Inflation and Innovation in a Schumpeterian Economy with North-South Technology Transfer

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December 2016

Abstract

This study analyzes the cross-country effects of inflation on innovation and international technology transfer via cash-in-advance (CIA) constraints on R&D investment. We consider a scale-invariant North-South quality-ladder model that features innovative R&D in the North and adaptive R&D in the South. We find that a higher inflation in the South causes a permanent decrease in the rate of international technology transfer, a permanent increase in the North-South wage gap, and a temporary decrease in the rate of Northern innovation. A higher inflation in the North causes a temporary decrease in the rate of Northern innovation, a permanent decrease in the North-South wage gap, and an ambiguous effect on the rate of international technology transfer depending on the relative size of the two economies. We also calibrate the model to China-US data and find that the cross-country welfare effect of inflation is quantitatively significant from the North to the South, but not from the South to the North. Specifically, permanently decreasing inflation to achieve the Friedman rule in the US leads to a welfare gain of 3.28% in the US and a welfare gain of 3.31% in China. However, permanently decreasing inflation to achieve the Friedman rule in China leads to much smaller welfare gains of 0.34% in China and 0.17% in the US.

JEL classification: O30, O40, E41, F43
Keywords: inflation, economic growth, R&D, North-South product cycles, FDI

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1 Introduction

In this study, we explore the cross-country effects of inflation on innovation and international technology transfer via foreign direct investment (FDI) in a scale-invariant North-South quality-ladder growth model that features innovative R&D in the North and adaptive R&D in the South. Multinational firms invest in adaptive R&D in the South to transfer the production of the highest quality products from the North to the South in order to take advantage of the lower Southern wage rate. To model money demand, we impose cash-in-advance (CIA) constraints on R&D investment, which is costly and subject to cash requirements in reality; see for example Chu *et al.* (2015) for a discussion of empirical evidence. Early empirical studies, such as Hall (1992) and Opler *et al.* (1999), show a positive and significant relationship between R&D expenditures and cash flows in US firms. From 1980 to 2006, the average cash-to-assets ratio in US firms increased substantially, and Bates *et al.* (2009) argue that this trend is partly driven by the firms' increasing R&D expenditures. Brown and Petersen (2011) show that firms smooth their R&D expenditures by maintaining a buffer stock of liquidity in the form of cash reserves. Berentsen *et al.* (2012) argue that information frictions and limited collateral value of R&D capital require firms to finance R&D projects with cash. Falato and Sim (2014) use firm-level data in the US to show that firms' cash holdings increase (decrease) significantly in response to a rise (cut) in R&D tax credits. These results suggest that due to financial frictions, firms need to use cash to finance their R&D investment. We capture these cash requirements on R&D by imposing CIA constraints on innovative R&D in the North and adaptive R&D in the South. Within this monetary growth-theoretic framework, we derive the following results.

A higher inflation in the South causes a permanent decrease in the rate of international technology transfer via the Southern CIA constraint on adaptive R&D. A higher inflation in the South also has the following general-equilibrium effects: a permanent increase in the North-South wage gap, and a temporary decrease in the rate of innovation in the North. Intuitively, a higher inflation in the South raises the cost of adaptive R&D, which in turn reduces the incentives for international technology transfer. As a result, less products are manufactured by Southern firms and more products are produced by Northern firms. The higher demand for production labor in the North reduces R&D labor, which in turn decreases the rate of Northern innovation but only temporarily due to the semi-endogenous-growth property of the model. Finally, given that a higher inflation in the South has a direct negative effect on the demand for Southern R&D labor, it depresses the wage rate in the South relative to the North.

A higher inflation in the North causes a temporary decrease in the rate of Northern innovation via the CIA constraint on innovative R&D in the North. A higher inflation in the North also has the following general-equilibrium effects: a permanent decrease in the North-South wage gap, and an ambiguous effect on the rate of technology transfer from the North to the South depending on the relative size of the two economies. Specifically, we find that if the Southern population size is sufficiently large (small), then an increase in the inflation rate in the North would cause a permanent decrease (increase) in the rate of technology transfer from the North to the South. Intuitively, a higher inflation in the North raises the cost of innovative R&D, which in turn reduces the incentives for innovation. As a result, the rate of innovation decreases temporarily. Given that a higher inflation in the

North has a direct negative effect on the demand for Northern R&D labor, it depresses the wage rate in the North relative to the South. As for the effects on the rate of international technology transfer, there are two opposing effects. On the one hand, it reduces the long-run level of aggregate quality, which reduces the difficulty of adaptive R&D due to the property of increasing R&D difficulty in the semi-endogenous growth model.[1] This is a positive effect on international technology transfer. On the other hand, the higher inflation in the North also reduces the incentives for adaptive R&D because there are less benefits from FDI due to the smaller North-South wage gap. This negative effect on international technology transfer via adaptive R&D labor in the South is relatively strong when the Southern labor force is large. Therefore, the overall effect of a higher inflation in the North on technology transfer would be negative (positive) if the Southern population size is sufficiently large (small).

We also calibrate the model to China-US data in order to conduct a quantitative investigation on the cross-country effects of inflation via the CIA constraints. We find that permanently decreasing inflation to achieve the Friedman rule (i.e., a zero nominal interest rate) in the US would raise the wage gap between the US and China by 0.471% (percent change) and surprisingly decrease the flow of technology transfer from the US to China by 1.257% (percent change). Decreasing inflation in the US also leads to welfare gains that are equivalent to a permanent increase in consumption of 3.281% in the US and 3.312% in China. These significant welfare gains are due to a large increase in the level of technology by 8.068%. Therefore, the cross-country welfare effect of inflation is quantitatively significant from the North to the South.

On the other hand, permanently decreasing inflation to achieve the Friedman rule in China would reduce the wage gap between the US and China by 0.469% and increase the flow of technology transfer from the US to China by 1.492%. Also, it leads to relatively small welfare gains of 0.338% in China and 0.174% in the US. These small welfare gains are partly due to the small increase in the level of technology by 0.361%. In other words, reducing inflation in China leads to a much smaller increase in the level of technology than reducing inflation in the US. This finding is due to innovation originating from the North.[2]

In the literature on inflation and economic growth, Stockman (1981) and Abel (1985) analyze a CIA constraint on capital investment in a monetary version of the Neoclassical growth model. Subsequent studies in this literature explore the effects of monetary policy in variants of the capital-based growth model. This study instead associates more closely with a related literature on inflation and *innovation-driven* growth. In this literature, Marquis and Reffett (1994) is the seminal study that analyzes the effects of inflation via a CIA constraint on consumption in a variant of the variety-expanding model in Romer (1990).[3] In contrast, we explore the effects of inflation in a Schumpeterian quality-ladder model as in Chu and Cozzi (2014) and Chu and Lai (2013).[4] However, the present study differs from all these

[1] See Venturini (2012) for empirical evidence based on US manufacturing industry data that supports the semi-endogenous growth model with increasing R&D difficulty.

[2] According to the OECD, at the beginning of this century OECD countries performed over 90% of global R&D. Although this share is gradually declining, it remains over 70% in 2014.

[3] Chu, Lai and Liao (2012) provide an analysis of the CIA constraint on consumption in a hybrid growth model in which economic growth in the long run is driven by both variety expansion and capital accumulation.

[4] See also Chu and Ji (2016) and Huang *et al.* (2015), who analyze the effects of monetary policy in a Schumpeterian model with endogenous market structure.

studies by considering an open-economy two-country model, which enables us to explore the cross-country effects of the CIA constraints on innovation and international technology transfer. In this open-economy model, we find that inflation in a country could lead to a sizable welfare effect in another country, which is an important finding that cannot be obtained in a closed-economy analysis. Chu *et al.* (2015) also analyze the effects of inflation in an open-economy Schumpeterian model, but they consider an environment with two Northern economies in the absence of North-South product cycles and technology transfer via FDI that characterize the interesting interaction between developed and developing economies. To our knowledge, this is the first study that explores the effects of inflation in the presence of North-South product cycles and technology transfer via FDI. Within this novel monetary growth-theoretic framework, we discover some interesting effects of the CIA constraints on innovation and international technology transfer.

The rest of this study is organized as follows. Section 2 presents the model. Section 3 solves the steady-state equilibrium. Section 4 analyzes the effects of inflation. The final section concludes.

2 A North-South monetary Schumpeterian model

The North-South quality-ladder growth model is based on Dinopoulos and Segerstrom (2010). The North-South R&D-based growth model originates from the seminal study by Grossman and Helpman (1991).[5] The model in Dinopoulos and Segerstrom (2010) is a recent vintage of this class of models and has the advantage of being free of scale effects by featuring semi-endogenous growth.[6] In the Dinopoulos-Segerstrom model, multinational firms employ Northern R&D labor to invest in innovative R&D that improves the quality of products manufactured in the North. In order to take advantage of the lower production cost in the South, the multinational firms then employ Southern R&D labor to invest in adaptive R&D that transfers the production of the highest quality products from the North to the South. After the manufacturing process of a product is transferred to the South, the multinational firm faces the possibility of the product being imitated by domestic firms in the South. To facilitate a realistic calibration to data, we generalize the Dinopoulos-Segerstrom model by introducing several parameters. Furthermore, to introduce money demand, we incorporate CIA constraints on innovative R&D in the North and adaptive R&D in the South. Then, we analyze the effects of inflation in the two countries on innovation and international technology transfer.

[5] Dinopoulos and Segerstrom (2010) provide a review of the subsequent development in this literature that focuses on the effects of intellectual property rights. See also Iwaisako *et al.* (2011) and Tanaka and Iwaisako (2014) for recent contributions.

[6] See Jones (1999) for a discussion of scale effects in R&D-based growth models. The semi-endogenous-growth version of the quality-ladder model originates from Segerstrom (1998) and Li (2003).

2.1 Households

In each country, there is a representative household. The lifetime utility function of the household in the North is given by

$$U^N = \int_0^\infty e^{-(\rho - g_L)t} \ln c_t^N dt, \tag{1}$$

where c_t^N denotes per capita consumption in the North at time t, and the parameter $\rho > 0$ determines subjective discounting. The population size, which is also the size of the representative household, in the North is L_t^N, which increases at an exogenous population growth rate $g_L > 0$. To ensure that lifetime utility is bounded, we impose the following parameter restriction: $\rho > g_L$. For simplicity, we make a common assumption that $\{\rho, g_L\}$ are the same in the two countries. Total population in the world is $L_t = L_t^N + L_t^S$. We use $s \equiv L_t^S / L_t$ to denote the share of world population in the South and $1 - s \equiv L_t^N / L_t$ to denote the share of world population in the North.

The household in the North maximizes (1) subject to the following asset-accumulation equation:

$$\dot{A}_t^N + \dot{M}_t^N = (i_t^N - g_L) A_t^N - g_L M_t^N + i_t^N B_t^N + W_t^N + D_t^N + T_t^N - P_t^N c_t^N.$$

P_t^N is the price of consumption goods denominated in units of domestic currency in the North. A_t^N is the nominal value of financial assets owned by each member of the household, and i_t^N is the nominal interest rate in the North. M_t^N is the nominal value of domestic currency held by each member of the household. B_t^N is the nominal value of domestic currency borrowed by R&D entrepreneurs to finance their R&D investment in the North, and the rate of return on B_t^N is the domestic nominal interest rate i_t^N.[7] There is a constraint on how much money that each person can lend to R&D entrepreneurs, and the constraint is $B_t^N \leq M_t^N$.[8] Each member of the household supplies one unit of labor to earn a nominal wage W_t^N. D_t^N is the nominal value of a profit from the R&D sector.[9] T_t^N is the nominal value of a lump-sum transfer (or tax if $T_t^N < 0$) from the government to each person in the North.

For convenience, we reexpress the asset-accumulation equation in real terms (denominated in units of consumption goods).[10]

$$\dot{a}_t^N + \dot{m}_t^N = (r_t^N - g_L) a_t^N - \left(\pi_t^N + g_L\right) m_t^N + i_t^N b_t^N + w_t^N + d_t^N + \tau_t^N - c_t^N. \tag{2}$$

[7] It can be easily shown as a no-arbitrage condition that the rate of return on B_t^N must be equal to i_t^N. The intuition can be explained as follows. The opportunity cost for the household to hold cash is the nominal interest rate. Therefore, in order for the household to be willing to lend cash to firms, it must be the case that firms pay the nominal interest rate in return. If firms pay less than the nominal interest rate, the household would not lend any cash to firms. If they pay more than the nominal interest rate, the household would want to lend an infinite amount of cash to firms.

[8] In the case of an additional CIA requirement on consumption, the CIA constraint in the North becomes $P_t^N c_t^N + B_t^N \leq M_t^N$. Given that we focus on inelastic labor supply for tractability, the CIA constraint on consumption would have no effect on the equilibrium allocations, except for the real money balance.

[9] See Section 2.4 for a discussion.

[10] Derivations are available upon request.

a_t^N is the real value of financial assets per capita, and $r_t^N = i_t^N - \pi_t^N$ is the real interest rate in the North. π_t^N is the inflation rate of P_t^N in the North. m_t^N is the real value of domestic currency per capita. b_t^N is the real value of domestic currency borrowed by domestic R&D entrepreneurs, and the constraint becomes $b_t^N \leq m_t^N$. w_t^N is the real wage rate. d_t^N is the real value of R&D profit. τ_t^N is the real value of the lump-sum transfer from the government.

We follow Dinopoulos and Segerstrom (2010) to assume that there is a global financial market. In this case, the real interest rates in the two countries must be equal such that $r_t^N = r_t^S = r_t$.[11] From standard dynamic optimization, the familiar Euler equation is[12]

$$\frac{\dot{c}_t^N}{c_t^N} = \frac{\dot{c}_t^S}{c_t^S} = r_t - \rho, \tag{3}$$

which implies that the growth rate of consumption is the same across countries.

2.2 Consumption goods

Consumption goods are produced by perfectly competitive firms that aggregate a unit continuum of intermediate goods $Y_t(j)$ using the following CES aggregator:

$$C_t = \left\{ \int_0^1 [Y_t(j)]^{\frac{\sigma-1}{\sigma}} dj \right\}^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\sigma > 1$ is the elasticity of substitution between intermediate goods. The resource constraint on C_t is

$$C_t = c_t^N L_t^N + c_t^S L_t^S = \left[c_t^N (1-s) + c_t^S s\right] L_t, \tag{5}$$

where $c_t^N L_t^N$ is total consumption in the North and $c_t^S L_t^S$ is total consumption in the South. P_t^N is the price of consumption goods denominated in units of currency in the North. P_t^S is the price of consumption goods denominated in units of currency in the South. Given zero transportation cost, the law of one price holds such that $P_t^N = \epsilon_t P_t^S$, where ϵ_t is the nominal exchange rate. For convenience, we will express all variables in real terms denominated in units of consumption goods that have the same value in the two countries. From profit maximization, we derive the conditional demand function for $Y_t(j)$ as

$$Y_t(j) = p_t(j)^{-\sigma} C_t \tag{6}$$

for $j \in [0, 1]$. $p_t(j)$ is the price of $Y_t(j)$.

[11] The nominal interest rates in the two countries would be different if inflation rates differ across countries. However, even when the nominal interest rates differ across countries, there is no incentive for the household to hold foreign currency. The reason is that given the same real interest rate across countries as a result of the global financial market, differences in the nominal interest rates are due to differences in the inflation rates, which in turn equal percent changes in the nominal exchange rate because the law of one price holds in our model as we discuss below. Therefore, a small transaction cost on foreign exchange would discourage the household from holding foreign currency.

[12] The representative household in the South also performs an analogous dynamic optimization.

2.3 Intermediate goods

There is a unit continuum of differentiated intermediate goods $j \in [0, 1]$. Some of these intermediate goods are produced in the North, and each of these Northern industries is temporarily dominated by a quality leader until the arrival of the next innovation.[13] The production function of intermediate goods manufactured by a quality leader in the North is

$$Y_t(j) = z^{n_t(j)} L^N_{y,t}(j) \equiv Y^N_t(j), \tag{7}$$

where the parameter $z > 1$ is the step size of a quality improvement, and $n_t(j)$ is the number of quality improvements that have occurred in industry j as of time t. The firm employs $L^N_{y,t}(j)$ units of labor in the North for production. Given $z^{n_t(j)}$, the marginal cost of production for the industry leader is $w^N_t / z^{n_t(j)}$.[14] We follow Dinopoulos and Segerstrom (2010) to assume that new quality leaders are always able to charge the unconstrained monopolistic price because the closest competitors choose to immediately exit the market in equilibrium.[15] In this case, the monopolistic price charged by industry leaders is

$$p_t(j) = \frac{\sigma}{\sigma - 1} \frac{w^N_t}{z^{n_t(j)}} \equiv p^N_t(j). \tag{8}$$

To take advantage of the lower labor cost in the South, industry leaders in the North invest in adaptive R&D in the South in order to shift the manufacturing process to the South. If the adaptive R&D project of a Northern leader is successful, then a Southern affiliate of the Northern leader would start producing the intermediate goods. The production function of intermediate goods manufactured by the foreign affiliate of a Northern quality leader is

$$Y_t(j) = z^{n_t(j)} \delta L^F_{y,t}(j) \equiv Y^F_t(j), \tag{9}$$

where we have introduced $\delta > 0$ as a labor-productivity parameter, which captures the productivity of Southern labor relative to Northern labor. The Southern affiliate employs $L^F_{y,t}(j)$ units of labor in the South for production, and the marginal cost of production is $w^S_t / [\delta z^{n_t(j)}]$, which is assumed to be less than $w^N_t / z^{n_t(j)}$.Given the marginal cost of production, the unconstrained monopolistic price is given by

$$p_t(j) = \frac{\sigma}{\sigma - 1} \frac{w^S_t}{\delta z^{n_t(j)}} \equiv p^F_t(j). \tag{10}$$

The Southern affiliate produces the intermediate goods until the arrival of the next innovation in the North or until the current innovation is imitated by other firms in the South. When the next innovation arrives, the manufacturing process shifts back to the North. To ensure that this return of production to the North occurs, we follow Dinopoulos and Segerstrom (2010) to assume $w^S_t / \delta > w^N_t / z$, so that new quality leaders are able to drive out Southern affiliates of previous quality leaders.

Technologies of Southern affiliates may be imitated by other Southern firms subject to an exogenous imitation rate ϕ. When this imitation occurs, the intermediate goods are

[13] This is known as the Arrow replacement effect in the literature; see Cozzi (2007a) for a discussion.

[14] It is useful to note that we here adopt a cost-reducing view of quality improvement.

[15] See Dinopoulos and Segerstrom (2010) for a detailed discussion.

produced by competitive firms in the South. The production function of intermediate goods produced by competitive firms in the South is

$$Y_t(j) = z^{n_t(j)}\delta L_{y,t}^S(j) \equiv Y_t^S(j), \tag{11}$$

and the perfectly competitive price is given by the marginal cost of production:

$$p_t(j) = \frac{w_t^S}{\delta z^{n_t(j)}} \equiv p_t^S(j). \tag{12}$$

Southern competitive firms produce the intermediate goods until the next innovation arrives at which point the manufacturing process shifts back to the North.

Let's define the aggregate quality index across industries $j \in [0,1]$ as

$$Q_t \equiv \int_0^1 q_t(j)dj,$$

where $q_t(j) \equiv \left[z^{n_t(j)}\right]^{\sigma-1}$. Then, we can derive the labor demands for an average-quality product produced by a Northern leader as

$$\tilde{L}_{y,t}^N = Q_t \left(\frac{\sigma}{\sigma-1}w_t^N\right)^{-\sigma} C_t, \tag{13}$$

by a Southern affiliate as

$$\tilde{L}_{y,t}^F = \delta^{\sigma-1}Q_t \left(\frac{\sigma}{\sigma-1}w_t^S\right)^{-\sigma} C_t, \tag{14}$$

and by Southern competitive firms as

$$\tilde{L}_{y,t}^S = \delta^{\sigma-1}Q_t \left(w_t^S\right)^{-\sigma} C_t. \tag{15}$$

Using these expressions, we can then express the labor demand for product j as

$$L_{y,t}^o(j) = \frac{q_t(j)}{Q_t}\tilde{L}_{y,t}^o, \tag{16}$$

where $o = \{N, F, S\}$. The amount of monopolistic profit earned by a Northern leader is

$$\Pi_t^N(j) = \frac{w_t^N}{\sigma-1}\frac{q_t(j)}{Q_t}\tilde{L}_{y,t}^N, \tag{17}$$

and the amount of monopolistic profit earned by a Southern affiliate is

$$\Pi_t^F(j) = \frac{w_t^S}{\sigma-1}\frac{q_t(j)}{Q_t}\tilde{L}_{y,t}^F. \tag{18}$$

2.4 Innovative and adaptive R&D

Innovative R&D is performed by entrepreneurs in the North. If an R&D entrepreneur employs Northern labor $L_{r,t}^N(j)$ to engage in innovative R&D in industry j, then she is successful in inventing the next higher-quality product in the industry with an instantaneous probability given by

$$\varphi_t^N(j) = \frac{1}{\gamma}\left[\frac{L_{r,t}^N}{Q_t}\right]^{\beta^N}\left[\frac{L_{r,t}^N(j)}{q_t(j)}\right]^{1-\beta^N}, \tag{19}$$

where the parameter $\gamma > 0$ inversely measures innovation productivity. $q_t(j)$ captures the effect of increasing innovation difficulty, and it removes the scale effect in the innovation process of the quality-ladder model as in Segerstrom (1998). Here we introduce a positive R&D spillover effect,[16] and the parameter $\beta^N \in [0, 1)$ measures the degree of this R&D externality.[17] The expected benefit from investing in innovative R&D is $v_t^N(j)\varphi_t^N(j)dt$, where $v_t^N(j)$ is the real value of the expected discounted profits generated by an innovation and $\varphi_t^N(j)dt$ is the entrepreneur's probability of having a successful innovation during the infinitesimal time interval dt. To facilitate the wage payment to R&D labor in the North, the entrepreneurs borrow domestic currency[18] from the domestic household.[19] The cost of borrowing is determined by the nominal interest rate i_t^N in the North. Therefore, the total cost of innovative R&D is $\left(1 + i_t^N\right) w_t^N L_{r,t}^N(j)dt$. The profit-maximizing condition of R&D is

$$(1 - \beta^N)\varphi_t^N(j)v_t^N(j) = (1 + i_t^N)w_t^N L_{r,t}^N(j). \tag{20}$$

Given (20), the amount of R&D profit in the North is[20]

$$d_t^N(j) = \beta^N \varphi_t^N(j)v_t^N(j).$$

Adaptive R&D in the South is performed by local entrepreneurs and the Southern affiliates of Northern industry leaders. If the Southern affiliate of a Northern leader in industry j employs Southern labor $L_{r,t}^F(j)$ to engage in adaptive R&D, then the Northern firm is successful in shifting the production to the Southern affiliate with an instantaneous probability

[16] See for example Jaffe (1986), Bernstein and Nadiri (1988, 1989) and Los and Verspagen (2000) for empirical evidence on the presence of R&D spillovers across firms.

[17] The scaling by Q_t in (19) is to ensure a steady-state value of $\varphi_t^N(j)$.

[18] Given that this is wage payment to workers in the domestic economy, the wage payment is naturally paid in domestic currency. Furthermore, there is no incentive for the entrepreneurs to borrow foreign currency and convert it into domestic currency even when the nominal interest rates differ across countries because uncovered interest rate parity holds in our model.

[19] Due to the static nature of the R&D sector in the model, we cannot consider the case in which R&D entrepreneurs accumulate cash holdings. However, even if we allow entrepreneurs to accumulate cash, inflation would have the same positive effect on the cost of R&D as in our current setting in which entrepreneurs borrow cash from the household because the opportunity cost of using cash to finance R&D is determined by the nominal interest rate in both cases.

[20] Positive profit in the R&D sector can be justified by the presence of a fixed factor input $K^N(j)$, which is implicitly normalized to unity. For example, this fixed factor input may be the entrepreneurial talent of R&D entrepreneurs in the specific industry. Given that not everyone possesses this entrepreneurial talent, there is no free entry in this industry generating a monopolistic rent that is captured by the entrepreneurs.

given by

$$\varphi_t^F(j) = \frac{1}{\alpha}\left[\frac{L_{r,t}^F}{Q_t^N}\right]^{\beta^F}\left[\frac{L_{r,t}^F(j)}{q_t(j)}\right]^{1-\beta^F}, \tag{21}$$

where the parameter $\alpha > 0$ inversely measures adaptation productivity. $q_t(j)$ captures the effect of increasing adaptation difficulty, and it removes the scale effect in the adaptation process as in Dinopoulos and Segerstrom (2010). Here we introduce a positive spillover effect of adaptive R&D, and the parameter $\beta^F \in [0, 1)$ measures the degree of this R&D externality.[21] The expected *net* benefit for the Northern leader to invest in adaptive R&D is $\left[v_t^F(j) - v_t^N(j)\right]\varphi_t^F(j)dt$, where $v_t^F(j)$ is the real value of the expected discounted profits generated by the Southern affiliate and $\varphi_t^F(j)dt$ is the probability of having a successful adaptation during the infinitesimal time interval dt. To facilitate the wage payment to R&D labor in the South, the Southern affiliate borrows domestic currency from the domestic household, and the cost of borrowing is determined by the nominal interest rate i_t^S in the South. Therefore, the total cost of adaptive R&D is $\left(1 + i_t^S\right) w_t^S L_{r,t}^F(j)dt$. Given that the net benefit of adaptive R&D is increasing in $L_{r,t}^F(j)$, the Southern affiliate engages in a positive finite amount of adaptive R&D if and only if the following equilibrium condition holds:

$$(1 - \beta^F)\varphi_t^F(j)[v_t^F(j) - v_t^N(j)] = (1 + i_t^S)w_t^S L_{r,t}^F(j). \tag{22}$$

Given (22), the amount of R&D profit in the South is[22]

$$d_t^F(j) = \beta^F \varphi_t^F(j)\left[v_t^F(j) - v_t^N(j)\right]. \tag{23}$$

Finally, Southern affiliates face the risk of imitation (with an exogenous probability $\phi > 0$) by other firms in the South.

2.5 Stock market

The no-arbitrage condition that determines the value of $v_t^N(j)$ is given by[23]

$$r_t = \frac{\Pi_t^N(j) - \left(1 + i_t^S\right) w_t^S L_{r,t}^F(j) - d_t^F(j) + \dot{v}_t^N(j) - \varphi_t^N(j)v_t^N(j) + \varphi_t^F(j)\left[v_t^F(j) - v_t^N(j)\right]}{v_t^N(j)}.$$

This condition equates the real interest rate r_t to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profits net of adaptive R&D expenditure and rent,[24] (b) any potential capital gain $\dot{v}_t^N(j)$, (c) the expected capital loss $-\varphi_t^N(j)v_t^N(j)$ from creative destruction, and (d) the expected change in asset value $\varphi_t^F(j)\left[v_t^F(j) - v_t^N(j)\right]$ when adaptive R&D is successful. Using (22) and (23), we simplify the no-arbitrage condition to a more familiar expression given by

$$r_t = \frac{\Pi_t^N(j) + \dot{v}_t^N(j) - \varphi_t^N(j)v_t^N(j)}{v_t^N(j)}. \tag{24}$$

[21] The scaling by Q_t^N (to be defined in Section 3.1) in (21) is to ensure a steady-state value of $\varphi_t^F(j)$.

[22] Once again, positive profit is the rent captured by local entrepreneurs who own a fixed factor input $K^S(j)$, which is normalized to unity.

[23] It is useful to note that the following $\Pi_t^N(j)$ refers to the profit after the arrival of the next innovation.

[24] Recall that R&D rent is not captured by Northern leaders or their Southern affiliates.

The no-arbitrage condition that determines the value of $v_t^F(j)$ is given by

$$r_t = \frac{\Pi_t^F(j) + \dot{v}_t^F(j) - [\varphi_t^N(j) + \phi]v_t^F(j)}{v_t^F(j)}. \tag{25}$$

This condition equates the real interest rate r_t to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profits in the South, (b) any potential capital gain $\dot{v}_t^F(j)$, (c) the expected capital loss $-\varphi_t^F(j)v_t^F(j)$ from creative destruction, and (d) the expected capital loss $-\phi v_t^F(j)$ from imitation.

The value of a successful innovation $v_t^N(j)$ in industry j is linearly increasing in $\Pi_t^N(j)$, which in turn is linearly increasing in $q_t(j)$ as shown in (17). Together with $L_{r,t}^N(j)$ being linearly increasing in $q_t(j)$, the arrival rate of innovation $\varphi_t^N(j)$ is independent of $q_t(j)$. Therefore, we follow the standard treatment in this class of models to focus on the symmetric equilibrium in which $\varphi_t^N(j) = \varphi_t^N$.[25] Similarly, the property that $v_t^F(j)$ and $L_{r,t}^F(j)$ are linearly increasing in $q_t(j)$ implies that $\varphi_t^F(j)$ is independent of $q_t(j)$. Therefore, we focus on the symmetric equilibrium in which $\varphi_t^F(j) = \varphi_t^F$.

2.6 Monetary authority

The monetary policy instrument in the North (South) is the domestic inflation rate π_t^N (π_t^S), which is exogenously chosen by the Northern (Southern) monetary authority. Given π_t^N (π_t^S), the nominal interest rate in the North (South) is endogenously determined according to the Fisher identity $i_t^N = \pi_t^N + r_t$ ($i_t^S = \pi_t^S + r_t$), where r_t is the global real interest rate. Then, the growth rate of the nominal money supply per capita in the North (South) is endogenously determined by $\dot{M}_t^N/M_t^N = \pi_t^N + \dot{m}_t^N/m_t^N$ ($\dot{M}_t^S/M_t^S = \pi_t^S + \dot{m}_t^S/m_t^S$). The Northern (Southern) monetary authority returns the seigniorage revenue as a lump-sum transfer that has a real value of $\tau_t^N = (\dot{M}_t^N + g_L M_t^N)/P_t^N$ ($\tau_t^S = (\dot{M}_t^S + g_L M_t^S)/P_t^S$) to each member of the domestic household in the North (South).

Due to the semi-endogenous-growth property of this model, the long-run growth rate of total consumption C_t is exogenously given by $g_L\sigma/(\sigma-1)$. Therefore, from the Euler equation (3), the real interest rate in the steady state is also exogenous and given by $r = \rho + g_L/(\sigma-1)$. Consequently, there is an one-to-one relationship between the nominal interest rate and the inflation rate in the long run such that $i^N = \pi^N + \rho + g_L/(\sigma-1)$ and $i^S = \pi^S + \rho + g_L/(\sigma-1)$.[26]

2.7 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t^N, c_t^S, C_t, Y_t^N(j), Y_t^F(j), Y_t^S(j), L_{y.t}^N(j), L_{y.t}^F(j), L_{y.t}^S(j), L_{r.t}^N(j), L_{r.t}^F(j)\}_{t=0}^{\infty}$, a time path of prices $\{w_t^N, w_t^S, p_t^N(j), p_t^F(j), p_t^S(j), v_t^N, v_t^F, \epsilon_t\}_{t=0}^{\infty}$ and a time path of monetary policies $\{i_t^N, i_t^S\}_{t=0}^{\infty}$. Also, at each instance of time,

[25] See Cozzi (2007b) for a discussion on the possibility of multiple equilibria in the Schumpeterian growth model. Cozzi *et al.* (2007) provide theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian growth model.

[26] Empirical evidence supports a positive long-run relationship between inflation and the nominal interest rate; see for example Mishkin (1992) and Booth and Ciner (2001).

- the representative household in the North maximizes lifetime utility taking $\{r_t, i_t^N, w_t^N\}$ as given;
- the representative household in the South maximizes lifetime utility taking $\{r_t, i_t^S, w_t^S\}$ as given;
- competitive consumption-good firms produce C_t to maximize profit taking $\{p_t^N(j), p_t^F(j), p_t^S(j)\}$ as given;
- quality leaders in the North choose $p_t^N(j)$ and produce $Y_t^N(j)$ to maximize profit taking w_t^N as given;
- affiliates in the South choose $p_t^F(j)$ and produce $Y_t^F(j)$ to maximize profit taking w_t^S as given;
- competitive intermediate goods firms produce $Y_t^S(j)$ to maximize profit taking $\{p_t^S(j), w_t^S\}$ as given;
- R&D entrepreneurs in the North employ $L_{r.t}^N(j)$ to do innovative R&D taking $\{i_t^N, w_t^N, v_t^N\}$ as given;
- quality leaders in the North and their affiliates in the South employ $L_{r.t}^F(j)$ to do adaptive R&D taking $\{i_t^S, w_t^S, v_t^F\}$ as given;
- the market-clearing condition for consumption goods holds;
- the market-clearing conditions for labor hold in both countries; and
- finally, the nominal exchange rate is determined by the law of one price such that $\epsilon_t = P_t^N / P_t^S$.

3 Steady-state equilibrium

In this section, we proceed to solve the steady-state equilibrium in the following steps. First, we derive the steady-state number of each type of industries and the steady-state expression of the quality index. Then, we derive the steady-state labor market conditions in the two countries. Finally, we put all these conditions together to derive the steady-state equilibrium rates of technology transfer and innovation.

3.1 Industry composition and quality dynamics

In the intermediate goods sector, there are three types of industries in which intermediate goods are produced respectively by Northern quality leaders, Southern affiliates, and Southern competitive firms. We use $\{\theta^N, \theta^F, \theta^S\}$ to denote the steady-state measure of these three types of industries. To solve for these three endogenous variables, we use the following conditions. First, the measure of all industries adds up to one.

$$\theta^N + \theta^F + \theta^S = 1. \tag{26}$$

In the steady state, the flows in and out of each type of industry must be equal. The flow into industries θ^S dominated by Southern competitive firms is $\theta^F \phi$ given by the measure of industries in which Southern affiliates' technologies are imitated. The flow out of industries θ^S dominated by Southern competitive firms is $\theta^S \varphi^N$ given by the measure of these competitive industries experiencing the arrival of new innovations in the North. Therefore, the second condition is

$$\theta^F \phi = \theta^S \varphi^N. \tag{27}$$

The flow into industries θ^F dominated by Southern affiliates is $\theta^N \varphi^F$ given by the measure of industries in the North experiencing successful R&D adaptation. The flow out of industries θ^F dominated by Southern affiliates is the sum of (a) $\theta^F \varphi^N$ given by the measure of these industries experiencing the arrival of new innovations in the North and (b) $\theta^F \phi$ given by the measure of industries in which Southern affiliates' technologies are imitated. Therefore, the third condition is

$$\theta^N \varphi^F = \theta^F (\varphi^N + \phi). \tag{28}$$

Solving (26), (27) and (28) yields

$$\theta^N = \frac{\varphi^N}{\varphi^N + \varphi^F}, \tag{29}$$

$$\theta^F = \frac{\varphi^N}{\varphi^N + \phi} \frac{\varphi^F}{\varphi^N + \varphi^F}, \tag{30}$$

$$\theta^S = \frac{\phi}{\varphi^N + \phi} \frac{\varphi^F}{\varphi^N + \varphi^F}. \tag{31}$$

The aggregate quality index across industries $j \in [0, 1]$ is

$$Q_t \equiv \int_0^1 q_t(j) dj = \int_0^1 \lambda^{n_t(j)} dj, \tag{32}$$

where $\lambda \equiv z^{\sigma-1}$ is a composite parameter that is increasing in the quality step size z. This quality index can be decomposed into the following three components:

$$Q_t = Q_t^N + Q_t^F + Q_t^S = \int_{\theta^N} q_t(j) dj + \int_{\theta^F} q_t(j) dj + \int_{\theta^S} q_t(j) dj. \tag{33}$$

Lemma 1 provides the steady-state expression for the share of each of these three components of aggregate quality.

Lemma 1 *In the steady state, the three components of aggregate quality can be expressed as*

$$\frac{Q_t^N}{Q_t} = \frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F}, \tag{34}$$

$$\frac{Q_t^F}{Q_t} = \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi}\frac{\varphi^F}{\lambda\varphi^N + \varphi^F}. \tag{35}$$

$$\frac{Q_t^S}{Q_t} = \frac{\phi}{\lambda\varphi^N + \phi}\frac{\varphi^F}{\lambda\varphi^N + \varphi^F}. \tag{36}$$

Proof. See Appendix A. ■

3.2 Northern labor market

The market-clearing condition for labor in the North is given by

$$L_t^N = L_{y,t}^N + L_{r,t}^N = \int_{\theta_t^N} L_{y,t}^N(j)dj + \int_0^1 L_{r,t}^N(j)dj. \tag{37}$$

The amount of labor employed for production by Northern quality leaders is

$$L_{y,t}^N = \int_{\theta_t^N} \frac{q_t(j)}{Q_t}\tilde{L}_{y,t}^N dj = \frac{Q_t^N}{Q_t}\tilde{L}_{y,t}^N, \tag{38}$$

where the first equality uses (16). The amount of labor employed for innovative R&D is

$$L_{r,t}^N = \gamma\varphi_t^N Q_t, \tag{39}$$

which uses (19) and the symmetry condition $\varphi_t^N(j) = \varphi_t^N$. We define x_t^N as the average quality per Northern worker such that

$$x_t^N \equiv \frac{Q_t}{L_t^N}.$$

Finally, substituting (34), (38) and (39) into (37) yields the steady-state Northern labor-market condition expressed in per-capita terms given by

$$1 = \frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F}\frac{\tilde{L}_{y,t}^N}{L_t}\frac{1}{1-s} + \gamma\varphi^N x^N, \tag{40}$$

where we also have used $L_t^N = (1-s)L_t$.

3.3 Southern labor market

The market-clearing condition for labor in the South is given by

$$L_t^S = L_{y,t}^S + L_{y,t}^F + L_{r,t}^F = \int_{\theta_t^S} L_{y,t}^S(j)dj + \int_{\theta_t^F} L_{y,t}^F(j)dj + \int_{\theta_t^N} L_{r,t}^F(j)dj. \tag{41}$$

The amount of labor employed for production by Southern competitive firms is

$$L_{y,t}^S = \int_{\theta_t^S} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^S dj = \frac{Q_t^S}{Q_t} \tilde{L}_{y,t}^S, \tag{42}$$

where the first equality uses (16). The amount of labor employed for production by Southern affiliates is

$$L_{y,t}^F = \int_{\theta_t^F} \frac{q_t(j)}{Q_t} \tilde{L}_{y,t}^F dj = \frac{Q_t^F}{Q_t} \tilde{L}_{y,t}^F, \tag{43}$$

where the first equality also uses (16). The amount of labor employed for adaptive R&D by Southern affiliates is

$$L_{r,t}^F = \alpha \varphi_t^F Q_t^N = \alpha \varphi_t^F \frac{Q_t^N}{Q_t} Q_t, \tag{44}$$

where the first equality uses (21) and the symmetry condition $\varphi_t^F(j) = \varphi_t^F$. Substituting (34)-(36) and (42)-(44) into (41) yields the steady-state Southern labor market condition expressed in per-capita terms given by

$$1 = \frac{\varphi^F}{\lambda\varphi^N + \varphi^F} \left(\frac{\phi}{\lambda\varphi^N + \phi} \frac{\tilde{L}_{y,t}^S}{L_t^S} + \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi} \frac{\tilde{L}_{y,t}^F}{L_t^S} + \alpha\lambda\varphi^N \frac{Q_t}{L_t^S} \right), \tag{45}$$

where $Q_t/L_t^S = x^N L_t^N/L_t^S = x^N(1-s)/s$ and

$$\frac{\phi}{\lambda\varphi^N + \phi} \frac{\tilde{L}_{y,t}^S}{L_t^S} + \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi} \frac{\tilde{L}_{y,t}^F}{L_t^S} = \underbrace{\left[\frac{\phi}{\lambda\varphi^N + \phi} \left(\frac{\sigma}{\sigma - 1} \right)^{\sigma} + \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi} \right]}_{\equiv \Phi(\phi)} \frac{\tilde{L}_{y,t}^F}{L_t} \frac{1}{s},$$

which uses (14), (15) and $L_t^S = sL_t$. It is useful to note that $\Phi(\phi)$ is increasing in ϕ.

3.4 Innovation and technology transfer

We first derive the growth rate of the quality index. Differentiating (32) with respect to time yields

$$\dot{Q}_t = \int_0^1 \left[\lambda^{n_t(j)+1} - \lambda^{n_t(j)} \right] \varphi_t^N dj = (\lambda - 1) \varphi_t^N Q_t. \tag{46}$$

Then, taking the log of $x_t^N = Q_t/L_t^N$ and differentiating with respect to time yields

$$\frac{\dot{x}_t^N}{x_t^N} = \frac{\dot{Q}_t}{Q_t} - \frac{\dot{L}_t^N}{L_t^N} = (\lambda - 1) \varphi_t^N - g_L. \tag{47}$$

In the steady state, x_t^N is stationary implying that the steady-state arrival rate of innovation is

$$\varphi^N = g_L / (\lambda - 1), \tag{48}$$

which is determined by exogenous parameters in this semi-endogenous growth model. As discussed in Dinopoulos and Segerstrom (2010), the law of motion in (47) implies that any increase (decrease) in the steady-state level of x^N must be associated with a temporary increase (decrease) in φ_t^N during the transition path. Therefore, if a parameter increases (decreases) x^N in the long run, it must have increased (decreased) φ_t^N in the short run.

Using (24) and (25), one can show that the balanced-growth values of assets are[27]

$$v_t^N(j) = \frac{\Pi_t^N(j)}{\rho + \varphi^N}, \tag{49}$$

$$v_t^F(j) = \frac{\Pi_t^F(j)}{\rho + \varphi^N + \phi}. \tag{50}$$

Substituting (17), (19) and (49) into (20) yields the following steady-state innovative R&D condition:

$$\frac{(\sigma-1)(\rho+\varphi^N)\left(1+i^N\right)\gamma}{1-\beta^N} = \frac{\tilde{L}_{y,t}^N}{Q_t} = \frac{1}{(1-s)x^N}\frac{\tilde{L}_{y,t}^N}{L_t}, \tag{51}$$

where the second equality is obtained by multiplying $\tilde{L}_{y,t}^N/Q_t$ by $1 = (L_t/L_t)(L_t^N/L_t^N)$. Similarly, substituting (18), (19), (20), (21) and (50) into (22) yields the following steady-state adaptive R&D condition:

$$(\sigma-1)(\rho+\varphi^N+\phi)\left[\frac{\left(1+i^S\right)\alpha}{1-\beta^F} + \frac{\left(1+i^N\right)\gamma\omega}{1-\beta^N}\right] = \frac{\tilde{L}_{y,t}^F}{Q_t} = \frac{1}{(1-s)x^N}\frac{\tilde{L}_{y,t}^F}{L_t}, \tag{52}$$

where $\omega \equiv w_t^N/w_t^S$ is the relative wage between the two countries. Using (13) and (14), we derive

$$\frac{\tilde{L}_{y,t}^F}{L_t} = \delta^{\sigma-1}\omega^\sigma \frac{\tilde{L}_{y,t}^N}{L_t}. \tag{53}$$

Substituting (51) and (52) into (53) yields the following steady-state relative-wage condition:

$$\frac{\rho+\varphi^N}{\rho+\varphi^N+\phi}(\delta\omega)^\sigma - \delta\omega = \delta\left(\frac{1-\beta^N}{1-\beta^F}\right)\frac{\left(1+i^S\right)\alpha}{\left(1+i^N\right)\gamma}, \tag{54}$$

which is an implicit function determining the steady-state equilibrium value of the relative wage $\omega(i^N, i^S)$. It can be shown using (54) that $\omega(i^N, i^S)$ is decreasing in i^N and increasing in i^S. Given $\sigma > 1$, it is easy to show that $\delta\omega > 1$. Then, to ensure that $z > \delta\omega$,[28] we impose the following parameter restriction:

$$\frac{\rho+\varphi^N}{\rho+\varphi^N+\phi}z^\sigma - z > \delta\left(\frac{1-\beta^N}{1-\beta^F}\right)\frac{\left(1+i^S\right)\alpha}{\left(1+i^N\right)\gamma}. \tag{P1}$$

[27] Derivations are available upon request.
[28] $z > \delta\omega$ is equivalent to $w^S/\delta > w^N/z$.

Substituting (51) into (40) to eliminate $\tilde{L}^N_{y,t}/L_t$ yields the *Northern steady-state condition* given by

$$1 = \gamma x^N \left[\frac{(\sigma-1)\left(\rho+\varphi^N\right)}{1-\beta^N} \frac{\lambda\varphi^N}{\lambda\varphi^N+\varphi^F} \left(1+i^N\right) + \varphi^N \right]. \tag{55}$$

The Northern steady-state condition contains two endogenous variables $\{x^N, \varphi^F\}$[29] and is positively sloped in the (x^N, φ^F) space with a positive x^N-intercept. Substituting (52) into (45) to eliminate $\tilde{L}^F_{y,t}/L_t$ yields the *Southern steady-state condition* given by

$$1 = \frac{x^N\varphi^F(1-s)/s}{\lambda\varphi^N+\varphi^F} \left\{ (\sigma-1)\left(\rho+\varphi^N+\phi\right) \left[\frac{\left(1+i^S\right)\alpha}{1-\beta^F} + \frac{\left(1+i^N\right)\gamma}{1-\beta^N} \omega(\underset{-}{i^N}, \underset{+}{i^S}) \right] \Phi(\phi) + \alpha\lambda\varphi^N \right\}. \tag{56}$$

The Southern steady-state condition also contains two endogenous variables $\{x^N, \varphi^F\}$[30] and is negative sloped in the (x^N, φ^F) space with no intercepts. Finally, (55) and (56) are the two conditions that implicitly solve for the steady-state equilibrium values of $\{x^N, \varphi^F\}$.[31] Graphically, x^N and φ^F are determined by the intersection of the North curve and the South curve in Figure 1.

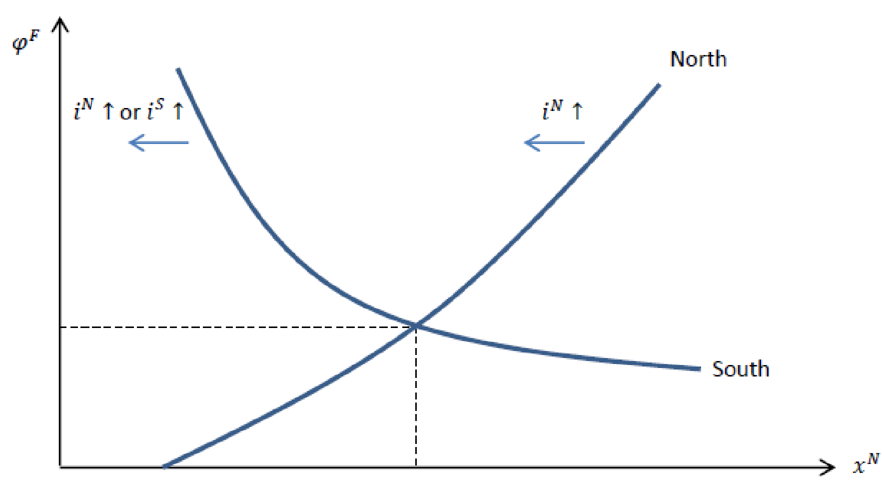

Figure 1: The steady-state equilibrium

3.5 Social welfare

In this section, we derive the steady-state level of social welfare in each country, which we will use to simulate the welfare effects of the CIA constraints in the quantitative analysis.

[29] Recall that $\varphi^N = g_L/(\lambda-1)$ and $i^N = \pi^N + \rho + g_L/(\sigma-1)$ are determined by exogenous parameters in the steady state.

[30] Recall that $\omega(i^N, i^S)$ in (54) and $i^S = \pi^S + \rho + g_L/(\sigma-1)$ are also determined by exogenous parameters in the steady state.

[31] These conditions are the same as the ones in Dinopoulos and Segerstrom (2010) when $\delta = 1$ and $\beta^N = \beta^S = i^N = i^S = 0$.

Imposing balanced growth on (1) yields the steady-state welfare of the Northern household given by

$$U^N = \frac{1}{\rho - g_L}\left(\ln c_0^N + \frac{g_c}{\rho - g_L}\right), \tag{57}$$

where $g_c = g_L/(\sigma - 1)$ is determined by exogenous parameters due to semi-endogenous growth. Therefore, the steady-state welfare is determined by the balanced-growth level of consumption. Substituting the lump-sum transfer τ_t^N from the government into (2) yields

$$c_t^N = (r_t - \dot{a}_t^N/a_t^N - g_L)a_t^N + i_t^N b_t^N + w_t^N + d_t^N.$$

Therefore, the balanced-growth level of consumption c_0^N is given by the sum of (a) asset income $(\rho - g_L)a_0^N$, (b) interest income $i^N b_0^N$,[32] (c) wage income w_0^N, and (d) R&D profit income d_0^N. An analogous derivation applies to the steady-state welfare of the Southern household. To determine a_0^N and a_0^S, we need to impose an assumption on the distribution of assets. Following Dinopoulos and Segerstrom (2010), we assume that the asset from innovative R&D in the North is owned by the Northern household whereas the asset from adaptive R&D in the South is owned by the Southern household. Under this assumption, we show in Lemma 2 that the balanced-growth levels of consumption can be expressed as $c_0^N = w_0^N I^N$ and $c_0^S = w_0^S I^S$, where $\{I^N, I^S\}$ denote income as a ratio of real wages because the different types of income are proportional to $\{w_0^N, w_0^S\}$.

Lemma 2 *The balanced-growth level of consumption can be expressed as*

$$c_0^N = w_0^N I^N = \left(\Psi L_0^N x^N\right)^{\frac{1}{\sigma-1}} I^N, \tag{58}$$

$$c_0^S = w_0^S I^S = \frac{\left(\Psi L_0^N x^N\right)^{\frac{1}{\sigma-1}}}{\omega} I^S, \tag{59}$$

where L_0^N is exogenous and $\{\Psi, I^N, I^S\}$ are given by

$$\Psi = \underbrace{\frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}}_{\text{Northern leaders}} + \underbrace{\frac{\lambda\varphi^N}{\lambda\varphi^N + \phi}\frac{\varphi^F}{\lambda\varphi^N + \varphi^F}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\delta\omega)^{\sigma-1}}_{\text{Southern affiliates}} + \underbrace{\frac{\phi}{\lambda\varphi^N + \phi}\frac{\varphi^F}{\lambda\varphi^N + \varphi^F}(\delta\omega)^{\sigma-1}}_{\text{Southern competitive firms}},$$

$$I^N = \underbrace{\frac{(\rho - g_L)\left(1 + i^N\right)\gamma x^N}{1 - \beta^N}\left(\frac{\lambda\varphi^N}{\lambda\varphi^N + \varphi^F} + \frac{\lambda\varphi^N}{\lambda\varphi^N + \phi}\frac{\varphi^F}{\lambda\varphi^N + \varphi^F}\right)}_{\text{asset income}} + \underbrace{i^N\varphi^N\gamma x^N}_{\text{interest income}} + \underbrace{1}_{\text{wage income}} + \underbrace{\frac{\beta^N\varphi^N\left(1 + i^N\right)\gamma x^N}{1 - \beta^N}}_{\text{R\&D profit income}},$$

[32]Interest income $i^N b^N$ appears in the budget of the household because together with R&D labor income (captured by wage income w^N), it represents the factor income from R&D that is paid to the household.

$$I^S = \underbrace{\frac{(\rho - g_L)(1+i^S)\alpha x^N}{1-\beta^F}\left(\frac{\lambda\varphi^N}{\lambda\varphi^N+\phi}\frac{\varphi^F}{\lambda\varphi^N+\varphi^F}\right)\frac{1-s}{s}}_{\text{asset income}} + \underbrace{i^S\varphi^F\alpha x^N\frac{\lambda\varphi^N}{\lambda\varphi^N+\varphi^F}\frac{1-s}{s}}_{\text{interest income}}$$

$$+ \underbrace{1}_{\text{wage income}} + \underbrace{\frac{\beta^F\varphi^F\left(1+i^S\right)\alpha x^N}{1-\beta^F}\left(\frac{\lambda\varphi^N}{\lambda\varphi^N+\varphi^F}\frac{1-s}{s}\right)}_{\text{R\&D profit income}}.$$

Proof. See Appendix A. ■

The intuition of the above expressions can be explained as follows. Recall that real wages are given by $w_0^N = \left(\Psi L_0^N x^N\right)^{\frac{1}{\sigma-1}}$ and $w^S = \left(\Psi L_0^N x^N\right)^{\frac{1}{\sigma-1}}/\omega$; therefore, the term Ψ captures the quality contributions of Northern leaders, Southern affiliates, and Southern competitive firms to consumption through the real wage. As for the terms I^N and I^S, they represent the contributions of the different sources of income to consumption.

4 Inflation and the CIA constraints

This section explores the effects of inflation via the CIA constraints. Section 4.1 analyzes the effects of inflation in the two countries on the rates of innovation and international technology transfer. Section 4.2 calibrates the model to data for a quantitative analysis.

4.1 Qualitative analysis

In this section, we explore the effects of inflation. A higher inflation in the South (i.e., an increase in π^S and i^S) affects only the Southern steady-state condition in (56). Specifically, it shifts the South curve to the left in Figure 1. As a result, both φ^F and x^N decrease along with an increase in ω as implied by (54). Intuitively, a higher nominal interest rate i^S in the South raises the cost of adaptive R&D and reduces the rate of international technology transfer φ^F. The decrease in the number of products manufactured by Southern affiliates implies more products being produced by Northern firms. The higher demand for production labor causes a reallocation of labor in the North from R&D to production. The decrease in innovative R&D in the North decreases the rate of innovation in the short run and leads to a lower average quality per worker x^N in the long run. Finally, given that the increase in π^S and i^S has a direct negative effect on the demand for Southern R&D labor, it depresses the wage rate in the South relative to the North. We summarize these results in Proposition 1.

Proposition 1 *A higher inflation in the South leads to (a) a permanent decrease in the rate of technology transfer from the North to the South, (b) a permanent increase in the North-South wage gap, and (c) a temporary decrease in the rate of innovation in the North.*

Proof. See Appendix A. ■

A higher inflation in the North (i.e., an increase in π^N and i^N) affects both the Northern and Southern steady-state conditions in (55) and (56). Specifically, it shifts both the South curve and the North curve to the left in Figure 1. As a result, the effect on φ^F is ambiguous, and x^N decreases along with a decrease in ω as implied by (54). Intuitively, an increase in the nominal interest rate i^N in the North raises the cost of innovative R&D. As a result, the rate of innovation decreases in the short run, and the average quality per worker x^N decreases in the long run. Given that the increase in π^N and i^N has a direct negative effect on the demand for Northern R&D labor, it depresses the wage rate in the North relative to the South.

As for the effect of i^N on the rate of international technology transfer φ^F, there are two opposing effects. To see this, we use $\varphi_t^F(j) = \varphi_t^F$ and (44) to derive

$$\varphi_t^F = \frac{1}{\alpha Q_t^N} L_{r,t}^F = \frac{1}{\alpha x_t^N} \frac{L_{r,t}^F}{(1-s)L_t} \frac{Q_t}{Q_t^N}, \tag{60}$$

where the second equality uses $x_t^N = Q_t/L_t^N$ and $L_t^N = (1-s)L_t$. In the steady state, Q_t^N/Q_t is given by (34), and hence, (60) can be reexpressed as

$$\frac{\lambda\varphi^N\varphi^F}{\lambda\varphi^N + \varphi^F} = \frac{1}{\alpha x^N} \frac{L_{r,t}^F}{(1-s)L_t}, \tag{61}$$

where the left-hand side is monotonically increasing in φ^F. From (61), we see that the Northern nominal interest rate i^N affects φ^F via the quality level per worker x^N and the number of adaptive R&D workers $L_{r,t}^F$. On the one hand, an increase in i^N reduces x^N and has a positive effect on φ^F by decreasing the difficulty of adaptive R&D. On the other hand, the increase in i^N also reduces the incentives for adaptive R&D by changing the asset values. To see this, we combine (49) and (50) to derive

$$\frac{v_t^F(j)}{v_t^N(j)} = \frac{\rho+\varphi^N}{\rho+\varphi^N+\phi} \frac{\Pi_t^F(j)}{\Pi_t^N(j)} = \frac{\rho+\varphi^N}{\rho+\varphi^N+\phi} \left(\delta \frac{w_t^N}{w_t^S}\right)^{\sigma-1}, \tag{62}$$

where the second equality uses (17)-(18) and then (13)-(14). Recall that the increase in i^N reduces the relative wage $\omega = w_t^N/w_t^S$; therefore, it also reduces $v_t^F(j)/v_t^N(j)$. In other words, the decrease in the North-South wage gap makes adaptive R&D less attractive relative to innovative R&D. This leads to a decrease in adaptive R&D in the South, which in turn has a negative effect on the rate of international technology transfer φ^F. This negative effect of i^N via the number of adaptive R&D workers in the South is relatively strong when the Southern population size s is large. Therefore, the overall effect of i^N on φ^F would be negative if s is sufficiently large, and vice versa. We summarize these results in Proposition 2.

Proposition 2 *An increase in the nominal interest rate in the North leads to (a) a temporary decrease in the rate of innovation in the North, (b) a permanent decrease in the North-South wage gap, and (c) a permanent decrease (increase) in the rate of technology transfer to the South if Southern population size is sufficiently large (small).*

Proof. See Appendix A. ■

4.2 Quantitative analysis

In this section, we provide a quantitative analysis on the effects of inflation via the CIA constraints. Specifically, we explore their welfare implications. Therefore, the purpose of this section is to provide an illustrative numerical experiment to quantify the welfare effects of inflation via the CIA constraints. For the parameter values, we either set them to conventional values in the literature or calibrate them using empirical moments from aggregate data of China and the US. We consider China as the South and the US as the North.

In the above qualitative analysis, we obtain the pattern of production shifting back to the North upon the arrival of new innovations by imposing $z > \delta\omega$ using the parameter restriction in (P1). The condition $z > \delta\omega$ allows the model to deliver a realistic pattern of offshoring and reshoring between the US and China.[33] For the quality step size z, we consider a conventional value of 1.2. For ω, we consider recent data from the Federal Reserve Economic Data on relative income between the US and China, and this value is 5.94 between 2010 and 2013. Then, we choose a value of $\delta = 0.2$ such that the condition $z > \delta\omega$ holds.

For the discount rate ρ, we follow Acemoglu and Akcigit (2012) to set it to 0.05. For the imitation rate ϕ, we set it to a value of 0.03. In the model, it is α/γ (rather than the individual values of α and γ) that determines the values of variables in equilibrium.[34] We calibrate α/γ by matching the relative wage ω from the model to the data discussed above. For the substitution elasticity σ, we calibrate it by using the innovation arrival rate φ^N of 0.06. The calibrated value of σ is 3.16, which is within the range of empirical estimates in Broda and Weinstein (2006). For the remaining parameters, we calibrate them to data from 1995 to 2013. For the population growth rate g_L, we set it to the average growth rate of the number of R&D scientists and engineers in the US, and this value is 0.029.[35] For the relative Southern population size s, we set it to 0.82 based on data from the Penn World Table on the population size of China and the US. We calibrate the values of the R&D externality parameters $\{\beta^N, \beta^S\}$ by using the R&D shares of GDP in the US and in China. According to the OECD Research and Development Statistics, the average R&D share of GDP is 0.011 in China and 0.026 in the US. Finally, we calibrate i^S and i^N using average inflation rates in China and the US, and π^S is 3.06% and π^N is 2.41% according to the Federal Reserve Economic Data. Under these calibrated parameter values, the equilibrium values of $\{x^N, \varphi^F\}$ are respectively 0.63 and 0.03, and the equilibrium values of $\{r,\ g_c\}$ are respectively 0.063 and 0.013. We provide a summary of the calibrated parameter values in Table 1.

Table 1: Calibrated parameter values

ρ	z	ϕ	α/γ	σ	i^S	i^N	g_L	s	β^F	β^N	δ
0.050	1.200	0.030	4.280	3.163	0.094	0.088	0.029	0.818	0.354	0.874	0.200

[33] For example, in a survey, the Boston Consulting Group (2011) document that "[t]ransportation goods such as vehicles and auto parts, electrical equipment including household appliances, and furniture are among seven sectors that could create 2 to 3 million jobs as a result of manufacturing returning to the U.S."

[34] x^N is the only variable affected by γ, but the equilibrium value of γx^N is independent of γ. Given that it is the value of γx^N that matters, we simply normalize γ to one when reporting the value of x^N.

[35] There are different ways to calibrate g_L. It can be calibrated to the population growth rate, the labor-force growth rate and the growth rate of R&D labor. Given that innovation is the most important element of the model, we calibrate g_L to the growth rate of R&D labor.

Given these calibrated parameter values, we consider the following experiments: (a) decreasing inflation in the US to achieve a zero nominal interest rate (i.e., $i^N = 0$), and (b) decreasing inflation in China to achieve a zero nominal interest rate (i.e., $i^S = 0$). The results are reported in Table 2. We find that a permanent decrease in inflation in the US would raise the wage gap ω by 0.471% (percent change) and decrease international technology transfer φ^F by 1.257% (percent change). Here φ^F decreases despite an increase in adaptive R&D because of the increase in the quality index x^N, which makes technology transfer more difficult. The effect of x^N on φ^F dominates because s is not sufficiently large despite the rather large population in China. The decrease in i^N leads to a welfare gain of 3.281% in the US and a welfare gain of 3.312% in China.[36] From Section 3.5, we see that the percent change in c_0^N is equal to the percent change in w_0^N plus the percent change in I^N. Table 2 shows that when i^N decreases, w_0^N increases by 3.689% whereas c_0^N increases by 3.281%, implying that I^N decreases slightly by 0.408%. Therefore, the quantitatively significant welfare gain as a result of the decrease in i^N is mostly due to the large increase in wage, which in turn is due to the large increase in the level of technology x^N by 8.068% (percent change).

A permanent decrease in inflation in China would reduce the wage gap ω by 0.469% and increase technology transfer φ^F by 1.492%. Also, it leads to a welfare gain of 0.338% in China and a welfare gain of 0.174% in the US. In this case, the welfare gains in the two countries are relatively small because the increase in wage is small, which in turn is due to the small increase in the level of technology x^N by 0.361%. In other words, although decreasing inflation in either China or the US leads to an increase in innovation and the technology level, inflation in the US has much larger effects on innovation and global welfare.

Table 2: Simulation

i^N	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.088	5.942	0.626	0.0299	-	-	-	-
0	5.970	0.677	0.0295	3.689%	3.219%	3.281%	3.312%
i^S	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.094	5.942	0.626	0.0299	-	-	-	-
0	5.914	0.628	0.0303	0.078%	0.548%	0.174%	0.338%

4.3 Sensitivity analysis: increasing US R&D share of GDP

Starting from this subsection, we perform sensitivity analysis by considering a number of robustness checks. First, we examine data on R&D share of GDP in the US. As Comin (2004) argues, data on R&D expenditures reported by firms may not capture all the resources devoted to innovation-related activities in the US. Here we consider a rough exercise by doubling the R&D share of GDP in the US from 0.026 to 0.052. Table 3 reports the recalibrated parameter values, whereas Table 4 reports the new simulation results. In this case, the R&D externality parameter β^N decreases to 0.74, which in turn implies a slightly smaller welfare gain from decreasing inflation in either country. The welfare gains of decreasing inflation in

[36] Welfare changes are all expressed in the usual equivalent variation in consumption.

the US remain quantitatively significant at 2.90% in the US and 3.22% in China. Therefore, we still find that inflation can cause a quantitatively significant cross-country welfare effect from the US to China.

Table 3: Recalibrated parameter values

ρ	z	ϕ	α/γ	σ	i^S	i^N	g_L	s	β^F	β^N	δ
0.050	1.200	0.030	2.104	3.163	0.094	0.088	0.029	0.818	0.354	0.744	0.200

Table 4: Simulation from recalibrated parameter values

i^N	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.088	5.942	1.238	0.0311	-	-	-	-
0	5.970	1.334	0.0308	3.601%	3.132%	2.901%	3.224%
i^S	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.094	5.942	1.238	0.0311	-	-	-	-
0	5.914	1.242	0.0315	0.076%	0.546%	0.162%	0.335%

4.4 Sensitivity analysis: increasing the innovation-arrival rate

In our benchmark calibration, we have chosen a conservatively low value for the innovation arrival rate. In this subsection, we consider a larger innovation innovation-arrival rate φ^N of 0.10 and recalibrate the rest of the parameters. Here we use our benchmark R&D share of 0.026 in order to isolate the effect of φ^N. Table 5 reports the recalibrated parameter values, whereas Table 6 reports the new simulation results. In this case, the R&D externality parameter β^N increases to 0.93, which in turn implies an even larger welfare gain from decreasing inflation. Specifically, the welfare gains of decreasing inflation in the US become 5.69% in the US and 5.86% in China.

Table 5: Recalibrated parameter values

ρ	z	ϕ	α/γ	σ	i^S	i^N	g_L	s	β^F	β^N	δ
0.050	1.200	0.030	4.168	2.397	0.101	0.095	0.029	0.818	0.130	0.925	0.200

Table 6: Simulation from recalibrated parameter values

i^N	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.095	5.942	0.438	0.0526	-	-	-	-
0	5.964	0.477	0.0522	6.174%	5.803%	5.691%	5.858%
i^S	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.101	5.942	0.438	0.0526	-	-	-	-
0	5.921	0.439	0.0530	0.092%	0.456%	0.191%	0.260%

4.5 Sensitivity analysis: decreasing the innovation-arrival rate

From the previous analysis, we see that the magnitude of the welfare effects of inflation depends on the R&D externality parameter, which in turn is the most sensitive to the assumed value of the innovation-arrival rate. Therefore, we now consider an unusually low value of 0.02 for the innovation arrival rate φ^N, which implies a time between innovation arrivals of 50 years. Table 7 reports the recalibrated parameter values, whereas Table 8 reports the new simulation results. In this case, the R&D externality parameter β^N decreases to 0.50, which in turn implies a smaller welfare gain from decreasing inflation. Even in this case, the welfare gains of decreasing inflation in the US remain non-negligible at 1.20% in the US and 0.98% in China. Therefore, our finding remains robust that inflation can indeed cause a quantitatively significant welfare effect across countries.

Table 7: Recalibrated parameter values

ρ	z	ϕ	α/γ	σ	i^S	i^N	g_L	s	β^F	β^N	δ
0.050	1.200	0.030	4.385	5.915	0.087	0.080	0.029	0.818	0.413	0.497	0.200

Table 8: Simulation from recalibrated parameter values

i^N	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.080	5.942	1.556	0.0092	-	-	-	-
0	5.977	1.669	0.0089	1.486%	0.911%	1.204%	0.979%
i^S	ω	x^N	φ^F	$\Delta \ln w_0^N$	$\Delta \ln w_0^S$	$\Delta \ln c_0^N$	$\Delta \ln c_0^S$
0.087	5.942	1.556	0.0092	-	-	-	-
0	5.907	1.565	0.0095	0.044%	0.638%	0.083%	0.469%

5 Conclusion

In this study, we have analyzed the effects of inflation via CIA constraints on R&D in a Schumpeterian economy with North-South product cycles. We show that inflation affects innovation, technology transfer and the allocation of manufacturing activities across countries. Calibrating the model to China-US data, we find that the cross-country welfare effect of inflation is quantitatively significant from the US to China, but less so from China to the US. The reason is that innovation originates from the North in the model, which until recently is a reasonable approximation to reality as OECD countries perform the majority of global R&D. However, as China and other developing countries become more innovative, the effect of Southern inflation on global welfare is likely to become more significant.

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Appendix A

Proof of Lemma 1. As in Dinopoulos and Segerstrom (2010), the dynamics of the quality indices is given by

$$\begin{aligned}\dot{Q}_t^N &= \int_{\theta_t^N} \left[\lambda^{n_t(j)+1} - \lambda^{n_t(j)}\right] \varphi_t^N dj + \int_{\theta_t^F + \theta_t^S} \lambda^{n_t(j)+1} \varphi_t^N dj - \int_{\theta_t^N} \lambda^{n_t(j)} \varphi_t^F dj \\ &= (\lambda - 1)\varphi_t^N Q_t^N + \lambda \varphi_t^N \left(Q_t^F + Q_t^S\right) - \varphi_t^F Q_t^N,\end{aligned}$$

$$\dot{Q}_t^F = \int_{\theta_t^N} \lambda^{n_t(j)} \varphi_t^F dj - \int_{\theta_t^F} \lambda^{n_t(j)} \varphi_t^N dj - \int_{\theta_t^F} \lambda^{n_t(j)} \phi dj = \varphi_t^F Q_t^N - \varphi_t^N Q_t^F - \phi Q_t^F,$$

$$\dot{Q}_t^S = \int_{\theta_t^F} \lambda^{n_t(j)} \phi dj - \int_{\theta_t^S} \lambda^{n_t(j)} \varphi_t^N dj = \phi Q_t^F - \varphi_t^N Q_t^S.$$

Let's define $Q_t^{FS} \equiv Q_t^F + Q_t^S$, which implies $\dot{Q}_t^{FS} = \varphi_t^F Q_t^N - \varphi_t^N Q_t^{FS}$. Setting $\dot{Q}_t^N / Q_t^N = \dot{Q}_t^{FS} / Q_t^{FS}$ yields (34), using $Q_t^{FS} = Q_t - Q_t^N$. Setting $\dot{Q}_t^F / Q_t^F = \dot{Q}_t^S / Q_t^S$ yields $Q_t^S / Q_t = \left(Q_t^F / Q_t\right) \left[\phi / \left(\lambda \varphi_t^N\right)\right]$, noting $Q_t^{FS} / Q_t^N = \left(1 - Q_t^N / Q_t\right) / (Q_t^N / Q_t) = \varphi_t^F / \left(\lambda \varphi_t^N\right)$. Applying this to $Q_t^F / Q_t + Q_t^S / Q_t = 1 - Q_t^N / Q_t$ and using (34), equations (35) and (36) follow. ■

Proof of Lemma 2. Time arguments are omitted for convenience. Using $\tau^N = (\dot{M}^N + g_L M^N) / P^N$ and $\dot{M}^N / M^N = \pi^N + \dot{m}^N / m^N$, we derive $\tau^N = \left(\pi^N + g_L\right) m^N + \dot{m}^N$. Substituting this condition into the balanced-growth version of (2) yields

$$c^N = (\rho - g_L)\, a^N + i^N w^N \varphi^N \gamma x^N + w^N + \frac{\beta^N w^N \varphi^N \left(1 + i^N\right) \gamma x^N}{1 - \beta^N}, \tag{A1}$$

where we have used $r^N = \rho + g_L / (\sigma - 1)$, $\dot{a}^N / a^N = g_L / (\sigma - 1)$, $m^N = b^N = \int_0^1 w^N L_r^N(j) dj / L^N = w^N \varphi^N \gamma x^N$, and $d_t^N = \int_0^1 \beta^N \varphi^N v^N(j)\, dj / L^N = \beta^N w^N \varphi^N \left(1 + i^N\right) \gamma x^N / \left(1 - \beta^N\right)$. Following Dinopoulos and Segerstrom (2010), we assume that the Northern household finances innovative R&D in equilibrium. That is, $L^N a^N = \int_{\theta^N + \theta^F} v^N(j) dj$. Given that $v^N(j) = \left(1 + i^N\right) w^N L_r^N(j) / \left[\left(1 - \beta^N\right) \varphi^N\right]$ from (20), we have

$$a^N = \frac{\left(1 + i^N\right) \gamma x^N w^N}{1 - \beta^N} \left(\frac{\lambda \varphi^N}{\lambda \varphi^N + \varphi^F} + \frac{\lambda \varphi^N}{\lambda \varphi^N + \phi} \frac{\varphi^F}{\lambda \varphi^N + \varphi^F}\right), \tag{A2}$$

which uses (19), (39) and Lemma 1. Using (A1) and (A2), we can show that $c^N = w^N I^N$, where I^N is defined in Lemma 2. By incorporating (8), (10) and (12) into the aggregate price index $\left\{\int \left[p_t(j)\right]^{1-\sigma} dj\right\}^{1/(1-\sigma)} = 1$, we can show that the real wage in the North is $w^N = \left(\Psi L^N x^N\right)^{\frac{1}{\sigma - 1}}$, which uses Lemma 1 and $x^N = Q / L^N$. Ψ is defined in Lemma 2, and we have derived (58). Applying analogous derivations to the Southern asset condition, one can also derive (59) by noting that $m^S = b^S = \int_{\theta^N} w^S L_r^F(j) dj / L^S$, $d_t^S = \int_{\theta^N} \beta^F \varphi^F [v^F(j) - v^N(j)] dj / L^S$, and $L^S a^S = \int_{\theta^F} \left[v^F(j) - v^N(j)\right] dj$, which comes from the assumption that the Southern household finances adaptive R&D and that $v^F(j) - v^N(j) = \left(1 + i^S\right) w^S L_r^F(j) / \left[\left(1 - \beta^F\right) \varphi^F\right]$ from (22). ■

Proof of Proposition 1. It is easy to graphically show from (54) that ω increases with i^S, proving (b). Given this, an increase in i^S leads to a downward shift in the South curve (56), whereas it has no effect on the North curve (55). Applying a simple graphical analysis to Figure 1, we find that an increase in i^S leads to permanent decreases in φ^F and x^N. This proves (a) and also (c) because a permanent decrease in x^N must be associated with a temporary decrease in the innovation rate φ_t^N below its steady-state level $\varphi^N = g_L/(\lambda - 1)$ given the dynamics in (47). ■

Proof of Proposition 2. Graphical analysis with (54) implies that ω decreases with i^N, proving (b). An increase in i^N leads to a downward shift in both the North and South curves, (55) and (56), given that we can easily show from (54) that $\left(1+i^N\right)\omega$ increases with i^N. Thus, an increase in i^N leads to a decrease in x^N, implying a temporary decrease in the innovation rate φ_t^N given the dynamics in (47) and proving (a). As for (c), we solve (55) and (56) for φ^F to obtain[37]

$$\varphi^F = \lambda\varphi^N \frac{s}{1-s} \frac{\frac{1+i^N}{1-\beta^N} + \frac{1}{\sigma-1}\frac{\varphi^N}{\rho+\varphi^N}}{\frac{(1+i^N)\delta^{\sigma-1}\omega^\sigma\Phi(\phi)}{1-\beta^N} - \frac{1}{\sigma-1}\frac{\varphi^N}{\rho+\varphi^N}\left(\frac{s}{1-s} - \frac{\alpha\lambda}{\gamma}\right)}. \tag{A3}$$

Differentiating (A3) with respect to i^N, we find that $d\varphi^F/di^N > (<)\ 0$ holds if the following inequality holds:[38]

$$\delta^{\sigma-1}\omega^\sigma\Phi(\phi)\left(\frac{\sigma\frac{(1+i^S)\alpha}{(1-\beta^F)\gamma} - \frac{\varphi^N}{\rho+\varphi^N}\omega}{\frac{\sigma}{\sigma-1}\left(\frac{1-\beta^N}{1-\beta^F}\right)\frac{(1+i^S)\alpha}{(1+i^N)\gamma} + \omega}\right) > (<) \frac{\varphi^N}{\rho+\varphi^N}\left(\frac{s}{1-s} - \frac{\alpha\lambda}{\gamma}\right). \tag{A4}$$

Given that the right-hand side of (A4) is monotonically increasing in s, $d\varphi^F/di^N > (<)\ 0$ becomes more likely to hold as s decreases (increases). Given that s has an upper bound $\tilde{s}$,[39] which ensures $\varphi^F > 0$, we can show that the inequality $<$ in (A4) must hold as $s \to \tilde{s}$ implying that $d\varphi^F/di^N < 0$ for a sufficiently large s. As $s \to 0$, the right-hand side of (A4) becomes negative. Therefore, $d\varphi^F/di^N > 0$ holds if the left-hand side of (A4) is positive, which is guaranteed by $\sigma\alpha\delta/\gamma > z\varphi^N/(\rho+\varphi^N)$ given that $z > \delta\omega$. ■

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[37] Here we have used the following condition derived from (54):

$$\delta\left[\frac{\left(1+i^S\right)\alpha}{1-\beta^F} + \frac{\left(1+i^N\right)\gamma\omega}{1-\beta^N}\right] = \frac{\rho+\varphi^N}{\rho+\varphi^N+\phi}\frac{\left(1+i^N\right)\gamma\left(\delta\omega\right)^\sigma}{1-\beta^N}.$$

[38] Here have used the following condition derived from (54):

$$\frac{d\left(1+i^N\right)\omega^\sigma}{di^N} = \omega^\sigma\left(\frac{\frac{\sigma-1}{\sigma}\omega}{\left(\frac{1-\beta^N}{1-\beta^F}\right)\frac{(1+i^S)\alpha}{(1+i^N)\gamma} + \frac{\sigma-1}{\sigma}\omega}\right).$$

[39] This is defined by

$$\frac{\left(1+i^N\right)\delta^{\sigma-1}\omega^\sigma\Phi(\phi)}{1-\beta^N} = \frac{1}{\sigma-1}\frac{\varphi^N}{\rho+\varphi^N}\left(\frac{\tilde{s}}{1-\tilde{s}} - \frac{\alpha\lambda}{\gamma}\right).$$