Selling to Consumers with Intransitive Indifference*

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Abstract

We are indifferent between two cups of coffee when one differs from the other in having only one more grain of sugar. But such an indifference is not transitive, because eventually, after many enough grains of sugar are added, we will become able to tell one cup is sweeter than the other. When consumers feature intransitive indifference, putting a bad deal alongside a good deal can boost the sale of the latter by helping consumers to better appreciate it. When sellers compete for these consumers, they tend not to undercut each other, because undercutting often go un-appreciated. Instead, sellers segregate into providers of good deals and bad deals, with the formers free-riding the latters in helping consumers better appreciate their good deals, and the latters free-riding the formers in making consumers less hesitant to buy.

KEYWORDS: intransitive indifference, monopoly pricing, price dispersion, oligopoly

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1 Introduction

We are indifferent between two cups of coffee when one differs from the other in having only one more grain of sugar. But such an indifference is not transitive, because eventually, after many enough grains of sugar are added, we will become able to tell one cup is sweeter than the other. Ever since Luce (1956), economists have developed various utility representations of a decision maker featuring intransitive indifference,[1] but never explored how they may affect classical economic analyses. This paper makes a first step in filling this gap. Specifically, it studies sellers' equilibrium behavior when consumers feature intransitive indifference.

In exploring the implications of intransitive indifference, we are forced to make an explicit choice over two competing interpretations. When a coffee drinker expresses indifference between two cups of coffee, A and B, is he merely having too dull an instrument to distinguish two alternatives that he otherwise would have strict preferences over, or is he genuinely indifferent between the two? To paraphrase the question, suppose he is told that A actually contains less sugar than B does, would he be so health-conscious that he then strictly prefers A to B, or would he remain indifferent on the basis that difference too small to detect is also immaterial to health? Let's call the corresponding interpretations the *dull-instrument* and the *genuine-indifference* interpretations, respectively.

Some authors apparently favor the dull-instrument interpretation. For example, Luce (1956) envisions that a decision maker's expressed preferences may change when he has access to better instruments, and Fishburn (1970a) speaks of the possibility that a coffee drinker breaking his earlier indifference after further taste tests a few moments later.

Other authors, however, seem to favor the genuine-indifference interpretation. For example, when Jamison and Lau (1973) and Fishburn (1975) characterize possible choice functions that may arise from intransitive indifference, they implicitly have in mind a health-conscious coffee drinker who, when choosing among three cups of coffee—with B indistinguishably sweeter than A, C indistinguishably sweeter than B, but C distinguishably sweeter than A—may nevertheless choose B on the basis that it is dominated by none. He does so notwithstanding the fact that he should have inferred from these pairwise comparisons that B contains more sugar than A does, presumably because he is

[1] See, among others, Fishburn (1970a, 1970b) and Beja and Gilboa (1992).

genuinely indifferent.[2]

In this paper, we explicitly espouses the dull-instrument interpretation. This is not to say that we consider the genuine-indifference interpretation implausible. But we conjecture that little change in classical economic analyses arises from the latter kind of intransitive indifference. For example, suppose a coffee drinker buying a cup of Arabica coffee is genuinely indifferent if up to 5% of the Arabica coffee bean is replaced by the cheaper Robusta coffee bean, then the optimal strategy of a monopolist is to simply offer him at the same price a cup of coffee that is 95% Arabica and 5% Robusta.

We hence have in mind a husband shopping for a gift for his wife (or a wife for her husband) during a business trip. He cares about her utility, and hence is not genuinely indifferent between two scarfs that look almost the same to him. Not willing to ask his wife on the phone and inadvertently give away the surprise, he has to make a decision given the limited sense data he has, limited by his inability to discern small differences in quality. Traditional game-theoretic tools continue to be applicable in analyzing the strategic interaction between sellers and such a consumer, but ample new results arise from the specific information structure of this game.

Behind many of these new results is the phenomenon that the presence of an inferior product can help a consumer appreciate a better product. To see this, let's return to the example two paragraphs above, where there are three alternatives, with A indistinguishably better than B, B indistinguishably better than C, but A distinguishably better than C. Suppose B is the consumer's outside option, and the seller is trying to sell A to him. In the absence of C, the consumer is not able to appreciate the superiority of A. By letting him examine the inferior C, the seller can convince him that A indeed is the best. This phenomenon is the driving force behind, for example, the result that sellers of good deals coexist with sellers of bad deals, with the former free-riding the presence of the latter (see below). We shall review evidences of this phenomenon from the psychology and marketing literature in the next subsection.

In modelling a consumer featuring intransitive indifference, we follow the decision-theory literature and introduce the notion of *just noticeable difference* (jnd). For example,

[2]Kamada (2016), to our best knowledge, is the first to point out this implicit interpretation underlying the kind of choice functions studied by Jamison and Lau (1973) and Fishburn (1975). He then proceeds to characterize an alternative kind of choice functions (called *sophisticated* choice functions) that are more in line with the dull-instrument interpretation.

suppose a coffee drinker cannot distinguish the sweetness of any two cups of coffee that differ by fewer than 100 grains of sugar, then 100 grains of sugar is his jnd. Much of the decision-theory literature postulates a *deterministic* jnd, which is admittedly unrealistic. In this paper, we follow the discussion in Gilboa (2009) and postulate a probabilistic jnd instead—as we add more and more sugar to cup B, the probability that the coffee drinker can tell that B is sweeter than A increases gradually instead of jumping from 0 to 1. One way to model such a probabilistic jnd is to assume that how sharp the coffee drinker's taste buds are depends on the quality of his previous-night's sleep, which in turn contains much randomness beyond the grasp of the coffee drinker himself as well as the others. Equivalently, we can also think of a continuum of consumers, each having a different jnd. While the overall distribution of these jnd's is commonly known, the jnd of any specific consumer is not (and not even to that consumer himself).

Section 2 provides further details of our model. In Section 3, we first study the case of a monopolist marketing a single brand of some indivisible products to these consumers. Under some regularity condition that guarantees that enough consumers have non-trivial jnds, there exists a unique equilibrium, which is necessarily in mixed strategies—the monopolist randomizes between offering a good deal and a bad deal, and consumers randomize between buying and not when they cannot compare the offered deal with their outside option. Non-degenerate price distribution from a single seller thus can arise for a purpose different from screening. When consumers' jnd's increase, the monopolist's equilibrium profit decreases. Intuitively, consumers are aware of their inability to compare, and hence are justifiably suspicious. When more consumers are suspicious, it is more difficult for the monopolist to sell, which explains its lower profit.

In Section 4, we study the case when the monopolist can at negligible costs market a second brand. The common knowledge that it can do so changes the strategic interaction between it and its suspicious consumers, and as a result its equilibrium profit may be even lower.

However, we show in Section 5 that, if the monopolist can at negligible costs market a sufficiently large number of brands, almost all of which are not meant to make any sale, its equilibrium profit must be close to the full surplus. Intransitive indifference plays a crucial role in this result. Intuitively, when consumers have difficulties in appreciating a genuinely good deal due to dull instruments, putting some bad deals alongside with the good deal can help these consumers make the necessary comparison. The reason why many different bad deals are needed to approximate full surplus extraction is that consumers with different jnd's have to be helped by different bad deals. This result can potentially address a puzzle posted in the marketing literature, namely that sellers are observed to place inexplicably many different brands on the shelf, notwithstanding the well-known effect that too many choices can potentially bring consumers headaches.[3]

The most interesting case arises when there are two *ex ante* identical sellers, each marketing one brand to these consumers, which we study in Section 6. Compared with the earlier case with a monopolist marketing two different brands, in this case the same number of brands are marketed, but by different sellers, and hence arguably the market is more competitive. However, we show that competition does not always increase consumers' surplus (Proposition 9). The intuition is that, when consumers feature intransitive indifference, competing sellers tend not to undercut each other, because undercutting often goes un-appreciated by consumers with dull instruments. Instead, in equilibrium, one seller specializes in offering a good deal, while another in offering a bad deal. The former free-rides the latter because the latter helps some consumers to appreciate the good deal the former offers. The latter also free-rides the former because the existence of a good deal make consumers less hesitant to buy. In short, *free-riding*, instead of undercutting, is the keyword in understanding sellers' behavior when consumers feature intransitive indifference. With the opportunity to free-ride the bad-deal seller, the good-deal seller does not need to sweeten its deal too much to make sale, which limits the benefit of competition for the consumers.

Section 7 concludes.

1.1 Related Literature

Evidences that a bad deal can help consumers better appreciate a good deal can be found both in the mass media and in the psychology and marketing literature. In a case study reported on the *Wall Street Journal*,[4] when Willimans-Sonoma brought the

[3]Mochon (2013) articulates the puzzle as follows: "Indeed, one Best Buy store surveyed displayed 114 different TVs in their store. While preference heterogeneity likely accounts in part for this vast array of options, it is unlikely that consumers' preferences are so refined that they require 15 different models of 32-inch televisions, suggesting that something else may be at play here."

[4]"Paul Lee: The Williams-Sonoma Bread Maker: A Case Study", *Wall Street Journal*, 10 April 2013, https://blogs.wsj.com/accelerators/2013/04/10/paul-lee-the-williams-sonoma-bread-maker-a-case-study/

very first bread maker to the market at US$275 in the 1990s, sales was very bad. After the introduction of a slightly better model at twice the price, sale of the original model skyrocketed. In the psychology literature, Kim, Novemsky, and Dhar (2012) find that the probability of a sale increases when two types of gums are priced at 620 and 640 Korean wons, respectively, than when both are priced at 630 Korean wons. In the marketing literature, Mochon (2013) finds that a brand is more likely to be purchased when it is presented with a competitor's brand than when it is presented alone.

Our results that a monopolist would in equilibrium permit a non-degenerate distribution of quality-adjusted-price—either by randomizing in the one-brand case, or by setting different prices for different brands in the two-brands case—resemble that of Salop (1977). However, the mechanisms are different. In Salop (1977), consumers differ in their price elasticities, and the monopolist would like to charge those who have lower price elasticities a higher price. If consumers who have lower price elasticities also have higher search costs, then the monopolist can achieve this goal by permitting a non-degenerate price distribution, because consumers who search less will on average pay a higher price, effecting the sorting pattern the monopolist wants. In our model, consumers differ in their ability to discern nearby quality-adjusted-prices, but their ability is not correlated with any other characteristic. As such, a non-degenerate price distribution arises not as a screening device of the monopolist, but rather as an equilibrium phenomenon: had the monopolist always priced low, consumers will be too willing to purchase even when they cannot tell for sure it is a good deal, generating incentives for the monopolist to price high; but had the monopolist always priced high, consumers will be too unwilling to purchase, generating incentives for the monopolist to price low.

Our model is also similar to Rubinstein (1993) in that consumers differ neither in their preferences nor in the information they possess, but rather in their ability to process information. Like Salop (1977) and our model, Rubinstein's (1993) monopolist also permits a non-degenerate price distribution. However, in terms of the underlying mechanism, Rubinstein (1993) is closer to Salop (1977) than to us. In particular, in Rubinstein (1993), consumers differ in their ability to process information, with less able consumers also being more costly to serve, and hence a non-degenerate price distribution serves as the monopolist's screening device to preclude these consumers. In our model, no correlation of this kind is assumed, and hence the equilibrium non-degenerate price distribution has nothing to do with screening.[5]

Our result that a non-degenerate price distribution arises in equilibrium in the multi-sellers case resembles that of Salop and Stiglitz (1977), Varian (1980), and Rob (1985). However, our findings that competition (between two identical sellers) does not drive down average price (compared to the case with a monopolist marketing up two different brands) and consequently does not necessarily benefit consumers (Proposition 9) does not have a natural counterpart in these previous studies.

Our paper also contributes to an emerging literature on consumers who have difficulties in comparing prices. One popular way to model such difficulties is called categorization, where consumers partition prices into a few categories, and react to different prices within the same category in the same way (see, e.g., Chen, Iyer, and Pazgal (2010) and Gul, Pesendorfer, and Strzalecki (2017)). Since a partition structure dictates that consumers' indifference must be transitive, some of the phenomena that arise only in an environment with intransitive indifference (such the good-deal seller free-riding the bad-deal seller) hence cannot arise in this literature.

Another approach to model consumers' difficulties in comparing prices is to postulate that they may simply ignore a competing seller, either because of inattention (de Clippel, Eliaz, and Rozen (2014)), or because the competing seller frames its price in a format that these consumers find too unfamiliar (Piccione and Spiegler (2012)). A recurring theme of this literature is that, when multiple sellers compete, consumers' surplus is decreasing in their ability to compare prices. This common finding resonates with some of our results (see Proposition 10). However, since the settings in these studies are so different from each other, it is not easy to tell whether some common force is at work. We leave this question for future research.

Finally, our paper is also related to Natenzon (forthcoming), who studies an environment where the choice between two options may be affected by the presence of a third one. We share the premise that consumers only obtain imperfect information about their true utilities from different options. We differ however in both our focus and the driving force. Our paper focuses on sellers' strategic choices of their products' true utilities, while

[5]See also Piccione and Rubinstein (2003), where consumers differ *both* in their preferences and in their ability to process information. Again, when these two characteristics are correlated in certain way, the monopolist can use a non-degenerate price distribution (or more precisely a deterministic price sequence that looks random to some consumers) as a screening device to separate consumers with different willingness to pay.

Natenzon (forthcoming) assumes an exogenously fixed distribution of these true utilities. In Natenzon (forthcoming), some pairs of options are inherently easier to compare, in the sense that consumers obtain more precise information about the ranking with these pairs than within other pairs. Such asymmetry is absent in our paper, where every pair is *a priori* similar, and contextual inference comes solely from true utility gap.

2 The Model

There is a unit mass of consumers,[6] each demands one and only one unit of an indivisible good, provided by a single seller called the monopolist. Consumers have identical reservation price, R. The cost of production per unit is c. Conditional on trade at price p, a consumer's payoff is $R - p$, and the monopolist's payoff is $p - c$. Their payoffs are 0 if there is no trade. We assume $R > c$, and hence gain of trade is certain. Consumers are heterogeneous only in their ability to discern two (quality-adjusted) prices.

We shall carefully distinguish two different concepts: a consumer's ability to *discern* two prices, and his ability to *compare* them. A consumer's ability to *discern* two prices depends on how sharp his receptors are. An analogy is that, when we are given two cups of coffee, whether we can tell which one is sweeter depends on how sharp our taste buds are. It may seem to the reader that discerning two numbers is very different in nature from discerning two cups of coffee. After all, any one who can count should be able to tell which of two numbers is higher. However, "prices" in our model are always intended to be shorthands for "quality-adjusted prices". Since discerning two quality-adjusted prices inevitably involves discerning the two corresponding products by their quality, it can be quite similar in nature to discerning two cups of coffee.[7]

Specifically, each consumer is of a different type $d \in [0, \infty)$. When a type-d consumer is given two prices, p_1 and p_2, he is able to discern them if and only if $|p_1 - p_2| > d$; i.e., if and only if the two prices are far enough apart. The type d is hence an inverse measure of

[6]As explained in the Introduction, an equivalent model is that there is a single consumer, with his jnd determined by the quality of his previous-night's sleep, which in turn contains much randomness beyond his grasp.

[7]It will be interesting to explicitly model a seller as choosing both quality and price—with the problem of intransitive indifference arising only in the quality dimension—instead of collapsing them into a one-dimensional variable called quality-adjusted price (as we do in this first-pass exercise). Such a model will be more realistic, but also more complicated due to the higher dimensionality. We leave the exploration of this alternative model for future research.

the consumer's ability to discern two prices. If the consumer can discern the two prices, we write $p_1 \widehat{\succ} p_2$ (respectively, $p_2 \widehat{\succ} p_1$) if he feels that p_1 is higher (respectively, lower) than p_2. If he cannot discern the two prices, we write $p_1 \widehat{\sim} p_2$ (i.e., $p_1 \widehat{\sim} p_2$ if and only if $p_1 \widehat{\nsucc} p_2$ and $p_2 \widehat{\nsucc} p_1$).

A consumer who cannot discern two prices (possibly because of poor receptors) may nevertheless still be able to *compare* them, especially if he receives certain aid. An especially interesting aid he may receive is the existence of a third price, p_3. For example, if he cannot discern p_1 and p_2 (i.e., $p_1 \widehat{\sim} p_2$) and cannot discern p_2 and p_3 (i.e., $p_2 \widehat{\sim} p_3$), but nevertheless feels that p_1 is higher than p_3 (i.e., $p_1 \widehat{\succ} p_3$), then by some simple logical deduction he should be able to infer that p_1 is actually higher than p_2. We shall assume that consumers are always able to make such kind of inference. In other words, while a consumer may have poor receptors, his rationality is undamaged.

Formally, we follow Kamada (2016) and construct *inferred ordering*, $\succ$, from the more primitive $\widehat{\succ}$ as follows: $p_1 \succ p_2$ if and only if at least one of the following holds:[8]

1. $p_1 \widehat{\succ} p_2$;

2. $\exists p_3$ such that $p_1 \widehat{\succ} p_3$ but $p_2 \widehat{\nsucc} p_3$;

3. $\exists p_3$ such that $p_3 \widehat{\succ} p_2$ but $p_3 \widehat{\nsucc} p_1$.

We write $p_1 \sim p_2$ if and only if $p_1 \nsucc p_2$ and $p_2 \nsucc p_1$.

An implicit assumption here is that p_3, which helps the consumer compare p_1 and p_2, has to come with an actual (as in contrast to fictitious) alternative. Without such an implicit assumption, our model would readily collapse into a traditional one. This is because, for example, a consumer with $d = 5$, though not being able to discern $p_1 = 10$ and $p_2 = 7$, would (had the implicit assumption relaxed) be able to compare them if he manages to imagine a fictitious product with price $p_3 = 4$.[9] One can think of a consumer's ability to discern two prices as already including his limited ability to imagine fictitious alternatives.

[8] Kamada (2016) shows that further iterations of this logic would not result in new inferences. More precisely, if we define $\succ^*$ using the same method (with $\succ$ in place of $\widehat{\succ}$), the new binary relation $\succ^*$ will remain the same as $\succ$.

[9] With $d = 5$, the consumer's primitive sense data are $p_1 \widehat{\nsucc} p_2$ and $p_2 \widehat{\nsucc} p_3$ but $p_1 \widehat{\succ} p_3$. From these primitive sense data the consumer can derive $p_1 \succ p_2$, inferring that p_1 is higher than p_2.

While the information a consumer possesses at the time of purchase is ordinal, his payoff remains cardinal. For example, he may find two different brands on the shelf, with prices p_1 and p_2, which he cannot discern (i.e., $p_1 \widehat{\sim} p_2$), while he feels that both are lower than his reservation price R (i.e., $R \widehat{\succ} p_1$ and $R \widehat{\succ} p_2$). These are the only primitive sense data he possesses at the time of purchase. However, his payoff of purchasing the first brand will still be $R - p_1$, as in any traditional model. In other words, the consumer's receptors are poor only at the time of purchase, but not when he actually consumes the good at home. It will be interesting to study consumers who feature intransitive indifference even at the time of actual consumption. But that will be a different model, and we shall leave that for future research.

We assume that both consumers and sellers are risk neutral. Therefore, a consumer makes his purchase decision based on expected prices, where expectation is taken conditional on the primitive sense data he possesses, and his knowledge of the sellers' pricing strategies. Similarly, a seller maximizes expected profit, where the profit from selling one unit of the good at the price p is $p - c$.

We assume that *consumers do not know their own types* (and hence, in particular, a consumer's purchasing strategy is independent of his type). It is commonly known that the distribution, F, of types among consumers has support $\mathbb{R}_+$,[10] and admits a density function, f, which in turn satisfy the following assumption:

Assumption 1 *The density function f is weakly decreasing and satisfies the monotone hazard rate property; i.e., $f/(1-F)$ is weakly increasing.*

Examples of a density function satisfying Assumption 1 include that of an exponential distribution.[11]

If F is a point mass at 0, then our model collapses to a traditional one, where in equilibrium a monopolist would set price $p = R$, and the consumer would purchase for sure. Gain of trade will be realized for sure, and the monopolist captures all the surplus. We shall refer to this outcome as the *first best* for short.

[10] Throughout this paper, we assume that F has full support on $\mathbb{R}_+$. This assumption is not necessary for any of our results. We can easily handle distributions with finite supports of the form $[0, D]$, where $D < \infty$, at the expenses of slightly messier proofs.

[11] As explained in Footnote 10, we can easily handle distributions with finite support at the expenses of messier proofs. Examples of a finite-support density function satisfying Assumption 1 include that of a uniform distribution.

It turns out that the first best remains an equilibrium outcome when the probability mass nearby 0 is sufficiently close to 1. The intuition is that consumers with d close to 0 impose a discipline on the monopolist, discouraging it from raising its price beyond R and exploiting consumers with larger d. In order for our model to generate results that are qualitatively different from a traditional one, we need F to be sufficiently different from a point mass at 0. The dividing line turns out to be the following condition.[12]

Assumption 2 *The density at $d = 0$ is sufficiently small; specifically, $f(0)(R - c) < 1$.*

Throughout this paper, our solution concept is the standard perfect Bayesian equilibrium, which we shall simply refer to as the *equilibrium.*

3 A Monopolist Marketing A Single Brand

In this section, we start with the simplest possible case where there is a monopolist who markets only a single brand. We can think of the costs of marketing a second brand as being prohibitively high, an assumption that we shall relax in the next two sections. The monopolist's strategy is a distribution of the (quality-adjusted) price p. A consumer's strategy is his probability of purchasing the good conditional on his primitive sense data (recall that he does not know his own type and hence cannot contingent his purchase probability on it). Since he is faced with only two prices (p, and his reservation price, R) at the time of purchase, $\succ$ is the same as $\widehat{\succ}$. Utility maximization dictates that the consumer purchases with probability 1 (respectively, with probability 0) when his primitive sense data is $R \widehat{\succ} p$ (respectively, $p \widehat{\succ} R$). Therefore, the consumer's strategy can be reduced to his purchase probability when his his primitive sense data is $p \widehat{\sim} R$, which we shall denote by q.

Given the consumer's strategy q, the monopolist's profit as a function of its (pure-

[12]While we shall provide more intuition for the meaning of Assumption 2 in Section 3, let's promptly point out one particular implication of this assumption. For any finite-support F (see Footnote 10) that satisfies Assumptions 1 and 2, the upper limit of its support, D, is larger than $R - c$. This is because, since f is weakly decreasing, we have $F(R - c) \leq f(0)(R - c) < 1$, and hence $D > R - c$. An implication of this observation is that, even when a product is priced as low as its marginal production cost, c, some consumers will still not be able to discern its price and his reservation price, R.

strategy) price is

$$\pi(p;q) = \begin{cases} (p-c)[1-F(p-R)]q & \text{if } p \geq R \\ (p-c)\left(F(R-p)+[1-F(R-p)]q\right) & \text{if } p \leq R \end{cases},$$

where, in the case of $p \leq R$ for example, $F(R-p)$ is the mass of consumers who can discern p and R and hence would purchase with probability 1, and $1-F(R-p)$ is the mass of those who cannot and hence would purchase with probability q.

Since the profit function has a kink at $p = R$, we maximize it over the upper sub-ranges $[R, \infty)$ and the lower sub-range $[c, R]$ separately, and then compare the maximized profit over each sub-range. In the proof of Proposition 1 below, we show that a unique maximizer exists in each sub-range, which we denote by $\overline{p}$ and $\underline{p}(q)$, respectively. Note that the maximizer in the upper sub-range does not depend on q, which can be readily verified by inspection of the profit function in that sub-range. The first best will be an equilibrium outcome iff $\overline{p} = R = \underline{p}(1)$. In the Appendix, we show that this is indeed the case if Assumption 2 is violated. In this sense a distribution F that violates Assumption 2 is not different enough from a point mass at 0.

To understand why Assumption 2 guarantees that the first best cannot be an equilibrium outcome, it suffices to understand why $\overline{p} > R$ under this assumption. Suppose consumers purchases for sure even when they cannot discern p and R; i.e., suppose $q = 1$. Suppose the monopolist raises its price from $p = R$ to $p = R + \epsilon$. For ϵ small, almost no consumer can detect the raise, and hence almost everyone pays ϵ more to the monopolist. There are approximately $\epsilon f(0)$ consumers whose receptors are very sharp (i.e., with $d < \epsilon$), who will be able to detect the raise and hence refuse to purchase. The lost profit from this small group of consumers amounts to $\epsilon f(0)(R-c)$. Under Assumption 2, the lost profit is smaller than the grain from exploiting the rest of the consumers (i.e., $\epsilon f(0)(R-c) < \epsilon$), and hence the monopolist cannot resist the temptation of secretly raising its price beyond R.

Indeed, for q sufficiently close to 1, consumers with poor receptors are so trusting and so willing to purchase that it is better for the monopolist to choose the high price $\overline{p}$ to exploit these consumers. On the contrary, when q is sufficiently close to 0, consumers with poor receptors are so untrusting and so unwilling to purchase that the only way to do business with them is to choose the low price $\underline{p}(q)$ in the hope of convincing them that the deal is good. Neither case can be an equilibrium, because consumers' best response against $\overline{p}$ is $q = 0$ and that against $\underline{p}(q)$ is $q = 1$. In equilibrium, q must take some intermediate value q^* so that the monopolist is willing to randomize between $\overline{p}$ and $\underline{p}(q^*)$, and the monopolist must randomize in a way that makes the consumers willing to randomize between purchasing or not.

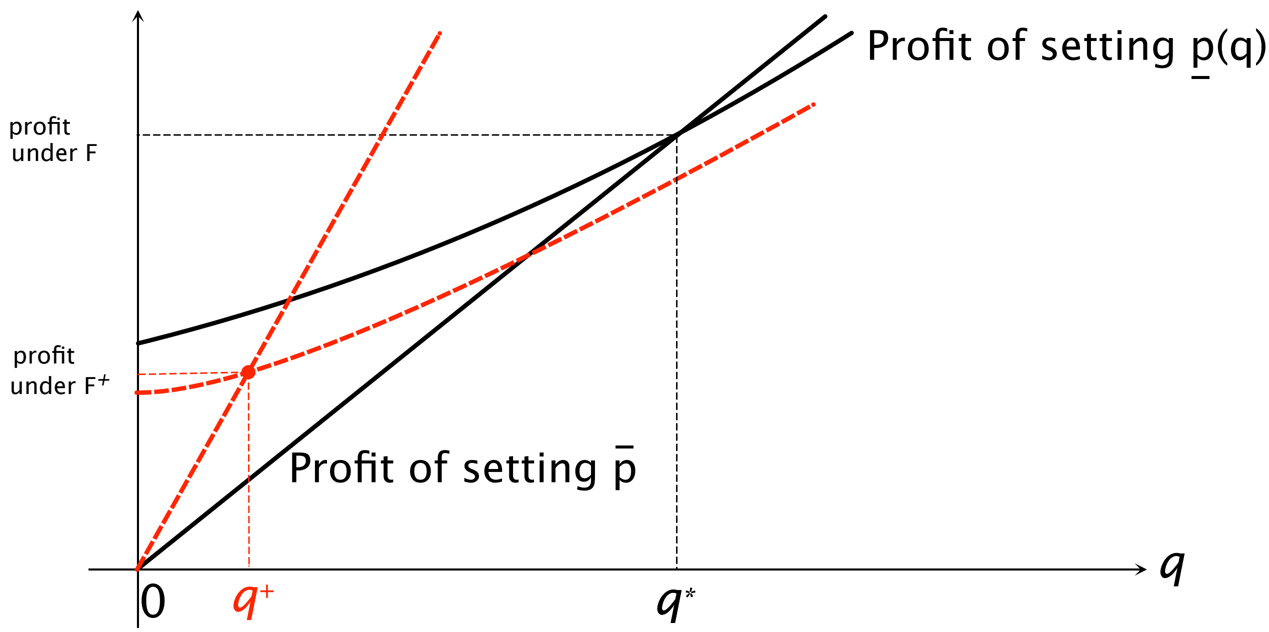

Figure 1: the monopolist's profits in the one-brand case

Figure 1 illustrates how the equilibrium is uniquely determined. In Figure 1, the solid line passing through the origin represents the monopolist's profit if it sets the high price $\overline{p}$, which in turn increases linearly in q. The solid curve represents the monopolist's profit if it sets the low price $\underline{p}(q)$, which is convex in q because the monopolist re-optimizes its price when it faces a different q. The convex curve is strictly above the linear line at $q = 0$, and is strictly below at $q = 1$. The shapes of the two profit functions dictate that they cross once and only once at q^*, at which point the monopolist is willing to randomize between the high and the low prices.

Proposition 1 *In the case of a monopolist marketing up to only one brand, the unique equilibrium is a mixed-strategy equilibrium, where*

- *consumers purchase with a probability q^* that is strictly between 0 and 1/2 when they cannot discern the price of the product and their reservation price R, and*

- *the monopolist randomizes between a high price $\overline{p}^*$ that is strictly above the consumers' reservation price R, and a low price $\underline{p}^*$ that is strictly between R and the marginal production cost c.*

In equilibrium, gain of trade is not always realised. Some consumers (those with $d \geq \overline{p}^* - R$) sometimes (when the monopolist sets the high price $\overline{p}^*$) obtain strictly negative surplus. But on average consumers obtain strictly positive surplus, meaning that the monopolist does not extract the full surplus even conditional on trade.

From Proposition 1, we can also obtain a very rough estimate of how much gain of trade is lost in equilibrium due to intransitive indifference. Notice that every time the monopolist sets a high price, the consumer either can tell that it is a bad deal (in which case there will be no sale), or cannot tell (in which case he purchases with probability $q^* < 1/2$). Therefore, conditional on a high price, gain of trade is realised with probability at most $1/2$. Suppose the monopolist randomized between the high and low prices with roughly equal probabilities. Then at least about $1/4$ of gain of trade is lost in equilibrium due to intransitive indifference.

It will be interesting to see how the monopolist's equilibrium profit changes when consumers become less able to discern two prices. To answer that question, we compare the monopolist's equilibrium profit under two different distributions of consumers' types, F and $F^{\dagger}$, where both satisfy Assumptions 1 and 2, but $F^{\dagger}$ first-order stochastically dominates (FOSD) F. In a market featuring $F^{\dagger}$, consumers have higher types and hence are less able to discern two prices. Our next proposition says that consumers' inability to discern prices actually hurts the monopolist.

Proposition 2 *In the case of a monopolist marketing up to only one brand, the monopolist's equilibrium profit decreases with an FOSD shift in F.*

Figure 1 provides a pictorial proof of Proposition 2. When the distribution of consumers' types undergoes an FOSD shift from F to $F^{\dagger}$, consumers are less able to discern prices. This raises the monopolist's profit from setting a high price, because consumers are less able to tell a bad deal. This results in an anti-clockwise tilt of the linear line. On the contrary, the monopolist's profit from setting a low price is now lower, because consumers are also less able to tell a good deal. This results in a downward shift of the convex curve.

In the new equilibrium, consumers are less trusting ($q^\dagger < q^*$), and the monopolist's profit is lower.

This may seems a bit surprising. After all, consumers who are less able to discern two prices seem vulnerable to exploitation, and hence should be welcomed by the monopolist. Such an intuition is incomplete, however. Recall that a consumer with difficulty in discerning two prices is a person equipped with some poor receptors. Although his instruments are poor, he is by no means irrational. He is aware that his instruments poor, and is rationally untrusting when he finds a product's (quality-adjusted) price indiscernable from his reservation price. A monopolist fares worse when more of its consumers are untrusting, because they cannot be easily convinced even when it is indeed offering them a good deal.

While consumers' inability to discern prices hurts the monopolist, it does not always benefit the consumers. Indeed, it is easy to see that the effect of an FOSD shift in F on consumer surplus is necessarily non-monotonic. Consider again the distributions F and $F^\dagger$, where the latter dominates the former in FOSD sense. In particular, this implies $F(R-c) \geq F^\dagger(R-c)$. At the limit when $F^\dagger(R-c) \searrow 0$, consumers almost surely cannot identify a good deal even when one exists (because the monopolist will never price below c, and hence $R - \underline{p}$ cannot be larger than $R - c$), and recall that whenever consumers cannot identify a good deal they walk home with 0 surplus. Therefore, consumer surplus decreases with an FOSD shift from F to $F^\dagger$.

On the other hand, consider yet another distribution $F^{\dagger\dagger}$, which also satisfies Assumptions 1 and 2, and is dominated by F in FOSD sense. In particular, this implies $f(0) \leq f^{\dagger\dagger}(0)$. At the limit when $f^{\dagger\dagger}(0) \nearrow 1/(R-c)$, the first best is an equilibrium outcome, where consumers walk home with 0 surplus.[13] Therefore, consumer surplus increases with an FOSD shift from $F^{\dagger\dagger}$ to F.

Proposition 3 *In the case of a monopolist marketing up to only one brand, consumers' inability to discern prices does not always benefit the consumers. Indeed, the effect of an FOSD shift in F on consumer surplus is necessarily non-monotonic.*

[13]Continuity holds at the limit, meaning that when $f^{\dagger\dagger}(0) \approx 1/(R-c)$ while still satisfying Assumption 2, consumers still walk home with approximately 0 surplus. See the proof of Proposition 3 for details.

4 A Monopolist Marketing Two Different Brands

In this section, we continue to study the case where there is only one seller, the monopolist. However, we now assume that the costs of marketing a second brand is negligible, while those of marketing more than two brands remain prohibitively high. We shall show that this improvement in the monopolist's marketing ability may paradoxically hurt its profit.

Formally, we assume that the costs of marketing a second brand are commonly known to be 0. The monopolist can market either a single brand or two different brands. In the case the monopolist markets two different brands, it can set potentially a different (quality-adjusted) price for each brand. We shall name those two brands "brand 1" and "brand 2", with associated prices p_1 and p_2, respectively. Every consumer continues to demand one and only one unit of the good.

By allowing prices to take the value of ∞, we can wlog proceed as if both brands are on the shelf. Marketing a single brand would then correspond to the case where $p_1 < \infty = p_2$, whereas marketing two different brands would correspond to the case where both prices are finite.

Some care should be taken in describing a game with two brands on the shelf. It makes more sense to think of the brands as "anonymous", in the sense that the consumers' purchase decisions can only depend on the inferred ordering, $\succ$, of R, p_1, and p_2, but otherwise cannot depend on the brand names. This precludes, for example, the strategy of purchasing brand 1 with probability 4/5 when $p_2 \succ R \sim p_1$, while purchasing brand 2 with only probability 1/3 when $p_1 \succ R \sim p_2$. Similarly, it precludes the strategy of purchasing brand 1 and brand 2 with probabilities 1/10 and 9/10, respectively, when $R \succ p_1 \sim p_2$. In other words, brand names are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to consumers. All a consumer can learn about a specific brand is already summarized by the inferred ordering $\succ$.

Since brand names are just artificial constructs that are meaningless to consumers, we shall follow the convention that "brand 2" is the brand with a higher price; i.e., $p_2 \geq p_1$ by our convention. This convention does not preclude positive sales for brand 2 in equilibrium. This is because, even though consumers know that brand 2 is more expensive than brand 1, a consumer who cannot compare p_1 and p_2 cannot tell which brand is brand 2 (recall the assumption of anonymity above), and hence may end up purchasing brand 2 by chance.

Under the convention of $p_2 \geq p_1$, there can only be 11 different configurations of primitive sense data a consumer may possibly encounter.[14] For 8 out of these 11 configurations, there is an unambiguous lowest-price option in the resulting inferred ordering $\succ$, and utility maximization dictates that the consumer chooses this lowest-price option.[15] Among the remaining 3 configurations, the consumer's best response is also straightforward when the primitive sense data are $R \mathrel{\widehat{\succ}} p_2 \mathrel{\widehat{\sim}} p_1 \mathrel{\widehat{\prec}} R$ (in which case the inferred ordering $\succ$ is the same as $\widehat{\succ}$): in this case, anonymity dictates that the best the consumer can do is to purchase each brand with probability 1/2.

Therefore, the only two non-trivial cases are

1. the *single-contender* case, where the primitive sense data are $p_2 \mathrel{\widehat{\succ}} R \mathrel{\widehat{\sim}} p_1 \mathrel{\widehat{\prec}} p_2$, and hence brand 1 is the only possible good deal for the consumer; and

2. the *all-tied* case, where the primitive sense data are $p_2 \mathrel{\widehat{\sim}} R \mathrel{\widehat{\sim}} p_1 \mathrel{\widehat{\sim}} p_2$, and hence both brands 1 and 2 are possibly good deal for the consumer.

How the consumer behave in these two cases will be determined in equilibrium. Let's denote by $q_1 \in [0, 1]$ the probability that a consumer purchases brand 1 (i.e., the single contender) in the single-contender case, and by $q_2/2 \in [0, 1/2]$ the probability that he purchases each contender in the all-tied case.

The reader may wonder why the monopolist may ever be hurt by its ability to market a second brand at negligible costs. Couldn't it guarantee at least its equilibrium profit in the one-brand case by simply marketing a single brand? The answer is no. If consumers anticipate that the monopolist markets two different brands, then the event that it markets a single brand will be an off-equilibrium event, and consumers' off-equilibrium beliefs in such an event can be quite different from their equilibrium beliefs in the one-brand case.

[14]There are 3 configurations where the primitive sense data already form a linear order; for example, $p_2 \mathrel{\widehat{\succ}} R \mathrel{\widehat{\succ}} p_1$. For each of these 3 configurations, the inferred ordering $\succ$ is the same as $\widehat{\succ}$. There are 4 configurations where exactly one pair of prices are indiscernible; for example, $p_2 \mathrel{\widehat{\succ}} R \mathrel{\widehat{\sim}} p_1 \mathrel{\widehat{\prec}} p_2$. For each of these 4 configurations, the inferred ordering $\succ$ is still the same as $\widehat{\succ}$. There are 3 configurations where exactly two pairs of prices are indiscernible; for example, $p_1 \mathrel{\widehat{\sim}} R \mathrel{\widehat{\sim}} p_2 \mathrel{\widehat{\succ}} p_1$. For each of these 3 configurations, the inferred ordering $\succ$ becomes a linear order; for example, from $p_1 \mathrel{\widehat{\sim}} R \mathrel{\widehat{\sim}} p_2 \mathrel{\widehat{\succ}} p_1$ the consumer obtains $p_2 \succ R \succ p_1$. Finally, there is 1 configuration where all three pairs of prices are indiscernible. For this configuration, the inferred ordering $\succ$ is also the same as $\widehat{\succ}$.

[15]There are 6 out of 11 configurations where the inferred ordering $\succ$ is a linear order (see Footnote 14), and hence an unambiguous lowest-price option exists. The other two configurations where an unambiguous lowest-price option exists are $R \mathrel{\widehat{\prec}} p_2 \mathrel{\widehat{\sim}} p_1 \mathrel{\widehat{\succ}} R$ and $p_1 \mathrel{\widehat{\prec}} p_2 \mathrel{\widehat{\sim}} R \mathrel{\widehat{\succ}} p_1$.

We present an equilibrium with such a flavor below in Proposition 4. We then provide an example of F such that the monopolist's profit in the equilibrium described in Proposition 4 is lower than its equilibrium profit in the one-brand case.

Proposition 4 *In the case of a monopolist marketing up to two different brands, there exists an equilibrium where*

- *consumers refuse to purchase whenever there is no apparent least-price option (i.e.,* $q_1^* = q_2^* = 0$*),*
- *the monopolist markets two different brands, with prices being mirror images of each other around* R *(i.e.,* $p_1^* < R < p_2^*$ *and* $(p_1^* + p_2^*)/2 = R$*), and*
- *given consumers' behavior (i.e.,* $q_1^* = q_2^* = 0$*),* (p_1^*, p_2^*) *is the unique maximizer of the monopolist's profit over all pairs of prices that are mirror images of each other around* R*.*

In this equilibrium, the monopolist's prices are deterministic. Yet gain of trade is still not always realized. Consumers purchase only if they can compare R *and* p_1^**, and hence they always obtain strictly positive surplus conditional on a purchase.*

To further elaborate on the point we made in the paragraph immediately before Proposition 4, let's consider what would happen if the monopolist deviates from its equilibrium behavior by marketing a single brand. Specifically, suppose the monopolist, instead of setting prices (p_1^*, p_2^*) as described in Proposition 4, deviates and sets $p_2 = \infty$ and randomizes between $p_1 = \overline{p}^*$ and $p_1 = \underline{p}^*$ as in Proposition 1. Given the consumers' equilibrium strategy $q_1^* = q_2^* = 0$, the monopolist cannot make any sale when the random price p_1 takes the value of $\overline{p}^*$. Even when the random price p_1 takes the value of $\underline{p}^*$, the monopolist makes a sale only if the consumer can discern p_1 and R. Its profit from such a deviation is hence much lower than its equilibrium profit in the one-brand case.

The reason behind this dismal deviation profit is that consumers are very untrusting in the single-contender case ($q_1^* = 0$). In the equilibrium described in Proposition 4, the single-contender case is an off-equilibrium event, and in such an event consumers' off-equilibrium beliefs can be quite pessimistic.

Note that, in the equilibrium described in Proposition 4, the monopolist never makes any sale from brand 2. The only role of brand 2 is to convince consumers that brand 1 is a good deal (it is). Specifically, consumers with types $R - p_1^* \leq d < p_2^* - p_1^*$, although unable to *discern* p_1^* and R, are nevertheless able to *compare* the two. The primitive sense data received by these consumers are $R \,\widehat{\sim}\, p_2^* \,\widehat{\succ}\, p_1^* \,\widehat{\sim}\, R$, which induce the inferred ordering of $p_2^* \succ R \succ p_1^*$, convincing them to purchase brand 1.

Paradoxically, this helping hand from the second brand can backfire. In the one-brand case, it cannot be an equilibrium for consumers to be totally untrusting. If consumers were totally untrusting (i.e., if $q = 0$), the monopolist would not be able to make any sale unless it set a price strictly lower than R, but then consumers should be totally trusting (i.e., $q = 1$) instead. This is no longer the case when there is a second brand that can be priced above R purely to help some consumers to compare prices. Now consumers who still cannot compare prices despite this helping hand *can* justifiably remain totally untrusting (i.e., $q_1^* = q_2^* = 0$), worrying that they may inadvertently purchase the over-priced brand 2.

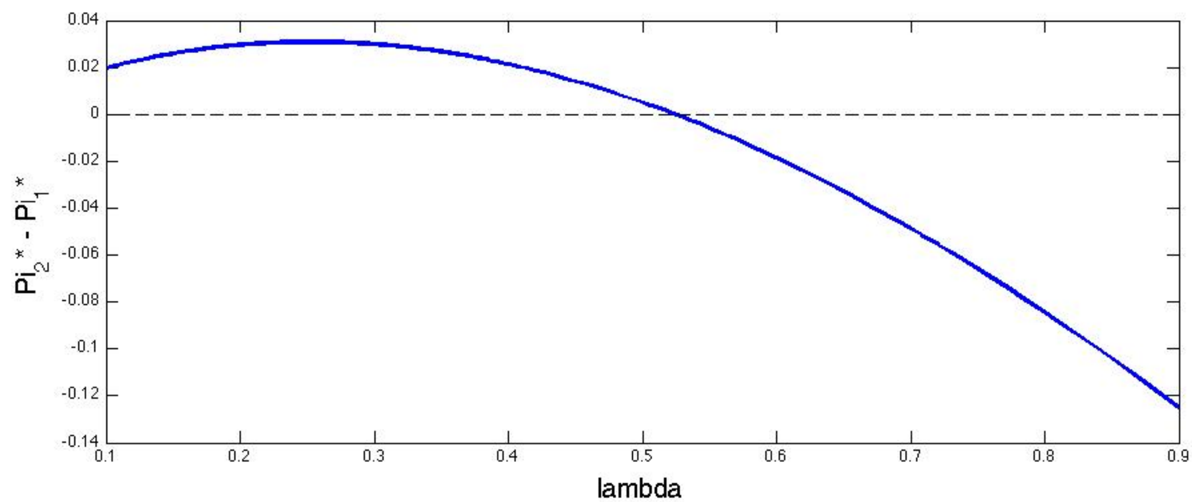

Figure 2: equilibrium profits in the one-brand and two-brand cases

That consumers are more untrusting in the equilibrium described in Proposition 4 than in the unique equilibrium described in Proposition 1 is the main reason why the monopolist's profit can be lower in the former than in the latter. As an illustration, we compute the monopolist's profit in each of these two equilibria by letting $R = 1$, $c = 0$, and letting F be a member of the exponential class; i.e., $F(d) = 1 - e^{-\lambda d}$. Let Π_1^* denote the monopolist's profit in the equilibrium described in Proposition 1, and Π_2^* that in the equilibrium described in Proposition 4. In Figure 2, we plot $\Pi_2^* - \Pi_1^*$ against λ, the parameter of the exponential distribution. Note that F satisfies Assumption 2 only if $\lambda < 1$.

As λ increases towards 1, consumers' ability to discern prices improves.[16] In the one-brand case, this generates more discipline on the monopolist, decreases its incentive to set a high price, and increases its incentive to set a low price. Consumers who cannot discern prices, by free-riding those who can, can hence afford to be more trusting, resulting in a higher q^* (see the proof of Proposition 2). Meanwhile, consumers in the equilibrium described in Proposition 4 remain totally untrusting. As a result, $\Pi_2^* - \Pi_1^*$ becomes negative as λ increases towards 1.

Proposition 5 *A monopolist's ability to market a second brand at negligible costs may paradoxically hurt its profit. Specifically, there exists an equilibrium (as described in Proposition 4) where its profit may be even lower than its equilibrium profit when marketing a second brand is prohibitively costly.*

While there is a unique equilibrium in the one-brand case, there are multiple equilibria in the two-brand case, with the one described in Proposition 4 being merely one of them. For the sake of completeness, we fully characterize an important sub-class of equilibria, namely the pure-pricing-strategy equilibria, in Proposition 6 below. These are equilibria where the monopolist plays pure pricing strategies, in contrast to the mixed pricing strategy played in the unique equilibrium in the one-brand case. Note that the equilibrium described in Proposition 4 is an example of pure-pricing-strategy equilibrium, where the monopolist sets prices $(p_1, p_2) = (p_1^*, p_2^*)$ with probability 1.

Proposition 6 *In the case of a monopolist marketing up to two different brands, there is an* $q_2^{max} \in (0, 1)$ *such that*

- *every pure-pricing-strategy equilibrium features an* $q_2^* \leq q_2^{max}$*;*
- *there exists a strictly increasing function* $p_1(\cdot)$ *that maps* $[0, q_2^{max}]$ *into* (c, R) *such that, in the pure-pricing-strategy equilibrium featuring* $q_2^* \in [0, q_2^{max}]$*, the monopolist sets deterministic prices* $p_1^* = p_1(q_2^*)$ *and* $p_2^* = 2R - p_1^*$*;*
- *comparing any two pure-pricing-strategy equilibria, monopolist's profit is higher and consumer surplus is lower in the one with a higher* q_2^**; and*
- *if, in addition to Assumption 2,* f *further satisfies* $f(0)(R - c) > 1/2$*, then*

[16] Formally, an exponential distribution with a smaller λ dominates one with a larger λ in the FOSD sense.

– *for every $q_2^* \in [0, q_2^{max}]$, there exists a pure-pricing-strategy equilibrium featuring that specific q_2^*; and*

– *in the pure-pricing-strategy equilibrium featuring $q_2^* = q_2^{max}$, the monopolist's profit is higher than its equilibrium profit in the one-brand case.*

In other words, every pure-pricing-strategy equilibrium resembles the one described in Proposition 4, in the sense that the monopolist markets two different brands, with prices being mirror images of each other (i.e., $p_1^* < R < p_2^*$ and $(p_1^* + p_2^*)/2 = R$). As a result, the single-contender case is always an off-equilibrium event, rendering the exact description of q_1^* payoff-irrelevant. Each pure-pricing-strategy equilibrium described in Proposition 6 hence is more precisely an equivalent class of equilibria featuring the same q_2^* but different q_1^*'s.

Comparing different pure-pricing-strategy equilibria, consumers are more trusting in those featuring higher q_2^*. When consumers are more trusting, the monopolist's profit is higher, at the expenses of consumer surplus. The equilibrium described in Proposition 4 is hence the worse for the monopolist and the best for consumers among all pure-pricing-strategy equilibria.

Consumers, however, will never be totally trusting in any pure-pricing-strategy equilibrium. This is shown by the fact that q_2^* is capped from above by an upper bound q_2^{max} that is strictly smaller than 1. Therefore, once again, the first best cannot be achieved.

Finally, one may wonder whether there exists any (mixed-pricing-strategy) equilibrium where the monopolist does not market brand 2 at all; i.e., $p_2 = \infty$ with probability 1. Such an equilibrium, if exists, must look exactly like the unique equilibrium in the one-brand case as described in Proposition 1; i.e., the monopolist plays a mixed-pricing-strategy and randomizes between $p_1 = \overline{p}^*$ and $p_1 = \underline{p}^*$, and a consumer purchases with probability $q_1 = q^*$ when he finds himself in the single-contender case. Let's call such an equilibrium, if exists, the single-brand equilibrium.[17]

When the monopolist can market up to two different brands at negligible costs, the single-brand equilibrium may not exist. The reason is that, whenever the monopolist is to set $p_1 = \underline{p}^*$—which is a good deal for consumers because $\underline{p}^* < R$—it would lament the

[17]The single-brand equilibrium is more precisely an equivalent class of equilibria featuring different q_2^*. This is because $p_2 = \infty$ with pobability 1 implies that the all-tied case is an off-equilibrium event, and hence many different q_2^*'s can be supported by appropriately chosen off-equilibrium beliefs.

fact that too few consumers can *discern* $\underline{p}^*$ and R and hence appreciate this good deal, and would have incentives to bring in the second brand in order to help more consumers to *compare* $\underline{p}^*$ and R.

Characterizing exactly when the single-brand equilibrium fails to exist turns out to be both tedious and non-illuminating. This is because there are many possible deviations involving "bringing in the second brand", and the single-brand equilibrium will fail to exist as long as one of these deviations is profitable. We shall hence provide only an easy sufficient condition for the non-existence of the single-brand equilibrium, focusing on only one particular deviation, namely the deviation to setting $(p_1, p_2) = (p_1^*, p_2^*)$, where (p_1^*, p_2^*) is as defined in Proposition 4. In the following proposition, $\Pi 1^*$ and Π_2^* are as defined in the paragraph immediately before Proposition 5.

Proposition 7 *In the case of a monopolist marketing up to two different brands, the single-brand equilibirum does not exist whenever* $\Pi_2^* > \Pi_1^*$.

Proof: This is because the monopolist's equilibrium profit in the single-brand equilibrium, if exists, must equal to Π_1^*, while its deviation profit is at least Π_2^* if it deviates to setting $(p_1, p_2) = (p_1^*, p_2^*)$, where (p_1^*, p_2^*) is as defined in Proposition 4.[18] ■

For example, we can see from Figure 2 that, when $R = 1$, $c = 0$, and F is an exponential distribution with parameter $\lambda < 1/2$, the single-brand equilibrium does not exist.

5 A Monopolist Marketing Many Brands

While a monopolist's ability to market a second brand at negligible costs may paradoxically hurt its profit (Proposition 5), we can however prove that, its ability to market at negligible costs a sufficiently large number of brands, almost all of which are not meant to make any sale, will necessarily help its profit.

Formally, we assume that the costs of marketing the first n brands are commonly known to be 0, while those of of marketing more than n brands remain prohibitively high. We shall show that, for n sufficiently large, the monopolist's profit in *any* equilibrium is

[18] Recall from Footnote 17 that the single-brand equilibrium can feature many different q_2^*. The deviation profit is exactly Π_2^* if $q_2^* = 0$, and is strictly higher than Π_2^* if $q_2^* > 0$. (See Proposition 6.) Note that q_1^* is irrelevant in calculating the deviation profit.

arbitrarily close to its first-best profit $R - c$, and hence is higher than its equilibrium profit in the one-brand case. Since the proof is short and constructive, we include it here in the main text and let it help explain the underlying intuition.

Proposition 8 *For any $\varepsilon > 0$, there exists $\overline{n}$ such that, for any $n \geq \overline{n}$, in the case of a monopolist marketing up to n different brands, the monopolist can achieve a profit higher than $R - c - \varepsilon$ in any equilibrium.*

Proof: Pick any δ small enough and $\overline{n}$ big enough so that $(R - c - \delta)F(\overline{n}\delta) > R - c - \varepsilon$. Suppose the monopolist is to market $n \geq \overline{n}$ brands, with prices $p_1 = R - \delta$, $p_2 = R + \delta$, $p_3 = R + 2\delta$, ..., and $p_n = R + (n-1)\delta$, respectively. A consumer with $d < \delta$ will be able to discern p_1 and R and hence tell that p_1 is lower than R, and will purchase brand 1 at price p_1. A consumer with $d \in [\delta, 2\delta)$ cannot discern p_1 and R, and cannot discern R and p_2, but is able to discern p_1 and p_2, and hence can infer that p_1 is lower than R, and will also purchase brand 1 at price p_1. More generally, a consumer with $d \in [(k-1)\delta, k\delta)$, $k \in \{2, \ldots, n\}$, cannot discern p_1 and R, and cannot discern R and p_k, but is able to discern p_1 and p_k, and hence can infer that p_1 is lower than R, and will purchase brand 1 at price p_1. The monopolist's profit is hence at least $(p_1 - c)F(n\delta) \geq (R - c - \delta)F(\overline{n}\delta) > R - c - \varepsilon$. ■

6 Two Sellers Marketing One Brand Each

In Section 4, we study the case of a single seller marketing up to two different brands. In this section, we study the case where the ability to market the second brand comes from a second seller instead of from the original seller. Specifically, we study the case where there are two identical sellers, each marketing up to only one brand. Arguably the marketing capacity available to the society is the same, in the sense that the costs of marketing two or fewer brands are negligible, while those of marketing more than 2 brands are prohibitively high. The only change from the setting in Section 4 to the current setting is how this marketing capacity is distributed. Which this marketing capacity is evenly distributed between two identical sellers instead of being concentrated in the hands of one, the market is more competitive. We shall, however, show that more competition does not always benefit consumers.

We shall name the two sellers "seller 1" and "seller 2", with associated prices p_1 and p_2 for their respective brands. Every consumer continue to demand one and only one unit of the good. Following the second half of Section 4, we focus on pure-pricing-strategy equilibria, meaning those equilibria where each seller plays a pure pricing strategy.

As in Section 4, we assume that brands are "anonymous", in the sense that brand names are artificial constructs that are for the convenience of we analysts only, but are otherwise meaningless to consumers. All a consumer can learn about a specific brand is already summarized by the inferred ordering $\succ$. As such, and since sellers play pure pricing strategies, we can follow the convention in Section 4 that "brand 2" is the brand with a higher equilibrium price; i.e., $p_2^* \geq p_1^*$ by our convention. As before, this convention does not preclude an asymmetric equilibrium where the two sellers set different (quality-adjusted) prices. This is because, even though consumers know that brand 2 is more expensive than brand 1, a consumer who cannot compare p_1 and p_2 cannot tell which brand is brand 2, and hence may end up purchasing brand 2 by chance.

As in Section 4, to describe a consumer's strategy, it suffices to describe his behavior in the single-contender case and the all-tied case, which are defined in exactly the same way as in Section 4. Let's continue to denote by $q_1 \in [0, 1]$ the probability that a consumer purchases brand 1 (i.e., the single contender) in the single-contender case, and by $q_2/2 \in [0, 1/2]$ the (necessarily common) probability that he purchases each contender in the all-tied case.

The reader may wonder why more competition does not always benefit consumers. Wouldn't competing sellers undercut each other and lead to lower prices as in the traditional Bertrand model? The answer is no. For starter, when consumers feature intransitive indifference, undercutting one's opponent does not enable it to capture the whole market, because many consumers are not able to tell that its (quality-adjusted) price is lower than its opponent's. This reduces one's incentives to undercut its opponent. Indeed, if sellers are anticipated to undercut each other aggressively, consumers will become fairly trusting, and will be fairly willing to purchase even when they cannot compare prices. Sellers hence will have incentives to raise their prices in order to exploit these trusting consumers, invalidating the original anticipation.

When consumers feature intransitive indifference, sellers actually free-ride instead of undercut each other. There are two different kinds of free-riding behavior, and are respectively adopted by the two sellers. In a pure-pricing-strategy equilibrium, one seller will under-price its brand relative to R, while the other will over-price its. Seller 2 free-rides seller 1's low price, which keeps consumers trusting, and sets a high price to exploit these trusting consumers.[19] Seller 1, on the other hand, free-rides seller 2's high price, which enables some consumers to recognize the good deal offered by seller 1—those consumers who cannot *discern* p_1 and R, but can *compare* them with the help of p_2—and avoids the need to set an even lower price to win over these consumers.[20] As a result, both sellers manage to alleviate some downward pressure on their prices by free-riding each other, albeit free-riding in very different manners.

A pure-pricing-strategy equilibrium in this two-seller case ends up being very similar to one in the two-brand case studied in Section 4, in the sense that the prices of the two brands are mirror images of each other around R. Consumers purchase either when they can compare prices—in which case they purchase the lower-priced brand and obtain strictly positive surplus—or when they find themselves in the all-tied case—in which case they randomize between purchasing or not, and randomize between the two brands when they do purchase, and obtain zero surplus on average. Consumers' surplus hence depends solely on how low brand 1 is priced, same as in a pure-pricing-strategy equilibrium in the two-brand case. It turns out that the free-riding logic mentioned in the previous paragraph will push p_1 so high that consumers' surplus in this two-seller case is even

[19]More formally, $p_2 > R$ can be a best response for seller 2 only when $q_2 > 0$ (because seller 2 can make a sale at a price $p_2 > R$ only when the consumer finds himself in the all-tied case), and $q_2 > 0$ can be part of the consumer's best response only when $p_1 \leq R$ (otherwise $(p_1 + p_2)/2 \geq p_1 > R$ and hence the consumer's best response must feature $q_2 = 0$).

[20]More formally, consder the case where $q_1 = 0 < q_2$ (which, as we shall argue soon, can be assumed wlog in our search for pure-pricing-strategy equilibria). Suppose seller 2 sets $p_2 = \infty$, effectively dropping out from the market. Then seller 1's best response is to set $p_1 = \underline{p}(0)$, where $\underline{p}(\cdot)$ is as defined in Section 3. Suppose seller 2 now lowers its price, but not too low as to offer consumers a genuine good deal. Specifically, suppose seller 2 lowers its price from ∞ to some finite $p_2 \in (R, 2R - p_1)$. Such a move of seller 2, instead of imposing competitve pressure on seller 1, actually *raises* seller 1's profit from $(p_1 - c)F(R - p_1)$ to $(p_1 - c)\{F(p_2 - p_1) + q_2[1 - F(p_2 - p_1)]\}$. More importantly for consumers, seller 1 would now have incentives to even further *raise* its price, because

$$\begin{aligned}\left.\frac{\partial \pi_1}{\partial p_1}\right|_{p_1=\underline{p}(0)} &= \{F(p_2 - p_1) + q_2[1 - F(p_2 - p_1)]\} - (p_1 - c)(1 - q_2)f(p_2 - p_1) \\ &= q_2 + (1 - q_2)f(p_2 - p_1)\left[\frac{F(p_2 - p_1)}{f(p_2 - p_1)} - \frac{F(R - p_1)}{f(R - p_1)}\right] \\ &> 0,\end{aligned}$$

where the second equality makes use of the first-order condition that characterizes $p_1 = \underline{p}(0)$, and the inequality makes use of Assumption 1. Intuitively, with the help of a finite p_2, more consumers can appreciate the good deal that seller 1 is offering them. With a bigger consumer base, seller 1 now has incentives to raise its price.

lower than that in the pure-pricing-strategy equilibrium described in Proposition 4.

Proposition 9 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium,*

- *the two sellers set prices that are mirror images of each other around R; specifically, $(p_1^*, p_2^*) = (R - x^*, R + x^*)$, where $x^* > 0$ is the unique solution to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + R - c;$$

- *consumers' surplus is strictly lower than in the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different brands.*

In the special case where F belongs to the exponential class, consumers' surplus in any pure-pricing-strategy equilibrium is especially easy to calculate. First, using the equation in Proposition 9, we have

$$x^* + R - c = \frac{1 - F(2x^*)}{f(2x^*)} = \frac{\exp(-\lambda(2x^*))}{\lambda \exp(-\lambda(2x^*))} = \frac{1}{\lambda},$$

which gives us $x^* = 1/\lambda - (R - c)$. Consumers with $d < 2x^*$ are able to compare p_1^* and R (either by directly *discerning* them, or by *comparing* them with the help of p_2^*) and hence will walk home with surplus $R - p_1^* = x^*$. Consumers with $d \geq 2x^*$, on the other hand, cannot compare p_1^* and R, and hence will purchase both brands 1 and 2 with the same probability if they ever do any purchases. These consumers hence walk home with zero surplus, given that p_1^* and p_2^* are mirror images of each other around R. On average, consumers' surplus is hence

$$x^* F(2x^*) = x^*[1 - \exp(-\lambda(2x^*))] = \left[\frac{1}{\lambda} - (R - c)\right][1 - \exp(-2 + 2\lambda(R - c))],$$

which is strictly decreasing in λ. Recall that, in the special case of the exponential class, a smaller λ represents an FOSD shift in F, meaning that consumers are less able to discern prices. We hence have the conclusion that consumers' surplus is increasing in their *in*ability to discern prices, echoing similar results in Piccione and Spiegler (2012) and de Clippel, Eliaz, and Rozen (2014). Intuitively, when more consumers have difficulty in discerning prices, there are two opposite effects on consumers' surplus. On the one hand, fewer consumers can recognize the good deal offered by seller 1, which reduces consumers' surplus. On the other hand, seller 1 is pressured to further lower its price in order to convince consumers that its deal is good, which increases consumers' surplus. Depending on the distribution of consumers' types, it is possible that the second effect dominates the first effect.

Proposition 10 *Suppose F belongs to the exponential class. In the case of two sellers marketing up to only one brand each, consumers' pure-pricing-strategy-equilibrium surplus increases with and FOSD shift in F.*[21]

A pure-pricing-strategy equilibrium, however, may not exist. Indeed, one can prove that it does not exist if $f(0)(R-c) < 1/3$. In order to make sure that Proposition 9 above is not a characterization of the empty set, we numerically demonstrate the existence of a pure-pricing-strategy equilibrium given some values of parameters R and c, and some distribution F that satisfies Assumptions 1 and 2.

Specifically, let $R = 1$, $c = 0$, and F be the exponential distribution with parameter $\lambda = 0.8$. Plugging these into the equation in Proposition 9, one readily computes that $x^* = 0.25$. By Proposition 9, a pure-pricing-strategy equilibrium, if exists, must feature $(p_1^*, p_2^*) = (R - x^*, R + x^*) = (1 - 0.25, 1 + 0.25) = (0.75, 1.25)$. We can plot seller 1's profit $\pi_1(p_1, p_2)$ (respectively, seller 2's profit $\pi_2(p_1, p_2)$) as a function of its price p_1 (respectively, p_2), given the opponent's price $p_2 = 1.25$ (respectively, $p_1 = 0.75$), and given any (q_1, q_2). A pure-pricing-strategy equilibrium exists iff there exists some (q_1, q_2) such that $p_1 = 0.75$ and $p_2 = 1.25$ are global optima of π_1 and π_2, respectively. A familiar argument we once

[21] As explained in Footnote 10, we can easily handle distributions with finite supports (such as uniform distributions) at the expenses of messier proofs. In the special case where F belongs to the uniform class $U[0, D]$, the equation in Proposition 9 becomes

$$x^* + R - c = \frac{1 - F(2x^*)}{f(2x^*)} = D - 2x^*,$$

which gives us $x^* = (D - R + c)/3$. Therefore, consumers' surplus is

$$x^* F(2x^*) = x^* \frac{2x^*}{D} = \frac{2(D - R + c)^2}{9D},$$

which is strictly increasing in D if $D > R - c$, which in turned is guaranteed by Assumption 2 (see Footnote 12). Recall that, in the special case of the uniform class, a bigger D represents an FOSD shift in F. We hence once again arrive at the same conclusion that consumers' surplus is increasing in their *in*ability to discern prices.

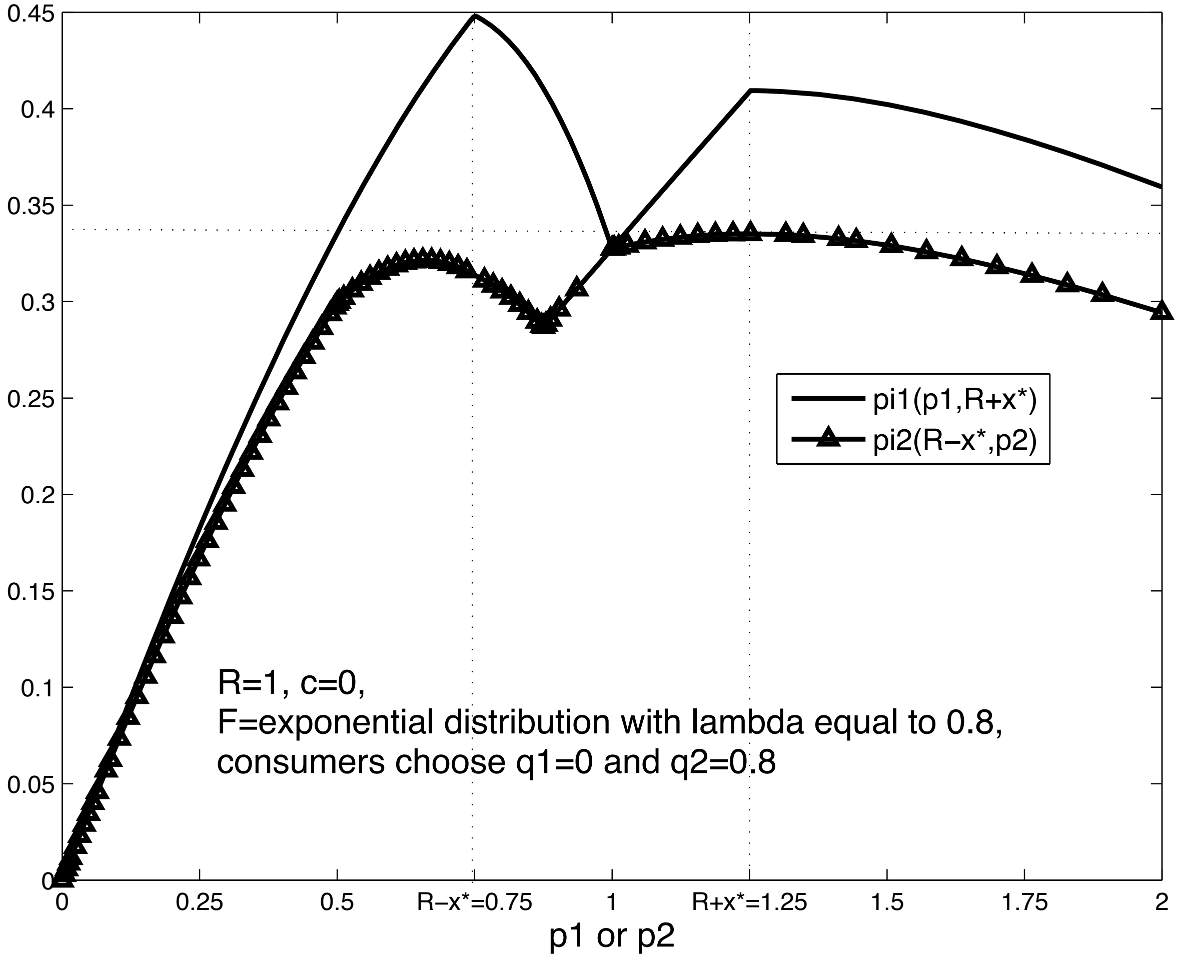

Figure 3: the solid line is seller 1's profit as a function of p_1, given opponent's price $p_2 = R + x^*$; the line with little triangles is seller 2's profit as a function of p_2, given opponent's price $p_1 = R - x^*$; both are drawn with $R = 1$, $c = 0$, F being the exponential distribution with parameter $\lambda = 0.8$ (which implies $x^* = 0.25$), and $(q_1, q_2) = (0, 0.8)$

used in the proof of Proposition 6 suggests that, in our search for such (q_1, q_2), it is wlog to let $q_1 = 0$, because this does not affect either seller's profit at its candidate-equilibrium price, while weakly lowers its profit at other prices.

This leaves us only one variable to tune with. As we tune q_2, the shapes of π_1 and π_2 change. In the neighborhood of $q_2 = 0.8$, $p_1 = 0.75$ and $p_2 = 1.25$ indeed become the global optima of π_1 and π_2, respectively, as shown in Figure 3, verifying the existence of a pure-pricing-strategy equilibrium for such R, c, and F.

Figure 3 is robust to perturbation in q_2, meaning that $(p_1^*, p_2^*, q_1^*, q_2^*) = (0.75, 1.25, 0, q_2^*)$

remains a pure-pricing-strategy equilibrium for an open set of q_2^* containing 0.8. Indeed, π_1 has a kink at $p_1 = 0.75$. As we perturb q_2, the left and right derivatives of π_1 at $p_1 = 0.75$ will be perturbed, and similarly for the values of π_1 at the two local optima, but $p_1 = 0.75$ will remain the unique global optimum. The case for π_2 is slightly different: its slope at $p_2 = 1.25$ remains flat regardless of the value of q_2. Indeed, this property is the geometric meaning of the equation in Proposition 9, which we used earlier to compute x^*. Therefore, as we perturb q_2, $p_2 = 1.25$ remains a local optimum. Since the value of π_2 at $p_2 = 1.25$ is strictly higher than that at the other local optimum, $p_2 = 1.25$ will remain the unique global optimum upon perturbation of q_2.

It is also worth highlighting some interesting features of Figure 3 that are not mentioned in Proposition 9. First, the two sellers earn different profits, with the one over-pricing its products earning strictly less than the one under-pricing. While seller 2 may envy seller 1, it cannot mimic the latter by also under-pricing its products. If it were to do so, the best way to do it is to undercut seller 1, which is shown by the fact that the left local optimum of π_2 is located on the left of 0.75. However, by doing so, seller 2 actually will earn even less than free-riding seller 1 and exploiting the trusting consumers.

Second, sellers' profits as functions of own prices are not quasi-concave. This explains why it is difficult to provide interesting sufficient conditions for the existence of a pure-pricing strategy equilibrium beyond numerical examples such as the one depicted in Figure 3.

7 Conclusion

In this paper, we made a first step in exploring the implications of intransitive indifference in some classical economic analyses. Many of these implications are driven by the phenomenon that the presence of an inferior product can help a consumer appreciate a better product—a phenomenon that is the signature of intransitive indifference.

To keep our first-pass exercise tractable, we have made the simplifying assumption that quality and price can be collapsed into a one-dimensional variable called quality-adjusted price. In future research, it will be desirable to disentangle the two, with intransitive indifference arising in the quality dimension but not in the price dimension. New questions arising in this more realistic setting include how sellers' competition in the quality dimension interacts with their competition in the price dimension, and how consumers' inability to discern small differences in quality affect the equilibrium prices.

It will also be desirable to consider multiple dimensions of quality, with intransitive indifference arising in different degrees in each dimension. We conjecture that many informal concepts in the marketing literature can be formalized and refined in this setting. Two examples of such concepts that immediately come to mind are the *range* and *frequency effects* (Huber, Payne, and Puto (1982)). Consider two products, A and B, each is better than the other in a different dimension. For example, A scores 10 in dimension I, but scores only 6 in dimension II; similarly, B scores 10 in dimension II, but scores only 6 in dimension I. A seller who wants to promote product A would like to manipulate consumers into downplaying dimension II (the dimension along which product A is weaker) and paying more attention to dimension I (the dimension along which product A is stronger). One way to do it, according to the marketing literature, is to add to the choice set a third product C that scores 2 in dimension II. This increases the range of scores along dimension II from 4 points ($10 - 6 = 4$) to 8 points ($10 - 2 = 8$), and hence gives consumers the illusion that every incremental point along dimension II is less important (the range effect), thus increases the sales of product A. Another way to promote product A is to add to the choice set a third product C that scores 8 in dimension I. The range of scores along dimension I does not change, but the increased frequency gives consumers the illusion that every incremental point along dimension I is more important (the frequency effect), thus also increases the sales of product A. While range and frequency effects are often explained in terms of manipulation and illusion, we conjecture that they may also be explained in terms of intransitive indifference, and hence be given a rigorous foundation based on economic theory.

Appendix A: Omitted Proofs in Section 3

The following lemma will be used in the proof of Proposition 1.

Lemma 1 *Consider the function* $\pi(p) = (p-c)[1-F(p-R)]$. *Then* π *is quasi-concave and has a unique maximizer in the sub-range* $p \in [R, \infty)$.

Proof: At any $p \in (R, \infty)$, we have $\pi(p) > 0$ and

$$\frac{d\pi}{dp} = f(p-R)\left[\frac{1-F(p-R)}{f(p-R)} - (p-c)\right].$$

By Assumption 1, the term in the square parentheses is strictly decreasing in p. This proves quasi-concavity of π and the existence of a unique maximizer in the sub-range $p \in [R, \infty)$. ■

Proof of Proposition 1: By Lemma 1, for any $q > 0$, π is quasi-concave and has a unique maximizer in the sub-range $p \in [R, \infty)$. Let $\overline{p} \geq R$ denote this unique maximizer. Apparently $\overline{p}$ does not depend on q.[22] Moreover, we have

$$\overline{p} > R \quad \text{iff} \quad \left.\frac{\partial \pi}{\partial p}\right|_{p=R+} > 0 \quad \text{iff} \quad f(0)(R-c) < 1.$$

In the sub-range $p \in (c, R)$,

$$\begin{aligned} \frac{\partial \pi}{\partial p} &= q + F(R-p)(1-q) - (p-c)f(R-p)(1-q), \\ \frac{\partial^2 \pi}{\partial p^2} &= -2f(R-p)(1-q) + (p-c)f'(R-p)(1-q) < 0, \end{aligned}$$

where the last inequality follows from Assumption 1. Therefore, π is strictly concave in p, and has a unique maximizer in the sub-range $p \in [c, R]$. Let $\underline{p}(q) \leq R$ denote this unique maximizer. Since

$$\frac{\partial^2 \pi}{\partial p \partial q} = 1 - F(R-p) + (p-c)f(R-p) > 0,$$

$\underline{p}$ is increasing in q (strictly so if $\underline{p} \in (c, R)$). It attains its upper bound R iff

$$\left.\frac{\partial \pi}{\partial p}\right|_{p=R-} = q - f(0)(R-c)(1-q) \geq 0 \quad \text{iff} \quad f(0)(R-c) \leq \frac{q}{1-q}.$$

[22] If $q = 0$, $\pi \equiv 0$ for any $p \in [R, \infty)$, and hence $\overline{p}$ remains a maximizer.

Let $\overline{q}$ be the unique solution of $f(0)(R-c) = \overline{q}/(1-\overline{q})$. Note that $0 < \overline{q} < 1/2$ by Assumption 2.

At $q = 0$, $\pi = 0$ for any $p \in [R, \infty)$, and hence $\pi(\overline{p}, q) = 0 < \pi\left(\underline{p}(q); q\right)$. At any $q \in [\overline{q}, 1]$, we have $\underline{p}(q) = R$, and hence $\pi(\overline{p}; q) > \pi(R; q) = \pi\left(\underline{p}(q); q\right)$. At any $q \in (0, \overline{q})$, $\pi(\overline{p}; q)$ is linear in q, while $\pi\left(\underline{p}(q); q\right)$ is convex in q:

$$\frac{d\pi\left(\underline{p}(q); q\right)}{dq} = \frac{\partial\pi\left(\underline{p}(q); q\right)}{\partial q} = \left(\underline{p}(q) - c\right)\left[1 - F\left(R - \underline{p}(q)\right)\right],$$

which is increasing in q. Therefore, $\pi(\overline{p}; \cdot)$ crosses $\pi\left(\underline{p}(\cdot); \cdot\right)$ once and only once, and crosses from below. Let $q^* \in (0, \overline{q})$ denote the unique solution of $\pi(\overline{p}, q) = \pi\left(\underline{p}(q); q\right)$.

Any equilibrium must have $q = q^*$. Indeed, if $q < q^*$ in equilibrium, $\underline{p}(q) < R$ will be the unique maximizer of π in the sub-range $p \in [c, \infty)$, leading to $q = 1$ as the consumers' best response, a contradiction. Similarly, if $q > q^*$ in equilibrium, $\overline{p} > R$ will be the unique maximizer of π in the sub-range $p \in [c, \infty)$, leading to $q = 0$ as the consumers' best response, a contradiction again. At $q = q^*$, the monopolist is indifferent between setting the price at $\overline{p}$ and at $\underline{p}(q^*)$. In equilibrium it must randomize between these two in a way that makes consumers willing to randomize between purchasing and not purchasing when they cannot discern the price of the product and their reservation price.

Let α be the probably that the monopolist sets the high price $\overline{p}$ in equilibrium. Conditional on the event that a consumer cannot discern the product price and his reservation price, the conditional expectation of the product price is

$$\frac{\alpha\left[1 - F\left(\overline{p} - R\right)\right]\overline{p} + (1-\alpha)\left[1 - F\left(R - \underline{p}(q^*)\right)\right]\underline{p}}{\alpha\left[1 - F\left(\overline{p} - R\right)\right] + (1-\alpha)\left[1 - F\left(R - \underline{p}(q^*)\right)\right]}.$$

In order for the consumer to be indifferent between purchasing and not purchasing, this conditional expectation must be the same as his reservation price, or equivalently,

$$\alpha = \frac{\left(R - \underline{p}(q^*)\right)\left[1 - F\left(R - \underline{p}(q^*)\right)\right]}{\left(\overline{p} - R\right)\left[1 - F\left(\overline{p} - R\right)\right] + \left(R - \underline{p}(q^*)\right)\left[1 - F\left(R - \underline{p}(q^*)\right)\right]}.$$

Consumers on average obtain strictly positive surplus because they break even either when they feels the price of the good is above their reservation price, or when they cannot discern the two, yet with strictly positive probability (more precisely, with probability $(1-\alpha)F\left(R-\underline{p}(q^*)\right) > 0$) they obtain a strictly positive surplus of $R - \underline{p}(q^*) > 0$. ■

Proof of Proposition 2: Suppose $F^\dagger$ is a distribution that also satisfies Assumptions 1 and 2, and dominates F in the FOSD sense. Let's write the distribution explicitly as an argument of the profit function. Then $\pi(p;q,F^\dagger) > \pi(p;q,F)$ for any $q > 0$ and any $p > R$ (this is because when the monopolist sets a price higher than consumers' reservation price, it will fare better if more consumers cannot discern these two prices and hence cannot tell for sure that this is a bad deal), and hence $\pi(\overline{p}^\dagger;q,F^\dagger) > \pi(\overline{p};q,F)$ for any $q > 0$, where $\overline{p}^\dagger$ is the unique maximizer of $\pi(p;q,F^\dagger)$ in the range $p \in [R,\infty)$.

On the other hand, $\pi(p;q,F^\dagger) < \pi(p;q,F)$ for any q and any $p < R$ (this is because when the monopolist sets a price lower than consumers' reservation price, it will fare worse if more consumers cannot discern these two prices and hece cannot appreciate this good deal), and hence $\pi(\underline{p}^\dagger(q);q,F^\dagger) > \pi(\underline{p}(q);q,F)$ for any q, where $\underline{p}^\dagger(q)$ is the unique maximizer of $\pi(p;q,F^\dagger)$ in the range $p \in [c,R]$.

Recall that $\pi\left(\overline{p};\cdot,F\right)$ and $\pi\left(\underline{p}(\cdot);\cdot,F\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^* \in (0,1/2)$. Similarly $\pi\left(\overline{p};\cdot,F^\dagger\right)$ and $\pi\left(\underline{p}(\cdot);\cdot,F^\dagger\right)$ are both increasing functions of q, with the former crossing the latter once and only once and from below at some $q^\dagger \in (0,1/2)$. The facts that $\pi\left(\overline{p};\cdot,F\right)$ lies pointwise below $\pi\left(\overline{p};\cdot,F^\dagger\right)$ and that $\pi\left(\underline{p}(\cdot);\cdot,F\right)$ lies pointwise above $\pi\left(\underline{p}(\cdot);\cdot,F^\dagger\right)$ hence implies $q^* \geq q^\dagger$. The monopolist's equilibrium profit under distribution $F^\dagger$ is hence

$$\begin{aligned}\pi\left(\underline{p}^\dagger(q^\dagger);q^\dagger,F^\dagger\right) &\leq \pi\left(\underline{p}^\dagger(q^\dagger);q^\dagger,F\right)\\ &\leq \pi\left(\underline{p}^\dagger(q^\dagger);q^*,F\right)\\ &\leq \pi\left(\underline{p}(q^*);q^*,F\right),\end{aligned}$$

where the first inequality follows from the fact that the monopolist benefits from having more consumers appreciating its good deal, second inequality follows from the fact that $\pi(p;q,F)$ is strictly increasing in q for any $p > c$, and the third inequality follows from the optimality of $\underline{p}(q^*)$ given q^* and F. This proves that the monopolist's equilibrium profit decreases with a FOSD shift in F. ■

Proof of Proposition 3: Start with any arbitrary distribution F that satisfies Assumptions 1 and 2. By Proposition 1 consumer surplus is strictly positive. Construct an *increasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $f_n(0) \nearrow 1/(R-c)$. Note that, along this sequence, we have F_n dominates F_{n+1} in the FOSD sense for all n. We shall prove that, for n sufficiently large, consumer surplus in an economy featuring F_n is lower than that in an economy featuring F. To this end, it suffices to prove that consumer surplus converges to 0 as $n \to \infty$.

Let $\overline{p}_n^*$, $\underline{p}_n^*$, and q_n^* be the corresponding equilibrium variables in an economy featuring F_n. Recall that $\overline{p}_n^*$ maximizes $\pi_n(p;q_n^*) = (p-c)[1-F_n(p-R)]q_n^*$ over the sub-range $[R,\infty)$, and that $q_n^* > 0$. Therefore, $\overline{p}_n^*$ solves the following first-order condition:

$$\overline{p}_n^* - c = \frac{1-F_n\left(\overline{p}_n^* - R\right)}{f_n\left(\overline{p}_n^* - R\right)} \leq \frac{1-F_n(0)}{f_n(0)} = \frac{1}{f_n(0)} \searrow R-c,$$

where the inequality follows from Assumption 1. We hence have $\overline{p}_n^* \searrow R$, which implies

$$\lim_{n\to\infty} \pi_n\left(\overline{p}_n^*;q\right) = q(R-c).$$

We next prove that $q_n^* \nearrow 1/2$. Recall from the proof of Proposition 1 that $\pi_n\left(\overline{p}_n^*;\cdot\right)$ crosses $\pi_n\left(\underline{p}_n(\cdot);\cdot\right)$ from below at $q_n^* < 1/2$. Therefore, it suffices to prove that, for any $q < 1/2$, $\pi_n\left(\overline{p}_n^*;q\right) < \pi_n\left(\underline{p}_n(q);q\right)$ for n sufficiently large.

Recall from the proof of Proposition 1 that $\underline{p}_n(q) < R$ for all $q < \overline{q}_n$, where $\overline{q}_n$ is the unique solution of $f_n(0)(R-c) = \overline{q}_n/\left(1-\overline{q}_n\right)$. Apparently $\overline{q}_n \nearrow 1/2$. Therefore, for any $q < 1/2$, we have $\underline{p}_n(q) < R$ for n sufficiently large. Moreover, for such q and n, we have

$$\pi_n\left(\underline{p}_n(q);q\right) > \pi_n(R;q) = q(R-c) = \lim_{m\to\infty} \pi_m(\overline{p}_n^*;q),$$

where the inequality follows from the fact that $\underline{p}_n(q)$ is the unique maximizer of $\pi(\cdot;q)$ in the sub-range of $[c,R]$. For any $m > n$, $\pi_m\left(\underline{p}_m(q);q\right) > \pi_n\left(\underline{p}_n(q);q\right)$ (recall the proof of Proposition 2). Therefore, we have

$$\lim_{m\to\infty} \pi_m\left(\underline{p}_m(q);q\right) \geq \pi_n\left(\underline{p}_n(q);q\right) > \lim_{m\to\infty} \pi_m(\overline{p}_n^*;q),$$

and hence we have $\pi_m\left(\underline{p}_m(q);q\right) > \pi_m\left(\overline{p}^*_m;q\right)$ for m sufficiently large. Since $q < 1/2$ is arbitrary, we hence have $q^*_n \nearrow 1/2$ as claimed.

Finally, we prove that $\underline{p}^*_n \nearrow R$. Since $\underline{p}^*_n = \underline{p}_n(q^*_n)$, it maximizes $\pi_n(p;q^*_n) = (p-c)\left(F_n(R-p) + [1-F_n(R-p)]q^*_n\right)$ over the sub-range $[c,R]$, and hence solves the following first-order condition:

$$\underline{p}^*_n - c = \frac{F_n\left(R-\underline{p}^*_n\right)}{f_n\left(R-\underline{p}^*_n\right)} + \frac{q^*_n}{(1-q^*_n)\,f_n\left(R-\underline{p}^*_n\right)} > \frac{q^*_n}{(1-q^*_n)\,f_n\left(R-\underline{p}^*_n\right)} \geq \frac{q^*_n}{(1-q^*_n)\,f_n(0)}.$$

Take limit on both sides, we have

$$\lim_{n\to\infty} \underline{p}^*_n - c \geq R - c = \lim_{n\to\infty} \frac{q^*_n}{(1-q^*_n)\,f_n(0)},$$

and hence $\underline{p}^*_n \nearrow R$ as claimed.

That consumer surplus converges to 0 as $n \to \infty$ now follows from $\underline{p}^*_n \nearrow R$.

Similarly, we can construct a *decreasing* sequence of distributions $\{F_n\}_{n\geq 0}$ satisfying Assumptions 1 and 2 such that $F_0 = F$ and $F_n(R-c) \searrow 0$. Note that, along this sequence, we have F_n dominates F_{n-1} in the FOSD sense for all n. As explained in the main text, consumer surplus converges to 0 as $n \to \infty$, and hence consumer surplus in an economy featuring F_n is lower than that in an economy featuring F for n large enough. ∎

Appendix B: Omitted Proofs in Section 4

We first prove three lemmas that will be used in the proofs of both Proposition 4 and Proposition 6. Define

$$\mathbf{P} := \{(p_1, p_2) \,|\, p_1 \le R,\ p_2 = 2R - p_1\}.$$

Lemma 2 *Consider the case of a monopolist marketing up to two different brands. Suppose consumers' strategy* (q_1, q_2) *is such that* $q_1 = 0$*. Then, for every* (p_1, p_2) *such that* $p_1 < R$ *and* $p_1 \le p_2 < 2R - p_1$*, there exists* $(p'_1, p'_2) \in \mathbf{P}$ *such that the monopolist makes strictly higher profit (i.e.,* $\pi(p_1, p_2) < \pi(p'_1, p'_2)$*).*

Proof: Fix any $p_1 < R$. If the monopolist is to set $p_2 \in [R, 2R - p_1)$, its profit will be

$$\begin{aligned}
\pi(p_1, p_2) &= (p_1 - c)F(p_2 - p_1) + q_2\left(\frac{p_1 + p_2}{2} - c\right)[1 - F(p_2 - p_1)] \\
&< \left(R - \frac{p_2 - p_1}{2} - c\right)F(p_2 - p_1) + q_2\,(R - c)\,[1 - F(p_2 - p_1)] \\
&= \pi\left(R - \frac{p_2 - p_1}{2}, R + \frac{p_2 - p_1}{2}\right),
\end{aligned}$$

where the strict inequality follows from $p_1 < R - (p_2 - p_1)/2$.

If the monopolist is to set $p_2 \in [(p_1 + R)/2, R)$, its profit will be

$$\pi(p_1, p_2) = (p_1 - c)F(R - p_1) + q_2\left(\frac{p_1 + p_2}{2} - c\right)[1 - F(p_2 - p_1)],$$

which is weakly increasing in p_2 (strictly so if $q_2 > 0$), and hence according to the last paragraph is also strictly worse than $(p'_1, p'_2) = (R - (R - p_1)/2, R + (R - p_1)/2) \in \mathbf{P}$.

If the monopolist is to set $p_2 \in [p_1, (p_1 + R)/2)$, its profit will be

$$\begin{aligned}
\pi(p_1, p_2) = {} & (p_1 - c)F(p_2 - p_1) \\
& + \left(\frac{p_1 + p_2}{2} - c\right)[F(R - p_2) - F(p_2 - p_1)] \\
& + (p_1 - c)\,[F(R - p_1) - F(R - p_2)] \\
& + q_2\left(\frac{p_1 + p_2}{2} - c\right)[1 - F(R - p_1)] \\
\le {} & \left(\frac{p_2 + p_2}{2} - c\right)(F(R - p_1) + q_2\,[1 - F(R - p_1)])\,.
\end{aligned}$$

However, if the monopolist is to set prices $(p'_1, p'_2) = (p_2, 2R - p_2) \in \mathbf{P}$, its profit will be

$$\begin{aligned}
\pi(p'_1, p'_2) &= \pi(p_2, 2R - p_2) \\
&= (p_2 - c)F(2R - 2p_2) + q_2(R - c)[1 - F(2R - 2p_2)] \\
&> \left(\frac{p_1 + p_2}{2} - c\right)F(2R - 2p_2) + q_2\left(\frac{p_1 + p_2}{2} - c\right)[1 - F(2R - 2p_2)] \\
&> \left(\frac{p_2 + p_2}{2} - c\right)(F(R - p_1) + q_2\left[1 - F(R - p_1)\right]) \\
&\geq \pi(p_1, p_2),
\end{aligned}$$

where the second inequality follows from $2R - 2p_2 > 2R - 2(p_1 + R)/2 = R - p_1$. ■

Lemma 3 *Consider the case of a monopolist marketing up to two different brands. Suppose consumers' strategy (q_1, q_2) is such that $q_1 = 0$. Suppose, furthermore, either $q_1 = 0$ or $f(0)(R-c) \geq 1/2$. Then, for every (p_1, p_2) such that $p_1 < R$ and $p_2 > 2R - p_1$, there exists $(p'_1, p'_2) \in \mathbf{P}$ such that the monopolist makes weakly higher profit (i.e., $\pi(p_1, p_2) \leq \pi(p'_1, p'_2)$).*

Proof: Fix any $p_1 < R$. If the monopolist is to set $p_2 > 2R - p_1$, its profit will be

$$\begin{aligned}
\pi(p_1, p_2) = (p_1 - c)F(R - p_1) & \\
&+ (p_1 - c)\left[F(p_2 - p_1) - F(p_2 - R)\right] \\
&+ q_2\left(\frac{p_1 + p_2}{2} - c\right)\left[1 - F(p_2 - p_1)\right].
\end{aligned}$$

Partial-differentiating wrt p_2, we have

$$\begin{aligned}
\frac{\partial \pi}{\partial p_2} &= (p_1 - c)\left[f(p_2 - p_1) - f(p_2 - R)\right] \\
&\quad + q_2\left[\frac{1 - F(p_2 - p_1)}{2} - \left(\frac{p_1 + p_2}{2} - c\right)f(p_2 - p_1)\right] \\
&\leq \frac{q_2 f(p_2 - p_1)}{2}\left[\frac{1 - F(p_2 - p_1)}{f(p_2 - p_1)} - 2\left(\frac{p_1 + p_2}{2} - c\right)\right] \\
&\leq \frac{q_2 f(p_2 - p_1)}{2}\left[\frac{1}{f(0)} - 2(R - c)\right] \\
&\leq 0,
\end{aligned}$$

where the first and the second inequalities follow from Assumption 1, and the third inequality follows from the supposition that either $q_2 = 0$ or $f(0)(R - c) \geq 1/2$. Therefore, (p_1, p_2) is weakly worse than $(p_1', p_2') = (p_1, 2R - p_1) \in \mathbf{P}$. ∎

Lemma 4 *Consider the case of a monopolist marketing up to two different brands. Consider the following constrained maximization problem: the monopolist is to maximize its profit by seting (p_1, p_2), subject to the constraints that $(p_1, p_2) \in \mathbf{P}$, and given the consumers' strategy (q_1, q_2). The monopolist's problem has a unique solution that depends only on q_2 but not on q_1. There exists a strictly increasing function $p_1(\cdot)$, with $p_1(0) > c$ and $p_1(1) = R$, such that, for any $q_2 \in [0, 1]$, the monopolist's unique solution is $p_1 = p_1(q_2)$ and $p_2 = 2R - p_1(q_2)$.*

Proof: Fix any (q_1, q_2). For any $(p_1, p_2) \in \mathbf{P}$, the monopolist's profit is

$$\pi = (p_1 - c)F(2R - 2p_1) + q_2(R - c)[1 - F(2R - 2p_1)],$$

which apparently depends only on q_2 but not on q_1.

Differentiating π wrt p_1, we have

$$\begin{aligned}\frac{d\pi}{dp_1} &= F(2R - 2p_1) - 2(p_1 - c)f(2R - 2p_1) + 2q_2(R - c)f(2R - 2p_1) \\ &= f(2R - 2p_1)\left[\frac{F(2R - 2p_1)}{f(2R - 2p_1)} - 2(p_1 - c) + 2q_2(R - c)\right]. \end{aligned} \tag{1}$$

By Assumption 1, F/f is weakly increasing, and hence $F(2R-2p_1)/f(2R-2p_1)$ is weakly decreasing in p_1. Therefore, the term inside the square brackets is strictly decreasing in p_1. This shows that π is quasi-concave in p_1, and hence admits a unique maximizer, denoted by $p_1(q_2)$.

Since the term inside the square brackets is strictly positive at $p_1 = c$ and strictly negative (unless $q_2 = 1$) at $p_1 = R$, we have $p_1(q_2) \in (c, R)$ for all $q_2 \in [0, 1)$. As for $q_2 = 1$, the term inside the square brackets is strictly positive at any $p_1 < R$ and is 0 at $p_1 = R$. Therefore, we have $p_1(1) = R$.

Finally, since an increase in q_2 strictly increases the term inside the square brackets, $p_1(\cdot)$ is strictly increasing in q_2. ∎

Proof of Proposition 4: If the monopolist always sets prices that are mirror images of each other around R, then $q_2 = 0$ is apparently a best response of the consumers contingent on the all-tied case. Moreover, the single-contender case will never arise, and hence we are free to specify a consumer's (off-equilibrium) belief contingent on such an event. In particular, one possible (off-equilibrium) belief contingent on the single-contender case is that p_1 is strictly higher than R, while at the same time the consumer's type d is strictly larger than $p_1 - R$. Against such a belief, $q_1 = 0$ is the consumer's best response contingent on the single-contender case.

On the other hand, if $q_1 = q_2 = 0$, a consumer will never make a purchase unless he finds at least one brand priced strictly below R. The monopolist will then make 0 sales if it sets $p_1 \geq R$ (recall that $p_2 \geq p_1$ by our convention). Therefore the monopolist's optimal pricing strategy must have $p_1 < R$. By Lemmas 2 and 3, for every (p_1, p_2) such that $p_1 < R$ and $p_1 \leq p_2 \neq 2R - p_1$, there exists $(p_1', p_2') \in \mathbf{P}$ such that the monopolist makes weakly higher profit. Therefore, solutions of the constrained maximization problem described in Lemma 4 are also the monopolist's *un*constrained best responses. By Lemma 4, the constrained maximization problem admits a unique solution, namely $p_1^* = p_1(0)$ and $p_2^* = 2R - p_1(0)$, where $p_1(0) \in (c, R)$. ■

We prove Proposition 6 through a series of lemmas.

Lemma 5 *In the case of a monopolist marketing up to two different brands, in any pure-pricing-strategy equilibrium, $(p_1^* + p_2^*)/2 \geq R$.*

Proof: Suppose not. Then the monopolist's equilibrium profit is bounded from above by $(p_1^* + p_2^*)/2 - c < R - c$, because any consumer who purchases brand 2 with positive probability must also purchase brand 1 with the same probability.

Note that in equilibrium consumers know the monopolist's pricing strategy. Therefore, $q_2^* = 1$, as a consumer's unique best response is to purchase either brand randomly when he finds himself in the all-tied case. Given $q_2^* = 1$, however, the monopolist can profit from deviating to $p_1 = p_2 = R$, because its profit will increase to $R - c$, a contradiction. ■

Lemma 6 *In the case of a monopolist marketing up to two different brands, in any pure-pricing-strategy equilibrium, if $(p_1^* + p_2^*)/2 > R$, then $p_1^* = R$, $p_2^* = \infty$, and $q_1^* > 0$.*

Proof: Suppose there is a pure-pricing-strategy equilibrium where $(p_1^* + p_2^*)/2 > R$. Then $q_2^* = 0$ (when a consumer finds himself in the all-tied case, he rationally refrains from purchasing any brand). Apparently $p_1^* \le R$, otherwise we would have had $q_1^* = 0$ as well, and the monopolist's equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $p_1 = p_2 = (R + c)/2$.

We claim that, if $q_1^* = 1$, then $p_2^* = \infty$. The presumption of $(p_1^* + p_2^*)/2 > R$ implies $p_2^* > 2R - p_1^*$. For any $p_2 > 2R - p_1^*$, given $q_1^* = 1$ and $q_2^* = 0$, the monopolist's profit is

$$\pi(p_1^*, p_2) = (p_1^* - c)F(p_2 - p_1^*),$$

which is maximized by setting $p_2 = \infty$.

Suppose $p_1^* < R$. Then $q_1^* = 1$ indeed (a consumer who finds himself in the one-contender case would know for sure that the only contender is a good deal). Then, according to the claim in the above paragraph, we have $p_2^* = \infty$. However, by setting $p_1 < R$ and $p_2 = \infty$, given $q_1^* = 1$, the monopolist's profit is $p_1 - c$ (a consumer will purchase brand 1 for sure regardless whether he can discern p_1 and R), which is increasing in p_1. Hence the only possible candidate for p_1^* is R.

It remains to prove that $p_2^* = \infty$. Note that $q_1^* = 0$ is not an equilibrium, otherwise the monopolist"s equilibrium profit would have been 0, and would have profited strictly from deviating to, say, $p_1 = p_2 = (R + c)/2$. For any $q_1^* > 0$, the monopolist's profit is

$$\pi(p_1^*, p_2) = \pi(R, p_2) = q_1^*(R - c)F(p_2 - R),$$

which is maximized by setting $p_2 = \infty$. ■

Corollary 1 *In the case of a monopolist marketing up to two different brands, in any pure-pricing-strategy equilibrium,* $(p_1^* + p_2^*)/2 = R$.

Proof: By Lemma 5, it suffices to show that $(p_1^* + p_2^*)/2 > R$ is impossible. By Lemma 6, if $(p_1^* + p_2^*)/2 > R$, then $p_1^* = R$, $p_2^* = \infty$, and $q_1^* > 0$. Given $p_2^* = \infty$ and $q_1^* > 0$, the monopolist's profit from setting any $p_1 \ge R$ is

$$\pi(p_1, p_2^*) = \pi(p_1, \infty) = q_1^*(p_1 - c)\left[1 - F(p_1 - R)\right].$$

By the same argument as in the proof of Proposition 1, there exists some $\overline{p} > R$ such that $\pi\left(\overline{p}, \infty\right) > \pi(R, \infty) = \pi(p_1^*, p_2^*)$, a contradiction. ■

Lemma 7 *Consider the case of a monopolist marketing up to two different brands. Consider the following constrained maximization problem: the monopolist is to maximize its profit by setting* (p_1, p_2)*, subject to* $R \leq p_1 \leq p_2$*, and given consumers' strategy* (q_1, q_2) *is such that* $q_1 = 0$*. A solution of the monopolist's problem is* $(p_1, p_2) = (\overline{p}, \overline{p})$*, where* $\overline{p} > R$ *is as defined in Proposition 1.*

Proof: For any (p_1, p_2) such that $R \leq p_1 \leq p_2$, the monopolist's profit is

$$\begin{aligned}
\pi(p_1, p_2) &= q_2\left(\frac{p_1 + p_2}{2} - c\right)[1 - F(p_2 - R)] \\
&\leq q_2(p_2 - c)[1 - F(p_2 - R)] \\
&\leq q_2\left(\overline{p} - c\right)\left[1 - F\left(\overline{p} - R\right)\right] \\
&= \pi\left(\overline{p}, \overline{p}\right),
\end{aligned}$$

where the second inequality follows from the definition of $\overline{p}$ in the proof of Proposition 1. ■

Proof of Proposition 6: By Corollary 1, all pure-pricing-strategy equilibria resemble the one described in Proposition 4, in the sense that $(p_1^* + p_2^*)/2 = R$. Therefore, if (q_1^*, q_2^*) is the consumers' equilbirium strategy, the monopolist's equilibrium strategy must also solves the constrained maximization problem described in Lemma 4. That is, any pure-pricing-strategy equilibrium must take the form of $(p_1^*, p_2^*, q_1^*, q_2^*) = \left(p_1(q_2^*), 2R - p_1(q_2^*), q_1^*, q_2^*\right)$.

Apparently, if the monopolist is to set (p_1, p_2) such that $(p_1 + p_2)/2 = R$, any (q_1, q_2) would be a consumer's best response (see the proof of Proposition 4). Therefore, a candidate equilibrium $(p_1^*, p_2^*, q_1^*, q_2^*) = \left(p_1(q_2^*), 2R - p_1(q_2^*), q_1^*, q_2^*\right)$ is a valid equilibrium as long as (p_1^*, p_2^*) is also the monopolist's *un*constrained optimal choice given (q_1^*, q_2^*).

Note that, if $\left(p_1(q_2^*), 2R - p_1(q_2^*), q_1^*, q_2^*\right)$ is a pure-pricing-strategy equilibrium, then $\left(p_1(q_2^*), 2R - p_1(q_2^*), 0, q_2^*\right)$ will also be a pure-pricing-strategy equilibrium. This is because (*i*) the single-contender case is an off-equilibrium event, and any q_1 can be supported by some pessimistic enough off-equilibrium beliefs (see the proof of Proposition 4), and (*ii*) lowering q_1 to 0 does not affect the monopolist's equilibrium profit, but weakly lowers its profit if it were to deviate. In what follows we shall hence focus on equilibria of the form

$(p_1^*, p_2^*, q_1^*, q_2^*) = \left(p_1(q_2^*), 2R - p_1(q_2^*), 0, q_2^*\right)$.

Let $\mathbf{Q_2}$ denote the compact[23] set of q_2's such that $\left(p_1(q_2), 2R - p_1^*(q_2), 0, q_2\right)$ is a pure-pricing-strategy equilibrium. From Proposition 4 we know that $\mathbf{Q_2}$ is not empty, and contains the point 0.

For any $q_2, q_2' \in \mathbf{Q_2}$ such that $q_2 < q_2'$, the monopolist's equilibrium profits are

$$\begin{aligned}
\pi^*(q_2) &= (p_1(q_2) - c)\, F\,(2R - 2p_1(q_2)) + q_2(R - c)\,[1 - F\,(2R - 2p_1(q_2))] \\
&< (p_1(q_2) - c)\, F\,(2R - 2p_1(q_2)) + q_2'(R - c)\,[1 - F\,(2R - 2p_1(q_2))] \\
&\leq (p_1(q_2') - c)\, F\,(2R - 2p_1(q_2')) + q_2'(R - c)\,[1 - F\,(2R - 2p_1(q_2'))] \\
&= \pi^*(q_2'),
\end{aligned}$$

where the second inequality follows from the definition of $p_1(\cdot)$. Therefore, different equilibria are rankable in terms of the monopolist's equilibrium profit, with higher q_2^* implies higher profit.

On the other hand, in any pure-pricing-strategy equilibrium, a consumer gets a positive surplus only when he has a type $d < R - p_1(q_2^*)$, which enables him to compare p_1^* and R and hence identify brand 1 out of the two brands. His expected surplus is hence

$$\overline{CS} = (R - p_1(q_2^*))\, F\,(2R - 2p_1(q_2^*))\,, \tag{2}$$

which is strictly decreasing in q_2^*. Therefore, different equilibria are also rankable in terms of consumer surplus, with higher q_2^* implies lower consumer surplus.

Let $q_2^{max} := \sup \mathbf{Q_2}$. We shall now prove that $q_2^{max} < 1$.

Recall from the proof of Proposition 1 that $\overline{p}$ is the unique solution of $\max_{p \geq R}(p - c)[1 - F(p - R)]$. Since $\overline{p} > R$, we have

$$(\overline{p} - c)\,[1 - F\,(\overline{p} - R)] > (R - c).$$

Suppose $q_2^{max} = 1$, then by compactness of $\mathbf{Q_2}$ there is a pure-pricing-strategy equilibrium featuring $q_2^* = 1$. In such an equilibrium, by Lemma 4, we must have $p_1^* = p_2^* = R$. The monopolist's equilibrium profit is hence $R - c$.

If the monopolist deviates to $p_1 = p_2 = \overline{p}$, its profit will increase to $(\overline{p} - c)\,[1 - F\,(\overline{p} - R)]$

[23] Compactness follows from the usual continuity argument.

thanks to $q_2^* = 1$, a contradiction.

In the remainder of this proof, suppose $f(0)(R-c) \geq 1/2$. Then, by Lemmas 2, 3, 4, and 7, when consumers' strategy (q_1, q_2) is such that $q_1 = 0$, the monopolist's profit is maximized either at $(p_1, p_2) = (p_1(q_2), 2R - p_1(q_2))$, or at $(p_1, p_2) = (\overline{p}, \overline{p})$.

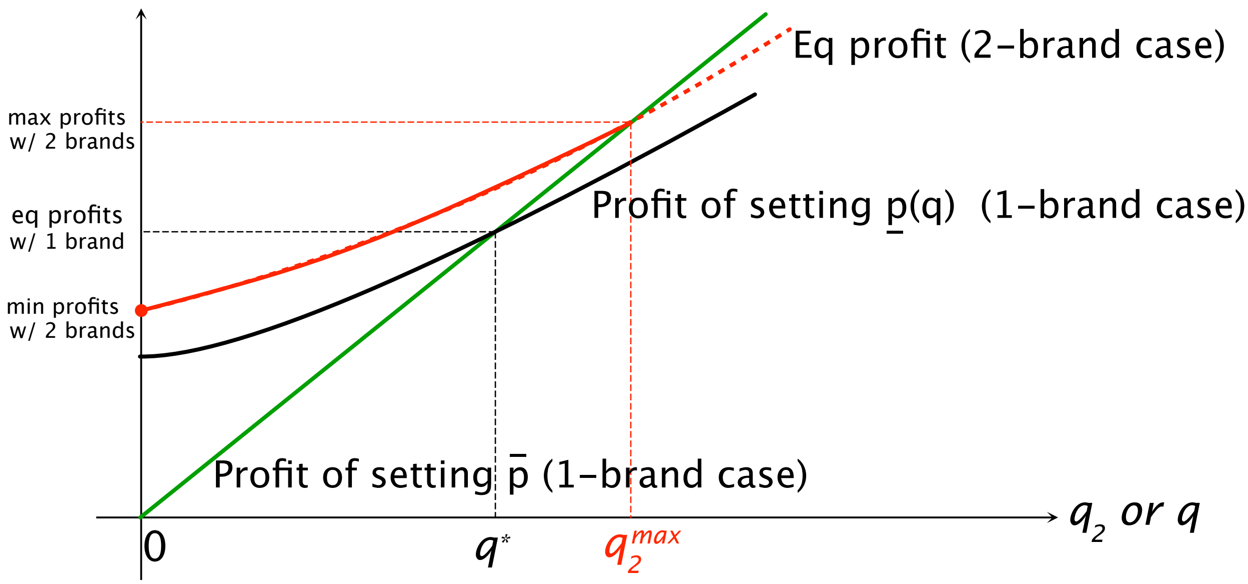

Figure 4: equilibrium profits in the one-brand and two-brand cases

In Figure 4, we plot the monopolist's profits at each of these two candidate maximizers as functions of q_2. The profit at $(p_1, p_2) = (\overline{p}, \overline{p})$ is depicted by the straight line passing through the origin. Incidentally, it is the same straight line in Figure 1, with q_2 replacing q in the expression of $\pi = q\,(\overline{p} - c)\,[1 - F\,(\overline{p} - R)]$.

The profit at $(p_1, p_2) = (p_1(q_2), 2R - p_1(q_2))$ is depicted by the upper convex curve, and has the expression of $\pi = (p_1(q_2) - c)\,F\,(2R - 2p_1(q_2)) + q_2(R - c)\,[1 - F\,(2R - 2p_1(q_2))]$. That it is strictly increasing and convex can be seen by totally differentiating it wrt q_2 using the Envelope Theorem, yielding

$$\frac{d\pi}{dp_2} = (R - c)\,[1 - F\,(2R - 2p_1(q_2))]\,,$$

which is strictly positive and strictly increasing in q_2.

The convex curve is strictly above the straight line at $q_2 = 0$, and is strictly below at $q_2 = 1$ (recall that $p_1(1) = R$). The shapes of the two profit functions dictate that the convex curve crosses the straight line once and only once, and crosses from above, at some $q_2^{max} \in (0, 1)$. Therefore, the candidate equilibrium $(p_1^*, p_2^*, q_1^*, q_2^*) = (p_1(q_2), 2R - p_1(q_2), 0, q_2)$ is a valid equilibrium if and only if $q_2 \in \left[0, q_2^{max}\right]$.

We also superimpose onto Figure 4 the monopolist's profit if it sets the low price $\underline{p}(q)$ in the one-brand case . It is depicted by the lower convex curve. Indeed, it is strictly below the upper convex curve at every $q < 1$, because it has the expression of

$$\begin{aligned}
\pi &= \left(\underline{p}(q) - c\right)\left(F\left(R - \underline{p}(q)\right) + q\left[1 - F\left(R - \underline{p}(q)\right)\right]\right) \\
&\leq \left(\underline{p}(q) - c\right)\left(F\left(2R - 2\underline{p}(q)\right) + q\left[1 - F\left(2R - 2\underline{p}(q)\right)\right]\right) \\
&\leq \left(\underline{p}(q) - c\right)F\left(2R - 2\underline{p}(q)\right) + q(R - c)\left[1 - F\left(2R - 2\underline{p}(q)\right)\right] \\
&\leq (p_1(q) - c)\, F\,(2R - 2p_1(q)) + q(R - c)\,[1 - F\,(2R - 2p_1(q))]
\end{aligned}$$

where the first two inequalities are strict if $\underline{p}(q) < R$, and the third inequality is strict if $\underline{p}(q) = R \neq p_1(q)$, and hence at least one of these inequalities is strict for every $q < 1$. Note that the last expression is exactly the same as the monopolist's profit at $(p_1, p_2) = (p_1(q_2), 2R - p_1(q_2))$ if we replace q with q_2, which proves that the upper and the lower convex curves touch only at $q = 1$.

It becomes apparent from Figure 4 that $q^* < q_2^{max}$, and that in the pure-pricing-strategy equilibrium featuring $q_2^* = q_2^{max}$, the monopolist's profit is higher than its equilibrium profit in the one-brand case. ■

Appendix C: Omitted Proofs in Section 6

We prove Proposition 9 through a series of lemmas.

Lemma 8 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium, if $p_1^* = p_2^* = p^*$, then $p^* < R$.*

Proof: Suppose $p^* > R$. Then we must have $q_2^* = 0$ (a consumer who finds himself in the all-tied case would rationally refuse to purchase either brand), resulting in 0 equilibrium profit for both sellers. Seller 1, for example, can profit from deviating to $p_1 = (R+c)/2 < R$. After such deviation, seller 1 can sell to at least $F((R-c)/2)$ consumers at a margin of $(R-c)/2$, which yields strictly positive profit, a contradiction.

Suppose $p^* = R$. Then $p_1 = R$ must be a best response against $p_2 = R$. When $p_2 = R$, seller 1's profit as a function of p_1 is

$$\pi_1(p_1) = \begin{cases} (p_1 - c)[1 - F(p_1 - R)]q_2/2 & \text{if } p_1 \geq R \\ (p_1 - c)\left(F(R - p_1) + [1 - F(R - p_1)]q_2/2\right) & \text{if } p_1 \leq R \end{cases}.$$

which is exactly the same profit function as that in the one-brand case (with q replaced by $q_2/2$, and p replaced by p_1). However, from the proof of , we know that $p_1 = R$ is never a best response regardless of $q_2/2$, a contradiction. ∎

Lemma 9 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium, we have $c < p_1^* < p_2^*$.*

Proof: By Lemma 8, it suffices to prove that there is no pure-pricing-strategy equilibrium with $p_1^* = p_2^* = p^* < R$. Suppose, on the contrary, such an equilibrium exists. Then we must have $q_2^* = 1$ (a consumer who finds himself in the all-tied case can guarantee a strictly positive surplus of $R - p^*$ by purchasing randomly from one of the two sellers). Note that, since the single-contender case is an off-equilibrium event, any q_1^* can be supported by some off-equilibrium belief. It is wlog to set $q_1^* = 0$, because that leaves the sellers' equilibrium profits intact, while makes their deviation profits weakly lower.

Given $(q_1^*, q_2^*) = (0, 1)$ and $p_1^* = p_2^* = p^*$, seller 1's equilibrium profit is

$$\pi_1^* = (p^* - c)/2.$$

If seller 1 deviates to $p_1 = R$, its profit will become

$$\pi_1 = (R - c)[1 - F(R - p^*)]/2.$$

We shall show that $\pi_1 > \pi_1^*$, which will be a contradiction. Let $H(p^*) := 2(\pi_1 - \pi_1^*)$, and differentiate it wrt p^*, we have

$$H'(p^*) = (R - c)f(R - p^*) - 1 \leq (R - c)f(0) - 1 < 0,$$

where the first and the second inequalities follows from Assumption 1 and 2, respectively. Therefore, $H(p^*) > H(R) = 0$ for any $p^* < R$, and hence $\pi_1 > \pi_1^*$ as claimed.

If $p_1^* \leq c$, seller 1 makes non-positive profit, and can strictly profit from deviating to $p_1 = (c + p_2^*)/2$, a contradiction. ■

Lemma 10 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium, we have $(p_1^* + p_2^*)/2 \leq R$.*

Proof: By Lemma 9, we have $c < p_1^* < p_2^*$. Suppose $(p_1^* + p_2^*)/2 > R$. Then $p_2^* > R$, and hence the only chance that seller 2 can make a sale is to sell to consumers who find themselves in the all-tied case. However, $(p_1^* + p_2^*)/2 > R$ also implies that any consumer who finds himself in the all-tied case would rationally refuse to purchase; i.e., $q_2^* = 0$. Therefore, seller 2 makes 0 equilibrium profit, and can strictly profit from deviating to $p_2 = (p_1^* + c)/2$, a contradiction. ■

Lemma 11 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium, we have $p_2^* > R$.*

Proof: By Lemma 9, we have $c < p_1^* < p_2^*$. Suppose $p_2^* \leq R$. Then $(p_1^* + p_2^*)/2 < R$, and hence any consumer who finds himself in the all-tied case would rationally purchase for sure; i.e., $q_2^* = 1$.

Divide consumers into two (disjoint and exhaustive) groups. The first group are consumers who purchases from seller 1 for sure; i.e., consumers with type d in the set

$$D_1 := \left\{d \,\middle|\, d < p_2^* - p_1^*\right\} \cup \left\{d \,\middle|\, \max\{p_2^* - p_1^*, R - p_2^*\} \leq d < R - p_1^*\right\}.$$

The second group are consumers who purchase from each seller with probability 1/2; i.e., consumers with type d in the set

$$D_2 := \left\{d \,\middle|\, p_2^* - p_1^* \le d < R - p_2^*\right\} \cup \left\{d \,\middle|\, d \ge R - p_1^*\right\}.$$

Seller 1's equilibrium profit is

$$\pi_1^* = (p_1^* - c)[Pr(D_1) + Pr(D_2)/2].$$

If seller 1 deviates to $p_1 = p_2^*$, it will share the market with seller 2, resulting in profit

$$\pi_1 = (p_2^* - c)/2 = (p_2^* - c)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(p_1^* - c)[Pr(D_1) + Pr(D_2)/2] \ge (p_2^* - c)[Pr(D_1)/2 + Pr(D_2)/2]. \tag{3}$$

On the other hand, seller 2's equilibrium profit is

$$\pi_2^* = (p_2^* - c)Pr(D_2)/2.$$

If seller 2 deviates to $p_2 = p_1^*$, it will share the market with seller 1, resulting in profit

$$\pi_2 = (p_1^* - c)/2 = (p_1^* - c)[Pr(D_1)/2 + Pr(D_2)/2].$$

Equilibrium requires that such deviation is not profitable; i.e.,

$$(p_2^* - c)Pr(D_2)/2 \ge (p_1^* - c)[Pr(D_1)/2 + Pr(D_2)/2]. \tag{4}$$

Adding (3) and (4), we have $p_1^* \ge p_2^*$, a contradiction. ■

Lemma 12 *In the case of two sellers marketing up to only one brand each, in any pure-pricing-strategy equilibrium, we have* $(p_1^*, p_2^*) = (R - x^*, R + x^*)$, *where* $x^* > 0$ *is the unique solution* *to*

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + R - c.$$

Proof: By Lemmas 10 and 11, we have $(p_1^* + p_2^*)/2 \leq R$ and $p_2^* > R$, which together imply $p_1^* < R < p_2^*$ and $q_1^* = 1$ (any consumer who finds himself in the single-contender case would rationally purchase for sure). Suppose $(p_1^* + p_2^*)/2 < R$. Then any consumer who finds himself in the all-tied case would rationally purchase for sure; i.e., $q_2^* = 1$. Against $p_1^* < R$ and $(q_1^*, q_2^*) = (1, 1)$, seller 2's profit for any $p_2 > R$ is

$$\pi_2 = (p_2 - c)[1 - F(p_2 - p_1^*)]/2,$$

which implies that p_2^* satisfies the FOC of

$$1 - F(p_2^* - p_1^*) = (p_2^* - c)f(p_2^* - p_1^*). \tag{5}$$

Similarly, against $p_2^* > R$ and $(q_1^*, q_2^*) = (1, 1)$, seller 1's profit for any $p_1 < R$ is

$$\pi_1 = (p_1 - c) - (p_1 - c)[1 - F(p_2^* - p_1)]/2,$$

which implies that p_1^* satisfies the FOC of

$$1 - [1 - F(p_2^* - p_1^*)]/2 = (p_1^* - c)f(p_2^* - p_1^*)/2. \tag{6}$$

Multiply (6) by 2, and subtract (5) from it, we have $2F(p_2^* - p_1^*) = (p_1^* - p_2^*)f(p_2^* - p_1^*) < 0$, a contradiction. This proves that $(p_1^* + p_2^*)/2 = R$.

Recall that seller 2 must make strictly positive equilibrium profit (otherwise it can strictly profit from deviating to $p_2 = (c + p_1)/2$), hence we must have $q_2^* > 0$ (because the only chance that seller 2 makes a sale is when a consumer finds himself in the all-tied case). Against $p_1^* < R$ and $q_2^* > 0$, for any $p_2 > R$, seller 2's profit is

$$\pi_2 = (p_2 - c)[1 - F(p_2 - p_1^*)]q_2^*/2,$$

which implies that p_2^* satisfies the same FOC as (5). Rewrite (5) as

$$\frac{1 - F(2x^*)}{f(2x^*)} = x^* + R - c, \tag{7}$$

where $x^* := R - p_1^* = p_2^* - R > 0$. Since the LHS of (7) is weakly decreasing in x^* by Assumption 1, while the RHS is strictly increasing in x^* without bound, a solution to (7) is unique if it exists. Existence of a strictly positive solution follows from the fact that, by Assumption 2, we have the LHS strictly bigger than the RHS at $x^* = 0$. ■

Proof of Proposition 9: The first half of Proposition 9 follows from Lemma 12. It remains to prove the second half.

By the first half of the Proposition 9, in any pure-pricing-strategy equilibrium, a consumer gets a positive surplus iff he has a type $d < 2x^*$ (in which case he can compare prices and is able to identify the lower-price brand 1). His expected surplus is hence

$$CS = x^* F(2x^*).$$

In the equilibrium described in Proposition 4 for the case of a monopolist marketing up to two different brands, by (2) in the proof of Proposition 6, consumers' surplus is

$$\overline{CS} = (R - p_1(0))F(2R - 2p_1(0)) = \overline{x}F(2\overline{x}),$$

where $\overline{x} := R - p_1(0)$ is the unique solution to the first-order condition

$$\frac{F(2\overline{x})}{f(2\overline{x})} = 2(R - c - \overline{x}) \tag{8}$$

by (1) in the proof of Lemma 4 (where we have simplified using $q_2 = 0$). To prove the second half of Proposition 9, it suffices to prove that $x^* < \overline{x}$.

Note that the LHS of (8) is strictly increasing and the RHS is strictly decreasing, and they are equal to each other when evaluated at $\overline{x}$. Therefore, to prove that $x^* < \overline{x}$, it suffices to prove that the LHS of (8) is strictly smaller than the RHS when they are evaluated at x^*.

That is, it suffices to prove that

$$\frac{F(2x^*)}{f(2x^*)} < 2(R - c - x^*). \tag{9}$$

Step 1: We first give a lower bound for $(R - c - x^*)$.

Let $\pi_1(p_1, p_2^*)$ and $\pi_2(p_1^*, p_2)$ be sellers 1's and 2's profits as functions of their own prices, where we have suppressed their dependence on (q_1, q_2). Recall from Figure 3 that $\pi_1(\cdot, R+x$ has a kink at $p_1 = R - x^*$. In order for $p = R - x^*$ to be a local optimum for $\pi_1(p_1, p_2^*)$, the right derivative at $p = R - x^*$ must be non-positive.

For $p_1 \in [R - x^*, R]$,

$$\begin{aligned}
\pi_1(p_1, p_2^*) &= (p_1 - c)\Big(F(R - p_1) + q_1\left[F(p_2^* - R) - F(R - p_1)\right] + \left[F(p_2^* - p_1) - F(p_2^* - R)\right] + \frac{q_2}{2}\left[1 - F(p_2^* - p_1)\right]\Big) \\
&= (p_1 - c)\Big(F(p_2^* - p_1) + \frac{q_2}{2}\left[1 - F(p_2^* - p_1)\right] - (1 - q_1)\left[F(p_2^* - R) - F(R - p_1)\right]\Big).
\end{aligned}$$

Therefore, for $p_1 \in (R - x^*, R)$,

$$\begin{aligned}
\frac{\partial \pi_1(p_1, p_2^*)}{\partial p_1} &= \Big(F(p_2^* - p_1) + \frac{q_2}{2}[1 - F(p_2^* - p_1)] - (1 - q_1)[F(p_2^* - R) - F(R - p_1)]\Big) \\
&\quad - (p_1 - c)\left[\left(1 - \frac{q_2}{2}\right) f(p_2^* - p_1) + (1 - q_1) f(R - p_1)\right] \\
&\geq \Big(F(p_2^* - p_1) + \frac{q_2}{2}[1 - F(p_2^* - p_1)] - [F(p_2^* - R) - F(R - p_1)]\Big) \\
&\quad - (p_1 - c)\left[\left(1 - \frac{q_2}{2}\right) f(p_2^* - p_1) + f(R - p_1)\right].
\end{aligned}$$

Local optimality of $p_1 = R - x^*$ hence requires that

$$\begin{aligned}
0 &\geq \left.\frac{\partial \pi_1(p_1, p_2^*)}{\partial p_1}\right|_{p_1 = p_1^* +} \\
&\geq \Big(F(2x^*) + \frac{q_2}{2}\left[1 - F(2x^*)\right]\Big) - \left[\left(1 - \frac{q_2}{2}\right) f(2x^*) + f(x^*)\right](R - c - x^*) \\
&= \left[F(2x^*) + \frac{q_2}{2} f(2x^*)(R - c + x^*)\right] - \left[\left(1 - \frac{q_2}{2}\right) f(2x^*) + f(x^*)\right](R - c - x^*) \\
&= F(2x^*) + q_2 f(2x^*)(R - c) - \left[f(2x^*) + f(x^*)\right](R - c - x^*),
\end{aligned}$$

where the first equality follows from (7). This implies

$$q_2 \leq \frac{[f(2x^*) + f(x^*)](R - c - x^*) - F(2x^*)}{f(2x^*)(R - c)}. \tag{10}$$

Rearranging terms, we can express (10) as a lower bound for $(R - c - x^*)$:

$$R - c - x^* \geq \frac{F(2x^*) + q_2 f(2x^*)(R - c)}{f(2x^*) + f(x^*)} \geq \frac{F(2x^*)}{f(2x^*) + f(x^*)}. \tag{11}$$

STEP 2: We next give an upper bound for $F(2x^*)$.

In order for $p_2 = R + x^*$ to be a global optimum for $\pi_2(p_1^*, p_2)$, we must have

$$\begin{aligned} & \pi_2(R - x^*, R + x^*) \geq \pi_2(R - x^*, R - x^*) \\ \iff \quad & \frac{q_2}{2}\left[1 - F(2x^*)\right](R - c + x^*) \geq \left(\frac{1}{2}F(x^*) + \frac{q_2}{2}\left[1 - F(x^*)\right]\right)(R - c - x^*) \end{aligned}$$

which requires

$$q_2 \geq \frac{F(x^*)(R - c - x^*)}{\left[1 - F(2x^*)\right](R - c + x^*) - \left[1 - F(x^*)\right](R - c - x^*)} \geq 0. \tag{12}$$

Combining (10) and (12), we have

$$\frac{F(x^*)(R - c - x^*)}{\left[1 - F(2x^*)\right](R - c + x^*) - \left[1 - F(x^*)\right](R - c - x^*)} \leq \frac{[f(2x^*) + f(x^*)](R - c - x^*) - F(2x^*)}{f(2x^*)(R - c)}.$$

Rearranging terms, we have

$$
\begin{aligned}
F(2x^*) &\leq [f(2x^*) + f(x^*)](R-c-x^*) - \frac{F(x^*)(R-c-x^*)f(2x^*)(R-c)}{[1-F(2x^*)](R-c+x^*) - [1-F(x^*)](R-c-x^*)} \\
&= f(2x^*)(R-c-x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{F(x^*)(R-c)}{[1-F(2x^*)](R-c+x^*) - [1-F(x^*)](R-c-x^*)}\right) \\
&\leq f(2x^*)(R-c-x^*) \\
&\quad \times \left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{x^* f(x^*)(R-c)}{[1-F(2x^*)](R-c+x^*) - [1-F(2x^*) + x^* f(2x^*)](R-c-x^*)}\right) \\
&= f(2x^*)(R-c-x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{x^* f(x^*)(R-c)}{2x^*[1-F(2x^*)] - x^* f(2x^*)(R-c-x^*)}\right) \\
&= f(2x^*)(R-c-x^*)\left(1 + \frac{f(x^*)}{f(2x^*)} - \frac{f(x^*)(R-c)}{2f(2x^*)(R-c+x^*) - f(2x^*)(R-c-x^*)}\right) \\
&= f(2x^*)(R-c-x^*)\left[1 + \frac{f(x^*)}{f(2x^*)}\left(1 - \frac{R-c}{2(R-c+x^*) - (R-c-x^*)}\right)\right] \\
&= f(2x^*)(R-c-x^*)\left[1 + \frac{f(x^*)}{f(2x^*)}\left(1 - \frac{R-c}{R-c+3x^*}\right)\right],
\end{aligned}
$$

where the second inequality follows from Assumption 1,[24] and the third equality follows from (7). Therefore, to prove (9), it suffices to prove that

$$\frac{f(x^*)}{f(2x^*)}\left(1 - \frac{R-c}{R-c+3x^*}\right) < 1. \tag{13}$$

Step 3: We shall now prove (13).

By (11), we have

$$
\begin{aligned}
\left[f(2x^*) + f(x^*)\right](R-c-x^*) &\geq F(2x^*) \\
&= 1 - f(2x^*)(R-c+x^*),
\end{aligned}
$$

where the equality follows from (7). Rearranging, we have

$$
\begin{aligned}
1 &\leq 2f(2x^*)(R-c) + f(x^*)(R-c-x^*) \\
&\leq 2f(0)(R-c) + f(0)(R-c) \\
&= 3f(0)(R-c), \qquad (14)
\end{aligned}
$$

[24] By Assumption 1, f is weakly decreasing. Hence $F(x^*) \geq x^* f(x^*)$ and $F(2x^*) - F(x^*) \geq x^* f(2x^*)$.

where the second inequality follows from Assumption 1.

By (7), we have

$$x^* = \frac{1 - F(2x^*)}{f(2x^*)} - (R - c) \leq \frac{1}{f(0)} - (R - c),$$

where the inequality follows from Assumption 1. Therefore,

$$\frac{x^*}{R - c + x^*} \leq \frac{1/f(0) - (R - c)}{R - c + 1/f(0) - (R - c)} = 1 - f(0)(R - c) \leq \frac{2}{3}, \tag{15}$$

where the last inequality follows from (14).

By (11) again, we have

$$R - c - x^* \geq \frac{F(2x^*)}{f(2x^*) + f(x^*)} \geq \frac{2x^* f(2x^*)}{f(2x^*) + f(x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\begin{aligned} R - c &\geq x^* \left(1 + \frac{2f(2x^*)}{f(2x^*) + f(x^*)}\right) \\ \frac{x^*}{R - c} &\leq \frac{f(2x^*) + f(x^*)}{3f(2x^*) + f(x^*)}. \end{aligned} \tag{16}$$

By Assumption 1,

$$0 \leq \left(\frac{f}{1 - F}\right)' = \frac{f'}{1 - F} + \left(\frac{f}{1 - F}\right)^2 = \frac{f}{1 - F}\left(\frac{f'}{f} + \frac{f}{1 - F}\right) = \frac{f}{1 - F}\left((\ln f)' + \frac{f}{1 - F}\right).$$

Therefore, for any $x \in [x^*, 2x^*]$,

$$\frac{d \ln f(x)}{dx} \geq -\frac{f(x)}{1 - F(x)} \geq -\frac{f(2x^*)}{1 - F(2x^*)},$$

where the second inequality follows from Assumption 1. Therefore,

$$\ln f(2x^*) - \ln f(x^*) = \int_{x=x^*}^{2x^*} \frac{d \ln f(x)}{dx} dx \geq -\frac{x^* f(2x^*)}{1 - F(2x^*)} = -\frac{x^*}{R - c + x^*} \geq -\frac{2}{3},$$

where the second equality follows from (7), and the last inequality follows from (15). We hence have

$$\frac{f(x^*)}{f(2x^*)} \leq \exp\left(\frac{x^*}{R-c+x^*}\right) \leq \exp(2/3), \tag{17}$$

and, by (16),

$$\frac{x^*}{R-c} \leq \frac{f(2x^*)+f(x^*)}{3f(2x^*)+f(x^*)} = \frac{1+f(x^*)/f(2x^*)}{3+f(x^*)/f(2x^*)} \leq \frac{1+\exp(2/3)}{3+\exp(2/3)}. \tag{18}$$

The upper bound (18) allows us to give an even tighter upper bound for $f(x^*)/f(2x^*)$ than (17):

$$\frac{f(x^*)}{f(2x^*)} \leq \exp\left(\frac{x^*}{R-c+x^*}\right) = \exp\left(\frac{x^*/(R-c)}{1+x^*/(R-c)}\right) \leq \exp\left(\frac{1+\exp(2/3)}{4+2\exp(2/3)}\right). \tag{19}$$

Plugging (16) and (19) into the LHS of (13), we then have

$$\begin{aligned}
\frac{f(x^*)}{f(2x^*)}\left(1-\frac{R-c}{R-c+3x^*}\right) &= \frac{f(x^*)}{f(2x^*)}\left(\frac{3x^*/(R-c)}{1+3x^*/(R-c)}\right) \\
&\leq \exp\left(\frac{1+\exp(2/3)}{4+2\exp(2/3)}\right) \times \left(\frac{3+3\exp(2/3)}{6+4\exp(2/3)}\right) \\
&= 0.9314 \\
&< 1,
\end{aligned}$$

as claimed. ■

Appendix D: When Assumption 2 is Violated

This appendix contains the complementary analysis for the case when Assumption 2 is violated. We start with a monopolist marketing a single brand.

Proposition 11 *Consider a monopolist marketing up to only one brand. In the case of* $f(0)(R-c) \geq 1$*, there exists a continuum of equilibria, indexed by different* $q^* \in [\overline{q}, 1]$ *with some* $\overline{q} \geq 1/2$*, where the monopolist sets the price* $p^* = R$*, and consumers purchase with probability* q^* *whenever they cannot discern the price of the product and their reservation price. Among these equilibria, the most efficient one is the one with* $q^* = 1$*, which also achieves the first best.*

Proof: Define $\overline{p}$ and $\underline{p}(q)$ as in the proof of Proposition 1; i.e., $\overline{p}$ is the unique maximizer of $\pi(p;q)$ in the sub-range $p \in [R, \infty)$, while $\underline{p}(q)$ is that in the sub-range $p \in [c, R]$. Since $\left.\frac{\partial \pi}{\partial p}\right|_{p=R+} = 1 - f(0)(R-c) \leq 0$, we have $\overline{p} = R$, and hence $\underline{p}(q)$ is also the unique maximizer of π in the whole range $p \in [c, \infty)$.

Define $\overline{q}$ as in the proof of Proposition 1; i.e., $\overline{q}$ is the point at which $\underline{p}(q)$ as an increasing function of q first reaches R (in other words, $\underline{p}(q) = R$ iff $q \geq \overline{q}$). Suppose $q^* < \overline{q} < 1$ in equilibrium. Then we must have $p^* = \underline{p}(q^*) < R$ in equilibrium as well. But then a consumer who is unable to discern p^* and R can still infer from his knowledge of the monopolist equilibrium strategy that $p^* = \underline{p}(q^*) < R$. His best response is hence to purchase the product for sure (i.e., $q^* = 1$), contradicting the presumption that $q^* < \overline{q} < 1$.

On the other hand, any $q^* \in [\overline{q}, 1]$ can be part of an equilibrium, with the monopolist's best response being $p^* = R$. ∎

We then move on to the case of a monopolist marketing up to two different brands. When Assumption 2 is violated, there are more pure-pricing-strategy equilibria. Not only that there are more pure-pricing-strategy equilibria where the monopolist markets two different brands (and sets prices that are mirror images of each other around R), there is also a pure-pricing-strategy equilibrium where the monopolist markets only one brand.

Proposition 12 *Consider a monopolist marketing up to two different brands. In the case of* $f(0)(R-c) \geq 1$*,*

- *there exists a strictly increasing function* $p_1(\cdot)$ *that maps* $[0,1]$ *into* $(c, R]$*, with* $p_1(1) = R$*,* *such that, for every $q_2^* \in [0,1]$, there exists a pure-pricing-strategy equilibrium featuring that specific q_2^*, in which the monopolist sets deterministic prices $p_1^* = p_1(q_2^*)$ and $p_2^* = 2R - p_1^*$;*

- *there also exists a pure-pricing-strategy equilibrium featuring $(p_1^*, p_2^*) = (R, \infty)$ and $(q_1^*, q_2^*) = (1, 0)$.*

Proof: Note that Lemmas 2, 3, 4, 5, 6, 7 (with "$\overline{p} > R$" replaced by "$\overline{p} = R$"), and Proposition 4 remain valid, because their proofs do not rely on Assumption 2. However, the proof of Corollary 1 falls apart, because it relies on an argument made in the proof of Proposition 1, which in turn relies on Assumption 2. Therefore, we continue to have pure-pricing-strategy equilibria of the form described in Proposition 6, but we can no longer use Corollary 1 to rule out pure-pricing-strategy equilibria featuring $(p_1^*, p_2^*) = (R, \infty)$.

Let q_2^{max} be defined as in the proof of Proposition 6. Since $f(0)(R - c) > 1/2$ when Assumption 2 is violated, the same argument as in the last part of the proof of Proposition 6 suggests that (*i*) for every $q_2^* \in [0, q_2^{max}]$, there exists a pure-pricing-strategy equilibrium featuring that specific q_2^*, in which the monopolist sets deterministic prices $p_1^* = p_1(q_2^*)$ and $p_2^* = 2R - p_1^*$ (where $p_1(\cdot)$ is defined in Lemma 4), and (*ii*), q_2^{max} is the maximum q_2 such that profit at $(p_1, p_2) = (p_1(q_2), 2R - p_1(q_2))$ is weakly higher than profit at $(p_1, p_2) = (\overline{p}, \overline{p})$. However, when Assumption 2 is violated, $\overline{p} = R$, and hence the former profit is weakly higher than the latter profit for every q_2 by the definition of $p_1(\cdot)$. Therefore, we have $q_2^{max} = 1$, which implies the first half of the proposition.

To prove the second half of the proposition, note that if $(p_1^*, p_2^*) = (R, \infty)$, then any q_1 is a best response, and any q_2 is a best response against some off-equilibrium belief. So it suffices to prove that $(p_1, p_2) = (R, \infty)$ is a best response against $(q_1^*, q_2^*) = (1, 0)$. The proof, however, is almost the same as that for $p = R$ being a best response against $q = 1$ in the one-brand case when Assumption 2 is violated (see the proof of Proposition 11), and hence is omitted. ∎

References

[1] Beja, A. and I. Gilboa (1992), "Numerical Representations of Imperfectly Ordered Preferences (A Unified Geometric Exposition)." *Journal of Mathematical Psychology*, 36: 426-449.

[2] Chen, Yuxin, Ganesh Iyer, and Amit Pazgal (2010), "Limited Memory, Categorization, and Competition." *Marketing Science*, 29(4): 650-670.

[3] de Clippel, Geoffroy, Kfir Eliaz, and Kareen Rozen (2014), "Competing for Consumer Inattention." *Journal of Political Economy*, 122(6):1203-1234.

[4] Fishburn, Peter C. (1970a), "Intransitive Indifference in Preference Theory: A Survey." *Operations Research*, 18: 207-228.

[5] Fishburn, Peter C. (1970b), "Intransitive Indifference with Unequal Indifference Intervals." *Journal of Mathematical Psychology*, 7: 144-149.

[6] Fisburn, Peter C. (1985), "Semiorders and Choice Functions." *Econometrica*, 43(5/6): 975-977.

[7] Gilboa, Itzhak (2009), *Theory of Decision under Uncertainty*, Cambridge University Press.

[8] Gul, Faruk, Wolfgang Pesendorfer, and Tomasz Strzalecki (2017), "Coarse Competitive Equilibrium and Extreme Prices." *American Economic Review*, 107(1): 109-137.

[9] Huber, Joel, John W. Payne, and Christopher Puto (1982), "Adding Asymmetrically Dominated Alternatives: Violations of Regularity and the Similarity Hypothesis." *Journal of Consumer Research*, 9(1): 90-98.

[10] Jamison, Dean T. and Lawrence J. Lau (1973), "Semiorders and the Theory of Choice." *Econometrica*, 41(5): 901-912.

[11] Kamada, Yuichiro (2016), "Naive and Sophisticated Choices under Intransitive Indifference." mimeo, UC Berkeley.

[12] Kim, Jongmin, Nathan Novemsky, and Ravi Dhar (2012), "Adding Small Differences Can Increase Similarity and Choice." *Psychological Science*, 24(2): 225-229.

[13] Luce, Duncan (1956), "Semiorders and a Theory of Utility Discrimination." *Econometrica*, 24(2): 178-191.

[14] Mochon, Daniel (2013), "Single-Option Aversion." *Journal of Consumer Research*, 41: 555-566.

[15] Natenzon, Paulo (forthcoming), "Random Choice and Learning." *Journal of Political Economy*.

[16] Piccione, Michele and Ariel Rubinstein (2003), "Modeling the Economic Interaction of Agents with Diverse Abilities to Recognize Equilibrium Patterns." *Journal of European Economic Association*, 1(1): 212-223.

[17] Piccione, Michele and Ran Spiegler (2012), "Price Competition under Limited Comparability." *Quarterly Journal of Economics*, 127(1): 97-135.

[18] Rob, Rafael (1985), "Equilibrium Price Distributions." *Review of Economic Studies*, 52(3): 487-504.

[19] Rubinstein, Ariel (1993), "On Price Recognition and Computational Complexity in a Monopolistic Model." *Journal of Political Economy*, 101(3): 473-484.

[20] Salop, Steven (1977), "The Noisy Monopolist: Imperfect Information, Price Dispersion and Price Discrimination." *Review of Economic Studies*, 44(3): 393-406.

[21] Salop, Steven and Joseph Stiglitz (1977), "Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion." *Review of Economic Studies*, 44(3): 493-510.

[22] Varian, Hal (1980), "A Model of Sales." *American Economic Review*, 70: 651-659.