Lambda\int_\theta^{\overline{\theta}}\int_0^\theta 2F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx. \end{aligned}$$

Notice that

$$\int_0^\theta s f_2(s)\, ds = \int_0^\theta\int_0^x s f_2(s|x)\, ds f_1(x)\, dx + \int_\theta^{\overline{\theta}}\int_0^\theta s f_2(s|x)\, ds f_1(x)\, dx$$

implying that

$$V^{seq}(\theta) - V^{sim}(\theta) \geq 0 \;\Leftrightarrow$$

$$\int_0^\theta F_2(s) s f_2(s)\, ds \geq \int_0^\theta\int_0^x F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx + \int_\theta^{\overline{\theta}}\int_0^\theta F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx.$$

Furthermore, using integration by parts, we have that

$$\int_0^\theta F_2(s) s f_2(s)\, ds = \frac{\theta F_2(\theta)^2}{2} - \int_0^\theta \left[\frac{F_2(s)^2}{2}\right] ds$$

and

$$\int_0^\theta\int_0^x F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx + \int_\theta^{\overline{\theta}}\int_0^\theta F_2(s|x) s f_2(s|x)\, ds f_1(x)\, dx =$$

$$\theta\left[\frac{N-2}{N-3}F_1(\theta) - \frac{N-1}{2(N-3)}F(\theta)^{2(N-2)}\right] - \int_0^\theta\left[\frac{N-2}{N-3}F_1(s) - \frac{N-1}{2(N-3)}F(s)^{2(N-2)}\right] ds.$$

Therefore,

$$V^{seq}(\theta) - V^{sim}(\theta) \geq 0 \;\Leftrightarrow L(\theta) \equiv \theta\pi(\theta) - \int_0^{\theta} \pi(s)\, ds \geq 0$$

where

$$\pi(s) = F_2(s)^2 - \frac{2(N-2)}{N-3} F_1(s) + \frac{N-1}{N-3} F(s)^{2(N-2)}.$$

To conclude the proof we need to show that there exists a unique θ^* such that $L(\theta) \geq 0 \Leftrightarrow \theta \geq \theta^*$. First, notice that $L(0) = 0 = \pi(0) = \pi(\overline{\theta})$ and that $L'(\theta) = \theta\pi'(\theta)$, where

$$\begin{aligned} \pi'(\theta) &= 2F_2(\theta) f_2(\theta) - \frac{2(N-2)}{N-3} f_1(\theta) + \frac{2(N-1)(N-2)}{N-3} F(\theta)^{2N-5} f(\theta) \\ &= \frac{2(N-2) f_1(\theta)}{N-3} \left\{ (N-3)\left[1 - F(\theta)\right] F(\theta)^{N-3} \left[N - 1 - (N-2) F(\theta)\right] - \left[1 - F(\theta)^{N-3}\right] \right\}. \end{aligned}$$

Hence,

$$\begin{aligned} sign\left(L'(\theta)\right) &= sign\left(\pi'(\theta)\right) \\ &= sign\left\{ (N-3)\left[1 - F(\theta)\right] F(\theta)^{N-3} \left[N - 1 - (N-2) F(\theta)\right] - \left[1 - F(\theta)^{N-3}\right] \right\} \end{aligned}$$

and we have

$$(N-3)\left[1 - F(\theta)\right] F(\theta)^{N-3} \left[N - 1 - (N-2) F(\theta)\right] - \left[1 - F(\theta)^{N-3}\right] |_{\theta=0} = -1$$

implying that $\lim_{\theta \downarrow 0} \pi'(\theta) < 0$. Since $N - 1 - (N-2) F(\theta) \geq 0$ and $-\left[1 - F(\theta)^{N-3}\right] \leq 0$, there is a unique value of $\theta \in \left(0, \overline{\theta}\right)$ for which $\pi'(\theta) = 0$. Therefore, since $\pi(\theta)$ is a continuous function, we have that $\pi(\theta) \leq 0$, and the inequality is strict for $\theta \in \left(0, \overline{\theta}\right)$. This, in turn, implies that $L\left(\overline{\theta}\right) = -\int_0^{\overline{\theta}} \pi(s)\, ds > 0$.

Summing up: $L(\theta)$ is continuous with $L(0) = 0$ and $L\left(\overline{\theta}\right) > 0$. Furthermore, since $\lim_{\theta \downarrow 0} L'(\theta) < 0$ and $L'(\theta)$ changes sign only once for $\theta \in \left(0, \overline{\theta}\right)$, there exists a unique $\theta^* \in \left(0, \overline{\theta}\right)$ such that $L(\theta) \geq 0 \Leftrightarrow \theta \geq \theta^*$. ■

Proof of Proposition 4: Let $Z_1^{(N)} \equiv Z_1$ be the highest of N values, $Z_2^{(N)} \equiv Z_2$ be the second-highest and so on. Also, let M_1 and M_2 be the distributions of Z_1 and Z_2 respectively, with corresponding densities m_1 and m_2. If $\Lambda > 0$, revenue non-equivalence between sequential and simultaneous auctions follows immediately from Proposition 1 and the supermartingale property. Next, we have that

$$\mathbb{E}\left[R^{\text{seq}}\right] = \int_0^{\overline{\theta}} \beta_1^*(\theta)\, m_1(\theta)\, d\theta + \int_0^{\overline{\theta}} \int_0^{x} \frac{\beta_2^*(\theta, x) f_1(\theta)}{F_1(x)} d\theta m_1(x)\, dx$$

and

$$\mathbb{E}\left[R^{sim}\right] = \int_0^{\overline{\theta}} \beta^*(\theta) \left[m_1(\theta) + m_2(\theta)\right] d\theta.$$

Hence,

$$\mathbb{E}\left[R^{sim}\right]-\mathbb{E}\left[R^{\text{seq}}\right]=$$

$$\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{\theta}sf_2(s)\,ds+\Lambda\int_0^{\theta}2F_2(s)\,sf_2(s)\,ds}{F_2(\theta)}\right][m_1(\theta)+m_2(\theta)]\,d\theta$$

$$-\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{\theta}\int_0^{x}sf_2(s|x)\,dsf_1(x)\,dx+\Lambda\int_0^{\theta}\int_0^{x}2F_2(s|x)\,sf_2(s|x)\,dxf_1(x)\,dx}{F_1(\theta)}\right]m_1(\theta)\,d\theta$$

$$-\int_0^{\overline{\theta}}\left[\frac{(1-\Lambda)\int_0^{x}\int_0^{\theta}\frac{sf_2(s|x)}{F_2(\theta|x)}dsf_1(\theta)\,d\theta+\Lambda\int_0^{x}\int_0^{\theta}2F_2(s|x)\,\frac{sf_2(s|x)}{F_2(\theta|x)}dsf_1(\theta)\,d\theta}{F_1(x)}\right]m_1(x)\,dx.$$

Notice that

$$\begin{aligned}\int_0^{\overline{\theta}}\int_0^{\theta}\frac{sf_2(s)}{F_2(\theta)}ds\,[m_1(\theta)+m_2(\theta)]\,d\theta &= 2\mathbb{E}[Z_3]\\ &= \int_0^{\overline{\theta}}\int_0^{\theta}\int_0^{x}\frac{sf_2(s|x)}{F_1(\theta)}dsf_1(x)\,dxm_1(\theta)\,d\theta\\ &\quad+\int_0^{\overline{\theta}}\int_0^{x}\int_0^{\theta}\frac{sf_2(s|x)}{F_2(\theta|x)}ds\frac{f_1(\theta)}{F_1(x)}d\theta m_1(x)\,dx\end{aligned}$$

implying that

$$\mathbb{E}\left[R^{sim}\right]-\mathbb{E}\left[R^{\text{seq}}\right]\leq 0\ \Leftrightarrow$$

$$\begin{aligned}\int_0^{\overline{\theta}}\int_0^{\theta}\frac{F_2(s)\,sf_2(s)}{F_2(\theta)}ds\,[m_1(\theta)+m_2(\theta)]\,d\theta &\leq \int_0^{\overline{\theta}}\int_0^{\theta}\int_0^{x}\frac{F_2(s|x)\,sf_2(s|x)}{F_1(\theta)}dsf_1(x)\,dxm_1(\theta)\,d\theta\\ &\quad+\int_0^{\overline{\theta}}\int_0^{x}\int_0^{\theta}\frac{F_2(s|x)\,sf_2(s|x)}{F_2(\theta|x)}ds\frac{f_1(\theta)}{F_1(x)}d\theta m_1(x)\,dx.\end{aligned}$$

Simplifying and re-arranging yields

$$\mathbb{E}\left[R^{sim}\right]-\mathbb{E}\left[R^{\text{seq}}\right]\leq 0\Leftrightarrow$$

$$N(N-1)(N-2)\int_0^{\overline{\theta}}\int_0^{\theta}[F_2(x)-F_2(x|\theta)]\,[1-F(x)]\,xF(x)^{N-3}f(x)\,dxf(\theta)\,d\theta\leq 0\Leftrightarrow$$

$$N(N-1)(N-2)\int_0^{\overline{\theta}}\theta F(\theta)^{2N-5}[1-F(\theta)]\left\{[1-F(\theta)]\,[N-1-(N-2)\,F(\theta)]-\frac{F(\theta)^{3-N}-1}{N-3}\right\}f(\theta)\,d\theta\leq 0.$$

where the last expression obtains by changing the order of integration.[34]

[34] Notice that $\lim_{N\to 3}\left[\frac{F(\theta)^{3-N}-1}{N-3}\right]=-\ln[F(\theta)]$.

Let $F(\theta) = p \in [0,1]$. Then, we can re-write the above expression as

$$N(N-1)(N-2)\int_0^1 F^{-1}(p)\, p^{2N-5}(1-p)\left\{(1-p)\left[N-1-(N-2)p\right] - \frac{p^{3-N}-1}{N-3}\right\} dp \leq 0 \qquad (22)$$

where $F^{-1}(p) = \inf\{x \in \mathbb{R} : F(x) \geq p\}$ is the generalized inverse distribution function and is non-decreasing. Notice that the sign of the expression on the left-hand-side of (22) depends on the sign of $\left\{(1-p)\left[N-1-(N-2)p\right] - \frac{p^{3-N}-1}{N-3}\right\}$. It's easy to see that this term can change sign at most once for $p \in [0,1)$. Hence, the desired result follows since $\frac{p^{3-N}-1}{N-3}$ grows faster with N than $(1-p)\left[N-1-(N-2)p\right]$. ■

A.3 Proofs of Section 5

Proof of Lemma 3: We have

$$\frac{\partial \widehat{\beta}_2(\theta,\sigma)}{\partial \sigma} = -\frac{2(N-1)\Lambda f(\sigma)\left[1-F_2(\sigma)\right]\int_0^\theta x F(x)^{N-2}\, dF(x)^{N-2}}{\left[1-F(\sigma)^{N-1}\right]^2 F(\theta)^{N-2}} < 0$$

where $F_2(\sigma) = F(\sigma)^{N-1} + (N-1)\left[1-F(\sigma)\right]F(\sigma)^{N-2}$. ■

Proof of Lemma 4: Immediate by inspection. ■

Proof of Lemma 5: We have

$$\beta_1^*(\theta) - \widehat{\beta}_1(\theta) = \frac{\int_0^\theta \left[\beta_2^*(s,s) - \widehat{\beta}_2(s,s) + \left.\frac{\partial \widehat{\beta}_2(s,\widetilde{\theta})}{\partial \widetilde{\theta}}\right|_{\widetilde{\theta}=s} \frac{1-F(s)}{f(s)}\right] f_1(s)\, ds}{F_1(\theta)}.$$

A sufficient condition for the above expression to be non-negative is

$$\begin{aligned}
\beta_2^*(s,s) - \widehat{\beta}_2(s,s) &\geq -\left.\frac{\partial \widehat{\beta}_2(s,\widetilde{\theta})}{\partial \widetilde{\theta}}\right|_{\widetilde{\theta}=s} \frac{1-F(s)}{f(s)} \\
&\Leftrightarrow \frac{2\Lambda\left\{\frac{1}{F(s)^{N-2}} - \frac{(N-1)[1-F(s)]}{1-F(s)^{N-1}}\right\}\int_0^s x F(x)^{N-2}\, dF(x)^{N-2}}{F(s)^{N-2}} \geq \\
&\quad \frac{2\Lambda(N-1)\left[1-F(s)\right]\left[1-F_2(s)\right]\int_0^s x F(x)^{N-2}\, dF(x)^{N-2}}{\left[1-F(s)^{N-1}\right]^2 F(s)^{N-2}} \\
&\Leftrightarrow \frac{1}{F(s)^{N-2}} - \frac{(N-1)\left[1-F(s)\right]}{1-F(s)^{N-1}} \geq \frac{(N-1)\left[1-F(s)\right]\left[1-F_2(s)\right]}{\left[1-F(s)^{N-1}\right]^2} \\
&\Leftrightarrow \frac{1-F_2(s)}{(N-1)\left[1-F(s)\right]F(s)^{N-2}} \geq \frac{1-F_2(s)}{1-F(s)^{N-1}} \\
&\Leftrightarrow 1 \geq F_2(s)
\end{aligned}$$

and this concludes the proof. ■

Proof of Proposition 5: Let m_1 and m_2 be defined as in the proof of Proposition 4. We know from Lemma 5 that first-round bidding is more aggressive with price announcement and this implies, trivially, that the first-round expected revenue is higher with price announcement. Hence, it suffices to show that the second-round expected revenue is also higher with price announcement; that is:

$$\int_0^{\overline{\theta}} \int_0^{y_1} \frac{\beta_2^* (\theta, y_1) f_1 (\theta) d\theta}{F_1 (y_1)} m_1 (y_1) dy_1 \geq \int_0^{\overline{\theta}} \widehat{\beta}_2 (\theta, \theta) m_2 (\theta) d\theta.$$

Substituting and re-arranging yields:

$$(1-\Lambda) \int_0^{\overline{\theta}} \int_0^{y_1} \frac{X (\theta) f_1 (\theta) d\theta}{F_1 (y_1)} m_1 (y_1) dy_1 + \Lambda \int_0^{\overline{\theta}} \int_0^{y_1} \frac{2\widehat{X} (\theta) f_1 (\theta) d\theta}{F (y_1)^{N-2} F_1 (y_1)} m_1 (y_1) dy_1 \geq$$
$$(1-\Lambda) \int_0^{\overline{\theta}} X (\theta) m_2 (\theta) d\theta + \Lambda \int_0^{\overline{\theta}} \frac{2 (N-1) [1 - F (\theta)]}{1 - F (\theta)^{N-1}} \widehat{X} (\theta) m_2 (\theta) d\theta$$

where $X (\theta) = \frac{\int_0^{\theta} x dF(x)^{N-2}}{F(\theta)^{N-2}}$ and $\widehat{X} (\theta) = \frac{\int_0^{\theta} x F(x)^{N-2} dF(x)^{N-2}}{F(\theta)^{N-2}}$.

Notice that

$$\int_0^{\overline{\theta}} \int_0^{y_1} \frac{X (\theta) f_1 (\theta) d\theta}{F_1 (y_1)} m_1 (y_1) dy_1 = \int_0^{\overline{\theta}} X (\theta) m_2 (\theta) d\theta$$

which further implies that

$$\int_0^{\overline{\theta}} \underbrace{\int_0^{y_1} \frac{\widehat{X} (\theta) f_1 (\theta) d\theta}{F_1 (y_1)}}_{\Omega(y_1)} m_1 (y_1) dy_1 = \int_0^{\overline{\theta}} \widehat{X} (\theta) m_2 (\theta) d\theta.$$

The result then follows since

$$\int_0^{\overline{\theta}} \frac{2\Omega (y_1)}{F (y_1)^{N-2}} m_1 (y_1) dy_1 \geq \int_0^{\overline{\theta}} \frac{2 (N-1) [1 - F (\theta)]}{1 - F (\theta)^{N-1}} \widehat{X} (\theta) m_2 (\theta) d\theta$$

as

$$\frac{1}{F (s)^{N-2}} \geq \frac{(N-1) [1 - F (s)]}{1 - F (s)^{N-1}} \Leftrightarrow 1 \geq F_2 (s)$$

which holds $\forall s \in \left[0, \overline{\theta}\right]$. ■

Proof of Proposition 6: We have

$$\begin{aligned} \widehat{\beta}_1 (y_1) &= \frac{\int_0^{y_1} \widehat{\beta}_2 (\theta, \theta) f_1 (\theta) d\theta}{F_1 (y_1)} - \frac{\int_0^{y_1} \left\{ \left. \frac{\partial \widehat{\beta}_2 (\theta, \widetilde{\theta})}{\partial \widetilde{\theta}} \right|_{\widetilde{\theta}=\theta} \frac{1-F(\theta)}{f(\theta)} \right\} f_1 (\theta) d\theta}{F_1 (y_1)} \\ &> \frac{\int_0^{y_1} \widehat{\beta}_2 (\theta, \theta) f_1 (\theta) d\theta}{F_1 (y_1)} = \mathbb{E} [p_2 | p_1] \end{aligned}$$

where the inequality follows from Lemma 3. ■

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Web Appendix (not for publication)

This appendix collects extensions the main model and related proofs omitted from the main text. Section B analyzes a model with common values and loss aversion over consumption only. Section C, instead, deals with loss aversion over both consumption and money in a model with independent private values.

B Common Values

In this section, I consider common-value sequential auctions. Suppose each bidder i observes a private signal t_i independently and identically distributed on the support $[0, \bar{t}]$ according to the distribution function $F(t_i)$, $i = 1, 2, ..., n$. Assume $F(\cdot)$ admits a continuous, positive density f everywhere on its support. The common value of the objects for sale is given by $V = \sum_{i=1}^{n} t_i$. I focus on sequential first-price auctions with announcement of the winning bids, but the same analysis applies also to second-price auctions via revenue equivalence.

Notice that at the time of the sale, buyers can only estimate the value of the good and they are well aware that the true value to them will be revealed only some time after the sale. Hence, as the value of the good for sale is subject to ex post risk, a loss-averse bidder can derive feelings of loss and/or gain from comparing the actual realized value of the good (if he gets it) with all the possible values the good could have taken with positive probability. In order to keep the analysis tractable, however, henceforth I shall assume that a bidder's reference point is given by the expected value of the good for sale rather than the full probabilistic distribution of its possible values.

Consider a bidder of type t who plans to bid as if his type were $\widetilde{t} \neq t$ when all other $N-2$ remaining bidders follow the posited equilibrium strategy $\beta_2(\cdot, y_1)$. His expected payoff is

$$\begin{aligned} EU_2\left(\widetilde{t}, t; y_1\right) &= F_2\left(\widetilde{t}|y_1\right)\left[t + y_1 + \frac{(N-2)\int_0^{\widetilde{t}} x f(x)\,dx}{F\left(\widetilde{t}\right)} - \beta_2\left(\widetilde{t}, y_1\right)\right] \\ &\quad -\Lambda F_2\left(\widetilde{t}|y_1\right)\left[1 - F_2\left(\widetilde{t}|y_1\right)\right]\left[t + y_1 + \frac{(N-2)\int_0^{\widetilde{t}} x f(x)\,dx}{F\left(\widetilde{t}\right)}\right]. \end{aligned} \tag{23}$$

The first term on the right-hand side of (23) captures classical expected consumption utility. The second term, instead, captures expected gain-loss utility. Notice that $t + y_1 + \frac{(N-2)\int_0^{\widetilde{t}} x f(x) dx}{F(\widetilde{t})}$ denotes the expected value of the item for a bidder with type t, conditional on the signal of the first-round winner being y_1 and on winning in the current round — that is, on all other $N-2$ remaining bidders having signals lower than $\widetilde{t}$.

Taking the FOC with respect to $\widetilde{t}$ yields

$$
\begin{aligned}
0 &= f_2\left(\widetilde{t}|y_1\right)\left[t+y_1+\frac{(N-2)\int_0^{\widetilde{t}} xf(x)\,dx}{F\left(\widetilde{t}\right)}-\beta_2\left(\widetilde{t},y_1\right)\right] \\
&+F_2\left(\widetilde{t}|y_1\right)\left[\frac{(N-2)\,\widetilde{t}f\left(\widetilde{t}\right)F\left(\widetilde{t}\right)-f\left(\widetilde{t}\right)(N-2)\int_0^{\widetilde{t}} xf(x)\,dx}{\left[F\left(\widetilde{t}\right)\right]^2}-\frac{\partial\beta_2\left(\widetilde{t},y_1\right)}{\partial\widetilde{t}}\right] \\
&-\Lambda f_2\left(\widetilde{t}|y_1\right)\left[1-F_2\left(\widetilde{t}|y_1\right)\right]\left[t+y_1+\frac{(N-2)\int_0^{\widetilde{t}} xf(x)\,dx}{F\left(\widetilde{t}\right)}\right] \\
&+\Lambda F_2\left(\widetilde{t}|y_1\right)f_2\left(\widetilde{t}|y_1\right)\left[t+y_1+\frac{(N-2)\int_0^{\widetilde{t}} xf(x)\,dx}{F\left(\widetilde{t}\right)}\right] \\
&-\Lambda F_2\left(\widetilde{t}|y_1\right)\left[1-F_2\left(\widetilde{t}|y_1\right)\right]\left[\frac{(N-2)\,\widetilde{t}f\left(\widetilde{t}\right)F\left(\widetilde{t}\right)-f\left(\widetilde{t}\right)(N-2)\int_0^{\widetilde{t}} xf(x)\,dx}{\left[F\left(\widetilde{t}\right)\right]^2}\right].
\end{aligned}
$$

Substituting $t=\widetilde{t}$ and re-arranging results in the following differential equation

$$
\begin{aligned}
\frac{\partial}{\partial t}\left\{F_2(t|y_1)\,\beta_2(t,y_1)\right\} &= f_2(t|y_1)\left[t+y_1+\frac{(n-2)\int_0^t xf(x)\,dx}{F(t)}\right]\left\{1-\Lambda\left[1-2F_2(t|y_1)\right]\right\} \\
&+f_2(t|y_1)\left[t-\frac{\int_0^t xf(x)\,dx}{F(t)}\right]\left\{1-\Lambda\left[1-F_2(\theta|y_1)\right]\right\}
\end{aligned}
$$

whose solution is

$$
\begin{aligned}
\beta_2^*(t,y_1) &= \frac{(1-\Lambda)\int_0^t\left[2s+y_1+\frac{(N-3)\int_0^s xf(x)dx}{F(s)}\right]dF(s)^{N-2}}{F(t)^{N-2}} \\
&+\frac{\Lambda\int_0^t\left[3s+2y_1+\frac{(2N-5)\int_0^s xf(x)dx}{F(s)}\right]F_2(s|y_1)\,dF(s)^{N-2}}{F(t)^{N-2}}.
\end{aligned}
$$

The next Lemma describes the effect that y_1 has on the second-round bidding function.

Lemma 6. *(Discouragement Effect IV) For $\Lambda>0$, there exists a threshold $\widehat{N}\geq 3$ such that if $N\geq\widehat{N}$ then there exists a threshold $\widehat{\Lambda}$ such that $\frac{\partial\beta_2^*(t,y_1)}{\partial y_1}<0$ if and only if $\Lambda>\widehat{\Lambda}$.*

Proof of Lemma 6: We have

$$
\begin{aligned}
\frac{\partial\beta_2^*(t,y_1)}{\partial y_1} &= \frac{(1-\Lambda)\int_0^t dF(s)^{N-2}}{F(t)^{N-2}} \qquad (24)\\
&+\frac{\Lambda\int_0^t\left\{2F_2(s|y_1)-\frac{(N-2)F(s)^{N-2}f(y_1)}{F(y_1)^{N-1}}\left[3s+2y_1+\frac{(2N-5)\int_0^s xf(x)dx}{F(s)}\right]\right\}dF(s)^{N-2}}{F(t)^{N-2}}.
\end{aligned}
$$

It's easy to see that the first term on the right-hand side of (24) simplifies to $1-\Lambda$, which is always positive. On the other hand, it is easy to verify that for N high enough the second term on the right-hand side of (24) is negative and decreasing in N. Let $\widehat{N}$ be the value such that

$$N \geq \widehat{N} \Rightarrow \frac{\Lambda \int_0^t \left\{ 2F_2(s|y_1) - \frac{(N-2)F(s)^{N-2}f(y_1)}{F(y_1)^{N-1}} \left[3s + 2y_1 + \frac{(2N-5)\int_0^s xf(x)dx}{F(s)} \right] \right\} dF(s)^{N-2}}{F(t)^{N-2}} < 0.$$

Then, the result follows since $\frac{\partial \beta_2^*(t,y_1)}{\partial y_1}$ is continuous in Λ. ■

Next, I derive the equilibrium bidding function in the first round. Consider a bidder with type t who plans to bid as if his type were $\tilde{t} \neq t$ when all other bidders follow the strategy β_1. His expected total utility is

$$\begin{aligned} EU_1\left(t,\tilde{t}\right) &= F_1\left(\tilde{t}\right)\left[\underbrace{t + \frac{(N-1)\int_0^{\tilde{t}} xf(x)\,dx}{F\left(\tilde{t}\right)}}_{V\left(t,\tilde{t}\right)} - \beta_1\left(\tilde{t}\right)\right] \\ &+ \int_{\tilde{t}}^{\overline{\theta}} F_2(t|y_1)\left[\underbrace{t + y_1 + \frac{(N-2)\int_0^t xf(x)\,dx}{F(t)}}_{V(t,y_1)} - \beta_2^*(t,y_1)\right] f_1(y_1)\,dy_1 \\ &- \Lambda\left[1 - F_1\left(\tilde{t}\right) - \int_{\tilde{t}}^{\overline{t}} F_2(t|y_1) f_1(y_1)\,dy_1\right]\left[F_1\left(\tilde{t}\right) V\left(t,\tilde{t}\right) + \int_{\tilde{t}}^{\overline{t}} F_2(t|y_1) V(t,y_1) f_1(y_1)\,dy_1\right]. \end{aligned}$$

Taking FOC with respect to $\tilde{t}$ yields

$$\begin{aligned} 0 &= f_1\left(\tilde{t}\right)\left[V\left(t,\tilde{t}\right) - \beta_1\left(\tilde{t}\right)\right] + F_1\left(\tilde{t}\right)\left[\frac{\partial V\left(t,\tilde{t}\right)}{\partial \tilde{t}} - \beta_1\left(\tilde{t}\right)'\right] - F_2\left(t|\tilde{t}\right)\left[V\left(t,\tilde{t}\right) - \beta_2^*\left(t,\tilde{t}\right)\right] f_1\left(\tilde{t}\right) \\ &- \Lambda\left[-f_1\left(\tilde{t}\right) + F_2\left(t|\tilde{t}\right) f_1\left(\tilde{t}\right)\right]\left[F_1\left(\tilde{t}\right) V\left(t,\tilde{t}\right) + \int_{\tilde{t}}^{\overline{t}} F_2(t|y_1) V(t,y_1) f_1(y_1)\,dy_1\right] \\ &- \Lambda\left[1 - F_1\left(\tilde{t}\right) - \int_{\tilde{t}}^{\overline{t}} F_2(t|y_1) f_1(y_1)\,dy_1\right]\left[f_1\left(\tilde{t}\right) V\left(t,\tilde{t}\right) + F_1\left(\tilde{t}\right)\frac{\partial V\left(t,\tilde{t}\right)}{\partial \tilde{t}} - F_2\left(t|\tilde{t}\right) V\left(t,\tilde{t}\right) f_1\left(\tilde{t}\right)\right]. \end{aligned}$$

Setting $t = \tilde{t}$ results in the following differential equation

$$\{F_1(t)\,\beta_1(t)\}' = \beta_2^*(t,t)\, f_1(t)$$

whose solution is

$$\beta_1^*(t) = \frac{\int_0^t \beta_2^*(s,s)\, f_1(s)\,ds}{F_1(t)}.$$

Then, we have the following result about the equilibrium price path:

Proposition 7. *(Afternoon Effect IV) The price sequence in a two-round sequential first-price* *auction with common values is a supermartingale if* $\frac{\partial \beta_2^*(t,y_1)}{\partial y_1} < 0$ *and a submartingale otherwise.*

Proof of Proposition 7: We have

$$p_1 = \beta_1^*(y_1) = \frac{\int_0^{y_1} \beta_2^*(t,t) f_1(t)\, dt}{F_1(y_1)}$$

and

$$\mathbb{E}[p_2|p_1] = \frac{\int_0^{y_1} \beta_2^*(t,y_1) f_1(t)\, dt}{F_1(y_1)}.$$

Hence,

$$p_1 - \mathbb{E}[p_2|p_1] > 0 \Leftrightarrow \frac{\partial \beta_2^*(t,y_1)}{\partial y_1} < 0. \quad \blacksquare$$

C Loss Aversion over Money

In this section, I consider sequential first-price auctions with price announcement. The analysis mirrors the one carried out in Section 3; the only, crucial, difference is that now I allow for the bidders to be loss-averse over money as well as consumption. Let $\eta^m > 0$ and $\lambda^m > 1$ be the the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion for the money dimension, respectively. Similarly, let $\eta^g > 0$ and $\lambda^g > 1$ be the the relative weight a consumer attaches to gain-loss utility and the coefficient of loss aversion over the consumption dimension, respectively. Finally, let $\Lambda^l = \eta^l\left(\lambda^l - 1\right)$ and assume $\Lambda^l \leq 1$, for $l \in \{g, m\}$.

Consider a bidder of type θ who plans to bid as if his type were $\widetilde{\theta} \neq \theta$ when all other $N-2$ remaining bidders follow the posited equilibrium strategy $\beta_2(\cdot, y_1)$. His expected payoff is

$$\begin{aligned} EU_2\left(\widetilde{\theta}, \theta; y_1\right) \quad = \quad & F_2\left(\widetilde{\theta}|y_1\right)\left[\theta - \beta_2\left(\widetilde{\theta}, y_1\right)\right] \\ & - F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g \\ & - F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2\left(\widetilde{\theta}, y_1\right)\Lambda^m \end{aligned} \tag{25}$$

where $F_2\left(\widetilde{\theta}|y_1\right)$ is the probability that the second highest valuation, among $N-1$, is less than $\widetilde{\theta}$ conditional on $Y_1 = y_1$ being the highest. Compared to expression (3) in the main text, there is an additional term, $-F_2\left(\widetilde{\theta}|y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2\left(\widetilde{\theta}, y_1\right)\Lambda^m$, in expression (25). This term captures the comparison, in the money dimension, between losing and winning the auction. Indeed, with probability $F_2\left(\widetilde{\theta}|y_1\right)$ the bidder will win the auction and asses this as a loss of $-\eta^m\lambda^m\beta_2\left(\widetilde{\theta}, y_1\right)\left[1 - F_2\left(\widetilde{\theta}|y_1\right)\right]$ given that he was expecting to lose the auction and hence pay nothing with probability $1 - F_2\left(\widetilde{\theta}|y_1\right)$; similarly, with probability $1 - F_2\left(\widetilde{\theta}|y_1\right)$ the bidder will lose the auction and asses this as a gain of $\eta^m\beta_2\left(\widetilde{\theta}, y_1\right)F_2\left(\widetilde{\theta}|y_1\right)$ given that he was expecting to win the auction and pay $\beta_2\left(\widetilde{\theta}, y_1\right)$ with probability $F_2\left(\widetilde{\theta}|y_1\right)$.

Taking FOC of (25) with respect to $\widetilde{\theta}$ yields

$$\begin{aligned} 0 &= f_2\left(\widetilde{\theta}|y_1\right)\left(\theta-\beta_2\left(\widetilde{\theta},y_1\right)\right)-\beta_2'\left(\widetilde{\theta},y_1\right)F_2\left(\widetilde{\theta}|y_1\right) \\ &\quad -f_2\left(\widetilde{\theta}|y_1\right)\left[1-2F_2\left(\widetilde{\theta}|y_1\right)\right]\theta\Lambda^g \\ &\quad -f_2\left(\widetilde{\theta}|y_1\right)\left[1-2F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2\left(\widetilde{\theta},y_1\right)\Lambda^m \\ &\quad -F_2\left(\widetilde{\theta}|y_1\right)\left[1-F_2\left(\widetilde{\theta}|y_1\right)\right]\beta_2'\left(\widetilde{\theta},y_1\right)\Lambda^m \end{aligned}$$

where β_2' is the derivative of β_2 with respect to its first argument.

Substituting $\theta=\widetilde{\theta}$ and re-arranging results in the following differential equation

$$\frac{\partial}{\partial\theta}\left\{\beta_2\left(\theta,y_1\right)F_2\left(\theta|y_1\right)\left[1+\Lambda^m\left(1-F_2\left(\theta|y_1\right)\right)\right]\right\}=f_2\left(\theta|y_1\right)\theta\left\{1-\Lambda^g\left[1-2F_2\left(\theta|y_1\right)\right]\right\} \tag{26}$$

together with the boundary condition that $\beta_2\left(0,y_1\right)=0$.

Because the different values are drawn independently, we have that

$$F_2\left(\theta|y_1\right)=\frac{F\left(\theta\right)^{N-2}}{F\left(y_1\right)^{N-2}}$$

and substituting into (26) yields

$$\beta_2^*\left(\theta,y_1;\Lambda^g,\Lambda^m\right)=\frac{\int_0^\theta x\left\{1-\Lambda^g\left[1-2\frac{F(x)^{N-2}}{F(y_1)^{N-2}}\right]\right\}dF\left(x\right)^{N-2}}{F\left(\theta\right)^{N-2}\left\{1+\Lambda^m\left[1-\frac{F(\theta)^{N-2}}{F(y_1)^{N-2}}\right]\right\}}. \tag{27}$$

It is easy to see that $\frac{\partial\beta_2^*(\theta,y_1;\Lambda^g,\Lambda^m)}{\partial\Lambda^m}<0$, implying that loss aversion on the money dimension pushes all bidders to behave less aggressively compared to the risk-neutral case. Furthermore, loss aversion over money does not affect does not invalidate the (negative) effect that y_1 has on the second-round bidding function:

Lemma 7. *(Discouragement Effect V) If* $\Lambda^g\leq 1$, *then* $\frac{\partial\beta_2^*(\theta,y_1;\Lambda^g,\Lambda^m)}{\partial y_1}<0\ \forall\theta$.

Proof of Lemma 7: We have

$$\begin{aligned}\frac{\partial\beta_2^*\left(\theta,y_1;\Lambda^g,\Lambda^m\right)}{\partial y_1} &= \frac{-\left[2\Lambda^g\left(1+\Lambda^m-\Lambda^m\left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\int_0^\theta F\left(x\right)^{2N-5}f\left(x\right)xdx\right]f\left(y_1\right)\left(N-2\right)^2}{F\left(y_1\right)^{N-1}F\left(\theta\right)^{N-2}\left[1+\Lambda^m\left(1-\left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]^2}+ \\ &\quad \frac{-\left[\Lambda^m F\left(\theta\right)^{N-2}\int_0^\theta F\left(x\right)^{N-3}f\left(x\right)x\left(1-\Lambda^g+2\Lambda^g\left(\frac{F(x)}{F(y_1)}\right)^{N-2}\right)dx\right]f\left(y_1\right)\left(N-2\right)^2}{F\left(y_1\right)^{N-1}F\left(\theta\right)^{N-2}\left[1+\Lambda^m\left(1-\left(\frac{F(\theta)}{F(y_1)}\right)^{N-2}\right)\right]^2} \\ &< 0. \qquad \blacksquare\end{aligned}$$

Next, I derive the equilibrium bidding function in the first round. Consider a particular bidder with type θ who plans to bid as if his type were $\widetilde{\theta}\neq\theta$ when all other $N-1$ bidders follow the equilibrium strategy β_1. Suppose that all bidders expect to follow strategy the equilibrium $\beta_2^*(\theta, y_1)$ in the second auction, regardless of what happens in the first one (sequential rationality). His expected total utility is

$$\begin{aligned}
EU_1\left(\theta,\widetilde{\theta}\right) &= F_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\left[\theta-\beta_2^*\left(\theta,y_1\right)\right] f_1\left(y_1\right) dy_1 \\
&-\Lambda^g\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right] \\
&-\Lambda^m\beta_1\left(\widetilde{\theta}\right) F_1\left(\widetilde{\theta}\right)\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right] \\
&-\Lambda^m\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right)\beta_2^*\left(\theta,y_1\right) f_1\left(y_1\right) dy_1\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}} F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1\right] \\
&-\Lambda^m F_1\left(\widetilde{\theta}\right)\int_{\widetilde{\theta}}^{\overline{\theta}}\left[\beta_1\left(\widetilde{\theta}\right)-\beta_2^*\left(\theta,y_1\right)\right] F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1 \\
&-\Lambda^m\int_{\widetilde{\theta}}^{\overline{\theta}}\int_{\widetilde{\theta}}^{y_1}\left[\beta_2^*\left(\theta,x\right)-\beta_2^*\left(\theta,y_1\right)\right] F_2\left(\theta|x\right) f_1\left(x\right) dx F_2\left(\theta|y_1\right) f_1\left(y_1\right) dy_1
\end{aligned} \tag{28}$$

where $F_1\left(\widetilde{\theta}\right)$ is the probability that the highest valuation, among $N-1$, is less than $\widetilde{\theta}$, and $F_2(\theta|y_1)$, Λ^g and Λ^m are defined as before. The first line in (28) is the sum of expected consumption utilities. The second line captures expected gain-loss utility on the product dimension: $F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta|y_1) f_1(y_1)\, dy_1$ is the sum of the probability with which a bidder of type θ expects to win the first auction given that he pretends to be of type $\widetilde{\theta}$ and the period-1 expectation of the probability with which he expects to win in the second auction given that he he pretends to be of type $\widetilde{\theta}$ in the first auction but expects to behave as his real type in the second one. Similarly, the third and fourth lines in (28) are expected gain-loss utility on the payment dimension: he expects to pay $\beta_1\left(\widetilde{\theta}\right)$ with probability $F_1\left(\widetilde{\theta}\right)$ (that is, if he wins the first auction), to pay $\int_{\widetilde{\theta}}^{\overline{\theta}} F_2(\theta|y_1)\beta_2^*(\theta, y_1) f_1(y_1)\, dy_1$ if he wins the second auction and to pay nothing otherwise. The fifth line captures the expected gain-loss utility from the comparison between winning the first auction at price $\beta_1\left(\widetilde{\theta}\right)$ and expecting to win the second auction at price $\beta_2^*(\theta, y_1)$. Finally, the sixth and last line captures expected gain-loss utility from the comparison between all possible prices that the bidder expects to pay with positive probability if he wins the second auction.[35]

[35]Notice that this last term would be equal to zero if the second-round strategy were history-independent.

Taking FOC of (28) with respect to $\widetilde{\theta}$ yields

$$
\begin{aligned}
0 \quad = \quad & f_1\left(\widetilde{\theta}\right)\left[\theta-\beta_1\left(\widetilde{\theta}\right)\right]-\beta_1'\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)-F_2\left(\theta|\widetilde{\theta}\right)\left[\theta-\beta_2^*\left(\theta,\widetilde{\theta}\right)\right]f_1\left(\widetilde{\theta}\right) \\
& -\Lambda^g\theta\left[f_1\left(\widetilde{\theta}\right)-F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right]\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^g\theta\left[F_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right]\left[-f_1\left(\widetilde{\theta}\right)+F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& -\Lambda^m\beta_1'\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\beta_1\left(\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\beta_1\left(\widetilde{\theta}\right)F_1\left(\widetilde{\theta}\right)\left[-f_1\left(\widetilde{\theta}\right)+F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& +\Lambda^m F_2\left(\theta|\widetilde{\theta}\right)\beta_2^*\left(\theta,\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\left[1-F_1\left(\widetilde{\theta}\right)-\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right] \\
& -\Lambda^m\int_{\widetilde{\theta}}^{\overline{\theta}}F_2\left(\theta|y_1\right)\beta_2^*\left(\theta,y_1\right)f_1\left(y_1\right)dy_1\left[-f_1\left(\widetilde{\theta}\right)+F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)\right] \\
& -\Lambda^m f_1\left(\widetilde{\theta}\right)\int_{\widetilde{\theta}}^{\overline{\theta}}\left[\beta_1\left(\widetilde{\theta}\right)-\beta_2^*\left(\theta,y_1\right)\right]F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1 \\
& -\Lambda^m F_1\left(\widetilde{\theta}\right)\left\{-\left[\beta_1\left(\widetilde{\theta}\right)-\beta_2^*\left(\theta,\widetilde{\theta}\right)\right]F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)+\int_{\widetilde{\theta}}^{\overline{\theta}}\beta_1'\left(\widetilde{\theta}\right)F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1\right\} \\
& -\Lambda^m\int_{\widetilde{\theta}}^{\overline{\theta}}\left[\beta_2^*\left(\theta,y_1\right)-\beta_2^*\left(\theta,\widetilde{\theta}\right)\right]F_2\left(\theta|\widetilde{\theta}\right)f_1\left(\widetilde{\theta}\right)F_2\left(\theta|y_1\right)f_1\left(y_1\right)dy_1.
\end{aligned}
$$

Substituting $\theta=\widetilde{\theta}$ and re-arranging results in the following differential equation

$$
\frac{d}{d\theta}\left\{\beta_1\left(\theta\right)F_1\left(\theta\right)\left[1+\Lambda^m\left(1-F_1\left(\theta\right)\right)\right]\right\}=f_1\left(\theta\right)\beta_2\left(\theta,\theta\right)+\Lambda^m\beta_2\left(\theta,\theta\right)f_1\left(\theta\right)\left[1-2F_1\left(\theta\right)\right]
$$

together with the boundary condition that $\beta_1(0)=0$. Solving the differential equation yields

$$
\beta_1^*\left(\theta;\Lambda^g,\Lambda^m\right)=\frac{\int_0^\theta\beta_2^*\left(s,s\right)\left\{1+\Lambda^m\left[1-2F_1\left(s\right)\right]\right\}f_1\left(s\right)ds}{F_1\left(\theta\right)\left\{1+\Lambda^m\left[1-F_1\left(\theta\right)\right]\right\}}. \tag{29}
$$

It is easy to see that $\Lambda^m\leq 1$ is a sufficient condition for β_1^* to be strictly increasing in θ. Moreover, notice that β_1^* depends on Λ^g only indirectly, through $\beta_2(s,s)$. On the other hand, β_1^* depends directly on Λ^m. This renders the comparison between the first-round price and the (conditional) expected second-round price somewhat more involved. Indeed, as the term $\Lambda^m\left[1-2F_1(s)\right]$ becomes negative when N is relatively small and s is close to $\overline{\theta}$, the effect of loss aversion over money on the price path can either go in the same direction as that of loss aversion over consumption, or in the opposite one. In particular, if $\Lambda^g=0$ equilibrium prices can either increase or decline depending on the number of bidders and how high the first-round price is.

Let $y_1=\beta_1^{-1}(p_1)$. Figure 4 displays the difference $\mathbb{E}\left[p_2|p_1\right]-p_1$ as a function of y_1 for three different values of N when $\Lambda^g=0$, $\Lambda^m=1$ and θ is distributed uniformly on $[0,1]$: $N=3$ (solid), $N=4$ (dashed), and $N=5$ (dotted). For $N\geq 4$, the difference $\mathbb{E}\left[p_2|p_1\right]-p_1$ is always negative.

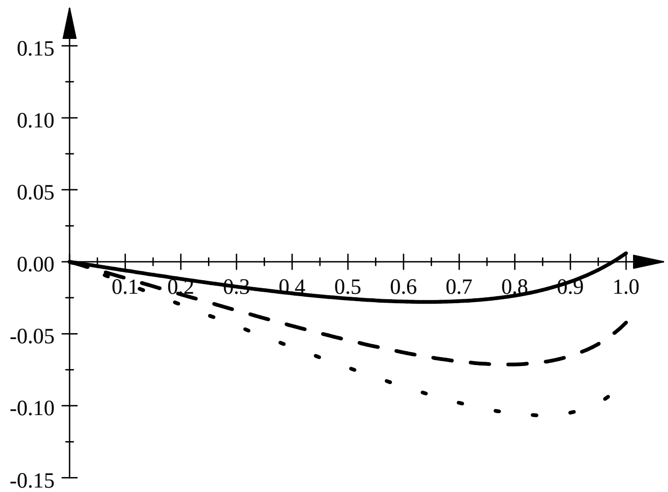

Figure 4: $\mathbb{E}[p_2|p_1] - p_1$ as a function of y_1 for three different values of N (3, 4, 5) when $\Lambda^g = 0$, $\Lambda^m = 1$ and θ is distributed uniformly on $[0, 1]$.

Yet, for $N = 3$, equilibrium prices decline if and only if $y_1 \leq \frac{\sqrt{30}\left(\sqrt{960\ln^2 2 - 1376\ln 2 + 493}\right)}{87 - 120(\ln 2)} \simeq 0.976$.

As the following Proposition shows, for $\Lambda^g > 0$ loss aversion over money does not eliminate the afternoon effect.

Proposition 8. *(Afternoon Effect V) If $\Lambda^g > 0$, then there exists a threshold $\widetilde{\Lambda}^m$ such that if $\Lambda^m < \widetilde{\Lambda}^m$ the price sequence in a two-round sequential first-price auction is a supermartingale and the afternoon effect arises in equilibrium.*

Proof of Proposition 8: We have

$$p_1 = \beta_1^*(y_1) = \frac{\int_0^{y_1} \beta_2^*(\theta, \theta)\{1 + \Lambda^m[1 - 2F_1(\theta)]\} f_1(\theta)\, d\theta}{F_1(y_1)\{1 + \Lambda^m[1 - F_1(y_1)]\}}$$

and

$$\mathbb{E}[p_2|p_1] = \frac{\int_0^{y_1} \beta_2^*(\theta, y_1) f_1(\theta)\, d\theta}{F_1(y_1)}.$$

Hence,

$$p_1 - \mathbb{E}[p_2|p_1] = \frac{\int_0^{y_1} \left\{\beta_2^*(\theta, \theta)\frac{1 + \Lambda^m[1 - 2F_1(\theta)]}{1 + \Lambda^m[1 - F_1(y_1)]} - \beta_2^*(\theta, y_1)\right\} f_1(\theta)\, d\theta}{F_1(y_1)}. \tag{30}$$

If $\Lambda^g > 0$, we know that the term on the right-hand-side of (30) is positive for $\Lambda^m = 0$. By continuity, then, there exists a threshold $\widetilde{\Lambda}^m$ such that $p_1 - \mathbb{E}[p_2|p_1] > 0$ if $\Lambda^m < \widetilde{\Lambda}^m$. ■