The formation of partnerships in social networks [*]

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Abstract

This paper analyzes the formation of partnerships in social networks. Agents randomly request favors and turn to their neighbors to form a partnership. If favors are costly, agents have an incentive to delay the formation of the partnership. In that case, for any initial social network, the unique Markov Perfect equilibrium results in the formation of the maximum number of partnerships when players become infinitely patient. If favors provide benefits, agents rush to form partnerships at the cost of disconnecting other agents and the only perfect initial networks for which the maximum number of partnerships are formed are the complete and complete bipartite networks. The theoretical model is tested in the lab. Subjects generally play according to their equilibrium strategy and the efficient outcome is obtained over 78% of the times. Decisions are affected by the complexity of the network. Two behavioral rules are observed during the experiment: subjects accept the formation of the partnership too often and reject partnership offers when one of their neighbors is only connected to them.

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1   Introduction

1.1   The formation of partnerships in social networks

The idea that *power* and *influence* of an individual in a network depend upon his or her *position* in the network is well-established in a variety of contexts cutting across disciplines. For instance, social network analysis suggests that the power of individuals cannot be explained by the individual's characteristics alone, but must be combined with the structure of his or her relationship with others -power arises from occupying advantageous positions in networks of relationships. In particular, network exchange theories focus on *exchange rates* which essentially represent the relative "bargaining power" of one individual in his or her bilateral exchange with neighbors in the network.[1]

We study the impact of network structures on the pattern of bilateral exchanges both theoretically as well as by means of laboratory experiments. Our setting is one in which individuals form partnerships to exchange *favors* with one another. Favors could be small – advice on a particular issue, a small loan, help on a school project or with baby-sitting, or large – sharing one's life with another person, or forming a professional partnership with other workers . The need for such favors arises randomly for any individual at any point of time. If an individual i needs a favor at any point, he turns to one of his neighbors j in the network to request the favor. The recipient of such a request can either grant the favor at some cost c (which is strictly less than the value of the favor v) or refuse to grant the favor. In the latter case, the link between i and j is broken. Individual i can then approach another neighbor to grant him the favor. If the favor is granted, the two players enter a reciprocal agreement to grant each other the favor and leave the network. The process is repeated in the next period when some individual chosen at random needs a favor.

Since there is a cost involved in granting the favor, an individual grants a favor only because she knows that she might need a favor tomorrow. In particular, our model is purely individualistic in the sense that we ignore completely the existence of any social norm to enforce cooperative behavior. This distinguishes our model from that of Jackson, Rodriguez-Barraquer and Tan (2012), whose work we discuss later. The absence of societal punishments implies that typical repeated-game theoretic considerations do not apply in our model. So, any individual i who is approached with a request for a favor by individual j will only grant the favor if he does not want to break the link with j. Since the cost of granting a favor is incurred in the current period, i will grant the favor if the expected discounted stream of future payoffs from maintaining the link with j exceeds c.

This is where the structure of the network comes into play. We illustrate this informally by means of a couple of examples. Suppose, for instance, that the network is a *line* on three individuals, with 1 and 3 being the extreme nodes while 2 is in the middle. Suppose that one of the extreme nodes, say 1, requires a favor in some period t, and requests his neighbor

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[1]For an overview of recent developments in network exchange theories, see Willer (1999).

2 to grant him the favor. Clearly, the only rational response is for 2 to reject the request since he does not lose anything by doing so, and saves c in the process. This is because 2 is *ensured* from any possibility of all his links being broken - once the link with 1 is broken, 2 and 3 will form a partnership which will never break up. On the other hand, suppose it is 2 who needs a favor and requests 1 to give him the favor. Notice that if 1 is sufficiently patient, then he should not refuse the request. For suppose he agrees to grant the favor to 2. In the next period, with a probability of a third, it is 3 who will need the favor and 2 will refuse leaving 1 and 2 to exchange favors indefinitely in the future. Since $v > c$, the current cost of granting the favor must be less than the future stream of discounted payoffs if i is not too impatient.

Consider another example where the initial network is a 4-person line, with 1 and 4 being the extreme nodes. Then, *two* matchings are possible. But, this requires that 2 and 3 do not form a partnership with each other since that would leave 1 and 4 unmatched. We argue that this pair will not form in equilibrium. Suppose that 2 approaches 3 with a request. Then 3 will refuse the request since she is sure that 4 will accept to grant him a favor in the future. (We will later refer to players 1 and 4 as 'captive' agents of players 2 and 3 since they have no other option and will always accept the formation of the partnership.)

In general, it is not difficult to see that not all requests will be granted and so the network will grow sparser over time. At any point in time, either a player will see all his requests rejected, and become isolated or a pair of players will agree to form a lasting partnership. The main focus of our paper is to examine whether the pattern of these bilateral links is *efficient* in the sense of forming the maximum number of matched pairs no matter what the initial network.[2] While this intuition is simple enough in the simple examples given above, it is far from obvious that some "wrong" pair of agents will not form a partnership, thus making the overall pattern of matches inefficient. However, we are able to show that in *all* networks with a finite number of nodes, the efficient pattern of bilateral links will indeed be established.

The intuition for this result derives from a generalization of the strategy of the players in the 4-person line, where we saw that agents 2 and 3 will never choose to form a partnership which disconnects the two other agents in the network. We establish that the unique optimal strategy of a player i is to accept the request of a player j if and only if, once the link ij is broken, player i does not belong to all maximum matchings of the graph $g \setminus ij$. This characterization of the optimal strategy (which is obtained by induction on the size of the graph) relies on the identification of all maximum matchings of a graph, and of the players who belong to all maximum matchings.[3] It immediately implies that a maximum matching

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[2]In our model, this is the only possible definition of efficiency since all pairs of agents generate the same surplus. As players become perfectly patient, maximizing the total number of pairs is equivalent to maximizing the sum of utilities of all agents.

[3]We call these nodes "essential" nodes of the graph. These nodes, termed "always efficient" nodes also play a role in the characterization of the equilibrium strategies in the model of non-stationary bargaining in networks of Abreu and Manea (2012). They also appear in the Edmunds Gallai decomposition of bipartite

is obtained in equilibrium, as players will never break a link which reduces the number of matches in the graph.

In this baseline model, the maximum number of pairs is formed because agents delay the formation of the partnership until the social network is such that there is a risk that they will not be able to find a partner in the long run. We contrast this model with another model where both agents immediately benefit from the formation of the partnership so that players rush to form partnerships. As opposed to the case of costly favors, the maximum matching in this alternative model with positive favors is not necessarily formed in equilibrium. In fact, the efficient number of pairs will be formed if and only if the initial network is *completely elementary*, in the sense that the number of matchings does not decrease by more than one whenever a pair leaves the graph.[4] We show that there are only two perfect completely elementary graphs: the complete network and the complete bipartite network with half of the players on each side of the network.

In the second part of the paper, we test whether players form efficient partnerships in social networks running a series of laboratory experiments. The experimental design mimics the game of partnership formation, but in a finite setting where, instead of receiving an expected discounted value, subjects obtain a fixed finite value when they form the partnership.[5] We consider five different settings with initial social networks of increasing complexity. We observe that a large fraction of the subjects (more than 75%) do indeed select the equilibrium action, and that the subjects' ability to compute and select the subgame perfect equilibrium action depends on the complexity of the network. We also note that, even when subjects do not perfectly employ their subject equilibrium strategy, the proportion of rounds for which the efficient maximum matching is obtained is very high – around 78% of all rounds.

We analyze the systematic departures from equilibrium behavior and discover that subjects err by accepting too often. In addition, we see that subjects who are more risk averse (as measured by a classical questionnaire on risk aversion) accept more often, in the fear of being left isolated at the end of the game. One instance where we observe that agents correctly reject the requests is when they have access to 'captive' agents who are only linked to them. We show that this 'captive agents' heuristic works very well and that players with captive agents are much more likely to play their subgame perfect equilibrium strategy. Finally, we note that the complexity of the network – and in particular the presence of cycles – greatly complicates the computation of the equilibrium behavior and results in subjects making more mistakes.

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graphs which was used by Corominas-Bosch (2004) and Polansky (2007) to characterize equilibrium strategies in a model where nodes on the same side of the market make simultaneous offers.

[4]The terminology of "elementary" networks is due to Lovasz and Plummer (1986).

[5]The value is computed so that the equilibrium behavior in the finite game is equal to the equilibrium behavior in the infinite game studied in the theoretical section of the paper.

1.2 Relation to Literature

Our model of partnership formation in social networks is related to two different strands of the literature. First, it has close connections to models of bargaining in networks, in particular models of bargaining in non-stationary networks where agents who leave the network are not replaced.[6] Corominas-Bosch (2004) and Polansky (2007) proposed the first models of bargaining in non-stationary buyer-seller networks, but under the assumption that all players on the same side of the market make simultaneous offers. Manea (2011), and Abreu and Manea (2012a) and (2012b) consider bargaining models where a pair of players is chosen at random to make offers. While Manea (2011) analyzes the stationary situation, where players are replaced after an offer is accepted, Abreu and Manea (2012a) and (2012b) analyze the situation where the network is non-stationary. Abreu and Manea (2012a) is indeed very closely connected to our model, and some insights are common in the two papers. They study an infinite horizon bargaining game in which pairs of players connected in an exogenous network are randomly matched to bargain over a unit of surplus. If the matched pair reach agreement, then they leave the network, and so the network becomes sparser over time just as in our model. Moreover, their focus is similar to ours in the sense that they too are interested in whether the maximum number of matchings will be attained in equilibrium. Of course, this will be possible only if the "right" pairs of agents reach agreement. Abreu and Manea (2012a) point out that efficient matching cannot be attained in general in Markov equilibria, even though Markov equilibria are shown to always exist in Abreu and Manea (2012b). However, they construct an ingenious system of punishments and rewards which ensure the existence of an efficient subgame perfect non-Markovian equilibrium. The context that we are modeling is very different from theirs, leading to very different formal models and proof techniques even though some of the structural properties of nodes (i.e. the "essentiality" of nodes which belong to all maximum matchings) appear to play an important role in the proofs of both papers. The main differences are that we suppose that players do not bargain over a continuous surplus, use a different definition of the value of a partnership, assume that links are broken after a rejection and that the same player can turn to all his neighbors in sequence to form a partnership. These differences imply different conclusions. We are able to show that there is a unique Markov equilibrium which is efficient. On the other hand, there will in general be multiple equilibria in Abreu and Manea (2012), but one non-Markovian equilibrium will be efficient.

Second, because we model the value of a partnership through reciprocal exchange of favors, our model is related to the literature on favor exchange.[7] Gentzkow and Möbius (2003), Bramoullé and Kranton (2007), Bloch, Genicot and Ray (2008), Karlan, Möbius, Rozenblat and Szeidl (2009) and Ambrus, Möbius and Szeidl (2014) are all papers which share the same basic structure as our model, where agents request favors or transfers at

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[6]See Manea (2016) for a recent survey.

[7]This literature is surveyed in Möbius and Rozenblat (2016).

different points in time, and favors or transfers are enforced through reciprocation in the future. The paper in the literature on favor exchange which is closest to our model is the paper by Jackson, Rodriguez-Barraquer and Tan (2012). In this paper, pairs of agents are matched randomly in any period, with one of the agents requiring a favor from the other. Contrary to our model, favors are link-specific and the agent can only obtain a favor form one of his neighbors. Pairs meet too infrequently to sustain bilateral exchange, However, the favor exchange network may be sustained through social pressures or punishments leading to possible loss of neighbors in the network. Despite the similarity in the settings, the primary focus of their model is very different from ours. In particular, very different forces sustain the socially efficient network in the two settings - social pressures in their case and individual incentives in our case.

There is a growing literature on experiments in networks which is related to the experimental part of this paper.[8] To the best of our knowledge, our paper is the first to propose an experimental test of the model of partnership formation in non-stationary networks. The most closely related paper is the paper by Charness, Corominas-Bosch and Frechette (2007) who test the Corominas-Bosch bargaining model and observe that, as in our experimental study, the proportion of efficient trade is very high and players' behavior seems to conform to the equilibrium behavior predicted by the theory.

2 The Model

2.1 Partnerships

We consider a society of n agents who are organized in a social network g. The social network evolves over time, as agents will delete links and leave the network. At any discrete time $t = 1, 2, ..$, one agent is chosen with probability $\frac{1}{n}$ to request a favor from a neighbor. If the favor is granted, the agent who receives the favor obtains a flow payoff of v and the agent who grants the favor pays a flow cost c. All agents discount the future using the same discount factor δ. We define the *value* of a *partnership* as the expected discounted payoff obtained by an agent when he has found a partner with whom he *reciprocates* favors,

$$V = \frac{v - c}{n(1 - \delta)}.$$

Partnerships are formed according to the following decentralized procedure. Suppose that an agent i needs a favor at date t. Two situations may arise:

- Either agent i is already in a partnership

- Or agent i is not yet in a partnership

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[8]See Choi, Gallo and Kariv (2016) for an up-to-date survey of this literature

In the former case, the favor is offered by agent i's partner. In the latter case, agent i turns to his direct neighbors in the current social network g_t and asks them sequentially for a favor. The agent needing the favor *chooses* the sequence in which to approach his neighbors for the favor. If neighbor j is approached by agent i, he responds by Yes or No to the offer. If agent j rejects the request from i, the link ij is destroyed, the new social network is $g_t \setminus ij$, and agent i turns to the next neighbor in his chosen sequence. If all agents reject i's request, the network at next period is

$$g_{t+1} = g_t \setminus i,$$

the network obtained from g by deleting i and all his links.

If agent j responds Yes, the partnership $\{ij\}$ is formed, and the two partners leave the social network, deleting all their links. Thus, the partnership forms as soon as a favor is granted. We let

$$g_{t+1} = g_t \setminus i, j,$$

denote the network obtained after agents i and j have left.

In general, a *strategy* for player i who needs a favor is to choose the sequence in which to approach his neighbors for the favor, given the history of the game, while a strategy for j who has been asked by i to grant a favor is to decide whether to grant the favor (Y) or not(N), again as a function of the history of the game. However, in what follows, we focus attention on *Markov* strategies which only depend on the current social network. More formally, let $S_i(g)$ be the set of all possible sequences over the neighbors of i in the network g. Then, if i needs a favor at time t and current network g_t, his strategy is a mapping from g_t to $S_i(g_t)$. If j is asked to grant a favor by i at network g_t, his strategy is a mapping from (g_t, i) to $\{Y, N\}$.

A Markov equilibrium is a collection of Markov strategies such that all agents choose their best responses to the strategies of the others.

An outcome of the partnership formation process is a list of partnerships formed or agents leaving the network at every time period as a function of the realization of "needs"– the list of agents who request a favor at each period. Given a fixed realization of needs, an outcome is *efficient* if it maximizes the sum of payments of all agents. Given that agents are homogeneous, when the discount factor goes to one, the sum of payments of all agents is maximized when the number of partnerships is maximized.

Definition 2.1 *A social network g supports efficient equilibria* if and only if, for all realizations of needs, the equilibrium outcomes of the process of partnership formation starting from g are efficient.

2.2 Matchings and bipartite graphs

In this subsection, we collect definitions in graph theory pertaining to matchings and bipartite graphs which will prove useful in our analysis.[9] Given a network g, a *matching M* is a collection of edges in g such that no pair of edges in M has a common vertex. A matching M is maximal if there is no matching $M' \supset M$ in g. A matching M is a *maximum matching* if there is no matching M' in g such that $|M'| > |M|$. For any graph g, we let $\mu(g)$ denote the *matching number* of graph g, i.e. the size of any maximum matching in g. If n is even, and $\mu(g) = \frac{n}{2}$, there exists a matching covering all the vertices in g. This is called a *perfect matching* and any graph admitting a perfect matching is a *perfect graph.*

A graph g is *bipartite* if the set of vertices can be partitioned into two subsets A and B such that there is no edge among any two vertices in A and no edge among any two vertices in B. A bipartite graph is complete if all vertices in A are connected to all vertices in B. If $|A| = |B|$, a partite graph is perfect if and only if it satisfies Hall's condition: for any subset $C \subseteq A$, the set of vertices in B which are connected to vertices in C, $f(C)$ satisfies $|f(C)| \geq |C|$.

3 Partnerships with costly favors

In this Section, we analyze the equilibrium of the process of partnership formation. We first illustrate the equilibrium in a simple four-player line. We then introduce the concept of essential players in the network, and prove a Lemma on the effect of link deletion on essentiality. Using this definition, we characterize the optimal behavior of agents in the partnership formation game. We then prove the main Theorem of this section, establishing that all social networks support efficient equilibria for δ sufficiently close to 1. Finally, we discuss equilibrium behavior when players are less patient in the complete network and in the line.

3.1 Equilibrium in a four player line L_4

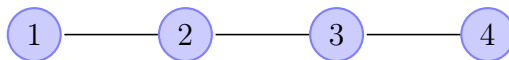Let $n = 4$ and suppose that $g = L_4$, the four-player line as illustrated in Figure 1.

Figure 1: The line L_4

The matching number of the line L_4 is two: the maximum number of partnerships formed is two. We claim that, when δ is sufficiently close to 1, in equilibrium, the maximum number

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[9]See Lovasz and Plummer (1986) for an excellent monograph on matchings and bipartite graphs.

of matchings is achieved in equilibrium. Suppose that player 1 has a need and approaches player 2. If player 2 rejects the offer, he becomes a peripheral agent in the line L_3. In L_3, if a peripheral agent requests a favor from the central agent, the central agent will decline the request, as he can form a partnership in the line L_2 and economize on the cost of giving the favor. Hence, with a positive probability, agent 2 ends up being disconnected if he rejects the request of player 1. For δ sufficiently close to 1, the cost of being disconnected exceeds the economy in the cost c, so agent 2 always accepts agent 1's request. Suppose that agent 2 has a request. If he meets agent 3, agent 3 declines the offer to form the partnership, as he can form a partnership later with agent 4 and economize on the cost c. If agent 2 meets agent 1, agent 1 always accepts the formation of the partnership.

In the line L_4, we can thus characterize the optimal response of agents for δ sufficiently close to 1 as follows:

- Agent 1 (4) accepts to form a partnership with agent 2 (3)

- Agent 2 (3) accepts to form a partnership with agent 1 (4)

- Agent 2 (3) declines to form a partnership with agent 3 (2)

Given this equilibrium behavior, the two partnerships 12 and 34 are always formed in equilibrium: the line L_4 supports efficient equilibria. We will now show that the construction of equilibrium can be extended to any graph g, and introduce the concept of *essential nodes* to characterize equilibrium behavior.

3.2 Essential nodes

A node i in graph g is called *essential* if it belongs to *all* maximum matchings of the graph g. It is called *inessential* otherwise. Clearly, all nodes are essential in a perfect graph. As illustrated in Figure 2, all nodes are inessential in the odd cycle C_3, and in the line L_5, nodes 2 and 4 are essential, but not nodes 1, 3 and 5. In the line L_5, node 3 is the most central node according to all measures of node centrality, but is inessential. This example shows that there is no connection between centrality and essentiality of nodes in a graph.

The line L_4

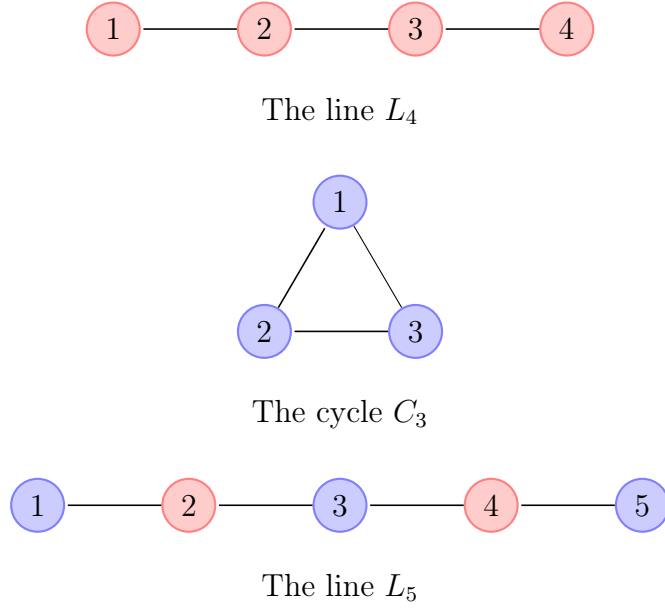The cycle C_3

The line L_5

Figure 2: Essential and inessential nodes

The next Lemma establishes properties on essential nodes which will prove useful in the characterization of equilibrium.

Lemma 3.1   *1. If i is an essential node in g, there exists $ij \in g$, such that j is inessential in $g \setminus i$.*

2. *If i is not an essential node in g and $ij \in g$, j is an essential node in $g \setminus i$.*

3. *If i is a essential node in g and j is inessential in g, i is essential in $g \setminus j$.*

4. *If i is an essential node in g , $jk \in g$, and $\mu(g) = \mu(g \setminus j, k) + 1$, then i is essential in $g \setminus j, k$.*

Proof:

1. Let M be a maximum matching in g and $E_1 = (ij)$ the edge covering i in M. Then $(E_2, ...E_M)$ is a maximum matching in $g \setminus i$ which does not contain j. Hence, j is not essential in $g \setminus i$.

2. Suppose by contradiction that j is not essential in $g \setminus i$. Then there exists a maximum matching of $g \setminus i$ with no edge covering j, $M = (E_1, ...E_M)$. Consider then the matching (M, ij) in g. This is a matching of cardinality $\mu(g \setminus i) + 1$, contradicting the fact that i is not essential in g.

3. Suppose by contradiction that there exists a maximum M matching of $g \setminus j$ where i is not covered. Because j is inessential in g, $\mu(g) = \mu(g \setminus j)$. So M has the same cardinality as a maximum matching in g and hence is a maximum matching of g, contradicting the fact that i is essential in g.

4. Suppose that i is inessential in $g \setminus j, k$. Then there exists a maximum matching M in $g \setminus j, k$ not covering i. As $\mu(g) = \mu(g \setminus j, k) + 1$, $M' = (M, jk)$ is a maximum matching of g not covering i, contradicting the fact that i is essential in g.

Lemma 3.1 shows that any essential node i must be connected to some node which is inessential in $g \setminus i$. On the other hand, all neighbors of an inessential node i are essential in $g \setminus i$. When an inessential agent is removed from the network, all essential agents remain essential. When a pair of agents leaves the network, without disrupting the total number of matchings, all essential agents remain essential as well.

3.3   Equilibrium behavior

With the help of Lemma 3.1, we now characterize the optimal response of agents in the game of partnership formation.

Proposition 3.2 *Let σ be a Markov equilibrium, and suppose j receives a request from i in the social network g. Then, j accepts the request iff j is inessential in $g \setminus i$ or $g \setminus i, k$ where k is the first agent if any in i's chosen sequence $s_i(g)$ to accept i's request if j refuses the request.*[10]

Proof: The proof is by induction on the number of agents in a connected component. For $n = 2$, both agents are essential and the statement is trivially satisfied. For $n = 3$, we distinguish between two cases;

- $g = L_3$

- $g = C_3$.

At $g = L_3$, the central agent rejects the request of the first peripheral agent to require a favor as he remains connected in the line L_2. On the other hand, the two peripheral agents become isolated if they reject the offer of the central agent and accept the request of the central agent. If $g = C_3$, the request of the first agent is always rejected as the other two agents remain connected and essential if that agent is removed from the network.

Suppose now that the statements are true for all $n' < n$ and consider a component with n agents.

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[10]The sequence $s_i(g)$ is of course part of the equilibrium strategy profile σ.

Let g' be the component formed if j rejects i's request. So, $g' = g \backslash i$, or $g' = g \backslash i, k$ depending on whether there is some neighbor k of i who accepts i's request following a refusal by j. In either case, $|g'| < n$, and we use the induction hypothesis to compute the continuation payoff of agent j if he rejects i's request.

Suppose first that j is inessential in g'. If j is chosen next period to have a request, by Lemma 3.1, (statement 2), all neighbors of j remain essential in $g' \backslash j$. By the induction hypothesis, the last agent in the sequence chosen by agent j, $s_j(g')$ must reject j's request. By backward induction, the agent preceding that agent in the sequence also rejects j's request, and all agents contacted in the sequence $s_j(g')$ also reject the request. Hence j obtains a continuation payoff of 0 with positive probability. For δ sufficiently close to 1, agent j thus has an incentive to accept i's request.

Suppose next that j is essential in g'. We claim that j's request will always be fulfilled. First suppose that j has a request next period, at $t + 1$, when the social network is g' and j is essential in g'. By Lemma 3.1 (statement 1), one of his neighbors, say k, becomes inessential in $g' \backslash j$. Let j choose a sequence $s_j(g')$ finishing with agent k. If no other agent in the sequence has accepted j's request, agent k will, by the induction hypothesis, because agent k is inessential in $g' \backslash j$.

Next suppose that j's first requests happens at some period $t' > t + 1$, when the social network is $g_{t'}$. If j is essential in $g_{t'}$, then the previous argument establishes that some neighbor of j in $g_{t'}$ must accept j's request.

So, suppose instead that j is inessential in $g_{t'}$. Let g'' be the first network in the sequence of networks between g' and $g_{t'}$ where j becomes inessential. Abusing notation, denote by g' the social network immediately preceding g'' along the equilibrium path, such that j is essential in g' but not in g'' where either (i) $g'' = g' \backslash k$ or (ii) $g'' = g' \backslash k, l$ for some k, l.

Suppose that (i) holds, and that k is essential in g'. Then, from statement (1) of Lemma 3.1 there is some l such that $k, l \in g'$ and l is inessential in g''. By the induction argument, agent l must accept k, a contradiction to k being isolated from g'. On the other hand, if k is inessential in g', from statement (3) of Lemma 3.1, j must be essential in g'', which is again a contradiction.

Suppose (ii) holds, so that $g'' = g' \backslash k, l$ for some $k, l \in g'$. It cannot be that $\mu(g'') = \mu(g')$ because we can add kl to a maximum matching in g'' and obtain a matching in g' of size $\mu(g'') + 1$. We use the following claim to show that $\mu(g'') + 1 = \mu(g')$.

Claim 3.3 *If along the equilibrium path, at some period t, a pair (k, l) of agents forms a partnership, then $\mu(g_t \backslash k, l) = \mu(g_t) - 1$.*

Proof of the Claim: Suppose that agent k places the request and agent l responds. Agent k must be essential in g_t. If agent k were inessential, all his neighbors would reject his claim. By the inductive step, player l must be inessential in the graph g' formed after his

rejection. Suppose first that all agents following l reject k's request in equilibrium so that $g' = g_t \setminus k$. As k is essential in g_t, $\mu(g_t \setminus k) = \mu(g_t) - 1$. Let M be a maximum matching of $g_t \setminus k$ not containing l. Then M is a maximum matching of $g \setminus k, l$ and $|M| = \mu(g_t) - 1$. Next suppose that there exist a sequence of agents following l who accept k's request and let $l_1, .., l_N$ denote the agents in the sequence. By the preceding argument, for the last agent in the sequence, $\mu(g_t \setminus k, l_N) = \mu(g_t) - 1$. We argue that whenever $\mu(g_t \setminus k, l_n) = \mu(g_t) - 1$, then $\mu(g_t \setminus k, l_{n-1}) = \mu(g_t) - 1$. If agent l_{n-1} accepts k's request, by the inductive step, he must be inessential in $\mu(g_t \setminus k, l_n)$. Pick a maximum matching M of $g_t \setminus k, l_n$ not containing l_{n-1}. This is a maximum matching of $g_t \setminus k, l_{n-1}$ and as $|M| = \mu(g_t) - 1$, $\mu(g_t \setminus k, l_{n-1}) = \mu(g_t) - 1$. This shows that $\mu(g_t \setminus k, l) = \mu(g_t) - 1$, concluding the proof of the Claim.

From statement (4) of lemma 3.1, if $\mu(g") = \mu(g') - 1$, then j must be essential in g''. Hence, we have shown that if j is essential at g_t, j must remain essential at all social networks along the equilibrium path, and hence j has an incentive to reject i's request. '

3.4 The main theorem

We now use the characterization of equilibrium behavior to prove our main theorem: when players are sufficiently patient, the maximum number of pairs are formed in equilibrium, and any network supports efficient equilibria.

Theorem 3.4 *There exists $\bar{\delta} > 0$ such that for all $\delta \geq \bar{\delta}$, all social networks support efficient equilibria.*

Proof: For a fixed realization of needs, an equilibrium is efficient if and only if the maximum number of pairs is formed in equilibrium and no agent delays the formation of a partnership. In the equilibrium characterized in subsection 3.3, at any period t either a partnership is formed or an agent is isolated from the network. Hence, there is no delay in the formation of partnerships. Furthermore, by Claim 3.3, along the equilibrium path, whenever a pair of agents forms a partnership, it does not disrupt the formation of partnerships by the remaining agents. Hence the total number of partnerships formed in equilibrium is $\mu(g)$ the maximum number of partnerships in the original social network. $\qquad\square$

3.5 Exact conditions for the efficient formation of partnerships

Theorem 3.4 establishes that all social networks support efficient equilibria for sufficiently large values of δ. However, the exact condition on parameters for which efficient equilibria are supported depends on the architecture of the social network. In this subsection, we explicitly compute this condition for two specific networks; the line L_n and the complete network K_n where n is an even number. Both networks are perfect so that the maximum number of matchings is equal to $\frac{n}{2}$.

3.5.1 Conditions for efficient partnership formation in the line L_n

We determine the condition for existence of an efficient equilibrium – where the maximum number of matchings is formed for the line L_n and any line L_k of length k. Let V^k denote the continuation value of a peripheral agent in a line with k agents. If k is even, $V^k = V$ as we consider an efficient equilibrium ; if k is odd, $V^k < V$ and we provide an explicit computation below. When player j receives a request from agent i, he accepts the request if and only if

$$V \geq -c + \delta V^k,$$

where k is the size of the component containing j after the link ij is severed. Clearly, if k is even, j always has an incentive to reject i's request. This guarantees that, whenever a partnership ij is formed, $\mu(g \setminus i, j) = \mu(g) - 1$ so that the total number of matchings formed in equilibrium is equal to the matching number of L_n. We now compute the continuation value of a peripheral agent, say agent 1, in the line L_k where k is odd.

If an agent outside the component L_k has a need, agent 1's value is δV^k. If agent 1 has a need, his request is rejected and his value is 0. If any other inessential agent $j = 3, 5, ..., k$ has a need, this request is rejected and the component containing agent 1 becomes an even line so that agent 1's value is δV. If an essential agent $j = 2, ..., k - 1$ has a request, in an efficient equilibrium, the request will be accepted by one of his neighbors. With probability $\frac{1}{2}$, the neighbor is to the left of agent j and the component containing agent 1 becomes an even line so that agent 1's value is δV. With probability $\frac{1}{2}$, the request is accepted by an agent to the right of agent j, and agent 1 becomes a peripheral agent in an odd line of size $j - 1$. If agent 2 has a request and addresses it to agent 1, agent 1 accepts it and pays the cost c. Hence, we write the value as

$$V^k = \frac{1}{n}\left[(n-k)\delta V^k + \frac{3(k-1)\delta V}{4} - \frac{c}{2} + \sum_{j=0}^{\frac{k-1}{2}} \frac{\delta V^{2j+1}}{2}\right]. \tag{1}$$

As $V^k < V$ for all k, we observe that V^k is *increasing* in k: the value of a peripheral agent in an odd line increases with the size of the line. This implies that the condition for existence of an efficient equilibrium in the line is the most stringent when $k = n - 1$. Hence the condition for existence of an efficient equilibrium is

$$V \geq -c + \delta V^{n-1},$$

where V^{n-1} is defined recursively through equation (1)

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3.5.2 Conditions for efficient partnership formation in the complete network K_n

In any complete network K_k, the continuation value is identical for all agents as they are all symmetric in the continuation network. Let W^k denote the continuation value of any agent in the complete network K_k. We claim that, whenever i requests a need from a sequence of agents $j_1, ..j_{k-1}$, all agents but the last agent j_{k-1} are going to reject the request. If k is odd, K_{k-1} is an even complete graph, and in an efficient equilibrium, all agents obtain a value V and reject the request. If k is even, in an efficient equilibrium, the last agent accepts the request, so that the continuation graph is the even complete graph K_{k-2}. All agents preceding j_{k-1} thus have an incentive to reject the request, anticipating that the graph K_{k-2} will be formed.

An efficient equilibrium thus exists if and only if

$$V \geq -c + \delta W^k,$$

where W^k is the value of an agent, say agent 1, in an odd complete graph K_k, which is computed as follows.

If an agent outside the component K_k has a need, agent 1's value is δW^k. If agent 1 has a need, his request is rejected and he obtains a value 0. If any other agent has a need, his request is rejected and agent 1 obtains a value δV. Hence

$$W^k = \frac{1}{n}[(n-k)\delta W^k + (k-1)\delta V].$$

We thus obtain

$$W^k = \frac{(k-1)\delta V}{n - \delta(n-k)},$$

which is increasing in k so that the most stringent equilibrium condition is

$$V \geq -c + \delta W^{n-1}.$$

Interestingly, we observe that, as $V > V^k$ for all odd k, $V^k < W^k$ for all odd k and hence $V^{n-1} < W^{n-1}$. The continuation value of an agent in an odd complete graph is always larger than in an odd line of the same cardinality. This implies that it is *easier* to sustain efficient partnership formation in the line than in the complete graph. Hence, an increase in the number of links in the social networks may be detrimental to the efficient formation of partnerships. Increasing the number of social links increases the number of potential matchings, but may also increase the continuation value of agents after a link is severed, making it more difficult to sustain the efficient formation of partnerships.

4 Partnerships with positive favors

4.1 Positive favors

We now consider a model where the formation of a partnership results in positive values for both agents. When an agent responds to a request, he obtains a positive flow payoff $w > 0$ rather than incurring a negative cost $c < 0$. The equilibrium response of an agent is obvious: every agent has an incentive to accept the formation of a partnership immediately. As opposed to the model in the previous section, where agents try to delay the formation of a partnership, when agents obtain joint values, they want to rush to form partnerships. This behavior may result in the inefficient formation of matches. For example, in the line L_4, if agent 2 approaches agent 3 with a request, agent 3 accepts immediately, and the pair (23) is the only partnership formed, short of the maximum number of matches which is equal to 2. Hence, when agents rush to form partnerships, not every social network supports efficient equilibria, and our objective in this Section is to characterize those social networks for which the maximum number of matchings is always formed in equilibrium.

4.2 Elementary social networks

Following Lovasz and Plummer (1986), we call a social network g *elementary* if any edge in g appears in some maximum matching. The line L_4 is not elementary because the edge 23 does not appear in any maximum matching. On the other hand, the line L_5 is elementary. Any cycle C_k is elementary. Any complete graph K_k is elementary. Lovasz and Plummer (1986) provide the following characterization of elementary social networks.

Lemma 4.1 *(Lovasz and Plummer) A social network g is elementary if and only if for all $ij \in g$, $\mu(g \setminus i, j) = \mu(g) - 1$.*

Proof: Pick an edge $ij \in g$. If g is elementary, there exists a maximum matching containing ij, $M = (ij, M')$. As M' is a maximum matching of $g \setminus i, j$, $\mu(g \setminus i, j) = |M'| = |M| - 1 = \mu(g) - 1$. Conversely, pick a maximum matching M' of $\mu(g \setminus i, j)$, then as $\mu(g) = \mu(g \setminus i, j) + 1$, $M = (M', ij)$ is a maximum matching of g and thus any edge of g appears in some maximum matching.

When a network is elementary, whenever a pair of agents i, j leaves the network, the maximum number of pairs formed in $g \setminus i, j$ is equal to the matching number of g minus one, so that the formation of the partnership (ij) does not result in the disruption of the matchings in the graph. However, this argument only works if one considers the formation of a single partnership (ij) and not of a sequence of partnerships $(ij_1)(j_1j_2)....$ Consider for example the cycle C_6. This cycle is elementary: when a partnership (ij) forms, the remaining network is the line L_4 and the matching number of the line L_4 satisfies $\mu(L_4) = 2 = \mu(C_6) - 1$. However,

as we argued earlier, the line L_4 is not elementary: the formation of the partnership (23) results in a single matching formed. Hence, in order to guarantee that the maximum number of partnerships is formed at some social network g we require that *the social network formed after any sequence of pairs have left is itself elementary*: this is a very strong property and we call graphs satisfying this condition *completely elementary*.

A social network g is *completely elementary* if after any sequence of pairs $(ij_1), (j_1, j_2), .., (j_k j)$ leaves the network, the resulting network $g \setminus i, j_1, j_2, .., j_k, j$ is elementary. Any complete network K_k is completely elementary. The line L_5 is completely elementary because if a pair leaves, it results either in the formation of the line L_3 which is elementary or in the formation of the lines L_2 and L_1 which are elementary. The line L_7 is not completely elementary because if a pair leaves, it may result in the formation of the line L_4 which is not elementary. The cycles C_5 and C_7 are completely elementary as the formation of a partnership results in the formation of the line L_3 and L_5 which are themselves completely elementary. But the cycle C_9 which leads to the formation of the line L_7 is not completely elementary. We now state

Proposition 4.2 *When favors provide positive values to both agents, a social network g supports efficient equilibria if and only if it is completely elementary.*

4.3   Perfect networks supporting efficient equilibria

We now provide an alternative characterization of completely elementary networks when the social network g admits a perfect matching. Hence, we focus attention on networks with an even number of nodes such that $\mu(g) = \frac{n}{2}$. We will prove that a perfect network supports efficient equilibria if and only if it is either formed of components which are complete or complete bipartite graphs.

Theorem 4.3 *When favors provide values to both agents, a perfect social network g supports efficient equilibria if and only if it is the disjoint union of perfect components which are either complete or complete bipartite.*

Proof: (Sufficiency) Consider a perfect component g which is either complete bipartite or complete bipartite. and an edge ij in g. If g is the complete graph K_k, then $g \setminus i, j$ is the complete graph K_{k-2} and hence is a perfect complete graph. If g is the perfect complete bipartite graph $B_{k,k}$, then $g \setminus i, j$ is the perfect complete bipartite graph $B_{k-1,k-1}$. Because any perfect complete or perfect complete bipartite graph is elementary, the perfect component g is completely elementary. The full network formed of the disjoint union of perfect components is also completely elementary and hence supports efficient equilibria.

(Necessity) The proof is by induction on the number of nodes in a connected component g of the original network. If $|g| = 2$, the statement is vacuous and always satisfied. If

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$|g| = 4$ and the network is completely elementary, it must either be the complete network K_4 or the cycle C_4. The cycle C_4 is equal to the complete bipartite graph $B_{2,2}$. The other two connected networks of size 4 (up to a a permutation of the players) is the line L_4 and the network formed by the cycle and one additional link. As we argued earlier, the line L_4 is not elementary (the link 23 in red does not appear in any maximum matching. The other network, illustrated below, is not elementary because one of the links (in red) does not appear in any maximum matching.

The line L_4
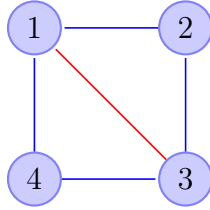
The cycle C_4 plus one link

Now consider a connected component g of size $2k$ which is perfect and completely elementary. For any $ij \in g$, $g \setminus i, j$ is completely elementary. In addition, as ij belongs to some maximum matching, $\mu(g \setminus i, j) = \mu(g) - 1$ so that $g \setminus i, j$ is perfect. By the induction hypothesis, $g \setminus i, j$ is the disjoint union of components which are either complete or complete bipartite.

Claim 4.4 *If g is perfect and completely elementary, then $g \setminus i, j$ is connected for all $ij \in g$.*

Proof of the Claim: Suppose by contradiction that there exists i, j such that $g \setminus i, j$ contains different components. Because $g \setminus i, j$ is perfect, all components must be even. Furthermore, each agent in g must be connected to at least two other agents. Suppose that this were not the case, and some agent j is only linked to another agent i. Pick any other agent k such that $ik \in g$ (which exists since $|g| > 2$) and consider the graph $g \setminus i, k$. Agent j becomes isolated in the graph $g \setminus i, k$ contradicting the fact that $g \setminus i, k$ is a perfect graph. Because g is connected and $g \setminus i, j$ contains different components, there must exist two agents k and l belonging to two different components h_1 and h_2 such that either $ik, jl \in g$ or $il, jk \in g$. Without loss of generality, suppose that $ik, jl \in g$ and consider the graph $g \setminus i, k, j, l$. Because g is perfect and completely elementary, $g \setminus i, k, j, l$ is perfect and only contains even components. But because $|h_1|$ and $|h_2|$ are even $k \in h_1, l \in h_2$ and $i, j \notin h_1 \cup h_2$, $g \setminus i, k, j, l$ contains two components with odd sizes $|h_1| - 1$ and $|h_2| - 1$, a contradiction.

Next, consider a link $ij \in g$. By Claim 4.4, $g \setminus i, j$ is connected. By the induction hypothesis, it is either a complete graph K_{2k-2} or a complete bipartite graph $B_{k-1,k-1}$. Suppose first

that $g \setminus i, j = K_{2k-2}$. Because $|g| > 4$, for any $kl \in g$, the graph $g \setminus k, l$ is a complete graph K_{k-2} and not a complete bipartite graph. Now we show that all agents are connected to all other agents in g. Pick an agent i. He is connected to at least two other agents j and k in g. Consider two other nodes l, m. Because $g \setminus i, j$ is a complete graph, $lm \in g$. So $g \setminus lm$ is a complete graph and $jk \in g$. But then $g \setminus jk$ is a complete graph, showing that i is connected to all agents $l \neq j, k$, so that i is connected to all other agents in g.

Next suppose that $g \setminus i, j$ is a complete bipartite graph $B_{k-1,k-1}$. The set of agents $N \setminus i, j$ can thus be decomposed into two subsets A and B such that there is no edge among agents in A and among agents in B. Because $|g| > 4$, for any $k, l \in g$, the graph $g \setminus k, l$ must be a complete bipartite graph $B_{k-1,k-1}$ and not a complete graph. Now, for $k, l \neq i, j$, $g \setminus k, l$ is a complete bipartite graph, so that agents i and j cannot be on the same side. We thus partition the set of players into two sets of equal cardinality $A \cup \{i\}$ and $B \cup \{j\}$ such that there is no edge among agents on the same side. The graph g is necessarily bipartite To prove that g is a complete bipartite graph, consider i and pick one agent in A, k and one agent in B, l. Because $g \setminus k, j$ is a complete bipartite graph, i is connected to all agents in B. Because $g \setminus k, l$ is a complete bipartite graph, i is connected to all agents in $B \setminus l \cup \{j\}$. Hence i is connected to all agents in $B \cup \{j\}$ completing the proof of the Theorem.

5   Experimental Design

In order to test the behavior of agents in the game of partnership formation in social networks, we design a laboratory experiment in the model with costly favors.[11] The objective of the experiment is to check whether boundedly rational agents will play equilibrium strategies when facing incentives in real social-network interactions, and to what extent different networks support efficient outcomes.

Unlike the infinite process described in Section 2, the experiment must stop in finite time. We assume that once agents form partnerships and leave the network, they immediately collect the total value of the partnership and will not request or grant favors anymore. Only those agents who are not yet in a partnership are chosen with equal probability to request a favor from one of their neighbors. The process ends when no new partnership can be formed in the network. The value obtained by an agent in a partnership is either $v - c$ (if the agent grants the favor) or v (if he requests the favor). Agents who are not in a partnership at the end of the process receive a value of 0. We calibrate the values of v and c so that, in the particular networks we consider in the experiment, the equilibrium behavior in the finite game coincides with the equilibrium of the game of partnership formation of Section 2 when the discount factor δ converges to 1.

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[11]Behavior in the model of positive favors is obvious, so we do not feel that an experiment will be helpful there.

5.1 Initial Social Networks in the Experiment

We choose five initial social networks in the experiment which are depicted in Figure 3. The number of nodes, links and complexity of the network structure increase from social network 1 to social network 5. The first two social networks are the lines L_4 and L_5. The other three social networks are more complex and involve cycles with four agents in social network 3, five agents in social network 4, and seven agents in social network 5.

In the experiment, subjects go through the initial social networks 1 to 5 in sequence. They play the game with each initial social network five times so play a total of 25 times. There are also 2 practice periods on social network 1 at the beginning of the experiment.

At the beginning of each period, subjects are randomly re-matched into groups. Given any initial network, each period starts with one agent, say i, being randomly chosen to request a favor from one of his neighbors, agent j. The sequence in which agent i approaches his neighbors is chosen at random by the computer.[12] Agent j then decides whether to accept the offer or not. If the offer is accepted, the partnership is formed and both agents leave the social network. If this agent decides to reject the offer, the link between i and j is destroyed. Agent i then requests a favor from his next neighbor in the sequence. If all neighbors reject his request, agent i will be cut off from the social network. Once agents leave or are cut off from the social network, the computer will then randomly select the next agent from the remaining social network and the same process is executed again. The social network evolves until each agent either has a partner or is isolated. Each period thus involves a sequence of decisions, with each decision made in a specific network by the selected subject. For each decision, subjects who made it and who proposed the request are informed of the result and their respective payoffs, and others in their group are shown the changes in the social network on the computer screen.

5.2 Individual difference tests

Belief elicitation
Subjects should make decisions in the game according to their beliefs about the rationality and the behavior of other agents in the social network. In order to take into account these beliefs in the analysis of decisions, we have asked subjects about the decision of other agents in a simple situation. In the context of the 3-agent line L_3, subjects are asked to give an estimate of the proportion of central agents who actually accepted the request from one of

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[12]In the theoretical model, the proposer chooses the sequence in which neighbors are approached. But whether the sequence is chosen endogenously or exogenously does not affect the equilibrium response of the agents. Since the analysis focuses on equilibrium responses and matchings formed, the two models with endogenous and exogenous sequences are equivalent.
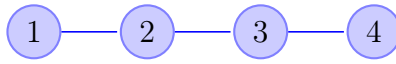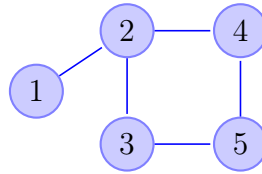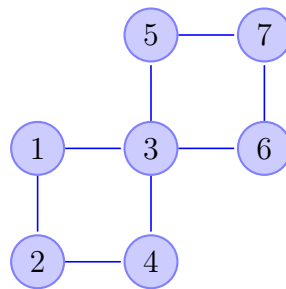
Social network 1
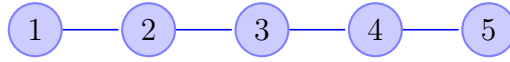
Social network 2
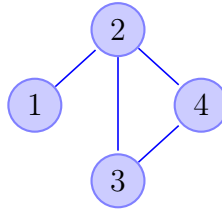
Social network 3

Social network 4

Social network 5

Figure 3: Initial social networks 1-5 in the experiment

the extreme nodes. In this situation, the agent should always reject the request, as he obtains either v or $v - c$ after the rejection. The actual proportion of acceptance from central agents is 14.6%. The average estimated proportion of acceptance is 20.8%, and half of the subjects believe that it is smaller than 10%. In addition, only 15.6% of subjects estimate that the proportion of acceptance is equal to or higher than 50%. This question is not incentivized in our experimental design.[13]

Cognitive ability
In our experiment, cognitive abilities are elicited with the CRT test (Frederick, 2005). This test is designed to assess an individual's ability to move from an intuitive and spontaneous wrong decision to a reflective and deliberative right one. Subjects are asked to answer three questions in the CRT test, which are listed as follows:

- Question 1: A bat and a ball cost €11. The bat costs €10 more than the ball. How much does the ball cost?

- Question 2: If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets?

- Question 3: In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake?

Although the CRT test is relatively short and simple to perform compared to other cognitive tests, its results are positively related with rational thinking performance (Toplak, West and Stanovich, 2011). On average, 1.75 questions are answered correctly by subjects in the CRT test for this study.

Risk elicitation
Rejection is a risky decision when the acceptance of the request gives $v - c$ for sure. So, acceptance ans rejection decisions should be related to subjects' attitudes towards risk. We elicited this attitude following a procedure introduced by Eckel *et al* (2012). The procedure consists of a choice among six lotteries in the form of a coin flip that gives a low or a high payoff with equal probability. The lotteries are arrayed from a safe one with a certain payoff of 18 experiment points to a highly risky one with a high payoff of 54 points and a negative low payoff of -2 points. Expected return increases along with higher variance as one moves from the safest to the riskiest lottery. The variance that a subject is willing to accept gives a proxy of his risk preference. Therefore, we can estimate each subject's level of risk attitude

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[13]This is not a true elicitation of subjects' beliefs about other agents' behavior as the answer to this question depends also on the rationality of the subject questioned. Nevertheless, the answer to this question may explain deviation from the equilibrium strategy.

by looking at his choice among six lotteries: lottery 1 through 4 represent decreasing levels of risk aversion, lottery 5 indicates risk neutrality, and lottery 6 corresponds to risk seeking individuals.

Table 1: Six lotteries in risk test

Lottery	Payoff (experiment points)				Risk Preference	Percentage
	Low (50%)	High (50%)	Expected	Variance		
1	18	18	18	0	Highly risk averse	20.14%
2	14	26	20	36	Very risk averse	22.86%
3	10	34	22	144	Risk averse	17.1%
4	6	42	24	324	Slightly risk averse	7.49%
5	2	50	26	576	Risk neutral	15.31%
6	-2	54	26	784	Risk loving	17.1%

5.3 Experimental Procedure

In each experimental session, subjects are randomly assigned ID numbers and seats in front of the corresponding terminal in the laboratory. The experimenter reads the instructions aloud. Subjects are given the opportunities to ask questions, which are answered in private. We check the subjects' understanding of the instructions by asking them to answer 7 incentivized review questions at their own pace. After answering one review question, each subject is shown whether his answer is correct, as well as the right answer. After going over all review questions, subjects go through 27 periods in the social-network experiment, including 2 practice periods. Afterwards, subjects are asked to report their beliefs on other agents' behaviors, and take the CRT and risk tests. At the end of the experiment, each subject fills out a demographic survey on the computer, and is then paid in private. Each session lasts approximately 80 minutes, with 15 minutes devoted to the instructions. The experiment is programmed in Java.

In the experiment, we set the parameters at $v = 20$ experimental points and $c = 8$ points. Therefore, in a given period, a subject will obtain a payoff of 20 points by requesting a favor or 12 points by granting a favor. If the subject has no partner, he will earn 0 points. There are 21 subjects in each session. As there are 4 or 5 subjects per group from the initial social networks 1 to 4, for the corresponding period one subject will be randomly chosen not to play and be paid 10 points. At the end of the experiment, 10 out of 25 periods are randomly chosen to be paid. In addition, a subject could earn 2 points per review question and per CRT question answered correctly. He will also earn the payoff resulting from the draw for the lotteries he chose in the risk test. The exchange rate is 10 experiment points for €1

for all sessions. Each subject also receives a participation fee of €3. The average earning (including participation fee) is equal to €21.

All sessions were conducted in French at GATE-LAB, the Experiment Economics Laboratory in Lyon between April and September 2015. The subjects are students from an engineering department, Ecole Centrale de Lyon, a business school EM Lyon, and the University of Lyon. No one participated more than once. We ran 6 independent sessions. In total, 126 subjects participated in the experiment and we collected 1842 decisions.The English translations of the experimental instructions can be found in the Appendix.

6   Results

In this section, we analyze the results of the experiments, focusing on two main questions. First, we study individual behavior and analyze whether agents play the subgame perfect equilibrium strategy, which is also referred to as risk-neutral best response (BR) here. We also identify which factors could explain the deviation from the best response behavior. Second, we analyze whether, at the aggregate level, the social interaction among real agents results in efficient outcomes in different social networks.

6.1   Individual Behavior

To check whether subjects choose the BR, we consider all networks which may arise during the experiment and break down behavior in the various networks, defining different situations that subjects are faced with when making decisions in each graph.[14] Overall, there are 28 possible graphs and 105 possible situations in the experiment.

For each situation, we compute the BR of the agent using the characterization results of Section 2. We also calculate the expected value of acceptance, which is always 12 with certainty, and the expected value of rejection for each situation. For example, in Figure 3, suppose that agent 2 in the initial social network 1 receives a request from agent 3. Agent 2 can make the following calculation using backward induction. He will earn 12 for sure by accepting the request; however, if agent 2 declines the offer, the link between agents 2 and 3 will be destroyed, and agent 3 would make a request to agent 4, who is expected to accept the offer. The network would then evolve to L_2 where agent 2 has 50% chance of earning 20 by making a request that should be accepted by a rational agent 1, and 50% chance of earning 12 by accepting the request from agent 1. The expected value is thus $0.5 \times 20 + 0.5 \times 12 = 16$.

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[14]For instance, in the initial network 1 or line L_4, there are 3 different situations: one where an extreme agent requests a favor from a central agent, one where a central agent requests a favor from an extreme agent and one where a central agent requests a favor from the other central agent.

Hence agent 2 should reject the offer. Generally, the difference between the expected values of rejection and acceptance is defined as follows:

EV.difference = Expected value of rejection − Expected value of acceptance.

Note that when EV.difference > 0, BR is to reject, and when EV.difference < 0, BR is to accept.[15]

Due to strategy uncertainty, subjects may not behave according to the expected value difference calculated under the assumption that other agents play BR. For instance, in the previous example, it is possible that agent 1 mistakenly rejects the offer, making the payoff after rejection equal to $0.5 \times 0 + 0.5 \times 12 = 6$ for agent 2. Considering the possibility of agent 1's mistake, agent 2 may instead accept the offer as he earns less than 12 by rejection. We thus decided to check if subjects make decisions based on the difference between the actual payoffs after rejection and acceptance. The actual payoff after rejection is computed, for each situation, as the average actual payoff after rejection. The difference between actual payoff after rejection and acceptance is therefore defined as follows:

Real.difference = Real gain of rejection − Real gain of acceptance.

Finally, in order to assess the complexity of each situation, we compute the steps of reasoning a person has to consider when making the decision, i.e., the number of successive decisions in the longest path of the extensive form game starting from this situation.[16] For instance, when agent 2 receives a request from agent 3 in the previous example, the complexity for agent 2 is equal to 2. We also calculate the complexity of the social network by taking the average of complexity in all possible situations at a given initial graph, and find that the complexity increases from 2 steps of reasoning on average for the initial social network 1 to 8 steps of reasoning for the initial social network 5.

6.1.1 Basic Findings

We first examine whether subjects behave according to the subgame perfect equilibrium of the partnership formation game. Overall, we find that the proportion of best responses is equal to 79.5%. Even if we exclude the simple situations where the decision maker only has one link, the proportion of best response remains as high as 66.7%. Table 2 presents the proportions of rejection for EV.difference > 0 (Real.difference > 0) and EV.difference < 0 (Real.difference < 0) in each session, respectively. On average, the proportion of rejection

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[15]Given our design, we did not have any situation with indifference in the experiment.

[16]Other measures of complexity of the situation can also be computed, such as the total number of nodes in the extensive form of the game or the total number of terminal nodes, etc. We find that all of these measures are highly correlated with each other.

is as high as 67.2% (59.5%) when EV.difference > 0 (Real.difference > 0) and as low as 12.8% (26.5%) when EV.difference < 0 (Real.difference < 0) (the proportion of acceptance is 87.2% (73.5%) correspondingly). In other words, a majority of subjects play equilibrium strategies, with 67.2% of rejection when BR is to reject and 87.2% of acceptance when BR is to accept.
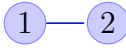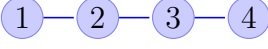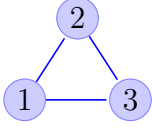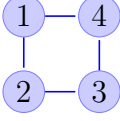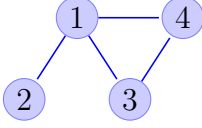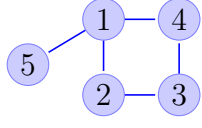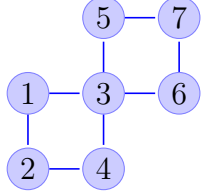
Table 2: Proportions of rejection

	EV.difference > 0	EV.difference < 0	Real.difference> 0	Real.difference < 0
Session 1	0.691	0.106	0.583	0.263
Session 2	0.703	0.148	0.588	0.297
Session 3	0.655	0.089	0.556	0.239
Session 4	0.528	0.140	0.492	0.231
Session 5	0.768	0.143	0.708	0.290
Session 6	0.683	0.139	0.646	0.273
Average	0.672	0.128	0.595	0.265

Even though subjects generally conform to the theoretical prediction, we find that their choices vary greatly in different situations. Table 3 presents some graphs which arise frequently during the experiment. These graphs are ordered from the simple two-agent line to the most complex seven-agent network, labeled as $g_1, \ldots g_8$ in sequence. In each graph, we compute the expected value difference, real earning difference, the proportion of best response as well as the number of observations for each possible situation. It can be seen from Table 3 that subjects perform differently when the social networks are lines (e.g., 85.7% of best response for g_1 to g_3) and when social networks have cycles (e.g., 24% and 51.3% of best responses in g_4 and g_5, respectively). Their rational reaction also changes with different positions in a given graph (e.g., in g_6, 88.9% of best response when agent 3 or 4 requests to 1 and 50% conversely) or when different neighbors place requests (e.g., in g_7, 80% of rational acceptance for agent 2 when 3 requests a favor and 44.4% when 1 makes the request).

In particular, we observe that subjects tend to follow two behavioral patterns, which are presented in Figure 4.

Table 3: Proportions of best response in selected graphs

| #	Graph	Situation	EV.diff.	Real.diff.	BR	# of obs.
g_1:	1 — 2	1 (2) → 2 (1)	-12	-12	0.983	478
g_2:	1 — 2 — 3 — 4	1 (4) → 2 (3)	-4.7	-4.44	0.649	154
		2 (3) → 3 (2)	4	3.82	0.733	60
		2 (3) → 1 (4)	-12	-12	1	63
g_3:	1 — 2 — 3 — 4 — 5	2 (4) → 3	-4.7	-0.27	0.7	20
		1 (5) → 2 (4)	4	1.57	0.529	87
		3 → 2 (4)	4	3.2	0.769	26
		2 (4) → 1 (5)	-12	-12	1	27
g_4:		2 (3) → 1	4	-1.33	0.24	25
g_5:		2 (4) → 1	4	1.6	0.513	39
g_6:		2 → 1	-1.33	-4	0.39	41
		3 (4) → 1	4	1.33	0.889	27
		3 (4) → 4 (3)	-4.7	-4	0.676	37
		1 → 3 (4)	4	3.5	0.5	32
		1 → 2	-12	-12	1	13
g_7:		5 → 1	4	-0.52	0.784	37
		2 (4) → 1	4	0.57	0.824	34
		3 → 2 (4)	-4.7	0.8	0.8	35
		1 → 2 (4)	-0.35	0.8	0.444	18
		2 (4) → 3	4	-0.11	0.359	39
		1 → 5	-12	-12	1	12
g_8:		1 (4, 5, 6) → 3	4	0.76	0.8	30
		2 (7) → 1, 4 (5, 6)	- 3.34	1.14	0.667	24
		3 → 1 (4, 5, 6)	-1.8	-1.71	0.714	7
		1, 4 (5, 6) → 2 (7)	4	0.95	0.414	29
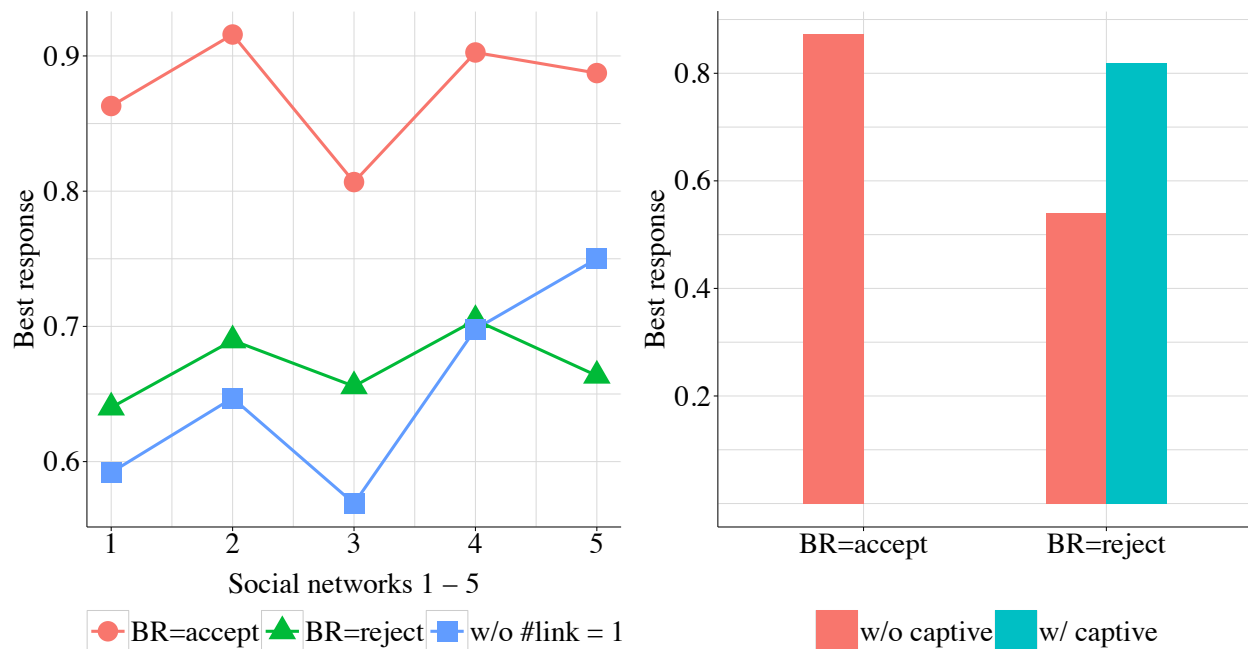
Figure 4: Two behavior patterns in the experiment

First, subjects are more likely to accept than to reject (left panel of Figure 4). On average, 65.6% of requests are accepted by subjects. We also find a higher rate of rational acceptance (acceptance when BR is to accept), which is 87.2%, compared to 67.2% for rational rejection (rejection when BR is to reject). However, the high rate of acceptance is probably due to the fact that rational acceptance includes the simple situations when the decision maker has only one link. If we exclude these situations, the proportion of rational acceptance is only 64.7%. We conduct probit regressions to control for this fact,[17] and still find a significantly higher rate of rational acceptance than rational rejection.[18] There are two plausible explanations for this tendency to accept: (1) to reject is a risky choice and subjects tend to accept because of risk aversion; (2) there is a cognitive cost to calculate expected value of rejection, and so subjects will make the immediate acceptance decision instead. We will explore these two explanations in the next subsection.

Second, subjects tend to rationally reject when they have a *captive agent* not making the

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[17]In the regressions, the dependent variable is the probability of best response. The primary independent variable is the dummy variable "BR Reject" which is equal to 1 if BR is to reject. Control variables include the dummy variable "One link" which is equal to 1 if the subject has only one link, as well as other variables introduced in the regressions in the next subsection. The regression results are presented in Table 7 in Appendix.

[18]In Table 7, the coefficients for dummy variable "BR Reject" are negative, $p < 0.01$, $p < 0.05$, $p < 0.1$.

request (right panel of Figure 4). An agent is said to be a *captive agent* if he has only one connection in the social network. So one should expect that a captive agent will always accept the request from his only neighbor. When a subject has a captive agent he should reject the current offer as he is guaranteed to earn 20 by requesting a favor from his captive agent. We find that 81.9% of requests are rationally rejected when subjects have captive agents not making the request, compared to 54.1% of rational rejection by subjects who do not have captive agents. (This effect is proved to be significant through regression results in the next subsection).

6.1.2   Determinants of behavior

We now analyze in detail departures from equilibrium behavior related to the characteristics of the current social network and situation. We also control for factors related to individuals. In order to systematically check how these factors affect the strategies of the subjects, we conduct probit regressions. The results are presented in Table 4 and Table 5. The dependent variable is the probability of best response when BR is to reject (in Table 4) and when BR is to accept (in Table 5), respectively. Independent variables include "EV.difference", the difference between expected value after rejection and acceptance in specifications (1) through (3), "Real.difference", the difference between actual payoffs after rejection and acceptance in specifications (4) through (6). Specifications (1) and (4) only include variables related to the characteristics of the social network and situation: a dummy variable "Cycle" which is equal to 1 if subject is in the cycle, a dummy variable "Steps of reasoning" which is equal to 1 if an agent has to consider more than 3 steps of reasoning in the extensive form of the game he faced. In the case of the regressions when BR is to reject (in Table 4), we add an additional dummy variable "Captive" which is equal to 1 if the subject has a captive agent who is not the one who requests a favor from him. Specifications (2) and (5) control for individual differences: the number of correct answers in the "CRT" test , the measure of "Risk" preference for a subject where smaller value indicates higher level of risk aversion, and the proxy for the subject's "Belief" about other agents' rationality where a lower percentage represents a higher estimation of the rationality of other agents. Finally, specifications (3) and (6) add individual "Experience", the number of decisions the subject has already made, in order to capture a learning effect.

We first check the relation between best response and expected value difference as well as real earning difference. Figure 5 presents the proportion of rejection in each situation for each expected value difference (left panel) and real earning difference (right panel), respectively. For risk neutral rational agents, the proportion of rejection should be equal to zero when EV.difference < 0 and equal to one when EV.difference > 0.

Table 4: Probit regressions: Probability to best respond when BR is to reject

	(1)	(2)	(3)	(4)	(5)	(6)
EV.difference	-0.030	-0.029	-0.043			
	(0.115)	(0.117)	(0.112)			
Real.difference				-0.007	-0.008	-0.001
				(0.010)	(0.008)	(0.009)
Steps of Reasoning	-0.168***	-0.177***	-0.143***	-0.175***	-0.184***	-0.145***
	(0.043)	(0.044)	(0.041)	(0.038)	(0.039)	(0.034)
Cycle	0.010	0.005	-0.033	0.016	0.011	-0.032
	(0.062)	(0.058)	(0.059)	(0.066)	(0.061)	(0.063)
Captive	0.400***	0.397***	0.395***	0.404***	0.401***	0.394***
	(0.040)	(0.041)	(0.042)	(0.037)	(0.040)	(0.040)
Risk		0.016**	0.018**		0.016**	0.018**
		(0.007)	(0.007)		(0.007)	(0.007)
CRT		0.014	0.011		0.014	0.011
		(0.021)	(0.021)		(0.020)	(0.021)
Belief		-0.002**	-0.002***		-0.002**	-0.002***
		(0.001)	(0.001)		(0.001)	(0.001)
Experience			0.014***			0.014***
			(0.003)			(0.004)
No. of observations	724	724	724	724	724	724

Note: standard errors in parentheses are clustered at the session level; coefficients are marginal effects. *** p<0.01, ** p<0.05, * p<0.1.

Table 5: Probit regressions: Probability to best respond when BR is to accept

	(1)	(2)	(3)	(4)	(5)	(6)
EV.difference	-0.033***	-0.033***	-0.032***			
	(0.002)	(0.002)	(0.003)			
Real.difference				-0.024***	-0.024***	-0.024***
				(0.002)	(0.002)	(0.002)
Steps of Reasoning	-0.101***	-0.109***	-0.074*	-0.034	-0.048	0.011
	(0.033)	(0.030)	(0.038)	(0.042)	(0.044)	(0.060)
Cycle	-0.062	-0.068	-0.058	-0.028	-0.040	-0.018
	(0.048)	(0.044)	(0.045)	(0.044)	(0.045)	(0.050)
Risk		-0.008***	-0.009***		-0.007**	-0.007***
		(0.002)	(0.002)		(0.003)	(0.003)
CRT		0.016**	0.015**		0.016**	0.016*
		(0.008)	(0.008)		(0.008)	(0.008)
Belief		-0.001	-0.001		-0.001	-0.001
		(0.000)	(0.000)		(0.001)	(0.001)
Experience			0.005**			0.009***
			(0.003)			(0.003)
No. of observations	1,118	1,118	1,118	1,104	1,104	1,104

Note: standard errors in parentheses are clustered at the session level; coefficients are marginal effects. *** p<0.01, ** p<0.05, * p<0.1.
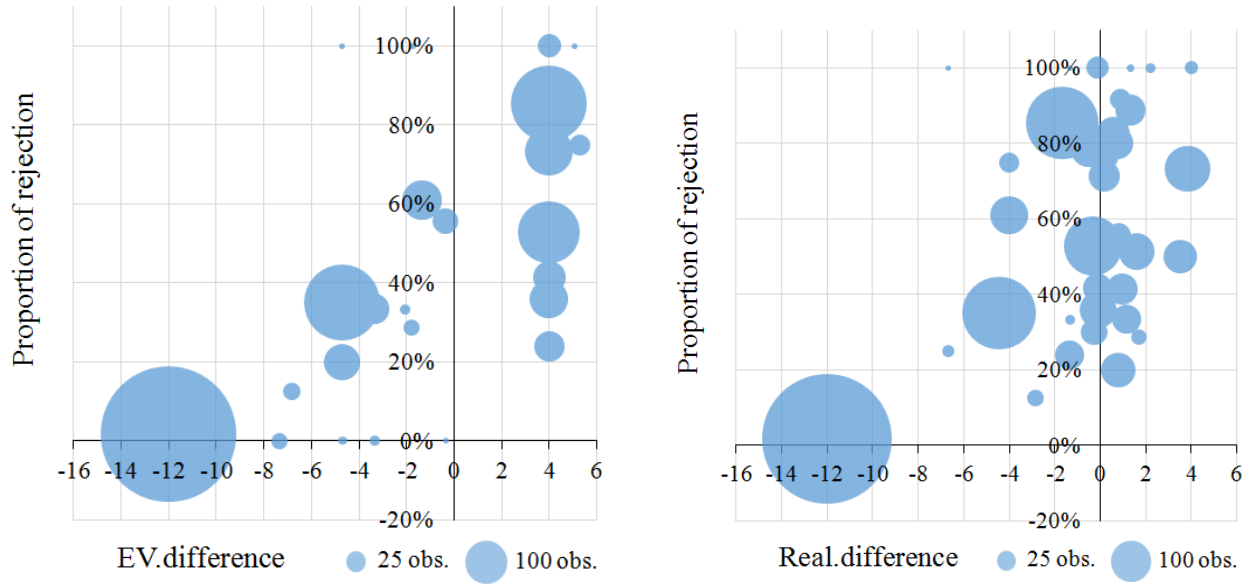
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Figure 5: Proportion of rejection in each situation by EV.difference and Real.difference

It can be seen from Figure 5 that although subjects do not play completely according to the theoretical prediction, they are more likely to reject when the expected value difference or the real earning difference increases. In particular, this relation is almost linear when EV.difference < 0 and Real.difference < 0.

Regression results in Table 5 support this finding. A subject's probability of playing a best response (acceptance) will decrease by about 3.2 percentage points when the expected value difference increases by 1 unit (p < 0.01 for coefficients "EV.difference"), and will decrease by about 2.4 percentage points when the real earning difference increases by 1 unit (p < 0.01 for coefficients "Real.difference"). We note that when BR is to accept (EV.difference < 0), the subject's best response increases linearly with the expected value difference and the real earning difference, indicating that subjects tend to play equilibrium strategies when the cost of deviation from rational acceptance increases.

However, Figure 5 shows that when BR is to reject (EV.difference > 0), there is a striking heterogeneity among subjects' best responses according to different decision situations, even for situations with the same expected value difference. In fact, the coefficients of "EV.difference" in specifications (1) through (3) and the coefficients of "Real.difference" in specifications (4) through (6) in Table 4 are all negative and insignificant. This indicates that other characteristics of the situation, such as complexity of the network or the structure of the network are likely to play a role in situations when BR is to reject.

We next present in Figure 6 the proportion of best responses as a function of the steps of reasoning when BR is to accept or to reject (left panel), and when the current social network is in a line or contains a cycle (right panel).
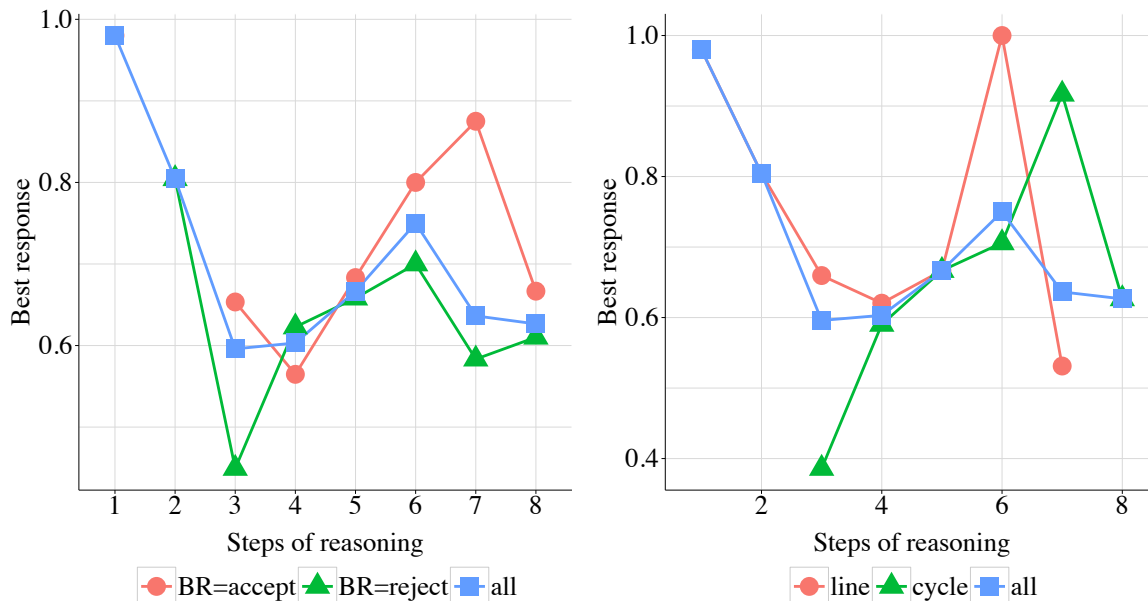

Figure 6: Proportion of best response by situation complexity

It can be seen from Figure 6 that the proportion of best response decreases with the complexity at first and then increases slightly when the number of steps of reasoning is higher than 3. We also observe a high volatility of best responses for some steps of reasoning. This is probably due to the small number of observations, especially when the situation becomes more complex -we only have a third of the total observations corresponding to situations where the number of steps of reasoning is higher than 3. Overall, we find that the proportion of best responses is high when the situation is less complex. The proportion of rational rejections is 73.1% (91.8% of rational acceptance) when the number of steps of reasoning is smaller than 3, and is 62.9% (64.1% of rational acceptance) otherwise. Results of the regressions in Table 4 and Table 5 further show that when it takes more than 3 steps of reasoning, the probability of best responses decreases by about 14.3 percentage points when BR is to reject and by about 7.4 percentage points when BR is to accept. However, the effect is only significant if we do not control for learning effect and real earning difference.

On average, it takes 5 steps for a subject to figure out his subgame perfect equilibrium strategies when the network contains a cycle, but it only requires 2 steps in a line. However, when we control for the complexity of the situation, the network structure - whether the

subject is in a line or a network containing a cycle - has no significant effect on the probability of best response (coefficients for "Cycle" are insignificant in Table 4 and Table 5). On the other hand, when facing the same level of complexity, the probability of best response increases by at least 39.4 percentage points if the subjects have captive agents when BR is to reject ($p < 0.01$ for coefficients "Captive" in Table 4). This result further supports the previous finding that this heuristics helps subjects adopt equilibrium strategies.

We next argue that risk aversion, cognitive ability, subject's belief about other participants' rationality can also help explain behavior heterogeneity. Regression results show that subjects with higher levels of risk aversion are less likely to rationally reject (the coefficients for "Risk" are 0.016 to 0.018 at $p < 0.05$ in Table 4) and are more likely to rationally accept the favor (the coefficients for "Risk" are -0.009 to -0.007 at $p < 0.05$ in Table 5). That is, risk aversion makes subjects more likely to accept, especially in situations when BR is to reject. Even in the "safe rejection" situations when subjects will earn at least 12 by rejection, highly risk averse subjects- those who choose low but certain payoff in the risk test- still tend to accept (56.6% of rejection by high risk averse subjects v.s. 70.3% of rejection by other types of subjects).

On the other hand, subjects have to invest their cognitive abilities and cognitive efforts to calculate the expected value of rejection so as to find out their optimal choices. We use the number of correct answers in the CRT test as the proxy for subjects cognitive abilities and cognitive efforts. We find that subjects with a better answer in the CRT test are more likely to play equilibrium strategies. This effect is only significant ($p < 0.05$ for coefficient "CRT" in Table 5) and sometimes marginally significant ($p < 0.1$ for coefficient "CRT" in specification (3) of Table 5) when BR is to accept.

Subjects' beliefs about others' rationality may also affect their tendency to best respond. Regression results in Table 4 and Table 5 show that the coefficients of variable "Belief" are negative in all specifications, and in particular, they are significant when BR is to reject ($p < 0.05$ for coefficients "Belief" in Table4). The result indicates that subjects holding a stronger belief about strategy uncertainty (i.e., weaker belief about others' rationality) are more likely to choose the safe "acceptance", when rejection is in fact their subgame perfect equilibrium strategies under the assumption of rationality for other agents.

Lastly, as the experiment is repeated for 25 periods, from the simple social network to the complex ones, it is interesting to ask whether previous experiences affect individual choices, and more importantly, whether subjects learn to best respond over time. We check the effect of a subject's own decision experience on best response by controlling for situation characteristics as well as individual difference, and find a significant learning effect. It can be seen from Table 4 and Table 5 that on average, with one more decision a subject has made, the probability of best response increase sby 1.4 percentage points when BR is to

reject (p < 0.01 for coefficients "Experience" in Table 4), and by at least 0.5 percentage points when BR is to accept (and p < 0.05 for coefficients "Experience" in Table 5).

6.2   Aggregate outcomes

In this subsection, we analyze whether aggregate behavior leads to efficient outcomes in the experiment. We first look at the number of matched pairs for each of the five initial social networks formed in the experiment. If all subjects follow the subgame perfect equilibrium strategies, as shown in Section 2, the maximum number of matches in the initial social network will be achieved. We therefore compute an efficiency index (EI) as follows:

$$\text{EI} = \frac{\text{Number of actual matched pairs}}{\text{Maximum number of matched pairs}}.$$

Notice that, in some social networks, the maximum number of matched pairs can still be formed when agents do not play their equilibrium strategies. For example, in the social network 5, even when some agents make mistakes by accepting the offer when they should reject, there will still be 3 matches formed at the end. On the contrary, in social network 1, if any of the agents does not play his best response, the efficient outcome cannot be achieved. As a result, social networks differ by the sensitivity of the number of matches formed as a function of the behavior of agents. Taking this fact into account, we consider random agents who randomly reject and accept the request in each situation with equal probability, and compute the number of matched pairs formed by these random agents. This gives us a benchmark with which to compare the efficiency level obtained by real agents. We compute a relative efficiency index (REI) as follows:

$$\text{REI} = \frac{\text{Number of actual matched pairs - Number of randomly matched pairs}}{\text{Maximum number of matched pairs - Number of randomly matched pairs}}.$$

Table 6 presents the outcome efficiency for each of five initial social networks, including the number of actual matched pairs , the number of randomly matched pairs , the maximum number of matched pairs, the efficiency index as well as the relative efficiency index. The proportion of best response is also computed for each initial social network. On average, the efficiency index is as high as 0.90 and the relative efficiency index is 0.75. We also find that 78% of times (493 out of 630 total outcomes) the maximum number of matched pairs is achieved. More interestingly, in the most complex seven-agent social network, all groups achieve efficient outcomes in the last period. For each initial social network, the number of matched pairs established by random agents is also lower than that achieved by actual agents.

Table 6: Outcome efficiency for initial social networks

Period	Network 1	Network 2	Network 3	Network 4	Network 5
1	1.63	1.96	1.53	1.96	2.78
2	1.73	1.92	1.53	1.88	2.89
3	1.60	1.96	1.63	1.96	2.94
4	1.63	1.88	1.53	1.96	2.89
5	1.77	1.88	1.57	1.96	3.00
Average	1.67	1.92	1.56	1.94	2.90
Random agents	1.09	1.41	1.13	1.51	2.25
Max.# of pairs.	2	2	2	2	3
EI	0.84	0.96	0.78	0.97	0.97
REI	0.64	0.86	0.49	0.88	0.87
Best response	0.828	0.821	0.756	0.804	0.774

In addition, the efficiency index (relative efficiency index) is 0.84 (0.64) and 0.78 (0.49) in social networks 1 and 3, lower than those in social networks 2, 4 and 5. We also observe that the efficiency level achieved by random agents is also lower in these two networks. In fact, social networks 1 and 3 have an even number of agents, whereas the rest has an odd number of agents. Therefore, the low level of efficiency in these two networks is partly due to the fact that they are more sensitive to mistakes in the agents' behavior. As a result, even though the proportion of best response in social network 1 is higher than in all other networks, the efficiency level is lower.

7 Conclusion

This paper analyzes the formation of partnerships in social networks. Agents randomly request favors and turn to their neighbors to form a partnership where they commit to provide the favor when requested. If favors are costly, agents have an incentive to delay the formation of the partnership. In that case, we show that for any initial social network, the unique Markov Perfect equilibrium results in the formation of the maximum number of partnerships when agents become infinitely patient. If favors provide benefits, agents rush to form partnerships at the cost of disconnecting other agents and the only perfect initial networks for which the maximum number of partnerships are formed are the complete and complete bipartite networks. The theoretical model is tested in the lab. Experimental results show that a large fraction of the subjects (75%) play according to their subgame perfect equilibrium strategy and reveals that the efficient maximum matching is formed over 78% of the times. When subjects deviate from their best responses, they accept to form

partnerships too early. The incentive to accept when it is optimal to reject is positively correlated with the subjects' risk aversion, and players employ simple heuristics – like the presence of a captive partner – to decide whether they should accept or reject the formation of a partnership.

We are aware of a number of limitations of our model and experimental study and would like to focus our attention to two important questions in future work. First, we would like to extend the model to the study of partnerships of more than two agents. While this extension does not pose any conceptual difficulty, it requires to define generalized matchings of more than two agents, and requires to use more complex tools from graph theory. The second extension is to allow for heterogeneity in the value of partnerships, letting the value of the partnership depend on the pair ij, v_{ij}. Computing the optimal behavior of agents in non0stationary networks with heterogeneous values is probably a very complex task, but we are hopeful that it is tractable and that our model and experimental results could be generalized to a model with heterogeneous players.

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A   Supplementary results

Table 7 presents the probit regression results. The dependent variable is the probability to play a best response. Independent variables include the dummy variable "BR Reject" which is equal to 1 if BR is to reject, the dummy variable "One link" which is equal to 1 if the subject has only one link, " Abs(EV.difference)", the absolute difference between expected value of rejection and acceptance in specifications (1) through (3), "Abs(Real.difference)", the absolute difference between actual values after rejection and acceptance in specifications (4) through (6), the dummy variable "Captive" which is equal to 1 if the subject has a captive agent who is not the one who requests a favor to him, a dummy variable "Cycle" which is equal to 1 if subject is in the cycle, the number of "Steps of reasoning" a agent has to consider in the extensive form of the game he faced, the interaction term between "Captive" and "Cycle", the interaction term between "Captive" and "Steps of reasoning", the number of correct answers in the "CRT" test a subject made, the measure of "Risk" preference for a subject, and the proxy for subject's "Belief" about other agents' rationality, as well as subject's own "Experience".

Figure 7 shows the proportion of rejection conditional on the number of rejections a subject has chosen before. We exclude the last four decisions as they contain few observations, and we see in Figure 7 that subjects tend to reject if they chose more rejections before. In particular, once they have chosen a rejection, they are more likely to reject the favor later (27.1% of rejection when never rejected v.s. 36.8% of rejection when made at least one rejection before). Furthermore, in order to check whether subjects learn to play optimally, we present in Figure 4 the proportion of best responses over an individual own experience, i.e., the number of decisions the person has made so far.[19] We exclude the last four decisions and find that on average the proportion of best responses increases slightly as subjects gain more experiences. We also observe a high volatility of the subject's best response over time, especially in situations when BR is to reject. The volatility of rational behavior is probably due to the fact that subjects have to make the decision in a different situation every time.

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[19]We also check subjects' rejection patterns and best responses over their previous experiences of not only making decisions, but also observing decisions as the person made the request or as others not involved in the game. We observe a similar behavior.

Table 7: Probit regressions: Probability to best respond

	(1)	(2)	(3)	(4)	(5)	(6)
One link	-0.013	-0.021	-0.012	0.450***	0.442***	0.387***
	(0.038)	(0.038)	(0.042)	(0.039	(0.041)	(0.046)
BR Reject	-0.104***	-0.107***	-0.113***	-0.0811**	-0.083**	-0.083**
	(0.031)	(0.029)	(0.030)	(0.039)	(0.036)	(0.037)
Abs(EV.difference)	0.053***	0.054***	0.051***			
	(0.005)	(0.006)	(0.006)			
Abs(Real.difference)				-0.007	-0.006	-0.000
				(0.005)	(0.004)	(0.004)
Steps of Reasoning	-0.097***	-0.106***	-0.081***	-0.042	-0.051*	-0.037
	(0.028)	(0.031)	(0.029)	(0.027)	(0.030)	(0.028)
Cycle	-0.009	-0.014	-0.032	-0.020	-0.025	-0.046
	(0.032)	(0.030)	(0.033)	(0.034)	(0.032)	(0.036)
Captive	0.242***	0.242***	0.239***	0.205***	0.205***	0.206***
	(0.013)	(0.015)	(0.014)	(0.012)	(0.012)	(0.010)
Risk		0.000	0.001		0.001	0.001
		(0.003)	(0.003)		(0.003)	(0.003)
CRT		0.016	0.015		0.016	0.015
		(0.011)	(0.012)		(0.011)	(0.012)
Belief		-0.001**	-0.001**		-0.001*	-0.001*
		(0.001)	(0.001)		(0.001)	(0.001)
Experience			0.008***			0.009***
			(0.002)			(0.002)
No. of observations	1,842	1,842	1,842	1,828	1,828	1,828

Note: standard errors in parentheses are clustered at the session level; coefficients are marginal effects. *** p<0.01, ** p<0.05, * p<0.1.
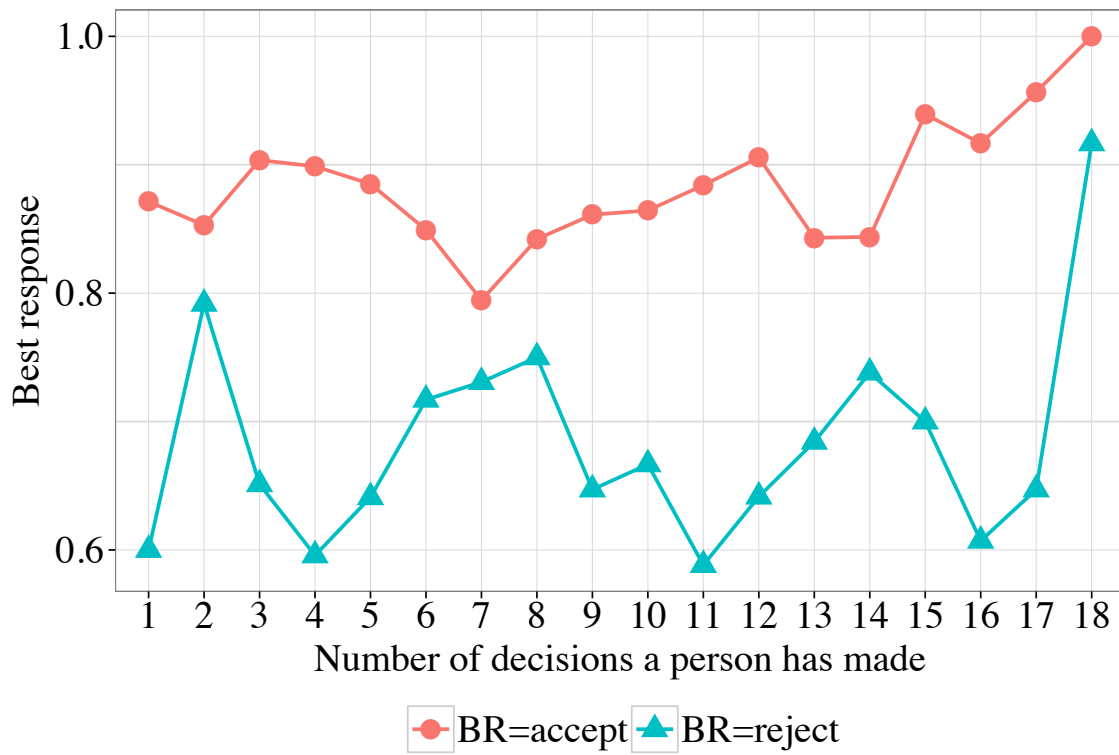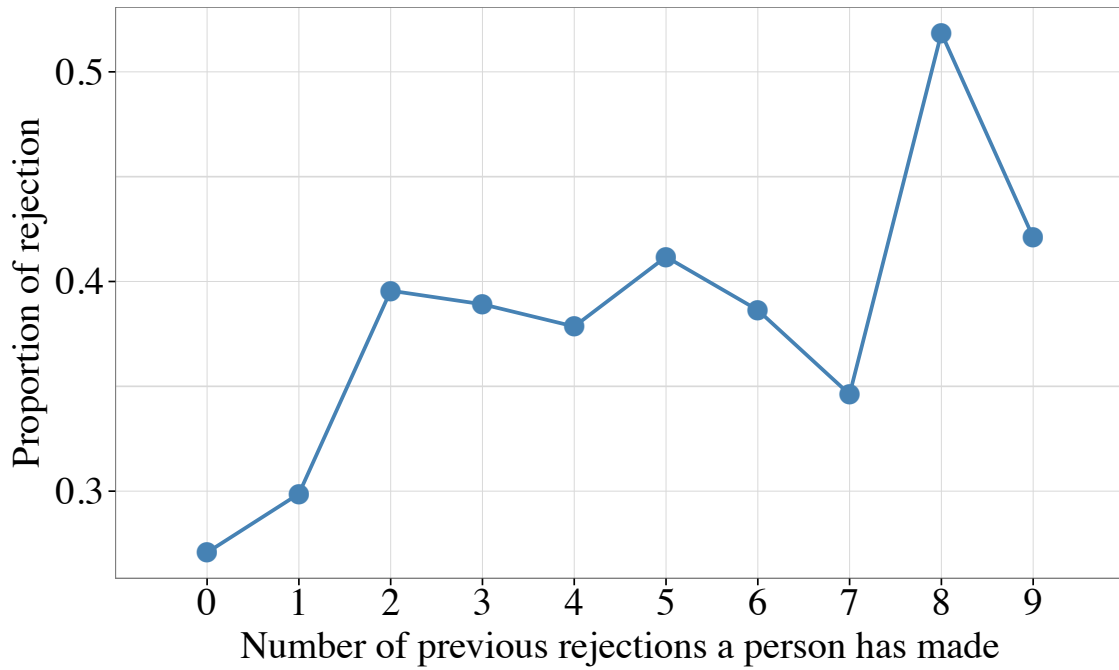
Figure 7: Individual choices over their own experiences

We also compute the number of rounds for the partnership formation process to achieve the equilibrium outcome. Table 8 presents the number of rounds to equilibrium for each of the five social networks. It can be seen from Table 8 that social network 1 only needs 2.1 rounds on average to reach equilibrium, but it takes 4.7 rounds on average for social network 5. These results indicate that when we have more players, it will take more rounds to reach the equilibrium outcome. The more links a given social network has, the more rounds it takes to achieve the equilibrium outcome.

Table 8: Number of rounds to equilibrium for initial social networks

Period	Network 1	Network 2	Network 3	Network 4	Network 5
1	2.1	2.9	2.5	3.3	5.0
2	2.1	2.8	2.5	3.3	4.8
3	2.1	2.8	2.6	3.0	4.6
4	2.0	2.9	2.2	3.3	4.8
5	2.3	3.1	2.2	3.3	4.6
Average	2.1	2.9	2.4	3.2	4.7

B   Experimental Instructions

We would like to thank you for having agreed to participate in this economics experiment. During this experiment, you will earn a given sum of money. Your earnings are stated in experimental currency unit (ECU). At the end of the session they will be converted to euros using the following rate of conversion :

1 ECU = 0,1 Euros

So **10 ECU= 1 Euros**

Besides the earnings you will make during the experiment, you will receive a 3 Euros participation fee. Your earnings will be paid using a bank transfer in a maximum of 4 weeks . All the decisions which you will take during this experiment are anonymous. You will never have to identify yourself on the computer.

The experiment consists of several periods. At the beginning of every period the groups of players are randomly formed. The links between the members of the same group are represented in the form of a graph. In a graph a player can form a pair with his direct neighbors but not with the other players. The number of players and the structure of the graph change every five periods. The first two periods of the first sequence are trial periods

which are not taken into account to determine your earnings. This experiment contains a total of 27 periods.

Example 1 :
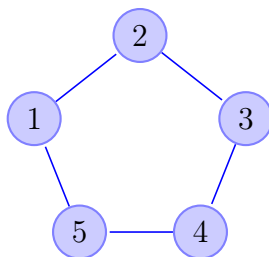
In this group of 5 players, player #1 can form a pair with players #2 and #5 but not with players #3 and #4.

A player is chosen randomly among every group to be the claimant. All the players in the graph have an equal chance to be chosen. A neighbor chosen randomly among the neighbors of the claimant is requested to form a pair with the claimant. All the neighbors of the claimant have an equal chance to be chosen.

If this chosen neighbor accepts to form a pair with the claimant, then:

- The pair leaves the graph: all the links that linked the pair to the rest of the graph are deleted. The period ends for the two players of the pair.

- The claimant earns 20 ECU.

- The neighbor that accepted to form the pair with the claimant earns 20 - 8 ECU, so 12 ECU.

- If there is another possibility of forming a new pair in the remaining graph, another player is chosen randomly among the remaining players to be the new claimant.
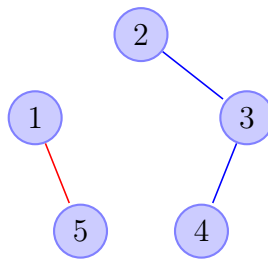
If this chosen neighbor refuses to form a pair with the claimant, then:

- The link that linked the claimant to this neighbor is deleted.

- A neighbor is chosen randomly among the remaining neighbors to form a pair with the claimant.

- If the claimant has no remaining neighbors, then if there is another possibility of forming a new pair in the remaining graph, another player is chosen among the remaining players to be the new claimant.

Example 2 : In the graph of example 1. We suppose that player #5 is chosen to be the claimant. We suppose that among the neighbors of player #5 (in this case player #1 and #4), player #1 is chosen to form a pair with player #5.
If player #1 accepts to form the pair with player #5.

- Players #1 and #5 are no longer linked to the remaining graph.

- Player #5 earns 20 ECU for this period.

- Player #1 earns 12 ECU for this period.

- A new claimant is randomly chosen among the players of the remaining graph formed by players #2, #3 and #4.

If player #1 refuses to form a pair with player #5.

- Players #1 and #5 are no longer linked in the graph.

- Player #4 has the opportunity to form a pair with player #5.
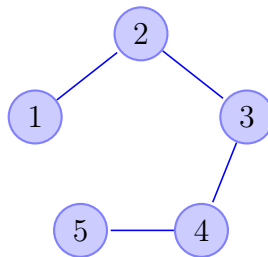
In the end, a claimant that forms a pair earns 20 ECU. A neighbor player who was chosen to form a pair and he accepts, earns 12 ECU. A player that doesn't belong to any pair at the end of a period, earns 0 ECU.
After the last period of the last sequence, 10 periods will be drawn randomly among the periods except the trial periods. The earnings obtained for these 10 periods will determine your earnings for this experiment. Every period has an equal chance to be drawn.

You are 21 participants in the room. When the number of players in the group is 4 or 5, there is then a participant that is randomly chosen, that won't be able to participate during one period. In this case, his earning for this period is 10 ECU.

It is totally forbidden to communicate between each other during the experiment. Any communication may cause the exclusion of the participant from the experiment without compensation. We kindly ask you to reread carefully these instructions and answer the questionnaire which is going to appear on your screens. Every correct answer to this questionnaire will yield a profit of 2 ECU . If you have questions - now or during the experiment, kindly call us by pressing your call button. We shall come to answer you in private.

A series of questions will be given to you after the 27 periods of the experiment. Some of these questions will allow you to win additional earnings