Youth Unemployment and Jobless Recoveries: A Risk-based Explanation

Indrajit Mitra   Yu Xu[*]

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Abstract

We quantitatively analyze an equilibrium job-matching model in the presence of time-varying discount rates and persistent aggregate shocks to labor productivity. In our model workers and firms learn about an unobservable, idiosyncratic component of match productivity. We obtain three results. First, the unemployment rate of young, inexperienced workers is more sensitive to economic conditions than older, experienced workers. Second, labor productivity shocks are amplified and propagated more strongly in states with higher discount rates. We find this effect to be quantitatively large. Third, our model features jobless recoveries which are more pronounced after recessions in which risk premia is higher. We provide empirical evidence for these findings.

1 Introduction

The unemployment rate of young workers is much more sensitive to economic conditions than that of prime-age workers. Figure 1 shows youth unemployment rates increase sharply relative to that of older workers at the onset of a recession and is slower to revert to mean levels. The experience of OECD countries has been similar during and after the recent crisis.[1] Unemployment early in an individual's career has been documented to result in a large and long-lasting drop in lifetime earnings (Topel and Ward, 1992; Mroz and Savage, 2006). In spite of the significant negative welfare consequences of youth unemployment, a successful explanation of the business cycle dynamics of youth unemployment which is in agreement with the data is lacking. A quantitative understanding of the latter also improves our understanding of aggregate unemployment fluctuations. Young workers contribute disproportionately to the amplitude and persistence of aggregate unemployment fluctuations not only because their unemployment rate is more volatile than the average worker, but also because at any given point in time, the young constitute a significant fraction of the unemployed population.[2]

In this paper we build on a standard labor-search model and add two key ingredients. First, we assume that firms and workers learn about an unobservable component to their productivity from observed output. Second, we assume that in determining the present value of retaining a worker or hiring a new one, firms discount expected future cash flows from the match using a discount rate which varies with macro-economic conditions.[3] These two ingredients generate three new results. First, we show that compared to older, more experienced workers, the unemployment rate of young workers is higher and more sensitive to economic conditions. To the best of our knowledge, our paper is the first to generate a large, empirically realistic difference between the volatility of youth unemployment rates and that of

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[1]The youth unemployment rate in Italy was above 38% in 2016, compared to about 20% in 2007.

[2]Between 1951 – 2016, on average, more than a third of unemployed workers in the US were less than 24 years old.

[3]For evidence on time-varying discount rates, we point the reader to the large literature in asset pricing. For a review see Cochrane (2011).
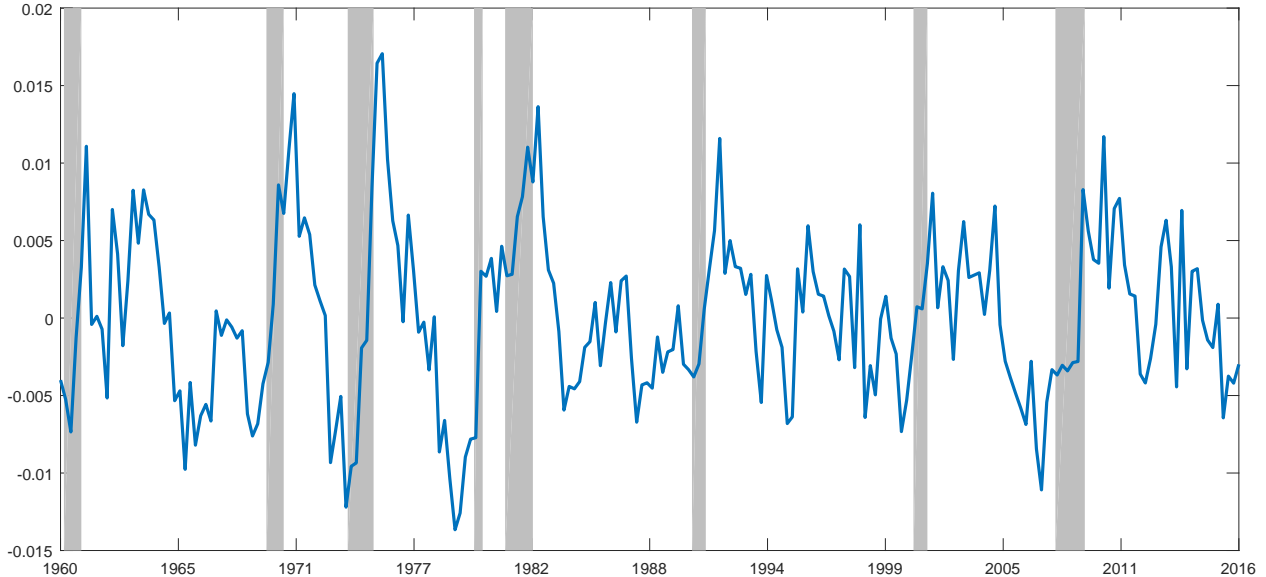
Figure 1: Difference in unemployment rates, Young minus Old. Difference in seasonally adjusted unemployment rates of young (20-24 year old) and old (35-44 year old) workers. Each series is constructed by the BLS from the Current Population Survey (CPS). The quarterly data shown here is computed by averaging deseasonalized monthly numbers and the trend is removed using an HP filter with smoothing parameter 1600. The grey bands are NBER recessions.

older workers.[4] Second, the difference in unemployment rates between young and old workers is more sensitive to labor productivity when discount rates are high. This finding has the implication that intervention policies which reduce risk-premia, have large and long-lasting effects on the dynamics of unemployment levels of all workers, especially younger ones. Third, our model features jobless recoveries which are more pronounced after recessions with higher risk premia. During such episodes, average unemployment levels remain well above normal levels long after measured labor productivity has recovered. We test the first two predictions of our model in US data and find empirical support. Figure 2 shows evidence of our third prediction during and after the Great Recession.

Our assumption of firms and workers learning about match-quality from observed output generates ex-post heterogeneity across matches. High output realizations lead to upward

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[4]See Rios-Rull (1996), Gomme, Rogerson, Rupert, and Wright (2005), Hansen and Imrohoroglu (2009), and Jaimovich and Siu (2009) for attempts using life-cycle models which fail to generate empirically realistic differences between young and old workers.

revisions in expected future output, while low realizations lead to downward revisions and an increase in the likelihood of the match being dissolved. In our model we make the stark assumption that all differences across workers is unobservable. In reality, of course, workers and firms are also heterogenous along many observable dimensions. We view our model as providing a simple benchmark to which some of these other differences (such as differences in worker ability) could be added.

Our assumption that an existing worker or a potential hire's value depends on time-varying discount rates is in contrast to canonical labor models in the search literature which attribute changes in a firm's labor policy occurring solely in response to changes in labor productivity. We assume that this discount rate used by firm owners is identical to the one they use in discounting future cash flows from other assets they own. In other words, financial markets are informative about future labor market conditions. Our calibrated model shows that discount rate variations avoid the need to assume unrealistically large variations in labor productivity and play a large quantitative role in explaining the volatility and persistence of unemployment fluctuations of young and old workers over the business cycle.

The effect of each of the above two ingredients in isolation has been studied by prior literature. Our model of job-matching builds on early work by Jovanovic (1979), while Hall (2017) studies the effect of variable discount rates on aggregate unemployment dynamics. Our contribution is to quantify the interaction of labor market heterogeneity and time-varying risk-premia, and to demonstrate their effects on the composition of the workforce over the business cycle.

The intuition for our first result is as follows. In our model, older workers have a lower unemployment rate compared to younger workers because of a survivorship effect: they have had time and also more attempts to be matched to agreeable positions. In contrast, the separation probability of younger workers is higher as they sort through jobs to be better matched. When the economy transitions into a recession, the job-finding probability declines

uniformly for all workers.[5] However, because the young started the period with a higher unemployment rate, the flow out of unemployment declines more for this group relative to older workers. Consequently, they experience a bigger increase in the unemployment rate compared to the old.

Our second result, namely that the difference in unemployment rates between young and old workers is more sensitive to labor productivity when discount rates are high can be understood as follows. When risk premia is high, a drop in labor productivity is bad not only because future cash flows are expected to be lower, but also because a high (and persistent) discount rate further reduces the net present value of these cash flows. The net effect is a large reduction in hiring and an increase in firing, especially of younger workers.

Our result on jobless recoveries is due to cleansing of poor matches. In a recession with higher risk premium, more bad matches are terminated. This leads to an improvement of the average quality of the remaining pool of workers. This leads to an increase in the output per worker which can more than offset the drop in exogenous productivity. Unemployment is still quite high even as measured productivity has more than recovered. Through the lens of our quantitative model, we find labor market recoveries are slower after deeper recessions.

A growing literature examines the effect of labor market frictions, such as rigid wages, on the returns of the aggregate stock market and the cross-section of asset returns. Our paper is the first to show that financial markets are informative about the composition of the labor force in a quantitative dynamic model. Although there are many potential causes for differences in the employment dynamics of young and old workers, we focus on learning about the quality of worker-firm match because it allows us to stay as close as possible to the standard paradigm of labor market search, while still generating non-trivial results. We view this as providing a useful benchmark to which various other frictions (for example, financial frictions) could be added.

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[5]To simplify our analysis, we assume that the job-finding probability for unemployed workers is independent of age.

Literature Review

Our paper belongs to the labor search and matching literature of McCall (1970), Mortensen (1970), Diamond (1981), Diamond (1982a), Diamond (1982b), Pissarides (1985), and Mortensen and Pissarides (1994). For a comprehensive list of references, we refer the reader to Rogerson, Shimer, and Wright (2005). While most of this literature assumes exogenous separation rates, in our model separation rates endogenously arise as a result of firms and employees learning about match-quality. We build on early work by Jovanovic (1979) and adapted to a labor-search framework by Moscarini (2005). While these papers analyze the behavior of wage dynamics and unemployment in the steady-state, we focus on an analysis of aggregate shocks.

Our paper builds on the analysis of Hall (2017) who shows that accounting for variation in discount rates can potentially allow the canonical search models to quantitatively match the large volatilities of labor market quantities observed in the data – a shortcoming of search-friction based models pointed out by Shimer (2005). However, in contrast to Hall (2017) which focuses on explaining the dynamics of aggregate labor market quantities, we focus on the heterogenous response of unemployment rates of young and old workers to aggregate shocks. In a companion paper Mitra and Xu (2017), we show that the sensitivity of unemployment rates for young and old workers additionally depend on the risk characteristics of the respective industries in which they are employed. In particular, unemployment rates, especially that for young workers, are more sensitive to business cycle conditions in industries that exhibit higher cyclicality in firm values (i.e. "high beta" industries).

Our paper also contributes to the literature on business cycle dynamics of youth unemployment. For an extensive study on the macro-economic causes of youth unemployment see the NBER volume by Freeman and Wise (1982) which contains the influential work by Clark and Summers (1981). Clark and Summers (1981), Rios-Rull (1996), Gomme, Rogerson, Rupert, and Wright (2005) and Guvenen, Schulhofer-Wohl, Song, and Yogo (2017) document

the much higher volatility of young workers compared to older workers in US data; Jaimovich and Siu (2009) establishes that this fact extends to G7 countries. On the quantitative side, Rios-Rull (1996), Gomme, Rogerson, Rupert, and Wright (2005), Hansen and Imrohoroglu (2009), and Jaimovich and Siu (2009) are more recently examples which focus on life-cycle considerations to explain the difference in unemployment volatility at various life-cycle stages. In contrast, our paper views large time-variation in the value of a match between an employee and a firm driven by changes in the cost of capital of the firm as the main driver of the spread in volatility between young and old workers.

There is a literature which examines the impact of search frictions on asset prices. Petrosky-Nadeau, Zhang, and Kuehn (2017) is an example. There is also a growing literature which examines the effect of labor market frictions on the cross-section of expected returns. Danthine and Donaldson (2002), Uhlig (2007), and Favilukis and Lin (2015) among others study the effect of sticky wages on the cost of capital of firms. Belo, Lin, and Bazdresch (2014) study changes in a firm's risk premium arising from changes in a firm's hiring rate. In contrast to these papers, we analyze the effect of firm valuation on labor policies of the firm and the composition of its labor force.

The rest of this paper is organized as follows. In Section 2 we describe our model. In Section 3 we present quantitative results of our model, and in Section 4 we present our empirical findings. Section 5 concludes. All proofs are relegated to the Appendix.

2 The Model

In this section, we present a labor search model with unobserved match quality and time varying risk premia. We first introduce the macroeconomic environment before describing the labor market choices of firms and workers.

2.1 The Economy

The economy runs in discrete over and infinite horizon. Operating in this economy is a representative household comprised of a unit mass of ex-ante identical workers and a large number of capitalists who create and operate firms. The large number of capitalists ensures free entry for firm creation. In addition, there is a government that levies lump sum taxes in order to provide unemployment benefits.

Firms are created when vacancies, which are posted by capitalists, are successfully matched to workers. The process of matching is imperfect along two dimensions. First, labor markets are subject to search frictions so that it takes time to fill vacancies. In particular, a total of $m(U, V)$ meetings take place between prospective workers and vacancies when U unemployed workers search for jobs and V vacancies are available. Following den Haan, Ramey, and Watson (2000), we parameterize the matching function to have the following form:[6]

$$m(U, V) = \frac{UV}{(U^\iota + V^\iota)^{\frac{1}{\iota}}}, \iota > 0. \tag{1}$$

The contact rate between unemployed workers and vacancies depends on labor market tightness $\Theta \equiv V/U$. In particular, the probability of an unemployed worker meeting a vacancy is given by $f(\Theta) = m(U, V)/U = (1 + \Theta^{-\iota})^{-\frac{1}{\iota}}$, while the probability of a vacancy meeting a prospective (unemployed) worker is $g(\Theta) = m(U, V)/V = (1 + \Theta^\iota)^{-\frac{1}{\iota}}$.

The second labor market imperfection is that workers need not be matched to their ideal jobs or, equivalently, not all vacancies are filled by ideal candidates. In particular, matches can differ in their quality $\nu_{it} \in \{H, L\}$, which could either be of high (H) or low (L) type. A match i generates output

$$y_{it} = e^{z_t + \mu(\nu_{it}) - \frac{1}{2}\sigma^2 + \sigma\varepsilon_{it}} \tag{2}$$

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[6]This parameterization has the convenient property that the resulting meeting probabilities automatically lie between 0 and 1.

in period t, where aggregate productivity z_t is observable and follows an AR(1) process:

$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}, \tag{3}$$

with autocorrelation ρ_z, volatility σ_z and normally distributed innovations $\varepsilon_{z,t} \sim \mathcal{N}(0,1)$. Output y_{it} is also subject to a match specific component $\mu(\nu_{it})$ that is higher when the match is of high quality (i.e. $\mu(H) > \mu(L)$), as well as a match specific shock $\varepsilon_{it} \sim \mathcal{N}(0,1)$. Following Moscarini (2005), we assume that match quality ν_{it} is not directly unobservable by either party. Instead, it must be inferred from observed output from the match. However, the presence of idiosyncratic output shocks ε_{it} means that the match quality type ν_{it} is never perfectly observed. Inferences regarding match quality types are Bayesian in nature: time series observations for the aggregate state ω_t and output y_{it} generates a filtration $\mathscr{F}_{it}$ from which parties update their beliefs from an initial common prior p_0 to form posterior beliefs $p_{it} = \mathbb{P}(\nu_{it} = H \,|\, \mathscr{F}_{it})$ regarding the quality of the match. In turn, the presence of idiosyncratic productivity shocks will lead to cross-sectional differences in beliefs p_{it} regarding the quality of different matches.

Neither firms nor workers can commit to a wage contract. The presence of search frictions creates a surplus which is split between workers and firms according to a generalized bargaining rule in which workers have bargaining power $\eta \in [0,1]$. Key in the determination of wages is the valuation of the surplus from a match. For this, we assume that there is perfect risk-sharing between members of the representative household, so that idiosyncratic risks are not priced and both workers and capitalists are symmetric in their assessment of systematic risks.[7] We assume that investors face complete asset markets for payoffs that depend only on aggregate outcomes. It then follows that there exists a unique stochastic discount factor (SDF) $\Lambda_{t,t+1}$ whose changes are driven purely by aggregate shocks. We assume that the SDF

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[7]This is a standard assumption in the labor search literature. For example, see Shimer (2010) for a textbook treatment.

is given by:

$$\Lambda_{t,t+1} = \exp\left\{-r_f - \frac{1}{2}x_t^2 - x_t\varepsilon_{z,t+1}\right\}, \tag{4}$$

where the risk free rate r_f is assumed to be constant, and x_t is the market price of risk for aggregate productivity shocks $\varepsilon_{z,t+1}$. We assume that the market price of risk varies over time according to an AR(1) process

$$x_t = (1 - \rho_x)\overline{x} + \rho_x x_t + \sigma_x\varepsilon_{x,t}, \tag{5}$$

with mean $\overline{x}$, autocorrelation ρ_x and volatility σ_x. For simplicity, innovations to the market price of risk $\varepsilon_{x,t} \sim \mathcal{N}(0,1)$ are assumed to be orthogonal to aggregate productivity innovations $\varepsilon_{z,t}$ (alternatively, x_t can be thought of as the component of market price of risk that is orthogonal to aggregate productivity). We do not take a stance on the microfoundations behind the stochastic discount factor.[8] Rather, we take asset prices as given and instead focus on the implications of time varying risk premia for labor market outcomes. This approach is very much in line with Hall (2017) who finds that movements in risk premia can have large effects on labor market tightness over the business cycle.

Finally, we complete the description of the economy by describing the timing of events within each period t, which goes as follows:

(i) At the start of period t, there is a mass of $N_t = 1 - U_t \in [0,1]$ previously employed workers. These incumbent workers may differ in their belief regarding their match quality p_{it}. The distribution of incumbent match quality beliefs at the start of the period t is denoted by $\mathcal{P}_t$. That is, for a given set $A \subset [0,1]$, $\mathcal{P}_t(A) \in [0,1]$ gives the fraction of incumbent workers with belief p_{it} contained in A.

(ii) Nature draws the aggregate states z_t and x_t.

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[8]A standard list of candidates from the asset pricing literature include habit (Campbell and Cochrane, 1999), long run risk (Bansal and Yaron, 2004), and disaster risk (Rietz, 1988), amongst others.

(iii) Capitalists post a total of V_t vacancies. The mass, $U_t = 1 - N_t$, of prospective unemployed workers are then matched to vacancies according to the matching function (1). The initial match type $\nu_{i0} \in \{H, L\}$ is determined by nature and has a probability of $p_0 \in [0, 1]$ of being the high type, where p_0 is the initial prior. Hiring and firing decisions are then made conditional on both the aggregate state and match quality beliefs. Wages are then set according to a generalized Nash bargaining rule.

(iv) Output y_{it} is realized. Match quality beliefs are updated for the next period in a Bayesian manner.

(v) Wages are paid and consumption takes place. Unemployed workers receive unemployment benefits which are financed through lump sum taxation.

(vi) Matches exogenously separate with probability s.

The above timing of events is summarized graphically in Figure 3. With this precise timing in mind, we proceed to characterize the remaining details of the model.

2.2   Firms and workers' problem.

A matched firm-worker pair has two decisions: whether or not to continue with the match, and conditional on continuing, how to split the resulting match surplus. In addition, capitalists have a choice regarding whether or not to post vacancies. These decisions depend on the present value of a match which is determined by discount rates as well as the expected productivity of a match. The latter crucially depend on learning about the (unobserved) quality of the match.

Bayesian learning.   We define p_{it} to be the *start of period* belief of a high quality match $p_{it} = \mathbb{P}\left(\nu_{it} = H \,|\, \mathscr{F}_{it}\right)$ where, due to our timing convention, the filtration $\mathscr{F}_{it}$ used in the

learning problem is generated from all *past* observations of output from the match $\{y_{is}\}_{s<t}$, as well as all *past and present* observations for the aggregate states $\{z_s, x_s\}_{s \leq t}$.

The expected output from a match, conditional on match quality belief p_i and aggregate productivity z is $\mathbb{E}[y_i | p_i, z] = e^z \left[p_i e^{\mu(H)} + (1 - p_i) e^{\mu(L)} \right]$. Clearly, matches with higher match quality beliefs are expected to be more productive. Bayesian updating for the posterior match quality belief compares realized output against expected output and makes upward (downward) adjustments in beliefs in the event of a positive (negative) performance surprise. More specifically, the match quality belief for the next period is given by $p_{i,t+1} = \mathbb{P}(\nu_{it} = H | \mathscr{F}_{it}, y_{it}) = p'(y_{it}, p_{it}, z_t)$, where the posterior is given by the following expression:

$$p'(y, p, z) = \frac{pe^{-\frac{1}{2\sigma^2}\left[\log y - \left(z + \mu(H) - \frac{1}{2}\sigma^2\right)\right]^2}}{pe^{-\frac{1}{2\sigma^2}\left[\log y - \left(z + \mu(H) - \frac{1}{2}\sigma^2\right)\right]^2} + (1 - p)e^{-\frac{1}{2\sigma^2}\left[\log y - \left(z + \mu(L) - \frac{1}{2}\sigma^2\right)\right]^2}}. \tag{6}$$

Firms' problem. The value of a matched firm at the start of the period $F(p, z, x)$, after observing the aggregate state (z, x) but before observing the current period's output, is given by

$$F(p, z, x) = \max\left\{0, d(p, z, x) + (1 - s)\mathbb{E}\left[\Lambda(x, \varepsilon'_z)F(p', z', x') | p, z, x\right]\right\}, \tag{7}$$

where p denotes the start of period match quality belief. Firm value (7) reflects the option for the capitalist to walk away from the match, in which case the value will be worth zero. Should the capitalist stick with the match, he obtains expected dividends $d(p, z, x) = e^z \left[pe^{\mu(H)} + (1 - p)e^{\mu(L)} \right] - w(p, z, x)$, which is just the difference between expected output and wages. In addition, the firm is kept as a going concern so long as the firm does not exogenously separate at the end of the period (this occurs with probability $1 - s$). The future cashflows of the firm are discounted according to the SDF (4), and cashflow forecasts naturally take into account Bayesian updating of match quality beliefs (6).

It can be shown that the continuation value of the firm is increasing in p (see Proposition 1 in Appendix A). This implies that firms follow a threshold strategy when deciding whether

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or not to continue with the match. In particular, matches with match quality belief below a cutoff $\underline{p}(z, x)$ are dissolved, where the firing threshold $\underline{p}(z, x)$ is characterized as the solution to the following indifference condition:

$$0 = d(\underline{p}(z, x), z, x) + (1 - s)\mathbb{E}\left[\Lambda(z, \varepsilon_z')F(p', z', x') \,\middle|\, \underline{p}(\omega), \omega\right], \tag{8}$$

while matches with match quality belief at or above $\underline{p}(z, x)$ are continued. The threshold $\underline{p}(z, x)$ is symmetric between firms and workers meaning that workers will also find it more favorable to stick with the match (walk away) whenever p is greater (less than) the threshold $\underline{p}(z, x)$.

Vacancy creation. Capitalists can freely post vacancies subject to a per unit vacancy creation cost of $\kappa > 0$. The value of a vacancy in state (z, x) is given by

$$F_V(z, x) = g(\Theta(z, x))F(p_0, z, x)\mathbb{1}\left(p_0 \geq \underline{p}(z, x)\right) \tag{9}$$

and takes into account the probability $g(\Theta(z, x))$ of meeting a potential employee when the aggregate state is (z, x). The vacancy is worthless ex-post if either it fails to get matched to a potential worker, or if a matched worker's initial match quality p_0 is too low to clear the threshold $\underline{p}(z, x)$.

A capitalist's decision to post a vacancy depends on the value of a vacancy relative to the unit cost of posting a vacancy, and vacancies will be posted so long as the former remain greater. Since there is free entry, the equilibrium amount of vacancies posted, V, is determined as the solution to the following complementary slackness problem:

$$\kappa \geq F_V(z, x) \tag{10}$$

with equality if and only if equilibrium vacancies V is strictly positive, where the left hand

side of (10) is the unit cost of posting a vacancy, while the right hand side is the expected value of a vacancy (9).

Workers' problem. The value function for a worker that is employed at the start of the period, $J_e(p, z, x)$, is given by

$$J_e(p, z, x) = \max \left\{ J_{eu}(z, x), w(p, z, x) + \mathbb{E}\left[\Lambda(x, \varepsilon_z') \left[s J_u(z', x') + (1-s) J_e(p', z', x')\right] | p, z, x\right] \right\}.$$

(11)

A matched worker can either quit or stay with the match. In the latter case, the worker obtains wages $w(p, z, x)$, and the match continues so long as the exogenous separation shock (which occurs with probability s) does not materialize.

A newly unemployed worker has value function

$$J_{eu}(z, x) = b + \mathbb{E}\left[\Lambda(x, \varepsilon_z') J_u(z', x') | z, x\right].$$

(12)

That is, he obtains unemployment benefit b in the current period and searches for new jobs starting from the next period. Note that our timing convention does not allow for a newly unemployed worker to immediately search for a new job.

The value for an already unemployed worker who is searching for a new job is given by

$$J_u(z, x) = f(\Theta(z, x)) J_e(p_0, z, x) \mathbb{1}\left(p_0 \geq \underline{p}(z, x)\right) + \left[1 - f(\Theta(z, x))\right] J_{eu}(z, x).$$

(13)

With probability $f(\Theta(z, x))$, the unemployed worker is matched to a vacancy, and the worker becomes employed so long as p_0 clears the threshold $\underline{p}(z, x)$. Otherwise, the worker remains unemployed in which case he obtains unemployment benefits b and continues searching for a job next period.

Wages. Wages are determined by standard Nash bargaining in which case, the total match surplus $S(p, z, x) \equiv J_e(p, z, x) - J_{eu}(z, x) + F(p, z, x)$ is split between the worker and the firm with the worker surplus accounting for a share $\eta \in [0, 1]$ of the total match surplus. In particular, the worker obtains $J_e(p, z, x) - J_{eu}(z, x) = \eta S(p, z, x)$ while the firm gets $F(p, z, x) = \eta S(p, z, x)$. This characterizes wages:

$$
w(p, z, x) \tag{14}
$$
$$
= \eta \left\{ e^z \left[pe^{\mu(H)} + (1 - p)e^{\mu(L)} \right] + (1 - s)\mathbb{E}\left[\Lambda(x, \varepsilon'_z) F(p', z', x') \,|\, p, z, x \right] \right\}
$$
$$
- (1 - \eta) \left\{ \mathbb{E}\left[\Lambda(x, \varepsilon'_z) \left[sJ_u(z', x') + (1 - s)J_e(p', z', x') \right] \,|\, p, z, x \right] - J_u(z, x) \right\}.
$$

Equilibrium. The notion of equilibrium for the economy is standard: all value functions must satisfy their respective Bellman equations (cf equations (7), (11), (12), and (13)), wages must be set according to the Nash bargaining rule (14), and labor market tightness must be determined according to the free entry condition (10). Appendix A provides the formal definitions and proves that an equilibrium exists. It also provides a scheme for the numerical verification of equilibrium uniqueness.

2.3 Aggregate Quantities

Laws of motion. The aggregate dynamics of the economy are determined by the start of period aggregate employment $N_t \in [0, 1]$, the start of the period distribution of match quality $\mathcal{P}_t$, which we view as a probability with support on $[0, 1]$, as well as the exogenous aggregate state ω_t.

Start of period employment N_t evolves as follows:

$$
N_{t+1} = (1 - s) \left\{ (1 - N_t)f(z_t, x_t)\mathbb{1}\left(p_0 \geq \underline{p}(z_t, x_t) \right) + N_t \mathcal{P}_t \left([\underline{p}(z_t, x_t), 1] \right) \right\}. \tag{15}
$$

The law of motion (15) reflects the following: at the start of period t, there are N_t employed

with match quality distributed according to $\mathcal{P}_t$. Of the employed, those with match quality below $\underline{p}(z_t, x_t)$ separate from their jobs and so $N_t \mathcal{P}_t \left([\underline{p}(z_t, x_t), 1] \right)$ is the amount of employed at the end of the period out of those who were initially employed at the start of the period. In addition, there are $1 - N_t$ unemployed at the start of the period. Of these individuals, a fraction $f(z_t, x_t)$ are matched to vacancies which are then consummated as long as the initial prior p_0 is above the threshold $\underline{p}(z_t, x_t)$—this gives an employment of $(1 - N_t) f(z_t, x_t) \mathbb{1} \left(p_0 \geq \underline{p}(z_t, x_t) \right)$ at the end of the period stemming from those who were unemployed at the start of the period. Finally, a fraction s of employed workers separate for exogenous reasons between the end of period t and the start of the next period $t + 1$.

The evolution of the distribution match quality beliefs at the start of each period is as follows:

$$\mathcal{P}_{t+1}(A) = \int_{\underline{p}(z_t, x_t)}^1 \Gamma_A(p, z_t) \, \widetilde{P}_t(dp), \tag{16}$$

where $\mathcal{P}_{t+1}(A)$ gives the fraction of workers at the start of the next period with match quality beliefs lying within the set $A \subset [0, 1]$,

$$\begin{aligned} \Gamma_A(p, z) &= p \mathbb{P} \left(p'(e^{z + \mu(H) - \frac{1}{2}\sigma^2 + \sigma \varepsilon_z'}, p, z) \in A \,|p, z \right) \\ &\quad + (1 - p) \mathbb{P} \left(p'(e^{z + \mu(L) - \frac{1}{2}\sigma^2 + \sigma \varepsilon_z'}, p, z) \in A \,|p, z \right) \end{aligned} \tag{17}$$

is the probability that an individual with start of period match quality belief p will end up having posterior (6) in set A, and

$$\widetilde{\mathcal{P}}_t(dp) = \frac{(1 - N_t) f(z_t, x_t) \mathcal{P}_0(dp) + N_t \mathcal{P}_t(dp)}{(1 - N_t) f(z_t, x_t) \mathbb{1} \left(p_0 \geq \underline{p}(z_t, x_t) \right) + N_t \mathcal{P}_t \left([\underline{p}(z_t, x_t), 1] \right)} \tag{18}$$

is the aggregate distribution of match quality beliefs right after hiring and firing have taken place but before output has been observed, with $\mathcal{P}_0$ denoting a point mass at p_0. The law of motion (16) first computes the posteriors for individuals of a fixed initial match quality belief p according to the function $\Gamma_A(p, z)$. Here, posteriors may differ despite individuals being

ex-ante identical—different idiosyncratic output shocks lead to different posteriors ex-post. These posteriors are then aggregated over groups of workers with different initial match quality beliefs according to the distribution $\widetilde{\mathcal{P}}_t$ in order to arrive at the final distribution of match quality beliefs for the start of the next period.

Of interest for our analysis is the unemployment rate and distribution of match quality beliefs for workers of different cohorts and (potential) experience. A cohort of workers who initially entered the job market at (calendar) time τ will have zero experience $e = 0$ at the start of period τ. This group begins being fully employed $N_{\tau,0} = 1$ and have initial match quality belief distribution $\mathcal{P}_{\tau,0} = \mathcal{P}_0$.[9] These cohort statistics will subsequently evolve as this cohort of workers gain experience over time. The start of period employment rate $N_{\tau,e}$ and match quality distribution $\mathcal{P}_{\tau,e}$ for cohort τ with potential experience $e \geq 0$ at the start of the period can be respectively computed from the aggregate laws of motion (15) and (16) by treating potential experience e as the time variable t and using the initial conditions $N_{\tau,0} = 1$ and $\mathcal{P}_{\tau,0} = \mathcal{P}_0$.

Quantities. The total mass of workers who end up producing at the end of the period

$$N_{end,t} = (1 - N_t)f(z_t, x_t)\mathbb{1}\left(p_0 \geq \underline{p}(z_t, x_t)\right) + N_t\mathcal{P}_t\left([\underline{p}(z_t, x_t), 1]\right), \tag{19}$$

is given by the sum of incumbent workers that survived being fired and newly matched workers. Aggregate output Y_t is the sum of all individual output (2) across the mass $N_{end,t}$ of productive workers:

$$Y_t = e^{Z_t}N_{end,t}, \tag{20}$$

where aggregate labor productivity, defined as log output per worker

$$Z_t = z_t + \log\left(\int_{\underline{p}(z_t, x_t)}^1 \left[pe^{\mu(H)} + (1 - p)e^{\mu(L)}\right]\widetilde{\mathcal{P}}_t(dp)\right), \tag{21}$$

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[9] In our calibrations, the initial distribution $\mathcal{P}_0$ always lies above the firing threshold.

takes into account both exogenous productivity z_t as well as the endogenous productivity of employed workers as determined by the distribution of match quality beliefs, $\widetilde{\mathcal{P}}_t$. The Law of Large Numbers is implicit in the computation of aggregate labor productivity so that idiosyncratic output shocks ε_{it} are diversified away and do not directly show up in expression (21).

Employed workers are paid wages, which total

$$W_t = \bar{w}_t N_{end,t}, \tag{22}$$

where average wages are given by

$$\bar{w}_t = \int_{\underline{p}(z_t,x_t)}^{1} w(p, z_t, x_t)\, \widetilde{\mathcal{P}}_t(dp). \tag{23}$$

Unemployed workers receive unemployment benefits at the end of the period, which total

$$B_t = b(1 - N_{end,t}). \tag{24}$$

Finally, our timing assumptions imply that a total of $V_t = (1 - N_t)\Theta(z_t, x_t)$ vacancies are created at the start of the period. This is determined by the unemployment rate, $1 - N_t$, and market tightness $\Theta(z_t, x_t)$, all at the start of the period. The total amount of resources spent towards vacancy creation is given by κV_t.

3 Quantitative Analysis

We simulate our model at monthly frequency and calibrate it to match aggregate labor market and asset pricing moments.

3.1 Calibration

We use the parameters shown in Table 1. We choose the persistence $\rho_z = 0.9$ and volatility $\sigma_z = 0.01$ of the labor productivity process to match the persistence and volatility of the de-trended, quarterly series for non-farm business real output per person reported by the FRED.[10] We set the persistence of the market price of risk process to $\rho_x = 0.985$ to match the persistence of the price-dividend ratio at quarterly frequency.[11] We choose the parameter $\sigma_x = 0.043$ so that a two-standard deviation increase in x results in the market price of risk increasing by $0.043/\sqrt{1 - \rho_x^2} = 0.25$. Our choice of the mean market price of risk $\overline{x} = 0.22$, then implies that this two-standard deviation increase in x results in a Sharpe ratio of 0.72. We choose the risk-free rate $r_f = 0.0033$ (which is an annual rate of 1.8%) to match the data counterpart.

We follow the labor search literature and choose the values of the parameters κ, ι, b, and η to target the elasticity of wages to productivity and the first two moments of unemployment and vacancies. We choose the curvature parameter of the matching function $\iota = 1.7$, the cost of posting vacancies $\kappa = 2.8$. The results from our simulations and the data counterparts are shown in Table 2. In simulations, the average unemployment rate is 5.6% which is the same as in the data. The volatility of the unemployment rate in our model simulations of 0.87% per quarter is also close to 0.75% in the data.[12] The mean market tightness in our model is 0.50 compared to 0.54 in the data. The volatility of market tightness in our model is 9.7%. The volatility of market tightness as reported by the FRED using JOLTS data between 2001-2017 is 9.48%. We set the bargaining power of workers to $\eta = 0.05$. Our model implied wage elasticity is 0.51 which is close to the empirical estimate of 0.45 reported by Shimer (2005).

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[10]We use an HP filter with a bandwidth of 1600. The series is from 1960 – 2016.

[11]Our value for ρ_x is also close to the persistence of the habit process in Campbell and Cochrane (1999). These authors choose an annual persistence of 0.87 for the log consumption ratio for the habit model, which corresponds to a monthly value of 0.9885.

[12]We report the volatility of the cyclical component of unemployment rates without taking logs. The volatility of the log unemployment rate is 0.16 in the data and 0.17 in our model.

We choose the value of the unemployment benefits parameter $b = 1.7$. The implied unemployment benefit normalized by mean wages is 0.95. This value is the same as the one used by Hagedorn and Manovskii (2008) who interpret the parameter b as capturing the value of leisure in addition to unemployment benefit payments. Since our focus is not to solve Shimer's puzzle, we choose this high value of b to generate volatile labor markets.

Finally, there are four learning parameters: the match-specific productivity parameters μ_H and μ_L, the parameter σ which measures the informativeness of individual output about match-quality, and p_0 which measures the prior belief about initial match-quality.[13] We set $\mu_H = 0.756$. This is a normalization. We choose $\mu_L = -1.4$ to target the cross-sectional dispersion in plant-level total factor productivity of 1.92 measured by Syverson (2004). Our calibration produces an estimate of 1.94. The speed of learning, which depends on the ratio $(\mu_H - \mu_L)/\sigma$, impacts the expected tenure of a new match. Having fixed μ_H and μ_L we choose σ to target the expected duration of a new match of 52 months. In our calibrated model, the expected tenure is 53.5 months. Finally, we include the possibility of matches dissolving for reasons other than those we consider here. We set the probability of these exogenous separations $s = 0.45\%$.

3.2   Youth unemployment rate is more volatile than old

Our model does a good job in matching unconditional moments and the conditional dynamics of unemployment rates of young and old workers. In simulations, we define young (Y) workers as those with between 1–3 years of experience, while old (O) workers have between 13–23 years of experience. These definitions correspond to the age groups considered by the BLS in measuring the unemployment rates of different age groups (20-24 and 35-44 year olds) under the assumption that workers enter the labor force at age 21. In the model simulations, a

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[13]Note that the parameter σ in our model is a measure of the informativeness of individual output about match-quality and not the idiosyncratic volatility of firm output. A firm employs many workers, therefore, match-level idiosyncratic shocks average out within a firm.

cohort of workers who enters in period τ begin their careers being unemployed and without any work experience, and are allocated initial match quality belief $p_0 = 0.4$ when they find a job. Thereafter, we track the unemployment rate and distribution of match quality beliefs of each cohort over time. We consider the CPS equivalent of age-specific unemployment rates so that the period t unemployment rate of workers with $e \geq 0$ periods of potential work experience corresponds to the unemployment rate of the cohort of workers that entered in period $\tau = t - e$.

Unconditional results. Table 3 shows the unconditional model implied moments for the unemployment rates of young and old workers. In the model, the mean unemployment rate is 5.6% for old workers, while that for young workers is 12.9%. The unconditional volatility of the youth unemployment rate is 2.2%, which is about twice that of older workers whose corresponding volatility is 0.9%. These model-implied quantities are in line with the data.

Young, inexperienced workers have a higher unemployment rate relative to older workers because newly formed matches have a higher likelihood of being dissolved. This is because matches which have survived for a long time are of higher quality on average. As bad matches get dissolved, the resulting pool improves in match quality. This can be seen from Panel A of Figure 4 in which we compare the distribution of posterior beliefs for workers with 12 months of experience against that for workers with 180 months of experience. Panel B of Figure 4 shows hazard rates as a function of tenure in the stochastic steady-state. The hazard rate declines as poor matches are dissolved early on during the tenure of employment. According to our model, the first seven or eight years of experience are critical in reducing unemployment risk – additional experience doesn't make much of a difference.

Even though job finding probabilities are symmetric across groups in the model, young workers will on average still experience more volatile flows from unemployment to employment. This is a result of match qualities being poorer on average for young workers, and as a result, these workers will be unemployed more often and therefore be more exposed to fluctuations

in firms' hiring incentives.

Conditional results. Figure 5 shows unemployment rates in more detail by plotting the conditional unemployment rates for young $\mathbb{E}[u_t^Y | x_t = x, z_t = z]$ and old $\mathbb{E}[u_t^O | x_t = x, z_t = z]$ as a function of labor productivity z and the market price of risk x. Panels A and B show how the unemployment rates for Y and O vary with aggregate productivity (z), for different levels of the market price of risk, while Panel C shows the difference in unemployment rates between these two groups $(u^Y - u^O)$ as a function of z for three different levels of the market price of risk. We gleam two results concerning conditional unemployment dynamics from these plots.

First, u^Y is much more sensitive to changes in aggregate productivity than u^O, especially in low productivity states. The intuition for this result is the following. When aggregate labor productivity drops, firms' incentive to post vacancies decline. Fewer vacancies reduce the probability of an unemployed worker meeting a vacancy. In our model, in spite of this decline in job-finding probability being the same for workers of all ages, the young suffer a bigger drop in the flow out of unemployment because this group started the period with a higher unemployment rate relative to older workers. Consequently, the reduction in the flow out of unemployment for the young is higher than the old.

Second, this increased sensitivity of u^Y compared to u^O is higher in states with high risk-premia. For instance, when the market price of risk is at the stochastic steady-state value of 0.22, a two-standard deviation drop in labor-productivity around the mean is accompanied by u^Y increasing about 1% more than u^O. In contrast, the same change in labor productivity leads to twice the increase in $u^Y - u^O$ when the market price of risk is 0.42. The reason for a much sharper increase in u^Y when risk-premia is high is because in addition to a drop in future cash flows (from the drop in labor productivity), there is a further drop in value of these future cash flows because of higher discount rates. This further reduces firms' incentives to post new vacancies.

3.3 Heterogeneity and Risk premia amplify productivity shocks

In this section we show that variations in risk premia have a large effect on the sensitivity of labor markets to productivity shocks.

Panel A of Figure 6 shows the variation of total unemployment rate. From the figure we see that the response of labor markets to variations in labor productivity is amplified when risk premia is high. During normal times when risk-premia is at its steady-state value, the response of labor market quantities to variations in productivity is quite muted. For instance, the difference in the total unemployment rate across two states plus and minus two standard deviations of mean productivity is only 1.3% when risk-premium is at its steady-state value of 0.22. In comparison, when the annualized risk premia increases to 0.73, a difference in unemployment rates between the two productivity states is about 3.1%. From the figure, we also see that the asymmetry in the response to declines in productivity compared to upswings is magnified in states with high risk-premia.

Panel B of Figure 6 shows similar behavior for labor market tightness. When risk-premia is at its steady-state value, a drop in productivity from plus two to minus two standard deviations from the mean results in a decrease in tightness by 0.15. In comparison, when risk-premia is 0.73, the same productivity change results in market tightness decreasing by 0.26.

Time varying risk premia plays a potent role in our model. In models without discount rate variation, changes in the value of a new hire is driven entirely by changes in average labor productivity. In our setting, the effect of shocks to labor productivity depends on current macro-economic conditions as we show in Figure 7. A drop in productivity in a state with a high discount rate is bad not only because productivity is currently low and expected to be low in the near future but this decline is aggravated by a high discount rate which further reduces the net present value of hiring a new worker. The reduction in incentives for the firm to hire a new worker translates into a low job finding probability as illustrated in

Panel B of Figure 7.

Matches in our model are dissolved when the net present value of continuing a current match declines sufficiently enough where firms and workers are indifferent to dissolving the current match and searching for a new one. Panel A of Figure 8 shows the critical threshold value of posterior beliefs at which matches are dissolved. We see that this threshold declines monotonically as a function of average labor productivity z so that relatively poor matches are tolerated in states with high productivity. This is even more true in states where risk-premia is lower. Conversely, controlling for posterior beliefs on the quality of the match between a worker and a firm, the match is more likely to be dissolved when either labor productivity declines or risk-premia increases.

The probability of job loss on average in the cross-section is more ambiguous. This is because the rate at which matches are dissolved depends both on the location of the threshold and also on the mass of workers near the threshold. Panel B of Figure 8 shows that this might result in acyclical separation rates in the aggregate, even though an individual faces a counter-cyclical probability of job loss. At the onset of a recession, matches close to the threshold are dissolved which reduces the mass near the threshold. This cleansing, which is illustrated in Figure 9, can sometimes result in pro-cyclical separation rates, especially in deep recessions with high risk-premia as shown in Panel B.

3.4   Recessions and Jobless Recovery

In this section we use our model to investigate the response of labor markets and output to a one-time recessionary shock. In this experiment we assume that the economy starts out in the stochastic steady state at $t = 0$ and encounters a recession at $t = 1$. We compare results for two recessions with varying severity. The shock to aggregate labor productivity (z), is shown in Panel A of Figure 10 which shows a one-time two-standard deviation decline followed by the usual dynamics. The two recessions differ in the shock to discount rates as

shown in Panel B of Figure 10. The dotted black line represents a milder recession in which the market price of risk increases from the stochastic steady-state value of 0.22 to 0.42 before reverting back to normal. The dashed, red line shows a severe recession in which the price of risk increases to 0.73 before reverting back to normal.

Our two main results are shown in Figure 11. First, we see that when risk-premia is low, unemployment and output are strongly negatively correlated and move one-for-one; that is Okun's law (Okun, 1962) holds. When the discount rate x is close to its steady-state value, we find that a 1% decrease in unemployment is accompanied by a 2% increase in output in our simulations, exactly as in Okun's law. However, this relationship breaks down when risk-premia is high. For instance, in the severe recession, we see from Panel A of Figure 11, that when output is within 0.5% of its long run average value, the aggregate unemployment rate is still about 2% above mean. Such jobless recoveries in episodes of high risk-premia are in line with the findings of Calvo, Coricelli, and Ottonello (2014). These authors document jobless recoveries after financial crises in a large cross-section of countries. When we combine their finding with Muir (2017), who documents an increase in risk-premia after financial crises, we see that our model captures jobless recoveries in periods of high risk-premia.

Second, comparing Panels A and B of Figure 11, we see that the youth are more strongly impacted than the average worker in recessions which are accompanied by higher discount rates. In our simulations, when output has all but recovered from a severe recession in which the market price of risk increased to 0.73 and is within 0.5% of its average value, the youth unemployment rate is still about 5% above its average. In comparison, in this phase of the recovery, aggregate unemployment is only about 2% above mean.

The intuition for these results is as follows. A deeper recession lowers the value of keeping a worker with low expected output. So the separation threshold moves up as shown in panel A of Figure 12. This cleansing has the effect of improving the quality of existing matching which can be seen from panels B and C of Figure 12. The net effect of this is an increase in the average productivity of existing, employed workers. Figure 13 shows the response of

labor markets. We see that labor markets take longer to recover from a recessionary shock in which risk-premium was higher. Figure 14 which shows the response of output.

4 Empirical Evidence

In this section, we test the following predictions of the model regarding the sensitivity of unemployment rates:

1. Compared to older workers, the unemployment rate of young workers is more sensitivity to economic conditions.

2. Labor productivity shocks are amplified and propagated more strongly in states with higher discount rates.

We use quarterly unemployment rates for various age groups as constructed by the BLS from the Current Population Survey. Our definition of old workers are those between the ages of 35-44 years and young workers are those between 20-24 years. Labor productivity (z) is taken to be the nonfarm business sector real output per hour series from FRED. Following the asset pricing literature, we use dividend yields (D/P) to proxy for time varying discount rates, and we use the aggregate value weighted returns series from CRSP to construct this series. All series are first adjusted for seasonal effects using the Census Bureau's X13 program before being HP-filtered with smoothing parameter 1600 to remove long run trends. The final sample is quarterly and spans 1951Q1 to 2015Q4. Table 4 shows the summary statistics.

Table 5 reports the results for one-quarter ahead predictive regressions. All standard errors are Newey-West with 4 lags. Columns (1) through (3) report the results for youth unemployment, columns (4) to (6) report results for older workers, while the final three columns report results for the difference in unemployment rates between young and old workers. Columns (1) and (4) reports the sensitivity of young and old unemployment rates, respectively, to changes in labor productivity in a univariate regression. We see that both

unemployment rates are counter-cyclical, with youth unemployment being about twice as sensitive to changes in labor productivity as old unemployment. The coefficients from the univariate regressions imply that as labor productivity decreases from its 95th percentile value of 1.6% to its 5th percentile value of -2%, youth unemployment rates increase by 1.06% while unemployment rates for older workers increase by 0.48%.

Next, columns (2) and (5) show that youth unemployment rates are also more sensitive to changes in risk premia (as proxied by the dividend yield) relative to older workers. The coefficients from these univariate regressions indicate that youth unemployment is, on average, 1.4 times more sensitive to changes in risk premia relative to that of their older peers. An increase in the (cycle component) of dividend yields increase from its 5th percentile value of -0.15% to its 95th percentile value of 0.24% is associated with a 1.29% (0.73%) increase in youth (old) unemployment rates.

When both productivity and dividend yields are included as regressors, we see from the negative coefficient on the interaction term in columns (3) and (6) that unemployment rates are more sensitive to labor productivity shocks during periods when discount rates are high. Furthermore, the coefficient of the interaction term is larger for the young, indicating that the difference in young and older worker unemployment rates becomes more sensitive to productivity shocks when discount rates are high. The interaction term is large and economically significant. For example, in comparison to a setting in which dividend yields is at its median value, a 95th to 5th percentile decrease in labor productivity is associated with an additional increase of 1.87% (1.13%) in youth (old) unemployment rates when the dividend yield is at its 95th percentile.

5   Conclusion

Differences in the unemployment experience of young and old workers are closely linked to changes in discount rates in addition to changes in labor productivity. We show that the

interaction of labor market heterogeneity and time-varying discount rates generates rich dynamics and provides a possible explanation for higher youth unemployment compared to prime-age workers. We show that explicitly modeling heterogeneity in expected output across workers improves the quantitative fit of business cycle dynamics of aggregate unemployment and provides a possible explanation for jobless recoveries after severe recessions.

We have decided to focus on labor market dynamics by taking discount rate variations as given. One possible avenue of future research would be to further endogenize discount rates. This will allow us to study the asset pricing implications of business cycle variations in the demographic composition of the workforce.

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Table 1: Parameter values. This figure displays parameters for the baseline calibration. The model is calibrated at a monthly frequency.

Parameter	Symbol	Value
AR(1) coefficient of labor productivity, z	ρ_z	0.900
Volatility of labor productivity, z	σ_z	0.010
AR(1) coefficient of discount rate process, x	ρ_x	0.985
Volatility of discount rate process, x	σ_x	0.043
Log-productivity of H type	$\mu(H)$	0.756
Log-productivity of L type	$\mu(L)$	-1.400
Match specific output volatility	σ	5.456
Worker's bargaining power	η	0.050
Fixed cost of vacancy creation	κ	2.800
Curvature of matching function	ι	1.700
Initial prior for match quality belief	p_0	0.400
Exogenous separation probability (%)	s	0.450
Unemployment benefit parameter	b	1.700
Risk-free rate	r_f	0.0015
Market price of risk, intercept	λ_0	0.220
Market price of risk, slope	λ_1	1.000

Table 2: Labor market and asset pricing moments. Labor market moments in the model and in the data are quarterly averages of monthly series reported by the FRED and constructed by the BLS from the Current Population Survey (CPS) between Q1 1951–Q4 2016. Market tightness is from the JOLTS series between 2001 – 2017. In our model, young workers have between 1–3 years experience, while older workers have between 13–23 years experience. In the data young workers are between the ages of 20 – 24, while older workers are between 35 –44 years old. All variables are as deviations from an HP trend with smoothing parameter 1600.

Moment	Model	Data
Market tightness:		
Mean	0.50	0.54
Volatility (%)	9.7	9.2
Autocorrelation	0.90	0.94
Unemployment:		
Mean (%)	5.6	5.6
Volatility (%)	0.87	0.75
Autocorrelation	0.94	0.94
Correlation (unemployment, market tightness)	-0.93	-0.89
Job finding rate	0.41	0.45
Expected tenure at entry (months)	53.5	52
Elasticity of wages to productivity	0.51	0.45
Dispersion of plant output: 90 percentile/10 percentile	1.94	1.92
Risk-free rate	0.018	0.018
Sharpe ratio (mean)	0.22	0.22
Sharpe ratio (volatility)	0.25	

Table 3: Unconditional moments of unemployment rates of young and old workers Moments in the data are calculated from the unemployment rates for young (20–24 year old) and old (35 –44 years old) constructed by the BLS from the Current Population Survey (CPS). The raw series is first de-seasonalized and then de-trended using an HP filter with bandwidth 1600. Quarterly data is computed by averaging monthly numbers. The model is simulated at monthly frequency. Moments of quarterly simulated data are computed by averaging monthly numbers.

	Mean (Data)	Std. Dev. (Data)	Mean (Model)	Std. Dev. (Model)
u_Y (%)	9.8	1.2	12.9	2.2
u_O (%)	4.4	0.7	5.6	0.9

Table 4: Summary statistics. The sample is quarterly and is for the period 1951Q1-2016Q4. All series are deseasonalized and HP filtered with smoothing parameter 1600 (and therefore have zero means). Unemployment rates are taken from the BLS for the age groups 20-24 (young) and 35-44 (old). Labor productivity is log real output per hour, also taken from the BLS. The dividend price ratio is the ratio between quarterly dividends and the end of quarter stock price, and is computed using the value weighted aggregate market index taken from CRSP.

A. Summary statistics					
	std	5%	50%	95%	Obs
Unemployment rate, young	0.0118	-0.18	-0.001	0.0232	264
Unemployment rate, old	0.0069	-0.0099	-0.0009	0.0128	264
Labor productivity	0.0104	-0.0204	-0.0002	0.0162	264
Dividend price ratio	0.0011	-0.0015	-0.0001	0.0024	264

B. Correlations			
	Unemp, old	Productivity	Div. yield
Unemployment rate, young	0.94	-0.056	0.13
Unemployment rate, old		-0.01	0.13
Labor productivity			-0.38

Table 5: Unemployment, labor productivity, and discount rates. Predictive regressions of one-quarter ahead unemployment rates. The dependent variables are the unemployment rate of young (Y) workers who are individuals with ages between 20-24, the unemployment rate of old (O) workers who are between 35-44 years old, and the difference in unemployment rates. Right hand side variables are non-farm business sector real output per hour per worker (z_t), and the dividend price ratio ($p_t = D_t/P_t$). The data is quarterly between 1951Q1 to 2016Q4. All variables are de-seasonalized, and de-trended using an HP filter with bandwidth 1600. Standard errors are Newey-West with 4 lags. Numbers in parentheses are t-statistics.

	$u_{Y,t+1}$			$u_{O,t+1}$			$u_{Y,t+1} - u_{O,t+1}$		
z_t	-0.29		-0.17	-0.13		-0.06	-0.15		-0.11
	[2.38]		[1.45]	[1.83]		[0.88]	[3.04]		[2.13]
dp_t		3.31	2.06		1.88	1.27		1.43	0.79
		[3.44]	[2.47]		[3.43]	[2.65]		[3.07]	[1.82]
$z_t \times dp_t$			-216.48			-130.88			-85.6
			[3.29]			[3.25]			[3.06]
$const$	0	0	-0.0009	0	0	-0.0006	0	0	-0.0004
	[0.01]	[0]	[0.74]	[0.02]	[0.02]	[0.76]	[0.01]	[0.02]	[0.62]
N	263	263	263	263	263	263	263	263	263
R^2	0.063	0.091	0.182	0.041	0.086	0.17	0.074	0.07	0.149

Standard errors are Newey-West with 4 lags.
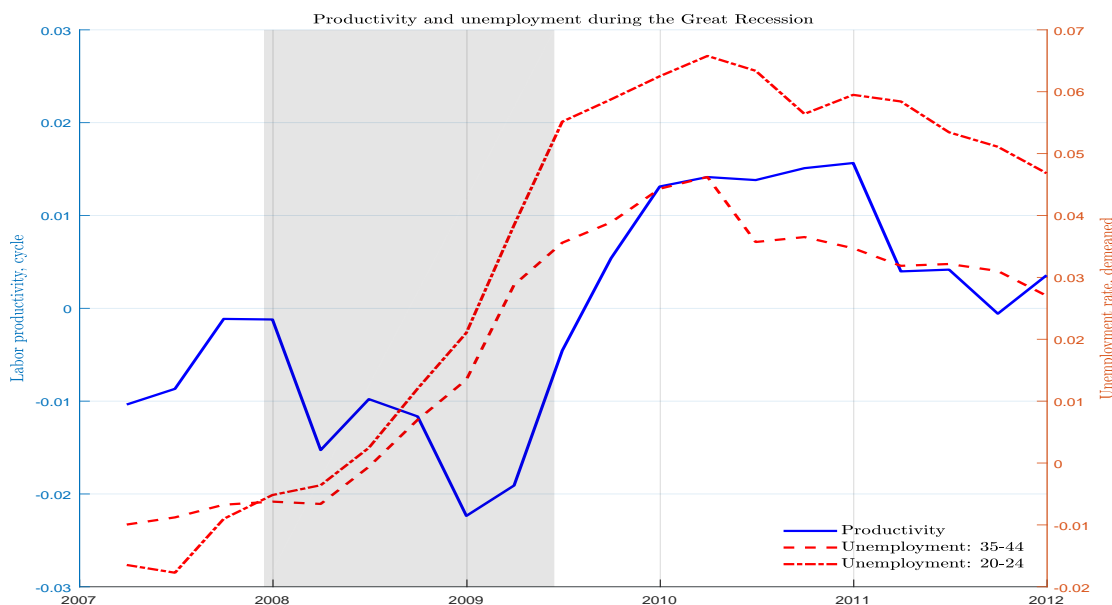
Figure 2: Unemployment rates of young, old, and labor productivity during and after the Great Recession The dotted red line shows the sharp increase in the unemployment rate of workers between 20–24 years (Y). The dashed, red line is for workers between 35–44 years. The blue line shows labor productivity and is measured as the seasonally adjusted output per worker per hour. The grey bands are NBER recessions.
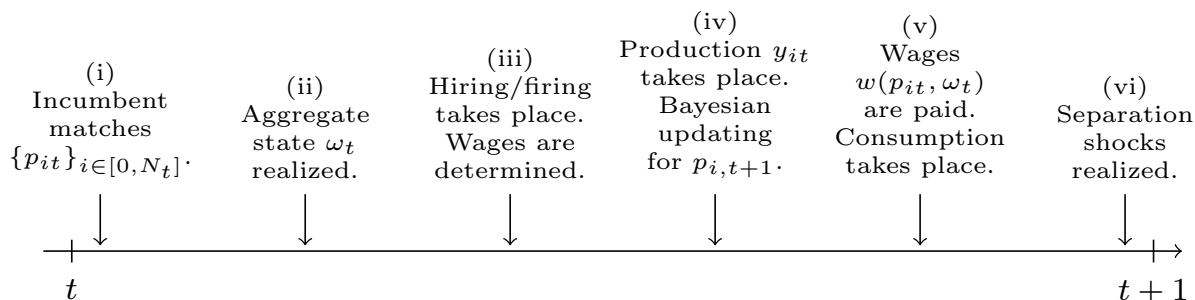

Figure 3: Timing of events within each period. (i) At the start of the period, the economy inherits N_t incumbent matches from the previous period. Matches may differ in their beliefs regarding the probability of the match being of high type. (ii) The aggregate state ω_t is realized. (iii) Matches are made in labor markets, hiring and firing decisions are made, and wages are determined. (iv) Production takes place and beliefs are updated. (v) Wages are paid, the government pays out unemployment benefits financed with lump sum taxation, and consumption takes place. (vi) Finally, idiosyncratic separation shocks and match quality switching shocks are realized.

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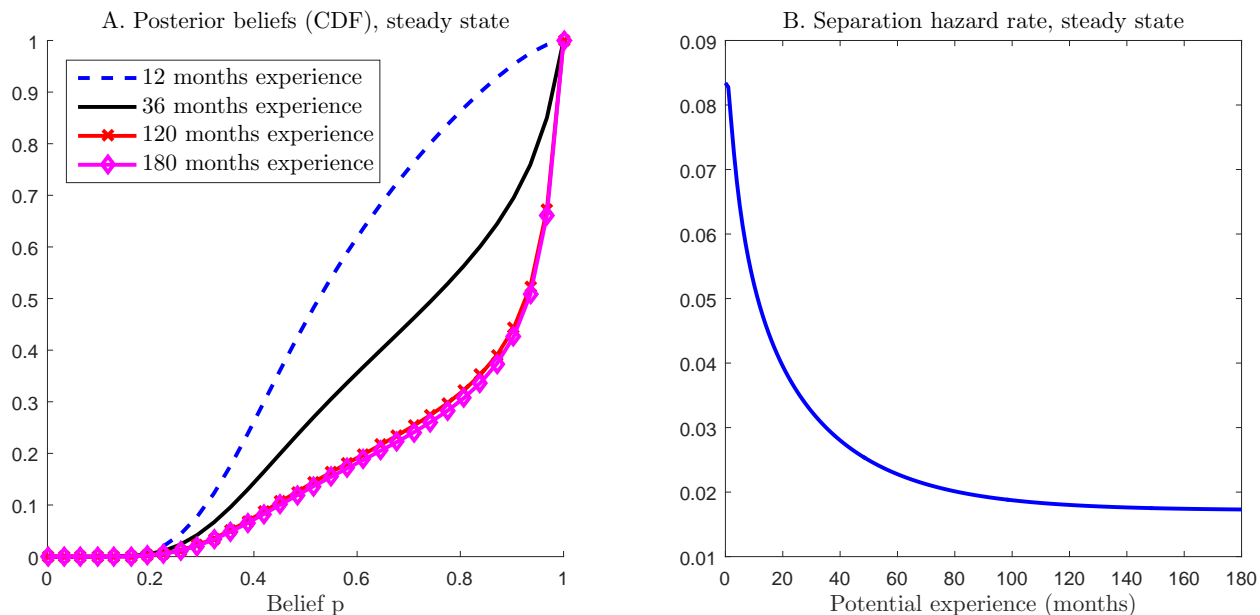
Figure 4: Match pool quality and hazard rates. Panel A shows that the cumulative distribution function of match quality beliefs as a function of experience in the stochastic steady-state. Panel B shows the hazard rate for match termination within the next month as a function of duration of match in the stochastic steady state.
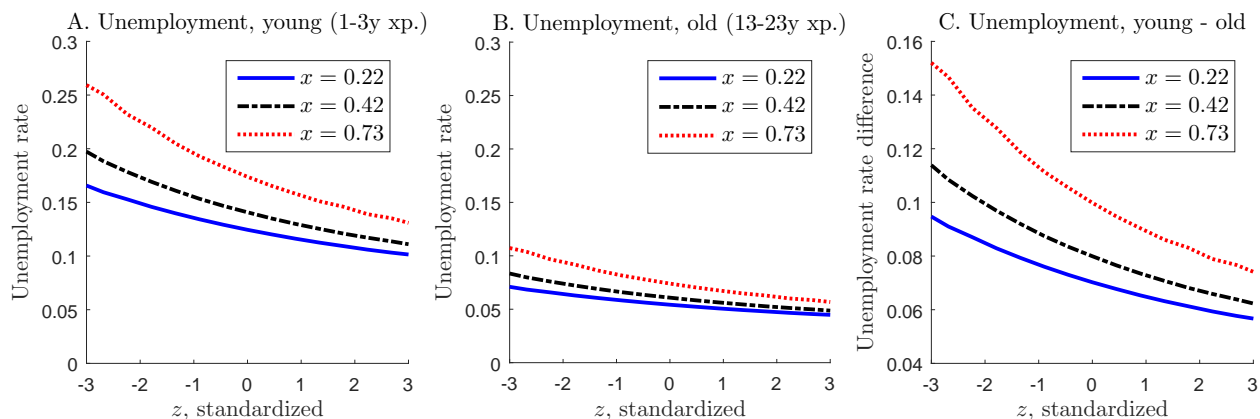

Figure 5: Unemployment rate of Y workers is more sensitive to fundamentals than O workers: Panels A and B show the variation of unemployment rates across aggregate states for young (Y) and old (O) workers, respectively, as a function of aggregate labor productivity for three different values of risk-premia. Panel C shows the difference in unemployment rate between Y and O workers. Labor productivity (z), is standardized by the unconditional volatility of the AR(1) process, $\sigma_z/\sqrt{1-\rho_z^2}$. The figure shows the unemployment rate at the start of each period, before that period's hiring and firing decisions have been made.
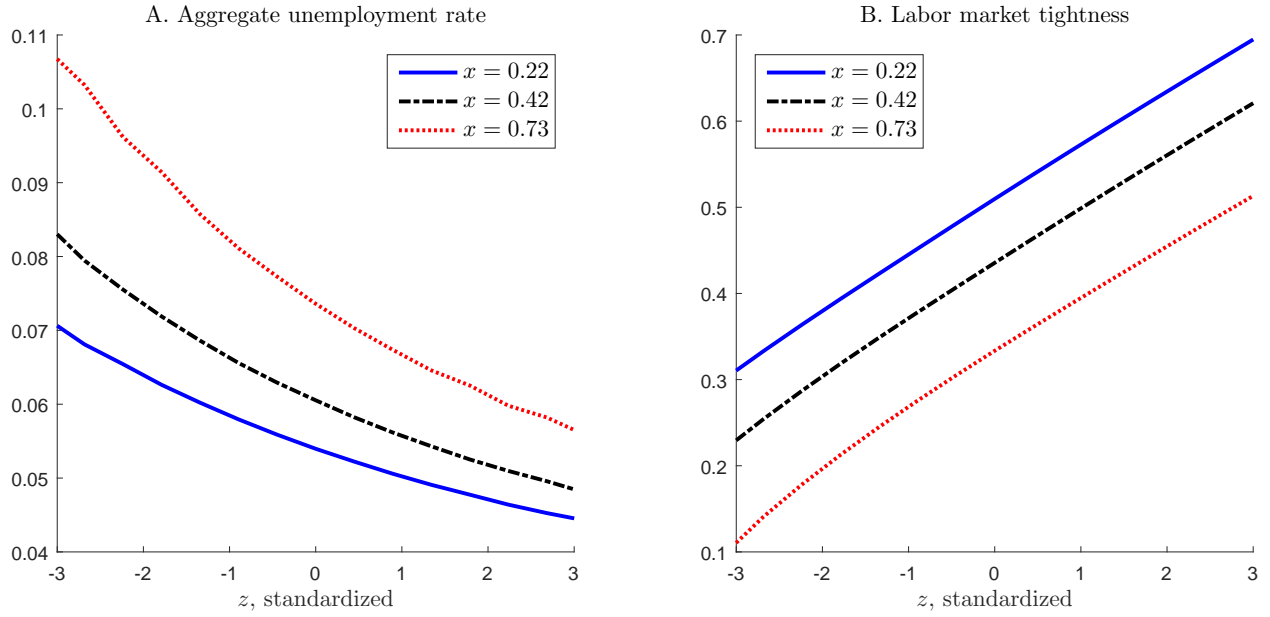
Figure 6: Labor market moments. Panel A shows the dependence of aggregate unemployment on labor productivity for three different levels of the market price of risk. Panel B shows the variation of labor market tightness for these same states.
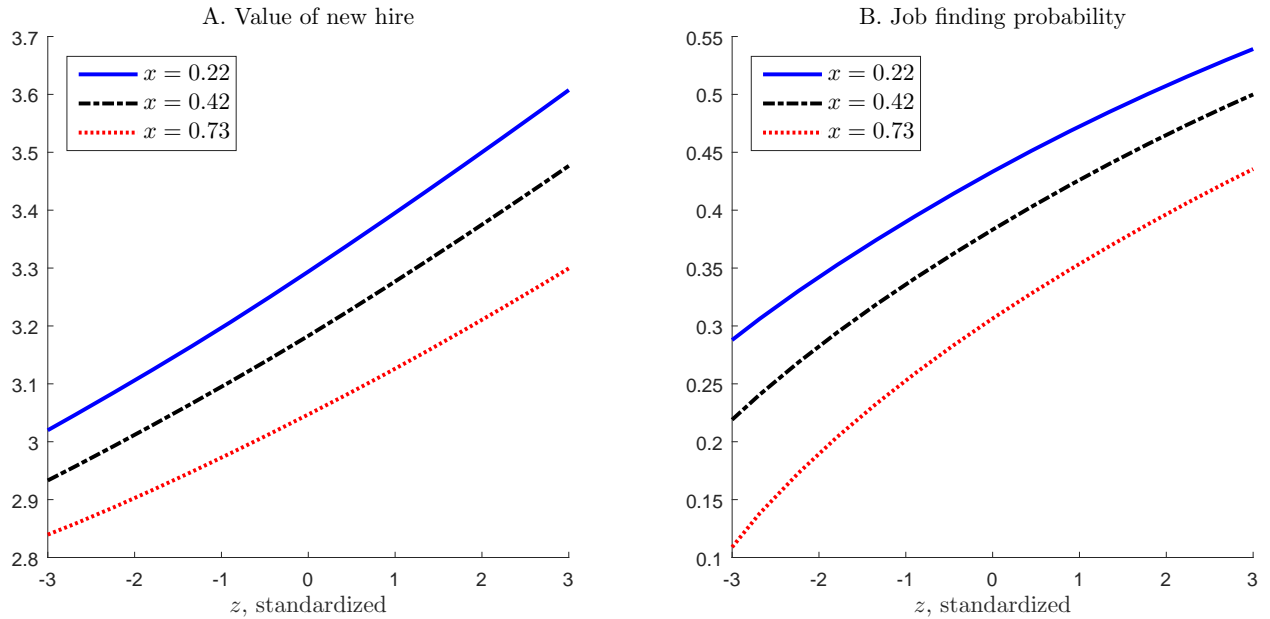

Figure 7: Value of new match and job finding probability. Panel A plots the value of a new hire $F(p_0, z, x)$ for three different levels of market price of risk. Panel B plots the corresponding job finding probabilities.
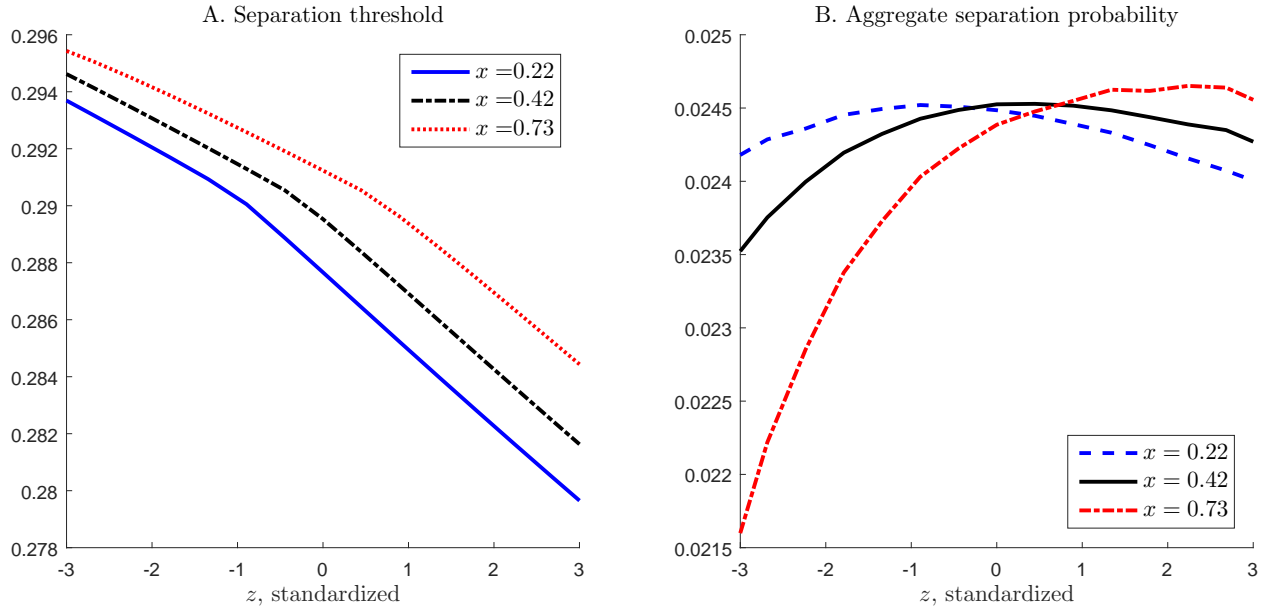
Figure 8: Match separations. Panel A plots the separation threshold $\underline{p}(z,x)$ for three different levels of market price of risk. Panel B plots the average conditional aggregate separation probabilities for each of the three levels of market price of risk.
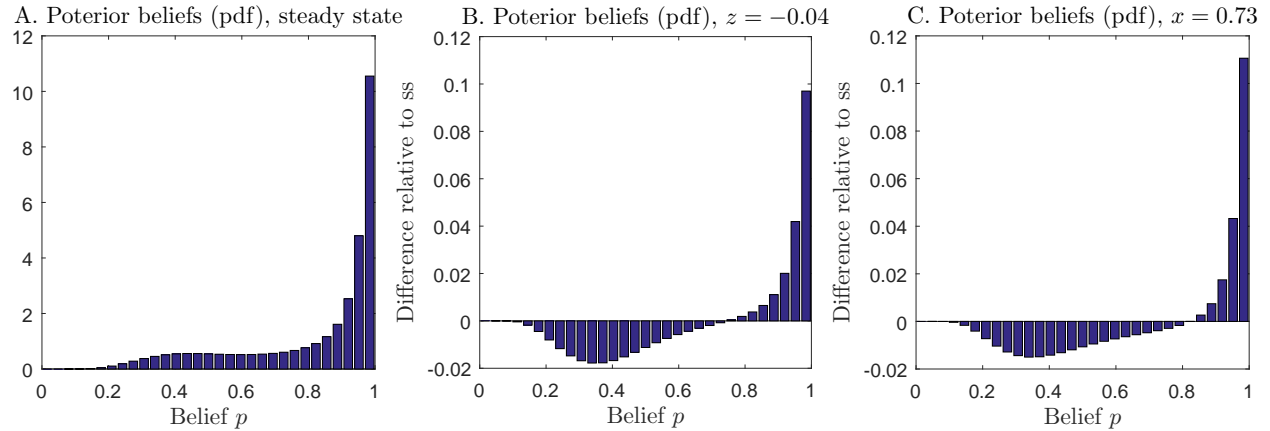

Figure 9: Match quality belief distribution. Panel A plots the cross-sectional match quality belief distribution at the stochastic steady state, $\mathbb{E}[\mathcal{P}_t]$. Panel B plots $\mathbb{E}[\mathcal{P}_t \,|\, z_t = -0.04] - \mathbb{E}[\mathcal{P}_t]$, the difference between the match quality distribution conditional on productivity being two standard deviations below its mean relative to its steady state value. Panel C plots $\mathbb{E}[\mathcal{P}_t \,|\, x_t = 0.73] - \mathbb{E}[\mathcal{P}_t]$, the difference between the match quality distribution conditional on the market price of risk being two standard deviations above its mean relative to the steady state distribution.
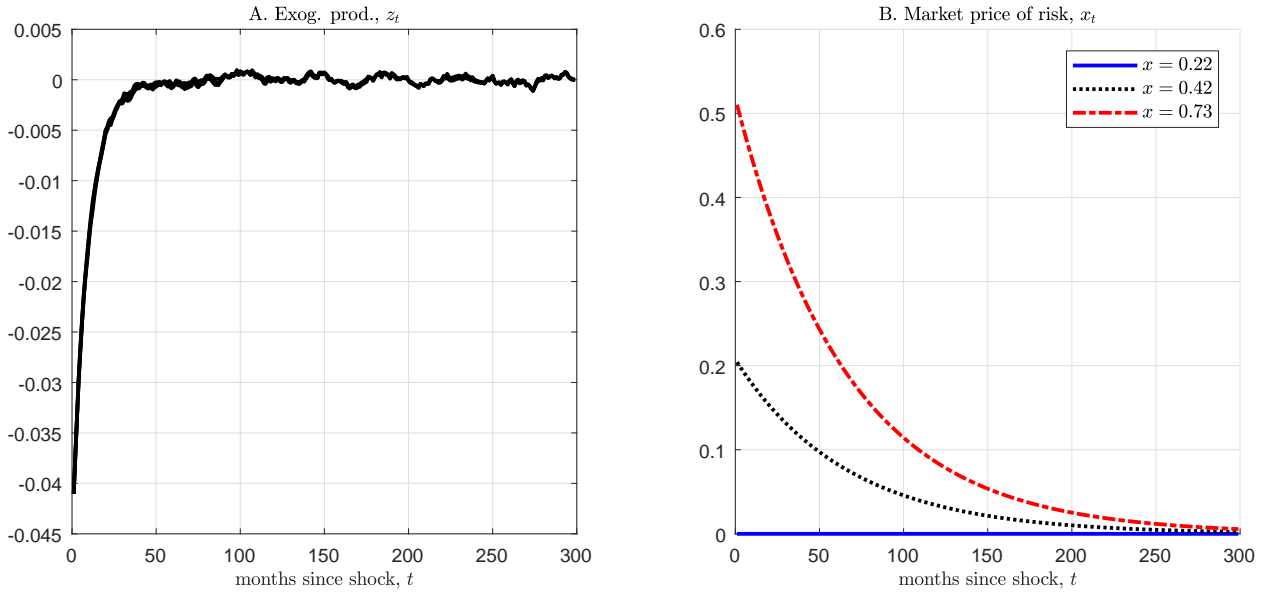
Figure 10: A one-time recessionary shock. This figure shows the average transition paths of productivity (Panel A) and the market price of risk (Panel B) following the recessionary shock.
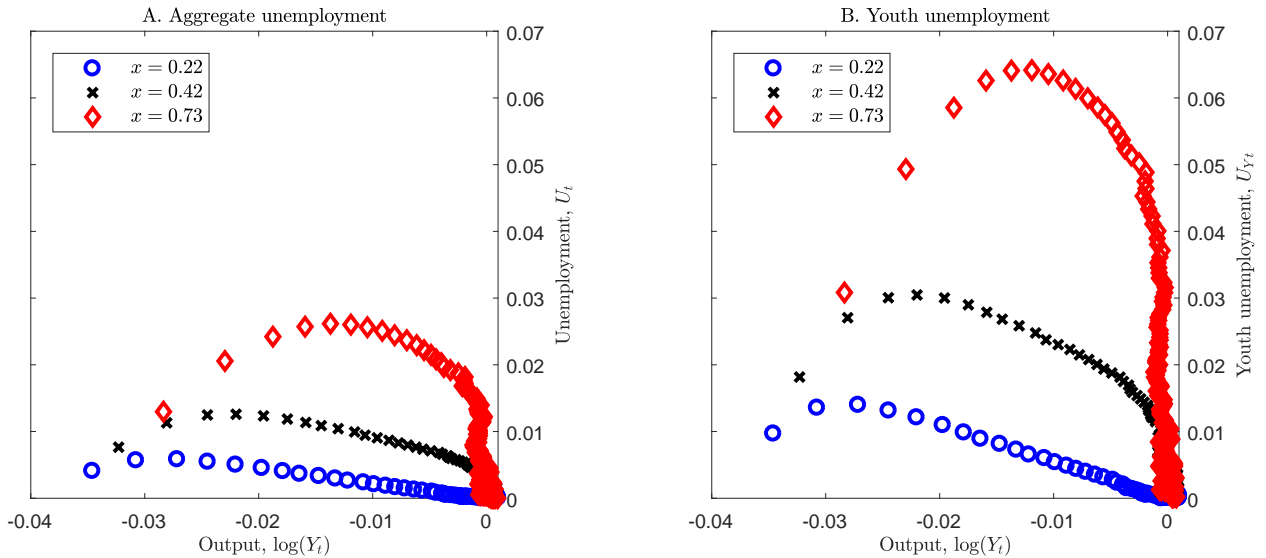

Figure 11: Jobless recoveries Panel A is a scatter plot of the values that log output (x-axis) and aggregate unemployment (y-axis) take along the transition path following a recessionary shock. The scatter plots are shown for three different levels of market price of risk. Panel B shows the corresponding scatter plots for youth unemployment rates.
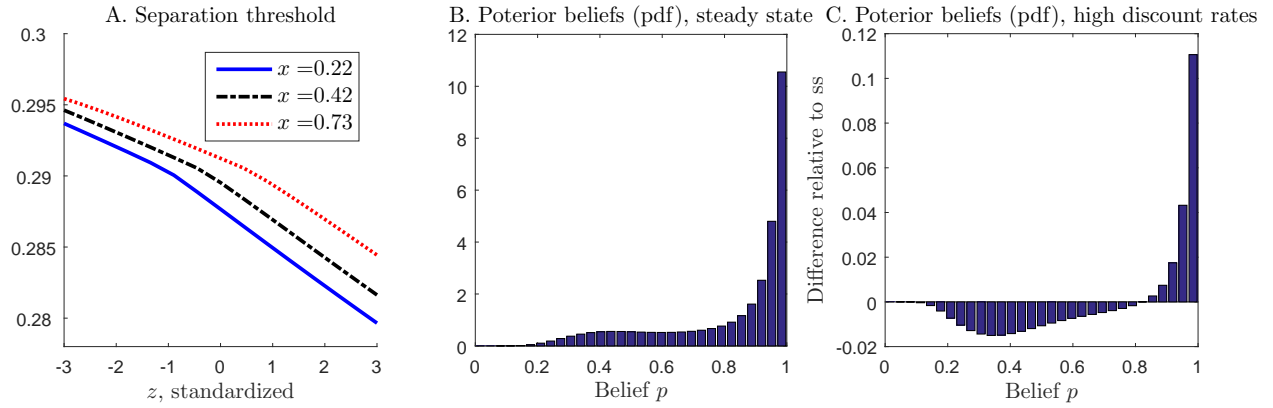
Figure 12: Cleansing Panel A shows the variation of the separation threshold at which matches dissolve as function of labor productivity (z) for three different values of the market price of risk (x). Panels B and C compare the distribution of posterior beliefs of surviving matches in the stochastic steady state and in the state with a high market price of risk. In all figures, labor productivity is normalized by the unconditional volatility of the AR(1) process, $\sigma_z/\sqrt{1-\rho_z^2}$.
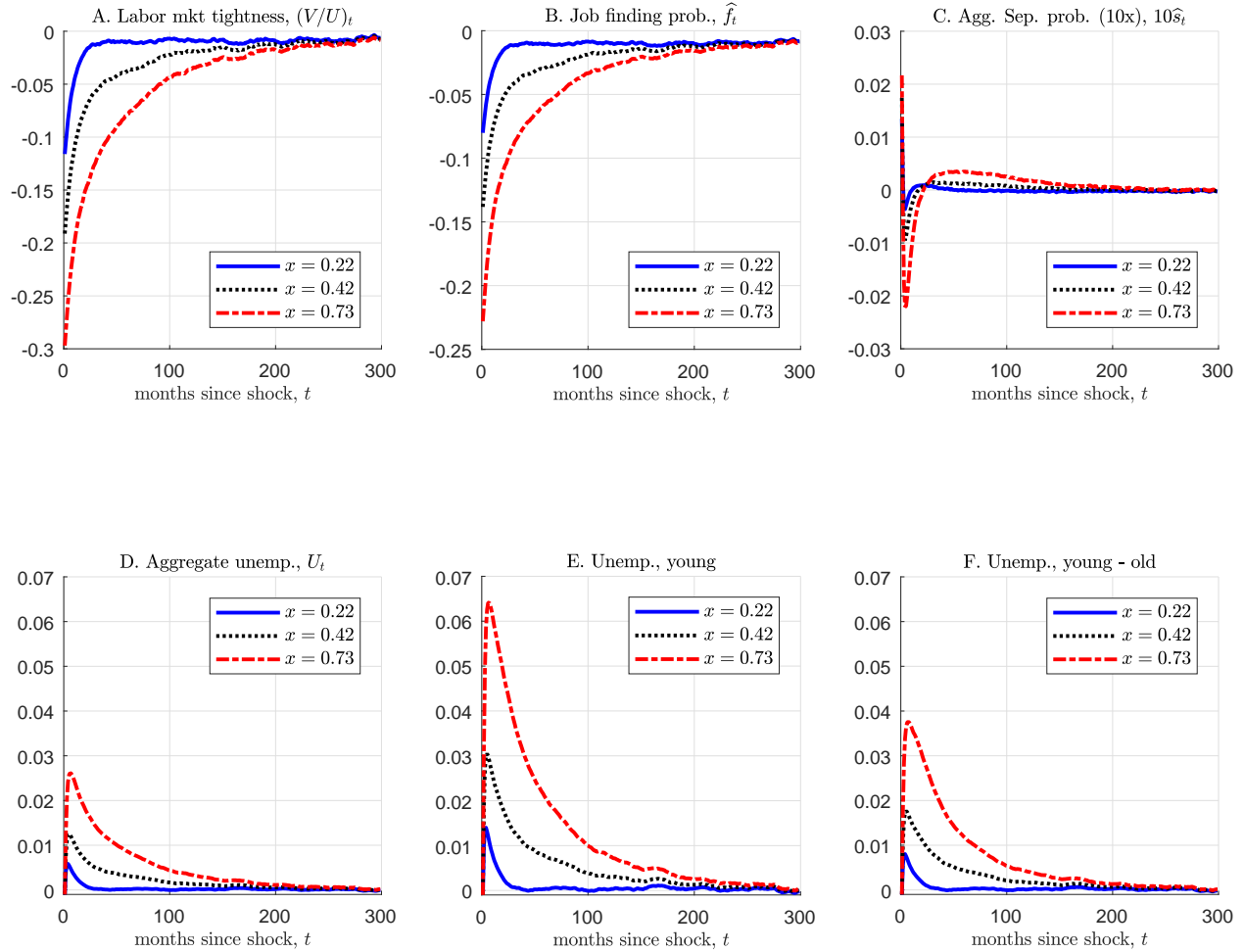
Figure 13: Labor market response Impulse response of labor market quantities (aggregate, young, and old) after a one-time recessionary shock shown in Figure 10. Panels A and D show that aggregate labor market changes are more pronounced when the market price of risk is higher. Panel B shows that job-finding rates are more sensitive to productivity shocks when the market price of risk is higher. Panel C shows cleansing of relatively poor matches upon impact of the shock. A comparison of Panels E and F shows the higher sensitivity of youth unemployment rates to this recessionary shock.
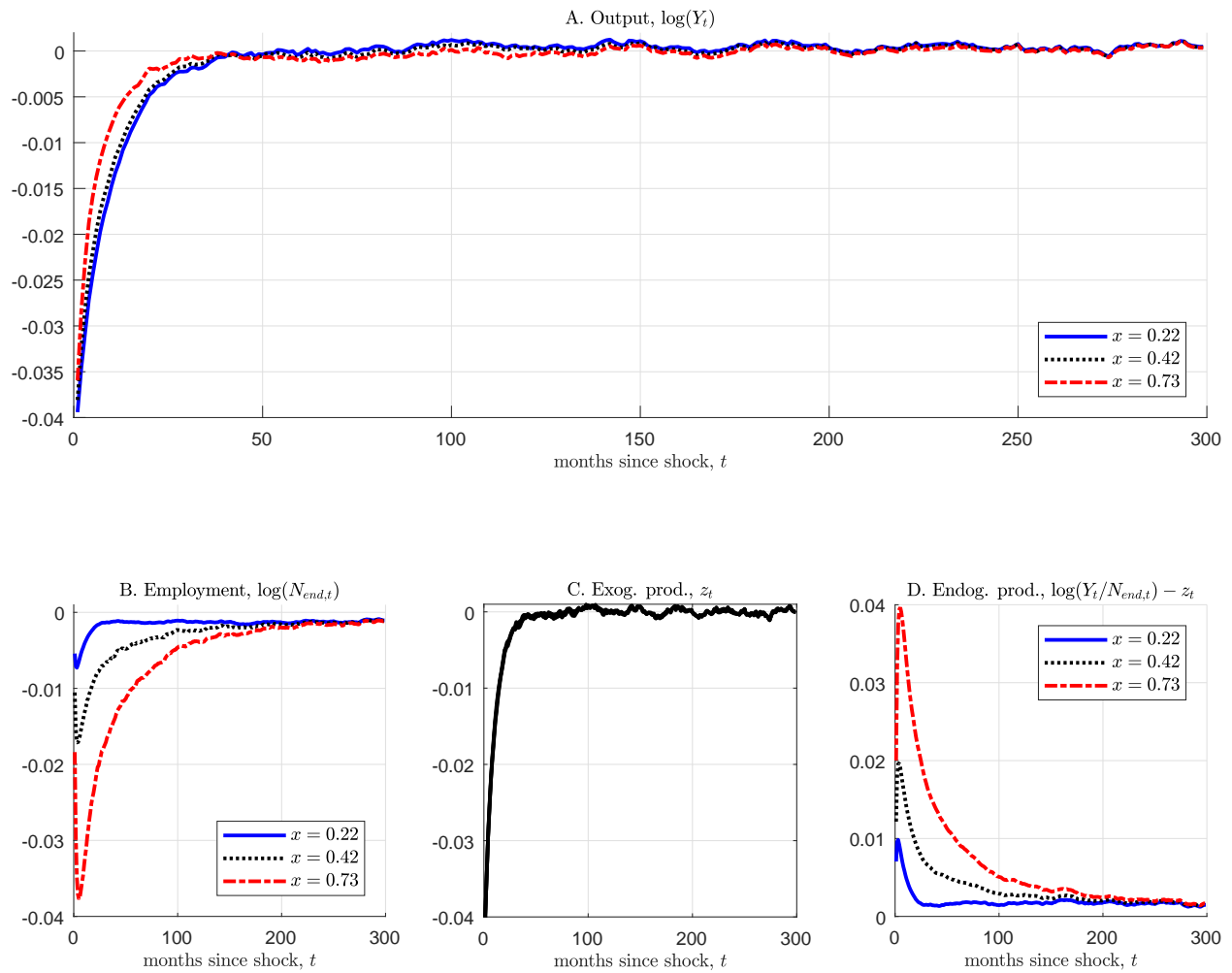
Figure 14: Output response Panel A shows the response of aggregate output after a one-time recessionary shock shown in Figure 10. Panel B shows the effect of this shock on total unemployment. Panel C plots the drop in exogenous labor productivity, while panel D shows the increase in endogenous productivity as a result of the dissolution of poor matches.

Appendix

A   Proofs

We characterize the equilibrium in a two step procedure. First, we characterize the surplus function taking market tightness as a parameter. Afterwards, we then characterize market tightness through the free entry condition for vacancy creation.

We work exclusively with the match surplus function $S : [0, 1] \times \Omega \mapsto \mathbb{R}_+$, which is defined by

$$S(p, \omega) = J_e(p, \omega) - J_{eu}(\omega) + F(p, \omega), \tag{A.1}$$

where $\omega = (z, x)$ denotes the exogenous state. It is sufficient to work with the surplus function alone as all other values of interest can be recovered from the surplus function (and equilibrium conditions).

Match surplus.   By combining the respective definitions (7), (11), and (12) for the value functions, and the Nash bargaining condition for worker's surplus, $J_e(p, \omega) - J_{eu}(\omega) = \eta S(p, \omega)$, we can show that the match surplus function satisfies the following Bellman equation:

$$S(p, \omega) \tag{A.2}$$
$$= \max \left\{ 0, \; \begin{array}{c} e^{z(\omega)} \left[pe^{\mu(H)} + (1 - p)e^{\mu(L)} \right] - b \\ + (1 - s)\mathbb{E} \left[\Lambda(\omega, \omega') \left(S(p', \omega') - f(\omega')\eta S(p_0, \omega') \right) | p, \omega \right] \end{array} \right\}.$$

Next, we simplify the above Bellman equation by noting that

$$f(\omega)\eta S(p_0, \omega) = \frac{\eta \kappa \Theta(\omega)}{1 - \eta} \tag{A.3}$$

must hold in equilibrium. To see this, observe that when the free entry condition (10) is slack, complementary slackness implies that no vacancies are posted in equilibrium so that $f(\omega) = \Theta(\omega) = 0$ and both the left and right hand side of (A.3) equal zero. On the other hand, when the free entry condition (10) is satisfied exactly, the Nash bargaining condition for a firm's share of the surplus, $F(p, \omega) = (1 - \eta)S(p, \omega)$, gives $\kappa = g(\omega)F(p_0, \omega) = (1 - \eta)g(\omega)S(p_0, \omega)$ which immediately imply (A.3).

By substituting (A.3) into the Bellman equation (A.2), we see that the surplus function S can be viewed as the fixed point of an operator, T, defined as follows:

$$T(S)(p, \omega) \tag{A.4}$$
$$= \max \left\{ 0, e^{z(\omega)} \left[pe^{\mu(H)} + (1 - p)e^{\mu(L)} \right] - \widehat{b}(\omega) + (1 - s)\mathbb{E} \left[\Lambda(\omega, \omega')S(p', \omega') | p, \omega \right] \right\},$$

where

$$\widehat{b}(\omega) \equiv b + (1 - s)\mathbb{E} \left[\Lambda(\omega, \omega')\frac{\eta \kappa \Theta(\omega')}{1 - \eta} | \omega \right]. \tag{A.5}$$

In the above expression, market tightness $\Theta = \Theta(\omega)$ is treated as a parameter, and the matching probability $g(\omega) = g(\Theta(\omega))$ is computing according to its definition $g(\Theta) = m(U, V)/V = (1 + \Theta^\iota)^{-\frac{1}{\iota}}$.

Proposition 1. *The operator T defined in (A.4) is a contraction mapping for any given profile of match probability $g(\omega)$ (equivalently market tightness $\Theta(\omega)$). Hence, the surplus function $S(p, \omega)$ is the unique fixed point of T. Furthermore, the surplus function is non-decreasing in p.*

Proof. It is easy to verify that T satisfies Blackwell's sufficiency conditions for a contraction mapping (see Theorem 3.3 of Stokey and Lucas (1999)). This immediately implies the existence of a unique fixed point for T. For the final claim, observe that $T(S)$ is non-decreasing in p whenever S is non-decreasing in p, hence the fixed point of T will also be non-decreasing in p (see Corollary 1 in Stokey and Lucas (1999)). $\qquad\square$

Equilibrium labor market tightness. Having already characterized the surplus function taking market tightness as given, we now characterize the equilibrium value of market tightness. To this end, we now recast the equilibrium in a form that is more convenient for this analysis.

Definition 1 (Equilibrium). *An equilibrium consists of a pair (g, S) where $g \in [0, 1]^{|\Omega|}$ is a probability vector for a firm getting matched to a worker in each of the $|\Omega|$ states, and $S = S(p, \omega)$ is a surplus function. The pair must satisfy the following:*

(i) *The surplus function must satisfy the fixed point problem $S = T_g(S)$, where T_g denotes the operator (A.4) with $g = g(\omega)$ taken as a parameter.*

(ii) *The matching probabilities $g = (g(\omega))$ must satisfy the following set of complementary slackness conditions:*

$$\kappa \geq g(\omega)(1 - \eta) \int_0^1 S(p, \omega) \, \mathcal{P}_0(dp_0), \ \forall \omega \in \Omega$$
$$0 = (1 - g(\omega)) \left[\kappa - g(\omega)(1 - \eta)S(p_0, \omega)\right], \ \forall \omega \in \Omega$$

We could equivalently state condition (ii) as

$$g = \Upsilon(g) \tag{A.6}$$

where the coordinates of $\Upsilon : [0, 1]^{|\Omega|} \mapsto [0, 1]^{|\Omega|}$ are defined as

$$\Upsilon(g)(\omega) = \frac{\kappa}{\max\left\{\kappa, (1 - \eta) \, S\left(p_0, \omega; g\right)\right\}} \tag{A.7}$$

Proposition 2 (Existence). *An equilibrium exists.*

Proof. Observe that Υ maps the unit cube $[0, 1]^{|\Omega|}$ into the unit cube $[0, 1]^{|\Omega|}$. Furthermore, the fact that S is continuous in g implies that Υ is a continuous map. Brouwer's fixed point theorem then guarantees that Υ has a fixed point. $\qquad\square$

Remark. *The above existence theorem only makes use of that fact that Υ is a continuous map. While Brouwer's fixed point theorem then guarantees that an equilibrium exists, there are no guarantees of uniqueness—all that we know from Brouwer's fixed point theorem is that (1) there is at least one equilibrium, and (2) when multiple equilibria exist, the number of equilibria is generically odd (cases with a non-odd number of equilibria are all pathological and can only occur for, possibly, a set of parameters of zero measure).*

Consider the partial ordering, $\succeq$, on $[0,1]^{|\Omega|}$ defined according to $x = (x_1, ..., x_{|\Omega|}) \succeq y = (y_1, ..., y_{|\Omega|})$ if and only if $x_i \geq y_i$ for all $1, ..., |\Omega|$. That is, the partial ordering $\succeq$ is defined coordinate-wise. We have the following:

Proposition 3 (Least and greatest equilibrium). *Under the den Haan, Ramey, and Watson (2000) parameterization for the matching function, $m(U, V) = UV/(U^\iota + V^\iota)^{\frac{1}{\iota}}$, the least and greatest fixed point of Υ, according to the partial order $\succeq$, can be computed by iterating $g_{n+1} = \Upsilon(g_n)$ from a starting point of $g_0 = \mathbf{0} = (0, ..., 0)$ and $g_0 = \mathbf{1} = (1, ..., 1)$, respectively.*

Proof. Under the den Haan, Ramey, and Watson (2000) parameterization for the matching function, $\Theta(g)$ is a strictly decreasing function of g. Hence, the operator T is weakly increasing in g. As a result, Υ is weakly decreasing in g. Since $\left([0,1]^{|\Omega|}, \succeq\right)$ is a complete lattice, Tarski's fixed point theorem then guarantees the existence of a least and greatest fixed point. The iterative procedure is guaranteed to locate the extremal fixed points because we initiate the algorithm from the extremal points of the unit cube and Υ is also a continuous operator (see for example, Echenique (2005)). $\qquad\square$

Remark (Numerical verification of uniqueness). *Proposition 3 allows us to numerically verify whether or not an equilibrium is unique. More specifically, the equilibrium is unique if the least and greatest fixed points of Υ agree. So far, we have not noticed cases of multiple equilibria in our numerical experiments.*

The separation threshold. The separation threshold is the point at which the surplus is worth zero.[14] That is, $\underline{p}(\omega)$ is the solution to the following indifference condition:[15]

$$\Psi(\underline{p}(\omega), \omega) = 0, \tag{A.8}$$

where

$$\Psi(p, \omega) = e^{z(\omega)} \left[pe^{\mu(H)} + (1-p)e^{\mu(L)} \right] - \widehat{b}(\omega) + (1-s)\mathbb{E}\left[\Lambda(\omega, \omega') S(p', \omega') \,|\, p, \omega \right]. \tag{A.9}$$

The solution to the indifference condition (A.8) is unique because Ψ is strictly increasing in p (to see this, note that the surplus function is increasing in p according to Proposition 1 and that the Bayesian posterior function (6) is also monotone increasing in the prior p).

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[14]This is equivalent to the characterization in terms of firm value (8) because, under Nash bargaining, the firm value is proportional to the surplus: $F(p, \omega) = (1 - \eta)S(p, \omega)$.

[15]In the event that $\Psi(p, \omega)$ lies uniformly above (below) 0, then $\underline{p}(\omega) = 0$ ($\underline{p}(\omega) = 1$).

Proposition 4 (Monotonicity of the firing threshold). *When (i) the autocorrelation of of aggregate productivity, ρ_z, is sufficiently close to zero, and (ii) the business cycle variation of the market price of risk, $\lambda(\omega)$, is sufficiently low, then the firing threshold is decreasing in aggregate productivity $z = z(\omega)$.*

Proof. First, consider the extreme case of iid aggregate shocks (i.e. $\rho_z = 0$) and a constant market price of risk. Under these assumptions, the conditional expectation operator in the definition of $\Psi(p, \omega)$ is independent of the current state ω so that $\Psi_z = e^{z(\omega)} \left[p e^{\mu(H)} + (1-p) e^{\mu(L)} \right] > 0$. In addition, we know that Ψ is always strictly increasing in p (regardless of assumptions for the underlying productivity and market price of risk process) so that $\Psi_p > 0$. But then $\underline{p}'(z) = -\frac{\Psi_z}{\Psi_p} < 0$ whenever $\underline{p}(z)$ is interior. Next, by continuity of the problem, we know that this statement will also hold in a sufficiently small neighbourhood around this iid case. $\qquad\square$

Remark (Non-monotonicity of firing thresholds). *Proposition 4 shows that any non-monotonicity in the firing threshold must be due to either persistence in the underlying productivity process or time varying risk premia, or both. Equivalently, the firing threshold can become non-monotonic only when the aggregate productivity process is sufficiently persistent (or perhaps explosive) under the risk-neutral measure. In numerical experiments, we noticed that the firing threshold can become non-monotonic under some parameter combinations whereby the market price of risk $\lambda(\omega)$ displays a sufficiently high level of variation.*