A Model of Safe Asset Determination[*]

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Abstract

What makes an asset a "safe asset"? We study a model where two countries each issue sovereign bonds to satisfy investors' safe asset demands. The countries differ in the float of their bonds and the fundamental resources available to rollover debts. A sovereign's debt is safer if its fundamentals are strong relative to other possible safe assets, not merely strong on an absolute basis. If demand for safe assets is high, a large float enhances safety through a market depth benefit. If demand for safe assets is low, then large debt size is a negative as rollover risk looms large.

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1 Introduction

US government debt is the premier example of a global safe asset. Investors around the world looking for a safe store of value, such as central banks, tilt their portfolios heavily towards US government debt. German government debt occupies a similar position as the safe asset within Europe. US and German debt appear to have high valuations relative to the debt of other countries with similar fundamentals, measured in terms of debt or deficit to income ratios. Moreover, as fundamentals in the US and Germany have deteriorated, these high valuations have persisted. Finally, as evident in the financial crises over the last five years, during times of turmoil, the value of these countries' bonds rise relative to the value of other countries' bonds in a flight-to-quality.

What makes US or German government debt a "safe asset"? This paper develops a model that helps understand the characteristics of an asset that make it safe, as well why safe assets display the phenomena described above. We study a model with many investors and two countries, each of which issues government bonds. The investors have a pool of savings to invest in the government bonds. Thus the bonds of one, or possibly both of the countries, will hold these savings and serve as a store of value. However, the debts are subject to rollover risk. The countries differ in their fundamentals, which measure their ability to service their debt and factor into their rollover risk; and debt sizes, which proxy for the financial market depth of the country's debt market. Our model links fundamentals and debt size to the valuation and equilibrium determination of asset safety.

In the model, an investor's valuation of a bond depends on the number of other investors who purchase that bond. If only a few investors demand a country's bond, the debt is not rolled over and the country defaults on the bond. For a country's bonds to be safe, the number of investors who invest in the bond must exceed a threshold, which is decreasing in the country's fundamentals (e.g., the fiscal surplus) and increasing in the size of the debt. The modeling of rollover risk is similar to Calvo [8] and Cole and Kehoe [11]. Investor actions are *complements* – as more investors invest in a country's bonds, other investors are incentivized to follow suit. Our perspective on asset safety emphasizes coordination, as opposed to

(exclusively) the income process backing the asset, as in conventional analyses of credit risk. In the world, the assets that investors own as their safe assets are largely government debt, money and bank debt. For these assets, valuation has a significant coordination component as in our model, underscoring the relevance of our perspective.

Besides the above strategic complementarity, the model also features strategic substitutability, as is common in models of competitive financial markets. Once the number of investors who invest in the bonds exceeds the threshold required to roll over debts, then investor actions become *substitutes*. Beyond the threshold, more demand for the bond that is in fixed supply drives up the bond price, leading to lower returns. Our model links the debt size to this strategic substitutability: for the same investor demand, a smaller debt size leads to a smaller return to investors.

The model predicts that relative fundamentals more so than absolute fundamentals are an important component of asset safety. Relative fundamentals matter because of the coordination aspect of valuation. Investors expect that other investors will invest in the country with better fundamentals, and thus relative valuation determines which country's bonds have less rollover risk and thus safety. This prediction helps understand the observations we have made regarding the valuation of US debt in a time of deteriorating fiscal fundamentals. In short, all countries' fiscal conditions have deteriorated along with the US, so that US debt has maintained and perhaps strengthened its safe asset status. The same logic can be used to understand the value of the German Bund (as a safe asset within the Euro area) despite deteriorating German fiscal conditions. The Bund has retained/enhanced its value because of the deteriorating fiscal conditions of other Euro area countries.

We further show that this logic can endogenously generate the negative β of a safe asset; that is, the phenomenon that safe asset values rise during a flight to quality. Starting from a case where the characteristics of one country's debt are so good that it is almost surely safe; a decline in world absolute fundamentals further reinforces the safe asset status of that country's debt, leading to an increase of its value. We can thus explain the flight-to-quality pattern in US government debt.

The model also predicts that debt size is an important determinant of safety. If the global demand for safe assets is high, then large debt size enhances safety. Consider an extreme example with a large debt country and a small debt country. If investors coordinate all of their investment into the small debt country, then the return on their investments will be low (or even turn negative). That is the quantity of world demand concentrating on a small float of bonds will drive bond prices up to a point that investors' incentives in equilibrium will be to coordinate investment toward the large debt. On the other hand, if global demand for safe assets is low, then investors will be concerned that the large debt may not attract sufficient demand to rollover the debt. In this case, investors will tend to coordinate on the small debt size as the safe asset.

Our model offers some guidance on when the US government may lose its dominance as a provider of the world safe asset. Many academics have argued that we are and have been in a global savings glut, which in the model corresponds to a high global demand for safe assets. In this case, US government debt is likely to continue to be the safe asset unless US fiscal fundamentals deteriorate significantly relative to other countries, or if another sovereign debt can compete with the US government debt in terms of size. Eurobonds seem like the only possibility of the latter, although there is considerable uncertainty whether such bonds will exist and will have better fundamentals than the US debt. However, if the savings glut ends and the world moves to a low demand for safe assets, then our model predicts that US debt may become unsafe. In this case, investors may shift safe asset demand to an alternative high fundamentals country with a relatively low supply of debt, such as the German Bund.

We use our model to investigate the benefits of creating "Eurobonds." We are motivated by recent Eurobond proposals (see Claessens et al. [10] for a review of various proposals). A shared feature of the many proposals is to create a common Euro-area-wide safe asset. Each country receives proceeds from the issuance of the "common bond" which is meant to serve as the safe asset, in addition to proceeds from the sale of an individual country-specific bond. By issuing a common Eurobond, all countries benefit from investors' need for a safe asset, as opposed to just one country (Germany) which is the de-facto safe asset in the absence of

a coordinated security design.

As our model features endogenous determination of the safe asset, it is well-suited to analyze these "Eurobonds" proposals formally. Suppose that countries issue α share of common bonds and $1 - \alpha$ share as individual bonds. We ask, how does varying α affect welfare, and the probability of safety for each country? Our main finding is that welfare is only unambiguously increased for α above a certain threshold. Above this threshold, the common-bond structure enhances the safety of both common bonds and individual bonds. Below the threshold, however, welfare can be increasing or decreasing, depending on the assumed equilibrium; and one country may be made worse off while another may be made better off by increasing α. We conclude that a successful Eurobond proposal requires a significant amount of coordination and volume / size of said Eurobonds.

Literature review. There is a literature in international finance on the reserve currency through history. Historians identify the UK Sterling as the reserve currency in the pre-World War 1 period, and the US Dollar as the reserve currency post-World War 2. There is some disagreement about the interwar period, with some scholars arguing that there was a joint reserve currency in this period. Eichengreen [15, 16, 17] discusses this history. Gourinchas et al. [22] present a model of the special "exorbitant privilege" role of the US dollar in the international financial system. A reserve currency fulfills three roles: an international store of value, a unit of account, and a medium of exchange (Krugman [36], Frankel [19]). Our paper concerns the store of value role. There is a broader literature in monetary economics on the different roles of money (e.g., Kiyotaki and Wright [33], Banerjee and Maskin [2], Lagos [38], Freeman and Tabellini [20], Doepke and Schneider [14]), and our analysis is most related to the branch of the literature motiving money as a store of value. Samuelson [44] presents an overlapping generation model where money serves as a store of value, allowing for intergenerational trade. Diamond [13] presents a related model but where government debt satisfies the store of value role. In this class of models, there is a need for a store of value, but the models do not offer guidance on which asset will be the store of value. For

example, it is money in Samuelson [44] and government debt in Diamond [13]. In our model, the store of value determination is endogenous.

Our paper also belongs to a growing literature on safe asset shortages. Theoretical work in this area explores the macroeconomic and asset pricing implications of such shortages (Holmstrom and Tirole [32], Caballero et al. [7], Caballero and Krishnamurthy [5], Maggiori [39], Caballero and Farhi [6]). There is also an empirical literature documenting safe asset shortages and their consequences (Krishnamurthy and Vissing-Jorgensen [34, 35], Greenwood and Vayanos [23], Bernanke et al. [4]). We presume that there is a macroeconomic shortage of safe assets, and our model endogenously determines the characteristics of government debt supply that satisfies the safe asset demand.

The element of rollover risk in our model is in the spirit of Calvo [8] and Cole and Kehoe [11]. Rollover risk is also an active research area in corporate finance, with prominent contributions by Diamond [12], and more recently, Morris and Shin [42], He and Xiong [29, 28], and He and Milbradt [26, 27]. We utilize global games techniques (Carlsson and van Damme [9]; Morris and Shin [40]; and others) to link countries' fundamentals to the determination of asset safety. In our economy agent actions can be strategic complements, as in much of this literature, but different from the literature (e.g., Rochet and Vives [43]) can also be strategic substitutes. In this sense, our paper is related to Goldstein and Pauzner [21], who derive the unique equilibrium in a bank-run model with strategic substitution effects. The strategic substitution effect in our model is however stronger than Goldstein and Pauzner [21] and can lead to multiple equilibria, similar to Hellwig et al. [31], Angeletos et al. [1]. In our analysis, when these strategic substitution effects are sufficiently strong, we construct an equilibrium that features regions of joint safety. This equilibrium in which investor strategies are non-monotone is new and a contribution to the global games literature. We label this equilibrium, which closely resembles a mixed-strategy equilibrium, a joint safety equilibrium. Last, a simplified version of the current model with an assumed equilibrium selection rule instead of global game techniques is given in He et al. [30].

In our model, debt size confers greater market depth in the sense that increased purchases

of the larger debt moves the price less. This notion of liquidity, that is, lower price impact, is one aspect of liquidity (see, e.g., Kyle [37]), but is different than the transactional liquidity studied in the over-the-counter search literature. In that literature, papers such as Vayanos and Weill [45] show that a larger float of debt can make it easier to buy or sell the asset. This occurs because it is easier to find trading partners when the float is larger. Thus, liquidity has a coordination element via ease of trading thanks to the larger float. This transactional notion of liquidity is not present in our model. Moreover, in our model, the coordination element is through rollover risk, which interacts with debt float. Note that the Vayanos and Weill [45] analysis could as well apply to risky assets as to safe assets. We are centrally interested in describing safe assets, which is why we study rollover risk and the feedback of liquidity into safety through rollover risk.[1]

2 Model

2.1 The Setting

Consider a two-period model with two countries, indexed by i, and a continuum of homogeneous risk-neutral investors, indexed by j. At date 0 each investor is endowed with one unit of consumption good, which is the numeraire in this economy. Investors invest in the bonds offered by these two countries to maximize their expected date 1 consumption, and there is no other storage technology available.

There is a large country, called country 1, and a small country, called country 2. We normalize the debt size of the large country to be one (i.e., $s_1 = 1$), and denote the debt size of the small country by $s \equiv s_2 \in (0, 1]$. Each country sells bonds at date 0 promising repayment at date 1. The size determines the total face value (in terms of promised repayment) of bonds that each country sells: the large (small) country offers 1 (s) units of sovereign bonds. Hence

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[1]Our paper complements the neoclassical asset pricing literature explaining differences in cross-country currency returns based on country size, such as Hassan [25]. This literature focuses on risk-sharing effects related to country size as reflected in GDP, whereas we focus on the coordination effects driven by the size of a country's debt.

the aggregate bond supply is $1 + s$. All bonds are zero coupon bonds. We can think of the large country as the US and the small country as Canada.

The aggregate measure of investors, which is also the aggregate demand for bonds, is $1 + f$, where $f > 0$ is a constant parameterizing the aggregate savings need. To save, we assume that investors place *market orders* to purchase sovereign bonds. In particular, since purchases are via *market orders*, the aggregate investor demand does not depend on the equilibrium price.[2] Denote by p_i the equilibrium price of the bond issued by country i. Since there is no storage technology available to investors, all savings of investors go to buy these sovereign bonds. This implies via the market clearing condition that

$$s_1 p_1 + s_2 p_2 = p_1 + s p_2 = 1 + f.$$

Country i has fundamentals denoted θ_i. Purely as a matter of notation we write the fiscal surplus as proportional to size and fundamentals, i.e., for country i it is $s_i \theta_i$. Then, country i has resources available for repayment consisting of the fiscal surplus $s_i \theta_i$ and the proceeds from newly issued bonds $s_i p_i$, for a total of $(s_i \theta_i + s_i p_i)$. We assume that a country defaults if and only if[3]

$$\underbrace{s_i \theta_i + s_i p_i}_{\text{total funds available}} < \underbrace{s_i}_{\text{debt obligations}} \tag{1}$$

If a country defaults at date 0, there is zero recovery and any investors who purchased the bonds of that country receive nothing.[4] If a country does not default, then each bond of that country pays off one at date 1. For simplicity, there is no default possibility at date 1, e.g., this assumption can be justified by a sufficiently high fundamental in period 1.

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[2]Market orders avoid the thorny theoretical issue of investors using the information aggregated by the market clearing price to decide which country to invest in, a topic extensively studied in the literature on Rational Expectations Equilibrium.

[3]One can think of the timing, as discussed in the text, as s_i is past debts that must be rolled over. This is a rollover risk interpretation, where we take the past debt as given. Here is another interpretation. The bonds are auctioned at date 0 with investors anticipating repayment at date 1. The date 0 proceeds of $s_i p_i$ are used by the country in a manner that will generate $s_i \theta_i + s_i p_i$ at date 1 which is then used to repay the auctioned debt of s_i. He et al. [30] discuss the difference between old debt and new debt in more detail.

[4]We study the case of positive recovery in Section 3.5.

The "fundamentals" of θ_i measures a country's surplus, i.e., the country's cushion against default. For most of our analysis we refer to θ_i as the country's fiscal surplus, which then increases the funds available to roll over the country's debt. Our model directly applies to the case where debt is in real terms and the default is driven by a lack of liquidity. But with some relabeling, there are other interpretations which are consistent with our modeling. We can interpret θ_i to include reputational costs associated with defaulting on debts, in which case the default equation, $s_i p_i < s_i(1 - \theta_i)$, can be read as one where default is driven by unwillingness-to-pay. For the case of foreign currency denominated debt, θ_i can include both the fiscal surplus and foreign reserves of the country. For the case where the debt is denominated in domestic currency, θ_i can include resources the central bank may be willing to provide to forestall a rollover crisis. In this case, such resources, provided via monetization of debt, may be limited by central bank's concerns over inflation or a devalued exchange rate (and its potential adverse effects on the country's real surplus).

Our model can also capture the realistic case in which default on domestic currency debt occurs through inflation rather than outright non-repayment of debt. Suppose that θ_i parameterizes the real backing of the currency, and a smaller θ_i implies higher inflation in the event of default. Then again, θ_i is a cushion against default, and the payout to bondholders is decreasing in θ_i. This case can be handled in our model by setting the recovery in default to be proportional to θ_i, an extension we consider in our NBER working paper version (w22271).[5]

Finally, our analysis links size, s_i, to the determination of safety. We note that this result does not arise mechanically through a relation that makes larger countries have a larger surplus. Formally, we will assume that θ_i is independent of s_i. In this case, we can cancel s_i on both sides of (1), so that it is evident that size does not directly enter the default condition. Country size matters in our model through its impact on the equilibrium prices (p_i).

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[5]For most of the analysis we set recovery to zero; but the model can be solved for case of positive recovery as we do in Section 3.5.

2.2 Multiple Equilibria in the Common Knowledge Case

We note that our model of sovereign debt features a multiple equilibrium crisis, in the sense of Calvo [8] and Cole and Kehoe [11]. Suppose that investors conjecture that other investors will not invest in the debt of a given country. Then the debt price p_i will be low, and the country is more likely to default, which further rationalizes the conjecture that other investors will not invest in the debt of the country. Our main analysis will use the global games approach of breaking common knowledge to pin down the equilibrium. But before diving into that analysis, it is useful to go through the common knowledge case to highlight the forces at work in the model.

We can rewrite (1) as,

$$p_i < (1 - \theta_i). \tag{2}$$

If the equilibrium price of country-i's debt is less than country-i's funding need, then the country defaults.

Let us take a special case where the aggregate international savings need f is large enough to rollover both countries' debts:

$$1 + f \geq (1 + s) \max \{1 - \theta_1, 1 - \theta_2\} \geq (1 - \theta_1) + s (1 - \theta_2).$$

The left-hand side of this expression is the total funding available from investors and the right-hand side is the sum of the funding needs of each country. Since in equilibrium returns on the country's debts have to be equalized, the middle inequality ensures that there exist equilibrium bond prices so that neither country defaults.

In this special case, there are three equilibria. Equilibrium A is that country 1 is safe (i.e., the safe asset) and country 2 defaults. The bond prices in this case are $p_1 = 1 + f$ and $p_2 = 0$, with investors earning a return of $\frac{1}{1+f}$. Equilibrium B is that country 2 is safe and country 1 defaults. The bond prices in this case are $s \cdot p_2 = 1 + f$ and $p_1 = 0$, with investors earning a return of $\frac{s}{1+f}$. Finally, equilibrium C is that both countries rollover their debts and are safe. Investors earn a return equal to $\frac{1+s}{1+f}$, which is the total repayments by both

countries divided by the funds that investors pay to buy the debts. The equilibrium bond prices are $p_1 = p_2 = \frac{1+f}{1+s}$.

Comparing across equilibrium A and B, we note that investors receive a higher return in equilibrium A. In equilibrium B, all of investors' savings chase a small quantity of debt, driving returns down. Investors' preference for equilibrium A, through the relative debt sizes of the two countries, provides some intuition for the size benefit which we have alluded to; its role in equilibrium selection will be explored more thoroughly in the next section.[6] There is a countervailing cost of size that is not evident in the common knowledge case, but will be clear in the full model. If f is small so that $1 + f < (1 - \theta_1) + s(1 - \theta_2)$, then only one country's debts can be rolled over, and it is reasonable that investors grow concerned that the large debt country will default and hence select equilibrium B. This mechanism manifests itself in our later analysis as well.

Finally, investors are best off in equilibrium C where both countries are safe. But this equilibrium can be unstable, especially when the abundance of global savings is not guaranteed, so f barely satisfies the two countries' borrowing needs. For illustration, take the case that the aggregate savings $1 + f$ is close to the aggregate funding needs $(1 - \theta_1) + s(1 - \theta_2)$, and both countries have similar fundamentals $\theta_1 \approx \theta_2$. To support this equilibrium, investors need to allocate fractions of exactly $\frac{1}{1+s}$ and $\frac{s}{1+s}$ of their portfolio to each country's debts. Suppose now that the fundamentals of the two countries, θ_1 and θ_2, start to diverge. It is reasonable to think that some investors will anticipate that other investors will tilt their portfolios towards the stronger country. But then the investors will follow suit, and equilibrium will shift to the stronger country (either A or B). The stability of the joint safety equilibrium depends in part on the relative fundamentals of the countries and whether $1 + f$ is large or small.

The rest of this paper follows the global games approach to link equilibrium selection to fundamentals. By doing so, we can thoroughly explore the economic intuitions that

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[6]The global games equilibrium selection is related to the notion of p-dominance, as the equilibrium with country 1 survival is $\frac{1}{1+s}$-dominant. More specifically, investing in country 1 is the unique best response for any individual if he believes other investors buy country 1's bond with probability at least $\frac{s}{1+s}$.

are hinted by the analysis in the common knowledge case. In He et al. [30] we posit an equilibrium selection rule based on maximizing investor welfare, and provide results that support the intuitions we have given in the common knowledge case. That analysis is far more rudimentary than the global games approach followed here.

2.3   Private Information and Equilibrium Selection

We assume that there is a publicly observable world-level fundamental index $\theta \in (0, 1)$. Our analysis focuses on a measure of relative strength between country 1 and country 2, which we denote by $\tilde{\delta}$ and is publicly unobservable. Specifically, conditional on the relative strength $\tilde{\delta}$, the fundamentals of these two countries satisfy

$$1 - \theta_1\left(\tilde{\delta}\right) = (1 - \theta)\exp\left(-\tilde{\delta}\right) \qquad \text{and} \qquad 1 - \theta_2\left(\tilde{\delta}\right) = (1 - \theta)\exp\left(\tilde{\delta}\right). \qquad (3)$$

Recall from (2) that $1 - \theta_i$ is the funding need of a country. Given $\tilde{\delta}$, the higher the θ, the greater the surplus of both countries and therefore the lower their funding need. And, given θ, the higher the $\tilde{\delta}$, the better are country 1 fundamentals relative to country 2, and therefore the lower is country 1's relative funding need.[7] Finally, the above specification implies that the funding need for each country is always positive.

We assume that the relative strength of country 1 has a support $\tilde{\delta} \in \left[-\overline{\delta}, \overline{\delta}\right]$. We do not need to take a stand on the distribution over the interval $\left[-\overline{\delta}, \overline{\delta}\right]$. Unless specified otherwise, we assume $\overline{\delta} < \ln\frac{1+f}{s(1-\theta)}$, which ensures that for the worst case scenario, financing need of the weaker country exceeds the total savings $1 + f$. This gives us the usual dominance regions when the fundamentals take extreme values.

Following the global games technique to pin down the equilibrium, we assume that country 1's relative strength $\tilde{\delta}$ is not publicly observable. Instead, each investor $j \in [0, 1]$ receives

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[7]The scale of $1 - \theta$ and exponential noise $e^{\tilde{\delta}}$ and $e^{-\tilde{\delta}}$ in (3) help in obtaining a simple closed-form solution in our model. The Appendix B.1 considers an additive specification $\theta_i = \theta + (-1)^i\tilde{\delta}$ and solves the case for $\sigma > 0$; we show that the main qualitative results hold in that setting.

a private signal

$$\delta_j = \tilde{\delta} + \epsilon_j,$$

where $\epsilon_j \sim \mathbb{U}\left[-\sigma, \sigma\right]$ and ϵ_j are independent across all investors $j \in [0, 1]$. Following the global games literature as in Morris and Shin [41] we will focus on the limit case where the noise vanishes, i.e., $\sigma \to 0$.[8]

Finally, note that although we do not need to take a stand on the (prior) distribution of $\tilde{\delta}$, for much of the analysis, it will make most sense to think of a distribution that places all of the mass around some point δ_0 and almost no mass on other points. This will correspond to a case where investor-j is almost sure that fundamentals are δ_0, but is unsure about what other investors know, and whether other investors know that investor-j knows fundamentals are δ_0, and so on. In other words, in the limiting case fundamental uncertainty vanishes and only strategic uncertainty about each investors' relative rank remains.

We focus on a symmetric single-survivor equilibrium in threshold strategies in this section. More specifically, we assume that all investors adopt the same threshold strategy in which each investor purchases country 1 bonds if and only if his private signal about country 1's relative strength is above a certain threshold, i.e. $\delta_j > \delta^*$; otherwise the investor purchases country 2 bonds, and these strategies result in an equilibrium where only one country does not default. We will later show in Proposition 2 that if we restrict agents to monotone strategies, i.e. strategies in which an agent's investment in a country is weakly increasing in the signal received about that country, the symmetric single-survivor equilibrium in threshold strategies is the unique equilibrium. Later in this paper, we study non-monotone strategies which are needed to (i) construct a single-survivor equilibrium in the case of positive recovery and (ii) to introduce a novel class of equilibria that feature joint safety.

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[8]Under global strategic complementarity, Frankel et al. [18] show that the resulting equilibrium in many-player-many-action models may depend on the specific distribution of ϵ_j (which we assume to be uniform here) even when taking the limit of $\sigma \to 0$. This noise-dependence feature may emerge in our model in which global strategic complementarity does not hold.

Deriving the equilibrium threshold.   In equilibrium, the marginal investor who receives the threshold signal $\delta_j = \delta^*$ must be indifferent between investing his money in either country. Based on this signal, the marginal investor forms belief about other investors' signals and hence their strategies. Denote by x the fraction of investors who receive signals that are above his own signal $\delta_j = \delta^*$, and as implied by threshold strategies will invest in country 1. It is well-known (e.g., Morris and Shin [41]) that in the limit of diminishing noise $\sigma \to 0$, the marginal investor forms a "diffuse" view about other investors' strategies, in that he assigns a uniform distribution for $x \sim \mathbb{U}[0,1]$.

Combined with the threshold strategy, the fraction of investors who purchase the bonds of country 1 is equal to the fraction of investors deemed more optimistic than the marginal agent, x. Thus, the total funds going to country 1 and 2 are $(1+f)\,x$ and $(1+f)\,(1-x)$, respectively. The resulting bond prices are thus

$$p_1 = (1+f)\,x \qquad \text{and} \qquad p_2 = \frac{(1+f)\,(1-x)}{s}.$$

We now calculate the expected return from investing in bond i, Π_i.

Expected return from investing in country 1.   Given x and its fundamental θ_1, country 1 does not default if and only if

$$p_1 - 1 + \theta_1 \;=\; (1+f)\,x - 1 + \theta_1 \geq 0 \iff x \geq \frac{1-\theta_1}{1+f}. \tag{4}$$

This is intuitive: country 1 does not default only when there are sufficient investors who receive favorable signals about country 1 and place their funds in country 1's bonds accordingly. The survival threshold $\frac{1-\theta_1}{1+f}$ is lower when country 1's fundamental, θ_1, is higher and when the total funds available for savings, f, are higher.

Of course, country 1's fundamental $1 - \theta_1 = (1-\theta)\,e^{-\tilde{\delta}}$ in (3) is uncertain. We take the limit as $\sigma \to 0$, so that the signal is almost perfect and the threshold investor who receives

a signal δ^* will be almost certain that[9]

$$1 - \theta_1 = (1 - \theta) e^{-\delta^*}. \tag{5}$$

Hence, in the limiting case of $\sigma \to 0$, plugging (5) into (4) we find that the large country 1 survives if and only if

$$x \geq \frac{1 - \theta_1}{1 + f} = \frac{(1 - \theta) e^{-\delta^*}}{1 + f}. \tag{6}$$

Here, either higher average fundamentals θ or a higher threshold δ^* make country 1 more likely to repay its debts.

Now we calculate the investors' return by investing in country 1. Conditional on survival, the realized return is

$$\frac{1}{p_1} = \frac{1}{(1 + f) x},$$

while if default occurs the realized return is 0. From the point of view of the threshold investor with signal δ^*, the chance that country 1 survives is simply the integral with respect to the uniform density dx from $\frac{(1-\theta)e^{-\delta^*}}{1+f}$ to 1:

$$\Pi_1 (\delta^*) = \int_{\frac{(1-\theta)e^{-\delta^*}}{1+f}}^{1} \frac{1}{(1 + f) x} dx = \frac{1}{1 + f} \left(\ln \frac{1 + f}{1 - \theta} + \delta^* \right). \tag{7}$$

The higher the threshold δ^*, the greater the chance that country 1 survives, and hence the higher the return by investing in country 1 bonds.

Expected return from investing in country 2. Denote the measure of investors that are investing in country 2 by $x' \equiv 1 - x$, that is the fraction of investors that are more pessimistic than the marginal agent, which again follows a uniform distribution over $[0, 1]$. If

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[9]In equilibrium, θ_1 depends on the realization of x, which is the fraction of investors with signals above δ^*. Given that the signal noise ϵ_j is drawn from a uniform distribution over $[-\sigma, \sigma]$, we have

$$x = \Pr \left(\tilde{\delta} + \epsilon_j > \delta^* \right) = \frac{\tilde{\delta} + \sigma - \delta^*}{2\sigma} \Rightarrow \tilde{\delta} = \delta^* + (2x - 1) \sigma.$$

which implies that $\theta_1 = \theta + (1 - \theta) \left(1 - e^{-\delta^* - (2x-1)\sigma} \right)$. Taking $\sigma \to 0$ we get (5).

the investor instead purchases country 2's bonds, he knows that country 2 does not default if and only if

$$sp_2 - s + s\theta_2 = (1 + f)\, x' - s + s\theta_2 \geq 0 \Leftrightarrow x' \geq \frac{s\,(1 - \theta_2)}{1 + f}, \tag{8}$$

Country 2 survives if the fraction of investors investing in country 2, x', is sufficiently high. The threshold is lower if the country is smaller, fundamentals are better, and the total funds available for savings are higher.

Similar to the argument in the previous section, in the limiting case of almost perfect signal $\sigma \to 0$, country 2 fundamental θ_2 in (8) is almost certain from the perspective of the threshold investor with signal δ^* (recall (3)):

$$1 - \theta_2 = (1 - \theta)\, e^{\delta^*}. \tag{9}$$

Plugging equation (9) into equation (8), we find that country 2 survives if and only if

$$x' \geq \frac{s\,(1 - \theta)\, e^{\delta^*}}{1 + f}. \tag{10}$$

Relative to (6), country size s plays a role. All else equal, the lower size s and the smaller country 2, the more likely that the country 2 survives.

Given survival, the investors' return of investing in country 2, conditional on x', is

$$\frac{1}{p_2} = \frac{s}{(1 + f)\, x'}; \tag{11}$$

while the return is zero if country 2 defaults. As a result, using (11), the expected return from investing in country 2 is

$$\Pi_2\,(\delta^*) = \int_{\frac{s(1-\theta)e^{\delta^*}}{1+f}}^{1} \frac{s}{(1 + f)\, x'} dx' = \frac{1}{1 + f} \cdot s \left(-\ln s + \ln \frac{1 + f}{1 - \theta} - \delta^* \right) \tag{12}$$

Note that if $s = 1$, we see that this profit is the same as for country 1 whose debt size is

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fixed at 1.

Expected return of investing in country 1 versus country 2. Figure 1 plots the return to investing in each country as a function of x (x') which is the measure of investors investing in country 1 (country 2). For illustration, we take the hypothetical equilibrium threshold $\delta^* = 0$, and study the payoffs from the perspective of the marginal investor with $\tilde{\delta} = \delta^* = 0$ so $\theta_1 = \theta_2 = \theta$. Consider the solid green curve first which is the return to investing in country 1. For x below the default threshold $\frac{1-\theta}{1+f}$, the return is zero. This default threshold is relatively high, since country 1 is large and hence it needs a large number of investors to buy bonds to ensure a successful auction. Across the threshold $\frac{1-\theta}{1+f}$, investor actions are strategic complements – i.e., if a given investor knows that other investors are going to invest in country 1, the investor wants to follow suit. Past the threshold, the return falls as the face value of bonds is constant and investors' demand simply bids up the price of the bonds. In this region, investor actions are strategic substitutes. The marginal investor's expected return from investing in country 1 is the integral of shaded area beneath the green solid line.

The dashed red curve plots the return to investing in country 2, as a function of x' which is the measure of investors investing in country 2. The default threshold for country 2, which is $\frac{s(1-\theta)}{1+f}$, is lower than for country 1 $(\frac{1-\theta}{1+f})$ because country 2 only needs to repay a smaller number of bonds. When $\delta^* = 0$, i.e., the marginal investor with signal $\delta^* = 0$ believes that both countries share the same fundamentals, the threshold return to investing in country 2 is $\frac{1}{1-\theta}$. This is the same as the threshold return to investing in country 1, as shown in Figure 1. While country 2 has a lower default threshold which implies a smaller strategic complementarity effect, past the threshold the return to investing in country 2 falls off quickly. That is, the strategic substitutes effect is more significant for country 2 than country 1. This is because country 2 has a small bond issue and hence an increase in demand for country 2 bonds increases the bond price (decreases return) more than the same increase in demand for country 1 bonds. We see this most clearly at the boundary where $x = x' = 1$,
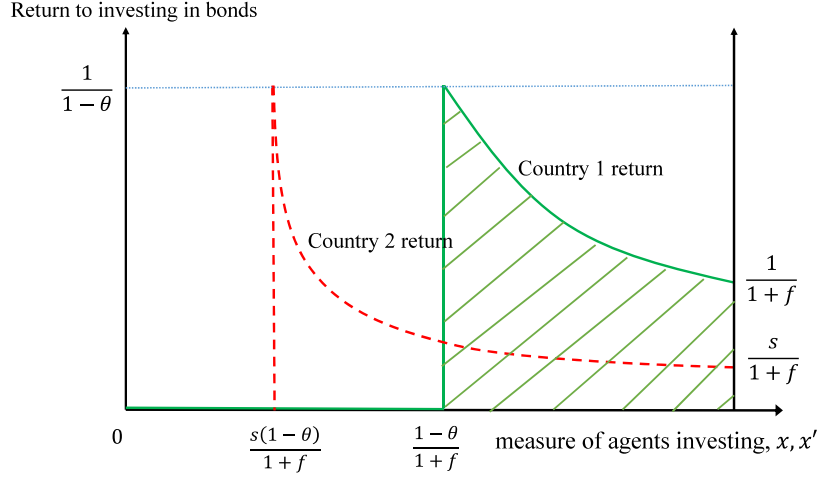
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Figure 1: **Returns of the marginal investor when investing in country 1 (2) as a function of x (x'):** The return to investing in country 1 (2) is the green solid (red dashed) line. We assume $\delta^* = 0$ so that the marginal investor with $\tilde{\delta} = \delta^* = 0$ believes that both countries have the same fundamentals. The bonds issued by the large country 1 (small country 2) only pay when $x > \frac{1-\theta}{1+f}$ $(x' > \frac{s(1-\theta)}{1+f})$. The return to country 1's bonds falls to $\frac{1}{1+f}$ when $x = 1$, while for country 2's bonds the return falls more rapidly to $\frac{s}{1+f}$ when $x' = 1$.

where the return to investing in the large country 1 is $\frac{1}{1+f}$, while the return to investing in country 2 is $\frac{s}{1+f}$.

To sum up, because the large country auctions off more bonds, it needs more investors to participate to ensure no-default. However, the very fact that the large country sells more bonds makes the large country a deeper financial market that can offer a higher return on investment. This tradeoff – size features more rollover risk but provides a more liquid savings vehicle – is at the heart of our analysis.

Equilibrium threshold δ^*. The equilibrium threshold δ^* is determined by the indifference condition for the threshold investor between investing in these two countries. Setting $\Pi_1(\delta^*) - \Pi_2(\delta^*) = 0$, plugging in (7) and (12), the equilibrium threshold signal δ^* is given

by

$$\delta^{*}\left(s, z\right) = \underbrace{-\frac{1-s}{1+s}}_{\text{liquidity, } (-)} \cdot z + \underbrace{\frac{-s \ln s}{1+s}}_{\text{rollover risk, } (+)} \qquad \text{where} \qquad z \equiv \ln \frac{1+f}{1-\theta} > 0. \quad (13)$$

Here, z measures aggregate funding conditions, which is greater if either more aggregate funds f are available or there is a higher aggregate fundamental θ. The "savings glut" which many have argued to characterize the world economy for the last decade is a case of high z.

From (13) we see that there are two effects of size. The first term is negative (for $s \in (0, 1)$) and reflects the liquidity or market depth benefit that accrues to the larger country, making country 1 safer all else equal. The second term is positive and reflects the rollover risk for country 1, whereby a larger size makes country 1 less safe. The benefit term is modulated by the aggregate funding condition z. We next discuss implications of our model based on the equation (13).

3   Model Implications

3.1   Determination of asset safety

Comparing the realized fundamental $\tilde{\delta}$ to the equilibrium threshold δ^{*} tells us which of the two countries will not default, and thus which country's debt will serve as the safe store of value. Consider the case where the distribution of $\tilde{\delta}$ places most of the mass around some point δ_0 and almost no mass on other points. This corresponds to a case where investor-j is almost sure that fundamentals are δ_0, but is unsure about what other investors know, and whether other investors know that investor-j knows fundamentals are δ_0. If $\delta_0 > \delta^{*}$ then country 1 debt is the safe asset, while if $\delta_0 < \delta^{*}$ then country 2 debt is the safe asset. Given that all investors know almost surely the value of δ_0, investors are then almost sure which debt is safe. Mapping this interpretation to thinking about the world, the model says today may be a day that U.S. Treasury bonds are almost surely safe, i.e., $\delta_0 >> \delta^{*}$. But there may

be a news story out that questions the fundamentals of the U.S. (e.g., negotiations regarding the debt limit), and while investor-j may know that it is still the case that $\delta_0 >> \delta^*$, the failure of common knowledge establishes a lower bound δ^* at which the U.S. Treasury bond will cease to be safe.

The following proposition gives the properties of the equilibrium threshold $\delta^*(s, z)$, as a function of country 2's relative size s and the aggregate funding condition z.

Proposition 1 *We have the following results for the equilibrium threshold $\delta^*(s, z)$ for the single-survivor equilibrium in threshold strategies:*

1. *The equilibrium threshold $\delta^*(s, z)$ is decreasing in the aggregate funding conditions z. Hence, country 1's bonds can be the safe asset for worse values of country 1 fundamentals $\tilde{\delta}$, if the aggregate fundamental θ or aggregate saving f is higher.*

2. *The equilibrium threshold $\delta^*(s, z) \leq 0$ for all $s \in (0, 1]$, if and only if $z \geq 1$. Hence, when the aggregate funding $z \geq 1$, the bonds issued by the larger country 1 can be the safe asset for worse values of country 1 fundamentals $\tilde{\delta}$.*

3. *When $s \to 0$ the equilibrium threshold $\delta^*(s, z)$ approaches its minimum, i.e., $\lim_{s \to 0} \delta^*(s, z) = \inf_{s \in (0, 1]} \delta^*(s, z) = -z < 0$. This implies that all else equal, country 1 is the safe asset over the widest range of fundamentals when country 2 is smallest.*

Proof. Result (1.) follows because of $\frac{\partial}{\partial z} \delta^*(s, z) = -\frac{1-s}{1+s} < 0$. To show result (2.), note that when $z = 1$ we have $\delta^*(s, z = 1) = \frac{s - s \ln s - 1}{1 + s} < 0$ for $s \in (0, 1]$. This inequality can be shown by observing (i) $(s - s \ln s - 1)' > 0$ and (ii) $(s - s \ln s - 1)_{s=1} = 0$. Result (3.) holds because

$$\delta^*(s, z) = -\frac{1-s}{1+s} z + \frac{-s \ln s}{1+s} > -\frac{1-s}{1+s} z > -z,$$

where the last inequality is due to $-\frac{1-s}{1+s} z$ being increasing in s for $z > 0$. ■

We illustrate these effects in Figure 2. The left Panel of Figure 2 plots δ^* as a function of s for the case of $z = 1$, which corresponds to strong aggregate funding conditions with abundant savings and/or good fundamentals. In this case, the equilibrium threshold $\delta^*(s)$ is
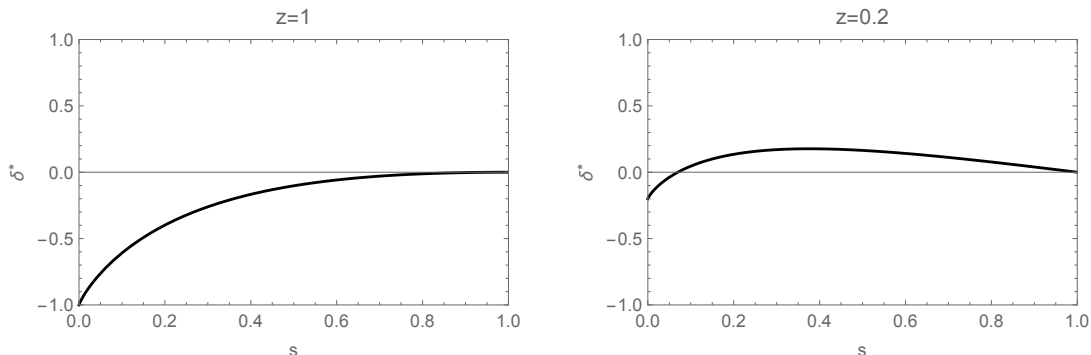
Figure 2: **Equilibrium threshold δ^* as a function of country 2 size s:** The left Panel is for the case of strong aggregate funding conditions with $z = 1$, and the right Panel is for the case of weak aggregate funding conditions with $z = 0.2$.

always negative, and is monotonically increasing in the small country size s. For small s close to zero, the large country is safe even for low possible values of the fundamental $\tilde{\delta}$, because in this case country 2 does not exist as an investment alternative. Then because all investors have no choice but to invest in country 1, the bonds issued by country 1 have minimal rollover risk. If we assume that the aggregate savings $1 + f$ are enough to cover country 1's financing shortfall $1 - \theta_1\left(\tilde{\delta}\right)$ even for the worst realization of $\tilde{\delta} = -\overline{\delta}$ then country 1 will always be safe in this case. This $s = 0$ case offers one perspective on why Japan has been able to sustain a large debt without suffering a rollover crisis. Many of the investors in Japan are so heavily invested in Japanese government, eschewing foreign alternative investments, making Japan's debt safe. In the model, when $s = 0$, investors have no elsewhere to go and are forced into a home bias. If this home bias in investment disappeared, then Japanese debt may no longer be safe.

The right Panel in Figure 2 plots δ^* for a case of weak aggregate funding conditions ($z = 0.2$), with insufficient savings and/or low fundamentals. Consistent with the first result in Proposition 1 we see that in this case the large country can be at a disadvantage. For medium levels of s (around 0.4), investors are concerned that there will not be enough demand for the large country bonds, exposing the large country to rollover risk. As a result, investors coordinate investment into the small country's debt. Note that this may be the case even if

the small country has worse fundamentals. For small s, the size disadvantage of the small country becomes a concern, and the large country is safe even with poor fundamentals (the third result in Proposition 1). For s large, we are back in the symmetric case. Comparing the right Panel with $z = 1$ to the left Panel with $z = 0.2$ highlights that the large country's debt size is an *unambiguous* advantage *only* when the aggregate funding conditions are strong; as the pool of savings shrink, the large debt size triggers rollover risk fears so that investors coordinate investment into the small country's debt.

3.2   Relative fundamentals

Our model emphasizes relative fundamentals as a central ingredient in debt valuation. To clarify this point, consider a standard model without coordination elements and without the safe asset saving need. In particular, suppose that the world interest rate is R^* and consider any two countries in the world with surpluses given by θ_1 and θ_2. Suppose that investors purchase these countries' bonds for $p_i s_i$ and receive repayment of $s_i \min(\theta_i, 1)$. Then,

$$p_1 = \frac{\mathbb{E}\left[\min(\theta_1, 1)\right]}{1 + R^*} \quad \text{and} \quad p_2 = \frac{\mathbb{E}\left[\min(\theta_2, 1)\right]}{1 + R^*},$$

so that bond prices depend on fundamentals, but not particularly on relative fundamentals $\theta_1 - \theta_2$. In contrast, in our model if country-i has the better fundamentals (relative to the equilibrium threshold δ^*), it attracts all the savings so that

$$p_i = 1 + f \quad \text{and} \quad p_{-i} = 0. \tag{14}$$

Valuation in our model becomes sensitive to relative fundamentals, as investors endogenously coordinate to buy bonds that they deem safer. In Section 3.5 we show that these forces also explain why a safe asset carries a negative β.

The importance of relative fundamentals helps us understand why, despite deteriorating US fiscal conditions, US Treasury bond prices have continued to be high: In short, all countries' fiscal conditions have deteriorated along with the US, so that US debt has maintained

and perhaps strengthened its safe asset status. The same logic can be used to understand the value of the German Bund (as a safe asset within the Euro-area) despite deteriorating German fiscal conditions. The Bund has retained/enhanced its value because of the deteriorating general European fiscal conditions.

3.3   Size and aggregate funding conditions

Our model highlights the importance of debt size in determining safety, and its interactions with the aggregate funding conditions. In the high aggregate funding regime, which the literature on the global savings glut has argued to be true of the world in recent history (see, e.g., Bernanke [3], Caballero et al. [7], and Caballero and Krishnamurthy [5]), higher debt size increases safety. US Treasury bonds are the world safe asset in part because US has maintained large debt issues that can accommodate the world's safe asset demands.

These predictions of the model also offer some insight into when US Treasury bonds may cease being a safe asset. If the world continues in the high savings regime, the US will only be displaced if another country can offer a large debt size and/or good relative fundamentals. This seems unlikely in the foreseeable future. On the other hand, if the world switches to the low savings regime, it is possible that US Treasury bonds become unsafe, and another country debt with a smaller debt size and good fundamentals, such as the German Bund, becomes the dominant safe asset.

3.4   Non-monotone strategies and joint safety equilibria

So far we have restricted the agents' strategy space to so-called "threshold" strategies, i.e., invest in country 1 if δ_j is above certain threshold; otherwise invest in country 2. This section discusses potential equilibria once this strategy space is expanded.

Denote the probability (or fraction) of investment in country 1 by an agent with signal δ_j by $\phi(\delta_j) \in [0, 1]$; the agent's strategy is monotone if $\phi(\delta_j)$ is monotonically increasing in his signal δ_j of the country 1's fundamental, i.e., $\phi(\delta) \geq \phi(\delta')$ if $\delta > \delta'$. Then we have the following proposition, which is proved in Appendix B.2:

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Proposition 2 *The single-survivor equilibrium with threshold strategies constructed in Eq. (13) is the unique equilibrium within the monotone strategy space.*

If we allow agents to choose among non-monotone strategies, i.e. $\phi(\delta_j)$ is non-monotone, then for large enough z it is possible to construct equilibria where both countries are safe for some values of the relative fundamental signal $\tilde{\delta}$ (while one country fails if $\tilde{\delta}$ is too low or too high). In the case where both countries do not default over some range of $\tilde{\delta}$, the equilibrium requires investors to "mix" because their investments are strategic substitutes. By allocating funds to both countries' debts in the right proportion, this mixing ensures no-arbitrage across the assets. In Appendix A.1 we construct such a joint safety equilibrium in which agents use "oscillating strategies" that are a tractable way of ensuring the no-arbitrage conditions when such are present, such as is the case in joint safety equilibria when both countries' bonds may pay out. As another application, oscillating strategies allow us also to derive a single-survivor equilibrium with strictly positive bond recovery, which requires no-arbitrage conditions across the non-defaulting and defaulting bond.

Under this oscillating strategy, agents invest in country 2 for sufficiently low δ_j. If the signal is slightly above an endogenous threshold δ_L, agents then invest in country 1, but go back to investing in country 2 for higher signals, oscillating back and forth. Oscillation stops when signals reach another endogenous threshold δ_H, above which agents always invest in country 1. An example of such a strategy can be found in the top panels and the bottom left panel of Figure 3 (they all depict the same strategy). The vertical black lines in these panels denote $\delta_L(=-0.37)$ and $\delta_H(=-0.12)$. The horizontal blue line graphs $\phi(\delta)$, the strategy as a function of δ, which takes values of 0 or 1 in non-monotone fashion. The oscillation intervals are of length of 2σ (in Figure 3, we have two full oscillations, i.e., $\delta_H - \delta_L = 4\sigma$). In the constructed joint safety equilibrium, oscillation occurs only in the region where both countries are safe given the realization of fundamental $\tilde{\delta}$ and equilibrium investment strategies.

Even though oscillating strategies are constructed from pure strategies $\phi(\delta) \in \{0, 1\}$, they feature investor indifference strictly inside the joint-safety regions, and thus closely mirror mixed strategies. The lower right panel of Figure 3 displays the expected payoff, $g(\delta)$, of
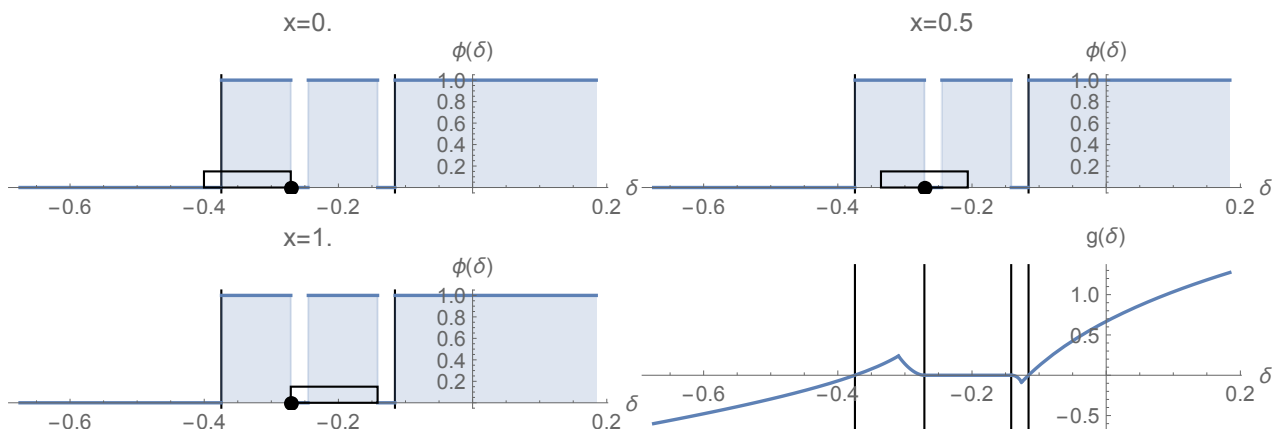
Figure 3: **Oscillating strategy for** $z = 1$ **and** $s = 0.25$**:** The joint safety region has boundaries $\delta_L(=-0.37)$ and $\delta_H(=-0.12)$, marked by black vertical lines. The blue horizontal lines are the investment strategy $\phi(\delta)$, which takes values of 0 or 1 in non-monotone fashion. An investor in the interior of the joint safety region (at the black dot$=-0.27$) considers a range of other investors' signals and their strategies (black rectangle) if he has the highest signal $(x = 0)$ [top left panel], median signal $(x = \frac{1}{2})$ [top right panel], lowest signal $(x = 1)$ [bottom left panel]. The bottom right panel plots the expected payoff $g(\delta)$ of investing in country 1 over country 2 as a function of the signal δ_j.

investing in country 1 over country 2. The black rectangles in the other panels indicate the region integrated over by an agent with a signal given by the black dot $(=-0.27)$ when computing $g(\delta)$. Regardless of the investor's relative position x, as indicated by the different position of the rectangle across the panels, the proportion of investors in country 1 stays constant and hence the integral $g(\delta)$ equals 0. Knowing that both countries will be safe and no arbitrage holds, investors are indifferent and the equilibrium prescribes them to oscillate between investing in country 1 and country 2 depending on their private signal realizations. Note that in the joint-safety region the fundamental $\tilde{\delta}$ (and hence the private signal δ_j) is no longer payoff relevant and instead the signal δ_j takes on the role of a randomization device.[10,11] However, for signal values close to the endogenous boundaries at which the

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[10]The realization of the signal δ_j thus can be thought of as serving the role of "coin-toss;" although the coin-toss is deterministic for each individual agent, we can aggregate these coin-tosses to follow an appropriate distribution to ensure each player's indifference. This is because every agent in this region knows that other agents whose private signals span an interval of 2σ follow an oscillating strategy in a manner so that the aggregate investment proportions is constant.

[11]What is the connection between our oscillating strategies and the purification scheme by Harsanyi [24]? In our model, the agents' choice of playing country 1 or 2 is a (non-monotone) function of their signal δ_i. In

oscillation frequency changes, incentives are strict, resulting in strategies that are at a corner, i.e., $\phi(\delta) \in \{0, 1\}$, as can be seen in the lower right panel of Figure 3. More importantly, these cornered strategies can be shown to be consistent with the equilibrium strategies outside the oscillation region $[\delta_L, \delta_H]$.[12] As before we are interested in the endogenous boundaries δ_L and δ_H when $\sigma \to 0$.

All key qualitative properties in Proposition 1 derived under the single-survivor equilibrium in threshold strategies are robust to considering the joint safety equilibrium in oscillation strategies, with minor modifications. It is also worth emphasizing that equilibria with oscillating strategies lead to the economically plausible situation that both countries' debts may be safe when the aggregate funding condition z is high. This possibility cannot emerge in the case of monotone strategies in which one country always survives and one country always defaults. The next proposition summarizes the results parallel to Proposition 1.

Proposition 3 *We have the following results for the joint safety equilibrium in oscillating strategies.*

1. *For sufficiently favorable aggregate funding conditions $z \geq \underline{z} > 0$ where $\underline{z}$ is derived in Appendix A.1, the joint safety equilibrium in oscillating strategies exists. joint safety supported by oscillation occurs on an interval*

$$[\delta_L, \delta_H] = \left[-z + (1+s)\ln(1+s) - s\ln s, \; z - \frac{1+s}{s}\ln(1+s) \right] \tag{15}$$

2. *The survival region of the larger country 1, $\left[\delta_L, \overline{\delta}\right]$, increases with the aggregate funding conditions z. However, a higher z also increases the survival region of the smaller country 2, $\left[-\overline{\delta}, \delta_H\right]$.*

Harsanyi [24], a given agent sees an epsilon-benefit to playing 1 or 2, with an appropriately chosen distribution of epsilon to ensure appropriate proportions for indifference. Hence we are imposing the epsilon-benefit in Harsanyi's purification scheme as to bear the specific relation to δ_i.

[12]Investors invest in one of the countries for extreme realizations of $\tilde{\delta}$, given the existence of upper- and lower-dominance regions. In equilibrium, oscillation in an endogenous interior region of of the support of δ naturally arises due to the "global games" insight: *"the uncertainty, however small it is, forces the players to take account of the entire class of a priori possible games."* (Carlsson and van Damme [9], p. 990).

3. When $z \geq \underline{z}$, the bonds issued by the larger country 1 are a safe asset for a wider range of fundamentals than the bonds issued by the smaller country 2.

4. All else equal, the larger country 1 is a safe asset for the lowest level of fundamentals when the debt size of country 2 goes to zero, i.e. $s \to 0$.

The first result shows that there is a simple closed form solution for the joint safety interval (δ_L, δ_H). Regarding the second result, recall that in the single-survivor equilibrium in threshold strategies studied in Proposition 1, a higher z *increases* the survival region of the larger country 1 and at the same time *decreases* the survival region of the smaller country 2. This is because only one country survives in the single-survivor equilibrium. In contrast, in the joint safety equilibrium, both counties may survive, and thus improved aggregate funding conditions makes both countries safer. The first and third result of Proposition 3 are similar to the results of Proposition 1, i.e., under sufficiently favorable aggregate funding conditions so that the joint safety equilibrium in oscillating strategies exists, the bonds of the larger country are safer than the bonds of the smaller country. The fourth result is identical to that of Proposition 1.

We have constructed two possible equilibria of the model, and as discussed, they have similar comparative statics if both exist. But is one equilibrium better than the other in a welfare sense? It will depend on the welfare criteria we apply. To start with, suppose that the welfare criterion is based only on the probability of survival (e.g., if default costs are very large); then the joint-safety equilibrium does achieve higher welfare as it results in (weakly) less default for any given value of δ. If the welfare criterion also includes the revenues that the countries raise from the bond auction, then the results are less clear. This is because under the single-survivor equilibrium, the surviving country receives all of the proceeds from the bond issuance, and can thus be hurt by joint-safety where it shares bond revenues with the other country. On the other hand, if we take an equal-weighted welfare function (which is appropriate if transfers are allowed), then the joint-safety equilibrium maximizes welfare because the transfers involved under joint-safety wash out.

3.5 Negative β safe asset

At the height of the US financial crisis, in the aftermath of the Lehman failure, the prices of US Treasury bonds increased dramatically in a flight to quality. Over a period in which the expected liabilities of the US government likely rose by several trillion dollars, the value of US government debt went up. We compute that from September 12, 2008 to the end of trading on September 15, 2008 the value of outstanding US government debt rose by just over \$70bn. Over the period from September 1, 2008 to December 31, 2008, the value of US government debt outstanding as of September 1 rose in value by around \$210bn. These observations indicate that US Treasury bonds are a "negative β" asset. In this section, we show that a safe asset in our model is naturally a negative β asset, and this β is closely tied to the strength of an asset's safety.

In our baseline model, in the single-survivor equilibrium with zero recovery, the price of a safe asset (the surviving bond) is equal to $\frac{1+f}{s_i}$ regardless of shocks. This stark result does not allow us to derive predictions for the β, which is the sensitivity of price to shocks. Let us introduce a positive recovery value in default per unit of face value, $0 < l_i < 1$. This essentially introduces total payouts from the defaulting country 1 or country 2 at the level of l_1 or sl_2, respectively. For simplicity, we do not allow l_i to be dependent on the country's relative fundamental $\tilde{\delta}$. However, l_i may depend on the average fundamental θ, to which we will introduce shocks later when calculating the β of the assets.

When recovery is strictly positive, there is a strong strategic substitution force that pushes investors to buy the defaulting country's debt if nobody else does so. This is because an infinitesimal investor would earn an unbounded return if she is the only investor in the defaulting country's bonds, given a strictly positive recovery. But this implies that threshold strategies are no longer optimal in any symmetric equilibrium, especially when the signal noise σ vanishes.

We thus focus on the strategy space of oscillation strategies to construct a single-survivor equilibrium for the case of positive recovery.[13] The basic idea, in the spirit of global games,

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[13]Thus, oscillating strategies can be seen as a building block to construct single-survivor equilibria in the

is as follows. Suppose that the relative fundamental of country 1, i.e., $\tilde{\delta}$, is sufficiently high so that country 1 survives for sure, irrespective of investors' strategies. This corresponds to the upper dominance region in global games. Then, investors given their private signals will follow an oscillation strategy so that on average there are $\frac{1}{1+l_2 s}$ $\left(\frac{l_2 s}{1+l_2 s}\right)$ measure of investors purchasing the bonds issued by country 1 (2). This way, the defaulting country 2 pays out $l_2 s$ while the safe country 1 pays out 1 in aggregate, and each investor receives the same return of

$$\frac{1}{(1+f)\frac{1}{1+l_2 s}} = \frac{l_2 s}{(1+f)\frac{l_2 s}{1+l_2 s}} = \frac{1+l_2 s}{1+f}.$$

For $\tilde{\delta}$'s that are below but close to the upper dominance region, we postulate that this oscillation strategy prevails in equilibrium, so that country 1 is the only safe country. On the lower dominance region (so $\tilde{\delta}$ is sufficiently low), investors follow an oscillation strategy so that on average there are $\frac{l_1}{l_1+s}$ $\left(\frac{s}{l_1+s}\right)$ measures of investors purchasing the bonds issued by country 1 (2). This way, defaulting country 1 pays out l_1 while the surviving country 2 pays out s in aggregate, and each investor receives the same return of

$$\frac{l_1}{(1+f)\frac{l_1}{l_1+s}} = \frac{s}{(1+f)\frac{s}{l_1+s}} = \frac{l_1+s}{1+f}.$$

Again, $\tilde{\delta}$'s that are above but close to the lower dominance region, we postulate that this oscillation strategy prevails in equilibrium so that country 2 is the safe country.

The logic of global games suggests that there will be an endogenous switching threshold δ^*, such that it is optimal for investors with private signals above δ^* to follow the oscillation strategies in which country 1 survives, while it is optimal for investors with private signals below δ^* to follow the oscillation strategies in which country 2 survives. When l_1, l_2 are

presence of no-arbitrage conditions. This is because oscillating strategies encompass both indifference and strict incentives, and consequently, are globally applicable.

sufficiently small, the closed-form solution for δ^* derived in Appendix B.3 is

$$\delta^* = \frac{[(1 - l_2)\, s - (1 - l_1)]\, z - (s + l_1) \ln (s + l_1) + (1 + sl_2) \ln (1 + l_2 s) + l_1 \ln l_1 - sl_2 \ln l_2}{(1 - l_1) + s\, (1 - l_2)}.$$
(16)

When setting $l_1 = l_2 = 0$, we recover $\delta^* = \frac{-(1-s)z - s \ln(s)}{1+s}$, our original zero-recovery monotone strategy result in (13).

For relative fundamental $\tilde{\delta} \in [\underline{\delta}, \delta^*)$, the price of each bond is given by

$$p_1 = \frac{l_1 (1 + f)}{l_1 + s} \qquad \text{and} \qquad p_2 = \frac{1 + f}{l_1 + s}, \tag{17}$$

while for the relative fundamental $\tilde{\delta} \in (\delta^*, \underline{\delta}]$, the resulting prices are

$$p_1 = \frac{1 + f}{1 + l_2 s} \qquad \text{and} \qquad p_2 = \frac{l_2 (1 + f)}{1 + l_2 s}. \tag{18}$$

Thus, this extension with a positive recovery allows us to determine the non-trivial endogenous bond prices for both countries (in the zero recovery case, those prices were zero or $(1 + f)/s_i$) by equalizing the returns across both countries. As bond prices of the two countries are linked via the cash-in-the-market pricing, the defaulting country's recovery can affect the price of the safe asset.

Consider the case where $\tilde{\delta} \in (\delta^*, \underline{\delta}]$, which corresponds to the case that country 1's bonds are safe. From (18) we see that both bond prices are unaffected by l_1. In contrast, through the cash-in-the-market pricing effect, when the recovery of country 2 (l_2) decreases, p_2 drops and p_1 increases. This observation implies that the safe asset in our model will behave as a negative β asset. To see this, suppose that as aggregate fundamentals deteriorate (say θ falls), recoveries in default of both bonds, l_1 and l_2, decrease. Then, country 1's bonds gain when aggregate fundamentals deteriorate, which makes it a negative β asset, while country 2's bonds lose.

In Appendix A.2, we formally derive the β in a world with shocks to θ. Figure 4 plots the β as a function of δ. As suggested by the intuition, the β for the country 1's bonds is
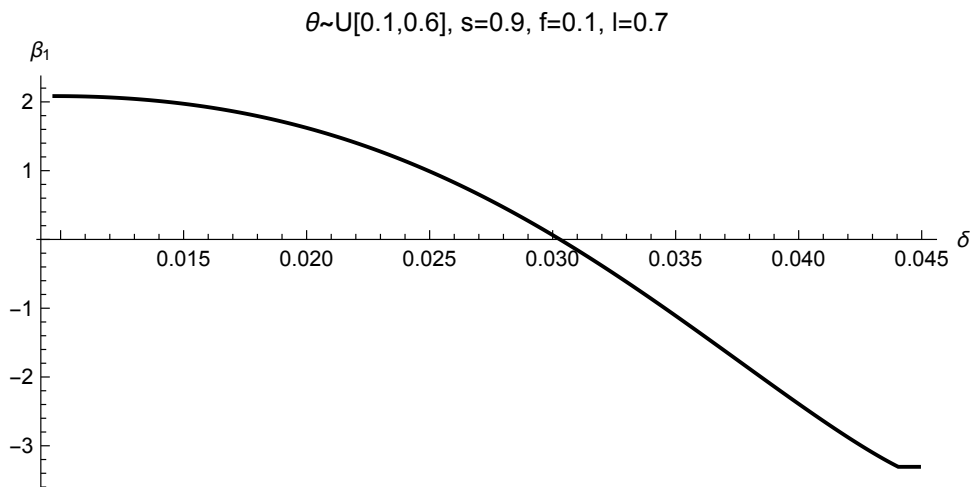
Figure 4: **Country 1 beta example:** $\beta_1 = \frac{Cov(p_1,\theta)}{Var(\theta)}$ for the bonds issued by country 1, as function of country 1's relative fundamental δ. For details, see Appendix A.2.

negative when the country 1's relative fundamental δ is high, i.e., when country 1 is the safe bond. Moreover, the higher the country 1's relative fundamental, the more negative the β of its bonds.

4 Coordination and Security Design

In this section, we characterize the benefits to coordinating through security design. We are motivated by the Eurobond proposals that have been floated over the last few years (see Claessens et al. [10], for a review of various proposals). A shared feature of these proposals is to create a common Euro-area-wide safe asset. More specifically, each country receives proceeds from the issuance of the "common bond," which is meant to serve as the safe asset. By issuing a common Euro-wide safe asset, all countries benefit from investors' flight to safety flows, as opposed to just the one country (Germany) which is the de-facto safe asset in the absence of a coordinated security design. Our model, in which the determination of asset safety is endogenous, is well-suited to analyze these issues formally. We are unaware of other similar models or formal analysis of this issue.

4.1 Main results

We assume that the two countries issue a common bond of size $\alpha(1+s)$ as well as individual country bonds of size $(1-\alpha)s_i$ where $s_1 = 1$ and $s_2 = s$, so that total world bond issuance in aggregate face-value is still $(1+s)$. Here, $\alpha \in [0,1]$ captures the size of common bond program. Denote by p_c the equilibrium price for the common bonds. Since the share of proceeds from the common bond issue flowing to country i is $\frac{s_i}{1+s}$, country i receives

$$\frac{s_i}{1+s} \cdot p_c \alpha(1+s) = s_i \alpha p_c$$

from the common bond auction. Country i also issues its individual bond of size $(1-\alpha)s_i$ at some endogenous price p_i, so total proceeds from both common and individual bond issuances to country i are $s_i(\alpha p_c + (1-\alpha)p_i)$. Then, country i avoids default whenever,

$$s_i(\alpha p_c + (1-\alpha)p_i) + s_i \theta_i > s_i, \tag{19}$$

which is a straightforward extension of the earlier default condition (1) to include the common bond proceeds as the first term inside the parentheses. We assume that default affects all of the country's obligations, so that a country's default leads to zero recovery on its individual bonds and its portion of common bonds. Hence, investors in common bonds receive repayments only from countries that do not default.

We model the bond auction as a two-stage game. In the first-stage, countries auction the common bonds and investors spend a total of $f - \hat{f}$ to purchase these bonds, so that the market clearing condition gives

$$f - \hat{f} = (1+s)\alpha p_c. \tag{20}$$

In this stage, $\tilde{\delta}$ is not yet observed and assumed to be distributed according to $pdf\left(\tilde{\delta}\right)$. In the second stage, investors use their remaining funds of $1 + \hat{f}$ to purchase individual country

bonds conditional on their signal $\delta_j = \tilde{\delta} + \varepsilon_j$. After both auctions, each country makes its own default decision. We discuss the robustness of the results to the timing assumption in Section 4.4 and in in more detail in Online Appendix C.

Motivated by the single-survivor equilibrium and joint safety equilibrium constructed in the base model, we derive the following equilibria for a setting with common bonds.

Proposition 4 *We consider two equilibria, a joint safety equilibrium, supported by oscillating strategies, and a single-survivor equilibrium, supported by threshold strategies. In both equilibria, the determination of asset safety depends on α as follows:*

1. *The single-survivor equilibrium exists for $\alpha \in [0, \alpha^*]$ with corresponding threshold $\delta^*(\alpha)$. If $\tilde{\delta} > \delta^*(\alpha)$, then country 1 is the safe asset and country 2 defaults, while if $\tilde{\delta} < \delta^*(\alpha)$ country 2 is the safe asset and country 1 defaults. Here, the upper bound $\alpha^* = e^{-z}(1 + s)$ solves $\delta^*(\alpha^*) = 0$.*

2. *The joint safety equilibrium exists for $\alpha \in [\alpha_{HL}, 1]$ with corresponding lower and upper thresholds $\delta_L(\alpha)$ and $\delta_H(\alpha)$: If $\tilde{\delta} \in [\delta_L(\alpha), \delta_H(\alpha)]$, then both countries' bonds are safe, while if $\tilde{\delta} < \delta_L(\alpha)$ ($\tilde{\delta} > \delta_H(\alpha)$) country 2 is safe and country 1 defaults (country 1 is safe and country 2 default). Here, the lower bound α_{HL} solves $\delta_L(\alpha_{HL}) = \delta_H(\alpha_{HL}) = \delta^*(\alpha_{HL})$ and is given in the Appendix.*

The two thresholds satisfy $\alpha_{HL} < \alpha^*$.

Figure 5 illustrates the statement of Proposition 4 for the cases of $s = 0.25$ (left Panel) and $s = 0.5$ (right Panel), both for $z = 1$. The black solid line plots the single-survivor equilibrium cutoff δ^* for $\alpha \in [0, \alpha^*]$. As $z = 1$, we are in the high savings case illustrated in the left Panel of Figure 2, and thus $\delta^*(0) < 0$. The joint safety equilibrium also exists, and overlaps with the single-survivor equilibrium on $[\alpha_{HL}, \alpha^*]$ (with possibly negative α_{HL}). In this equilibrium, joint safety is a possibility as long as both countries do not differ too much in fundamentals. The dashed-lines in Figure 5 indicate the upper/lower bounds of the joint safety region $[\delta_L(\alpha), \delta_H(\alpha)]$, where the region itself is indicated by the grey shading.
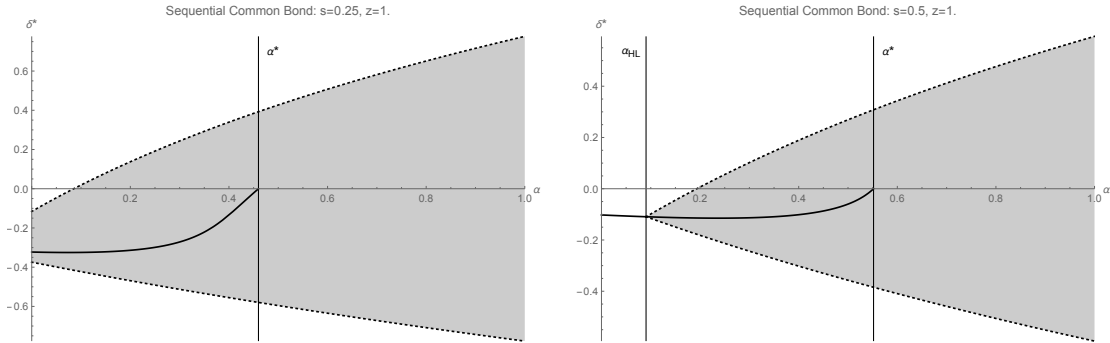
Figure 5: **Common bond equilibria:** δ^* (solid line), δ_H, δ_L (dotted lines) for the case of $s = 0.25$ and $s = 0.5$, as a function of α.

Focusing first on the right Panel with $s = 0.5$, we note that δ^* might decrease with α (one can see it graphically for small α's). This implies that the small country can actually be hurt by the introduction of small scale common bond issues. We discuss the intuition of this result later in Section 4.2. Next, we see that the joint safety region begins at $\alpha = \alpha_{HL} > 0$, and expands as a function of α. Intuitively, as α increases, the minimum funding of the small country increases, relaxing the winner-takes-all coordination game, which in turn allows the small country to be safe for a larger range of realizations of $\tilde{\delta}$. Next, the left Panel considers $s = 0.25$, thereby reducing the aggregate funding requirements for joint safety. This reduction in aggregate funding requirements is strong enough so that the joint safety equilibrium exists even for $\alpha = 0$ (i.e., even in the absence of common bonds).

To sum up, our analysis in this section suggests that increases in common bond issuance, i.e., increases in α, only unambiguously create welfare gains (when such gains are thought of in terms of reducing the instances of default) when $\alpha > \alpha^*$. In this case, increases in α raise the safety of both country 1 and country 2. For $\alpha < \alpha^*$ and in the single survivor equilibrium, a greater α reduces the safety of one country while increasing safety of the other country. Thus, small steps towards a fiscal union could be worse than no step. The rest of this section derives the equilibrium and results in Proposition 4, with proofs in Appendix A.3 and A.4, as well as discuss robustness of the timing assumption.

4.2 Single-survivor equilibrium

We first focus on the single-survivor equilibrium where only one country is safe for any $\tilde{\delta}$. Due to the absence of no arbitrage conditions in the second stage, this equilibrium can be constructed from threshold strategies. We will find the largest α so that this single-survivor equilibrium can exist, which we call α^*. Further, we explain why it is possible for δ^* to decrease with α in this equilibrium, i.e., why it is that common bonds may hurt the small country.

Stage 2. In the second stage, investors have remaining funds of $1 + \hat{f}$ to purchase individual bonds. Consider the marginal investor with signal δ^* who considers that a fraction x of investors have signals exceeding his. Country 1 does not default if and only if,

$$\alpha p_c + (1 - \alpha)p_1 + \theta_1 > 1.$$

Since, $f - \hat{f} = (1 + s)\alpha p_c$ by (20) and $(1 - \alpha)p_1 = x(1 + \hat{f})$, we rewrite this condition as,

$$\frac{f - \hat{f}}{1 + s} + x(1 + \hat{f}) + \theta_1 > 1 \Leftrightarrow x \geq \frac{1 - \theta_1 - \frac{1}{1+s}(f - \hat{f})}{1 + \hat{f}}.$$

We again take the limit as $\sigma \to 0$ and set $1 - \theta_1 = (1 - \theta)e^{-\delta^*}$. Additionally, as the return to the marginal investor in investing in country 1 is $\frac{1-\alpha}{(1+\hat{f})x}$ if the country does not default (and zero recovery in default), the expected return is (when $\hat{f} = f$ and $\alpha = 0$ one recovers the profit function in (7)):

$$\Pi_1(\delta^*) = \frac{1 - \alpha}{1 + \hat{f}} \ln \left(\frac{1 + \hat{f}}{(1 - \theta)e^{-\delta^*} - \frac{1}{1+s}\left(f - \hat{f}\right)} \right).$$

We repeat the same steps for the profits to investing in country 2 and find,

$$\Pi_2(\delta^*) = \frac{s(1 - \alpha)}{1 + \hat{f}} \ln \left(\frac{1 + \hat{f}}{s(1 - \theta)e^{\delta^*} - \frac{s}{1+s}\left(f - \hat{f}\right)} \right).$$

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We solve for the threshold $\delta^*(\hat{f}, \alpha)$ in the same way as before, which takes α and $\hat{f}$ as given:

$$\Pi_1\left(\delta^*\right) = \Pi_2\left(\delta^*\right) \Rightarrow \delta^*(\hat{f}, \alpha). \tag{21}$$

Stage 1. Next we derive $\hat{f}$ by considering Stage 1 in which investors make their investment decisions on common bonds before $\tilde{\delta}$ realizes. Under the assumed equilibrium where only one country is safe, the return to investing in the common bond, denoted by R_{com}, is,

$$R_{com} = \frac{1}{f - \hat{f}} \left[\int_{-\bar{\delta}}^{\delta^*} \alpha s \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta} + \int_{\delta^*}^{\bar{\delta}} \alpha \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta} \right]. \tag{22}$$

At the right-hand-side of (22), the denominator in front of the brackets is the total amount of funds invested in the common bond, while the term inside the brackets is the repayment on the common bonds in the cases of repayment only by country 2 and repayment only by country 1, respectively. The returns to keeping one dollar aside and investing in individual country bonds, denoted by R_{ind}, is,

$$R_{ind} = \frac{1}{1 + \hat{f}} \left[\int_{-\bar{\delta}}^{\delta^*} (1 - \alpha) s \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta} + \int_{\delta^*}^{\bar{\delta}} (1 - \alpha) \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta} \right]. \tag{23}$$

Again, the denominator in the front is the total amount of funds invested in individual bonds, while the term in parentheses is the repayment on individual bonds in the cases of repayment only by country 2 and repayment only by country 1. Note the similarity between the terms inside the brackets in (22) and (23). The similarity arises because along the nodes of country 2 defaulting or country 1 defaulting, the payoffs, state-by-state, to common bonds and individual bonds are αs_i and $(1-\alpha)s_i$. In equilibrium, the expected return from investing in common bonds in stage one must equal to that from waiting and investing in individual bonds in stage 2:

$$R_{com} = R_{ind} \Leftrightarrow \frac{\alpha}{f - \hat{f}} = \frac{1 - \alpha}{1 + \hat{f}} \Leftrightarrow f - \hat{f} = \alpha\left(1 + f\right), \tag{24}$$

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so that an issue $\alpha (1 + s)$ of common bonds attracts a fraction α of all available funds, $1 + f$. This implies that the common bond price is given by

$$p_c = \frac{f - \hat{f}}{\alpha (1 + s)} = \frac{1 + f}{1 + s}. \tag{25}$$

irrespective of our assumptions on the distribution of $\tilde{\delta}$, $pdf\left(\tilde{\delta}\right)$. We combine equations (21) and (24) to solve for the equilibrium threshold $\delta^*(\alpha)$ as a function of common bonds size α.

When does the single-survivor equilibrium exist?   We next consider the bound α^* so that the single-survivor equilibrium exists whenever $\alpha \in [0, \alpha^*]$. We assumed in our equilibrium derivation that only one country is safe (and the other country must default). However, inspecting (19) we see that as α rises, since $p_c > 0$, it may be that even a country that receives zero proceeds from selling its individual bonds can avoid default. But this would violate the equilibrium assumption that one country defaults for sure, leading to a contradiction.

Define $\theta_{def}\left(\delta\right) \equiv \max\left[\theta_1\left(\delta\right), \theta_2\left(\delta\right)\right]$, and let us look for the strongest possible country that is still assumed to default. What is the best fundamental that we can observe in a defaulting country? Clearly, the fundamental of the defaulting country when $\tilde{\delta} = \delta^*$. Then, the strongest country that is still assumed to default is given by $\theta_{def}\left(\delta^*\right)$. This country only defaults if

$$\theta_{def}\left(\delta^*\right) + \alpha p_c < 1 \Leftrightarrow \alpha \leq \frac{1 + s}{1 + f}\left[1 - \theta_{def}\left(\delta^*\left(\alpha\right)\right)\right].$$

Then, define α^* as the solution to

$$\alpha^* = \frac{1 + s}{1 + f}\left[1 - \theta_{def}\left(\delta^*\left(\alpha^*\right)\right)\right]. \tag{26}$$

In Appendix A.3, we show that the single-survivor equilibrium $\delta^*\left(\alpha\right)$ only exists for $\alpha \in [0, \alpha^*]$ where

$$\alpha^* = e^{-z}\left(1 + s\right),$$

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with $\delta^*(\alpha^*) = 0$. For any $\alpha > \alpha^*$, the single-survivor equilibrium does not exist.

The effect of introducing a small quantity of common bonds. Figure 5 shows that there are situations in which $\delta^*_\alpha(0) \equiv \left.\frac{\partial \delta^*(\alpha)}{\partial \alpha}\right|_{\alpha=0} < 0$, implying that the large country gains while the small country loses when a small fraction of common bonds are issued. Interestingly, this result is against the casual intuition that common bonds should bring safety to the small country.

This result is partly driven by the simple fact that the small country receives proportionally less common bonds proceeds. Note that common bonds decreases the default threshold, i.e., the proportion of investors required to make a country safe. Return to Figure 1, this implies that the vertical lines indicating the default threshold shift to the left for both countries, while holding the conditional returns fixed. The large country gains if, starting from $\delta^*(0)$, the new area from additional safety underneath the conditional return curve is greater than the new area for the small country. For s close to zero, almost all the common bond proceeds and thus the rollover risk reduction accrue to the large country, as the small country's vertical line almost coincides with the y-axis. As a result, introducing common bonds hurts, rather than enhances, the safety of the small country.

4.3   Joint safety equilibrium

We now construct a joint safety equilibrium supported by oscillating strategies in which both countries can be safe. We will further compute the minimum value of α, denoted by α_{HL}, for which this equilibrium exists. We find $\alpha_{HL} < \alpha^*$, and the resulting overlap implies that at least two equilibria exist for some parameters, as described in Proposition 4.

As discussed in Section 3.4, the possibility that both countries may be safe rules out monotone threshold strategies due to the presence of no arbitrage conditions in the second stage. Hence the equilibrium is constructed with oscillating strategies.

Stage 2. The construction of the stage 2 equilibrium is given in Appendix A.4.[14] There, for given values of $\hat{f}$, p_c and α, we derive the stage 2 equilibrium oscillating interval as

$$[\delta_L, \delta_H] = \left[-\ln \left\{ \frac{1}{1-\theta} \left[(1+\hat{f}) \frac{s^s}{(1+s)^{1+s}} + \alpha p_c \right] \right\}, \ln \left\{ \frac{1}{1-\theta} \left[\frac{1+\hat{f}}{(1+s)^{\frac{1+s}{s}}} + \alpha p_c \right] \right\} \right].$$

(27)

Of course, $\hat{f}$ and p_c are equilibrium values that are determined by stage 1 investment decisions.

Stage 1. With $[\delta_L, \delta_H]$ in hand, let us determine $\hat{f}$ and $p_c = \frac{f - \hat{f}}{\alpha(1+s)}$ (there are $\alpha(1+s)$ units of common bonds, and there is $f - \hat{f}$ money invested in them). Consider any $\alpha > 0$. Then, we know that the expected returns from investing in common bonds in stage 1 and investing in the best (i.e., surviving) individual country bonds in stage 2 have to be equalized. The expected return to investing in individual bonds is given by

$$R_{ind} = \frac{1-\alpha}{1+\hat{f}} \left[\underbrace{\int_{-\bar{\delta}}^{\delta_L} s \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta}}_{\text{Country 1}} + \underbrace{\int_{\delta_L}^{\delta_H} (1+s) \, pdf\left(\tilde{\delta}\right) d\tilde{\delta}}_{\text{Joint survival}} + \underbrace{\int_{\delta_H}^{\bar{\delta}} pdf\left(\tilde{\delta}\right) d\tilde{\delta}}_{\text{Country 2}} \right],$$

(28)

and the expected return for common bonds is given by

$$R_{com} = \frac{\alpha}{f - \hat{f}} \left[\int_{-\bar{\delta}}^{\delta_L} s \cdot pdf\left(\tilde{\delta}\right) d\tilde{\delta} + \int_{\delta_L}^{\delta_H} (1+s) \, pdf\left(\tilde{\delta}\right) d\tilde{\delta} + \int_{\delta_H}^{\bar{\delta}} pdf\left(\tilde{\delta}\right) d\tilde{\delta} \right].$$

(29)

Note the similarity between these last two expressions in (28) and (29). The similarity arises because the payoffs to common bonds and individual bonds are always αs_i and $(1 - \alpha) s_i$, state-by-state. Thus, equalizing returns we have

$$R_{ind} = R_{com} \Leftrightarrow \frac{\alpha}{f - \hat{f}} = \frac{1-\alpha}{1+\hat{f}} \Leftrightarrow f - \hat{f} = \alpha(1+f),$$

(30)

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[14]It is similar to Appendix A.1,which constructs the oscillating equilibrium discussed in Proposition 3.

so that as in the single-survivor equilibrium, common bonds of size $\alpha (1 + s)$ attract a proportion α of all funds, $(1 + f)$. The common bond price p_c is thus also the same as in (25):

$$p_c = \frac{f - \hat{f}}{\alpha (1 + s)} = \frac{1 + f}{1 + s}. \tag{31}$$

Plugging (30) and (31) into (27), we derive the joint safety interval

$$[\delta_L, \delta_H] = \left[- \ln \left\{ \frac{e^z}{1 + s} \left[\left(\frac{s}{1 + s} \right)^s (1 - \alpha) + \alpha \right] \right\}, \ln \left\{ \frac{e^z}{1 + s} \left[\left(\frac{1}{1 + s} \right)^{\frac{1}{s}} (1 - \alpha) + \alpha \right] \right\} \right] \tag{32}$$

The next proposition establishes conditions for the existence of the oscillating equilibrium.

Proposition 5 *Let $z \geq \ln (1 + s)$, so that there is sufficient funding for joint safety. For any given z, define α_{HL} as the solution to $\delta_H (\alpha_{HL}) = \delta_L (\alpha_{HL})$. Then, we have $\delta^* (\alpha_{HL}) = \delta_H (\alpha_{HL}) = \delta_L (\alpha_{HL})$ and $\alpha_{HL} < \alpha^*$.*

The first result states that at α_{HL}, the thresholds $\delta^*, \delta_H, \delta_L$ all coincide. On $[\alpha_{HL}, \alpha^*]$, both equilibria exists, with the joint safety equilibrium's joint safety region uniformly increasing. As α increases past α^*, the single-survivor equilibrium ceases to exists, while the joint safety equilibrium continues to exist.

4.4   Robustness to timing

This section discusses the robustness of the common bond results to the sequential timing assumption of first having the common bonds sold, then agents receiving their individual signals, and then having the individual bonds sold. In Online Appendix C, we construct a simultaneous three-asset single-survivor equilibrium supported by oscillating strategies. All three bonds are sold simultaneously after investors receive their individual signals. The equilibrium defines thresholds $\delta^*_{sim} (\alpha)$ that are close but not exactly equal to the above derived equilibrium thresholds $\delta^*_{seq} (\alpha)$.[15]

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[15]We construct the equilibrium by oscillating between the common bonds and the individual country bonds. For low and high values of δ, investment strategies oscillate in proportions α (common bond) and

The equilibrium is constructed by equalizing the expected returns on individual bonds and the common bond. For the sequential case, suppose counterfactually, that the marginal agent δ^*_{seq} could invest in the common bond in stage 2 at the stage 1 price $p_c = \frac{1+f}{1+s}$. Then, the expected return in the common bond would be

$$\Pi_c\left(\delta^*_{seq}\right) = \int_0^{x_{max}\left(\delta^*_{seq}\right)} \frac{\frac{s}{1+s}}{p_c}dx + \int_{x_{min}\left(\delta^*_{seq}\right)}^1 \frac{\frac{1}{1+s}}{p_c}dx = \int_0^{x_{max}\left(\delta^*_{seq}\right)} \frac{s}{1+f}dx + \int_{x_{min}\left(\delta^*_{seq}\right)}^1 \frac{1}{1+f}dx.$$

$$(33)$$

For the simultaneous case, the common bond price is given by $p_c\left(\delta, x\right) = \frac{(1+f)\rho_c(\delta,x)}{(1+s)\alpha}$ (where $\rho_c\left(\delta, x\right)$ is the endogenous fraction of agents investing in the common bond), and hence the expected return from common bonds, $\Pi_c\left(\delta\right)$ for the marginal agent δ^*_{seq}, is

$$\Pi_c\left(\delta\right) = \int_0^{x_{max}(\delta)} \frac{\frac{s}{1+s}}{p_c\left(\delta, x\right)}dx + \int_{x_{min}(\delta)}^1 \frac{\frac{1}{1+s}}{p_c\left(\delta, x\right)}dx = \int_0^{x_{max}(\delta)} \frac{s}{(1+f)}\frac{\alpha}{\rho_c\left(x\right)}dx + \int_{x_{min}(\delta)}^1 \frac{1}{(1+f)}\frac{\alpha}{\rho_c\left(x\right)}dx$$

$$(34)$$

which has to be set equal to $\Pi_i\left(\delta\right)$, $i \in \{1, 2\}$, as we are using oscillating strategies.

Let us compare (33) and (34). In the simultaneous case (34), the residual strategic uncertainty over x not only affects the survival probability of the countries (via the integral limits), but also affects the return conditional on survival (via the integrand). In contrast, only the effect on survival probability (via the integral limits) is present for the sequential case (33). The additional factor in (34) is $\frac{\alpha}{\rho_c(x)} \leq 1$ in the integrand (α is the no arbitrage investment proportion that should go to the common bonds when it is clear which country survives). This factor is needed to make common bonds less attractive in the change-over region, as its payoffs vary less with x than does the payoff of each individual bond. Essentially,

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$(1 - \alpha)$ (surviving individual bond). There is a changeover region defined by the interval $[\delta_L, \delta_H]$ on which investors invest in common bonds. The interval is pinned down as the solutions to $\Pi_2\left(\delta_L\right) = \Pi_c\left(\delta_L\right)$ and $\Pi_c\left(\delta_H\right) = \Pi_1\left(\delta_H\right)$. In the limit as $\sigma \to 0$, even though $\lim_{\sigma \to 0} \Pi_i\left(\delta_L\right) \neq \lim_{\sigma \to 0} \Pi_i\left(\delta_H\right)$ for $i \in \{c, 1, 2\}$, we nevertheless have $\delta_L \to \delta^*_{sim} \leftarrow \delta_H$ so we loosely use the term "threshold" for δ^*_{sim}. Unfortunately, we were unable to construct a tractable three-asset equilibrium for the joint safety case, for reasons explained in Online Appendix C.
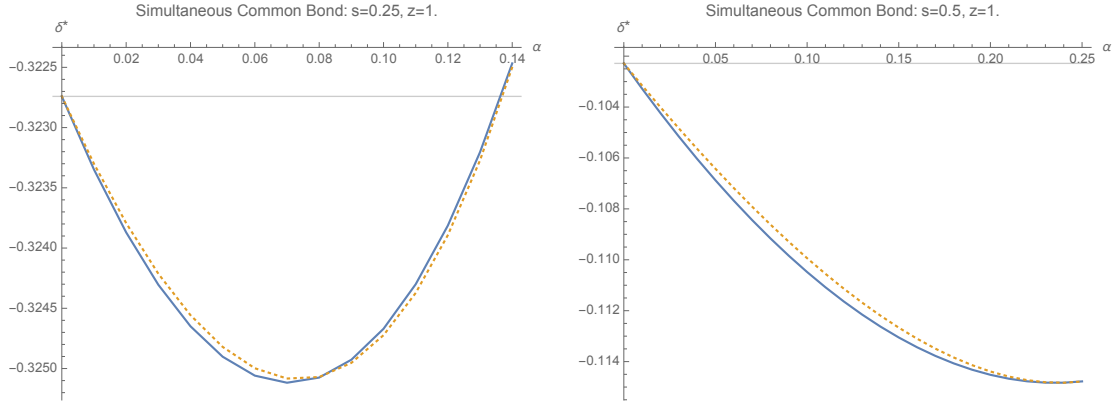
Figure 6: **Robustness of single-survivor common bond equilibrium to sequential timing assumption:** single-survivor sequential common bond equilibrium δ^*_{seq} (dashed yellow line) in comparison to the simultaneous common bond equilibrium δ^*_{sim} (solid blue line) for $(s = 0.25, z = 1)$ (left Panel) and $(s = 0.5, z = 1)$ (right Panel).

we require more investors to invest in the common bond than in the region where there is no uncertainty over which country will survive.

Numerically, we find that $\frac{\alpha}{\rho_c(x)}$ is extremely close to 1; consequently, the difference between δ^*_{seq} and δ^*_{sim} is insignificant. Figure 6 below compares the numerical results for δ^*_{seq} and δ^*_{sim} for parameters found in Figure 5. The graphs zoom in on the range of α for which the simultaneous three asset equilibrium can be derived. The simultaneous threshold δ^*_{sim} is plotted as the solid blue line, which is extremely close to the sequential equilibrium δ^*_{seq} (the dashed orange line) while the no common-bond benchmark $\delta^*_{\alpha=0}$ is plotted as the thin horizontal gray line.

5 Conclusion

US government debt is the world's premier safe asset currently because i) the US has good fundamentals *relative* to other possible safe assets, and ii) given that global demand for safe assets is currently high, the large float of US government debt is the best parking spot for all

of this safe asset demand. In short, there is nowhere else to go. We also derive endogenously the negative β, apparent in a flight to quality, of US government debt. Our analysis of endogenous asset safety also suggest that there can be gains from coordination, and that Eurobonds can exploit these gains by coordinating a security design across Europe.

Our analysis can be extended in other directions. We have taken debt size as well as fundamentals as fixed. But if there is a payoff for a country to ensure that its debt is viewed by investors as a safe asset, then a country is likely to make decisions to capture this payoff. Our investigations of this issue have turned up two results. When countries are roughly symmetric and when global demand for safe assets is high, countries will engage in a rat race to capture a safety premium. Starting from a given, smaller, debt size, and holding fixed the size decision of one country, the other country will have an incentive to increase its debt size since the larger debt size can confer increased safety. But then the first country will have an incentive to respond in a similar way, and so on so forth. In equilibrium, both countries will expand in a self-defeating manner to issue too much debt. The model identifies a second case, when countries are asymmetric and one country is the natural "top dog." In this case, the larger debt country will have an incentive to reduce debts to the point that balances rollover risk and retaining safety, while the smaller country will have an incentive to expand its debt size. Our investigations are suggestive that asymmetry leads to better outcomes than symmetry.

In closing, we emphasize again the main novelty of our analysis of safe assets. Our perspective on safety emphasizes coordination, as opposed to (exclusively) the income process backing the asset, as in conventional analyses of credit risk. In the world, the assets that investors own as their safe assets are largely government debt, money and bank debt. For these assets, valuation has a significant coordination component as in our model, underscoring the relevance of our perspective.

References

[1] **Angeletos, George-Marios, Christian Hellwig, and Alessandro Pavan**, "Dynamic global games of regime change: learning, multiplicity and timing of attacks," *Econometrica*, 2007, *75* (3), 711–756.

[2] **Banerjee, Abhijit V. and Eric S. Maskin**, "A Walrasian Theory of Money and Barter," *Quarterly Journal of Economics*, 1996, *111* (4), 955–1005.

[3] **Bernanke, Ben S.**, "The global savings glut and the US current account deficit," Technical Report, Speech delivered at the Sandbridge Lecture, Virginia Association of Economists, Richmond, VA mar 2005.

[4] _____ , **Carol Bertaut, Laurie Pounder DeMarco, and Steven Kamin**, "International Capital Flows and the Returns to Safe Assets in the United States, 2003-2007," *Board of Governors of the Federal Reserve System International Finance Discussion Papers*, 2011, *1014*.

[5] **Caballero, Ricardo J and Arvind Krishnamurthy**, "Global Imbalances and Financial Fragility," *American Economic Review*, 2009, *99* (2), 584–588.

[6] _____ **and Emmanuel Farhi**, "The Safety Trap," *Working Paper*, 2015.

[7] _____ , _____ , **and Pierre-Olivier Gourinchas**, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, 2008, *98* (1), 358–393.

[8] **Calvo, Guillermo A.**, "Servicing the Public Debt: The Role of Expectations," *American Economic Review*, 1988, *78* (4), 647–661.

[9] **Carlsson, Hans and Eric van Damme**, "Global Games and Equilibrium Selection," *Econometrica*, 1993, *61* (5), 989–1018.

[10] **Claessens, Stijn, Ashoka Mody, and Shahin Vallee**, "Paths to Eurobonds," *IMF Working Paper*, 2012, *12172.*

[11] **Cole, Harold L and T J Kehoe**, "Self-fulfilling debt crises," *Review of Economic Studies*, 2000, *67*, 91–116.

[12] **Diamond, D W**, "Debt Maturity and Liquidity Risk," *Quarterly Journal of Economics*, aug 1991, *106* (3), 709–737.

[13] **Diamond, Peter A.**, "National Debt in a Neoclassical Growth Model," *American Economic Review*, 1965, *55* (5), 1126–1150.

[14] **Doepke, Matthias and Martin Schneider**, "Money as a Unit of Account," *Working Paper*, 2013.

[15] **Eichengreen, Barry**, "The Euro as a Reserve Currency," *Journal of Japanese and International Economies*, 1998, *12* (4), 483–506.

[16] ____ , "The rise and fall of the dollar (or when did the dollar replace sterling as the leading reserve currency?)," *NBER Working Papers*, 2005, *11336.*

[17] ____ , *Exorbitant Privilege: the rise and fall of the Dollar and the future of the international monetary system*, Oxford University Press, 2011.

[18] **Frankel, D M, Stephen Morris, and Ady Pauzner**, "Equilibrium selection in global games with strategic complementarities," *Journal of Economic Theory*, 2003, *108* (1), 1–44.

[19] **Frankel, Jeffrey**, "On the dollar," in "The New Palgrave Dictionary of Money and Finance," London: MacMillan Press, 1992.

[20] **Freeman, Scott and Guido Tabellini**, "The Optimality of Nominal Contracts," *Economic Theory*, 1998, *11* (3), 545–562.

[21] **Goldstein, Itay and Ady Pauzner**, "Demand-Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, jun 2005, *60* (3), 1293–1327.

[22] **Gourinchas, Pierre-Oliver, Helene Rey, and Nicolas Govillot**, "Exorbitant Privilege and Exorbitant Duty," *Working Paper*, 2010.

[23] **Greenwood, Robin and Dimitri Vayanos**, "Bond supply and excess bond returns," *Review of Financial Studies*, 2014, *27* (3), 663–713.

[24] **Harsanyi, J.C.**, "Games with randomly disturbed payoffs: a new rationale for mixed-stratey equilibrium points," *International Journal of Game Theory*, 1973, *2*, 1–23.

[25] **Hassan, Tarek**, "Country size, currency unions, and international asset returns," *Journal of Finance*, 2013, *68* (6), 2269–2308.

[26] **He, Zhiguo and Konstantin Milbradt**, "Endogenous liquidity and defaultable bonds," *Econometrica*, 2014, *82* (4), 1443–1508.

[27] ⎯⎯ **and** ⎯⎯ , "Dynamic debt maturity," *Review of Financial Studies*, 2016, *29* (10), 2677–2736.

[28] ⎯⎯ **and Wei Xiong**, "Dynamic debt runs," *Review of Financial Studies*, 2012, *25*, 1799–1843.

[29] ⎯⎯ **and** ⎯⎯ , "Rollover Risk and Credit Risk," *Journal of Finance*, 2012, *67* (2), 391–429.

[30] ⎯⎯ **, Arvind Krishnamurthy, and Konstantin Milbradt**, "What makes US government bonds safe assets?," *American Economic Review Papers & Proceedings*, 2016, *106* (5), 519–523.

[31] **Hellwig, Christian, A Mukherji, and A Tsyvinski**, "Self-fulfilling currency crises: the role of interest rates," *American Economic Review*.

[32] **Holmstrom, Bengt and Jean Tirole**, "Private and Public Supply of Liquidity," *Journal of Political Economy*, feb 1998, *106* (1), 1–40.

[33] **Kiyotaki, Nobuhiro and Randall Wright**, "On Money as a Medium of Exchange," *Journal of Political Economy*, 1989, *97* (4), 927–954.

[34] **Krishnamurthy, Arvind and Annette Vissing-Jorgensen**, "The Aggregate Demand for Treasury Debt," *Journal of Political Economy*, 2012, *120* (2), 233–267.

[35] ⎯⎯ **and** ⎯⎯ , "The Impact of Treasury Supply on Financial Sector Lending and Stability," *Working Paper*, 2015.

[36] **Krugman, Paul R.**, "The international role of the dollar: theory and prospect," in "Exchange rate theory and practice," National Bureau of Economic Research, Inc., 1984, chapter NBER Chapt.

[37] **Kyle, Albert S**, "Continuous auctions and insider trading," *Econometrica*, nov 1985, *53* (6), 1315–1335.

[38] **Lagos, Ricardo**, "Asset prices and liquidity in an exchange economy," *Working Paper*, 2005.

[39] **Maggiori, Matteo**, "Financial Intermediation, International Risk Sharing, and Reserve Currencies," *Working Paper*, 2013.

[40] **Morris, Stephen and Hyun S Shin**, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, jun 1998, *88* (3), 587–597.

[41] ⎯⎯ **and** ⎯⎯ , "Liquidity black holes," *Working Paper*, 2003.

[42] ⎯⎯ **and** ⎯⎯ , "Illiquidity component of credit risk," *Working Paper*, 2010.

[43] **Rochet, Jean-Charles and Xavier Vives**, "Coordination failure and the lender of last resort: was Bagehot right after all?," *Journal of the European Economic Association*, dec 2004, *2* (6), 1116–1147.

[44] **Samuelson, Paul A.**, "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 1958, *66* (6), 467–482.

[45] **Vayanos, Dimitri and Pierre-Olivier Weill**, "A Search-Based Theory of the On-the-Run Phenomenon," *Journal of Finance*, 2008, *53* (3), 1361–1398.

A Main Appendix

A.1 Joint-safety Equilibrium with non-monotone strategies and zero recovery

We now construct a joint safety equilibrium with non-monotone strategies and joint safety on the endogenously determined interval $[\delta_L, \delta_H]$. Given this equilibrium, we will compute the minimum value of $z = \underline{z}$ for which this equilibrium exists. The possibility of joint safety means that our equilibrium construction using threshold strategies is no longer possible. In a region where both countries are known to be safe (recall we consider the limit where $\sigma \to 0$), investors must be indifferent between the two countries, thus equalizing bond returns. Outside the joint safety interval, i.e., $\tilde{\delta} \in [-\bar{\delta}, \delta_L) \cup (\delta_H, \bar{\delta}]$, we are back to the case where the signal is so strong that only one country is safe.

We conjecture the following non-monotone strategy whereby investment in country 1 and in country 2 alternates on discrete intervals of length $k\sigma$ and $(2-k)\sigma$, with $k \in (0,2)$. The investor j's strategy given his private signal δ_j is $\phi(\delta_j) \in \{0,1\}$:

$$\phi(\delta_j) = \begin{cases} 0, & \delta_j < \delta_L \\ 1, & \delta_j \in [\delta_L, \delta_L + k\sigma] \cup [\delta_L + 2\sigma, \delta_L + (2+k)\sigma] \cup [\delta_L + 4\sigma, \delta_L + (4+k)\sigma] \cup ... \\ 0, & \delta_j \in [\delta_L + k\sigma, \delta_L + 2\sigma] \cup [\delta_L + (2+k)\sigma, \delta_L + 4\sigma] \cup [\delta_L + (4+k)\sigma, \delta_L + 6\sigma] \cup ... \\ 1, & \delta_j > \delta_H \end{cases} \quad \text{(A.1)}$$

As we will show shortly, the non-monotone oscillation occurs only when both countries are safe, where the equilibrium requires proportional investment in each safe country to equalize returns across two safe bonds. Clearly, k determines the fraction of agents in investing in country 1 when oscillation occurs, to which we turn next.

Graphical intuition of the proof. Figure 3 supplies the intuition of the proof, for $\sigma > 0$. Figure 3 shows an investor (black dot) with a signal deep inside the joint safety region, $\delta_j = \delta_L + k\sigma$. Regardless of his relative position x, this investors knows that the proportions of investors in country 1 and 2 remain constant throughout, leading to joint safety. Thus, the investor is indifferent as the bottom right Panel shows, and follows the prescribed equilibrium oscillating strategy. Consider instead an investors on the edge of the joint safety region, $\delta_j = \delta_L$. As $0 < \sigma < \delta_H - \delta_L$, this investor knows for sure that country 2 will survive, regardless of x, but is uncertain if country 1 will survive (it survives for high x, but not for low x). To make this investor indifferent, the total amount of investment cannot be invariant to x in contrast to Figure 3: to balance the returns when country 1 does not survive (low x), it needs high returns when it does (high x). The highest returns, of course, are achieved when a country just survives, and thus the funding must change as a function of x to give indifference. However, as the signal of the agent in question increases, country 1 safety increases faster than its return drop, leading to (for $\sigma > 0$) a strict incentive to invest in country 1, as shown by the $g(\delta)$ function pushing above 0 for an interval $(\delta_L, \delta_L + k\sigma)$.

A.1.1   Fraction of agents in investing in country 1

Consider a region where all investors know that both countries are safe. In this case, the total investment in country 1 and 2 has to be $\frac{1+f}{1+s}$ and $\frac{s(1+f)}{1+s}$, respectively, to equalize returns. Take an agent with signal δ; introduce the function $\rho(\delta)$, which is the expected proportion of agents investing in country 1 given (own) signal δ. Then, given the assumed strategy for all agents and given that we are in the region where both countries are safe,

$$\rho(\delta) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma} dy = \frac{k\sigma}{2\sigma}.$$

We choose k so that $\rho(\delta) = \frac{1}{1+s} \iff k = \frac{2}{1+s}$. This is because in equilibrium the proportion investing in country 1 must be constant and equal to $\frac{1}{1+s}$ to equalize returns.

Recall that x denotes the fraction of agents with signal realizations above the agent's private signal δ; and x follows a uniform distribution on $[0, 1]$. For any value of δ and x,

$$\rho(\delta, x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma} dy = \begin{cases} 0, & \delta + 2\sigma x < \delta_L \\ \frac{\delta+2\sigma x-\delta_L}{2\sigma}, & \delta + 2\sigma x \in (\delta_L, \delta_L + k\sigma) \\ \frac{1}{1+s}, & \delta_H - (2-k)\sigma > \delta > \delta_L + k\sigma \end{cases} \tag{A.2}$$

When we evaluate δ at the marginal agent with signal $\delta = \delta_L$, we have

$$\rho(\delta_L, x) = \begin{cases} 0, & x = 0 \\ x, & x \in \left(0, \frac{1}{1+s}\right) \\ \frac{1}{1+s}, & x > \frac{1}{1+s} \end{cases} \tag{A.3}$$

where we observe that $\rho(\delta_L, x)$ is less than or equal to $\frac{1}{1+s}$.

A.1.2   Lower boundary δ_L

In the completely safe region discussed above (for δ exceeding δ_L sufficiently), investors were indifferent between both strategies. This is not the case for agent with signals around the threshold signal δ_L: as the agent knows investors with signal below are always investing in country 2, country 1 is a perceived default risk. We now calculate the return of investing in either country, from the perspective of the boundary agent δ_L.

For the boundary agent δ_L, the return from investing only in country 2 (i.e. $\phi = 0$) is given by

$$\Pi_2(\delta_L) = \int_0^1 \frac{s}{(1+f)(1-\rho(\delta_L, x))} dx \tag{A.4}$$

where we integrate over all x as country 2 is safe regardless of x. We will show consistency of this assumption with the derived equilibrium later. Thus, plugging in, we have

$$\Pi_2(\delta_L) = \frac{s}{1+f} \left[\int_0^{\frac{1}{1+s}} \frac{1}{1-x} dx + \int_{\frac{1}{1+s}}^1 \frac{1}{\frac{s}{1+s}} dx \right] = \frac{s}{1+f} \left[\ln \frac{1+s}{s} + 1 \right] < \frac{1+s}{1+f}. \tag{A.5}$$

where we used $s \ln \frac{1+s}{s} < 1$. Here, we see that payoff to investing in country 2 is lower than the expected payoff that would have realized if both countries were safe. This reflects the strategic substitution effect: because more people (in expectation) invest in the safe country 2, the return in country 2 is lower.

Now we turn to country 1. Since country 1 has default risk, we need to calculate the threshold $x = x_{min}$ so that country 1 becomes safe if there are $x > x_{min}$ measure of agents receiving better signals. To derive

x_{min}, we first solve for $\rho_1^{min}(\delta)$, which is the minimum proportion of agents investing in country 1 that are needed to make country 1 safe given fundamental δ. We have

$$\theta_1(\delta) + (1+f)\rho_1^{min}(\delta) = 1 \iff \rho_1^{min}(\delta) = \frac{1-\theta_1(\delta)}{1+f}$$

Define x_{min} as the solution to $\rho(\delta_L, x) = \rho_1^{min}(\delta_L)$. Given equation (A.3), we have that,

$$x_{min} = \frac{1-\theta_1(\delta_L)}{1+f}. \tag{A.6}$$

The expected return of investing in country 1 given one's own signal δ_L and the conjectured strategies $\phi(\cdot)$ of everyone else is given by,

$$
\begin{aligned}
\Pi_1(\delta_L) &= \int_{x_{min}}^1 \frac{1}{(1+f)\rho(\delta_L, x)} dx = \frac{1}{1+f}\left[\int_{x_{min}}^{\frac{1}{1+s}} \frac{1}{x} dx + \int_{\frac{1}{1+s}}^1 \frac{1}{1/(1+s)} dx\right] \\
&= \frac{1}{1+f}\left[\ln\frac{1}{1+s} - \ln x_{min} + s\right]. \tag{A.7}
\end{aligned}
$$

The boundary agent δ_L must be indifferent between investing in either country, i.e., $\Pi_2(\delta_L) = \Pi_1(\delta_L)$. Plugging in (A.4) and (A.7), we have

$$\frac{s}{1+f}\left[\ln\frac{1+s}{s} + 1\right] = \frac{1}{1+f}\left[\ln\frac{1}{1+s} - \ln x_{min} + s\right] \iff x_{min} = \frac{s^s}{(1+s)^{1+s}}. \tag{A.8}$$

We combine our two equations for x_{min}, (A.6) and (A.8), and use $1 - \theta_1(\delta_L) = (1-\theta)\exp(-\delta_L)$, to obtain:

$$\frac{s^s}{(1+s)^{1+s}} = \frac{(1-\theta)\exp(-\delta_L)}{1+f}.$$

Recall $z = \ln\frac{1+f}{1-\theta}$; we have

$$\delta_L(z) = -z + (1+s)\ln(1+s) - s\ln s \tag{A.9}$$

A.1.3   Upper boundary δ_H

The derivation is symmetric to the above. We have

$$\rho(\delta, x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma} dy = \begin{cases} \frac{1}{1+s}, & \delta - 2\sigma(1-x) < \delta_H - (2-k)\sigma \\ \frac{\delta+2\sigma x - \delta_H}{2\sigma} & \delta - 2\sigma(1-x) \in (\delta_H - (2-k)\sigma, \delta_H) \\ 1, & \delta - 2\sigma(1-x) > \delta_H \end{cases} \tag{A.10}$$

so that

$$\rho(\delta_H, x) = \begin{cases} \frac{1}{1+s}, & x < \frac{1}{1+s} \\ x, & x \in \left(\frac{1}{1+s}, 1\right) \\ 1, & x = 1 \end{cases} \tag{A.11}$$

which yields

$$\Pi_1(\delta_H) = \int_0^1 \frac{1}{(1+\hat{f})\rho(\delta_H, x) dy} dx = \frac{1}{1+f}[\ln(1+s) + 1] < \frac{1+s}{1+f},$$

where we integrated over all x as country 1 is always safe in the vicinity of δ_H.

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The default condition for country 2 is

$$s\theta_2\left(\delta_H\right) + (1+f)\left[1 - \rho_2^{max}\left(\delta_H\right)\right] = s \iff 1 - \rho_2^{max}\left(\delta_H\right) = s\frac{1 - \theta_2\left(\delta_H\right)}{1+f}$$

where $\rho_2^{max}\left(\delta\right)$ is the maximum amount of agents investing in country 1 so that country 2 does not default. Assume, but later verify, that at δ_H we have $1 - \rho_2^{max}\left(\delta_H\right) < \frac{s}{1+s}$, that is, country 2 would survive even if less than $\frac{s}{1+s}$ of investors invest in country 2. Define $x_{max}\left(\delta_H\right)$ as the solution to $\rho\left(\delta_H, x_{max}\right) = \rho_2^{max}\left(\delta_H\right)$; (A.11) implies that

$$1 - x_{max}\left(\delta_H\right) = s\frac{1 - \theta_2\left(\delta_H\right)}{1+f}. \tag{A.12}$$

As a result, the return to country 2 is,

$$
\begin{aligned}
\Pi_2\left(\delta_H\right) &= \int_0^{x_{max}\left(\delta_H\right)} \frac{s}{(1+f)\left(1 - \rho(\delta_H, x)\right)dy}dx = \frac{s}{1+\hat{f}}\left[\int_0^{\frac{1}{1+s}}\frac{1}{1 - \frac{1}{1+s}}dx + \int_{\frac{1}{1+s}}^{x_{max}\left(\delta_H\right)}\frac{1}{1 - x}dx\right] \\
&= \frac{s}{1+f}\left[\frac{1}{s} + \ln\frac{s}{1+s} - \ln\left(1 - x_{max}\left(\delta_H\right)\right)\right]
\end{aligned}
$$

Indifference at the boundary agent δ_H requires $\Pi_1\left(\delta_H\right) = \Pi_2\left(\delta_H\right)$, which yields $1 - x_{max}\left(\delta_H\right) = \frac{s}{(1+s)^{\frac{1+s}{s}}}$. Combining this result with (A.12) and $1 - \theta_2\left(\delta_H\right) = (1 - \theta)\exp\left(\delta_H\right)$, we solve,

$$\delta_H\left(z\right) = z - \frac{1+s}{s}\ln\left(1+s\right) \tag{A.13}$$

A.1.4 Verifying the equilibrium

We now verify the interior agents $\delta \in \left(\delta_L, \delta_H\right)$ have the appropriate incentives to play the conjectured strategy, and that our assumptions of country 1 (2) is always safe at δ_H (δ_L) are correct. As an investor with signal $\delta = \delta_L$ is indifferent, it is easy to show that agents with $\delta < \delta_L$ find it optimal to invest in country 2. Consider an investor with signal $\delta = \delta_L + k\sigma$ (i.e. let us consider the investors depicted by the black dot in Figure 3). Regardless of his relative position (as measured by x) in the signal distribution, this agent knows that a proportion $\frac{1}{1+s}$ of investors invest in country 1, thus making it safe for sure. Further, he knows that a proportion $\frac{s}{1+s}$ of investors invest in country 2, also making it safe. Therefore, this agent knows that (i) both countries are completely safe and that (ii) investment flows give arbitrage free prices. He is thus indifferent, and so is every investor with $\delta_L + k\sigma < \delta < \delta_H - (2 - k)\sigma$.

Next, we consider an investor with $\delta \in \left(\delta_L, \delta_L + k\sigma\right)$. We know that country 2 will always survive, and thus we have

$$\Pi_2\left(\delta\right) = \int_0^1 \frac{s}{(1+f)\int_{\delta - 2\sigma(1-x)}^{\delta + 2\sigma x}\frac{1 - \phi(y)}{2\sigma}dy}dx.$$

Note that for any x with $x \geq -\frac{\delta - \delta_L - k\sigma}{2\sigma}$ we are in the oscillating region; for x below we are in the increasing part. Let $\varepsilon \equiv \frac{\delta - \delta_L}{2\sigma} \in \left(0, \frac{1}{1+s}\right)$ so that so that $\delta = \delta_L + 2\sigma\varepsilon$. Thus, we have

$$1 - \rho\left(\delta, x\right) = \int_{\delta - 2\sigma(1-x)}^{\delta + 2\sigma x}\frac{1 - \phi(y)}{2\sigma}dy = \begin{cases} 1 - \varepsilon - x, & x \in \left(0, \frac{1}{1+s} - \varepsilon\right), \\ \frac{s}{1+s}, & x \in \left(\frac{1}{1+s} - \varepsilon, 1\right). \end{cases} \tag{A.14}$$

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Then, we have

$$\Pi_2\left(\delta\right) \;=\; \frac{s}{1+f}\left[\int_0^{\frac{1}{1+s}-\varepsilon}\frac{1}{1-\varepsilon-x}dx + \int_{\frac{1}{1+s}-\varepsilon}^1\frac{1}{\frac{s}{1+s}}dx\right] = \Pi_2\left(\delta_L\right) + \frac{s\left(\ln\left(1-\varepsilon\right) + \frac{1+s}{s}\varepsilon\right)}{1+f}$$

For investment in country 1, we know that, since $\delta > \delta_L$, we have $\rho_1^{min}\left(\delta\right) < \rho_1^{min}\left(\delta_L\right)$. First, note that

$$\rho\left(\delta,x\right) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi\left(y\right)}{2\sigma}dy = \begin{cases}\varepsilon+x, & x\in\left(0,\frac{1}{1+s}-\varepsilon\right)\\ \frac{1}{1+s}, & x\in\left(\frac{1}{1+s}-\varepsilon,1\right)\end{cases}$$

Let $x_{min}\left(\delta\right)$ be the measure of investors with higher signals than δ so that country 1 is safe. Since $\rho_1^{min}\left(\delta\right) = \frac{1-\theta_1(\delta)}{1+f}$, $x_{min}\left(\delta\right)$ is the lowest $x\in[0,1]$ such that

$$\rho\left(\delta,x\right) = \varepsilon + x \geq \rho_1^{min}\left(\delta\right).$$

Thus, we have

$$x_{min}\left(\delta\right) = x_{min}\left(\delta_L + 2\sigma\varepsilon\right) \quad=\quad \max\left\{\frac{1-\theta_1\left(\delta_L+2\sigma\varepsilon\right)}{1+f} - \varepsilon, 0\right\}. \tag{A.15}$$

The expected investment return from country 1 is

$$\begin{aligned}\Pi_1\left(\delta\right) \;&=\; \int_{x:\rho(\delta,x)\geq\rho_1^{min}(\delta)}\frac{1}{\left(1+f\right)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x}\frac{\phi(y)}{2\sigma}dy}dx\\ &=\; \Pi_1\left(\delta_L\right) + \frac{1}{1+f}\left\{\ln x_{min}\left(\delta_L\right) - \ln\left[\varepsilon + x_{min}\left(\delta_L+2\sigma\varepsilon\right)\right] + \left(1+s\right)\varepsilon\right\}\end{aligned}$$

Thus, to show that $\Pi_1\left(\delta_L+2\sigma\varepsilon\right) \geq \Pi_2\left(\delta_L+2\sigma\varepsilon\right)$, we need to show that the following inequality holds for $\varepsilon\in\left(0,\frac{1}{1+s}\right)$:

$$g_L\left(\varepsilon\right) \equiv \left(1+f\right)\left(\Pi_1-\Pi_2\right) = \ln x_{min}\left(\delta_L\right) - \ln\left[\varepsilon + x_{min}\left(\delta_L+2\sigma\varepsilon\right)\right] - s\ln\left(1-\varepsilon\right) \geq 0. \tag{A.16}$$

First, by using $\ln x_{min}\left(\delta_L\right) = s\ln s - \left(1+s\right)\ln\left(1+s\right)$ and $x_{min}\left(\delta_L + 2\sigma\frac{1}{1+s}\right) = 0$, we know the above inequality holds with equality at both end points $\varepsilon = 0$ and $\varepsilon = \frac{1}{1+s}$, i.e., $g_L\left(0\right) = g_L\left(\frac{1}{1+s}\right) = 0$. Second, it is easy to show that there exists a unique ε^* such that $\frac{1-\theta_1(\delta_L+2\sigma\varepsilon^*)}{1+f} = \varepsilon^*$, at which point (A.15) binds at zero. We further note that at $\varepsilon = 0$ we have $\frac{1-\theta_1(\delta_L)}{1+f} > 0$. Thus, in (A.15) we have $\varepsilon^* > 0$ and for $\varepsilon\in\left(0,\varepsilon^*\right)$ we have $x_{min}\left(\delta\right) = \frac{1-\theta_1(\delta_L+2\sigma\varepsilon)}{1+f} - \varepsilon > 0$, and for $\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right]$ we have $x_{min}\left(\delta\right) = 0$. Plugging in and taking derivative with respect to ε, we have

$$\frac{\partial}{\partial\varepsilon}\ln\left[\varepsilon + x_{min}\left(\delta_L+2\sigma\varepsilon\right)\right] = \begin{cases}\frac{-2\sigma\theta_1'\left(\delta_L+2\sigma\varepsilon\right)}{1-\theta_1\left(\delta_L+2\sigma\varepsilon\right)} &,\varepsilon\in\left(0,\varepsilon^*\right)\\ \frac{1}{\varepsilon} &,\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right]\end{cases}$$

Then, for (A.16), we have $g_L\left(\varepsilon\right)$ first rises and then drops:

$$g_L'\left(\varepsilon\right) = \begin{cases}\frac{2\sigma\theta_1'\left(\delta_L+2\sigma\varepsilon\right)}{1-\theta_1\left(\delta_L+2\sigma\varepsilon\right)} + \frac{s}{1-\varepsilon} > 0 &,\varepsilon\in\left(0,\varepsilon^*\right),\\ -\frac{1}{\varepsilon} + \frac{s}{1-\varepsilon} = \frac{(1+s)\varepsilon-1}{1-\varepsilon} < 0 &,\varepsilon\in\left[\varepsilon^*,\frac{1}{1+s}\right].\end{cases}$$

Combined with $g_L(0) = g_L\left(\frac{1}{1+s}\right) = 0$ we know that $g_L(\varepsilon) > 0, \forall \varepsilon \in \left(0, \frac{1}{1+s}\right)$, i.e., Thus, on $\varepsilon \in \left(0, \frac{1}{1+s}\right)$ the investors strictly want to invest in country 1.

We now consider the investors with $\delta \in (\delta_H - (2-k)\sigma, \delta_H)$. We know that country 1 will always survive, and thus we have

$$\Pi_1(\delta) = \int_0^1 \frac{1}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma}dy} dx.$$

Let $\varepsilon \equiv \frac{\delta_H - \delta}{2\sigma} \in \left(0, \frac{s}{1+s}\right)$ so that so that $\delta = \delta_H - 2\sigma\varepsilon$. Thus, we have

$$\rho(\delta, x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{\phi(y)}{2\sigma}dy = \begin{cases} \frac{1}{1+s}, & x \in \left(0, \frac{1}{1+s}+\varepsilon\right), \\ x-\varepsilon, & x \in \left(\frac{1}{1+s}+\varepsilon, 1\right). \end{cases} \tag{A.17}$$

Plugging in, we have

$$\Pi_1(\delta) = \frac{1}{1+f}\left[\int_0^{\frac{1}{1+s}+\varepsilon} \frac{1}{\frac{1}{1+s}}dx + \int_{\frac{1}{1+s}+\varepsilon}^1 \frac{1}{x-\varepsilon}dx\right] = \frac{1}{1+f}\left[1 + (1+s)\varepsilon + \ln(1-\varepsilon) + \ln(1+s)\right].$$

For investment in country 2, we know that, since $\delta < \delta_H$, we have $1 - \rho_2^{max}(\delta) < 1 - \rho_2^{max}(\delta_L) \iff \rho_2^{max}(\delta_L) < \rho_2^{max}(\delta)$. First, note that

$$1 - \rho(\delta, x) = \int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{1-\phi(y)}{2\sigma}dy = \begin{cases} \frac{s}{1+s}, & x \in \left(0, \frac{1}{1+s}+\varepsilon\right), \\ 1+\varepsilon-x, & x \in \left(\frac{1}{1+s}+\varepsilon, 1\right). \end{cases}$$

Let $x_{max}(\delta)$ be the measure of investors with higher signals than δ so that country 2 is safe. Since $1 - \rho_2^{max}(\delta) = s\frac{1-\theta_2(\delta)}{1+f}$, $x_{max}(\delta)$ is the highest $x \in [0,1]$ such that

$$1 - \rho(\delta, x) = 1 + \varepsilon - x \le 1 - \rho_2^{max}(\delta).$$

Thus, we have

$$x_{max}(\delta) = x_{max}(\delta_H - 2\sigma\varepsilon) = \min\left\{1 + \varepsilon - s\frac{1-\theta_2(\delta_H - 2\sigma\varepsilon)}{1+f}, 1\right\}. \tag{A.18}$$

The expected investment return from country 2 is

$$\begin{aligned} \Pi_2(\delta) &= \int_{x:\rho(\delta,x)\le\rho_2^{max}(\delta)} \frac{s}{(1+f)\int_{\delta-2\sigma(1-x)}^{\delta+2\sigma x} \frac{1-\phi(y)}{2\sigma}dy} dx \\ &= \frac{s}{1+f}\left[\frac{1+s}{s}\left(\frac{1}{1+s}+\varepsilon\right) - \ln\left[1+\varepsilon-x_{max}(\delta)\right] + \ln\left(\frac{s}{1+s}\right)\right] \end{aligned}$$

Differencing, we have

$$g_H(\varepsilon) = (1+f)\left[\Pi_1(\varepsilon) - \Pi_2(\varepsilon)\right] = \ln(1-\varepsilon) - s\ln s + (1+s)\ln(1+s) + s\ln\left[1+\varepsilon-x_{max}(\delta)\right]$$

with similar properties to $g_L(\varepsilon)$.

Finally, we need to pick σ appropriately so that there exists some natural number $N > 1$ so that $2N\sigma = \delta_H - \delta_L$. For this particular choice of $\sigma = \hat{\sigma}$, the limiting case of zero signal noise can be achieved when we take the sequence of $\sigma_n = \hat{\sigma}/n$ for $n = 1, 2, \dots$.

A.1.5   Equilibrium properties

First, with joint safety, the probability of survival for country 1 (or the probability of its bonds being the safe asset) is no longer one minus the probability of survival of country 2. Using $\tilde{\delta} \sim \mathbb{U}\left(-\overline{\delta}, \overline{\delta}\right)$, the probability of country 1 survival is

$$\Pr\left(\text{country 1 safe}\right) = \frac{\overline{\delta} - \delta_L}{2\overline{\delta}} = \frac{\overline{\delta} + z - (1+s)\ln(1+s) + s\ln s}{2\overline{\delta}}, \tag{A.19}$$

and the probability of country 2 survival is

$$\Pr\left(\text{country 2 safe}\right) = \frac{\delta_H + \overline{\delta}}{2\overline{\delta}} = \frac{\overline{\delta} + z - \frac{1+s}{s}\ln(1+s)}{2\overline{\delta}}.$$

As a result, the bonds issued by country 1 are more likely to be the safe assets than that issued by country 2 if the following condition holds:

$$s\ln s - (1+s)\ln(1+s) + \frac{1+s}{s}\ln(1+s) = s\ln s + \left(\frac{1}{s} - s\right)\ln(1+s) > 0. \tag{A.20}$$

This condition always holds: Define $F(s) \equiv s^2 \ln s + \left(1 - s^2\right)\ln(1+s)$, then $F(s) > 0$ holds for $s \in (0, 1)$. It is clear that $F(0) = 0$ while $F(1) = 0$. Simple algebra shows that

$$F'(s) = 2s\ln s - 2s\ln(1+s) + 1, \frac{1}{2}F''(s) = \ln s - \ln(1+s) + 1 - \frac{s}{1+s} = \ln\left(\frac{s}{1+s}\right) + 1 - \frac{s}{1+s}.$$

Let $y = \frac{s}{1+s} \in (0, 1)$; then because it is easy to show $\ln y + 1 - y < 0$ (due to concavity of $\ln y$), we know that $F''(s) < 0$. As a result, $F(s)$ is concave but $F(0) = F(1) = 0$. This immediately implies that $F(s) > 0$, which is our desired result. The condition is the same if we focus on sole survivals only instead of sole and joint survival, i.e., the bonds of country j are the only safe asset, the condition is exactly the same.

Country 1 has the highest likelihood of survival when $s \to 0$, which immediately follow from $-(1+s)\ln(1+s) + s\ln s$ is decreasing in s.

Obviously, the above equilibrium construction requires that $\delta_L(z) < \delta_H(z)$. Since $\delta_L(z)$ in (A.9) is decreasing in z while $\delta_H(z)$ in (A.13) is increasing in z, this condition $\delta_L(z) < \delta_H(z)$ holds if $z > \underline{z}$ so that $\delta_L(\underline{z}) = \delta_H(\underline{z})$ which gives $\underline{z}$:

$$-\underline{z} + (1+s)\ln(1+s) - s\ln s = \underline{z} - \frac{1+s}{s}\ln(1+s) \Rightarrow \underline{z} = \frac{1}{2}\left[\left(2 + s + \frac{1}{s}\right)\ln(1+s) - s\ln s\right]$$

A.2   Extension for a negative β asset

Suppose that θ, which proxies for the aggregate fundamental for both countries, is subject to shocks. For convenience, suppose that $\tilde{\theta}$ is drawn from the following uniform distribution $\tilde{\theta} \sim U\left[\underline{\theta}, \overline{\theta}\right]$, and recall $z\left(\tilde{\theta}\right) = \ln\frac{1+f}{1-\tilde{\theta}}$. Also, suppose that

$$l_i = l\tilde{\theta}, i \in \{1, 2\}$$

where $l > 0$ is a positive constant, so that recovery is increasing in the fundamental shock. Using (16), we calculate the threshold $\delta^*(\theta)$ as a function of the realization of $\tilde{\theta} = \theta$, to be

$$\delta^*(\theta) = \frac{[(1 - l\theta)s - (1 - l\theta)]z(\theta) - (s + l\theta)\ln(s + l\theta) + (1 + sl\theta)\ln(1 + l\theta s) + l\theta\ln(l\theta) - sl\theta\ln(l\theta)}{(1 - l\theta) + s(1 - l\theta)}$$

Note that $\frac{d}{d\theta}\delta^*(\theta) < 0$; that is, a higher θ, by reducing rollover risk, makes country 1 safer.

In this exercise we consider a distribution so that the relative fundamental δ is almost surely, $\delta > \delta^* (\mathbb{E}[\theta])$. This implies that ex-ante country 1 bonds are more likely to be safe. Also, define $\hat{\theta}(\delta)$ so that $\delta^* \left(\hat{\theta} \right) = \delta$ holds; this is the critical value of fundamental $\theta = \hat{\theta}$ so that country 1's bonds lose safety. We choose δ so that $\hat{\theta} > \underline{\theta}$, which implies that with strictly positive probability, country 1 defaults given a sufficiently low fundamental.

We are interested in the β of the bond price of each country with respect to the θ shock, i.e.,

$$\beta_i (\delta) = \frac{Cov \left(p_i \left(\tilde{\theta}; \delta \right), \tilde{\theta} \right)}{Var \left(\tilde{\theta} \right)} = \frac{\mathbb{E} \left[p_i \left(\tilde{\theta}; \delta \right) \cdot \tilde{\theta} \right] - \mathbb{E} \left[\tilde{\theta} \right] \mathbb{E} \left[p_i \left(\tilde{\theta}; \delta \right) \right]}{Var \left(\tilde{\theta} \right)}, \tag{A.21}$$

From equation (18), we know that

$$p_1 (\theta; \delta) = \begin{cases} \frac{(1+f)l\theta}{s+l\theta} & \text{if } \theta < \hat{\theta}(\delta) \text{ so country 1 defaults;} \\ \frac{1+f}{1+l\theta s} & \text{if } \theta \geq \hat{\theta}(\delta) \text{ so country 1 survives;} \end{cases}$$

and

$$p_2 (\theta; \delta) = \begin{cases} \frac{1+f}{s+l\theta} & \text{if } \theta < \hat{\theta}(\delta) \text{ so country 2 survives;} \\ \frac{(1+f)l\theta}{1+l\theta s} & \text{if } \theta \geq \hat{\theta}(\delta) \text{ so country 2 defaults.} \end{cases}$$

Given these pricing functions, it is straightforward to evaluate βs in (A.21). We vary country 1's relative strength δ and plot the βs for both bonds as a function of δ in Figure 4. We only plot the β for country 1's bonds, because $\beta_2 = -\beta_1/s$ in our model.[16]

A.3   single-survivor equilibrium with common bonds

In this appendix, we proof that $\delta^*(\alpha)$ is unique, $\delta^*(\alpha) \leq 0$, exists on $[0, \alpha^*]$, and has $\delta^*(\alpha^*) = 0$.

First, assume $s = 1$. Then, conjecture that $\delta^*(\alpha) = 0$ throughout by a simple symmetry argument. From (26), with $\theta_{def}(\delta^*(\alpha)) = \underline{\theta}$, we then have

$$\alpha^* = \frac{1+s}{1+f} (1 - \underline{\theta}) = e^{-z} (1 + s) \tag{A.22}$$

Next, assume $s < 1$ and $e^z > (1 + s)$ so that $\delta^*(0) < 0$. Then, let us conjecture $\delta^*(\alpha) \leq 0$ for $\alpha \in (0, \alpha^*)$.

Setting $\Pi_1(\delta^*) = \Pi_2(\delta^*)$ from (21) after substituting in for $\hat{f}$ from (24), $\delta^*(\alpha)$ is implicitly defined via

$$0 = h(\delta^*, \alpha) = \ln \left[e^z \frac{1-\alpha}{e^{-\delta^*} - \frac{\alpha}{1+s} e^z} \right] - s \ln \left[\frac{e^z}{s} \frac{1-\alpha}{e^{\delta^*} - \frac{\alpha}{1+s} e^z} \right] \tag{A.23}$$

Then, consider $\tilde{\delta} = \delta^*(\alpha)_+$. At this point, country 1 just survives, even though the funding gap (scaled by size) of country 2 is the best among all defaulting countries. Then, for the monotone cutoff strategy to be consistent, we need the default condition

$$\alpha \leq \frac{1+s}{1+f} [1 - \theta_2(\delta^*)] = \frac{1+s}{1+f} (1 - \underline{\theta}) e^{\delta^*} = e^{-z} (1 + s) e^{\delta^*}$$

---

[16]This is because cash-in-the-market-pricing implies that $p_1 + sp_2 = 1 + f$.

Suppose that the constraint is binding, which defines a loosest $\delta^*(\alpha)$ by

$$\hat{\delta}^*(\alpha) = z + \ln\left(\frac{\alpha}{1+s}\right) \iff e^{\hat{\delta}^*(\alpha)} = \frac{\alpha}{1+s}e^z \tag{A.24}$$

Assume that $\alpha < \alpha^* = \frac{1+s}{1+f}(1-\theta)$. Plugging in $\hat{\delta}^*(\alpha)$, we see that

$$h\left(\hat{\delta}^*(\alpha),\alpha\right) = \ln\left[e^z\frac{1-\alpha}{e^{-z}\frac{1+s}{\alpha} - \frac{\alpha}{1+s}e^z}\right] - s\ln\left[\frac{e^z}{s}\frac{1-\alpha}{e^z\frac{\alpha}{1+s} - \frac{\alpha}{1+s}e^z}\right] < 0 \tag{A.25}$$

as the second term explodes, i.e. $\ln[\cdot] = \infty$. Thus, it must be that $0 > \delta^*(\alpha) > \hat{\delta}^*(\alpha)$—the first part by our assumption that $\delta^* < 0$ and the second by the construction. However, we note that $\hat{\delta}^*(\alpha^*) = 0$ so that $\delta^*(\alpha^*) = 0$. This is possible as $(\delta,\alpha) = (0,\alpha^*)$ is a root of h – both sides are exploding at this point. The restriction above also implies that $0 < \delta^*_\alpha(\alpha^*) < \hat{\delta}^*_\alpha(\alpha^*) = \frac{1}{\alpha^*}$ so that $\delta^*(\alpha)$ has a bounded and positive derivative at α^*.

We next show that for a fixed $\alpha \in [0,\alpha^*]$, there exists unique $\delta^*(\alpha)$ that solves $h(\delta^*,\alpha)$. Fix α. Then, consider $h(\delta^*,\alpha)$ as a function of δ^*. Differentiating w.r.t. δ^*, we have

$$\frac{\partial h(\delta^*,\alpha)}{\partial \delta^*} = \frac{e^{-\delta^*}\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right) + se^{\delta^*}\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right)}{\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right)\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right)}$$

Then, given that we have $\alpha < \alpha^*$ and $\hat{\delta}^*(\alpha) < \delta^* < 0$ by assumption, we have

$$\left(e^{-\delta^*} - \frac{\alpha}{1+s}e^z\right) > \left(e^{-\delta^*} - \frac{\alpha^*}{1+s}e^z\right) = e^{-\delta^*} - 1 > 0$$

by assumption on the sign of δ^*. Next, we have

$$\left(e^{\delta^*} - \frac{\alpha}{1+s}e^z\right) > \left(e^{\hat{\delta}^*(\alpha)} - \frac{\alpha}{1+s}e^z\right) = \frac{\alpha}{1+s}e^z - \frac{\alpha}{1+s}e^z = 0$$

by the assumption on $\delta^* \in \left(\hat{\delta}^*(\alpha),0\right)$. Thus, we have $\frac{\partial h(\delta^*,\alpha)}{\partial \delta^*} > 0$. Finally, we know that $h\left(\hat{\delta}^*(\alpha),\alpha\right) < 0 < h(0,\alpha)$, so that a unique $\delta^*(\alpha) \in \left(\hat{\delta}^*(\alpha),0\right)$ exists.

What remains to be shown is that $\delta^*(\alpha)$ does not cross 0 before α^*. Suppose it does. Then, there exists an $\hat{\alpha} > 0$ but $\hat{\alpha} \neq \alpha^*$ such that $\delta^*(\hat{\alpha}) = 0$. Then, we have

$$h(0,\hat{\alpha}) = \ln\left[e^z\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] - s\ln\left[\frac{e^z}{s}\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] = (1-s)\ln\left[e^z\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] + s\ln s$$

Setting this equal to 0, we have

$$\ln\left[\frac{1-\hat{\alpha}}{1 - \frac{\hat{\alpha}}{1+s}e^z}\right] = \frac{-s\ln s}{1+s} - z \iff \frac{1 - e^{\left[\frac{-s\ln s}{1+s} - z\right]}}{\left[1 - \frac{1}{1+s}e^{\left[\frac{-s\ln s}{1+s}\right]}\right]} = \hat{\alpha}$$

Simplifying, we have

$$\hat{\alpha} = \frac{(1+s)\left(1 - s^{\frac{-s}{1+s}}e^{-z}\right)}{1 + s - s^{\frac{-s}{1+s}}}$$

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Then, notice that $\hat{\alpha} > \alpha^* \iff \dfrac{(1+s)\left(1 - s^{\frac{-s}{1+s}} e^{-z}\right)}{1 + s - s^{\frac{-s}{1+s}}} > e^{-z}(1+s)$, which simplifies to $1 > \alpha^*$. Thus, the function $\delta^*(\alpha)$ does not cross 0 before α^*.

A.4 Joint safety equilibrium with common bonds

Let us conjecture a non-monotone oscillating strategy as in A.1.

A.4.1 Lower boundary δ_L.

The definitions of $\rho(\delta, x)$ and $\rho(\delta_L, x)$ are as in Appendix A.1, and most of the result simply have $\hat{f}$ instead of f: as country 2 is safe to an agent with $\delta = \delta_L$, we have $\Pi_2(\delta_L) = \frac{s}{1+\hat{f}}\left[\ln\frac{1+s}{s} + 1\right] < \frac{1+s}{1+\hat{f}}$.

The common bonds change the safety condition for country 1 to

$$\theta_1(\delta) + \alpha p_c + \left(1 + \hat{f}\right)\rho_1^{min}(\delta) = 1 \iff \rho_1^{min}(\delta) = \frac{1 - \theta_1(\delta) - \alpha p_c}{1 + \hat{f}}$$

Define $x_{min}(\delta_L)$ as the solution to $\rho(\delta_L, x) = \rho_1^{min}(\delta_L)$. Given equation (A.3), we have that,

$$x_{min}(\delta_L) = \frac{1 - \theta_1(\delta_L) - \alpha p_c}{1 + \hat{f}} \tag{A.26}$$

Again, the expected return of investing in country 1 is given by $\Pi_1(\delta_L) = \frac{1}{1+\hat{f}}\left[\ln\frac{1}{1+s} - \ln x_{min}(\delta_L) + s\right]$. Indifference requires that $\Pi_2(\delta_L) = \Pi_1(\delta_L)$, which implies that

$$x_{min}(\delta_L) = \exp\left[s\ln s - (1+s)\ln(1+s)\right] \tag{A.27}$$

We combine the expressions for $x_{min}(\delta_L)$, (A.26) and (A.27), to solve for δ_L:

$$\delta_L = -\ln\left\{\frac{1}{1-\theta}\left[\left(1 + \hat{f}\right)\frac{s^s}{(1+s)^{(1+s)}} + \alpha p_c\right]\right\}. \tag{A.28}$$

A.4.2 Upper boundary δ_H.

The derivation of $\rho(\delta, x)$ and $\rho(\delta_H, x)$ follow Appendix A.1, , and most of the result simply have $\hat{f}$ instead of f. We have $\Pi_1(\delta_H) = \frac{\ln(1+s)+1}{1+\hat{f}}$ as country 1 is considered safe at $\delta_j = \delta_H$.

The default condition for country 2 is

$$s\theta_2(\delta) + s\alpha p_c + \left(1 + \hat{f}\right)[1 - \rho_2^{max}(\delta)] = s \iff [1 - \rho_2^{max}(\delta)] = s\frac{1 - \theta_2(\delta) - \alpha p_c}{1 + \hat{f}}$$

where $\rho_2^{max}(\delta)$ is the maximum amount of people investing in country 1 so that country 2 does not default. Define $x_{max}(\delta_H)$ as the solution to $\rho(\delta_H, x_{max}) = \rho_2^{max}(\delta_H)$. Given equation (A.11), we have that,

$$1 - x_{max}(\delta_H) = s\frac{1 - \theta_2(\delta_H) - \alpha p_c}{1 + \hat{f}} \tag{A.29}$$

Then the return to investing in country 2 is again given by $\Pi_2(\delta_H) = \frac{s}{1+\hat{f}}\left[\frac{1}{s} + \ln\frac{s}{1+s} - \ln(1 - x_{max}(\delta_H))\right]$. Indifference requires $\Pi_1(\delta_H) = \Pi_2(\delta_H)$, which implies that

$$1 - x_{max}(\delta_H) = \frac{s}{(1+s)^{\frac{1+s}{s}}} \tag{A.30}$$

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We combine the expressions for $x_{max}(\delta_H)$, (A.29) and (A.30), to solve for δ_H:

$$\delta_H \quad = \quad \ln\left\{\frac{1}{1-\theta}\left[\frac{1+\hat{f}}{(1+s)^{\frac{1+s}{s}}} + \alpha p_c\right]\right\} \tag{A.31}$$

The remainder of the proof, i.e., the verification argument, is exactly the same as in Appendix A.1 and hence omitted here.

A.4.3 Cutoff $\alpha_{HL} < \alpha^*$.

First, the assumption $e^z > (1+s) \iff (1+f) > (1-\theta)(1+s)$ guarantees that there is *some* realizations of $\tilde{\delta}$ that would allow joint safety. Consider the total funding requirement,

$$total\left(\tilde{\delta}\right) = (1-\theta_1) + (1-\theta_2)s = (1-\theta)\left(e^{-\tilde{\delta}} + s \cdot e^{\tilde{\delta}}\right) \tag{A.32}$$

This is minimized at $\tilde{\delta} = -\frac{1}{2}\ln s \geq 0$ for a total funding requirement of $total\left(-\frac{1}{2}\ln s\right) = (1-\theta)2\sqrt{s}$. Next, note that $1+s > 2\sqrt{s}$ so that $e^z > (1+s) > 2\sqrt{s}$.

Recall that $\alpha^* = e^{-z}(1+s)$. Then, assume that $z > \ln(1+s)$ so that $\alpha^* \in (0,1)$. Then, we have

$$\delta_H(\alpha^*) - \delta_L(\alpha^*) \quad = \quad \ln\left\{\frac{e^z}{1+s}\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha^*) + \alpha^*\right]\right\} + \ln\left\{\frac{e^z}{1+s}\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha^*) + \alpha^*\right]\right\}$$

$$= \quad \ln\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}\left(\frac{1}{\alpha^*} - 1\right) + 1\right] + \ln\left[\left(\frac{s}{1+s}\right)^{s}\left(\frac{1}{\alpha^*} - 1\right) + 1\right] > 0$$

where we used $\left(\frac{1}{1+s}\right)^{\frac{1}{s}} < 1$ and $\left(\frac{s}{1+s}\right)^{s} < 1$ and $\frac{1}{\alpha^*} > 1$ in the last line. Thus, at α^* the oscillating equilibrium already exists. It is easy to show that the the joint safety region $[\delta_L(\alpha), \delta_H(\alpha)]$ is expanding uniformly in α, and thus that $\alpha_{HL} < \alpha^*$.

Finally, define α_{HL} as the solution to

$$0 \quad = \quad \delta_H(\alpha_{HL}) - \delta_L(\alpha_{HL})$$

$$= \quad 2[z - \ln(1+s)] + \ln\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha_{HL}) + \alpha_{HL}\right] + \ln\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha_{HL}) + \alpha_{HL}\right]$$

Rearranging, we have

$$\left[\left(\frac{1}{1+s}\right)^{\frac{1}{s}}(1-\alpha_{HL}) + \alpha_{HL}\right]\left[\left(\frac{s}{1+s}\right)^{s}(1-\alpha_{HL}) + \alpha_{HL}\right] - e^{-2z}(1+s)^2 = 0$$

which is a quadratic equation in α_{HL}. We note that $e^{-2z}(1+s)^2 < 1 \iff 2[\ln(1+s) - z] < 0$, so that $\alpha_{HL} = 1$ makes the LHS positive. We also know that the LHS is increasing in α_{HL} for $\alpha_{HL} > 0$. Thus, there exists at most one positive root $\alpha_{HL} \in (0,1)$ under the assumption $z > \ln(1+s)$, and if not, both roots are negative. Solving for the larger root α_{HL}, and after some algebra, we can show that $\delta^*(\alpha_{HL}) = \delta_H(\alpha_{HL}) = \delta_L(\alpha_{HL})$.

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B Online Appendix B: Additional Results

B.1 Additive Fundamental Structure

We have considered the specification of $1 - \theta_i = (1 - \theta) \exp\left((-1)^i \tilde{\delta} \right)$ for country i's fundamental. We now show that results are qualitatively similar with the alternative additive specification

$$\theta_1 = \theta + \tilde{\delta}, \text{ and } \theta_2 = \theta - \tilde{\delta}.$$

As $x = \Pr\left(\tilde{\delta} + \epsilon_j > \delta^* \right) = \frac{\tilde{\delta} + \sigma - \delta^*}{2\sigma} \Rightarrow \tilde{\delta} = \delta^* + (2x - 1)\sigma$, we know that

$$\begin{aligned} \theta_1 &= \theta + \tilde{\delta} = \theta + \delta^* + (2x - 1)\sigma \\ \theta_2 &= \theta - \tilde{\delta} = \theta - \delta^* - (2x - 1)\sigma \end{aligned}$$

Given x, the large country 1 survives if and only if

$$p_1 - 1 + \theta_1 = (1 + f)x - 1 + \theta + \delta^* + (2x - 1)\sigma \geq 0 \Leftrightarrow x \geq \frac{1 - \theta - \delta^* + \sigma}{1 + f + 2\sigma}$$

which implies the expected return from investing in country 1 is

$$\Pi_1 = \int_{\frac{1 - \theta - \delta^* + \sigma}{1 + f + 2\sigma}}^1 \frac{1}{(1 + f)x} dx = \frac{1}{1 + f} \ln \frac{1 + f + 2\sigma}{1 - \theta - \delta^* + \sigma}.$$

For country 2, the bond is paid back if

$$\begin{aligned} (1 + f)x' - s + s\theta_2 &= (1 + f)x' - s + s[\theta - \delta^* - (2x - 1)\sigma] \geq 0 \\ \Leftrightarrow x' &\geq \frac{s(1 - \theta + \delta^* - \sigma)}{1 + f + 2s\sigma} \end{aligned}$$

which implies an expected return of

$$\Pi_2 = \int_{\frac{s(1 - \theta + \delta^* - \sigma)}{1 + f + 2s\sigma}}^1 \frac{s}{(1 + f)x'} dx' = \frac{s}{1 + f} \ln \frac{1 + f + 2s\sigma}{s(1 - \theta + \delta^* + \sigma)}$$

As a result, the equilibrium threshold δ^* is pinned by by the indifference condition

$$\ln \frac{1 + f + 2\sigma}{1 - \theta - \delta^* + \sigma} = s \ln \frac{1 + f + 2s\sigma}{s(1 - \theta + \delta^* + \sigma)}.$$

Letting $\sigma \to 0$ we obtain

$$\ln \frac{1 + f}{1 - \theta - \delta^*} = s \ln \frac{1 + f}{s(1 - \theta + \delta^*)}. \tag{B.1}$$

We no longer have close-form solution for δ^* in (B.1), as δ^* shows up in both sides. However, the solution is unique because LHS (RHS) is increasing (decreasing) in δ^*. Finally, to ensure $\delta^* < 0$ so that the larger country 1 is relatively safer, we require the same sufficient condition of $z = \ln \frac{1 + f}{1 - \theta} > 1$ in this alternative specification.

B.2 Uniqueness of the single-survivor equilibrium with threshold strategies within monotone strategies

First, let us define a few primitives. Let δ_j be a generic signal, and δ be the true state of the world. Further, let x denote the amount of pessimism of the investors, so that $x = 1$ is the most pessimistic agent (amongst all agents out there) and $x = 0$ is the least pessimistic agent. We then have $\delta(\delta_j, x) = \delta_j + 2\sigma\left(x - \frac{1}{2}\right)$. For most of the proofs, we assume wlog that the investor believes his signal to be the true signal, and thus all the action comes from movements in his relative position. As $\sigma \to 0$, fundamental uncertainty (that is movements in δ as a function of x) will vanish, whereas strategic uncertainty (relative ranking of investors as represented by x) remains.

Next, let us define $\phi(\delta_j)$ as the *proportion of funds* an investor with signal δ_j invests in country 1. Then define

$$\rho(\delta_j, x) = \frac{1}{2\sigma} \int_{\delta_j - 2\sigma(1-x)}^{\delta_j + 2\sigma x} \phi(y) \, dy$$

as the *aggregate proportion of investors in country 1* an investor with signal δ_j and level of pessimism x expects given the conjecture strategies $\phi(\cdot)$. Note that there is translation invariance

$$\rho(\delta_j, x) = \rho\left(\delta_j + \varepsilon, x - \frac{\varepsilon}{2\sigma}\right), \forall x \in \left(\frac{\varepsilon}{2\sigma}, 1\right)$$

Finally, define the (scaled by $1 + f$) *difference in expected returns* as

$$\Delta(\delta_j) = \int_0^1 \mathbf{1}_{\{\rho(\delta,x) \geq \rho_{min}(\delta)\}} \frac{1}{\rho(\delta,x)} dx - \int_0^1 \mathbf{1}_{\{\rho(\delta,x) \leq \rho_{max}(\delta)\}} \frac{s}{1 - \rho(\delta,x)} dx$$

Then, for any given conjectured difference function $\Delta(y)$, we must have

$$\phi(y) = \begin{cases} 1, & \Delta(y) > 0 \\ \in [0,1], & \Delta(y) = 0 \\ 0, & \Delta(y) < 0 \end{cases}$$

A *monotone strategy* is defined by $\phi'(y) \geq 0$ for all $y \in \left[-\overline{\delta}, \overline{\delta}\right]$, which implies that $\rho_\delta(\delta, x) \geq 0$ as well as $\rho_x(\delta, x) \geq 0$, i.e., $\rho(\delta, x)$ is monotone. This implies that we can write

$$
\begin{aligned}
\Delta(\delta_j) &= \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x) \geq \rho_{min}(\delta(\delta_j,x))\}} \frac{1}{\rho(\delta,x)} dx - \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x) \leq \rho_{max}(\delta(\delta_j,x))\}} \frac{s}{1 - \rho(\delta,x)} dx \\
&\approx \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x) \geq \rho_{min}(\delta_j)\}} \frac{1}{\rho(\delta,x)} dx - \int_0^1 \mathbf{1}_{\{\rho(\delta_j,x) \leq \rho_{max}(\delta_j)\}} \frac{s}{1 - \rho(\delta,x)} dx \\
&= \int_{x_{min}(\delta_j)}^1 \frac{1}{\rho(\delta_j,x)} dx - \int_0^{x_{max}(\delta_j)} \frac{s}{1 - \rho(\delta_j,x)} dx
\end{aligned}
$$

Country 1 survives if $\rho(\delta_j, x)$ is larger than $\rho_{min}(\delta(\delta_j, x))$. As the agent becomes more pessimistic relative to the other agents, i.e., x increases, the actual relative fundamental increases, and thus the threshold decreases:

$$
\begin{aligned}
\partial_x \rho_{min}(\delta(\delta_j, x)) &= \partial_x e^{-z} e^{-\delta(\delta_j, x)} = -e^{-z} e^{-\delta(\delta_j, x)} 2\sigma < 0 \\
\partial_{\delta_j} \rho_{min}(\delta(\delta_j, x)) &= -e^{-z} e^{-\delta(\delta_j, x)} < 0
\end{aligned}
$$

Thus, if $\rho(\delta, x)$ is monotone, there exists a unique threshold $x_{min}(\delta)$ above which country 1 is safe. Further,

by the implicit function theorem, we have

$$
\begin{aligned}
x'_{min}(\delta) &= -\frac{\rho_\delta(\delta, x) - \partial_\delta \rho_{min}\left(\tilde{\delta}(\delta, x)\right)}{\rho_x(\delta, x) - \partial_x \rho_{min}\left(\tilde{\delta}(\delta, x)\right)} \\
&= -\frac{\frac{\phi(\delta+2\sigma x)-\phi(\delta-2\sigma(1-x))}{2\sigma} + e^{-z}e^{-\tilde{\delta}(\delta,x)}}{\phi(\delta+2\sigma x) - \phi(\delta - 2\sigma(1-x)) + e^{-z}e^{-\tilde{\delta}(\delta,x)}2\sigma} \\
&= -\frac{1}{2\sigma}
\end{aligned}
$$

so that the pessimism threshold falls that makes country 1 safe. Similarly, we have

$$
\begin{aligned}
x'_{max}(\delta) &= -\frac{\rho_\delta(\delta, x) - \partial_\delta \rho_{max}\left(\tilde{\delta}(\delta, x)\right)}{\partial_x(\delta, x) - \partial_x \rho_{max}\left(\tilde{\delta}(\delta, x)\right)} \\
&= -\frac{\frac{\phi(\delta+2\sigma x)-\phi(\delta-2\sigma(1-x))}{2\sigma} + se^{-z}e^{\tilde{\delta}(\delta,x)}}{\phi(\delta+2\sigma x) - \phi(\delta - 2\sigma(1-x)) + se^{-z}e^{\tilde{\delta}(\delta,x)}2\sigma} \\
&= -\frac{1}{2\sigma}
\end{aligned}
$$

We can thus approximate

$$
x_{max}(\delta + \varepsilon) + \frac{\varepsilon}{2\sigma} \approx x_{max}(\delta) + x'_{max}(\delta)\varepsilon + \frac{\varepsilon}{2\sigma} = x_{max}(\delta) \quad \text{and} \quad x_{min}(\delta + \varepsilon) + \frac{\varepsilon}{2\sigma} \approx x_{min}(\delta)
$$

Finally, suppose a δ exists for which the investor expects *joint safety*, i.e., both countries to be safe for sure. Then, we must have $\phi(\delta) = \frac{1}{1+s}$ by the no arbitrage condition. A single-survivor equilibrium with threshold strategies is defined by a single-crossing condition on $\Delta = \Pi_1 - \Pi_2$ and a *non-flat part at* 0, where $\Delta(\delta) > 0$ implies $\phi = 1$ and $\Delta(\delta) < 0$ implies $\phi = 0$. Consider any other equilibrium. By dominance regions, we know that for high δ, $\phi = 1$ will eventually be optimal, and for very low δ, $\phi = 0$ will eventually be optimal.

Thus, any other equilibrium is either characterized by (1) a flat part $\Delta(\delta) = 0$, (2) multiple crossings $\Delta(\delta) = 0$ or (3) a combination of the two. In our joint safety equilibrium supported by oscillating strategy, (3) is the case, with a flat part in the middle.

B.2.1 Monotonicity and uniqueness of threshold equilibrium

A monotone strategy $\phi(\delta)$ requires $\Delta(\delta)$ to change signs only once. Thus, $\Delta(\delta)$ either crosses zero at a single point, or approaches it from below, stays flat on an interval $[\delta_L, \delta_H]$, and then rises above zero. Thus, at any point δ s.t. $\Delta(\delta) = 0$ we must have $\Delta'(\delta) \geq 0$. As we want to show that a threshold equilibrium is the only equilibrium possible, we now rule out any flat parts of Δ at zero.

To this end, suppose an interval $[\delta_L, \delta_H]$ exists on which $\Delta(\delta) = 0$.

Interior x_{min}, x_{max}**.** Suppose now that $x_{min}(\delta), x_{max}(\delta) \in (0, 1)$. This means that both countries are at risk of default, so there is no possibility of joint safety across all possible $x \in [0, 1]$ (it might exists for

some x if $x_{min}(\delta) < x_{max}(\delta)$). Take $\varepsilon \in (0, \delta_H - \delta_L)$. Then, we write

$$
\begin{aligned}
\Pi_1(\delta + \varepsilon) &= \int_{x_{min}(\delta+\varepsilon)}^{1} \frac{1}{\rho(\delta+\varepsilon, x)} dx \\
&= \int_{x_{min}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}}^{1+\frac{\varepsilon}{2\sigma}} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)} dx \\
&= \int_{x_{min}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)} dx + \int_{1}^{1+\frac{\varepsilon}{2\sigma}} \frac{1}{\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)} dx \\
&\approx \int_{x_{min}(\delta)}^{1} \frac{1}{\rho(\delta, x)} dx + \int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta+\varepsilon, x)} dx \\
&= \Pi_1(\delta) + \underbrace{\int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta+\varepsilon, x)} dx}_{\text{new pessimists}}
\end{aligned}
$$

Similarly, we have

$$
\begin{aligned}
\Pi_2(\delta + \varepsilon) &= \int_{0}^{x_{max}(\delta+\varepsilon)} \frac{s}{1-\rho(\delta+\varepsilon, x)} dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^{x_{max}(\delta+\varepsilon)+\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho\left(\delta+\varepsilon, x-\frac{\varepsilon}{2\sigma}\right)} dx \\
&\approx \int_{\frac{\varepsilon}{2\sigma}}^{x_{max}(\delta)} \frac{s}{1-\rho(\delta, x)} dx + \int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta, x)} dx - \int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta, x)} dx \\
&= \int_{0}^{x_{max}(\delta)} \frac{s}{1-\rho(\delta, x)} dx - \int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta, x)} dx \\
&= \Pi_2(\delta) - \underbrace{\int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta, x)} dx}_{\text{old optimists}}
\end{aligned}
$$

so that

$$
\begin{aligned}
\Delta(\delta_L + \varepsilon) &= \Pi_1(\delta_L + \varepsilon) - \Pi_2(\delta_L + \varepsilon) \\
&= \Pi_1(\delta_L) + \int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta_L+\varepsilon, x)} dx - \left[\Pi_2(\delta_L) - \int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta_L, x)} dx \right] \\
&= \int_{1-\frac{\varepsilon}{2\sigma}}^{1} \frac{1}{\rho(\delta_L+\varepsilon, x)} dx + \int_{0}^{\frac{\varepsilon}{2\sigma}} \frac{s}{1-\rho(\delta_L, x)} dx > 0
\end{aligned}
$$

But this implies that

$$
\phi(\delta_L + \varepsilon) = 1
$$

By monotonicity then, δ_L is the only point at which $\Delta(\delta) = 0$ and no flat parts can exist for $x_{min}, x_{max} \in (0, 1)$.

Cornered x_{min}, x_{max}. Next, suppose that at least one of the countries is going to survive regardless of x because of the assumed strategies. Wlog, let us focus on δ_L. First, let us rule out that $x_{min}(\delta_L) = 0$. Note that for any $\varepsilon > 0$, we have by the dominance boundaries $\Delta(\delta_L - \varepsilon) < 0$ and $\Delta(\delta_H + \varepsilon) > 0$, the

highest and lowest point of the all flat parts. Further note that $x_{min}(\delta_L) = 0$ implies that country 1 always survives in the eyes of an investor with signal δ_L. By construction we have $\rho(\delta, 0) = 0$— when the agent with signal δ_L is the most optimistic agent, he must believe by the conjecture on $\Delta(\delta)$ that everyone below him investors fully into country 2. But then this agent cannot believe that country 1 is safe regardless of x, as by assumption no country can survive without a minimum amount of investment.

Thus, at δ_L we must have $x_{max}(\delta_L) = 1$ and $x_{min}(\delta_H) = 0$—*country 2 always survives* given the strategies of the different agents. Then, we have the survival boundary of country 2 not changing, and thus again for $\varepsilon \in (0, \delta_H - \delta_L)$ we have

$$
\begin{aligned}
\Pi_2(\delta + \varepsilon) &= \int_0^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^{1 + \frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho\left(\delta + \varepsilon, x - \frac{\varepsilon}{2\sigma}\right)} dx \\
&= \int_{\frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta, x)} dx + \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx \\
&= \int_0^1 \frac{s}{1 - \rho(\delta, x)} dx + \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho(\delta, x)} dx \\
&= \Pi_2(\delta) + \underbrace{\int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx}_{\text{new pessimists}} - \underbrace{\int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho(\delta, x)} dx}_{\text{old optimists}}
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
0 = \Delta(\delta + \varepsilon) &= \Pi_1(\delta + \varepsilon) - \Pi_2(\delta + \varepsilon) \\
&= \Pi_1(\delta) + \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{1}{\rho(\delta + \varepsilon, x)} dx - \left[\Pi_2(\delta) + \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx - \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho(\delta, x)} dx\right] \\
&= \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{1}{\rho(\delta + \varepsilon, x)} dx - \int_{1 - \frac{\varepsilon}{2\sigma}}^1 \frac{s}{1 - \rho(\delta + \varepsilon, x)} dx + \int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho(\delta, x)} dx \\
&= \underbrace{\int_{1 - \frac{\varepsilon}{2\sigma}}^1 \left[\frac{1}{\rho(\delta + \varepsilon, x)} - \frac{s}{1 - \rho(\delta + \varepsilon, x)}\right] dx}_{\text{new pessimists}} + \underbrace{\int_0^{\frac{\varepsilon}{2\sigma}} \frac{s}{1 - \rho(\delta, x)} dx}_{\text{old optimists}}
\end{aligned}
$$

and there is now a *possibility* of a flat part. The intuition here is that we are balancing the returns that arise to the new most pessimistic investor (i.e. for high x) against the previous expected returns of the most optimistic investors (i.e. low x).

Taking derivatives around $\varepsilon = 0$, we have

$$
\begin{aligned}
\Delta\left(\delta+\varepsilon\right) \quad \approx \quad & \Delta\left(\delta\right)+\Delta'\left(\delta\right)\varepsilon \\
= \quad & \frac{1}{2\sigma}\left[\frac{1}{\rho\left(\delta+\varepsilon,1-\frac{\varepsilon}{2\sigma}\right)}-\frac{s}{1-\rho\left(\delta+\varepsilon,1-\frac{\varepsilon}{2\sigma}\right)}\right]_{\varepsilon=0}\varepsilon \\
& +\left[\int_{1-\frac{\varepsilon}{2\sigma}}^{1}\left[-\frac{\rho_{\delta}\left(\delta+\varepsilon,x\right)}{\rho\left(\delta+\varepsilon,x\right)^{2}}-\frac{s\left(-\rho_{\delta}\left(\delta+\varepsilon,x\right)\right)}{\left[1-\rho\left(\delta+\varepsilon,x\right)\right]^{2}}\right]dx\right]_{\varepsilon=0}\varepsilon \\
& +\frac{1}{2\sigma}\left[\frac{s}{1-\rho\left(\delta,\frac{\varepsilon}{2\sigma}\right)}\right]_{\varepsilon=0}\varepsilon \\
= \quad & \frac{1}{2\sigma}\left[\frac{1}{\rho\left(\delta,1\right)}-\frac{s}{1-\rho\left(\delta,1\right)}+\frac{s}{1-\rho\left(\delta,0\right)}\right]\varepsilon
\end{aligned}
$$

When $\delta = \delta_L$ we must have $\rho\left(\delta_L,0\right)=0$ by definition of δ_L. Then, the derivative $\Delta'\left(\delta_L\right)=0$ if

$$
\rho\left(\delta_L,1\right)=\frac{-1+\sqrt{1+4s}}{2s}>\frac{1}{1+s}
$$

which implies that least for some points on $\left(\delta_L,\delta_L+2\sigma\right)$ we must have $\phi\left(\delta\right)>\frac{1}{1+s}$.

By $x'_{min}\left(\delta\right)\le 0$ and $x'_{max}\left(\delta\right)\le 0$, as δ increases either we (i) move to a segment where $x_{min}\left(\delta\right),x_{max}\left(\delta\right)\in\left(0,1\right)$, an interior situation, or (ii) to a segment with $x_{min}\left(\delta\right)=0,x_{max}\left(\delta\right)=1$, a completely safe part.

But we know from the previous section that (i) immediately has $\Delta'\left(\delta\right)>0$, a violation of the premise that we are on a flat part for $\delta\in\left[\delta_L,\delta_H\right]$. Next, consider for (ii) any completely safe subset $J\subset\left(\delta_L,\delta_H\right)$ and $\delta\in J$. Then, we require $\rho\left(\delta,x\right)=\frac{1}{1+s},\forall x\in\left[0,1\right]$ by no arbitrage, which implies $\phi\left(\delta\right)=\frac{1}{1+s}$. But then we have a violation of monotonicity as $\rho\left(\delta_L,1\right)>\frac{1}{1+s}$. Thus, there cannot be any flat parts of $\Delta\left(\delta\right)$ at zero and the only equilibrium that survives is of the threshold form. By the construction in the paper, this threshold equilibrium is unique.

Existence of threshold equilibrium. Consider our unique candidate equilibrium

$$
\delta^{*}=-\frac{1-s}{1+s}z-\frac{s\ln s}{1+s}
$$

derived in the main text. Consider now $\delta_j<\delta^*$. Then, we have

$$
\Delta\left(\delta_j;\delta^{*}\right)=\int_{\rho(x)>\rho_{min}\left(\tilde{\delta}(x;\delta_j)\right)}\frac{1}{\left(1+f\right)\rho\left(x\right)}dx-s\int_{\rho(x)<\rho_{max}\left(\tilde{\delta}(x;\delta_j)\right)}\frac{1}{\left(1+f\right)\left(1-\rho\left(x\right)\right)}dx
$$

We know that $\Delta\left(\delta^{*};\delta^{*}\right)=0$. But by our setup, we know that moving $\delta_j<\delta^*$ lowers both $\rho_{min}\left(\delta\right)$ and $\rho_{max}\left(\delta\right)$. Thus, we need to look at the difference between the parts we are adding (region in which country 1 survives) and parts we are subtracting (region in which country 2 survives):

$$
\begin{aligned}
\Delta_{\delta_j}\left(\delta_j;\delta^{*}\right) \quad = \quad & -\rho'_{min}\left(\delta_j\right)\frac{1}{\left(1+f\right)\rho_{min}\left(\delta_j\right)}+s\rho'_{max}\left(\delta_j\right)\frac{1}{\left(1+f\right)\left(1-\rho_{max}\left(\delta_j\right)\right)} \\
= \quad & \frac{1}{\left(1+f\right)}-s\frac{1}{\left(1+f\right)}=\frac{1-s}{1+f}>0
\end{aligned}
$$

where we used

$$
\rho'_{min}\left(\delta_j\right)=-\rho_{min}\left(\delta_j\right)\quad\text{and}\quad\rho'_{max}\left(\delta_j\right)=-\left(1-\rho_{max}\left(\delta_j\right)\right)
$$

This is intuitive: as we increase δ_j, we are adding the most valuable states for country 1 (fixing $\rho(x)$) by evaluating at points set on which it will just survive, i.e., close to $\rho_{min}(\delta_j)$, and we are taking away the most valuable states for country 2 by evaluating at points set on which it will just default, i.e., close to $\rho_{max}(\delta_j)$.

B.3 single-survivor equilibrium with oscillating strategies because of positive recovery

Let us say that $s_1 = 1, s_2 = s$ and $l_i s_i$ to be the recovery given default of country i, so that it returns $\frac{l_i s_i}{y_i}$ per unit of dollar invested, where y_i is total investment in country i. Then if country 1 survives, to equalize return, we need

$$\frac{l_2 s}{y_2} = \frac{1}{y_1}, y_1 + y_2 = 1 + f \Rightarrow \frac{y_1}{y_2} = \frac{1}{l_2 s}.$$

This gives prices equal to

$$
\begin{aligned}
p_1 &= y_1 = \frac{(1+f)}{1 + l_2 s} \\
p_2 &= \frac{y_2}{s} = \frac{(1+f)\, l_2}{1 + l_2 s}
\end{aligned}
$$

Similarly, if country 2 survives, then

$$\frac{s}{y_2} = \frac{l_1}{y_1}, y_1 + y_2 = 1 + f \Rightarrow \frac{y_1}{y_2} = \frac{l_1}{s}$$

which results in prices

$$
\begin{aligned}
p_1 &= y_1 = \frac{(1+f)\, l_1}{l_1 + s} \\
p_2 &= \frac{y_2}{s} = \frac{(1+f)}{l_1 + s}
\end{aligned}
$$

Let

$$z = \ln \frac{1+f}{1-\theta} > 0$$

and fiscal surplus is given by

$$
\begin{aligned}
\theta_1 &= 1 - (1-\theta)\, e^{-\delta} = 1 - (1+f)\, e^{-z} e^{-\delta} \\
s\theta_2 &= s\left[1 - (1-\theta)\, e^{\delta}\right] = s\left[1 - (1+f)\, e^{-z} e^{\delta}\right]
\end{aligned}
$$

Define two constants $k_1 > 1$ and $k_2 > 1$ (which only occurs if $s < l_1$) so that

$$
\begin{aligned}
\frac{k_1}{2 - k_1} = \frac{1}{l_2 s} &\iff k_1 = \frac{2}{1 + l_2 s} > 1 \\
\frac{k_2}{2 - k_2} = \frac{s}{l_1} &\iff k_2 = \frac{2s}{s + l_1} > 1
\end{aligned}
$$

Then in the country-1-default region, $k_2 \sigma$ measure of agents invest in country 2, i.e. play $\phi = 0$, while $(2 - k_2)\sigma$ measure of agents play $\phi = 1$. Similarly in the country-2-default region, , $k_1 \sigma$ measure of agents play $\phi = 1$ while $(2 - k_1)\sigma$ measure of agents play $\phi = 0$.

Conjecture the following equilibrium strategy with cutoff δ^*

$$\phi(y) = \begin{cases} \cdots \\ 1, & y \in [\delta^* - 2\sigma, \delta^* - k_2\sigma] \\ 0, & y \in [\delta^* - k_2\sigma, \delta^*] \\ 1, & y \in [\delta^*, \delta^* + k_1\sigma] \\ 0, & y \in [\delta^* + k_1\sigma, \delta^* + 2\sigma] \\ 1, & y \in [\delta^* + 2\sigma, \delta^* + 2\sigma + k_1\sigma] \\ \cdots \end{cases}$$

In other words, two types of equilibria collide at δ^*. We conjecture that marginal investor at δ^* is indifferent, while the agents between $[\delta^* - k_2\sigma, \delta^*]$ strictly prefer $\phi = 0$, and symmetrically the agents between $[\delta^*, \delta^* + k_1\sigma]$ strictly prefer $\phi = 1$. Other agents in this economy are indifferent.

Let x denote the fraction of agents with signal realization above the agent's private signal δ_j, so that given x, the true fundamental is

$$\delta(x) = \delta_j - (1 - 2x)\sigma$$

Further, let $\rho(\delta_j, x)$ be the expected proportion agents investing in country 1 given x. Then, we have

$$\rho(\delta_j, x) = \begin{cases} 1 - \frac{k_2}{2}, & \delta + 2\sigma x < \delta^* + (2 - k_2)\sigma \\ x + cst, & else \\ \frac{k_1}{2} & \delta - 2\sigma(1 - x) > \delta^* - (2 - k_1)\sigma \end{cases}$$

where cst is picked so that $\rho(\delta_j, x)$ is continuous in x. We note that the slope is generically x as we are replacing $\phi = 0$ with $\phi = 1$ marginally. At $\delta_j = \delta^*$, we have

$$\rho(\delta^*, x) = \begin{cases} 1 - \frac{k_2}{2}, & x < 1 - \frac{k_2}{2} \\ x, & else \\ \frac{k_1}{2} & x > \frac{k_1}{2} \end{cases}$$

and we need

$$1 - \frac{k_2}{2} < \frac{k_1}{2}$$

Note that if we assume that $\rho_{min}(\delta), 1 - \rho_{max}(\delta) \in \left[1 - \frac{k_2}{2}, \frac{k_1}{2}\right]$ we have a 1-to-1 function between x and ρ that yields

$$x_{min} = \frac{1 - \theta_1(\delta^*)}{1 + f} = \frac{1 - \theta}{1 + f}e^{-\delta^*} \iff \ln x_{min} = -z - \delta^*$$

$$1 - x_{max} = s\frac{1 - \theta_2(\delta^*)}{1 + f} = s\frac{1 - \theta}{1 + f}e^{\delta^*} \iff \ln(1 - x_{max}) = \ln s - z + \delta^*$$

Note here that we are ignoring fundamental uncertainty. Otherwise, we need to take account of the fact that in the mind of the agent,

$$\rho_{min}(\delta(x)) = e^{-z}e^{-\delta(x)} = e^{-z}e^{-[\delta_j - (1 - 2x)\sigma]}$$

is the minimum investment in country 1 needed for it to survive conditional on x. For everything else below, we assume that $\rho_{min}(\delta(x)) = \rho_{min}(\delta_j)$. Next, note that

$$x = \text{Fraction of people with signal above agent}$$

so that $x = 1$ is the most pessimistic agent, and $x = 0$ is the most optimistic. As $\rho(\delta, x)$ is increasing in x,

we have

$$\begin{aligned}
x < x_{min} &\iff \text{Country 1 fails}\\
x > x_{min} &\iff \text{Country 1 survives}\\
x < x_{max} &\iff \text{Country 2 survives}\\
x > x_{max} &\iff \text{Country 2 fails}
\end{aligned}$$

Then, for the boundary agent, the expected return of investing in country 2 is given by

$$\begin{aligned}
\Pi_2\left(\delta^*\right) &= Return_2\left(survival\right) + Return_2\left(default\right)\\
&= \int_0^{x_{max}} \frac{s}{\left(1+f\right)\left(1-\rho\left(\delta^*,x\right)\right)}dx + \int_{x_{max}}^1 \frac{l_2 s}{\left(1+f\right)\left(1-\rho\left(\delta^*,x\right)\right)}dx\\
&= \int_0^{1-\frac{k_2}{2}} \frac{s}{\left(1+f\right)\left(1-\left(1-\frac{k_2}{2}\right)\right)}dx + \int_{1-\frac{k_2}{2}}^{x_{max}} \frac{s}{\left(1+f\right)\left(1-x\right)}\\
&\quad + \int_{x_{max}}^{\frac{k_1}{2}} \frac{l_2 s}{\left(1+f\right)\left(1-x\right)}dx + \int_{\frac{k_1}{2}}^1 \frac{l_2 s}{\left(1+f\right)\left(1-\frac{k_1}{2}\right)}dx\\
&= \left(1-\frac{k_2}{2}\right)\frac{s}{\left(1+f\right)\frac{k_2}{2}} + \frac{s}{1+f}\left[\ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}\right)\right]\\
&\quad + \frac{l_2 s}{1+f}\left[\ln\left(1-x_{max}\right) - \ln\left(1-\frac{k_1}{2}\right)\right] + \left(1-\frac{k_1}{2}\right)\frac{l_2 s}{\left(1+f\right)\left(1-\frac{k_1}{2}\right)}\\
&= \frac{s}{\left(1+f\right)}\left\{\left(\frac{1-\frac{k_2}{2}}{\frac{k_2}{2}}\right) + \left[\ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}\right)\right] + l_2 + l_2\left[\ln\left(1-x_{max}\right) - \ln\left(1-\frac{k_1}{2}\right)\right]\right\}
\end{aligned}$$

and the expected return of investing in country 1 is given by

$$\begin{aligned}
\Pi_1\left(\delta^*\right) &= \int_0^{x_{min}} \frac{l_1}{\left(1+f\right)\rho\left(\delta^*,x\right)}dx + \int_{x_{min}}^1 \frac{1}{\left(1+f\right)\rho\left(\delta^*,x\right)}dx\\
&= \int_0^{1-\frac{k_2}{2}} \frac{l_1}{\left(1+f\right)\left(1-\frac{k_2}{2}\right)}dx + \int_{1-\frac{k_2}{2}}^{x_{min}} \frac{l_1}{\left(1+f\right)x}dx\\
&\quad + \int_{x_{min}}^{\frac{k_1}{2}} \frac{1}{\left(1+f\right)x}dx + \int_{\frac{k_1}{2}}^1 \frac{1}{\left(1+f\right)\frac{k_1}{2}}dx\\
&= \left(1-\frac{k_2}{2}\right)\frac{l_1}{\left(1+f\right)\left(1-\frac{k_2}{2}\right)} + \frac{l_1}{1+f}\left[\ln\left(x_{min}\right) - \ln\left(1-\frac{k_2}{2}\right)\right]\\
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}\right)\right] + \left(1-\frac{k_1}{2}\right)\frac{1}{\left(1+f\right)\frac{k_1}{2}}\\
&= \frac{1}{1+f}\left\{l_1 + l_1\left[\ln\left(x_{min}\right) - \ln\left(1-\frac{k_2}{2}\right)\right] + \left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}\right)\right] + \left(\frac{1-\frac{k_1}{2}}{\frac{k_1}{2}}\right)\right\}
\end{aligned}$$

Note that

$$
\left(\frac{1-\frac{k_1}{2}}{\frac{k_1}{2}}\right) = \left(\frac{1}{\frac{k_1}{2}}-1\right) = 1 + sl_2 - 1 = sl_2
$$

$$
\left(\frac{1-\frac{k_2}{2}}{\frac{k_2}{2}}\right) = \left(\frac{1}{\frac{k_2}{2}}-1\right) = \frac{s+l_1}{s} - \frac{s}{s} = \frac{l_1}{s}
$$

Setting these equal, we have

$$
s\left\{\frac{l_1}{s} + \left[\ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}\right)\right] + l_2 + l_2\left[\ln\left(1-x_{max}\right) - \ln\left(1-\frac{k_1}{2}\right)\right]\right\}
$$
$$
= \left\{l_1 + l_1\left[\ln\left(x_{min}\right) - \ln\left(1-\frac{k_2}{2}\right)\right] + \left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}\right)\right] + sl_2\right\}
$$

Plugging in for k_1, k_2 and

$$
\frac{k_1}{2} = \frac{1}{1+l_2 s}
$$
$$
\frac{k_2}{2} = \frac{s}{s+l_1}
$$
$$
1 - \frac{k_1}{2} = \frac{l_2 s}{1+l_2 s}
$$
$$
1 - \frac{k_2}{2} = \frac{l_1}{s+l_1}
$$
$$
\ln\left(x_{min}\right) = -z - \delta^*
$$
$$
\ln\left(1-x_{max}\right) = -z + \delta^* + \ln s
$$

Setting these equal, we have

$$
s\left\{\left[\ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}\right)\right] + l_2\left[\ln\left(1-x_{max}\right) - \ln\left(1-\frac{k_1}{2}\right)\right]\right\}
$$
$$
= l_1\left[\ln\left(x_{min}\right) - \ln\left(1-\frac{k_2}{2}\right)\right] + \left[\ln\left(\frac{k_1}{2}\right) - \ln\left(x_{min}\right)\right]
$$
$$
\iff s\left\{-(1-l_2)\ln\left(1-x_{max}\right) + \left[\ln\left(\frac{k_2}{2}\right) - l_2\ln\left(1-\frac{k_1}{2}\right)\right]\right\}
$$
$$
= -(1-l_1)\ln\left(x_{min}\right) + \left[\ln\left(\frac{k_1}{2}\right) - l_1\ln\left(1-\frac{k_2}{2}\right)\right]
$$
$$
\iff s\left\{(1-l_2)(z-\delta^*-\ln s) + \left[\ln\left(\frac{s}{s+l_1}\right) - l_2\ln\left(\frac{l_2 s}{1+l_2 s}\right)\right]\right\}
$$
$$
= (1-l_1)(z+\delta^*) + \left[\ln\left(\frac{1}{1+l_2 s}\right) - l_1\ln\left(\frac{l_1}{s+l_1}\right)\right]
$$

Finally, solving for δ^*, we have

$$
\begin{aligned}
\delta^* &= \frac{s\left\{(1-l_2)(z-\ln s)+\left[\ln\left(\frac{s}{s+l_1}\right)-l_2\ln\left(\frac{l_2 s}{1+l_2 s}\right)\right]\right\}-(1-l_1)z-\left[\ln\left(\frac{1}{1+l_2 s}\right)-l_1\ln\left(\frac{l_1}{s+l_1}\right)\right]}{(1-l_1)+s(1-l_2)} \\
&= \frac{s\left\{(1-l_2)z-(1-l_2)\ln s+\ln s-\ln(s+l_1)-l_2\ln l_2-l_2\ln s+l_2\ln(1+l_2 s)\right\}}{(1-l_1)+s(1-l_2)} \\
&\quad +\frac{-(1-l_1)z+\ln(1+l_2 s)+l_1\ln(l_1)-l_1\ln(s+l_1)}{(1-l_1)+s(1-l_2)}
\end{aligned}
$$

so that finally

$$
\delta^* = \frac{[(1-l_2)s-(1-l_1)]z-(s+l_1)\ln(s+l_1)+(1+sl_2)\ln(1+l_2 s)+l_1\ln l_1-sl_2\ln l_2}{(1-l_1)+s(1-l_2)} \tag{B.2}
$$

Plugging in $l_1=l_2=0$, we have

$$
\delta^* = \frac{-(1-s)z-s\ln(s)}{1+s}
$$

our benchmark result absent recovery. This is the only single-survivor equilibrium supported by threshold strategies.

We want to show that from the perspective of δ^*, for an x small enough so that $\rho(\delta^*,x)=1-\frac{k_2}{2}$, does country 1 default? We know that $\rho_{min}(\delta^*)=e^{-z}e^{-\delta^*}$, so that

$$
\begin{aligned}
\rho_{min}(\delta^*) &> 1-\frac{k_2}{2} \\
\Longleftrightarrow \ln(\rho_{min}(\delta^*)) &> \ln\left(1-\frac{k_2}{2}\right) \\
\Longleftrightarrow -(\delta^*+z) &> \ln\left(\frac{l_1}{s+l_1}\right)
\end{aligned}
$$

which gives

$$
\begin{aligned}
&-[2(1-l_2)sz-(s+l_1)\ln(s+l_1)+(1+sl_2)\ln(1+l_2 s)+l_1\ln l_1-sl_2\ln l_2] \\
&> [(1-l_1)+s(1-l_2)][\ln l_1-\ln(s+l_1)]
\end{aligned}
$$

and ultimately yields

$$
F_1^*(l_1,l_2,s)]\equiv -2(1-l_2)sz-[1+s(1-l_2)]\ln l_1+sl_2\ln l_2+[1+s(2-l_2)]\ln(s+l_1)-(1+l_2 s)\ln(1+l_2 s)
$$

and the default condition is given by $F_1^*(l_1,l_2,s)\geq 0$. Assume $l_1=l_2=l$. Then, we have

$$
F_1^*(l,l,s)=-2(1-l)sz-[1-(1-2l)s]\ln l+[1+s(2-l)]\ln(s+l)-(1+ls)\ln(1+ls)
$$

We can show that $F_1^*(l,l,s)$ is always positive for small enough recovery l as the term $-[1-(1-2l)s]\ln l$ explodes, swamping any negative z effect.[17]

---

[17]Taking derivatives w.r.t. l and s, we have

$$
\begin{aligned}
\partial_l F_1^*(l,l,s) &= 2sz+s-\frac{(1+s)}{l}+\frac{1+(2-l)s}{s+l}+2s\ln l-s\ln(s+l)-s\ln(1+ls) \\
\partial_s F_1^*(l,l,s) &= \frac{1+(2-l)s}{s+l}-l\ln(1+ls)+(2-l)\ln(s+l)-2(1-l)z-l-(1-2l)\ln l
\end{aligned}
$$

Next, we want to show that from the perspective of δ^*, for an x large enough so that $\rho(\delta^*, x) = \frac{k_1}{2}$, does country 2 default? We know that $1 - \rho_{max}(\delta^*) = se^{-z}e^{\delta^*}$, so that

$$1 - \rho_{max}(\delta^*) > 1 - \frac{k_1}{2}$$

$$\iff \ln(1 - \rho_{max}(\delta^*)) > \ln\left(1 - \frac{k_1}{2}\right)$$

$$\iff \ln s - z + \delta^* > \ln\left(\frac{l_2 s}{1 + l_2 s}\right)$$

so that

$$[(1 - l_1) + s(1 - l_2)]\ln s - 2(1 - l_1)z - (s + l_1)\ln(s + l_1) + (1 + sl_2)\ln(1 + l_2 s) + l_1 \ln l_1 - sl_2 \ln l_2$$
$$> [(1 - l_1) + s(1 - l_2)][\ln l_2 + \ln s - \ln(1 + l_2 s)]$$

Define

$$F_2^*(l_1, l_2, s) \equiv -2(1 - l_1)z - (s + l_1)\ln(s + l_1) + (2 - l_1 + s)\ln(1 + l_2 s) + l_1 \ln l_1 - [s + (1 - l_1)]\ln l_2$$

and the default condition is given by $F_2^*(l_1, l_2, s) \geq 0$. Assuming equal recovery $l_1 = l_2 = l$, we have

$$F_2^*(l, l, s) = -2(1 - l)z - (s + l)\ln(s + l) + (2 - l + s)\ln(1 + ls) - [s + (1 - 2l)]\ln l$$

We can show that $F_2^*(l, l, s)$ is always positive for small enough recovery l as the term $-[s + (1 - 2l)]\ln l$ explodes, swamping any negative z effect.

Let us consider an interior agent, i.e., $\delta \in [\delta^* - k_2\sigma, \delta^* + k_1\sigma]$. Let

$$\delta(\varepsilon) = \delta^* + 2\varepsilon\sigma$$

with $\varepsilon \in \left[-\frac{k_2}{2}, \frac{k_1}{2}\right]$. Let us first consider investment in country 1. We have $\rho_{min}(\delta)$ as the default boundary, and actual investment is given by

$$\rho(\delta, x) = \begin{cases} 1 - \frac{k_2}{2}, & \delta^* + \varepsilon 2\sigma + 2\sigma x < \delta^* + (2 - k_2)\sigma \\ x + cst, & else \\ \frac{k_1}{2} & \delta^* + \varepsilon 2\sigma - 2\sigma(1 - x) > \delta^* - (2 - k_1)\sigma \end{cases} = \begin{cases} 1 - \frac{k_2}{2}, & 2\varepsilon\sigma + 2\sigma x < (2 - k_2)\sigma \\ x + cst, & else \\ \frac{k_1}{2} & 2\varepsilon\sigma - 2\sigma(1 - x) > -(2 - k_1)\sigma \end{cases}$$

which gives

$$\rho(\delta, x) = \begin{cases} 1 - \frac{k_2}{2}, & \varepsilon + x < 1 - \frac{k_2}{2} \\ x + \varepsilon, & else \\ \frac{k_1}{2} & \varepsilon + x > \frac{k_1}{2} \end{cases}$$

Note that we have $cst = \varepsilon$ by imposing continuity (which has to follow from $\rho(\delta, x)$ being an integral over strategies ϕ).

Let $x_{min}(\delta)$ be the lowest $x \in [0, 1]$ such that

$$\rho(\delta, x) = \varepsilon + x \geq \rho_{min}(\delta)$$

and we therefore have

$$x_{min}(\delta) = \max\{\rho_{min}(\delta) - \varepsilon, 0\}$$

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Similarly, let $x_{max}(\delta)$ be the highest $x \in [0,1]$ such that

$$1 - \rho(\delta, x) = 1 - \varepsilon - x \geq 1 - \rho_{max}(\delta)$$

and thus

$$1 - x_{max}(\delta) = \max\{1 - \rho_{max}(\delta) + \varepsilon, 0\}$$

The expected return of investing in country 1 is then given by

$$
\begin{aligned}
\Pi_1(\delta) &= \int_{x:\rho(\delta,x)<\rho_{min}(x)} \frac{l_1}{(1+f)\,\rho(\delta,x)} dx + \int_{x:\rho(\delta,x)\geq\rho_{min}(x)} \frac{1}{(1+f)\,\rho(\delta,x)} dx \\[2mm]
&= \int_0^{x_{min}(\delta)} \frac{l_1}{(1+f)\,\rho(\delta,x)} dx + \int_{x_{min}(\delta)}^1 \frac{1}{(1+f)\,\rho(\delta,x)} dx \\[2mm]
&= \int_0^{1-\frac{k_2}{2}-\varepsilon} \frac{l_1}{(1+f)\left(1-\frac{k_2}{2}\right)} dx + \int_{1-\frac{k_2}{2}-\varepsilon}^{x_{min}(\delta)} \frac{l_1}{(1+f)\,(x+\varepsilon)} dx \\[2mm]
&\quad + \int_{x_{min}(\delta)}^{\frac{k_1}{2}-\varepsilon} \frac{1}{(1+f)\,(x+\varepsilon)} dx + \int_{\frac{k_1}{2}-\varepsilon}^1 \frac{1}{(1+f)\,\frac{k_1}{2}} dx \\[2mm]
&= \frac{l_1}{1+f}\left[\frac{1-\frac{k_2}{2}-\varepsilon}{1-\frac{k_2}{2}} + \ln(x_{min}(\delta)+\varepsilon) - \ln\left(1-\frac{k_2}{2}\right)\right] \\[2mm]
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln(x_{min}(\delta)+\varepsilon) + \frac{1-\frac{k_1}{2}+\varepsilon}{\frac{k_1}{2}}\right] \\[2mm]
&= \frac{l_1}{1+f}\left[1 - \frac{\varepsilon}{1-\frac{k_2}{2}} + \ln(x_{min}(\delta)+\varepsilon) - \ln\left(1-\frac{k_2}{2}\right)\right] \\[2mm]
&\quad + \frac{1}{1+f}\left[\ln\left(\frac{k_1}{2}\right) - \ln(x_{min}(\delta)+\varepsilon) + \frac{1-\frac{k_1}{2}}{\frac{k_1}{2}} + \frac{\varepsilon}{\frac{k_1}{2}}\right] \\[2mm]
&= \Pi_1(\delta^*) + \frac{l_1}{1+f}\left[-\frac{\varepsilon}{1-\frac{k_2}{2}} + \ln(x_{min}(\delta)+\varepsilon) - \ln x_{min}(\delta^*)\right] \\[2mm]
&\quad + \frac{1}{1+f}\left[\ln x_{min}(\delta^*) - \ln(x_{min}(\delta)+\varepsilon) + \frac{\varepsilon}{\frac{k_1}{2}}\right] \\[2mm]
&= \Pi_1(\delta^*) + \frac{1}{1+f}\left\{\varepsilon\left(\frac{1}{\frac{k_1}{2}} - \frac{l_1}{1-\frac{k_2}{2}}\right) - (1-l_1)\left[\ln(x_{min}(\delta)+\varepsilon) - \ln x_{min}(\delta^*)\right]\right\} \\[2mm]
&= \Pi_1(\delta^*) + \frac{1}{1+f}\left\{\varepsilon\left[(1-l_1) - s(1-l_2)\right] - (1-l_1)\left[\ln(x_{min}(\delta)+\varepsilon) - \ln x_{min}(\delta^*)\right]\right\}
\end{aligned}
$$

Similarly, investing in country 2 gives

$$
\begin{aligned}
\Pi_2\left(\delta\right) &= \int_0^{x_{max}\left(\delta\right)} \frac{s}{\left(1+f\right)\left(1-\rho\left(\delta,x\right)\right)}dx + \int_{x_{max}\left(\delta\right)}^1 \frac{l_2 s}{\left(1+f\right)\left(1-\rho\left(\delta,x\right)\right)}dx \\
&= \int_0^{1-\frac{k_2}{2}-\varepsilon} \frac{s}{\left(1+f\right)\left(1-\left(1-\frac{k_2}{2}\right)\right)}dx + \int_{1-\frac{k_2}{2}-\varepsilon}^{x_{max}\left(\delta\right)} \frac{s}{\left(1+f\right)\left(1-x-\varepsilon\right)} \\
&\quad + \int_{x_{max}\left(\delta\right)}^{\frac{k_1}{2}-\varepsilon} \frac{l_2 s}{\left(1+f\right)\left(1-x-\varepsilon\right)}dx + \int_{\frac{k_1}{2}-\varepsilon}^1 \frac{l_2 s}{\left(1+f\right)\left(1-\frac{k_1}{2}\right)}dx \\
&= \frac{s}{1+f}\left[\frac{1-\frac{k_2}{2}-\varepsilon}{\frac{k_2}{2}} + \ln\left(\frac{k_2}{2}\right) - \ln\left(1-x_{max}\left(\delta\right)-\varepsilon\right)\right] \\
&\quad + \frac{s l_2}{1+f}\left[\ln\left(1-x_{max}\left(\delta\right)-\varepsilon\right) - \ln\left(1-\frac{k_1}{2}\right) + \frac{1-\frac{k_1}{2}+\varepsilon}{1-\frac{k_1}{2}}\right] \\
&= \Pi_2\left(\delta^*\right) + \frac{s}{1+f}\left\{\varepsilon\left(l_2\frac{1}{1-\frac{k_1}{2}} - \frac{1}{\frac{k_2}{2}}\right) + \left(1-l_2\right)\left[\ln\left(1-x_{max}\left(\delta^*\right)\right) - \ln\left(1-x_{max}\left(\delta\right)-\varepsilon\right)\right]\right\} \\
&= \Pi_2\left(\delta^*\right) + \frac{s}{1+f}\left\{\varepsilon\left[\frac{\left(1-l_1\right)-s\left(1-l_2\right)}{s}\right] + \left(1-l_2\right)\left[\ln\left(1-x_{max}\left(\delta^*\right)\right) - \ln\left(1-x_{max}\left(\delta\right)-\varepsilon\right)\right]\right\}
\end{aligned}
$$

Let us define

$$
\begin{aligned}
g\left(\varepsilon\right) &\equiv \left(1+f\right)\left[\Pi_1\left(\delta\right) - \Pi_2\left(\delta\right)\right] \\
&= \varepsilon\left[\left(1-l_1\right)-s\left(1-l_2\right)\right] - \left(1-l_1\right)\left[\ln\left(x_{min}\left(\delta\right)+\varepsilon\right) - \ln x_{min}\left(\delta^*\right)\right] \\
&\quad -s\left\{\varepsilon\left[\frac{\left(1-l_1\right)-s\left(1-l_2\right)}{s}\right] + \left(1-l_2\right)\left[\ln\left(1-x_{max}\left(\delta^*\right)\right) - \ln\left(1-x_{max}\left(\delta\right)-\varepsilon\right)\right]\right\} \\
&= -\left(1-l_1\right)\left[\ln\left(x_{min}\left(\delta^*+2\sigma\varepsilon\right)+\varepsilon\right) - \ln x_{min}\left(\delta^*\right)\right] \\
&\quad +s\left(1-l_2\right)\left[\ln\left(1-x_{max}\left(\delta^*+2\sigma\varepsilon\right)-\varepsilon\right) - \ln\left(1-x_{max}\left(\delta^*\right)\right)\right] \\
&\quad +\varepsilon\left\{\left[\left(1-l_1\right)-s\left(1-l_2\right)\right] - s\left[\frac{\left(1-l_1\right)-s\left(1-l_2\right)}{s}\right]\right\} \\
&= -\left(1-l_1\right)\left[\ln\left(x_{min}\left(\delta^*+2\sigma\varepsilon\right)+\varepsilon\right) - \ln x_{min}\left(\delta^*\right)\right] \\
&\quad +s\left(1-l_2\right)\left[\ln\left(1-x_{max}\left(\delta^*+2\sigma\varepsilon\right)-\varepsilon\right) - \ln\left(1-x_{max}\left(\delta^*\right)\right)\right]
\end{aligned}
$$

Taking the derivative w.r.t. ε, we have many different cases. The issue is if x_{min} or x_{max} start binding first. Regardless, close to $\varepsilon = 0$ we have neither x_{min} or x_{max} cornered, so that

$$
\begin{aligned}
\ln\left(x_{min}\left(\delta^*+2\sigma\varepsilon\right)+\varepsilon\right) &= \ln\left(\rho_{min}\left(\delta\left(\varepsilon\right)\right)\right) = -z - \delta\left(\varepsilon\right) = -z - \left(\delta^*+2\sigma\varepsilon\right) \\
\ln\left(1-x_{max}\left(\delta^*+2\sigma\varepsilon\right)-\varepsilon\right) &= \ln\left(1-\rho_{max}\left(\delta\left(\varepsilon\right)\right)\right) = s\ln s - z + \delta\left(\varepsilon\right) = s\ln s - z + \left(\delta^*+2\sigma\varepsilon\right)
\end{aligned}
$$

and thus for ε small we have

$$
g'\left(\varepsilon\right) = -\left(1-l_1\right)\left(-\right)2\sigma + s\left(1-l_2\right)2\sigma = 2\sigma\left[\left(1-l_1\right)+s\left(1-l_2\right)\right] > 0
$$

and indeed we have the incentives of the agents aligned with the conjectured strategies, at least around δ^*.

Next, we have to account for all the different cases – that is, we know that at some distance ε that x_{min}, x_{max} start binding at $0, 1$, respectively.

Let ε_{min} be the point at which x_{min} becomes cornered, that is

$$
\rho_{min}\left(\delta\right) = \varepsilon \iff e^{-z}e^{-\left(\delta^*+2\sigma\varepsilon\right)} = \varepsilon \iff 2\sigma\varepsilon + \ln\varepsilon = -z - \delta^*
$$

Note that $\rho_{min}(\delta) > 0$ so that there is no solution for $\varepsilon < 0$.

Similarly, let ε_{max} be the point at which x_{max} becomes cornered, that is

$$1 - \rho_{max}(\delta) = -\varepsilon \iff se^{-z}e^{\delta^* + 2\sigma\varepsilon} = -\varepsilon \iff 2\sigma(-\varepsilon) + \ln(-\varepsilon) = \ln s - z + \delta^*$$

Note that $1 - \rho_{max}(\delta) \geq 0$ so that there is no solution for $\varepsilon > 0$.

Positive ε. Consider positive ε. Thus, we only have to worry about x_{min} cornered. When x_{min} becomes cornered, then

$$\frac{\partial}{\partial\varepsilon} \ln(x_{min}(\delta^* + 2\sigma\varepsilon) + \varepsilon) = \frac{1}{\varepsilon}$$

Then, we have

$$g'(\varepsilon) = -(1 - l_1)\frac{1}{\varepsilon} + s(1 - l_2)2\sigma$$

The derivative is increasing in ε, and is largest at $\varepsilon = \frac{k_1}{2}$ at a value of

$$g'\left(\frac{k_1}{2}\right) = -(1 - l_1)(1 + l_2 s) + s(1 - l_2)2\sigma$$

For small enough σ, this is always negative.

Negative ε. Consider negative ε. Thus, we only have to worry about x_{max} cornered. When x_{max} becomes cornered, then

$$\frac{\partial}{\partial\varepsilon} \ln(1 - x_{max}(\delta^* + 2\sigma\varepsilon) - \varepsilon) = -\frac{1}{\varepsilon}$$

Then, we have

$$g'(\varepsilon) = (1 - l_1)2\sigma + s(1 - l_2)\left(-\frac{1}{\varepsilon}\right)$$

The derivative is again increasing in ε, and is largest at $\varepsilon = -\frac{k_2}{2}$ at a value of

$$g'\left(-\frac{k_2}{2}\right) = -(1 - l_2)(s + l_1) + (1 - l_1)2\sigma$$

For small enough σ, this is always negative.

For $s = 1$ and $l_1 = l_2 = l$, we have symmetric conditions.

The last thing we need to do is to check that

$$g\left(-\frac{k_2}{2}\right) = g(0) = g\left(\frac{k_1}{2}\right) = 0$$

To this end, we can also proof that as $\sigma \to 0$, indeed one country (which one depending on on which side of δ^* the realization of δ falls) will always default. This is equivalent to the interior assumption for x_{max}, x_{min} we made. For this to hold, we need the following restrictions

$$1 - \frac{k_1}{2} \leq 1 - \rho_{max}(\delta^*) \leq \frac{k_2}{2} \tag{B.3}$$

$$1 - \frac{k_2}{2} \leq \rho_{min}(\delta^*) \leq \frac{k_1}{2} \tag{B.4}$$

The first line says that as $\sigma \to 0$, if $\delta < \delta^*$ then a proportion $\frac{k_2}{2}$ of investors invests in country 2, and it survives. However, if $\delta > \delta^*$, then only a proportion $1 - \frac{k_1}{2}$ of investors invests in country 2, and it defaults.

Similar arguments hold for country 1, which is summarized by the second line.

This can be rewritten as

$$\ln\left(1 - \frac{k_1}{2}\right) \quad \leq \ln\left(1 - \rho_{max}\left(\delta^*\right)\right) \leq \quad \ln\left(\frac{k_2}{2}\right)$$

$$\ln\left(1 - \frac{k_2}{2}\right) \qquad \leq \ln \rho_{min}\left(\delta^*\right) \leq \qquad \ln\left(\frac{k_1}{2}\right)$$

which gives

$$\ln\left(\frac{l_2 s}{1 + l_2 s}\right) \quad \leq \ln s - z + \delta^* \leq \quad \ln\left(\frac{s}{s + l_1}\right)$$

$$\ln\left(\frac{l_1}{s + l_1}\right) \qquad \leq -z - \delta^* \leq \qquad \ln\left(\frac{1}{1 + l_2 s}\right)$$

equivalent to

$$\ln\left(\frac{l_2}{1 + l_2 s}\right) + z \quad \leq \delta^* \leq \quad \ln\left(\frac{1}{s + l_1}\right) + z$$

$$\ln\left(\frac{l_1}{s + l_1}\right) + z \quad \leq -\delta^* \leq \quad \ln\left(\frac{1}{1 + l_2 s}\right) + z$$

equivalent to

$$\ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z \quad \leq \delta^* \leq \quad -\ln\left(s + l_1\right) + z$$

$$-\ln\left(\frac{1}{1 + l_2 s}\right) - z \quad \leq \delta^* \leq \quad -\ln\left(\frac{l_1}{s + l_1}\right) - z$$

equivalent to

$$\ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z \quad \leq \delta^* \leq \quad -\ln\left(s + l_1\right) + z$$

$$\ln\left(1 + l_2 s\right) - z \quad \leq \delta^* \leq \quad \ln\left(s + l_1\right) - \ln\left(l_1\right) - z$$

so that finally

$$\max\left[\ln\left(l_2\right) - \ln\left(1 + l_2 s\right) + z, \ln\left(1 + l_2 s\right) - z\right] \leq \delta^* \leq \min\left[-\ln\left(s + l_1\right) + z, \ln\left(s + l_1\right) - \ln\left(l_1\right) - z\right] \quad \text{(B.5)}$$

The first term is binding on the RHS for $z > \ln\left(1 + l_2 s\right) - \frac{1}{2}\ln\left(l_2\right)$, and the first term is binding on the left hand side for $z < \ln\left(s + l_1\right) - \frac{1}{2}\ln\left(l_1\right)$.

C Online Appendix C: Robustness Common Bonds

Notational Convention: We will refer to Common Bonds (aka Eurobonds) as asset 0, their price per unit of face-value as p_0, and the proportion of investors investing in common bonds as ρ_0.

We maintain the main assumptions of the sequential setup: (i) there is an amount (face-value) $\alpha(1+s)$ of common bonds and an amount $(1-\alpha)s_i$ of individual bonds of country i available, (ii) each unit of common bonds (that is, per unit of face-value) is made up of $\frac{1}{1+s}$ units of country 1 bonds and $\frac{s}{1+s}$ units of country 2 bonds, and (iii) issuance proceeds of the common bonds accrue in proportions $\frac{1}{1+s}$ and $\frac{s}{1+s}$ to country 1 and 2, respectively.

We are looking for a simultaneous three asset equilibrium between assets 0,1, and 2 that has the *single-survivor* property, i.e., only one country survives. We will analyze the following oscillation strategy:

$$\underset{\text{proportion}}{\text{investment}}...\underset{\alpha}{|0|}\underset{(1-\alpha)}{2}\underset{\alpha}{|0|}\underset{(1-\alpha)}{2}\underset{h}{|\mathbf{0}|}\underset{(1-\alpha)}{1}\underset{\alpha}{|0|}\underset{(1-\alpha)}{1}\underset{\alpha}{|0|}...$$

We will sometimes refer to the central interval **0** as the central region, the changeover region or loosely the survival cutoff. The intuition of the strategy is as follows: when one country defaults for sure, the no arbitrage condition between the surviving country and the common bond requires investors to invests in proportions $(1-\alpha)$ and α into the surviving country's bonds and common bonds, respectively. Next, let us consider fundamentals close to the changeover region in which default risk of both countries appears. As the fundamental δ increases, country 2 becomes riskier and country 1 becomes safer. As a consequence, with common bonds being a portfolio of individual bonds, common bonds' value moves less than the individual country bonds. Thus, to achieve indifference, we would have to increase investment in common bonds to decrease common bond returns to a level on par with individual bonds around the central region when default risk starts affecting both countries. In particular, for any $\sigma > 0$, in such a region our strategy requires endogenous investment in the common bond on an interval $[\delta_L, \delta_H]$ of length $2\sigma \cdot h$, i.e., we have two degrees of freedom in the two points δ_L and δ_H, as described in more detail below. Importantly, for such a construction to be an equilibrium and still be tractable, we require that any such construction does not necessitate any further endogenous adjustment of the strategies away from the interval $[\delta_L, \delta_H]$. We term such a property *insulated* – an *insulated equilibrium* only depends on endogenous variables around the survival cutoff and does not require any further endogenous variables away from it.

Formally, let the (endogenous) width of the interval **0** be given by $2\sigma \cdot h$, while the intervals 1 and 2 have width $(1-\alpha)2\sigma$, and the intervals 0 have width $\alpha \cdot 2\sigma$. Further, let δ_L and δ_H denote the lower and upper end of interval **0**, so that $h = \frac{\delta_H - \delta_L}{2\sigma}$. Second, we note that when we take $\sigma \to 0$, we have $\delta_L \to \delta^* \leftarrow \delta_H$ as long as h remains finite. Thus, in the limit, we transform the two degrees of freedom from (δ_L, δ_H) to (δ^*, h). For any strategy to yield an insulated equilibrium we require $h > \alpha$.[18] Lastly, we note that in the

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[18]In case $h < \alpha$, we can still solve for δ^* and h, but realize that some of the payoffs $\Pi_i(\delta)$ away from δ_L and δ_H do not converge to indifference: at least for some $\delta < \delta_L$, we do not have indifference at oscillation widths $1-\alpha$ and α – this is easiest to see when we consider $\delta = \delta_L - (1-\alpha)2\sigma$; at this point there is still some influence of h as the no-arbitrage proportions, if indeed we assume play according to $1-\alpha$ and α away from δ_L, do not actually yield no arbitrage because of $h + (1-\alpha) < 1$ and so the proportions are off. Instead, we would need to build a sequence of intervals of endogenous width (similar to how we derived h) to make sure indifference holds at all δ's away from δ_L. But this any such equilibrium is not insulated anymore, as we now need to solve for an infinite number of endogenous intervals. Consequently, we are not succeeding at reducing the dimensionality of the problem significantly, and it remains intractable. If, however, the equilibrium fulfills $h > \alpha$, it is insulated, and the dimensionality reduces significantly to just (δ^*, h), making the model tractable. Some generalization of single-survivor equilibria can still be achieved in insulated form, but joint-safety equilibria immediately violate the insulated character of the equilibrium.

limit $\sigma \to 0$, we have

$$x_{min}\left(\delta_L\right) = x_{min}\left(\delta_H\right) + h$$
$$x_{max}\left(\delta_L\right) = x_{max}\left(\delta_H\right) + h$$

Suppose that country i is safe almost surely, and country $-i$ defaults almost surely. Then, no arbitrage between country i's bond (paying of 1 per unit of face-value) and the common bond 0 requires

$$\frac{1}{p_i} = \frac{\frac{s_i}{1+s}}{p_0}$$

The supply of each bond is $(1-\alpha)\,s_i$ and $\alpha\,(1+s)$, respectively. Let ρ_i be the proportion of money flowing to bond i. Then, we must have

$$(1-\alpha)\,s_i p_i = \rho_i\,(1+f)$$
$$\alpha\,(1+s)\,p_0 = \rho_0\,(1+f) = (1-\rho_i)\,(1+f)$$

where $\rho_0 = (1-\rho_i)$ and $\rho_{-i} = 0$. Plugging these into the no arbitrage condition, we have

$$(1+s)\,p_0 = s_i p_i \iff \frac{(1-\rho_i)\,(1+f)}{\alpha} = \frac{\rho_i\,(1+f)}{1-\alpha} \iff \rho_i = 1-\alpha$$

and $\rho_0 = (1-\rho_i) = \alpha$. Thus, regardless which country is considered "safe", as long as investors are certain of the safety of i they should invest their money in aggregate proportions $1-\alpha$ and α in the safe individual and common bonds, respectively. These no arbitrage investment proportions are incorporate via oscillation outside of the central interval $\mathbf{0}$ in proportions $\rho_i = 1-\alpha$ and $\rho_0 = \alpha$.

Finally, the default condition for country i is given by

$$\underbrace{\frac{s_i}{1+s}\left(1 - \rho_1 - \rho_2\right) +}_{\text{Common bond revenue}} \underbrace{\rho_i}_{\text{Individual bond revenue}} \geq s_i\frac{1-\theta_i}{1+f} = s_i e^{-z} e^{(-1)^i \delta}$$

Because the no-arbitrage proportions around the outside the central region are symmetric, we do not have separate cases for δ_L and δ_H. For δ_L, the cutoffs are $h, h+1-\alpha, \alpha, 1$, whereas for δ_H, the cutoffs are $0, 1-\alpha, \alpha-h, 1-h$. This abstractly leads to 5 different cases:

C1 $0 < h < \alpha < h+1-\alpha < 1$ equivalent to $0 < \alpha - h < 1 - \alpha < 1 - h$. We will ignore this case as we are concentrating on an insulated equilibrium with $h > \alpha$.

C2 $0 < h < h+1-\alpha < \alpha < 1$ equivalent to $0 < 1 - \alpha < \alpha - h < 1 - h$. We will ignore this case as we are concentrating on an insulated equilibrium with $h > \alpha$.

C3 $0 < \alpha < h < 1 < h+1-\alpha$ equivalent to $\alpha - h < 0 < 1 - h < 1 - \alpha$. This is a case consistent with an insulated equilibrium.

C4 $0 < \alpha < h < h+1-\alpha < 1$ equivalent to $\alpha - h < 0 < 1 - \alpha < 1 - h$. But this cases is impossible as $h+1-\alpha < 1 \iff h < \alpha$ which contradicts $\alpha < h$.

C5 $0 < h < \alpha < 1 < h+1-\alpha$ equivalent to $0 < \alpha - h < 1 - h < 1 - \alpha$. But this case is impossible as $1 < h+1-\alpha \iff \alpha < h$ which contradicts $h < \alpha$.

Thus, our analysis will focus solely on case C3.

Lower boundary δ_L

C3 $0 < \alpha < h < 1 < h + 1 - \alpha$

$$\rho_1\left(\delta_L, x\right) = \begin{cases} 0 & (0, h) \\ x - h & (h, 1) \end{cases}$$

$$\rho_2\left(\delta_L, x\right) = \begin{cases} 1 - \alpha & (0, \alpha) \\ 1 - x & (\alpha, 1) \end{cases}$$

$$\rho_1\left(\delta_L, x\right) + \rho_2\left(\delta_L, x\right) = \begin{cases} 1 - \alpha & (0, \alpha) \\ 1 - x & (\alpha, h) \\ 1 - h & (h, 1) \end{cases}$$

For interior equilibria, we need $x_{min}\left(\delta_L\right) \in (h, 1)$ and $x_{max}\left(\delta_L\right) \in (\alpha, 1)$.

Upper boundary δ_H

C3 $0 < \alpha < h < 1 < h + 1 - \alpha$ equivalent to $\alpha - h < 0 < 1 - h < 1 - \alpha$

$$\rho_1\left(\delta_H, x\right) = \begin{cases} x & (0, 1 - \alpha) \\ 1 - \alpha & (1 - \alpha, 1) \end{cases}$$

$$\rho_2\left(\delta_H, x\right) = \begin{cases} 1 - x - h & (0, 1 - h) \\ 0 & (1 - h, 1) \end{cases}$$

$$\rho_1\left(\delta_H, x\right) + \rho_2\left(\delta_H, x\right) = \begin{cases} 1 - h & (0, 1 - h) \\ x & (1 - h, 1 - \alpha) \\ 1 - \alpha & (1 - \alpha, 1) \end{cases}$$

For interior equilibria, we need $x_{min}\left(\delta_H\right) \in (0, 1 - \alpha)$ and $x_{max}\left(\delta_H\right) \in (0, 1 - h)$.

Simultaneous equations when $\sigma \to 0$

C3 $0 < \alpha < h < 1 < h + 1 - \alpha$ equivalent to $\alpha - h < 0 < 1 - h < 1 - \alpha$

$$
\begin{aligned}
\Pi_1\left(\delta_L\right) &= (1 - \alpha)\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_1\left(\delta_L, x\right)} dx\right] \\
&= (1 - \alpha)\left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{x - h} dx\right] \\
&= (1 - \alpha)\left[\ln\left(1 - h\right) - \ln\left(x_{min}\left(\delta_L\right) - h\right)\right] \\
\Pi_2\left(\delta_L\right) &= (1 - \alpha) s\left[\int_{0}^{x_{max}(\delta_L)} \frac{1}{\rho_2\left(\delta_L, x\right)} dx\right] \\
&= (1 - \alpha) s\left[\int_{0}^{\alpha} \frac{1}{1 - \alpha} dx + \int_{\alpha}^{x_{max}(\delta_L)} \frac{1}{1 - x} dx\right] \\
&= (1 - \alpha) s\left[\frac{\alpha}{1 - \alpha} + \ln\left(1 - \alpha\right) - \ln\left(1 - x_{max}\left(\delta_L\right)\right)\right]
\end{aligned}
$$

$$\Pi_1\left(\delta_H\right) = \left(1-\alpha\right)\left[\int_{x_{min}\left(\delta_H\right)}^{1} \frac{1}{\rho_1\left(\delta_H, x\right)} dx\right]$$

$$= \left(1-\alpha\right)\left[\int_{x_{min}\left(\delta_H\right)}^{1-\alpha} \frac{1}{x} dx + \int_{1-\alpha}^{1} \frac{1}{1-\alpha} dx\right]$$

$$= \left(1-\alpha\right)\left[\ln\left(1-\alpha\right) - \ln\left(x_{min}\left(\delta_H\right)\right) + \frac{\alpha}{1-\alpha}\right]$$

$$\Pi_2\left(\delta_H\right) = \left(1-\alpha\right)s\left[\int_{0}^{x_{max}\left(\delta_H\right)} \frac{1}{\rho_2\left(\delta_H, x\right)} dx\right]$$

$$= \left(1-\alpha\right)s\left[\int_{0}^{x_{max}\left(\delta_H\right)} \frac{1}{1-x-h} dx\right]$$

$$= \left(1-\alpha\right)s\left[\ln\left(1-h\right) - \ln\left(1 - x_{max}\left(\delta_H\right) - h\right)\right]$$

4 Possible cases: $x_{min}\left(\delta_L\right) \in \left(h, 1\right)$ and $x_{max}\left(\delta_L\right) \in \left(\alpha, h\right) \cup \left(h, 1\right)$, $x_{min}\left(\delta_H\right) \in \left(0, 1-h\right) \cup \left(1-h, 1-\alpha\right)$ and $x_{max}\left(\delta_H\right) \in \left(0, 1-h\right)$.

(a) $x_{max}\left(\delta_L\right) \in \left(\alpha, h\right)$ (which implies $x_{max}\left(\delta_H\right) = 0$) and $x_{min}\left(\delta_H\right) \in \left(0, 1-h\right)$ (which implies $x_{min}\left(\delta_L\right) \in \left(h, 1\right)$)

$$\frac{1}{1+s}h + x_{min}\left(\delta_L\right) - h = e^{-z}e^{-\delta_L} \iff x_{min}\left(\delta_L\right) = e^{-z}e^{-\delta_L} + \frac{s}{1+s}h$$

$$\frac{s}{1+s}x_{max}\left(\delta_L\right) + 1 - x_{max}\left(\delta_L\right) = s \cdot e^{-z}e^{\delta_L} \iff x_{max}\left(\delta_L\right) = \left(1+s\right)\left(1 - s \cdot e^{-z}e^{\delta_L}\right)$$

$$\frac{1}{1+s}h + x_{min}\left(\delta_H\right) = e^{-z}e^{-\delta_H} \iff x_{min}\left(\delta_H\right) = e^{-z}e^{-\delta_H} - \frac{1}{1+s}h$$

$$x_{max}\left(\delta_H\right) = 0$$

$$\Pi_0\left(\delta_L\right) = \alpha\left[\int_{x_{min}\left(\delta_L\right)}^{1} \frac{1}{\rho_0\left(\delta_L, x\right)} dx + s\int_{0}^{x_{max}\left(\delta_L\right)} \frac{1}{\rho_0\left(\delta_L, x\right)} dx\right]$$

$$= \alpha\left[\left(\int_{x_{min}\left(\delta_L\right)}^{1} \frac{1}{h} dx\right) + s\left(\int_{0}^{\alpha} \frac{1}{\alpha} dx + \int_{\alpha}^{x_{max}\left(\delta_L\right)} \frac{1}{x} dx\right)\right]$$

$$= \alpha\left(\begin{array}{c}\left[\frac{1-x_{min}\left(\delta_L\right)}{h}\right] \\ +s\left[1 + \ln\left(x_{max}\left(\delta_L\right)\right) - \ln\left(\alpha\right)\right]\end{array}\right)$$

$$\Pi_0\left(\delta_H\right) = \alpha\left[\int_{x_{min}\left(\delta_H\right)}^{1} \frac{1}{\rho_0\left(\delta_H, x\right)} dx + s\int_{0}^{x_{max}\left(\delta_H\right)} \frac{1}{\rho_0\left(\delta_H, x\right)} dx\right]$$

$$= \alpha\left[\left(\int_{x_{min}\left(\delta_H\right)}^{1-h} \frac{1}{h} dx + \int_{1-h}^{1-\alpha} \frac{1}{1-x} dx + \int_{1-\alpha}^{1} \frac{1}{\alpha} dx\right) + s \cdot 0\right]$$

$$= \alpha\left(\left[\frac{1-h-x_{min}\left(\delta_H\right)}{h} + \ln\left(h\right) - \ln\left(\alpha\right) + 1\right]\right)$$

(b) $x_{max}\left(\delta_L\right) \in \left(h, 1\right)$ (which implies $x_{max}\left(\delta_H\right) \in \left(0, 1-h\right)$) and $x_{min}\left(\delta_H\right) \in \left(0, 1-h\right)$ (which

implies $x_{min}(\delta_L) \in (h, 1))$

$$\frac{1}{1+s}h + x_{min}(\delta_L) - h = e^{-z}e^{-\delta_L} \iff x_{min}(\delta_L) = e^{-z}e^{-\delta_L} + \frac{s}{1+s}h$$

$$\frac{s}{1+s}h + 1 - x_{max}(\delta_L) = s \cdot e^{-z}e^{\delta_L} \iff x_{max}(\delta_L) = 1 + \frac{s}{1+s}h - s \cdot e^{-z}e^{\delta_L}$$

$$\frac{1}{1+s}h + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} \iff x_{min}(\delta_H) = e^{-z}e^{-\delta_H} - \frac{1}{1+s}h$$

$$\frac{s}{1+s}h + 1 - x_{max}(\delta_H) - h = s \cdot e^{-z}e^{\delta_H} \iff x_{max}(\delta_H) = 1 - \frac{1}{1+s}h - s \cdot e^{-z}e^{\delta_H}$$

$$\Pi_0(\delta_L) = \alpha \left[\int_{x_{min}(\delta_L)}^{1} \frac{1}{\rho_0(\delta_L, x)} dx + s \int_{0}^{x_{max}(\delta_L)} \frac{1}{\rho_0(\delta_L, x)} dx \right]$$

$$= \alpha \left[\left(\int_{x_{min}(\delta_L)}^{1} \frac{1}{h} dx \right) + s \left(\int_{0}^{\alpha} \frac{1}{\alpha} dx + \int_{\alpha}^{h} \frac{1}{x} dx + \int_{h}^{x_{max}(\delta_L)} \frac{1}{h} dx \right) \right]$$

$$= \alpha \left(\begin{array}{c} \left[\frac{1 - x_{min}(\delta_L)}{h} \right] \\ + s \left[1 + \ln(h) - \ln(\alpha) + \frac{x_{max}(\delta_L) - h}{h} \right] \end{array} \right)$$

$$\Pi_0(\delta_H) = \alpha \left[\int_{x_{min}(\delta_H)}^{1} \frac{1}{\rho_0(\delta_H, x)} dx + s \int_{0}^{x_{max}(\delta_H)} \frac{1}{\rho_0(\delta_H, x)} dx \right]$$

$$= \alpha \left[\left(\int_{x_{min}(\delta_H)}^{1-h} \frac{1}{h} dx + \int_{1-h}^{1-\alpha} \frac{1}{1-x} dx + \int_{1-\alpha}^{1} \frac{1}{\alpha} dx \right) + s \left(\int_{0}^{x_{max}(\delta_H)} \frac{1}{h} dx \right) \right]$$

$$= \alpha \left(\begin{array}{c} \left[\frac{1 - h - x_{min}(\delta_H)}{h} + \ln(h) - \ln(\alpha) + 1 \right] \\ + s \left[\frac{x_{max}(\delta_H)}{h} \right] \end{array} \right)$$

(c) $x_{max}(\delta_L) \in (\alpha, h)$ (which implies $x_{max}(\delta_H) = 0$) and $x_{min}(\delta_H) \in (1-h, 1-\alpha)$ (which implies $x_{min}(\delta_L) = 1$)

$$x_{min}(\delta_L) = 1$$

$$\frac{s}{1+s}x_{max}(\delta_L) + 1 - x_{max}(\delta_L) = s \cdot e^{-z}e^{\delta_L} \iff x_{max}(\delta_L) = (1+s)\left(1 - s \cdot e^{-z}e^{\delta_L}\right)$$

$$\frac{1}{1+s}(1 - x_{min}(\delta_H)) + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} \iff x_{min}(\delta_H) = \frac{(1+s)e^{-z}e^{-\delta_H} - 1}{s}$$

$$x_{max}(\delta_H) = 0$$

$$\Pi_0\left(\delta_L\right) = \alpha\left[\int_{x_{min}(\delta_L)}^1 \frac{1}{\rho_0\left(\delta_L, x\right)}dx + s\int_0^{x_{max}(\delta_L)}\frac{1}{\rho_0\left(\delta_L, x\right)}dx\right]$$

$$= \alpha\left[0 + s\left(\int_0^\alpha \frac{1}{\alpha}dx + \int_\alpha^{x_{max}(\delta_L)}\frac{1}{x}dx\right)\right]$$

$$= \alpha\left(s\left[1 + \ln\left(x_{max}\left(\delta_L\right)\right) - \ln\left(\alpha\right)\right]\right)$$

$$\Pi_0\left(\delta_H\right) = \alpha\left[\int_{x_{min}(\delta_H)}^1 \frac{1}{\rho_0\left(\delta_H, x\right)}dx + s\int_0^{x_{max}(\delta_H)}\frac{1}{\rho_0\left(\delta_H, x\right)}dx\right]$$

$$= \alpha\left[\left(\int_{x_{min}(\delta_H)}^{1-\alpha}\frac{1}{1-x}dx + \int_{1-\alpha}^1 \frac{1}{\alpha}dx\right) + s\cdot 0\right]$$

$$= \alpha\left(\left[\ln\left(1 - x_{min}\left(\delta_H\right)\right) - \ln\left(\alpha\right) + 1\right]\right)$$

(d) $x_{max}\left(\delta_L\right) \in (h, 1)$ (which implies $x_{max}\left(\delta_H\right) \in (0, 1-h)$) and $x_{min}\left(\delta_H\right) \in (1-h, 1-\alpha)$ (which implies $x_{min}\left(\delta_L\right) = 1$)

$$x_{min}\left(\delta_L\right) = 1$$

$$\frac{s}{1+s}h + 1 - x_{max}\left(\delta_L\right) = s\cdot e^{-z}e^{\delta_L} \iff x_{max}\left(\delta_L\right) = 1 + \frac{s}{1+s}h - s\cdot e^{-z}e^{\delta_L}$$

$$\frac{1}{1+s}\left(1 - x_{min}\left(\delta_H\right)\right) + x_{min}\left(\delta_H\right) = e^{-z}e^{-\delta_H} \iff x_{min}\left(\delta_H\right) = \frac{\left(1+s\right)e^{-z}e^{-\delta_H} - 1}{s}$$

$$\frac{s}{1+s}h + 1 - x_{max}\left(\delta_H\right) - h = s\cdot e^{-z}e^{\delta_H} \iff x_{max}\left(\delta_H\right) = 1 - \frac{1}{1+s}h - s\cdot e^{-z}e^{\delta_H}$$

$$\Pi_0\left(\delta_L\right) = \alpha\left[\int_{x_{min}(\delta_L)}^1 \frac{1}{\rho_0\left(\delta_L, x\right)}dx + s\int_0^{x_{max}(\delta_L)}\frac{1}{\rho_0\left(\delta_L, x\right)}dx\right]$$

$$= \alpha\left[0 + s\left(\int_0^\alpha \frac{1}{\alpha}dx + \int_\alpha^h \frac{1}{x}dx + \int_h^{x_{max}(\delta_L)}\frac{1}{h}dx\right)\right]$$

$$= \alpha\left(s\left[1 + \ln\left(h\right) - \ln\left(\alpha\right) + \frac{x_{max}\left(\delta_L\right) - h}{h}\right]\right)$$

$$\Pi_0\left(\delta_H\right) = \alpha\left[\int_{x_{min}(\delta_H)}^1 \frac{1}{\rho_0\left(\delta_H, x\right)}dx + s\int_0^{x_{max}(\delta_H)}\frac{1}{\rho_0\left(\delta_H, x\right)}dx\right]$$

$$= \alpha\left[\left(\int_{x_{min}(\delta_H)}^{1-\alpha}\frac{1}{1-x}dx + \int_{1-\alpha}^1 \frac{1}{\alpha}dx\right) + s\left(\int_0^{x_{max}(\delta_H)}\frac{1}{h}dx\right)\right]$$

$$= \alpha\left(\begin{array}{c}\left[\ln\left(1 - x_{min}\left(\delta_H\right)\right) - \ln\left(\alpha\right) + 1\right]\\ + s\left[\frac{x_{max}(\delta_H)}{h}\right]\end{array}\right)$$

Closed-form Approximations for $\alpha \approx 0$ Next, we approximate around $\alpha \approx 0$ to get some more analytical insights into the behavior of δ^* and h. To this end, we conjecture

$$h\left(\alpha\right) = h_0 + h_1\alpha + \frac{h_2}{2}\alpha^2$$

$$\delta^*\left(\alpha\right) = \delta_0 + \delta_1\alpha$$

As $\alpha \to 0$, to converge to the known solution of the two asset simultaneous game, we need

$$h_0 = 0$$

$$\delta_0 = \delta^* = \frac{-(1-s)z - s\ln s}{(1+s)}$$

Next, we take limits for each of the cases (except case CT2, which requires $\alpha \geq \frac{1}{2}$, so is not applicable), and impose $h_0 = 0$. First, note that $\lim_{\alpha \to 0} \Pi_0(\delta_L) = \lim_{\alpha \to 0} \Pi_0(\delta_H)$. Thus, we are looking for h_1, h_2 and δ_0, δ_1 that satisfy

$$\lim_{\alpha \to 0} \Pi_2(\delta_L) = \lim_{\alpha \to 0} \Pi_0(\delta_L) = \lim_{\alpha \to 0} \Pi_0(\delta_H) = \lim_{\alpha \to 0} \Pi_1(\delta_L)$$

Next, note that a local equilibrium requires $h(\alpha) \geq \alpha$, and thus for small α we require parameters such that $h_1 \geq 1$.

C3a We have

$$\lim_{\alpha \to 0} \Pi_0(\delta_L) = \lim_{\alpha \to 0} \Pi_0(\delta_H) = \frac{1 + s - e^{-\delta_0 - z} - e^{\delta_0 - z}s^2}{h_1}$$

$$\lim_{\alpha \to 0} \Pi_1(\delta_L) = \lim_{\alpha \to 0} \Pi_1(\delta_H) = -\ln\left[e^{-\delta_0 - z}\right]$$

$$\lim_{\alpha \to 0} \Pi_2(\delta_L) = -s\ln\left[1 - (1+s)\left(1 - e^{\delta_0 - z}s\right)\right] \neq 0 = \lim_{\alpha \to 0} \Pi_2(\delta_H)$$

For consistency $\lim_{\alpha \to 0} \Pi_1(\delta_L) = \lim_{\alpha \to 0} \Pi_1(\delta_H)$, we require

$$1 - (1+s)\left(1 - e^{\delta_0 - z}s\right) = 1 \iff e^{\delta_0 - z}s = 1 \iff e^z = e^{\delta_0}s$$

The indifference condition is

$$-s\ln\left[1 - (1+s)\left(1 - e^{\delta_0 - z}s\right)\right] = \frac{1 - e^{-\delta_0 - z}}{h_1} = -\ln\left[e^{-\delta_0 - z}\right]$$

and equating the first and third term requires $e^z = e^{-\delta_0}$. These conditions can only hold for $z = -\frac{1}{2}\ln s$, and are violated for general parameters. Thus, case C3a is not possible in equilibrium for small α.[19]

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[19]A more direct proof: C3a requires $0 < \alpha < h < 1 < h + 1 - \alpha$ and $x_{max}(\delta_L) \in (\alpha, h)$ (which implies $x_{max}(\delta_H) = 0$) and $x_{min}(\delta_H) \in (0, 1 - h)$ (which implies $x_{min}(\delta_L) \in (h, 1)$)

$$\frac{1}{1+s}h + x_{min}(\delta_L) - h = e^{-z}e^{-\delta_L} \iff x_{min}(\delta_L) = e^{-z}e^{-\delta_L} + \frac{s}{1+s}h$$

$$\frac{s}{1+s}x_{max}(\delta_L) + 1 - x_{max}(\delta_L) = s \cdot e^{-z}e^{\delta_L} \iff x_{max}(\delta_L) = (1+s)\left(1 - s \cdot e^{-z}e^{\delta_L}\right)$$

$$\frac{1}{1+s}h + x_{min}(\delta_H) = e^{-z}e^{-\delta_H} \iff x_{min}(\delta_H) = e^{-z}e^{-\delta_H} - \frac{1}{1+s}h$$

$$x_{max}(\delta_H) = 0$$

Note that $x_{max}(\delta_L) \to (1+s)\left(1 - s \cdot e^{-z}e^{\delta^*}\right) = 0$, so that $e^{\delta^*} = s^{-1}e^z$; further, note that $x_{min} \to e^{-z}e^{-\delta^*} \in (0, 1)$; plugging in, we have $e^{-2z} \cdot s \in (0, 1)$, which is not a contradiction, but when inspecting the indifference condition for investment yields a contradiction.

C3b We have

$$\lim_{\alpha \to 0} \Pi_0\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_0\left(\delta_H\right) = \frac{1 + s - e^{-\delta_0 - z} - e^{\delta_0 - z} s^2}{h_1}$$

$$\lim_{\alpha \to 0} \Pi_1\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_1\left(\delta_H\right) = -\ln\left[e^{-\delta_0 - z}\right]$$

$$\lim_{\alpha \to 0} \Pi_2\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_2\left(\delta_H\right) = -s\ln\left[e^{\delta_0 - z} s\right]$$

and the indifference condition is

$$-s\ln\left[e^{\delta_0 - z} s\right] = \frac{1 + s - e^{-\delta_0 - z} - e^{\delta_0 - z} s^2}{h_1\left(1 + s\right)} = -\ln\left[e^{-\delta_0 - z}\right] \iff \delta_0 = \frac{-\left(1 - s\right)z - s\ln s}{1 + s} = \delta^* \ \checkmark$$

Next, we have

$$h_1 = \frac{1 + s - e^{-\delta_0 - z} - e^{\delta_0 - z} s^2}{\left(\delta_0 + z\right)} = \frac{1 + s - e^{-\left(\frac{2s \cdot z - s\ln s}{1+s}\right)} - e^{\frac{-2z - s\ln s}{1+s}} s^2}{\left(\frac{2s \cdot z - s\ln s}{1+s}\right)}$$

where we used $\delta_0 + z = \frac{2s \cdot z - s\ln s}{1+s}$ and $\delta_0 - z = \frac{-2z - s\ln s}{1+s}$. The insulated equilibrium as constructed exists around $\alpha \approx 0$ if $h_1 > 1$.

C3c We have

$$\lim_{\alpha \to 0} \Pi_0\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_0\left(\delta_H\right) = 0$$

$$\lim_{\alpha \to 0} \Pi_1\left(\delta_L\right) = 0 \neq -\ln\left[\frac{\left(1 + s\right) e^{-\delta_0 - z} - 1}{s}\right] = \lim_{\alpha \to 0} \Pi_1\left(\delta_H\right)$$

$$\lim_{\alpha \to 0} \Pi_2\left(\delta_L\right) = -s\ln\left[1 - \left(1 + s\right)\left(1 - e^{\delta_0 - z} s\right)\right] \neq 0 = \lim_{\alpha \to 0} \Pi_2\left(\delta_H\right)$$

For consistency $\lim_{\alpha \to 0} \Pi_1\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_1\left(\delta_H\right)$, we require

$$\left(1 + s\right) e^{-\delta_0 - z} - 1 = s \iff e^z = e^{-\delta_0}$$

and for consistency $\lim_{\alpha \to 0} \Pi_1\left(\delta_L\right) = \lim_{\alpha \to 0} \Pi_1\left(\delta_H\right)$, we require

$$1 - \left(1 + s\right)\left(1 - e^{\delta_0 - z} s\right) = 1 \iff e^{\delta_0 - z} s = 1 \iff e^z = e^{\delta_0} s$$

These two conditions can only hold for $z = -\frac{1}{2}\ln s$, and are violated for general parameters. Thus, case C3c is not possible in equilibrium for small α.[20]

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[20]A more direct proof: C3c requires $0 < \alpha < h < 1 < h + 1 - \alpha$ and $x_{max}\left(\delta_L\right) \in \left(\alpha, h\right)$ (which implies $x_{max}\left(\delta_H\right) = 0$) and $x_{min}\left(\delta_H\right) \in \left(1 - h, 1 - \alpha\right)$ (which implies $x_{min}\left(\delta_L\right) = 1$)

$$x_{min}\left(\delta_L\right) = 1$$

$$\frac{s}{1 + s} x_{max}\left(\delta_L\right) + 1 - x_{max}\left(\delta_L\right) = s \cdot e^{-z} e^{\delta_L} \iff x_{max}\left(\delta_L\right) = \left(1 + s\right)\left(1 - s \cdot e^{-z} e^{\delta_L}\right)$$

$$\frac{1}{1 + s}\left(1 - x_{min}\left(\delta_H\right)\right) + x_{min}\left(\delta_H\right) = e^{-z} e^{-\delta_H} \iff x_{min}\left(\delta_H\right) = \frac{\left(1 + s\right) e^{-z} e^{-\delta_H} - 1}{s}$$

$$x_{max}\left(\delta_H\right) = 0$$

Thus, we have $x_{max}\left(\delta_L\right) \to \left(1 + s\right)\left(1 - s \cdot e^{-z} e^{\delta_L}\right) = 0$ and $x_{min}\left(\delta_H\right) = \frac{\left(1+s\right)e^{-z} e^{-\delta_H} - 1}{s} = 1$. But as $\delta_L \to \delta^* \leftarrow \delta_H$, so we require $e^{\delta^*} = s^{-1} e^z = e^{-z}$, which in turn requires the specific parameter restriction

C3d We have

$$\lim_{\alpha \to 0} \Pi_0(\delta_L) = \lim_{\alpha \to 0} \Pi_0(\delta_H) = \frac{s - e^{\delta_0 - z} s^2}{h_1}$$

$$\lim_{\alpha \to 0} \Pi_1(\delta_L) = 0 \neq -\ln\left[\frac{(1+s)e^{-\delta_0 - z} - 1}{s}\right] = \lim_{\alpha \to 0} \Pi_1(\delta_H)$$

$$\lim_{\alpha \to 0} \Pi_2(\delta_L) = \lim_{\alpha \to 0} \Pi_2(\delta_H) = -s \ln\left[e^{\delta_0 - z} s\right]$$

For consistency $\lim_{\alpha \to 0} \Pi_1(\delta_L) = \lim_{\alpha \to 0} \Pi_1(\delta_H)$, we require

$$(1+s)e^{-\delta_0 - z} - 1 = s \iff e^z = e^{-\delta_0}$$

But then for indifference we require

$$-s \ln\left[e^{\delta_0 - z} s\right] = \frac{s - e^{-\delta_0 - z} s^2}{h_1} = -\ln\left[\frac{(1+s)e^{-\delta_0 - z} - 1}{s}\right]$$

But we know the third term is equal to 0, so the first term requires $e^{\delta_0 - z} s = 1 \iff e^{\delta_0} s = e^z$ which can only hold for $z = -\frac{1}{2}\ln s$, and are violated for general parameters. Thus, case C3d is not possible in equilibrium for small α.[21]

Thus, we are left with only case C3b for small α, which fulfills the insulated equilibrium criterion for points (s, z) such that

$$\left\{ (s, z) : h_1(s, z) = \frac{1 + s - e^{-\left(\frac{2s \cdot z - s \ln s}{1+s}\right)} - e^{\frac{-2z - s \ln s}{1+s}} s^2}{\left(\frac{2s \cdot z - s \ln s}{1+s}\right)} \geq 1 \right\}$$

Figure C.1 maps the set of points (s, z) for which the insulated criterion is fulfilled.

Verifying the equilibrium. Note that, away from $\alpha = 0$, we have the expected returns at either end-point not equal, even as $\sigma \to 0$, because strategic uncertainty does not vanish:

$$\lim_{\sigma \to 0} \Pi_i(\delta_L) \neq \lim_{\sigma \to 0} \Pi_i(\delta_H)$$

To verify the equilibrium, we need to check that for any $\delta \in [\delta_L, \delta_H]$, indeed common bonds are the most attractive asset, for $\delta < \delta_L$, bond 2 is the most attractive asset, and for $\delta > \delta_H$, bond 1 is the most attractive

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$z = -\frac{1}{2}\ln s$.

[21]A more direct proof: C3d requires $0 < \alpha < h < 1 < h + 1 - \alpha$ and $x_{max}(\delta_L) \in (h, 1)$ (which implies $x_{max}(\delta_H) \in (0, 1 - h)$) and $x_{min}(\delta_H) \in (1 - h, 1 - \alpha)$ (which implies $x_{min}(\delta_L) = 1$)

$$x_{min}(\delta_L) = 1$$

$$\frac{s}{1+s} h + 1 - x_{max}(\delta_L) = s \cdot e^{-z} e^{\delta_L} \iff x_{max}(\delta_L) = 1 + \frac{s}{1+s} h - s \cdot e^{-z} e^{\delta_L}$$

$$\frac{1}{1+s}(1 - x_{min}(\delta_H)) + x_{min}(\delta_H) = e^{-z} e^{-\delta_H} \iff x_{min}(\delta_H) = \frac{(1+s)e^{-z} e^{-\delta_H} - 1}{s}$$

$$\frac{s}{1+s} h + 1 - x_{max}(\delta_H) - h = s \cdot e^{-z} e^{\delta_H} \iff x_{max}(\delta_H) = 1 - \frac{1}{1+s} h - s \cdot e^{-z} e^{\delta_H}$$

Thus, we have $x_{min}(\delta_H) \to \frac{(1+s)e^{-z} e^{-\delta_H} - 1}{s} = 1$, which requires $e^{\delta^*} = e^{-z}$; similarly, we have $x_{max} = 1 - s \cdot e^{-z} e^{\delta^*} \in (0, 1)$; plugging in, we have $1 - s \cdot e^{-2z} \in (0, 1)$ which does not give a contradiction, but when inspecting the indifference condition for investment yields a contradiction.
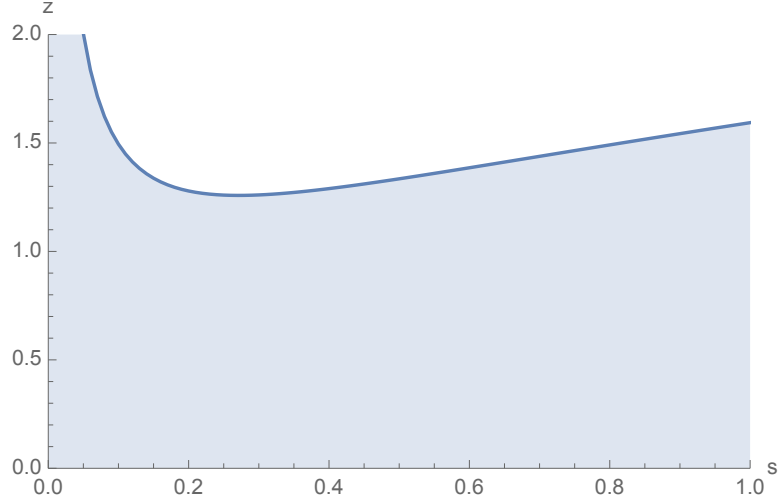
Figure C.1: **Existence of insulated simultaneous single-survivor common bond equilibrium for small α:** Set of points (s, z) for which an insulated single-survivor equilibrium exists in the common bonds case for $\alpha \approx 0$, i.e., $\{(s, z) : h_1(s, z) \geq 1\}$.

asset. For a given δ_L, δ_H, let

$$\delta \equiv \delta_L + 2\sigma\varepsilon$$

with $\varepsilon \in (0, h)$, so that $\varepsilon = 0$ yields δ_L and $\varepsilon = h$ yields δ_H. Then, for $\varepsilon \in [0, h]$, we have

$$\rho_1(\delta, x) = \begin{cases} 0 & (0, h - \varepsilon) \\ x + \varepsilon - h & (h - \varepsilon, h - \varepsilon + 1 - \alpha) \\ 1 - \alpha & (h - \varepsilon + 1 - \alpha, 1) \end{cases}$$

$$\rho_2(\delta, x) = \begin{cases} 1 - \alpha & (0, \alpha - \varepsilon) \\ 1 - (x + \varepsilon) & (\alpha - \varepsilon, 1 - \varepsilon) \\ 0 & (1 - \varepsilon, 1) \end{cases}$$

where of course if for example as in C3 we have $\alpha < h$, then some intervals are empty (i.e., $(0, \alpha - \varepsilon) = \emptyset$ for $\varepsilon \in (\alpha, h)$). For interior equilibria, we need $x_{min}(\delta) \in (h - \varepsilon, h - \varepsilon + 1 - \alpha)$ and $x_{max}(\delta) \in (\alpha - \varepsilon, 1 - \varepsilon)$.

  C3 $0 < \alpha < h < 1 < h + 1 - \alpha$

$$\rho_1(\delta, x) + \rho_2(\delta, x) = \begin{cases} 1 - \alpha & (0, \alpha - \varepsilon) \\ 1 - (x + \varepsilon) & (\alpha - \varepsilon, h - \varepsilon) \\ 1 - h & (h - \varepsilon, 1 - \varepsilon) \\ x + \varepsilon - h & (1 - \varepsilon, h - \varepsilon + 1 - \alpha) \\ 1 - \alpha & (h - \varepsilon + 1 - \alpha, 1) \end{cases}$$

Online Appendix C-10

Let us calculate expected returns as a function of ε.[22] To calculate expected returns, we have to conjecture a position of $x_{min}(\delta)$ and $x_{max}(\delta)$. For $\alpha \approx 0$, we can only be in case C3b, and our numerical results for our benchmark cases show that this case is applicable even when α increases. Thus, we only show the expected returns for this case:

C3b $0 < \alpha < h < 1 < h + 1 - \alpha$

$x_{min} \in (h - \varepsilon, 1 - \varepsilon)$ and $x_{max}(\delta) \in (h - \varepsilon, 1 - \varepsilon)$. Now the position of ε in relation to α and $h - \alpha$ matters, i.e., three intervals matter: $\varepsilon < \min\{h - \alpha, \alpha\}$, $\varepsilon \in (\min\{h - \alpha, \alpha\}, \max\{h - \alpha, \alpha\})$, and $\varepsilon > \max\{h - \alpha, \alpha\}$. Two sub-cases arise, which essentially define the relation of $h - \alpha$ to α:

(a) $\min\{h - \alpha, \alpha\} = h - \alpha \iff \alpha < h < 2\alpha$ (this is the applicable case for our benchmark cases $(s, z) = \left(\frac{1}{4}, 1\right)$ and $(s, z) = \left(\frac{1}{2}, 1\right)$, as numerically h is very close to α). Thus, the three intervals are $\varepsilon < h - \alpha$, $\varepsilon \in (h - \alpha, \alpha)$, and $\varepsilon > \alpha$. Note that $\varepsilon = \frac{h}{2}$ gives the midpoint δ_M, and the midpoint is part of interval $\frac{h}{2} \in (h - \alpha, \alpha)$.[23]

For $\varepsilon < h - \alpha = \min\{h - \alpha, \alpha\}$ so that $h - \varepsilon + 1 - \alpha > 1$ as well as $\alpha - \varepsilon > 0$, we have

$$\Pi_0(\delta) = \alpha\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_0(\delta, x)}dx + s\int_{0}^{x_{max}(\delta)} \frac{1}{\rho_0(\delta, x)}dx\right]$$

$$= \alpha\left[\int_{x_{min}(\delta)}^{1-\varepsilon} \frac{1}{1-(1-h)}dx + \int_{1-\varepsilon}^{1} \frac{1}{1-(x+\varepsilon-h)}dx\right]$$

$$+ \alpha \cdot s\left[\int_{0}^{\alpha-\varepsilon} \frac{1}{1-(1-\alpha)}dx + \int_{\alpha-\varepsilon}^{h-\varepsilon} \frac{1}{1-[1-(x+\varepsilon)]}dx + \int_{h-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(1-h)}dx\right]$$

$$= \alpha\left[\frac{1-\varepsilon-x_{min}(\delta)}{h} + \ln(h) - \ln(h-\varepsilon)\right]$$

$$+ \alpha \cdot s\left[\frac{\alpha-\varepsilon}{\alpha} + \ln(h) - \ln(\alpha) + \frac{x_{max}(\delta)+\varepsilon-h}{h}\right]$$

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[22]Note that $\varepsilon = \frac{1}{2}h$ gives the central interval **0** midpoint

$$\delta_M = \frac{\delta_H + \delta_L}{2} = \frac{\delta_H - \delta_L + 2\delta_L}{2} = \delta_L + \sigma \cdot h.$$

[23]Consider $\frac{h}{2} < h - \alpha \iff \alpha < \frac{h}{2} \iff 2\alpha < h$, which violates the assumptions. Next, consider $\frac{h}{2} > \alpha \iff h > 2\alpha$, which also violates the assumptions. Thus, only $\frac{h}{2} \in (h - \alpha, \alpha)$ is consistent with $\alpha < h < 2\alpha$.

$$\Pi_1(\delta) = (1-\alpha)\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_1(\delta,x)} dx\right]$$

$$= (1-\alpha)\left[\int_{x_{min}(\delta)}^{1} \frac{1}{x+\varepsilon-h} dx\right]$$

$$= (1-\alpha)\left[\ln(1+\varepsilon-h) - \ln(x_{min}(\delta)+\varepsilon-h)\right]$$

$$\Pi_2(\delta) = (1-\alpha)s\left[\int_{0}^{x_{max}(\delta)} \frac{1}{\rho_2(\delta,x)} dx\right]$$

$$= (1-\alpha)s\left[\int_{0}^{\alpha-\varepsilon} \frac{1}{1-\alpha} dx + \int_{\alpha-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(x+\varepsilon)} dx\right]$$

$$= (1-\alpha)s\left[\frac{\alpha-\varepsilon}{1-\alpha} + \ln(1-\alpha) - \ln(1-(x_{max}(\delta)+\varepsilon))\right]$$

For $\varepsilon \in (h-\alpha, \alpha)$ so that $h-\varepsilon+1-\alpha < 1$ and $\alpha-\varepsilon > 0$, we have

$$\Pi_0(\delta) = \alpha\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_0(\delta,x)} dx + s\int_{0}^{x_{max}(\delta)} \frac{1}{\rho_0(\delta,x)} dx\right]$$

$$= \alpha\left[\int_{x_{min}(\delta)}^{1-\varepsilon} \frac{1}{1-(1-h)} dx + \int_{1-\varepsilon}^{h-\varepsilon+1-\alpha} \frac{1}{1-(x+\varepsilon-h)} dx + \int_{h-\varepsilon+1-\alpha}^{1} \frac{1}{1-(1-\alpha)} dx\right]$$

$$+ \alpha \cdot s\left[\int_{0}^{\alpha-\varepsilon} \frac{1}{1-(1-\alpha)} dx + \int_{\alpha-\varepsilon}^{h-\varepsilon} \frac{1}{1-[1-(x+\varepsilon)]} dx + \int_{h-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(1-h)} dx\right]$$

$$= \alpha\left[\frac{1-\varepsilon-x_{min}(\delta)}{h} + \ln(h) - \ln(\alpha) + \frac{\varepsilon+\alpha-h}{\alpha}\right]$$

$$+ \alpha \cdot s\left[\frac{\alpha-\varepsilon}{\alpha} + \ln(h) - \ln(\alpha) + \frac{x_{max}(\delta)+\varepsilon-h}{h}\right]$$

$$\Pi_1(\delta) = (1-\alpha)\left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_1(\delta,x)} dx\right]$$

$$= (1-\alpha)\left[\int_{x_{min}(\delta)}^{h-\varepsilon+1-\alpha} \frac{1}{x+\varepsilon-h} dx + \int_{h-\varepsilon+1-\alpha}^{1} \frac{1}{1-\alpha} dx\right]$$

$$= (1-\alpha)\left[\ln(1-\alpha) - \ln(x_{min}(\delta)+\varepsilon-h) + \frac{\varepsilon+\alpha-h}{1-\alpha}\right]$$

$$\Pi_2(\delta) = (1-\alpha)s\left[\int_{0}^{x_{max}(\delta)} \frac{1}{\rho_2(\delta,x)} dx\right]$$

$$= (1-\alpha)s\left[\int_{0}^{\alpha-\varepsilon} \frac{1}{1-\alpha} dx + \int_{\alpha-\varepsilon}^{x_{max}(\delta)} \frac{1}{1-(x+\varepsilon)} dx\right]$$

$$= (1-\alpha)s\left[\frac{\alpha-\varepsilon}{1-\alpha} + \ln(1-\alpha) - \ln(1-(x_{max}(\delta)+\varepsilon))\right]$$

For $\varepsilon > \alpha = \max\{h - \alpha, \alpha\}$ so that $h - \varepsilon + 1 - \alpha < 1$ as well as $\alpha - \varepsilon < 0$, we have

$$\Pi_0(\delta) = \alpha \left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_0(\delta, x)} dx + s \int_{0}^{x_{max}(\delta)} \frac{1}{\rho_0(\delta, x)} dx \right]$$

$$= \alpha \left[\int_{x_{min}(\delta)}^{1-\varepsilon} \frac{1}{1 - (1-h)} dx + \int_{1-\varepsilon}^{h-\varepsilon+1-\alpha} \frac{1}{1 - (x + \varepsilon - h)} dx + \int_{h-\varepsilon+1-\alpha}^{1} \frac{1}{1 - (1-\alpha)} dx \right]$$

$$+ \alpha \cdot s \left[\int_{0}^{h-\varepsilon} \frac{1}{1 - [1 - (x + \varepsilon)]} dx + \int_{h-\varepsilon}^{x_{max}(\delta)} \frac{1}{1 - (1-h)} dx \right]$$

$$= \alpha \left[\frac{1 - \varepsilon - x_{min}(\delta)}{h} + \ln(h) - \ln(\alpha) + \frac{\varepsilon + \alpha - h}{\alpha} \right]$$

$$+ \alpha \cdot s \left[\ln(h) - \ln(\varepsilon) + \frac{x_{max}(\delta) + \varepsilon - h}{h} \right]$$

$$\Pi_1(\delta) = (1 - \alpha) \left[\int_{x_{min}(\delta)}^{1} \frac{1}{\rho_1(\delta, x)} dx \right]$$

$$= (1 - \alpha) \left[\int_{x_{min}(\delta)}^{h-\varepsilon+1-\alpha} \frac{1}{x + \varepsilon - h} dx + \int_{h-\varepsilon+1-\alpha}^{1} \frac{1}{1 - \alpha} dx \right]$$

$$= (1 - \alpha) \left[\ln(1 - \alpha) - \ln(x_{min}(\delta) + \varepsilon - h) + \frac{\varepsilon + \alpha - h}{1 - \alpha} \right]$$

$$\Pi_2(\delta) = (1 - \alpha) s \left[\int_{0}^{x_{max}(\delta)} \frac{1}{\rho_2(\delta, x)} dx \right]$$

$$= (1 - \alpha) s \left[\int_{0}^{x_{max}(\delta)} \frac{1}{1 - (x + \varepsilon)} dx \right]$$

$$= (1 - \alpha) s \left[\ln(1 - \varepsilon) - \ln(1 - (x_{max}(\delta) + \varepsilon)) \right]$$

Next, we numerically check $\Pi_0(\delta^*; \varepsilon) > \max\{\Pi_1(\delta^*; \varepsilon), \Pi_2(\delta^*; \varepsilon)\}$ for candidate equilibria (h, δ^*) for any $\varepsilon \in [0, h]$. This holds for all numerically solved for candidate equilibria.

(b) $h - \alpha > \alpha \iff h > 2\alpha > \alpha$ would be the other case, but we do not observe numerically any h that are twice the size of α. Calculations for this case, as well as for cases C3a C3c and C3d are available upon request.

The numerical results (h, δ^*_{sim}) as well as the comparison δ^*_{seq} for cases $(s = .25, z = 1)$ and $(s = .5, z = 1)$ are presented in Figure C.2. The left Panels show the equilibrium h as the solid blue line in comparison to the 45 degree line as the dashed yellow line, thus visualizing the insulated requirement $h > \alpha$. We restrict the graph to levels of α for which this condition holds. The right Panels then show the equilibrium δ^*_{sim} as the solid blue line in comparison to their sequential counterpart δ^*_{seq} as the dashed yellow line.
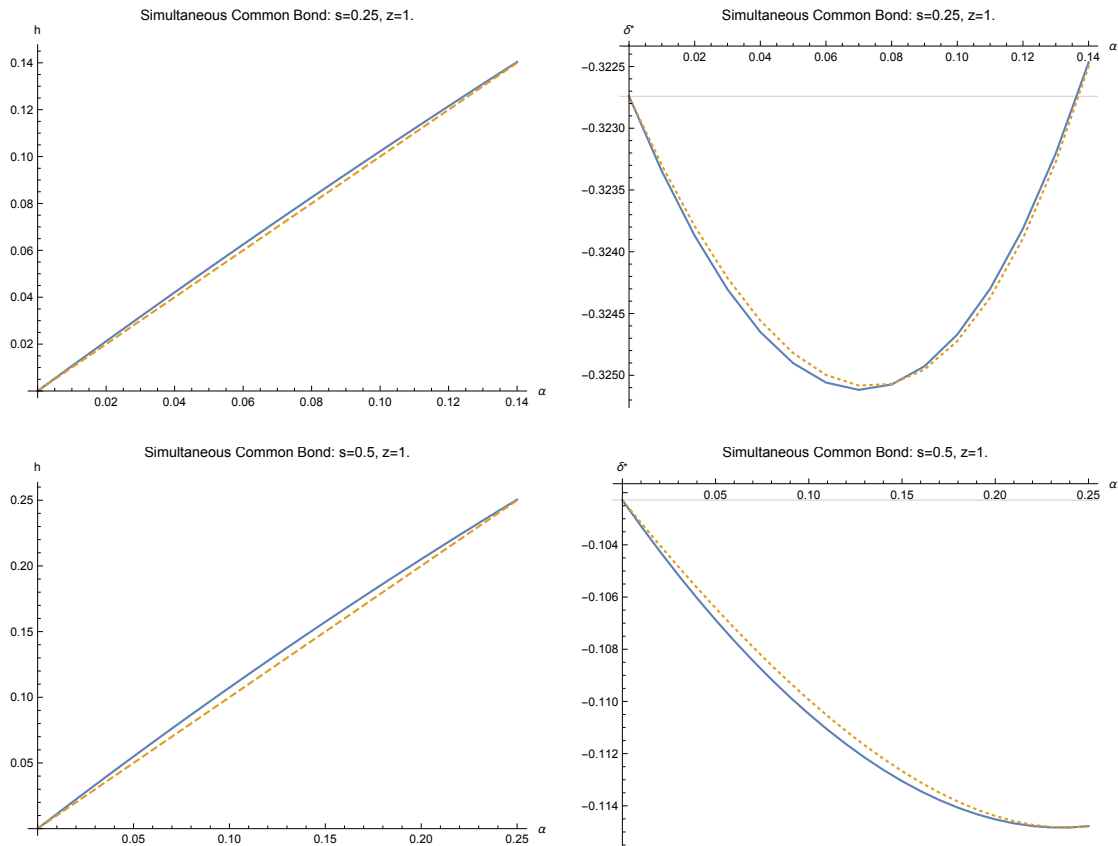
Figure C.2: **Robustness of single-survivor common bond equilibrium to sequential timing assumption:** Simultaneous equilibrium central interval width h (solid blue line) in comparison to 45 degree line (dashed yellow line) (left Panels); simultaneous equilibrium threshold δ^*_{sim} (solid blue line) in comparison to the sequential equilibrium threshold δ^*_{seq} (dashed yellow line) (right Panels).