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# Trend Inflation and Evolving Inflation Dynamics: A Bayesian GMM Analysis of the Generalized New Keynesian Phillips Curve

**Yasufumi Gemma*, Takushi Kurozumi**, and Mototsugu Shintani*****

**Abstract**

Inflation dynamics in the U.S. and Japan are investigated by estimating a "generalized" version of the Galí and Gertler (1999) New Keynesian Phillips curve (NKPC) with Bayesian GMM. This generalized NKPC (GNKPC) differs from the original only in that, in line with the micro evidence, each period some prices remain unchanged even under non-zero trend inflation. Yet the GNKPC has features that are significantly distinct from those of the NKPC. Model selection using quasi-marginal likelihood shows that the GNKPC empirically outperforms the NKPC in both the U.S. and Japan. Moreover, it explains U.S. inflation dynamics better than a constant-trend-inflation variant of the Cogley and Sbordone (2008) GNKPC. According to our selected GNKPC, when trend inflation fell after the Great Inflation of the 1970s in the U.S., the probability of no price change rose. Consequently, the GNKPC's slope flattened and its inflation-inertia coefficient decreased. As for Japan, when trend inflation turned slightly negative after the late 1990s (until the early 2010s), the fraction of backward-looking price setters increased; therefore, the GNKPC's inflation-inertia coefficient increased and its slope flattened.

**Keywords:** Inflation Dynamics; Trend Inflation; Generalized New Keynesian Phillips Curve; Bayesian GMM Estimation; Quasi-marginal Likelihood

**JEL classification:** C11, C26, E31

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# 1 Introduction

Inflation is an issue that has long been of central interest in economics. In particular, short-run inflation dynamics have continued to be the subject of intense investigation. In the literature, the so-called New Keynesian Phillips curve (henceforth NKPC) has been much analyzed from both theoretical and empirical points of view.[1] A canonical form of the NKPC can be obtained by either assuming zero trend inflation (e.g., Woodford, 2003; Galí, 2008) or introducing price indexation (e.g., Yun, 1996; Christiano, Eichenbaum, and Evans, 2005). While zero trend inflation can be optimal (e.g., Schmitt-Grohé and Uribe, 2010), it is not a realistic assumption because actual average inflation is not (necessarily) zero. Besides, the introduction of price indexation implies that all prices change in every period, which contradicts the micro evidence that some prices remain unchanged for several months, as argued by Woodford (2007). Against this background, there has been a surge of interest in the effects of non-zero trend inflation on the NKPC, particularly without price indexation. Specifically, recent studies have examined a generalized NKPC (henceforth GNKPC) and shown that the GNKPC is significantly distinct from the canonical NKPC in terms of its inflation dynamics, macroeconomic stability, and welfare and policy implications.[2]

This paper investigates inflation dynamics in the U.S. and Japan by estimating a "generalized" version of the Galí and Gertler (1999) NKPC. This GNKPC is based on Calvo's (1983) staggered price model with constant trend inflation and backward-looking price setters. It differs from the NKPC only in that, in line with the micro evidence, each period a fraction of prices remains unchanged even under non-zero trend inflation.[3] This difference,

[1] As shown by Roberts (1995), the NKPC can be derived from staggered price models à la Calvo (1983) and Taylor (1980) and price adjustment cost models à la Rotemberg (1982). Other prominent models used in the literature are state-dependent pricing models developed by Sheshinski and Weiss (1977), Caplin and Spulber (1987), Caplin and Leahy (1991), Dotsey, King, and Wolman (1999), Golosov and Lucas (2007), Nakamura and Steinsson (2010), and Midrigan (2011) among others.

[2] See, e.g., Ascari (2004), Ascari, Castelnuovo, and Rossi (2011), Ascari, Florio, and Gobbi (2017), Ascari and Ropele (2007, 2009), Cogley and Sbordone (2008), Coibion and Gorodnichenko (2011), Coibion, Gorodnichenko, and Wieland (2012), Hirose, Kurozumi, and Van Zandweghe (2017), Kobayashi and Muto (2013), Kurozumi (2014, 2016), Kurozumi and Van Zandweghe (2016a, b, 2017), Lago Alves (2014), and Shirota (2015). This strand of the literature has been reviewed by, for example, Ascari and Sbordone (2014).

[3] The NKPC can be derived by altering our model so that prices which remain unchanged in the model

however, causes the GNKPC to have two features that are significantly distinct from those of the NKPC. First, the driving force of inflation includes not only the current real marginal cost but also the expected growth rates of future demand and the expected discount rates on future profits under non-zero trend inflation. Besides, the real marginal cost reflects relative price distortion in addition to the real unit labor cost.[4] Second, the GNKPC's slope and inflation-inertia coefficient (i.e., its coefficients on the real marginal cost and past inflation) depend on the level of trend inflation as well as the probability of no price change and the fraction of backward-looking price setters.

Our paper adopts a Bayesian GMM approach to estimate the GNKPC.[5] The use of classical GMM in estimating NKPCs is known to have the drawback that some parameters can be unidentified or only weakly identified.[6] The Bayesian approach enables econometricians to exploit their prior knowledge in identifying the parameters.[7] Empirical performance of the GNKPC is then examined using quasi-marginal likelihood (henceforth QML), as suggested by Inoue and Shintani (2014). Taking account of a possible shift in trend inflation, the GNKPC is estimated during the Great Inflation (1966:Q1–1982:Q3) and thereafter (1982:Q4–2012:Q4) in the U.S., and during the Moderate Inflation (1981:Q4–1997:Q4) and thereafter (1998:Q1–2012:Q4) in Japan.

Three main findings of the paper are as follows. First, model selection (using QML) shows that the GNKPC empirically outperforms the NKPC of Galí and Gertler (1999) in both the U.S. and Japan. The improved fit of the GNKPC to U.S. and Japanese macroeconomic data

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are updated by indexing to trend inflation as in Yun (1996). Therefore, the GNKPC and the NKPC coincide only when trend inflation is zero. This implies that they are not nested and that the GNKPC does not necessarily literally generalize the NKPC.

[4] As stressed by Galí and Gertler (1999), the real unit labor cost is a theoretically justified measure of the real marginal cost in the NKPC, where there is no first-order effect of the relative price distortion. By contrast, in the GNKPC, this distortion has a first-order effect on the cost when trend inflation is non-zero. In this case, the log-linearized distortion is equal to a weighted sum of current and past inflation rates.

[5] For Bayesian GMM, see, e.g., Kim (2002) and Chernozhukov and Hong (2003).

[6] See, e.g., Mavroeidis (2005), Nason and Smith (2008), Kleibergen and Mavroeidis (2009), and Magnusson and Mavroeidis (2014). This strand of the literature has been reviewed by, for example, Mavroeidis, Plagborg-Møller, and Stock (2014).

[7] This is analogous to the use of full-information Bayesian methods in the estimation of dynamic stochastic general equilibrium models by, for example, Smets and Wouters (2007).

suggests the empirical importance of retaining some unchanged prices in each quarter in line with the micro evidence. Our finding for the U.S. is consistent with Hirose, Kurozumi, and Van Zandweghe (2017), who use a full-information Bayesian approach to compare a GNKPC and an NKPC in a dynamic stochastic general equilibrium (henceforth DSGE) model.[8] Compared with their system approach to estimate the fully specified DSGE model, an advantage of our approach to estimate the GNKPC is free from misspecification issues on any other equations in the system.

Second, the model selection also demonstrates that the GNKPC (with backward-looking price setters) explains U.S. inflation dynamics better than a constant-trend-inflation variant of the Cogley and Sbordone (2008) GNKPC.[9] Their paper estimates a GNKPC during the period 1960–2003, with drifting trend inflation and price indexation only to past inflation.[10] They find that such indexation plays no role, and thus conclude that there is no need for any backward-looking component in their GNKPC. The drifting trend inflation helps drive their conclusion because it makes the gap between actual and trend inflation less persistent. Our paper estimates the constant-trend-inflation variant of their GNKPC to examine whether the drifting trend inflation is the main driver of their result. The model selection shows that the role of price indexation to past inflation may have become negligible after the Great Inflation even in the absence of the drifting trend inflation. Moreover, compared with such indexation, the backward-looking price setters employed in our paper is a better specification of the backward-looking component in the GNKPC with constant trend inflation. These results suggest that the conclusion of Cogley and Sbordone may also depend on the specification of

[8] Hirose, Kurozumi, and Van Zandweghe (2017) estimate the two DSGE models with the GNKPC and with the NKPC during and after the Great Inflation. Restricting attention to a post-Great Inflation period, Ascari, Castelnuovo, and Rossi (2011) estimate three DSGE models with distinct NKPCs: the NKPC of Christiano, Eichenbaum, and Evans (2005), a variant of the Cogley and Sbordone (2008) GNKPC (with price indexation only to past inflation), and a GNKPC with price indexation only to trend inflation. Their model selection with marginal likelihood shows that the third model performs best, while the first does worst.

[9] Moreover, the model selection shows that backward-looking price setters played a non-negligible role in our GNKPC both during and after the Great Inflation, thus suggesting that the GNKPC requires such a backward-looking component to better explain U.S. inflation dynamics.

[10] To introduce the drifting trend inflation, Cogley and Sbordone (2008) assume subjective expectations à la Kreps (1998) instead of rational expectations. The price indexation to past inflation implies that all prices change in every period, whereas the backward-looking price setters do not.

backward-looking components of their GNKPC.

Third, our selected GNKPC indicates that when trend inflation fell after the Great Inflation in the U.S., the probability of no price change increased, while the fraction of backward-looking price setters remained almost unchanged. This increase in the probability of no price change is in line with the micro evidence reported by Nakamura *et al.* (2017) that the frequency of (regular) price changes declined after the Great Inflation.[11] The increased probability of no price change flattened the GNKPC's slope and decreased its inflation-inertia coefficient. This decrease in the inertia coefficient is consistent with the empirical result of Cogley, Primiceri, and Sargent (2010) that the persistence of the gap between actual and trend inflation declined after the Volcker disinflation. As for Japan, when trend inflation turned slightly negative after the Moderate Inflation, the fraction of backward-looking price setters increased, while the probability of no price change remained almost constant.[12] This caused the GNKPC's inflation-inertia coefficient to increase and its slope to flatten, thus obviating a severe deflation in the period 1998–2012.

The remainder of the paper proceeds as follows. Section 2 presents a GNKPC. Section 3 explains a methodology and data for estimating the GNKPC. Sections 4 and 5 show empirical results on inflation dynamics in the U.S. and Japan, respectively. Section 6 concludes.

# 2 Generalized New Keynesian Phillips Curve

This section presents a GNKPC. It is a "generalized" version of the Galí and Gertler (1999) NKPC. The GNKPC is derived from a Calvo (1983)-style staggered price model with constant trend inflation and backward-looking price setters. It differs from the NKPC only in that, in line with the micro evidence, each period a fraction of prices remains unchanged even when trend inflation is non-zero. Yet this difference causes the GNKPC to have features

[11] The concurrence of the increase in the probability of no price change and the fall in trend inflation in the estimated GNKPC is consistent with the literature on endogenous price stickiness, such as Ball, Mankiw, and Romer (1988), Levin and Yun (2007), and Kurozumi (2016).

[12] According to the model selection, backward-looking price setters played no role in the GNKPC during the Moderate Inflation, but their role became important thereafter.

that are significantly distinct from those of the NKPC.

## 2.1 Model

In the economy there are a representative final-good firm and a continuum of intermediate-good firms $f \in [0,1]$. The final-good firm produces homogeneous goods $Y_t$ under perfect competition. Given the final-good price $P_t$ and the intermediate-good prices $\{P_t(f)\}$, the firm chooses a combination of intermediate goods $\{Y_t(f)\}$ so as to maximize profit $P_t Y_t - \int_0^1 P_t(f) Y_t(f)\, df$ subject to the CES production technology $Y_t = \left[\int_0^1 (Y_t(f))^{(\theta-1)/\theta}\, df\right]^{\theta/(\theta-1)}$, where $\theta > 1$ denotes the elasticity of substitution between intermediate goods.

The first-order condition for profit maximization yields the final-good firm's demand curve for each intermediate good

$$Y_t(f) = Y_t \left(\frac{P_t(f)}{P_t}\right)^{-\theta}, \tag{1}$$

and thus the CES production technology leads to

$$P_t = \left[\int_0^1 (P_t(f))^{1-\theta}\, df\right]^{\frac{1}{1-\theta}}. \tag{2}$$

Each intermediate-good firm $f$ produces one kind of differentiated good $Y_t(f)$ under monopolistic competition. Given the real rental rate on capital $r_{k,t}$ and the real wage rate $W_t$, the firm chooses capital and labor inputs $\{K_t(f), l_t(f)\}$ so as to minimize cost $r_{k,t} K_t(f) + W_t l_t(f)$ subject to the Cobb–Douglas production technology $Y_t(f) = (K_t(f))^{\alpha} (Z_t l_t(f))^{1-\alpha}$, where $\alpha \in (0,1)$ denotes the capital elasticity of output and $Z_t$ represents the level of (neutral) technology. Its log level is assumed to follow a random walk process

$$\log Z_t = \log gy + \log Z_{t-1} + \varepsilon_t, \tag{3}$$

where $gy$ is the steady-state rate of technological change $Z_t/Z_{t-1}$ and $\varepsilon_t$ is white noise. Note that $gy$ corresponds to the steady-state rate of output growth $gy_t = Y_t/Y_{t-1}$. The process (3) implies that

$$\log \frac{y_t}{y_{t-1}} = \log \frac{gy_t}{gy} - \varepsilon_t, \tag{4}$$

where $y_t \equiv Y_t/Z_t$ denotes detrended output.

The first-order conditions for capital and labor inputs are given by $r_{k,t}K_t(f) = \alpha\, mc_t(f)Y_t(f)$ and $W_t l_t(f) = (1-\alpha)mc_t(f)Y_t(f)$, where $mc_t(f)$ denotes firm $f$'s real marginal cost. It can then be shown that the marginal cost is identical among all intermediate-good firms, i.e., $mc_t = mc_t(f) = (r_{k,t}/\alpha)^{\alpha}\left[(W_t/Z_t)/(1-\alpha)\right]^{1-\alpha}$. Moreover, aggregating the first-order condition for labor input over the firms $f$ and using the demand curve (1) leads to

$$mc_t = \frac{\int_0^1 W_t l_t(f)\, df}{(1-\alpha)\int_0^1 Y_t(f)\, df} = \frac{W_t l_t}{(1-\alpha)Y_t d_t} = \frac{ulc_t}{(1-\alpha)d_t}, \tag{5}$$

where $l_t \equiv \int_0^1 l_t(f)\, df$ is aggregate labor, $ulc_t \equiv W_t l_t/Y_t$ denotes the final-good-based real unit labor cost, and

$$d_t \equiv \int_0^1 \left(\frac{P_t(f)}{P_t}\right)^{-\theta} df \tag{6}$$

represents the relative price distortion. This distortion captures relative demand dispersion (i.e., $d_t = \int_0^1 (Y_t(f)/Y_t)\, df$) and measures the loss of intermediate goods in producing final goods (i.e., $Y_t \leq Y_t d_t = \int_0^1 Y_t(f)\, df$).[13]

One point to be noted in the marginal cost equation (5) is that, given the final-good-based real unit labor cost $ulc_t$ and the labor elasticity of output $1-\alpha$, an increase in the relative price distortion $d_t\,(\geq 1)$ reduces each intermediate-good firm's real marginal cost $mc_t$. One might have considered that a larger distortion in production would raise firms' marginal cost. However, as equation (5) indicates, each intermediate-good firm's real marginal cost $mc_t$ is equal to the intermediate-good-based real unit labor cost $\int_0^1 W_t l_t(f)\, df \Big/ \int_0^1 Y_t(f)\, df$ divided by the labor elasticity $1-\alpha$. Therefore, the distortion $d_t$ effectively acts to shift the basis of unit labor cost from final-good to intermediate-good production.

Intermediate-good firms set the prices of their products on a staggered basis à la Calvo (1983). In each period, a fraction $\lambda \in (0,1)$ of firms keeps prices unchanged, while the remaining fraction $1-\lambda$ sets prices in the following two ways. As in Galí and Gertler (1999), a fraction $\omega \in [0,1)$ of price-setting firms uses a backward-looking rule of thumb, while the remaining fraction $1-\omega$ optimizes prices.

[13] These follow from the demand curve (1), the final-good price equation (2), and Jensen's inequality.

The price set by the backward-looking rule of thumb is given by

$$P_t^r = P_{t-1}^a \pi_{t-1}, \tag{7}$$

where

$$P_t^a \equiv (P_t^r)^{\omega} (P_t^o)^{1-\omega} \tag{8}$$

and $P_t^o$ is the price chosen by optimizing firms in period $t$.

The optimized price $P_t^o$ is set to maximize the profit function $E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j}(P_t(f)/P_{t+j} - mc_{t+j})Y_{t+j|t}(f)$ with regard to $P_t(f)$ subject to the demand curve $Y_{t+j|t}(f) = Y_{t+j}(P_t(f)/P_{t+j})^{-\theta}$, where $E_t$ denotes the (rational) expectation operator conditional on information available in period $t$ and $Q_{t,t+j}$ is the real stochastic discount factor between period $t$ and period $t+j$, which satisfies $Q_{t,t+j} = \prod_{k=1}^{j} Q_{t+k-1,t+k}$. The first-order condition for the optimized price becomes

$$E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} \frac{Y_{t+j}}{Y_t} \prod_{k=1}^{j} \pi_{t+k}^{\theta} \left( p_t^o \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} - \frac{\theta}{\theta - 1} mc_{t+j} \right) = 0, \tag{9}$$

where $p_t^o \equiv P_t^o / P_t$ and $\pi_t \equiv P_t / P_{t-1}$ is the (gross) inflation rate of the final-good price.

Under the aforementioned price setting, the final-good price equation (2) and the relative price distortion equation (6) can be reduced to

$$1 = (1-\lambda)\Big[(1-\omega)(p_t^o)^{1-\theta} + \omega(p_t^r)^{1-\theta}\Big] + \lambda \pi_t^{\theta-1}, \tag{10}$$

$$d_t = (1-\lambda)\Big[(1-\omega)(p_t^o)^{-\theta} + \omega(p_t^r)^{-\theta}\Big] + \lambda \pi_t^{\theta} d_{t-1}, \tag{11}$$

where $p_t^r \equiv P_t^r / P_t$.

With one-period (nominal) bonds, their (gross) interest rate $r_t$ satisfies

$$1 = E_t \left( Q_{t,t+1} \frac{r_t}{\pi_{t+1}} \right). \tag{12}$$

For the steady state to be well defined, the following condition is assumed.

$$\lambda \pi^{\theta} < \min(1, \pi), \tag{13}$$

where $\pi$ is the steady-state value of $\pi_t$, that is, (gross) trend inflation.

## 2.2 Generalized New Keynesian Phillips curve for estimation

As shown in Appendix, under the assumption (13), equations (3), (4), (5), (7), (8), (9), (10), and (12) give rise to the GNKPC

$$\hat{\pi}_t = \gamma_b \hat{\pi}_{t-1} + \gamma_f E_t \hat{\pi}_{t+1} + \kappa\Big(\hat{ulc}_t - \hat{d}_t\Big) + \kappa_f \sum_{j=1}^{\infty} (\beta\lambda\pi^{\theta-1})^j \big(E_t \hat{gy}_{t+j} + \theta E_t \hat{\pi}_{t+j} - E_t \hat{r}_{t+j-1}\big)\,, \tag{14}$$

where hatted variables denote log-deviations from steady-state values and $\beta \in (0,1)$ is the subjective discount factor, which is the steady-state value of the growth-adjusted real stochastic discount factor $\beta_{t,t+1} \equiv Q_{t,t+1} Z_{t+1}/Z_t$. The coefficient $\gamma_b \equiv \omega/\phi$, where $\phi \equiv \lambda\pi^{\theta-1} + \omega[1 - \lambda\pi^{\theta-1}(1-\beta\pi)]$, represents the degree of intrinsic inflation inertia (generated in the presence of backward-looking price setters), and $\kappa \equiv (1-\lambda\pi^{\theta-1})(1-\beta\lambda\pi^{\theta})(1-\omega)/\phi$ is the so-called "slope" of the GNKPC (i.e., the elasticity of inflation with respect to real marginal cost). The others are given by $\gamma_f \equiv \beta\lambda\pi^{\theta}/\phi$ and $\kappa_f \equiv (\pi-1)(1-\lambda\pi^{\theta-1})(1-\omega)/\phi$.

In the GNKPC (14), two points are worth noticing. First, the real marginal cost $\hat{mc}_t$ reflects the relative price distortion $\hat{d}_t$ as well as the real unit labor cost $\hat{ulc}_t$, i.e., $\hat{mc}_t = \hat{ulc}_t - \hat{d}_t$.[14] Besides, not only the real marginal cost but also the expected growth rates of future demand and the expected discount rates on future profits drive inflation in the GNKPC under non-zero trend inflation (i.e., $\pi \neq 1$).

Second, the GNKPC's slope $\kappa$ and inflation-inertia coefficient $\gamma_b$ depend not only on the probability of no price change $\lambda$ and the fraction of backward-looking price setters $\omega$ but also on the level of trend inflation $\pi$ and the elasticity of substitution $\theta$.[15] Table 1 summarizes how the slope $\kappa$ and the inflation-inertia coefficient $\gamma_b$ are related with the model parameters $\lambda$, $\omega$, $\pi$, and $\theta$.

[14] The real unit labor cost is the only force that drives inflation in the NKPC of Galí and Gertler (1999) (see Section 2.3).

[15] The slope $\kappa$ and the inflation-inertia coefficient $\gamma_b$ depend on the subjective discount factor $\beta$ as well. As $\beta$ decreases, the slope $\kappa$ steepens and the inertia coefficient $\gamma_b$ increases. This is because only optimizing firms take account of the discount factor $\beta$. A decline in $\beta$ makes optimizing firms myopic, so that they respond more to the current and less to the future real marginal cost. Their increased sensitivity to the current steepens the slope $\kappa$, while their comparative indifference to the future reduces the inflation-expectation coefficient $\gamma_f$ and increases the inflation-inertia coefficient $\gamma_b$ in the presence of backward-looking price setters.

Table 1: Effects of model parameters on the GNKPC's slope and inflation-inertia coefficient

| | $\lambda$ | $\omega$ | $\pi$ | $\theta$ (if $\pi > 1$) | $\theta$ (if $\pi < 1$) |
|---|---|---|---|---|---|
| slope $\kappa \downarrow$ | $\uparrow$ | $\uparrow$ | $\uparrow$ | $\uparrow$ | $\downarrow$ |
| inflation inertia $\gamma_b \downarrow$ | $\uparrow$ | $\downarrow$ | $\uparrow$ | $\uparrow$ | $\downarrow$ |

Notes: The model parameters $\lambda$, $\omega$, $\pi$, and $\theta$ denote the probability of no price change, the fraction of backward-looking price setters, trend inflation, and the elasticity of substitution, respectively.

A flatter slope $\kappa$ is caused (ceteris paribus) by a higher probability $\lambda$, a larger fraction $\omega$, or higher trend inflation $\pi$. It is also generated by a higher elasticity $\theta$ if trend inflation is positive (i.e., $\pi > 1$) and by a lower elasticity $\theta$ if trend inflation is negative (i.e., $\pi < 1$). These factors except a larger fraction $\omega$ bring about a lower inflation-inertia coefficient $\gamma_b$, which is also caused by a smaller fraction $\omega$. In understanding these relationships, the following log-linearization of the final-good price equation (10) is particularly helpful.

$$0 = (1-\omega)(1-\lambda\pi^{\theta-1})\,\hat{p}_t^o + \omega(1-\lambda\pi^{\theta-1})\,\hat{p}_t^r + \lambda\pi^{\theta-1}\,(-\hat{\pi}_t)$$

This equation shows that the steady-state contributions to the current aggregate price level of optimizing firms ($\hat{p}_t^o$), backward-looking firms ($\hat{p}_t^r$), and firms keeping prices unchanged from the previous period ($-\hat{\pi}_t$) are given by $(1-\omega)(1-\lambda\pi^{\theta-1})$, $\omega(1-\lambda\pi^{\theta-1})$, and $\lambda\pi^{\theta-1}$, respectively. The slope $\kappa$ flattens when the contribution of optimizing firms decreases, since only these firms respond (directly) to the real marginal cost. The inflation-inertia coefficient $\gamma_b$ lowers when the contribution of backward-looking firms decreases. Because a higher probability of no price change $\lambda$ reduces the contributions of price-setting firms, that is, both optimizing and backward-looking firms, it flattens the slope and decreases the inflation-inertia coefficient. A smaller fraction of backward-looking price setters $\omega$ reduces the contribution of backward-looking firms while raising that of optimizing firms, thus decreasing the inertia coefficient and steepening the slope. These effects of $\lambda$ and $\omega$ on the slope and the inflation-inertia coefficient of the GNKPC are shared with the NKPC of Galí and Gertler (1999) (see Section 2.3). However, unlike in the NKPC, higher trend inflation $\pi$ flattens the slope and decreases the inertia coefficient in the GNKPC, since it reduces the contributions of both optimizing and backward-looking firms. Likewise, a higher elasticity of substitution $\theta$

reduces them under positive trend inflation (i.e., $\pi > 1$) and therefore flattens the slope and decreases the inertia coefficient. If trend inflation is negative (i.e., $\pi < 1$), a higher elasticity raises them, thus steepening the slope and increasing the inertia coefficient.

The relative price distortion $d_t$ appears in the GNKPC (14); yet there is no available data on the distortion (to the best of our knowledge). This issue is addressed by using the log-linearization of the distortion's law of motion (11),

$$\hat{d}_t = \lambda\pi^{\theta}\hat{d}_{t-1} + \kappa_d\hat{\pi}_t \quad \text{or} \quad \hat{d}_t = \kappa_d \sum_{j=0}^{\infty}(\lambda\pi^{\theta})^j\hat{\pi}_{t-j}, \tag{15}$$

where $\kappa_d \equiv \theta\lambda\pi^{\theta-1}(\pi-1)/(1-\lambda\pi^{\theta-1})$. By substituting this equation, the GNKPC can be rewritten as

$$\begin{aligned}\hat{\pi}_t = {} & \frac{\gamma_b}{1+\kappa_d\kappa}\hat{\pi}_{t-1} + \frac{\gamma_f}{1+\kappa_d\kappa}E_t\hat{\pi}_{t+1} + \frac{\kappa}{1+\kappa_d\kappa}\hat{ulc}_t - \frac{\kappa_d\kappa}{1+\kappa_d\kappa}\sum_{j=1}^{\infty}(\lambda\pi^{\theta})^j\hat{\pi}_{t-j} \\ & + \frac{\kappa_f}{1+\kappa_d\kappa}\sum_{j=1}^{\infty}(\beta\lambda\pi^{\theta-1})^j\left(E_t\hat{gy}_{t+j} + \theta E_t\hat{\pi}_{t+j} - E_t\hat{r}_{t+j-1}\right).\end{aligned} \tag{16}$$

This alternative representation of the GNKPC is convenient for estimation purposes in the ensuing sections.[16]

## 2.3 Galí and Gertler (1999) New Keynesian Phillips curve

To evaluate the empirical performance of the GNKPC, this paper also considers the NKPC of Galí and Gertler (1999)

$$\hat{\pi}_t = \gamma_{b,1}\hat{\pi}_{t-1} + \gamma_{f,1}E_t\hat{\pi}_{t+1} + \kappa_1\hat{ulc}_t, \tag{17}$$

where the coefficients $\gamma_{b,1}$, $\gamma_{f,1}$, $\kappa_1$ correspond respectively to $\gamma_b$, $\gamma_f$, $\kappa$ with $\pi = 1$.[17] This NKPC can be derived by altering the model so that intermediate-good firms which keep prices unchanged in the aforementioned setting instead update prices by indexing to trend

[16] The model also implies that $d_t = K_t^{\alpha}\left(Z_t l_t\right)^{1-\alpha}/Y_t$. It is thus theoretically possible to estimate the GNKPC (14) using this identity. However, the GNKPC (16) is chosen for estimation in this paper because there is no available data on $Z_t$. Note that total factor productivity is given by $Z_t^{1-\alpha}/d_t$ in the model.

[17] Note that the elasticity of substitution $\theta$ does not appear in the coefficients $\gamma_{b,1}$, $\gamma_{f,1}$, and $\kappa_1$. Indeed, $\gamma_{b,1} \equiv \omega/\phi_1$, $\gamma_{f,1} \equiv \beta\lambda/\phi_1$, $\kappa_1 \equiv (1-\lambda)(1-\beta\lambda)(1-\omega)/\phi_1$, and $\phi_1 \equiv \lambda + \omega[1-\lambda(1-\beta)]$.

inflation as in Yun (1996). The parameter $\lambda$ in the NKPC then represents the probability of trend inflation-indexed price setting.

One point to be emphasized here is that the GNKPC (16) and the NKPC (17) coincide only when trend inflation is zero (i.e., $\pi = 1$).[18] This suggests that they are not nested. Thus, to compare the empirical performance of the two non-nested models, QML is exploited in the ensuing sections, as suggested by Inoue and Shintani (2014).

# 3 Econometric Methodology and Data

This section explains our methodology and data for estimating both the GNKPC and the NKPC presented in the preceding section.

## 3.1 Bayesian GMM

This paper estimates the GNKPC (16) and the NKPC (17) with Bayesian GMM. In previous literature, classical GMM has often been used in estimating NKPCs. Yet a drawback of such estimation is that some parameters of NKPCs can end up unidentified or only weakly identified. This issue has been extensively discussed in a considerable number of studies, including Mavroeidis (2005), Nason and Smith (2008), Kleibergen and Mavroeidis (2009), Magnusson and Mavroeidis (2014), and Mavroeidis, Plagborg-Møller, and Stock (2014). These studies list a number of conditions that can help identify parameters of NKPCs.

Our paper adopts a Bayesian approach to GMM estimation of the GNKPC (16) and the NKPC (17) similarly to empirical work on DSGE models that utilizes full-information Bayesian methods.[19] The Bayesian GMM estimator enables econometricians to exploit their prior knowledge about model parameters, thereby mitigating the identification issue. Within the framework of Bayesian GMM, the classical GMM estimator can be viewed as the special case with a flat prior.

[18] When $\pi = 1$, the coefficients coincide (i.e., $\gamma_b = \gamma_{b,1}$, $\gamma_f = \gamma_{f,1}$, $\kappa = \kappa_1$, and $\kappa_f = 0$) and $\kappa_d = 0$ implies $\hat{d}_t = 0$ from the log-linearized law of motion of the relative price distortion (15). Hence, the GNKPC (16) and the NKPC (17) are mutually consistent under zero trend inflation.

[19] Recently, Bayesian GMM has been used to estimate a DSGE model by, for example, Gallant, Giacomini, and Ragusa (2017).

In the econometrics literature, the Bayesian GMM estimator belongs to the class of limited-information quasi-Bayesian estimators. Its asymptotic properties, such as consistency and asymptotic normality, have been established by Kim (2002) and Chernozhukov and Hong (2003). The latter authors emphasize a computational advantage of the Bayesian GMM estimator over the classical estimator, since the Markov Chain Monte Carlo (henceforth MCMC) method can be utilized even if its GMM objective function cannot be expressed in a simple form. Model selection procedures in the quasi-Bayesian framework have been developed by Kim (2014) and Inoue and Shintani (2014).

Let $\boldsymbol{\theta}$ denote an $m \times 1$ vector of model parameters to be estimated and $g_t(\boldsymbol{\theta})$ be an $n \times 1$ vector of moment functions that satisfies $E(g_t(\boldsymbol{\theta})) = 0$ at $\boldsymbol{\theta} = \boldsymbol{\theta}_0$. In our estimation, trend inflation $\pi$ is assumed to meet the condition $\log \pi = E \log \pi_t$ (i.e., $E\hat{\pi}_t = 0$), where $E$ is the unconditional expectation operator, and thus $g_t(\boldsymbol{\theta})$ is defined as

$$g_t(\boldsymbol{\theta}) = \begin{bmatrix} u_t \\ \mathbf{z}_t u_t \\ u_{\pi,t} \end{bmatrix}, \tag{18}$$

where $\mathbf{z}_t$ is an $(n-2) \times 1$ vector of instruments and $u_{\pi,t} = \log \pi_t - \log \pi$. Moreover, in the case of the GNKPC (16), $\boldsymbol{\theta} = [\pi \ \lambda \ \omega \ \theta]'$ and

$$\begin{aligned} u_t = {} & (\log \pi_t - \log \pi) - \frac{\gamma_b}{1+\kappa_d \kappa}(\log \pi_{t-1} - \log \pi) - \frac{\gamma_f}{1+\kappa_d \kappa}(\log \pi_{t+1} - \log \pi) \\ & - \frac{\kappa}{1+\kappa_d \kappa}\hat{ulc}_t + \frac{\kappa_d \kappa}{1+\kappa_d \kappa}\sum_{j=1}^{\infty}(\lambda \pi^{\theta})^j (\log \pi_{t-j} - \log \pi) \\ & - \frac{\kappa_f}{1+\kappa_d \kappa}\sum_{j=1}^{\infty}(\beta \lambda \pi^{\theta-1})^j \left[\hat{gy}_{t+j} + \theta(\log \pi_{t+j} - \log \pi) - \hat{r}_{t+j-1}\right]. \end{aligned}$$

Then, following Galí, Gertler, and López-Salido (2005), truncated sums are used to approximate the infinite sums of log-deviations of inflation, output growth, and the interest rate from their steady-state values.[20] In the case of the NKPC (17), $\boldsymbol{\theta} = [\pi \ \lambda \ \omega]'$ and

$$u_t = (\log \pi_t - \log \pi) - \gamma_{b,1}(\log \pi_{t-1} - \log \pi) - \gamma_{f,1}(\log \pi_{t+1} - \log \pi) - \kappa_1 \hat{ulc}_t.$$

[20] For the quarterly model, this paper employs 16 lags of the log-deviation of inflation and 16 leads of the log-deviations of inflation, output growth, and the interest rate. We also experimented with 12 lags and 12 leads, and with 20 lags and 20 leads, and confirmed that the results presented in this paper are not qualitatively affected.

This paper employs the efficient two-step GMM estimator.[21] This estimator maximizes the objective function $\hat{q}(\boldsymbol{\theta}) = -(1/2)g(\boldsymbol{\theta})'\,\hat{W}\,g(\boldsymbol{\theta})$ with regard to $\boldsymbol{\theta} \in \Theta$, where $g(\boldsymbol{\theta}) = (1/\sqrt{T})\sum_{t=1}^{T} g_t(\boldsymbol{\theta})$ and $\hat{W}$ is a consistent estimator of an $n \times n$ positive semidefinite optimal weighting matrix, which is based on the Newey and West (1987) HAC estimator and is given by $\hat{W} = [\Gamma_j(\tilde{\boldsymbol{\theta}}) + \sum_{j=1}^{J}(j/J)(\Gamma_j(\tilde{\boldsymbol{\theta}}) + \Gamma_j(\tilde{\boldsymbol{\theta}})')]^{-1}$, where $J$ is the lag length set by the Andrews (1991) automatic bandwidth selection method, $\Gamma_j(\boldsymbol{\theta}) = [1/(T-j)]\sum_{t=j+1}^{T} g_t(\boldsymbol{\theta})g_{t-j}(\boldsymbol{\theta})'$, and $\tilde{\boldsymbol{\theta}}$ is a first-step consistent estimator of the true value $\boldsymbol{\theta}_0$.[22]

Next, Bayesian methods are applied to GMM estimation. Following Chernozhukov and Hong (2003), the quasi-posterior distribution for $\boldsymbol{\theta}$ is defined as

$$\frac{\exp(\hat{q}(\boldsymbol{\theta}))\,p(\boldsymbol{\theta})}{\int_{\Theta}\exp(\hat{q}(\boldsymbol{\theta}))\,p(\boldsymbol{\theta})\,d\boldsymbol{\theta}},$$

where $p(\boldsymbol{\theta})$ is the prior distribution for $\boldsymbol{\theta}$. The Bayesian GMM estimator is typically defined as the mean of the quasi-posterior distribution. This estimator is particularly useful when the GMM objective function $\hat{q}(\boldsymbol{\theta})$ is not tractable, since the mean and credible intervals of the quasi-posterior distribution can be computed using the MCMC method.[23]

## 3.2 Quasi-Bayesian model selection

In conducting model selection, this paper follows Inoue and Shintani (2014) to use the QML defined as[24]

$$\int_{\Theta}\exp(\hat{q}(\boldsymbol{\theta}))\,p(\boldsymbol{\theta})\,d\boldsymbol{\theta}.$$

[21] Following most of the previous studies, including Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2001, 2005), our paper uses the two-step GMM estimator rather than the continuous updating GMM estimator proposed by Hansen, Heaton, and Yaron (1996).

[22] The HAC covariance matrix estimator of the moment functions (18) is employed for two reasons. First, the inflation gap $u_{\pi,t} = \log \pi_t - \log \pi$ in (18) is possibly serially correlated. Second, the use of the HAC estimator makes the resulting estimation valid not only for the exact specification of the GNKPC (16) but also for the case in which a disturbance (e.g., a mark-up shock) is incorporated in the GNKPC.

[23] In our estimation, $210,000$ MCMC draws that meet the assumption (13) were generated and the first $10,000$ draws were discarded. The Random-Walk Metropolis-Hastings algorithm was applied to generate draws from the quasi-posterior distribution. The scale factor for the jumping distribution in the algorithm was adjusted so that an acceptance ratio of around 25 percent would be obtained.

[24] The use of the QML in model selection is theoretically justified by Inoue and Shintani (2014), who show the consistency of model selection for procedures with the QML. Christiano, Eichenbaum, and Trabandt (2016) also utilize QML in the selection of DSGE models estimated with a minimum distance approach for impulse response functions.

Similarly to the model selection with marginal likelihood in full-information Bayesian estimation, a model with a higher QML is regarded as a better specification.

The QML is calculated using the modified harmonic mean method. This method computes the QML as the reciprocal of

$$E\left[\frac{w(\boldsymbol{\theta})}{\exp(\hat{q}(\boldsymbol{\theta}))\, p(\boldsymbol{\theta})}\right],$$

which is evaluated using MCMC draws, given a weighting function $w(\boldsymbol{\theta})$. This paper considers two alternative choices for the weighting function proposed in the literature. The first choice is suggested by Geweke (1999), who sets $w(\boldsymbol{\theta})$ to be the truncated normal density

$$w(\boldsymbol{\theta}) = \frac{\exp[-(1/2)(\boldsymbol{\theta}-\bar{\boldsymbol{\theta}})'\bar{V}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}-\bar{\boldsymbol{\theta}})]}{(2\pi)^{m/2}|\bar{V}_{\boldsymbol{\theta}}|^{1/2}} \frac{\mathbf{1}\{(\boldsymbol{\theta}-\bar{\boldsymbol{\theta}})'\bar{V}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}-\bar{\boldsymbol{\theta}}) \leq \chi^2_{m,\tau}\}}{\tau},$$

where $\bar{\boldsymbol{\theta}}$ is the quasi-posterior mean, $\bar{V}_{\boldsymbol{\theta}}$ is the quasi-posterior covariance matrix, $\pi$ in this subsection is the circular constant, $\mathbf{1}\{\cdot\}$ is an indicator function, $\chi^2_{m,\tau}$ is the $100\tau$th percentile of the chi-square distribution with $m$ degrees of freedom, and $\tau \in (0,1)$ is a constant.[25] The second choice is the one proposed by Sims, Waggoner, and Zha (2008). They point out that Geweke's choice may not work well when the posterior distribution is non-elliptical, and suggest the weighting function given by

$$w(\boldsymbol{\theta}) = \frac{\Gamma(m/2)}{2\pi^{m/2}|\hat{V}_{\boldsymbol{\theta}}|^{1/2}} \frac{f(r)}{r^{m-1}} \frac{\mathbf{1}\{\hat{q}(\boldsymbol{\theta}) + \log p(\boldsymbol{\theta}) > L_{1-q}\}}{\bar{\tau}},$$

where $\hat{V}_{\boldsymbol{\theta}}$ is the second moment matrix centered around the quasi-posterior mode $\hat{\boldsymbol{\theta}}$, $f(r) = [vr^{v-1}/(c_{90}^v/0.9 - c_1^v)]\,\mathbf{1}\{c_1 < r < c_{90}/(0.9)^{1/v}\}$, $v = \log(1/9)/\log(c_{10}/c_{90})$, $r = [(\boldsymbol{\theta} - \hat{\boldsymbol{\theta}})'\hat{V}_{\boldsymbol{\theta}}^{-1}(\boldsymbol{\theta}-\hat{\boldsymbol{\theta}})]^{1/2}$, $c_j$ is the $j$th percentile of the distance $r$, $L_{1-q}$ is the $100(1-q)$th percentile of the log quasi-posterior distribution, $q \in (0,1)$ is a constant, and $\bar{\tau}$ is the quasi-posterior mean of $\mathbf{1}\{\hat{q}(\boldsymbol{\theta}) + \log p(\boldsymbol{\theta}) > L_{1-q}\}\,\mathbf{1}\{c_1 < r < c_{90}/(0.9)^{1/v}\}$.

The ensuing analysis assesses the QML using various values of the truncation parameters, $\tau$ in the Geweke (1999) estimator and $q$ in the Sims-Waggoner-Zha (2008) estimator, for robustness. Following Herbst and Schorfheide (2015), this paper chooses $\tau = 0.5$ and 0.9 for the former estimator and $q = 0.5$ and 0.9 for the latter.

[25] Recall that $m$ denotes the number of model parameters to be estimated.

### 3.3 Data and estimation periods

Primary data for estimation of the GNKPC (16) are four quarterly time series: inflation $\pi_t$, the real unit labor cost $ulc_t$, output growth $gy_t$, and the interest rate $r_t$. As for instruments $\mathbf{z}_t$ in (18), this paper follows Galí, Gertler, and López-Salido (2001, 2005): four lags of inflation and two lags of wage inflation as well as of the other three variables that appear in the GNKPC, i.e., $ulc_t$, $gy_t$, and $r_t$.

For the U.S., the data on $\pi_t$ is based on the GDP implicit price deflator, and that on $ulc_t$ is the labor income share in the non-farm business sector, as in Galí and Gertler (1999) and Galí, Gertler, and López-Salido (2001, 2005). Those on $gy_t$ and $r_t$ are, respectively, the per-capita real GDP growth rate and the three-month Treasury bill rate. The wage inflation data is based on the hourly compensation for the non-farm business sector. To take account of a possible shift in trend inflation, estimation is performed separately for the Great Inflation period (1966:Q1–1982:Q3) and the period thereafter (1982:Q4–2012:Q4), as well as for the full sample period since 1966:Q1.[26]

For Japan, the data on $\pi_t$ is based on the CPI (excluding fresh foods), and that on $ulc_t$ is the labor income share obtained by dividing the nominal compensation of employees by nominal GDP.[27] The data on $gy_t$ is the growth rate of per-capita nominal GDP deflated with the CPI, while that on $r_t$ is the uncollateralized overnight call rate. The wage inflation data is based on the wage measure constructed in Sugo and Ueda (2008). Estimation is conducted separately for the Moderate Inflation period (1981:Q4–1997:Q4) and the period thereafter (1998:Q1–2012:Q4), as well as for the full sample period since 1981:Q4.[28]

[26] In estimating the GNKPC (16) with the quarterly data, this paper uses 16 leads of inflation, output growth, and the interest rate, so that the second sample period ends in 2012:Q4.

[27] Our measure of the labor income share differs quantitatively from that of Tsuruga and Muto (2008). Their measure takes into account compensation in self-employed firms, while it assumes that the labor income share for such firms is the same as for other firms. Although our measure takes no account of self-employed firms, it assumes that movements in the ratio of nominal compensation to nominal GDP are the same for the self-employed as for employees at other firms. Because the level of the labor income share does not matter to estimation of the GNKPC or the NKPC, there is no qualitative difference between the two measures inasmuch as each of them makes some assumptions regarding self-employed firms. Their measure thus seems to have no clear advantage over ours.

[28] Our estimation is not performed during the high inflation period from the early 1970s to the early 1980s, because there are limitations to the data prior to 1970 and this paper uses 16 lags of inflation in estimating

For the real unit labor cost, output growth, and the interest rate, the time series of their log-deviations from steady-state values $\{\hat{ulc}_t, \hat{gy}_t, \hat{r}_t\}$ are constructed separately for each sample period; they are all demeaned using their respective sample-period averages.

## 3.4 Prior distributions for parameters

In each estimation, the subjective discount factor $\beta$ is fixed at 0.99. All of the remaining parameters in the GNKPC (16) and the NKPC (17) are estimated. The prior distributions for the parameters are presented in Table 2.

Table 2: Prior distributions for parameters

| Parameter | | Distribution | Mean (U.S./Japan) | Std. dev. |
|---|---|---|---|---|
| $\bar{\pi}$ | annualized trend inflation rate | Normal | 3.67/0.65 | 1.50 |
| $\lambda$ | probability of no price change | Beta | 0.50/0.50 | 0.10 |
| $\omega$ | fraction of backward-looking price setters | Beta | 0.50/0.50 | 0.10 |
| $\theta$ | elasticity of substitution | Gamma | 9.32/7.67 | 1.00 |

Notes: $\bar{\pi} \equiv 400 \log \pi$. In the NKPC (17), $\lambda$ represents the probability of trend inflation-indexed price setting.

For the U.S., the prior for the annualized trend inflation rate $\bar{\pi}$ ($\equiv 400 \log \pi$) is centered around the full-sample-period average 3.67 with standard deviation of 1.5. The prior distributions for the probability of no price change $\lambda$ and for the fraction of backward-looking price setters $\omega$ are set to be the beta distributions with mean 0.5 and standard deviation of 0.1. For the prior for the elasticity of substitution $\theta$, the gamma distribution is chosen with mean 9.32 and standard deviation of unity. The prior mean of 9.32 follows the estimate by Ascari and Sbordone (2014).

For Japan, the prior distributions are the same as for the U.S., except that the prior mean of the trend inflation $\bar{\pi}$ is set at the full-sample-period average 0.65, while that of the substitution elasticity $\theta$ is chosen at 7.67, a value often used in the macroeconomics literature.

the GNKPC (16) with the quarterly data, so that the sample size for that period is not large enough. The second sample period ends in 2012:Q4, as is the case with the U.S.

# 4 Empirical Results for the United States

This section presents empirical results on U.S. inflation dynamics that can be seen to support our specification of the GNKPC.

## 4.1 Model selection for U.S. inflation dynamics

This section begins by comparing the empirical performance of the GNKPC and the NKPC. As noted in the preceding section, performance is examined with QML, which is computed using the two alternative modified harmonic mean estimators proposed by Geweke (1999) and by Sims, Waggoner, and Zha (2008).

Table 3: Log quasi-marginal likelihood (QML) of the GNKPC and the NKPC: United States

| | | Baseline | | $\omega = 0$ | |
|---|---|---|---|---|---|
| Period | Estimator of QML | GNKPC | NKPC | GNKPC | NKPC |
| Great Inflation | Geweke, $\tau = 0.5$ | $-9.17$ | $-10.95$ | $-15.70$ | $-13.05$ |
| (1966:Q1–1982:Q3) | Geweke, $\tau = 0.9$ | $-9.16$ | $-10.95$ | $-15.69$ | $-13.04$ |
| | SWZ, $q = 0.5$ | $-7.10$ | $-8.34$ | $-12.30$ | $-10.21$ |
| | SWZ, $q = 0.9$ | $-7.49$ | $-9.31$ | $-13.65$ | $-11.41$ |
| Post-Great Inflation | Geweke, $\tau = 0.5$ | $-18.11$ | $-20.11$ | $-20.81$ | $-22.44$ |
| (1982:Q4–2012:Q4) | Geweke, $\tau = 0.9$ | $-18.10$ | $-20.10$ | $-20.81$ | $-22.44$ |
| | SWZ, $q = 0.5$ | $-15.13$ | $-17.17$ | $-17.65$ | $-19.43$ |
| | SWZ, $q = 0.9$ | $-16.42$ | $-18.43$ | $-18.95$ | $-20.64$ |
| Full sample | Geweke, $\tau = 0.5$ | $-15.56$ | $-17.32$ | $-29.16$ | $-21.76$ |
| (1966:Q1–2012:Q4) | Geweke, $\tau = 0.9$ | $-15.56$ | $-17.31$ | $-29.11$ | $-21.70$ |
| | SWZ, $q = 0.5$ | $-12.56$ | $-14.59$ | $-23.55$ | $-19.04$ |
| | SWZ, $q = 0.9$ | $-13.83$ | $-15.82$ | $-25.95$ | $-20.13$ |

Note: In the second column, "Geweke" represents the Geweke (1999) modified harmonic mean estimator, while "SWZ" represents the Sims-Waggoner-Zha (2008) estimator.

Table 3 reports log QML for the GNKPC and the NKPC, with truncation parameter values of $\tau = 0.5, 0.9$ for the Geweke estimator, and $q = 0.5, 0.9$ for the Sims-Waggoner-Zha estimator. For both estimators with both parameter values, the third and fourth columns of the table show that the GNKPC has a higher QML than the NKPC during each of the three estimation periods. This result clearly indicates that the GNKPC provides a better specification for U.S. inflation dynamics than the NKPC, and that the GNKPC's improved

fit to the U.S. macroeconomic data can be attributed to retaining some unchanged prices in each quarter in line with the micro evidence. As noted in Introduction, recent studies have shown that GNKPCs are significantly distinct from canonical NKPCs in terms of their inflation dynamics, macroeconomic stability, and welfare and policy implications. Therefore, our finding suggests that GNKPCs should be preferred to canonical NKPCs for the analysis of the U.S. economy.

Table 3 also reports log QML in the cases where the GNKPC and the NKPC have no backward-looking price setters (i.e., $\omega = 0$). For both estimators with both truncation parameter values, the third to sixth columns of the table show that both the GNKPC and the NKPC have a higher QML during all the three estimation periods in the presence of backward-looking price setters than in their absence. This result indicates that backward-looking price setters played a non-negligible role in accounting for U.S. inflation dynamics during and after the Great Inflation. Thus, to better explain U.S. inflation dynamics, such a backward-looking component needs to be included in the GNKPC along with some unchanged prices in each quarter.[29]

Our empirical result on the non-negligible role of backward-looking price setters in the GNKPC may be considered as contrasting with Cogley and Sbordone (2008) (henceforth CS). Under the assumption of subjective expectations based on the anticipated utility model of Kreps (1998), CS incorporate drifting trend inflation in a GNKPC with price indexation only to past inflation. This GNKPC is crucially different from ours in that all prices change every period as long as there is the price indexation. CS follow Sbordone (2002) and Cogley and Sargent (2005) to conduct limited-information estimation of their GNKPC for the period

[29] Hirose, Kurozumi, and Van Zandweghe (2017) use a full-information Bayesian approach to compare a GNKPC and an NKPC in a DSGE model during the Great Inflation (1966:Q1–1979:Q2) and thereafter (1982:Q4–2008:Q4). Their model selection using marginal likelihood shows that the model with the GNKPC empirically outperforms that with the NKPC, in line with our result. Yet it suggests no role for backward-looking price setters in the model with the GNKPC, which contrasts somewhat with our result. This difference can be ascribed to the fact that our model selection is conducted only for the specification of inflation dynamics, while theirs is done for the full system of the model. In their model, habit formation in consumption preferences gives rise to output persistence and thus inflation persistence through the GNKPC; therefore, it enables their model to simultaneously explain both U.S. output and inflation dynamics without the need of backward-looking price setters, which generate only inflation persistence.

1960:Q1–2003:Q4. CS's estimate indicates no role of the price indexation to past inflation and thus they conclude that there is no need for any backward-looking component in their GNKPC. One of the drivers of this conclusion is the drifting trend inflation, which makes the gap between actual and trend inflation less persistent.

To better grasp the difference between CS's and our results, this paper estimates a GNKPC that is almost the same as theirs except that trend inflation is constant, for their sample period 1960:Q1–2003:Q4, as well as for the two subsample periods before 1982:Q3 and after 1982:Q4. Specifically, instead of (13), assume that $\lambda_{cs}\pi^{\theta(1-\omega_{cs})} < \min(1, \pi^{1-\omega_{cs}})$, where $\lambda_{cs} \in (0,1)$ denotes the probability of past inflation-indexed price setting and $\omega_{cs} \in [0,1]$ represents the degree of price indexation to past inflation. The GNKPC can then be derived as

$$\begin{aligned}\hat{\pi}_t &= \gamma_{b,cs}\hat{\pi}_{t-1} + \gamma_{f,cs}E_t\hat{\pi}_{t+1} + \kappa_{cs}\hat{ulc}_t \\ &\quad + \kappa_{f,cs}\sum_{j=1}^{\infty}(\beta\lambda_{cs}\tilde{\pi})^j\left[E_t\hat{gy}_{t+j} + (\theta-1)(E_t\hat{\pi}_{t+j} - \omega_{cs}E_t\hat{\pi}_{t+j-1}) - (E_t\hat{r}_{t+j-1} - E_t\hat{\pi}_{t+j})\right],\end{aligned} \tag{19}$$

where $\gamma_{b,cs} \equiv \omega_{cs}/\phi_{cs}$, $\gamma_{f,cs} \equiv \beta\pi^{1-\omega_{cs}}/\phi_{cs}$, $\kappa_{cs} \equiv (1-\lambda_{cs}\tilde{\pi})(1-\beta\lambda_{cs}\pi^{\theta(1-\omega_{cs})})/(\lambda_{cs}\tilde{\pi}\phi_{cs})$, $\kappa_{f,cs} \equiv (\pi^{1-\omega_{cs}}-1)(1-\lambda_{cs}\tilde{\pi})/(\lambda_{cs}\tilde{\pi}\phi_{cs})$, $\tilde{\pi} \equiv \pi^{(\theta-1)(1-\omega_{cs})}$, and $\phi_{cs} \equiv 1 + \omega_{cs}\beta\pi^{1-\omega_{cs}}$. Note that this GNKPC takes no account of the relative price distortion $\hat{d}_t$ as in CS.[30]

In estimating the CS-GNKPC (19), $\boldsymbol{\theta}$ and $u_t$ in the moment functions (18) are given by $\boldsymbol{\theta} = [\pi\ \lambda_{cs}\ \omega_{cs}\ \theta]'$ and

$$\begin{aligned}u_t &= (\log\pi_t - \log\pi) - \frac{\gamma_{b,cs}}{1+\tilde{\omega}_{cs}\kappa_{f,cs}}(\log\pi_{t-1} - \log\pi) - \frac{\gamma_{f,cs}}{1+\tilde{\omega}_{cs}\kappa_{f,cs}}(\log\pi_{t+1} - \log\pi) \\ &\quad - \frac{\kappa_{cs}}{1+\tilde{\omega}_{cs}\kappa_{f,cs}}\hat{ulc}_t - \frac{\kappa_{f,cs}}{1+\tilde{\omega}_{cs}\kappa_{f,cs}}\sum_{j=1}^{\infty}(\beta\lambda_{cs}\tilde{\pi})^j\left[\hat{gy}_{t+j} + (\theta-\tilde{\omega}_{cs})(\log\pi_{t+j} - \log\pi) - \hat{r}_{t+j-1}\right],\end{aligned}$$

where $\tilde{\omega}_{cs} \equiv \omega_{cs}\beta\lambda_{cs}\tilde{\pi}(\theta-1)$. The prior distributions for $\lambda_{cs}$ and $\omega_{cs}$ are set to be the same as those for $\lambda$ and $\omega$ that are presented in Table 2.[31]

[30] We confirmed that the results presented in this paper are qualitatively robust even when the relative price distortion $\hat{d}_t$ is present in the GNKPC (19) as theory suggests.

[31] As in the estimation of our GNKPC (16), this paper also uses 16 leads of inflation, output growth, and the interest rate in estimating the CS-GNKPC (19).

Table 4: Log quasi-marginal likelihood (QML) of GNKPCs: United States

| Period | Estimator of QML | CS-GNKPC | $\omega_{cs} = 0$ | GNKPC |
|---|---|---|---|---|
| Great Inflation | Geweke, $\tau = 0.5$ | $-17.39$ | $-22.83$ | $-12.49$ |
| (1960:Q1–1982:Q3) | Geweke, $\tau = 0.9$ | $-17.39$ | $-22.83$ | $-12.48$ |
| | SWZ, $q = 0.5$ | $-14.72$ | $-19.60$ | $-9.49$ |
| | SWZ, $q = 0.9$ | $-15.99$ | $-20.92$ | $-10.68$ |
| Post-Great Inflation | Geweke, $\tau = 0.5$ | $-17.64$ | $-16.68$ | $-14.12$ |
| (1982:Q4–2003:Q4) | Geweke, $\tau = 0.9$ | $-17.64$ | $-16.67$ | $-14.11$ |
| | SWZ, $q = 0.5$ | $-14.29$ | $-13.21$ | $-11.06$ |
| | SWZ, $q = 0.9$ | $-15.56$ | $-14.54$ | $-12.30$ |
| Full sample | Geweke, $\tau = 0.5$ | $-24.32$ | $-30.26$ | $-14.37$ |
| (1960:Q1–2003:Q4) | Geweke, $\tau = 0.9$ | $-24.33$ | $-30.26$ | $-14.38$ |
| | SWZ, $q = 0.5$ | $-21.15$ | $-27.41$ | $-11.02$ |
| | SWZ, $q = 0.9$ | $-22.46$ | $-28.68$ | $-12.28$ |

Notes: In the second column, "Geweke" represents the Geweke (1999) modified harmonic mean estimator, while "SWZ" represents the Sims-Waggoner-Zha (2008) estimator. The third to fifth columns report log QML for the CS-GNKPC (19), the CS-GNKPC with $\omega_{cs} = 0$, and our GNKPC (16), respectively.

Table 4 reports log QML for the CS-GNKPC (19), the CS-GNKPC with $\omega_{cs} = 0$, and our GNKPC (16).[32] The third and fourth columns of the table show that the CS-GNKPC (19) has a higher QML during the first subsample period 1960:Q1–1982:Q3 (and the full sample period) in the presence of price indexation to past inflation than in its absence (i.e., $\omega_{cs} = 0$), whereas it has a slightly lower QML during the second subsample period 1982:Q4–2003:Q4. This result indicates that the role of price indexation to past inflation may have become negligible in the CS-GNKPC after the Great Inflation even in the absence of the drifting trend inflation introduced by CS. The table also demonstrates that our GNKPC (16) has the largest QML among the three GNKPCs in all the three estimation periods, for the full sample and both subsamples. This result implies that, compared with the price indexation to past inflation used in CS, the backward-looking price setters employed in our paper provide a better specification of the backward-looking component in the GNKPC with constant trend inflation. Therefore, these results suggest that the conclusion of CS may depend on the specification of backward-looking components of their GNKPC in addition to the drifting

[32] Note that the estimation here sets the prior mean of the annualized trend inflation rate $\bar{\pi}$ at its average over the full sample period here (1960:Q1–2003:Q4), i.e., 3.68.

trend inflation.

## 4.2 Quasi-posterior estimates of the GNKPC selected for the U.S.

The preceding subsection has shown that, among those considered, the GNKPC (16) (with backward-looking price setters) is the best representation of U.S. inflation dynamics both during and after the Great Inflation. The present subsection thus analyzes this GNKPC in detail.

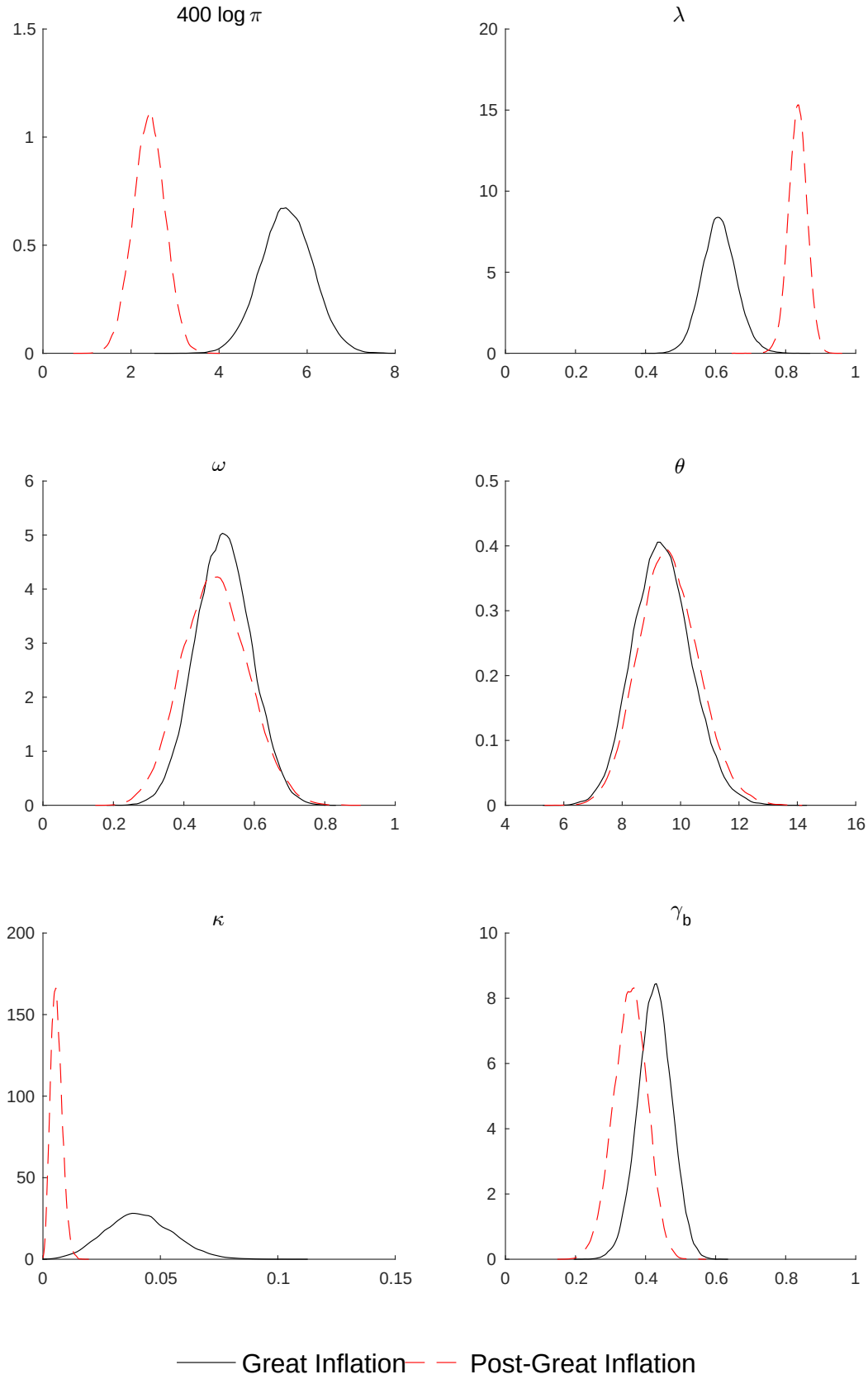
For each of the model parameters and reduced-form coefficients of the GNKPC, its quasi-posterior mean and 90 percent credible interval are reported in Table 5. The quasi-posterior mean estimates show that when the annualized trend inflation rate $\bar{\pi}$ ($\equiv 400 \log \pi$) fell from 5.54 percent during the Great Inflation to 2.41 percent during the Post-Great Inflation, the probability of no price change $\lambda$ increased from 0.61 to 0.83, while both the fraction of backward-looking price setters $\omega$ and the elasticity of substitution $\theta$ remained almost unchanged. The GNKPC's slope $\kappa$ diminished from 0.04 to 0.01, and its inflation-inertia coefficient $\gamma_b$ decreased from 0.43 to 0.36. These evolutions are also detected in the quasi-posterior distributions for the model parameters and reduced-form coefficients illustrated in Figure 1.

Table 5: Quasi-posterior estimates of the GNKPC selected for U.S. inflation dynamics

| Model parameter or reduced-form coefficient | Great Inflation (1966:Q1–1982:Q3) | | Post-Great Inflation (1982:Q4–2012:Q4) | | Full sample (1966:Q1–2012:Q4) | |
|---|---|---|---|---|---|---|
| | Mean | 90% interval | Mean | 90% interval | Mean | 90% interval |
| $\bar{\pi}$ | 5.537 | $[4.568, 6.504]$ | 2.411 | $[1.810, 3.005]$ | 4.700 | $[3.529, 5.870]$ |
| $\lambda$ | 0.611 | $[0.532, 0.693]$ | 0.834 | $[0.790, 0.877]$ | 0.760 | $[0.701, 0.817]$ |
| $\omega$ | 0.513 | $[0.384, 0.644]$ | 0.490 | $[0.341, 0.645]$ | 0.643 | $[0.505, 0.774]$ |
| $\theta$ | 9.380 | $[7.818, 11.066]$ | 9.597 | $[7.973, 11.347]$ | 9.487 | $[7.885, 11.210]$ |
| $\gamma_b$ | 0.426 | $[0.347, 0.504]$ | 0.356 | $[0.275, 0.432]$ | 0.432 | $[0.366, 0.493]$ |
| $\gamma_f$ | 0.575 | $[0.497, 0.654]$ | 0.643 | $[0.567, 0.723]$ | 0.568 | $[0.507, 0.635]$ |
| $\kappa$ | 0.041 | $[0.019, 0.065]$ | 0.006 | $[0.002, 0.010]$ | 0.006 | $[0.003, 0.010]$ |
| $\kappa_f$ | 0.002 | $[0.001, 0.003]$ | 0.000 | $[0.000, 0.000]$ | 0.000 | $[0.000, 0.001]$ |
| $\kappa_d$ | 0.120 | $[0.064, 0.194]$ | 0.193 | $[0.081, 0.357]$ | 0.158 | $[0.066, 0.287]$ |

Here, three points are worth mentioning. First, the increase in the probability of no price

Figure 1: Quasi-posterior distribution for the model parameters and reduced-form coefficients of the GNKPC selected for U.S. inflation dynamics



change in the estimated GNKPC is in line with the micro evidence reported by Nakamura *et al.* (2017) that the frequency of regular price changes declined after the Great Inflation. Besides, the concurrence of the increase in the probability of no price change and the fall in trend inflation in the GNKPC is consistent with the literature on endogenous price stickiness, such as Ball, Mankiw, and Romer (1988), Levin and Yun (2007), and Kurozumi (2016). Moreover, it is the increased probability of no price change that flattens the slope of the GNKPC. As previously indicated in the discussion of Table 1, theoretical factors behind a flattening of the GNKPC's slope are a higher probability of no price change, a larger fraction of backward-looking price setters, higher trend inflation, and a larger elasticity of substitution under positive trend inflation.

Second, the decrease in the inflation-inertia coefficient of the estimated GNKPC after the Great Inflation is in line with the empirical result of Cogley, Primiceri, and Sargent (2010) that the persistence of the gap between actual and trend inflation fell after the Volcker disinflation. The decrease in the inertia coefficient is also ascribed to the increase in the probability of no price change. As shown in Table 1, theoretical factors behind a lower inflation-inertia coefficient are a higher probability of no price change, a smaller fraction of backward-looking price setters, higher trend inflation, and a larger elasticity of substitution under positive trend inflation.

Third, the probability of no price change in the estimated GNKPC is consistent with the most recent micro evidence. The quasi-posterior distribution for the probability of no price change after the Great Inflation is centered around 80 percent, implying a duration of about 5 quarters (15 months). This duration is in line with the micro evidence provided by Kehoe and Midrigan (2015), who report that the implied duration of regular price changes is 14.5 months.[33]

[33] The duration of regular price changes reported by Kehoe and Midrigan (2015) is longer than that by Klenow and Kryvtsov (2008) and Nakamura and Steinsson (2008), who show a figure of 7 to 11 months. This difference is because the latter two studies identify temporary price increases as regular price changes, which unsurprisingly shortens the duration.

# 5 Empirical Results for Japan

Empirical results on Japan's inflation dynamics that offer further support for our GNKPC are provided in this section.

## 5.1 Model selection for Japan's inflation dynamics

This section also begins by comparing the empirical performance of the GNKPC and the NKPC. Table 6 reports log QML using the Geweke (1999) estimator with $\tau = 0.5, 0.9$ and the Sims-Waggoner-Zha (2008) estimator with $q = 0.5, 0.9$. For both estimators with both parameter values, the third and fourth columns of the table show that the GNKPC has a higher QML than the NKPC during each of the three estimation periods. As is the case with the U.S., this result indicates that the GNKPC is a better specification for Japan's inflation dynamics than the NKPC, and that the GNKPC's improved fit to the Japanese macroeconomic data can be achieved through retaining some unchanged prices in each quarter in line with the micro evidence. Therefore, our finding suggests the use of GNKPCs rather than canonical NKPCs in analysis of Japan's economy.

Table 6: Log quasi-marginal likelihood (QML) of the GNKPC and the NKPC: Japan

| | | Baseline | | $\omega = 0$ | |
|---|---|---|---|---|---|
| Period | Estimator of QML | GNKPC | NKPC | GNKPC | NKPC |
| Moderate Inflation | Geweke, $\tau = 0.5$ | $-15.32$ | $-17.20$ | $-12.26$ | $-13.66$ |
| (1981:Q4–1997:Q4) | Geweke, $\tau = 0.9$ | $-15.32$ | $-17.19$ | $-12.26$ | $-13.66$ |
| | SWZ, $q = 0.5$ | $-13.36$ | $-15.15$ | $-9.97$ | $-11.50$ |
| | SWZ, $q = 0.9$ | $-14.53$ | $-16.32$ | $-11.15$ | $-12.63$ |
| Deflation | Geweke, $\tau = 0.5$ | $-13.62$ | $-14.39$ | $-15.67$ | $-16.64$ |
| (1998:Q1–2012:Q4) | Geweke, $\tau = 0.9$ | $-13.63$ | $-14.40$ | $-15.67$ | $-16.65$ |
| | SWZ, $q = 0.5$ | $-12.40$ | $-13.09$ | $-14.19$ | $-15.16$ |
| | SWZ, $q = 0.9$ | $-13.32$ | $-14.03$ | $-15.25$ | $-16.19$ |
| Full sample | Geweke, $\tau = 0.5$ | $-15.74$ | $-17.11$ | $-14.07$ | $-15.48$ |
| (1981:Q4–2012:Q4) | Geweke, $\tau = 0.9$ | $-15.75$ | $-17.09$ | $-14.08$ | $-15.48$ |
| | SWZ, $q = 0.5$ | $-14.45$ | $-15.94$ | $-12.87$ | $-14.21$ |
| | SWZ, $q = 0.9$ | $-15.44$ | $-16.86$ | $-13.82$ | $-15.15$ |

Note: In the second column, "Geweke" represents the Geweke (1999) modified harmonic mean estimator, while "SWZ" represents the Sims-Waggoner-Zha (2008) estimator.

Table 6 also reports log QML in the cases where the GNKPC and the NKPC have no backward-looking price setters (i.e., $\omega = 0$). For both estimators with both truncation parameter values, the third to sixth columns of the table show that the GNKPC and the NKPC have a higher QML during the Moderate Inflation and the full sample period in the absence of backward-looking price setters than in their presence, whereas they have a lower QML during the Deflation. This result indicates that backward-looking price setters played no role in accounting for Japan's inflation dynamics during the Moderate Inflation, but that their role became important during the Deflation.

## 5.2 Quasi-posterior estimates of the GNKPC selected for Japan

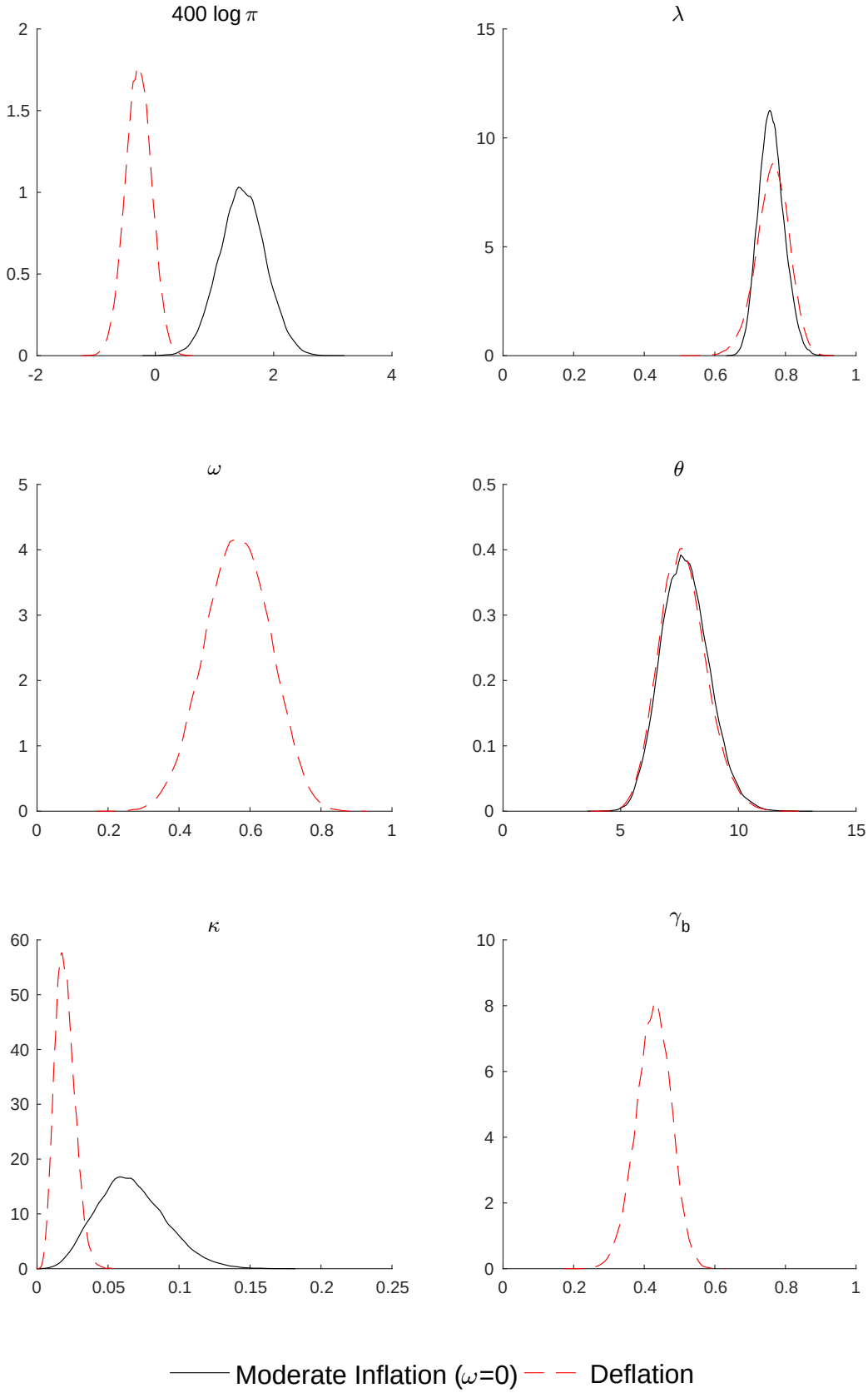
The preceding subsection has shown that the best representation of Japan's inflation dynamics among those considered is the GNKPC without backward-looking price setters during the Moderate Inflation and the GNKPC with them during the Deflation. This subsection thus investigates in detail the GNKPC selected in each of the two periods.

Table 7: Quasi-posterior estimates of the GNKPC selected for Japan's inflation dynamics

| Model parameter or reduced-form coefficient | Moderate Inflation (1981:Q4–1997:Q4) | | Deflation (1998:Q1–2012:Q4) | | Full sample (1981:Q4–2012:Q4) | |
|---|---|---|---|---|---|---|
| | Mean | 90% interval | Mean | 90% interval | Mean | 90% interval |
| $\bar{\pi}$ | 1.471 | $[0.836, 2.113]$ | $-0.285$ | $[-0.652, 0.082]$ | 0.458 | $[-0.400, 1.307]$ |
| $\lambda$ | 0.760 | $[0.704, 0.822]$ | 0.764 | $[0.684, 0.836]$ | 0.804 | $[0.760, 0.851]$ |
| $\omega$ | 0 | – | 0.565 | $[0.414, 0.713]$ | 0 | – |
| $\theta$ | 7.744 | $[6.150, 9.465]$ | 7.651 | $[6.091, 9.376]$ | 7.700 | $[6.105, 9.423]$ |
| $\gamma_b$ | 0 | – | 0.426 | $[0.342, 0.505]$ | 0 | – |
| $\gamma_f$ | 0.994 | $[0.992, 0.995]$ | 0.572 | $[0.494, 0.654]$ | 0.991 | $[0.989, 0.993]$ |
| $\kappa$ | 0.067 | $[0.030, 0.109]$ | 0.020 | $[0.009, 0.032]$ | 0.048 | $[0.024, 0.074]$ |
| $\kappa_f$ | 0.001 | $[0.001, 0.002]$ | $-0.000$ | $[-0.000, 0.000]$ | 0.000 | $[-0.000, 0.001]$ |
| $\kappa_d$ | 0.106 | $[0.049, 0.185]$ | $-0.008$ | $[-0.022, 0.002]$ | 0.041 | $[-0.030, 0.125]$ |

For each of the model parameters and reduced-form coefficients of the selected GNKPC, its quasi-posterior mean and 90 percent credible interval are reported in Table 7. The quasi-posterior mean estimates show that when the annualized trend inflation rate $\bar{\pi}$ ($\equiv 400 \log \pi$) declined from 1.47 percent during the Moderate Inflation to $-0.29$ percent during

Figure 2: Quasi-posterior distribution for the model parameters and reduced-form coefficients of the GNKPC selected for Japan's inflation dynamics



the Deflation, the fraction of backward-looking price setters $\omega$ increased from 0 to 0.57. Meanwhile, both the probability of no price change $\lambda$ and the elasticity of substitution $\theta$ remained almost constant. The increase in the fraction of backward-looking price setters causes the GNKPC's inflation-inertia coefficient $\gamma_b$ to increase from 0 to 0.43, along with the decline in trend inflation. It also causes the GNKPC's slope $\kappa$ to diminish from 0.07 to 0.02. These evolutions are also observed in the quasi-posterior distributions for the model parameters and reduced-form coefficients illustrated in Figure 2. Owing to the increase of the inflation-inertia coefficient and the flattening of the slope, severe deflation was not observed during the Deflation from 1998 through 2012.

The quasi-posterior distribution for the probability of no price change is centered around 75 percent, implying a duration of about 4 quarters (12 months). This duration is longer than the micro evidence provided by previous studies, for example, Kurachi, Hiraki, and Nishioka (2016). To reconcile model-implied duration with micro evidence, real rigidity may be needed in the GNKPC in addition to nominal rigidity; alternatively, studies with micro data may need to identify regular price changes by excluding temporary price changes along the lines suggested by Kehoe and Midrigan (2015).

# 6 Conclusion

This paper has investigated inflation dynamics in the U.S. and Japan by estimating a "generalized" version of the Galí and Gertler (1999) NKPC with Bayesian GMM. The GNKPC differs from the NKPC only in that, in line with the micro evidence, each period a fraction of prices remains unchanged even under non-zero trend inflation. Yet this difference causes the GNKPC to have features that are significantly distinct from those of the NKPC. Model selection using QML has shown that the GNKPC empirically outperforms the NKPC in both the U.S. and Japan. It has also demonstrated that the GNKPC explains U.S. inflation dynamics better than a constant-trend-inflation variant of the Cogley and Sbordone (2008) GNKPC. Our selected GNKPC has indicated that when trend inflation fell after the Great Inflation in the U.S., the probability of no price change rose—a finding that is consistent

with the micro evidence reported by Nakamura *et al.* (2017) as well as with the literature on endogenous price stickiness, including Ball, Mankiw, and Romer (1988). Consequently, the GNKPC's slope flattened, while its inflation-inertia coefficient decreased in line with the empirical result of Cogley, Primiceri, and Sargent (2010). As for Japan, when trend inflation turned slightly negative after the Moderate Inflation, the fraction of backward-looking price setters increased. This caused the inflation-inertia coefficient to increase and the slope to flatten in the GNKPC, thus obviating a severe deflation during the Deflation period of 1998–2012.

Our estimates of the probability of no price change and the fraction of backward-looking price setters (or equivalently, the inflation-inertia coefficient) in the GNKPC may be considered somewhat high. A possible approach to address this issue is to incorporate variable elasticity demand curves into our model. Kurozumi and Van Zandweghe (2016b) show that introducing such demand curves along the lines of Kimball (1995), Dotsey and King (2005), Levin *et al.* (2008), and Kurozumi and Van Zandweghe (2016a) generates not only real rigidity but also inflation persistence through strategic complementarity in price setting under non-zero trend inflation.[34] This extension of our paper would be one fruitful avenue for future research.

[34] The real rigidity generated by variable elasticity demand curves takes the place of the nominal rigidity represented by the probability of no price change when trend inflation declines. See also Shirota (2015).

# Appendix

This appendix presents the derivation of the GNKPC (14). Using $\beta_{t,t+1} \equiv Q_{t,t+1} Z_{t+1}/Z_t$, $y_t \equiv Y_t/Z_t$, and $gy \exp(\varepsilon_t) = Z_t/Z_{t-1}$ (from eq. (3)), the optimizing price-setting equation (9) and the stochastic discount factor equation (12) can be rewritten as

$$0 = E_t \sum_{j=0}^{\infty} (\lambda \pi^{\theta})^j \left[ \prod_{k=1}^{j} \beta_{t+k-1,t+k} \frac{y_{t+k}}{y_{t+k-1}} \left( \frac{\pi_{t+k}}{\pi} \right)^{\theta} \right] \left( \pi^{-j} p_t^o \prod_{k=1}^{j} \frac{\pi}{\pi_{t+k}} - \frac{\theta}{\theta - 1} mc_{t+j} \right),$$

$$1 = E_t \left( \frac{\beta_{t,t+1}}{gy \exp(\varepsilon_{t+1})} \frac{r_t}{\pi_{t+1}} \right).$$

Log-linearizing these equations as well as equations (4), (5), (7), (8), and (10) around the steady state with trend inflation $\pi$ under the assumption (13) and combining the resulting equations leads to the GNKPC (14).

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