Of Cities and Slums*

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Abstract

The emergence of slums is a frequent feature of a country's path toward urbanization, structural transformation, and development. Based on salient micro and macro evidence from Brazilian labor, housing, and education markets, we construct a simple dynamic model to examine the conditions for slums to emerge. We use the model to determine whether slums are barriers or stepping-stones for the ascension of low-skilled households and for the development of the country as a whole, exploring the dynamic interaction of slums, housing costs and sectoral productivities with the human capital formation and structural transformation of a country. We calibrate our model to Brazilian data, and use it to conduct counterfactual experiments. We find that cracking down on slums could slow down the acquisition of human capital in the low-end of the distribution, the growth of cities proper (outside slums) and induce even larger slums in the future. We find that the impact of housing costs in the city depends crucially on the human capital distribution of the country. Finally, procuring slum-dwelling children some access to schools in the city would eventually lead to larger cities and smaller slums.

Keywords: *Human capital formation; School locations; Occupations; Structural transformation.*

JEL Codes: O15, O18, O64, R23, R31.

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"The new residents brought garbage, bins, mongrel dogs... poverty to desire wealth...legs for waiting for buses, hands for hard work, pencils for state schools, courage to turn the corner and...asses for the police to kick..." Paulo Lins, CITY OF GOD: A NOVEL

1 Introduction

Structural transformation and urbanization are hallmarks of the development of countries.[1] Most developed countries have displaced nearly all their workers from agriculture and other primary sectors toward manufacturing and services. With agriculture being predominantly a rural sector, and manufacturing and many service sectors being predominantly urban activities, then barriers to urbanization can easily translate into barriers to growth and development as implied by Lewis (1954) long ago. Indeed, many developing countries lie behind the richer ones partly because they allocate a high share of workers to agriculture, where they tend to be relatively unproductive.[2]

To be sure, urbanization has rarely been a smooth process. The story of the world's leading cities –e.g., London, Paris, New York, Tokyo– cannot be told without paying attention to the rise, expansion and eventual fall of their slums, as well as to the lives of their dwellers and the advancement of their descendants.[3] More recently, since World War II, many developing countries have transitioned from rural to urban economies. In a matter of two generations, the urban population in South Korea moved from only 28% in 1960 to 93% in 2010. Non-agricultural employment also expanded rapidly, from 38% to 82% in those same years.[4] Seemingly, in Brazil the fraction of the population living in urban areas increased from 36% in 1950 to 85% in 2010, with non-agricultural employment tracking this movement closely, from 36% in 1950 to 83% in 2010. Beneath these apparently common patterns lie drastic differences in the behavior of output per worker. On the one hand, like a handful of other countries, Korea has moved consistently toward the world frontier. From having less than 7% of the U.S. level, the per capita income in Korea reached 65% in 2010. On the other hand, like many other developing countries, Brazil has remained in the low- or middle-income category, with a relative per capita income that moved from 14% in 1960 to just 27% in 2010.[5] Furthermore, in Brazil, like in many other countries, urbanization has been largely driven by the growth of urban slums while in Korea they have all but disappeared.

In this paper, we study the causes and consequences of urban slums along the structural transformation of countries. Specifically, we explore the conditions on education, labor and housing markets that lead to the emergence and persistence of urban slums as equilibrium outcomes. Then, we explore whether slums are barriers or stepping-stones for the ascension of low-skilled households and for the development of the country as a whole. To answer these questions, we construct a simple growth model with endogenous skill formation, structural transformation and urbanization. The model is based on salient micro and macro aspects of the Brazilian data. We use the model to analyze the interaction between the country's distribution of human capital, sectoral productivities and housing costs that lead to the emergence of slums. Then, we examine the resulting evolution of human capital formation and structural transformation under different initial conditions and alternative housing and education conditions. Our calibrated model replicates the observed rise of slums, urbanization, sectoral employment and education distribution observed in Brazil from 1950 to 2010. We use the calibrated model to conduct counterfactual experiments, such as policies that

[1] See for example the Nobel lecture of Kuznets (1973).

[2] See Duarte and Restuccia (2010), Herrendorf, Rogerson and Valentinyi (2013), Herrendorf and Valentinyi (2012), Herrendorf and Schoellman (2017) and Silva and Ferreira (2011).

[3] For example, see Anbinder (2001) for an engaging account of the life stories of some residents of Five Points, one of New York's most prominent slum in the 19^{th} century.

[4] Data taken from the Groningen Growth and Development Centre (GGDC) database, Timmer et al. (2014).

[5] Data taken from the Penn World Table 9. http://febpwt.webhosting.rug.nl/Home.

crack-down on slums, reduce housing barriers, or integrate the schools of cities and slums.

We document a number of salient aspects of the Brazilian data on structural transformation, urbanization, slum formation as well as earnings and education in different locations. We use macro data, such as the evolution through the years of the population across locations, i.e., urban and rural and slums and cities proper, and across employment sectors. We also use micro data, such as relative income differences, housing costs, access to employment and education opportunities, and education outcomes (in the form of inter-generational transitions of schooling attainment levels). Three features stand out in the Brazilian experience. First, living in a slum imperfectly circumvents the housing costs of the city. It gives the adults in a household access to the urban labor markets at large, but this access comes with direct costs, as highlighted by our opening quote from Lins. More importantly, it reduces the schooling options for children to the slum itself or its near vicinity. Second, the location of households has a very large impact on the education attainment of their children. We find that marginal urban areas are much worse than the main city but far superior than rural areas. Third, city housing costs are much higher than in rural areas, precluding some of the low-skilled households from entering the city.

We construct a stylized model around these micro observations. The model can be used to analyze *(i)* structural transformation, *(ii)* urban development, *(iii)* the country's distribution of human capital, and, to some extent, *(iv)* social mobility. Ours is a discrete-time, infinite-horizon economy populated by dynasties of two-period-lived individuals with a cross-section distribution of skills that endogenously evolves over time. There are two goods, agricultural and non-agricultural goods (which encompasses a construction sector), and three occupations: an unskilled rural occupation, and two urban occupations, qualified and adaptable. Qualified occupations require a minimum skill level; adaptable occupations can be composed of one or two groups of workers at the extremes of the skill distribution, depending on an endogenous urban configuration. The market-clearing price of goods and the earnings across occupations and skills are driven by non-homothetic preferences as used in recent models of structural transformation. In equilibrium, the skill population is endogenously sorted across the locations of the country, and the human capital formation of children is determined by the average human capital in each location. Altruistic parents take into account the human capital formation of their children at the time they choose their location of residence. To live in the city proper, a household needs to pay for a house, a fixed cost whose level is determined in equilibrium. Slums offer the option of entering urban labor markets while avoiding housing costs, but this option involves a utility cost, which varies directly with the household's earnings, and inferior schooling options for the children.

We provide conditions for an equilibrium in the economy to always exist and perfectly sort households by skill levels across rural areas, urban slums and cities. With respect to urban occupations, there can be two equilibrium configurations: economies with only high-skill urban jobs and economies with both high- and low-skill urban jobs. With respect to urban residential configurations, there can also be two different configurations: economies with only cities (i.e., empty slums), and economies with cities and slums. We examine the conditions under which the different configurations arise and highlight the role of the country's sectoral productivities, human capital distribution and housing costs to generate low-skill urban jobs with or without slums. We also discuss how housing costs and education concerns shape the urban configuration of countries. Finally, we stress the importance of urban segmentation of schools for the persistence of low-skill urban jobs and slums.

A calibration of our model replicates the observed rise of slums, the expansion of the urban population and the changes in the distribution of workers across employment sectors and education levels observed in Brazil from 1950 to 2010. We then use the model as a basis for counterfactual exercises on potential policies that drive housing decisions and schooling outcomes. First, we find that cracking down on slums could slow down the acquisition of human capital in the low-end of the distribution and reduce the size of cities proper (outside slums) and induce even larger slums in the future. Second, we find that housing costs can reshape the urban configuration of a country, but their impact crucially depends on the human capital distribution of the country. Third, we show that procuring slum-dwelling children with some access to schools in the city would initially exacerbate the formation of slums, but would eventually lead to larger cities and smaller slums as a result of a higher skill formation in the lower-end of the urban distribution.

Our paper is connected to the extensive literature on structural transformation, by focusing on the reallocation of workers from agricultural occupations to urban occupations.[6] Recently, Duarte and Restuccia (2010) study the role of sectoral labor productivity in structural transformation for the trajectory of aggregate productivity of 29 economies. Duarte and Restuccia (2010) note that the catch-up of productivity (relative to the U.S.) in manufacturing can account for about half of the productivity gains. As a counterpart, the low productivity –and lack of catching up– of the service sector explains cases of stagnation and decline, which is consistent with our emphasis on the expansion of low-skill services to explain the low growth in productivity in the Brazilian economy. In this aspect, our work is closest to Silva and Ferreira (2015), who look at six Latin American countries in the period of 1950-2003. Silva and Ferreira (2015) use a four-sector model (agriculture, manufacturing, modern services and traditional services), and conclude that the expansion and poor productivity of the traditional services sector is a major source of the slowdown in productivity growth after the mid-1970s in Latin America. By highlighting the expansion of low-skilled workers in urban occupations, we provide a contrapositive result to that of Buera and Kaboski (2012) and Buera, Kaboski and Rogerson (2015), who find that the growth in output per worker for developed and fast-growing developing countries is mostly accounted for by the expansion of high-skill service sectors. With our model, we examine the conditions under which a country's urbanization and structural transformation is directed to high-skill or to low-skill urban jobs.

Our paper is also connected to the literature on urbanization and development.[7] Much empirical work has studied the forces that "pull" migrants to urban destinations, e.g., better economic opportunities and better amenities and public services, including schools, as well as the forces that "push" migrants away from rural areas, e.g., low productivity in agriculture, environmental changes, and lack of access to basic public services. For Brazil, Lall et al. (2009) find that wage differences are the main factor driving migration but also that access to basic public services matters a lot. Indeed, Lall et al. (2009) find that poor households are willing to accept lower wages in order to get access to better amenities.[8] These findings are consistent with the equilibrium of our model, where the marginal migrants, in both the slums and the cities, would sacrifice some income in order to access better schools for their children.

We emphasize the role of urbanization in a country's accumulation of human capital as in Lucas (2004). Our most substantial difference with Lucas (2004) is that the learning opportunities in urban areas are fragmented between cities and slums. Thus, our paper is related to Benabou (1996), Durlauf (1996), Fernandez and Rogerson (1998), Fogli and Guerrieri (2017) and others, who examine the fragmentation of schools within urban areas. While part of our analysis and policy counterfactuals are similar to those papers, our goal here is on the causes and consequences of slums along the development of countries. For example, with Fogli and Guerrieri (2017) we share the interest in the allocation of urban households between poor and richer urban areas and the implications for the children's human capital formation. But a key margin in our analysis is also on the size of urban areas, which in our model is endogenous. For us, the key issues are not only in

[6] For a recent review of the structural transformation literature, see the handbook chapter by Herrendorf, Rogerson and Valentiny (2014).

[7] For a recent review of urbanization and development, see the handbook chapter by Brueckner and Lall (2015).

[8] Along the same lines, for Nepal, Dudwick et al. (2011) find that migrants are most attracted to destinations with better access to schools, hospitals and markets.

the allocation of households between the city and its slums, but also between these two locations and the countryside.

On whether slums are barriers or stepping stones, the experience of developed countries would suggest that slums are stepping stones, a temporary phase in the urbanization of countries. Slums were pervasive during the Industrial Revolution in European cities and in the surge of American cities, e.g., London and New York. Before giving way to formal and even posh parts of their cities, those slums were the playground of children whose descendants were to become some of the country's most prominent academic, cultural, and entrepreneurial leaders.[9] In contrast, on the basis of the more recent experience of developing countries, authors such as Marx, Stocker and Suri (2013) conclude that slums are poverty traps driven by policy failures, government neglect, housing restrictions, low human capital accumulation, and low levels of public and private investments.[10] In this vein, Cavalcanti and Da Mata (2014) construct a structural general equilibrium static model and show how urban poverty, rural-urban migration and land-use regulations impacted the growth of slums in Brazil between 1980 and 2000.

In this paper, we complement the work of these authors in multiple dimensions. First, we empirically explore the intergenerational transition probabilities in education attainment for households located in rural areas, urban slums and cities. We indeed find evidence that relative to the city, slums act as barriers for the human capital accumulation of their children. However, relative to the rural areas, slums provide a valuable stepping stone, one which can substantially improve the odds that, after two or three generations, some of the descendants of a low-skill household would attain high levels of education and earnings. Second, we construct a simple dynamic general equilibrium model that can be used to explore the conditions under which the path of structural transformation involves the formation of slums. We show that a simple calibration of the model can replicate salient features of the Brazilian experience. Third, we use the model to conduct policy counterfactuals, such as restricting the formation of slums, reducing the cost of housing and integrating the schools of the city with those of the slums. In doing so, we explore the implications not only for the different individuals, but also for the country as a whole.

In our model, the emergence of slums is driven by housing costs in the cities. Therefore, we connect to a rich literature on the reallocation costs of labor across cities, e.g., Demset et al., (2016), and Hsieh and Moretti (2015), and on the sorting of skills across multiple cities, e.g., Eeckhout et al. (2014). Differing from those papers, however, we focus on the urbanization that takes place in the early stages of development, when much of their labor is still in agriculture. To do so, we abstract from the rich geographic aspects in some of those papers and instead center our attention on the location decisions of rural-urban migrants with low human capital.

The paper is organized as follows. Section 2 underlines a number of salient facts about the structural transformation, urbanization and the emergence of slums in Brazil. Section 3 sets out our model and defines an equilibrium. Section 4 characterizes the equilibrium allocations, explores the conditions for the emergence of slums, and analyzes the impact of slums on the structural transformation of a country. Section 5 calibrates our model to the Brazilian experience from 1950 to 2010, and explores a number of counterfactual experiments. Section 6 concludes. An appendix includes additional details of our data, provides a brief historical overview of the slums in Brazil, and contains the proofs.

[9] See the discussions in Frankenhoff (1967), Turner (1969) and Glaeser (2011). For vivid descriptions of the slums in New York, see Riis (1970) and Anbinder (2001).

[10] See the extensive policy discussions in Annez et al. (2009), Hammam (2013) and Lall et al. (2007).

2 Brazil: Structural Change and the Emergence of *Favelas*

In this section we review the growth, structural transformation and urbanization patterns in Brazil. After listing our sources of data,[11] we notice that, as for other developing countries, the emergence of slums is a prominent feature of Brazil. We also use micro data to examine the characteristics of individuals living in slums, known in Brazil as *favelas*. We analyze the access of slum residents to urban labor and education markets. We highlight the substantial differences in education outcomes of children growing up in rural areas, cities and urban slums in Brazil. Finally, we explore the relevance of housing cost differences between the cities, the slums and the countryside.

2.1 Data

From the *Brazilian Census*, we collect our data on the distribution of the Brazilian population across rural and urban areas, levels of education, personal incomes and sectors of employment. Since 1950, the Census has been conducted every ten years by the Brazilian Institute of Geography and Statistics (IBGE.)[12]

Our data on slums comes from two sources. The first source is the Brazilian Census itself. In 1991 and 2000, the IBGE included a question about whether households live in a "subnormal agglomerate." The IBGE defines a subnormal agglomerate as a set of 51 or more housing units characterized by the absence of a proper ownership title and at least one of the following aspects: *(i)* irregular traffic routes or land plots of irregular size or shape; or *(ii)* lack of essential public services, such as garbage collection, a sewage system, electricity and public lighting. This definition is very close to the definition of "slums" employed by the UN Habitat, which does not require the household to be in an agglomerate.[13]

Our second source of data for slums in Brazil is the *Favela Census*, conducted by the state government of Rio de Janeiro in 2010.[14] This Census is a unique initiative that collects information on the households residing in three large slums in Rio: Complexo do Alemão, Complexo do Manguinhos and Complexo do Rocinha, three of the biggest slums in the city of Rio de Janeiro. Figure 1 locates these three favelas in the map of Rio de Janeiro. Rocinha, which is much closer than the others to affluent neighborhoods in Rio (e.g., Leblon, Ipanema, Jardim Botânico), has historically been one of the most prominent slum in Rio. However, much of the expansion of the slum population in Rio in the last decades has taken place in favelas located in the outskirts of the city, like Complex of Alemão and Manguinhos. We collect data on schooling and employment choices of children and adults from all households living in these three favelas. The large number of respondents in the three slums allows us a very reliable description of the labor and education choices of adults and children living in urban slums in Brazil.[15]

Data on the intergenerational transitions of education levels for households are taken from the social mobility supplement of the Household Survey, *Pesquisa Nacional por Amostra de Domicílio, (PNAD).*[16] For 1988 and 1996, the PNAD included a special supplement with information about the education levels of the parents of the household head and spouse. From it, we trace the intergenerational transition probabilities of education levels for different urban and rural regions.

[11] Additional details are in Appendix A.

[12] See *www.ibge.gov.br/english/.* The Census for 1990 was conducted in 1991.

[13] The UN Habitat defines a slum household as a group of individuals living under the same roof and lacking one or more of the following amenities: (i) access to improved water; (ii) access to improved sanitation; (iii) sufficient living area; (iv) durability of housing; or (v) security of tenure.

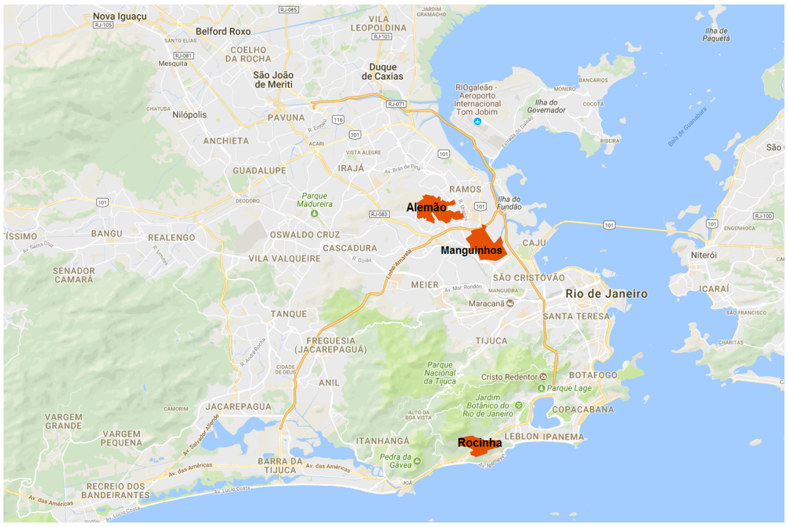
[14] For more details see *www.emop.rj.gov.br/trabalho-tecnico-social/censos-comunitarios.*

[15] For Complexo do Alemão we have the 69,586 responses out of an estimated population of 89,912. For Complexo do Manguinhos we have 27,073 responses out of 31,535 residents. For Complexo do Rocinha the numbers are 73,410 respondents out of an estimated population of 98,319.

[16] The PNAD is conducted every year since 1976.

Finally, employment by sectors and other aggregate data were taken from the Groningen Growth and Development Centre (*GGDC*) database.[17]

Figure 1: Three Major Favelas in Rio de Janeiro



2.2 Structural Transformation: Urban Low-Skill Workers and Slums

Figure 2 displays the most relevant macro aspects of the structural transformation and urbanization of Brazil for the last sixty years. The two blue lines indicate the labor productivity or output per worker (total value added divided by total number of workers) in agriculture (dashed line) and non-agriculture (solid line); both series are scaled in the left vertical axis. The two series in red are the employment share outside agriculture (dashed line) and the urban share of the population (red boxes.) The black dots indicate the share of the urban population living in slums. All of these three series are scaled in the right vertical axis.

Figure 2 shows that at a superficial level, Brazil conforms with the usual notions for urbanization and structural transformation. The country transitions from being a predominantly rural and agricultural economy to an urban economy, with most workers employed outside agriculture.[18] Employment outside agriculture steadily increases from 36% in 1950, to 60% in 1980, and to 84% in 2010. The share of the urban population follows closely all along. In terms of sectoral productivity and output growth, from 1950 to 1980, Brazil also exhibited the standard patterns highlighted in the literature. Both agricultural and non-agricultural labor productivities were growing over time, sometimes quite rapidly, on average above 2.5% for both sectors. Labor productivity outside agriculture was consistently much higher than the labor productivity in agriculture. Thus, the sustained reallocation of workers from agriculture to non-agriculture was a significant factor for the overall growth of Brazil.[19]

More interestingly, Figure 2 also shows two clear departures from the standard view of structural transformation and urbanization. First, around the year 1980, labor productivity outside agriculture has consistently declined, sometimes rapidly. The decline is substantial, an average of 0.5% per year from 1980 to 2010.[20] We argue that a major driver for this decline is a composition effect, as the

[17] Timmer et al. (2014)

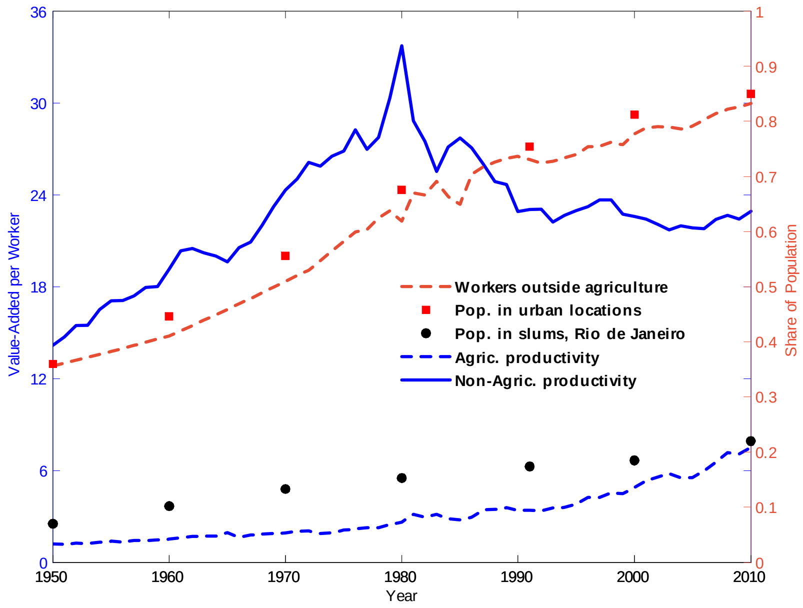
[18] Services sectors, which are not displayed here to simplify the already loaded figure, account for most of the increase in the employment outside agriculture.

[19] Silva and Ferreira (2011) find that 45% of the 1950-1980 growth in Brazil is accounted for by labor reallocation across sectors.

[20] The pattern of a persistent decline in output per worker outside agriculture is observed in many other developing countries in Latin America, Africa and Asia. For the countries that we have in the GGDC database, only Chile does not display it in Latin America, and only Botswana does not display it in Africa. Multiple Central and South Asian countries also display a persistent decline in the output per worker outside agriculture.

expansion of employment outside agriculture has been driven by an increase in the mass of workers with low levels of schooling and skills. Second, a significant share of the expansion of the urban population is accounted for by marginalized housing, or, in its extreme form, slums.[21]

Figure 2: Brazil: Labor Productivities and Employment and Population Shares

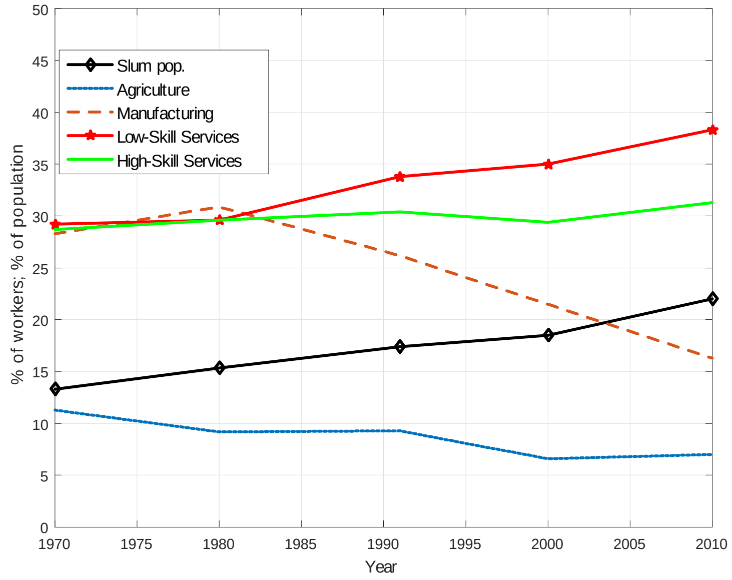


Source: Groningen Growth and Development Centre (GGDC) database.

The remarkable decline and stagnation in the labor productivity outside agriculture can be explained, at least partially, by the relative expansion of low-skill service sectors. Figure 3 shows the urban employment shares in urban areas across agriculture (dotted blue line), manufacturing (dashed orange line), high-skill services (solid green line) and low-skill services (red line with diamond marks.) Here, we follow the classification in Silva and Ferreira (2011): Manufacturing includes the construction sector; low-skill services are sectors that include personal services, e.g., housekeeping, retail, transportation, restaurants, etc.; high-skill service sectors include health, education, government and financial services. In any event, Figure 3 clearly shows that, contrary to what has been observed for developed and fast-growing developing countries, during the last forty years the share of high-skill service sectors in urban employment has remained flat at around 30%. Manufacturing shows a steep decline, from around 30% of urban employment in the 1970s and 1980s, to 15% in 2010. Agriculture is small and declining. The counterpart is a substantial increase in the employment of low-skill services. Notice also that the expansion of low-skill services is closely tracked by the expansion of the urban population living in slums, albeit in this figure it is only for the city of Rio.

[21] To be sure, favelas have had a long history in Brazil as we briefly summarize in Appendix B.

Figure 3: Brazil: Urban Sectoral Employment and Slum Shares in Rio



Sources: GGDC database, IBGE and Favela Census of Rio.

Table 1 explores further the relationship between slums and the expansion of low-skill service sectors in urban areas. The table shows data for 1991 and 2000 for São Paulo and Rio. In 2000, around half the workers living in the slums of both cities were in the low-skill service sectors. The equivalent shares were appreciably lower for those living in the cities proper. On the contrary, the shares of workers living in the cities working in high-skill sectors are twice as high as the shares for those living in slums.

Table 1: Brazil: Employment Distribution by Sector and Location (%)

	1991				2000			
	São Paulo		Rio de Janeiro		São Paulo		Rio de Janeiro	
	Slums	City	Slums	City	Slums	City	Slums	City
Agriculture	0.9	1.2	1.1	1.4	0.6	0.4	0.4	0.6
Manufacturing	41.2	31.1	29.3	18.5	32.1	24.4	22.9	15.5
Low-Skill Services	42.8	33.8	48.7	34.0	47.7	39.6	53.6	39.5
High-Skill Services	14.2	32.9	20.3	45.7	18.0	33.8	20.6	41.5
Not well defined	1.1	1.0	0.7	0.4	1.6	1.9	2.7	2.8

Source: Brazilian Census.

The expansion of urban slums has been fueled in Brazil by waves of rural-urban migration during all the twentieth century. As in other countries, rural-urban migration has been massive in Brazil.[22] Just for the years 1960 to 1970, the World Bank (2008) estimates that around 40 million people migrated to the larger cities. This massive migration is reflected in the high shares of immigrants in Rio in the census data for 1960: 52.2% of the slums population in Rio were migrants; in the city of Rio proper the share is 38.3%. After expanding mostly in Rio, Brazil's capital until 1960, slums became a national and widespread phenomenon after World War II.[23] Table 2 shows that the shares of urban slums are substantial for the other major cities.[24] Notice that the slums in both Rio and São Paulo have grown rapidly in the last twenty years. This is most remarkable in São Paulo, now the largest and richest city, where the share more than doubled in twenty years, reaching 23% in

[22]For more details about urbanization and rural-urban migration in developing countries, see Brueckner and Lall (2015) and Lall et al. (2006).

[23]See Pearlman (2010).

[24]Unfortunately, for the other major cities we only have slum data from the Census for the years 1991 and 2000.

2010 from 9.2% in 1991. In absolute terms, in 2010, São Paulo and Rio de Janeiro had 2.1 and 1.7 million people living in slums in their respective metropolitan regions.

Table 2: Brazilian Main Cities: Urban Population Living in Slums (%)

	Cities				
Year	Rio de Janeiro	São Paulo	Belo Horizonte	Belém	Salvador
1950	7.0	–	–	–	–
1960	10.2	–	–	–	–
1970	13.3	–	–	–	–
1991	17.4	9.2	14.2	25.8	10.1
2000	18.5	11.1	12.3	34.6	9.6
2010	22.0	23.2	–	–	–

Source: Brazilian Census.

At the macro level, urban slums are pervasive, growing and persistent over time. At the household level, we can also document that slums are persistent, i.e., not just a temporary port of entry for rural families that quickly transit to the city. Instead, upon entry, a significant fraction of those families –and their descendants– stay in the slums. Table 3 contains the fractions of migrants living in cities and slums; the share of natives, or non-migrants, can be inferred by difference, as migrants are defined as those individuals whose family was not living in the location where they were residing at the time they were surveyed, 1991.

Table 3: Brazil: Migrants in Cities and Slums, 1991

	São Paulo	Rio de Janeiro	Belo Horizonte	Belém	Salvador
	A. Cities				
Migrants, total:	38.3%	27.7%	42.8%	28.0%	42.8%
from Rural Areas	11.0%	4.8%	9.1%	9.2%	9.1%
from Urban Areas	27.3%	22.9%	33.7%	18.8%	33.7%
	B. Slums				
Migrants, total:	48.2%	29.8%	43.5%	29.5%	32.8%
from Rural Areas	19.5%	10.8%	20.7%	16.4%	13.9%
from Urban Areas	28.7%	19.0%	22.8%	13.1%	18.9%

Source: Brazilian Census

Table 3 shows three interesting patterns. First, both cities and slums attract a significant mass of newcomers from either rural areas or other urban areas. The shares of migrants are high for all urban areas, with São Paulo being the most notable because of its size and growth. Second, the share of rural migration is substantially higher for slums than for cities. This is consistent with a significant fraction of low-skilled migrants for whom slums are the best or only option. Third, the fractions of non-migrants in all urban locations are high, higher than 50% for all cities, and higher than 70% for both Rio and Belém. These fractions provide lower bounds for the probability that someone born in a slum remains his life dwelling in a slum.[25]

The aggregate of these residential decisions allocate the country's population across rural areas, cities and slums. For Brazil in 2000, Table 4 shows the resulting distribution by school attainment levels. The differences across locations are stark. A clear first order stochastic ordering emerges: Cities are higher than the slums, and the slums are higher than the rural areas. In the table, columns 2 and 3 show the distribution for the rural and urban areas for the country as a whole;

[25] The actual probabilities that someone born in a city or in a slum stays, respectively, in a city or in a slum, can be higher for two reasons: (i) the populations in both cities and slums have been growing over time, and (ii) migrants in cities or slums areas can come from other cities or slums, respectively.

for São Paulo and Rio, respectively, columns 4 and 5, and columns 6 and 7, show the distribution between slums and cities proper. With the exception of the last row, all numbers are in percentage terms of each location's population.

Table 4: Population Distribution by Years of Schooling, 2000 (%)

	Brazil		São Paulo		Rio de Janeiro	
Years	Rural	Urban	Slums	City	Slums	City
0	31.3	11.6	14.4	5.4	12.4	3.8
1 to 4	50.2	33.3	42.7	27.5	39.1	21.5
5 to 8	12.5	23.3	30.2	23.6	31.0	21.0
9 to 11	4.9	20.9	10.4	23.2	15.4	29.1
12 or +	1.0	10.9	2.3	20.2	2.2	24.6
Average (years)	2.9	6.5	4.8	8.1	5.3	9.0

Source: Brazilian Census.

A quick look at the average years of education reveals the large regional disparities. While in the rural areas the average is less than 3 years, in the slums of São Paulo and Rio, the averages are much higher, 4.8 and 5.3, respectively. For the cities proper, the averages are substantially higher, 8.1 and 9.0 years of education. Table 4 also shows that the share of individuals with zero or very little schooling (groups with 0, or 1 to 4 years of schooling) is much higher in rural areas. The rural areas disproportionally host the many Brazilian workers with little or no formal education:[26] The rural-to-urban ratio between the shares of individuals with 0 years is 3-to-1; for those between 1 and 4 years of schooling, the ratio is 2-to-1. On the other extreme, urban areas disproportionally host the highly-educated workers. There is a 5-to-1 urban-to-rural ratio between the shares individuals with 9 to 11 of education; for workers with more than 12 years of schooling, that ratio is 10-to-1. Table 4 also shows that the relative difference between the cities and the slums is also very large. For instance, the share of households with no education in slums of Rio is only one third of the ratio in the rural areas.

Table 4 can be summarized as follows: Rural areas are populated mostly by households with very low education: 80% of the rural population has 4 years of schooling or less. Slums are populated with relatively more educated households: 70% of them have 1 to 8 years of schooling. Cities are populated by much more educated households: 70% or more of them have 5 years of education and a significant share has 12 or more.

2.3 The Workings of a Slum: Urban Labor and School Markets

Employment and education opportunities have long been emphasized as factors that pull households toward urban areas. For the country as whole, employment opportunities drive the allocation of human capital across the different occupations available in the different locations, while the access to schools of different households determine the evolution of the cross-section of skills over time. In this section, we explore whether these factors operate for the urban slums in Brazil.

We first explore whether living in a slum gives access to urban labor markets. Table 5 reports the job location for the three slums for which we have micro data, Alemão, Manguinhos and Rocinha. The table shows that slum residents work mostly outside the slum, i.e., in the main city of Rio, where the majority of job opportunities can be found. Almost around 4 in 5 of the slum dwellers work outside the slum, albeit a significant percentage remain in the close vicinity. The share of those working well outside the slum is highest, 71.1%, for Rocinha, a favela favorably

[26] Brazil has come a long way. In 1970, average schooling in the countryside was less than one year, and 64% of the adult population had no formal education. In urban areas, the average education level was only 3.4 years and 28% of the urban adult population had no formal education.

located in the proximity to affluent neighborhoods where retail stores and well-off households employ many low-skill workers. Yet, despite Alemão and Manguinhos being much farther away from rich neighborhoods, the majority of their inhabitants works also in the city.

Table 5: Job Location of People Living in Three Slums in Rio (%)

	Alemão	Manguinhos	Rocinha	All
Inside slums	22.7	22.4	22.0	22.4
In the close vicinity	15.7	19.3	6.9	13.9
Outside slums	61.6	58.4	71.1	63.7

Source: Favela Census of Rio de Janeiro.

In terms of labor market outcomes, Table 6 compares the earnings of workers with similar education levels but residing in different locations. For Brazil as a whole, the second column shows that across all education levels, urban workers earn 30-40% more than rural ones. For Rio de Janeiro, columns 3 and 4 compare the earnings of those in the city and in the slums with those in the Brazilian countryside, respectively. Columns 5 and 6 of the table do the same for São Paulo. The table suggests that a rural worker with 0 to 3 years of schooling could double his income if he moves to the city. The implied earnings gain is similarly high for those opting for the slums.[27][28] All in all, both Tables 5 and 6 suggest that, albeit possibly imperfect and costly, living in slums gives workers access to the labor markets in the city.

Table 6: Households Income Ratios by Education and Location, 2000

	Brazil	Rio de Janeiro		São Paulo	
Education	Urban/Rural	City/Rural	Slum/Rural	City/Rural	Slum/Rural
0	1.3	2.1	1.6	2.5	2.0
1 to 3	1.4	1.9	1.4	2.1	1.6
4	1.3	1.6	1.1	1.8	1.2
5 to 8	1.3	1.6	1.0	1.7	1.0
9 to 11	1.4	1.6	0.8	1.8	0.9
12 or +	1.3	1.4	0.5	1.5	0.5

Source: Brazilian Census.

We now explore whether living in a slum gives access to urban schools. With respect to what we found for labor markets, here we find a substantial difference: Living in a slum does not give access to the schools in the city proper. Table 7 reports the location of the schools attended by children of primary school age. The vast majority, around 90%, of the children in Alemão and Manguinhos go to schools there or in the near vicinity. Children of Rocinha seem to have much better access to the city's schools, because of the better location of that favela.

Table 7: Location of schools attended by children living in Slums in Rio, (%)

	Alemão	Manguinhos	Rocinha	All
Inside slums	86.3	55.9	43.3	61.8
Outside but <1km away	8.9	21.3	0.5	10.2
Outside between 1-3km way	0.0	12.3	26.0	12.8
Outside >3km	1.5	7.8	30.2	13.2
Could not locate school	3.3	2.7	0.0	2.0

Source: Favela Census of Rio de Janeiro

[27] This is also true for most large cities in Brazil, with the exception of Salvador, where incomes are about the same for people living inside slums and in rural areas.

[28] The implied gains are much lower, even negative, for workers with high levels of education. These lower ratios may be explained by the fact that some high-skilled workers can earn high incomes in rural areas, e.g., the town's doctor, lawyers, and school principals. Unobserved negative factors that explain why some highly schooled individuals end up living in a slum may be also associated with their lower incomes.

For us, the most interesting aspect in the workings of slums is the asymmetry between giving access to the city labor markets for the adults while secluding the children to the schools of the slum. Then, the key question is how the opportunities for human capital formation in the favelas compare with those in rural areas and in the cities proper. This is the issue we discuss next.

2.4 Locations and Intergeneration Mobility in Schooling Attainment

We now document the enormous impact that location has for the education attainment of the children, especially for those of low-education households. Specifically, we find that children growing up in urban slums have much worse education outcomes than those growing up in the main cities. We also find that children growing up in slums have substantially better education outcomes than their comparable peers in rural area. While the effects are also present for children with better-educated parents, here we focus on the groups of households with 0 and 1 to 4, because they represent most of the actual –and potential– migrants from rural areas.

Data from the 1996 supplement of the PNAD allow us to link the education of adults with the education attainment of their parents. We can directly separate those living in urban and in rural locations. Unfortunately, the PNAD data does not provide an explicit indicator for residence in a slum. Then, to proxy for slum dwellers, we divide the respondents in Rio de Janeiro into two groups: those with an income level in the 35^{th} percentile or lower and all the rest. The first group is our best proxy for households in Rio's slums, because the overwhelming majority of those in the Favela Census of Rio have incomes below the 35^{th} percentile within the population of Rio.[29]

Figure 4 presents the probability distributions for the attainment of schooling for children growing up in the different regions. As before, the education attainments are grouped into categories of 0, 1 to 4, 5 to 8, 9 to 11, and 12 or more years of education. Each color indicates a region: Blue bars are for children in rural areas and red bars for those in urban areas. Green bars are for the children in poor areas of Rio (proxying for slums) and purple bars are for the children in richer areas of Rio. Panel (a) of Figure 4 considers children whose fathers have 0 years of formal schooling, and panel (b) does the same for those whose fathers have 1 to 4 years of education. Appendix C contains the same information for the children with better-educated parents.

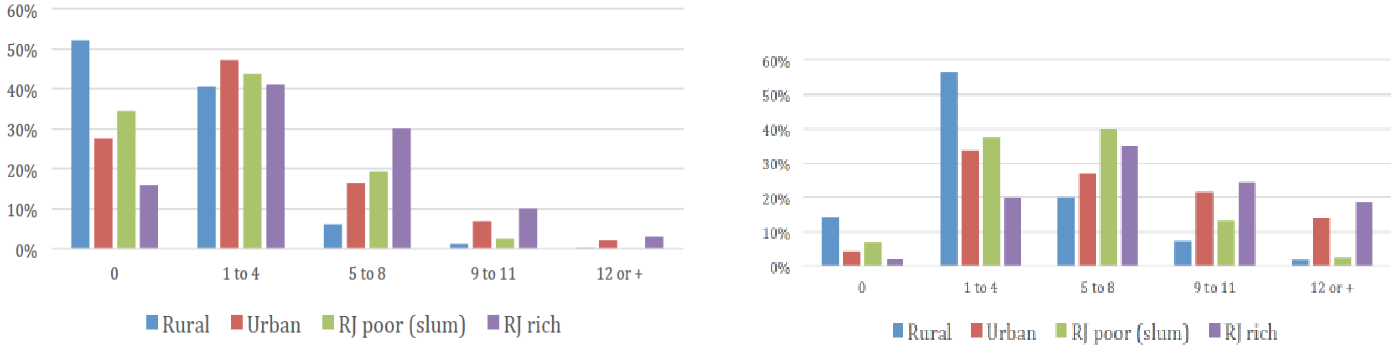
The differences in the education attainment for the children of low-educated households are dramatic. First of all, from Figure 4 (a) we see that out of all the children of illiterate fathers who grow up in rural areas, more than 52% end up being illiterate themselves. In urban areas that probability is much lower, less than 28%. The same comparison between children in the slums of Rio and those in the city proper is also striking: 34% vs. than 17%. The reproduction of illiterate workers is very high in rural areas, substantially lower in urban slums and much lower within the formal confines of cities. Second, there are also significant differences in the probability of attaining higher education levels for these children. Essentially, only cities provide any prospects of attaining 9 or more years of education. Yet, slums provide a significant 20% chance of attaining education levels between 5 and 8 years. Using additional information from the 1996 PNAD supplement, not included in Figure 4, we found that for children whose fathers have no formal education, the average years of schooling is 2.7 years in the rural areas and 4.5 years in the urban areas. In Rio de Janeiro and São Paulo, these averages are 5.58 and 3.94, respectively. Using our proxy for formal cities, the averages are much higher, 9.91 and 9.23, respectively.[30]

[29] We use father-son pairs, but the results are very similar if we use mother-daughter pairs or other combinations of parents and children of either or both genders. Here we opted for fathers-sons simply to avoid additional aspects of single-parent families.

[30] See Ferreira and Veloso (2003) for a more extensive exploration of the intergenerational transition probabilities in the schooling attainment levels.

Figure 4: Brazil: Education Attainment Probabilities, Different Locations, 1996

(a) Children with fathers with 0 years of schooling *(b) Children with fathers with 1-4 years of schooling*



Panel (b) in Figure 4 shows differences and similarities for the children of parents with some primary education. The most notable difference is that most of these children attain at least the education levels of their parents, i.e. 1-4 years of education. Also, the probabilities of attaining the higher levels of education are also much higher for these other children. The most remarkable similarity is that the same ranking of urban dominating the rural areas, cities dominating slums, and, slums dominating the rural areas, is also valid for these households. Moreover, Appendix C shows the same ordering applies for children of much better-educated fathers, those with 5-8, 9-11 and 12+ years of education. For instance, in urban areas, 62.8% of parents with 12 or more years of schooling have children with the same level of education, while in rural areas the corresponding figure is only 30.6%.

In sum, on whether slums are traps or stepping stones, the evidence in Brazil is as follows: For the children of low-educated households, moving to an urban slum is a stepping stone relative to staying in the countryside. However, relative to reaching the city proper, living in a slum imposes barriers that slowdown the accumulation of human capital and impairs upward mobility. Our model uses both aspects to rationalize the emergence and persistence over time of slums and of low-skill urban jobs.

2.5 Housing Costs: Barriers to Entry in Cities

Relocating from a rural to an urban area entails multiple costs: the loss of family and social connections, temporary unemployment, the higher uncertainty and risks of living in unknown and dangerous urban areas, etc.[31] Many of these costs are temporary, resolved within one or two years for many migrants, especially the young and middle-age ones. But a higher cost of living is a permanent cost of living in urban areas. A key component of the cost-of-living differences across regions is the cost of housing.

To characterize those housing costs differences in Brazil, Table 8 takes data from the Census in 1991 and reports the relative cost differences of renting an apartment with different number of bedrooms. We concentrate on rent differences because renting is probably the relevant option for poorer migrants, and makes the comparison with the relative earnings off Table 6 more direct.[32] Table 8 shows the urban-to-rural rents ratio (column 2), and the city-to-rural and city-to-slum ratios for the cities of Rio (columns 3 and 4) and São Paulo (columns 5 and 6.) For Brazil as a whole, rents in urban areas were between twice and three times as expensive as in rural areas. The cities of Rio and São Paulo, where the better labor market opportunities and schools are located, were even more expensive, up to 6.5 times for a three-bedroom apartment. Relative to the slums, housing in

[31] These costs can be particularly high for low-skilled families in Brazil, as amply discussed by Pearlman (2010) and Pinto (1997).

[32] Using housing prices would also entail considering bubbles and other investment and financing aspects relevant for the urban real estate markets in Brazil.

the cities is also more expensive, but the city-slum ratios are on average half of the city-rural ratios. Therefore, slums are in fact a relevant option for low-income households to work and live in urban areas.

Table 8: Ratio of Monthly Rents, 1991

	Brazil	Rio de Janeiro		São Paulo	
# Bedrooms	Urban/Rural	City/Rural	City/Slum	City/Rural	City/Slum
1	2.0	3.2	1.8	3.5	1.5
2	2.3	3.5	1.9	4.7	2.0
3	2.9	4.8	2.7	6.5	2.6
Average	2.3	3.6	2.3	4.1	1.9

Source: Brazilian Census

The costs differences are not minor. For example, consider a hypothetical rural household, composed of parents with just 2 years of formal education and multiple children. Assume that in the rural area they spend 25% of their income renting a three-bedroom residence. According to Table 6, if they move to the city of São Paulo, they could increase their income by 2.1; yet, to do that they would need to spend 6.5 times in housing. A simple calculation shows that the move to São Paulo would entail a drop of 47.5% of their consumption, which was already low. For this household, a more attractive alternative may be to move to a slum in São Paulo: If so, even if their earnings are multiplied only 1.6 times (Table 6), their housing costs would go up 2.5 (=6.5/2.6) times (Table 8), and the resulting consumption loss is only 20%. This loss could be compensated by the better schooling options for the children. Moreover, the household's children, may soon start working, and their higher earnings could easily compensate the temporary net loss in consumption.

Beyond this simple and hypothetical case, the evidence is unambiguous in that the urban expansion in Brazil is characterized by both a relative high cost of housing in the cities and a high fraction of Brazilian households living in urban slums.

3 Model

We consider a simple model that can be used to analytically examine a country's structural transformation, urban development, human capital formation and intergenerational social mobility. Our attention is focused on the allocation of individuals –and their skills– across locations, production sectors and occupations and on the dynamic implications of those decisions for the skill formation of future generations. In this section, we lay out the environment and then define a competitive equilibrium. In the next section, we examine the equilibrium allocations and provide a number of simple but key analytical results on the emergence and persistence of slums.

3.1 The Environment

We consider a discrete-time, infinite-horizon economy populated by dynasties of two-period-lived overlapping generations (OLG) of individuals. Time periods are indexed by $t = 1, 2, 3, ...$ Individuals differ in their skills. The population in every period has total mass normalized to one and is described by a probability measure, μ_t, with support over all positive reals. We index each skill level by $z \in \mathbb{R}_+$. The evolution over time of the distribution of skills, $\{\mu_t\}_{t=1}^{\infty}$ is determined endogenously, from the equilibrium allocation of households across locations in each period t, as explained below.

Adults choose occupations, locations and consumption. There are *three locations*: *rural areas, slums (favelas)* and *city*, which we index $l = R, F, C$, respectively. There are *two production sectors*: *agriculture* and *non-agriculture* (manufacturing and services), which we index $i = A, N$.

There are *three occupations* or *job* or types: *unskilled, qualified* and *adaptable,* which we index $j = u, q, a$, respectively.

Preferences: The utility of a household at time t is defined over the consumption of goods and over the expected human capital or skill formation of their children. Specifically, the utility of an adult at t is given by:

$$V_t = \bar{u}_t \left(c_t^A, c_t^N\right) + \beta E_t \left[z_{t+1}\right],$$

the sum of a term that depends on the consumption of goods and another term that depends on expected future skill or human capital of the adult's child. The first term is given by

$$\bar{u}_t \left(c_t^A, c_t^N\right) = u \left[\left(c_t^A - \bar{c}^A\right)^\alpha \left(c_t^N\right)^{1-\alpha}\right],$$

where $u(\cdot)$ is a strictly increasing and weakly concave function of a final good $\left(c_t^A - \bar{c}^A\right)^\alpha \left(c_t^N\right)^{1-\alpha}$, composed of agricultural goods (food) and non-agricultural goods, respectively. As a driver of structural transformation, we assume standard Stone-Geary non-homothetic preferences, defined by the parameter $0 < \alpha < 0$ and a minimum consumption floor of agricultural goods (food), $\bar{c}^A > 0$.

We assume this form of impure altruism, i.e., paternalistic preferences à la Fernandez and Rogerson (1998) for tractability. The parameter $\beta \geq 0$ governs the weight that parents put on their children's expected skills for the next period, z_{t+1}. Here $E_t(\cdot)$ is the expectation over the realization of skills z_{t+1} given the location choice of the household, as explained below.

Supply of Skills: All workers are endowed with one unit of time, which they can choose to provide in three different forms of labor services: First, regardless of their skills z, everyone can provide *one* unit of *unskilled* labor. Second, workers possessing skills above a minimum qualification requirement, $z_{\min} > 0$, can opt to supply one unit of *qualified* labor; workers below that qualification threshold have a zero supply of those skills. Third, all individuals can supply *adaptable* labor services in direct proportion to their skills. More formally, the mutually exclusive occupation choices for adults in this economy are:

$$\begin{array}{ll} \text{unskilled:} & h^u(z) = 1 \text{ for all } z \in \mathbb{R}_+; \\ \text{qualified:} & h^q(z) = \begin{cases} 0 & \text{if } z < z_{\min}, \\ 1 & \text{otherwise;} \end{cases} \\ \text{adaptable:} & h^a(z) = z \text{ for all } z \in \mathbb{R}_+. \end{array}$$

The resulting labor services from the occupation choices of workers are employed in the production of consumption goods.

Production of Goods: We assume that both agricultural and non-agricultural consumption goods are produced using only labor. For simplicity, we assume that agricultural goods are produced only in rural areas and non-agricultural goods are produced only in urban areas. We also assume that labor inputs are fixed in each location but goods are fully tradable across the three locations of the country.

We assume that agricultural productivity X_t^A and non-agricultural productivity X_t^N evolve exogenously. The country's aggregate output of production of agricultural goods Y_t^A is

$$Y_t^A = X_t^A L_t^u, \tag{1}$$

where L_t^u is the aggregate units of rural labor, which here is also unskilled labor exclusively. On the other hand, production of non-agricultural goods requires both qualified and adaptable labor, i.e.,

$$Y_t^N = X_t^N \left(L_t^q\right)^\eta \left(L_t^a\right)^{1-\eta}, \tag{2}$$

where $0 \leq \eta \leq 1$ is a share parameter. Here, L_t^q and L_t^a are, respectively, the aggregate supply of qualified and adaptable labor, which we assume are located in urban areas.

Locations: Occupations and Housing Costs. Households choose among three locations, one rural and two urban, considering the implications of those decisions in terms of housing costs, labor market opportunities and implications for the skill formation of their children. In our model, the trade-offs are simple and stark: For households living in rural areas, the provision of unskilled labor is the only occupation available. For households living in urban areas –cities or slums– their occupation choices may be between providing adaptable or qualified labor. In this way, we capture the integration of urban labor markets between slums and cities as we documented for Brazil in the previous section.

Our assumptions about housing costs in the three regions is as follows: First, as a normalization, living in the rural area entails no direct housing costs. Second, living in the city entails a fixed cost, paying for one unit of housing. We assume for simplicity that the consumption of housing does not deliver utility and only gives access to the city. Constructing a house requires $\xi_t > 0$ units of non-agriculture goods.[33] Hence, housing prices p_t^h, are endogenously determined by the structural transformation of the country and the size of the city, as explained below. Finally, we assume that while living in a slum entails no direct housing costs, it entails utility costs. The utility costs of living in a slum can represent time and goods committed –or lost– due to transportation, crime, lack of property rights and protections, and missing infrastructure and other public services. We capture these utility costs by assuming that households living in a slum lose a fraction τ_t of their net-consumption of goods.[34][35] These modeling choices are parsimonious and capture the big picture for the housing costs and labor and occupation choices for most workers, especially for workers away from the top-end of the skill distribution. In a simple and transparent way, the model captures how housing cost differences matter for structural transformation and the country's overall formation of skills. On the one hand, urban-rural housing cost differences constitute barriers to structural transformation, as they restrict the movement from agricultural to non-agricultural production sectors. On the other hand, higher housing costs of the city relative to the utility costs of living in slums fragment the urban schooling markets, impacting the formation of skills for the next generation of workers, as we now explain.

Locations and Production of Skills. In any period time t, the adult population in the economy is described by a probability distribution $\mu_t(\cdot)$ over the skill levels z. The location decisions of all households split μ_t into subpopulations $\mu_t^l(\cdot)$, positive measures that add up to the country's population,

$$\mu_t(\cdot) = \sum_{l=R,F,C} \mu_t^l(\cdot).$$

The country's current distribution μ_t and location decisions of all households determine the formation of skills for the children growing up in each location. We model this as follows: Let

$$Z_t^l \equiv \left[\frac{\int_0^\infty z^\rho \mu_t^l(dz)}{\int_0^\infty \mu_t^l(dz)} \right]^{1/\rho}$$

be the average skills of the adults living in location $l = R, F, C$. Here, ρ is a parameter that determines the curvature in this average and will determine the behavior of human capital externalities

[33] We can easily extend the model to allow for conglomeration costs, by having the costs of houses to be increasing in the size of the city σ_t^C, e.g., $\xi_t = \xi\left(\sigma_t^C\right)$ where $\xi'\left(\sigma_t^C\right) > 0$.

[34] This utility cost is also a proportional cost in terms of net-of-subsistence consumption. Having it in this way, neither the relative demand of goods for all households nor the aggregation of consumption and market-clearing conditions are affected by the costs of slum dwelling.

[35] As with housing costs, we can easily accommodate conglomeration costs of slums by letting τ_t to depend on the size of the slum population σ_t^F, i.e. $\tau_t = \tau\left(\sigma_t^F\right)$, where $\tau'\left(\sigma_t^F\right) > 0$.

and the persistence and mobility of skill levels across generations. In any event, Z_t^l determines the exposure to ideas of each child in each location. We assume that each child draws a skill level from a distribution that is shifted by Z_t^l. i.e.,

$$z' \sim Q\left(\cdot|Z_t^l\right).$$

We assume that $Q\left(\cdot|Z_t^l\right)$ has a continuous density with full support in the non-negative reals $[0,\infty)$, and is increasing in Z_t^l in the *first order stochastic sense*. In our numerical implementation of the model, we will assume that $Q\left(\cdot|Z_t^l\right)$ is a Gamma distribution with mean Z_t^l.

For the country as a whole, the population of adults in the next period, $\mu_{t+1}(\cdot)$, will be composed by the children that grew up in all three regions in the previous period, i.e.,[36]

$$\mu_{t+1}(\cdot) = \sum_{l\in\{R,F,C\}} \int_0^\infty Q\left(\cdot \mid Z_t^l\right)\mu_t^l(dz). \tag{3}$$

In what follows, we make the following assumptions:

Assumption 1: *The initial distribution μ_0 has a continuous density and full support over $[0,\infty)$. For any $Z>0$, the transition probability measures $Q(\cdot|Z)$ also have these two properties. In addition, for some $G\geq 1$,*

$$E\left[z'|Z_t^l\right] = GZ_t^l.$$

This assumption ensures that the support for all probability measures $\{\mu_t\}$ remains unbounded from above for all t. The assumption ensures that equilibrium location and occupation choices can be readily characterized by thresholds over the real line, and we can abstract from proportions of individuals within skills in mass points.

An equilibrium in this environment allocates households across the three different regions. We need to define what value would be attained by a household who moves to a region that in equilibrium is empty, even it doing so is out of equilibrium for everyone. To that end, we make a "home-schooling" assumption:

Assumption 2: *Define an empty region l, as one in which its population has measure zero, i.e., $\mu_t^l(\mathbb{R}_+)=0$. Then, if a child grows up there, his skills next period will be entirely driven by the skills of his household, i.e. $z'\sim Q(\cdot|z)$.*

Finally, we will make the assumption that the agricultural productivity X_t^A is strictly above the minimum required to provide for the subsistence level $\bar{c}^A$ for everyone in all periods.

Assumption 3: *For all t, $X_t^A > \bar{c}^A$.*

Under this assumption, we ensure that total agricultural output can always suffice for everyone to subsist, i.e., $Y_t^A\left(\mu_t^R\right) = X_t^A\int\mu_t^R(dz) > \bar{c}^A$. Since in equilibrium all workers can attain wages equal to X_t^A, Assumption 3 guarantees that all households will be away from the corner of spending all their income on food, making the aggregation of the demand for goods straightforward.

3.2 Equilibrium

We now define competitive equilibria. In any period t, the aggregate state variable is given by $S_t = \left(X_t^A, X_t^N, \mu_t\right)$, the two exogenous sectoral productivities and the predetermined skills of the adult population. Setting agricultural goods as the numeraire, $p_t^A=1$, the price system $\{p_t\}_{t=0}^\infty$ is composed of time-sequences of $p_t = \left(p_t^N, w_t^u, w_t^q, w_t^a, p_t^h\right)$, the vector of the price of non-agricultural goods, the unitary prices of unskilled, qualified and adaptable labor and the price of housing in the city, all of them in units of agricultural goods. Given $\{p_t\}_{t=0}^\infty$, households of all skills z in all periods decide their *consumption* levels, $c_t(z) = \left\{c_t^A(z), c_t^N(z)\right\}$, and their *occupations* and *locations*,

[36]More formally, for any Borel set $B\subset\mathbb{R}_+$, $\mu_{t+1}(B) = \sum_{l\in\{R,F,C\}}\int_0^\infty Q\left(B\mid Z_t^l\right)\mu_t^l(dz)$.

which we denote by dichotomic variables, $\{\varphi_t^u(z), \varphi_t^q(z), \varphi_t^a(z)\}$ and $\{\lambda_t^R(z), \lambda_t^F(z), \lambda_t^C(z)\}$, respectively. Leading up to its formal definition, we now describe the individual optimization choices and market-clearing conditions for a competitive equilibrium.

First, consider the market for consumption goods. Both goods are produced by competitive firms that take goods prices and wages as given to maximize profits. Agricultural firms hire unskilled labor to maximize

$$\max_{L_t^u} \ \{p_t^A Y_t^A - w_t^u L_t^u\}.$$

Since agriculture is our numeraire, $p_t^A = 1$, and since $Y_t^A = X_t^A L_t^u$, then the equilibrium wages for unskilled workers are simply given by

$$w_t^u = X_t^A. \tag{4}$$

Non-agricultural firms hire qualified and adaptable labor to maximize:

$$\max_{L_t^q,\, L_t^a} \ \{p_t^N Y_t^N - w_t^q L_t^q - w_t^a L_t^a\},$$

subject, respectively, to the production function (1) or (2). Since $Y_t^N = X_t^N (L_t^q)^\eta (L_t^a)^{1-\eta}$, the equilibrium wage conditions have very familiar forms in terms of aggregate quantities of these two types of labor:

$$w_t^q = \eta p_t^N X_t^N \left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}, \tag{5}$$

and

$$w_t^a = (1-\eta)\, p_t^N X_t^N \left(\frac{L_t^q}{L_t^a}\right)^{\eta}. \tag{6}$$

Now, consider households in different locations choosing their occupation. For rural households, the only active labor market is for unskilled labor, hence, $\varphi_t^u(z) = 1$ if and only if the household resides in R. For urban dwellers, either in cities or slums, the two possible options are qualified or adaptable jobs. Therefore, if endowed with skills z above the minimum qualification level $z_{\min}$, and facing wages w_t^q and w_t^a, an urban worker would choose to be a qualified worker unless z is higher than an endogenous threshold w_t^q / w_t^a, i.e.,

$$\varphi_t^q(z) = 1 \text{ iff } z_{\min} \le z \le \frac{w_t^q}{w_t^a}. \tag{7}$$

The households supplying adaptable labor are those with skill levels below qualification, $z < z_{\min}$, a sector we will call *low-skill urban jobs*, or those with skill levels above some endogenous threshold, so that their earnings are higher than just providing qualified labor, a sector we will call *high-skill urban jobs.* This is:

$$\varphi_t^a(z) = 1 \text{ if either } z < z_{\min} \text{ or } z > \frac{w_t^q}{w_t^a}. \tag{8}$$

Obviously, $\varphi_t^a(z) + \varphi_t^q(z) = 1$ for all z. In any case, the earnings of urban households are given by $y_t^U(z) \equiv \varphi_t^a(z)\, w_t^a z + \varphi_t^q(z)\, w_t^q$.

These occupation decisions, $\{\varphi_t^u(z), \varphi_t^q(z), \varphi_t^a(z)\}$, define labor aggregates

$$L_t^u = \int \varphi_t^u(z)\, \mu_t(dz), \tag{9}$$

$$L_t^q = \int_{z_{\min}}^{\infty} \varphi_t^q(z)\, \mu_t(dz), \tag{10}$$

$$L_t^a = \int \varphi_t^a(z)\, z \mu_t(dz). \tag{11}$$

Notice the difference in the aggregation formulas. The first one is just the sum of all workers in rural areas; the second is the sum of all qualifying workers who opt to provide qualified labor. The third one is the sum of the skills z of all the workers in adaptable jobs.

Next, consider the *demand for goods* of a household. Let total expenditures in goods be denoted by e_t, i.e., $e_t = y_t^U(z)$ unless the household lives in the city, in which case, $e_t = y_t^U(z) - p_t^h$. The optimal consumption $\{c_t^A, c_t^N\}$ is defined by the utility maximization

$$\max_{\{c_t^A, c_t^N\}} u\left[\left(c_t^A - \bar{c}^A\right)^\alpha \left(c_t^N\right)^{1-\alpha}\right] \quad \text{s.t.} \quad c_t^A + p_t^N c_t^N \leq e_t.$$

The implied demand system is

$$c_t^A(z) = \bar{c}^A + \alpha\left[e_t(z) - \bar{c}^A\right], \text{ and } c_t^N(z) = \frac{1-\alpha}{p_t^N}\left[e_t(z) - \bar{c}^A\right]. \tag{12}$$

The indirect utility function for consumption can be written simply as a function of the optimized amount of final consumption,

$$u\left[\theta\left(p_t^N\right)\left(e_t - \bar{c}^A\right)\right],$$

where $\theta\left(p_t^N\right) \equiv (\alpha)^\alpha (1-\alpha)^{1-\alpha} \left(p_t^N\right)^{\alpha-1}$ derives from our Stone-Geary preferences.

Finally, consider the *location choices* of households, that maximize their utility, considering housing costs, labor market opportunities and schooling prospects for their children, i.e.,

$$V_t(z) = \max\left\{V_t^R(z),\, V_t^F(z),\, V_t^C(z)\right\}, \tag{13}$$

where, the three different options have the following values:

$$V_t^R(z) = u\left[\theta\left(p_t^N\right)\left(w_t^u - \bar{c}^A\right)\right] + \beta E\left[z'|Z_t^R\right], \tag{14}$$

$$V_t^F(z) = u\left[\theta\left(p_t^N\right)\left(y_t^U(z) - \bar{c}^A\right)(1-\tau_t)\right] + \beta E\left[z'|Z_t^F\right], \tag{15}$$

$$V_t^C(z) = u\left[\theta\left(p_t^N\right)\left(y_t^U(z) - p_t^h - \bar{c}^A\right)\right] + \beta E\left[z'|Z_t^C\right]. \tag{16}$$

The term w_t^u in (14) is the expenditure in goods for rural households, the term $(1-\tau_t)$ in (15) captures the direct reduction in utility from living in a slum, and the term $y_t^U(z) - p_t^h$ in (16) is the expenditure in goods net of housing costs in the city. The solution to this discrete choice can be described by dichotomic variables $\{\lambda_t^R(z),\, \lambda_t^F(z),\, \lambda_t^C(z)\}$ that take the value 1 if the household opts to live in region $l = R, F, C$, respectively, and the value 0 otherwise.

Individual location decisions shape the value of living in each location because they impact the skill formation for the children residing there. Formally, the location decisions $\{\lambda_t^R(z),\, \lambda_t^F(z),\, \lambda_t^C(z)\}$ partition the country's population μ_t across the three regions such that

$$\mu_t^l(B) = \int_B \lambda_t^l(z)\mu_t(dz), \tag{17}$$

for any (Borel) set $B \subset \mathbb{R}$. Each of these populations will have total mass $\sigma_t^l \equiv \int_B \mu_t^l(dz)$, and by construction, $\sigma_t^R + \sigma_t^F + \sigma_t^C = 1$. In particular, the exposure to skills and ideas for the youth growing up in region l would be

$$Z_t^l \equiv \left[\frac{\int_0^\infty z^\rho \mu_t^l(dz)}{\sigma_t^l}\right]^{1/\rho}, \tag{18}$$

i.e., an average of the skills of the adults living in that region. Given the country's state variable S_t, regional variables μ_t^l, σ_t^l, Z_t^l are endogenously determined in equilibrium, as detailed further below.

It is convenient to define a number of aggregate variables before formally defining an equilibrium. In units of agricultural goods, the country's total income is given by $Y_t = w_t^u L_t^u + w_t^q L_t^q + w_t^a L_t^a$.

Simply by using the wage conditions (4), (5) and (6), it is easy to show that $Y_t = Y_t^A + p_t^N Y_t^N$, where $Y_t^A = X_t^A L_t^u$ and $Y_t^N = X_t^N \left(L_t^a\right)^{1-\eta} \left(L_t^q\right)^{\eta}$. Define $E_t = Y_t - p_t^h \sigma_t^C$ as the country's aggregate expenditures in consumption goods, i.e. aggregate income minus aggregate housing expenditures. Since $p_t^h = p_t^N \xi_t$, then $E_t = Y_t - p_t^N \xi_t \sigma_t^C$. Using this equation, and aggregating over (12), the aggregate demand for agricultural and non-agricultural goods is given by

$$C_t^A = \bar{c}^A + \alpha \left[E_t - \bar{c}^A\right], \tag{19}$$

$$C_t^N = \frac{1-\alpha}{p_t^N} \left[E_t - \bar{c}^A\right]. \tag{20}$$

The formal definition of an equilibrium in this economy is as follows:

Definition 1 ***Competitive Equilibrium:*** *Given an initial skill distribution μ_0 and exogenous productivity sequences $\left\{X_t^A, X_t^N\right\}_{t=0}^{\infty}$, a competitive equilibrium is a* price system, $\left\{p_t^N, w_t^u, w_t^q, w_t^a\right\}_{t=0}^{\infty}$, *and* allocations *of individual location, occupation and consumption decisions, $\left\{\lambda_t^R(z), \lambda_t^F(z), \lambda_t^C(z)\right\}_{t=0}^{\infty}$, $\left\{\varphi_t^u(z), \varphi_t^q(z), \varphi_t^a(z)\right\}_{t=0}^{\infty}$, $\left\{c_t^A(z), c_t^N(z)\right\}_{t=0}^{\infty}$, and aggregate quantities of outputs, consumptions, exposures to ideas and location sizes $\left\{Y_t^A, Y_t^N, C_t^A, C_t^N, Z_t^R, Z_t^F, Z_t^C, \sigma_t^F, \sigma_t^C\right\}_{t=0}^{\infty}$ such that:*

1. *Individual choices are optimal: Given prices, the individual demands of goods $\left\{c_t^A(z), c_t^N(z)\right\}_{t=0}^{\infty}$ are given by (12); the occupation choices $\left\{\varphi_t^u(z), \varphi_t^q(z), \varphi_t^a(z)\right\}_{t=0}^{\infty}$ are given by (7, 8); and given the exposure levels $\left\{Z_t^R, Z_t^F, Z_t^C\right\}_{t=0}^{\infty}$, location choices $\left\{\lambda_t^R(z), \lambda_t^F(z), \lambda_t^C(z)\right\}_{t=0}^{\infty}$ solve the location problem (13).*

2. *Aggregate variables are consistent with individual choices, i.e., equations (*17), (18), (9), (10), (11), (19) and (20) *hold.*

3. *The goods and housing markets clear:*

$$Y_t^A = C_t^A; \tag{21}$$

$$Y_t^N = C_t^N + \xi_t \sigma_t^C. \tag{22}$$

4. *The law of motion of the population of skills follows condition (3).*

The next section characterizes the equilibrium allocations and provides comparative statics.

4 Urban and Employment Configurations

We now characterize the equilibrium allocation of households across locations and occupations. To this end, we first take as given an exogenous partition of the population between urban and rural regions, and characterize the internal urban configurations, i.e., the assignment of workers across urban jobs and of households between the city and the slum. Second, we use the conditional configuration of urban areas to characterize the endogenous partition of the country's entire population between rural and urban areas and analyze the conditions under which slums arise in equilibrium. Third, using the functional forms that we use later to analyze the experience of Brazil, we examine the interaction of sectoral productivities, skill distributions and housing costs and the resulting equilibrium formation of slums and of low-skill urban jobs. We also describe the different dynamic paths for structural transformation and urbanization implied by the model.

4.1 An Exogenous Rural-Urban Divide

In this section, we assume that the country's overall population is exogenously partitioned between a rural and an urban population, μ^R, μ_t^U, such that $\mu_t^R + \mu_t^U = \mu_t$. In what follows, we assume that μ_t^U has a positive mass of workers with skills above qualification, $z > z_{\min}$, and that its support is unbounded from above. From Assumption 1, μ_t satisfies those conditions so it is always possibly to pick a selection μ_t^U that satisfies them too. It will become evident that this has to be case in equilibrium. We proceeds with these conditions in the background, and also defer all proofs to Appendix B.

4.1.1 Occupation Choices and Production in Urban Areas

Given an urban-rural divide, occupation choices in the urban areas are independent of the partition between cities and slums. An urban worker can earn $w_t^a z$ as a provider of adaptable labor, or, if his skill level z is above the qualification threshold $z_{\min}$, he could earn w_t^q as a qualified worker. The optimal occupation choices are straightforward, and the resulting aggregate units of qualified and adaptable labor are

$$\begin{aligned} L_t^q &= \int_{z_{\min}}^{w_t^q/w_t^a} \mu_t^U(dz), \\ L_t^a &= \int_0^{z_{\min}} z\mu_t^U(dz) + \int_{w_t^q/w_t^a}^{\infty} z\mu_t^U(dz). \end{aligned}$$

Define $z_t^H \equiv w_t^q/w_t^a$ to be the threshold for the skills z above which an urban worker prefers to provide adaptable labor over qualified labor. Plugging the previous expressions for L_t^q and L_t^a in the formulas for equilibrium for w_t^q and w_t^a, (5) and (6), we can write

$$z_t^H = \frac{\eta}{1-\eta}\left[\frac{\int_0^{z_{\min}} z\mu_t^U(dz) + \int_{z_t^H}^{\infty} z\mu_t^U(dz)}{\int_{z_{\min}}^{z_t^H} \mu_t^U(dz)}\right]. \tag{23}$$

The following result characterizes the labor market equilibrium in urban areas:

Lemma 1 *Let μ_t^U be a strictly positive measure characterizing the urban population and assume its support is unbounded from above. Then, there exists a unique $z_t^H > z_{\min}$ that solves equation (23).*

Denote $L_t^q(\mu_t^U) \equiv \int_{z_{\min}}^{z_t^H} \mu_t^U(dz)$ and $L_t^a(\mu_t^U) \equiv \int_0^{z_{\min}} z\mu_t^U(dz) + \int_{z_t^H}^{\infty} z\mu_t^U(dz)$ the implied aggregate units of qualified and adaptable labor, consistent with the market-clearing z_t^H that solves the fixed point (23). Total output of non-agricultural goods is

$$Y_t^N(\mu_t^U) = X_t^N\left[L_t^q(\mu_t^U)\right]^{\eta}\left[L_t^a(\mu_t^U)\right]^{1-\eta}.$$

Similarly, total output of agricultural goods is given by $Y_t^A(\mu_t^U) = X_t^A \int \left[\mu_t - \mu_t^U\right](dz)$.

Aggregate production of agricultural goods is driven by the productivity X_t^A and the mass of workers in rural areas. Non-agricultural goods output can serve to summarize the aggregate skill content of the urban population μ_t^U. Total urban output, $Y_t^N(\mu_t^U)$, can increase because of (a) higher productivity X_t^N; (b) more individuals live in the city; or (c) because individuals have higher levels of skills, e.g. the supply of adaptable labor in the city from a fat-tailed distribution of highly skilled individuals that pushes up the productivity of less-skilled individuals as qualified workers.

Definition 2 *We say that an urban population μ_t^0 is higher than another population μ_t^1 in the* ***total output sense*** *if $Y_t^N(\mu_t^0) > Y_t^N(\mu_t^1)$.*

This ranking of the skill distribution captures all variations in the measure μ_t^U that can lead to an expansion of non-agricultural output. With it, we can provide a comparative static result on the relative price of non-agricultural goods, which is a key determinant of the endogenous rural-urban divide of the country discussed in the next section.

Using the market-clearing condition for both goods, the equilibrium relative price of non-agricultural goods is:

$$p_t^N = \frac{1-\alpha}{\alpha} \frac{Y_t^A\left(\mu_t^U\right) - \bar{c}^A}{Y_t^N\left(\mu_t^U\right) - \xi_t \sigma_t^C}, \tag{24}$$

where $\sigma_t^C = \int \mu_t^C(dz)$ is the size of the city and $\xi_t \sigma_t^C$ is the absorption of non-agricultural goods by the construction sector to supply housing for the city. The following simple lemma summarizes the behavior of the relative price p_t^N of non-agricultural goods.

Lemma 2 *Given a skill distribution μ_t, consider an exogenous population selection μ_t^R, μ_t^F, μ_t^C such that $Y_t^N\left(\mu_t^U\right) > \xi_t \sigma_t^C$. Then, the market clearing price p_t^N given by (24) is strictly positive. Moreover:* ***(a)*** *Given μ_t^U, p_t^N is strictly increasing in the size of the city, σ_t^C; and* ***(b)*** *given the size of the city, σ_t^C, the relative price p_t^N is decreasing in μ_t^U in the total output sense defined above.*

This statement is general in the sense that it captures all the variations in μ_t^U, an infinite dimensional object. For our purposes, however, we can narrow our analysis in terms of a threshold. As shown in the next section, in equilibrium there will be perfect sorting in skills between the urban and rural areas. There would be a finite threshold $0 < z_t^R < \infty$ such that, those households with $z \leq z_t^R$ would live in rural areas and those with $z > z_t^R$ would live in urban areas. Denoting F_t the c.d.f. associated with the measure μ_t, the aggregate supply of both forms of labor services can be written entirely in terms of the thresholds z_t^R and z_t^H:

$$L^q = F_t\left(z^H\right) - F_t\left(\max\left\{z_t^R, z_{\min}\right\}\right) \text{ and } L^a = \int_{z_t^R}^{\max\left\{z_t^R, z_{\min}\right\}} z\mu_t(dz) + \int_{z_t^H}^{\infty} zf(z)\,dz,$$

where $\max\left\{z_t^R, z_{\min}\right\}$ allows for the possibility that some qualified workers opt to remain in the rural areas. The determination of the urban occupation threshold z^H can be written more narrowly in terms of the two thresholds:

$$z_t^H = \frac{\eta}{1-\eta} * \left[\frac{\int_{z^R}^{\max\left\{z_t^R, z_{\min}\right\}} z\mu_t(dz) + \int_{z^H}^{\infty} z\mu_t(dz)}{F_t\left(z^H\right) - F_t\left(\max\left\{z_t^R, z_{\min}\right\}\right)}\right]. \tag{25}$$

Closed-form solutions for the optimal occupation split, $z^H\left(z^R\right)$, and for the resulting output of non-agricultural goods, $Y^N\left(z^R\right)$, cannot be provided, but we can prove the following characterization:

Proposition 1 *For any $z_t^R > 0$ such that $X_t^A F_t\left(z_t^R\right) > \bar{c}^A$, there exists a unique equilibrium occupation threshold $z_t^H < \infty$ defined by the fixed point of (25). Moreover:* ***(i)*** *If $z_t^R < z_{\min}$, then z_t^H is strictly decreasing in z_t^R;* ***(ii)*** *if $z_t^R > z_{\min}$, then z_t^H is strictly increasing in z_t^R; and* ***(iii)*** *the aggregate non-agricultural output, $Y^N\left(z_t^R\right)$, is always strictly decreasing in z_t^R.*

An interesting aspect of this proposition is the non-monotonicity of the occupation threshold in terms of the size of the city. In particular, given the population distribution of skills μ_t, the growth of the city can have a very different impact in the supply of adaptable and qualified jobs, and the structure of earnings within urban areas, depending on the equilibrium configuration of urban employment. For future reference we say an allocation has **urban low-skill jobs** when $z_t^R < z_{\min}$, because in this case some urban dwellers are below qualification and their only option is to provide low-skill adaptable labor. Similarly, we say that an allocation has **high-skill urban jobs only** when $z_t^R > z_{\min}$. In this case, the marginal rural migrant into urban areas is educated enough to provide qualified labor, and the urban labor market equilibrium directs only the very highly-skilled towards adaptable labor occupations.

Figure 5 displays these two occupation configurations for urban areas, depending on whether z_t^R is above or below the qualification threshold $z_{\min}$. Panel (a) is the first case discussed above, where $z_t^R > z_{\min}$ and adaptable labor is only provided by a set of high-skilled workers. Panel (b) is the second case, when the marginal urban worker is not qualified, $z_t^R < z_{\min}$, and adaptable jobs are provided by both low- and high- skilled workers. The relative importance of these two groups would depend on the measures $\mu_t\left(\left[z_t^R, z_{\min}\right]\right)$ and $\mu_t\left(\left[z_t^H, \infty\right)\right)$. Indeed, in the previous section we showed that the first group is important for Brazil, as many low-skilled workers have moved to the urban areas. In contrast, the second group is probably much more important for developed countries.

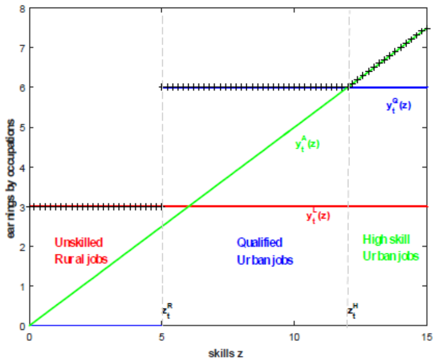


Figure 5a: High-Skill Urban Jobs Only

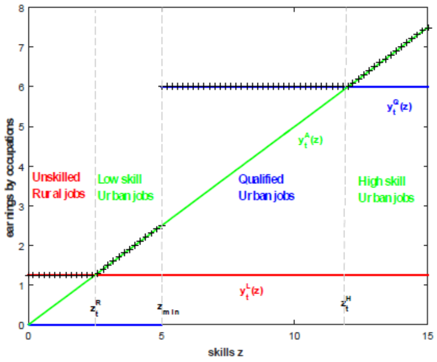


Figure 5b: Low- & High-Skill Urban Jobs

From Proposition 1, the impact of urban growth on the occupations and earnings depends on the equilibrium configuration. First, consider the case when there is only high-skill urban jobs, $z_t^R > z_{\min}$. *Ceteris paribus*, the growth of urbanization by a reduction in z_t^R would lead to an expansion of aggregate qualified labor. The general equilibrium response must be to reduce z_t^H, expanding adaptable labor by being less selective in the set of high-skilled workers doing those jobs. Second, consider the other case, an urban configuration with low-skill jobs, $z_t^R < z_{\min}$, and adaptable labor being provided both by low- and high-skilled workers. In this case, a reduction in z_t^R leads to an expansion of aggregate adaptable labor, and the general equilibrium response would be to expand qualified labor by rising z^H. We view this second case as capturing a key aspect of the structural change and urbanization in Brazil and multiple other developing countries.

Finally, the implication that $Y^N\left(z_t^R\right)$ is globally decreasing in the cut-off z_t^R will be used below, when we analyze the joint determination of the equilibrium price p_t^N and the urban-rural threshold z_t^R. To fix ideas, the threshold z_t^R must decrease with a higher value of p_t^N as this price pushes down the value of staying in the rural areas. The impact of z_t^R on p_t^N can be more complex, as it depends on whether the marginal urban resident lives in the city, and bears the resource cost of housing, or lives in the slum, and does not.

4.1.2 The Make-Up of Urban Areas

We now examine the allocation of an urban population between the city proper and the slum. As in the previous section, we take the total urban population μ_t^U as exogenously given, and explore how individual household decisions divide that population between a measure μ_t^C of city dwellers and a measure μ_t^F of slum dwellers.

We first derive a perfect sorting property: Under the postulate that cities are better than slums as places to raise children, then necessarily there has to be perfect separation between these two urban locations: Households with skills in a lower subinterval, which can be empty, would live in the slums. Households with skills in the upper-end of the distribution would live in the city. This separation is a direct result from both consumption and children's education being normal goods for all households.

Lemma 3 ***Urban Segregation:*** *Consider an economy whose population is exogenously partitioned between a rural population μ_t^R and an urban population μ_t^U, the latter with a support unbounded from above. Assume also that the price $p_t^N > 0$ is given, and that urban labor markets clear, i.e., z_t^H is given by (25). Assume also that the population μ_t^U is arbitrarily partitioned into a non-empty city and a non-empty slum sub-populations in a way that the implied exposure of ideas is better in the city, i.e., $Z_t^F \leq Z_t^C$. Then, there exists a threshold $z_t^F \geq 0$ such that living in a slum is utility maximizing for all households with $z \leq z_t^F$ and living in a city is utility maximizing for all those with $z > z_t^F$.*

This simple result is very useful, since it collapses the assignment of the urban population μ_t^U between measures μ_t^F and μ_t^C to simply finding a partition in their supports, i.e. determining the value of a threshold $0 < z_t^F < \infty$. To determine this threshold, define the functions that express the value of living in the slum or living in the city *for the marginal household* for whom $z = z_t^F$:

$$\begin{aligned} \mathcal{V}_t^F(z_t^F) &= u\left\{\theta\left(p_t^N\right)(1-\tau)\left[y_t^U\left(z_t^F\right)-\bar{c}^A\right]\right\}+\beta G Z_t^F\left(z_t^F\right), \\ \mathcal{V}_t^C(z_t^F) &= u\left\{\theta\left(p_t^N\right)\left[y_t^U\left(z_t^F\right)-\bar{c}^A-p_t^H\right]\right\}+\beta G Z_t^C\left(z_t^F\right). \end{aligned}$$

Here p_t^N is taken as given and $p_t^H = p_t^N \xi_t$, but the school quality are determined by the partition between the city and the slum, i.e. $Z_t^F\left(z_t^F\right) = \left[\int_{\bar{c}^A/w_t^a}^{z_t^F} z^\rho \mu_t^U(dz) \div \sigma_t^F\right]^{\frac{1}{\rho}}$ and $Z_t^C\left(z_t^F\right) = \left[\int_{z_t^F}^{\infty} z^\rho \mu_t^U(dz) \div \sigma_t^C\right]$, where $\sigma_t^C = \int_{z_t^F}^{\infty} \mu_t^U(dz)$ and $\sigma_t^F = \int_{\bar{c}^A/w_t^a}^{z_t^F} \mu_t^U(dz)$. Likewise, the function $y_t^U\left(z_t^F\right)$ is derived from wages w_t^a and w_t^q that arise from solving z_t^H by (25).

The low values of the threshold when $z_t^F < \bar{c}^A/w_t^a$, are not in the domains of $\mathcal{V}_t^F(\cdot)$ nor $\mathcal{V}_t^C(\cdot)$, since consumption would be below subsistence. For such low-skilled workers it is simply not feasible to live outside the countryside. For the higher values in which $\bar{c}^A < y_t^U\left(z_t^F\right) < \bar{c}^A + p_t^H$, paying for city housing is unfeasible, but living in a slum is feasible. For the points in which both urban options are feasible, the functions $\mathcal{V}_t^F(z_t^F)$ and $\mathcal{V}_t^C(z_t^F)$ are both continuous. It is easy to prove existence of an equilibrium, i.e. a threshold such that $\mathcal{V}_t^F(z_t^F) = \mathcal{V}_t^C(z_t^F)$.

Proposition 2 ***Existence of a Perfectly Sorted Urban Equilibrium.*** *Consider an economy whose population is exogenously partitioned between a rural population μ_t^R and an urban population μ_t^U, the latter with support unbounded from above. Assume also that the price $p_t^N > 0$ is given, that z_t^H is determined by (23), so that urban labor markets clear. Then, there exist a finite threshold $z_t^F \in (\frac{\bar{c}^A}{w_t^a}, \infty)$, such that location F is utility maximizing for all urban households with $z \in [0, z_t^F)$ and location C is utility maximizing for all urban households $z \in [z_t^F, \infty)$.*

Increasing the threshold z_t^F makes the city more and more exclusive. The schooling prospects and overall utility for all those remaining in the city, including the marginal household, are increasing in z_t^F. Indeed, $\lim_{z_t^F \nearrow \infty} Z_t^C = \infty$ and $\lim_{z_t^F \nearrow \infty} \mathcal{V}_t^C(z_t^F) = \infty$. On the other hand, as we increase the threshold z_t^F, the population of the slums increases and, by assumption, the schooling prospects there remain bounded, $\lim_{z_t^F \nearrow \infty} Z_t^F < \infty$. So, even if $\mathcal{V}_t^F(z_t^F)$ can grow without bound, it is easy to see that $\mathcal{V}_t^C(z_t^F) > \mathcal{V}_t^F(z_t^F)$ for all z_t^F high enough, because the cost of of living in a slum is directly associated with the consumption level of the household, as opposed to the cost of housing of the city, which is a fixed cost.

Because of externalities that operate in opposite directions, establishing uniqueness is a more elusive task. There are two types of concerns. First, one might entertain equilibria that do not satisfy the inequality $Z_t^F \leq Z_t^C$. This possibility is of low concern. If at all, an equilibrium with $Z_t^F > Z_t^C$ could only be sustained by a very irregular (and empirically irrelevant) urban configuration, where the slums are populated by middle-skilled households, and the city is populated by low- and high-skilled groups. A second, more interesting concern would be multiple thresholds z_t^F that solve the fixed point problem $\mathcal{V}_t^F(z_t^F) = \mathcal{V}_t^C(z_t^F)$. This concern is a more pressing one since both $Z_t^F\left(z_t^F\right)$ and $Z_t^C\left(z_t^F\right)$ are increasing in z_t^F: As we increase the threshold z_t^F, we are simultaneously pushing up the the maximum skill level in the slum and the minimum skill level in the city. Therefore, the difference $Z_t^C\left(z_t^F\right) - Z_t^F\left(z_t^F\right)$ can be initially decreasing and then increasing with respect to z_t^F.[37] Multiple crossings for the condition $\mathcal{V}_t^F(z_t^F) = \mathcal{V}_t^C(z_t^F)$ can lead to equilibria with different partitions of the urban population μ_t^U, with varying relative sizes and differences in the exposure to ideas between the city and the slums. However, for our calibration of the Brazilian economy, the non-monotonicity of $Z_t^C\left(z_t^F\right) - Z_t^F\left(z_t^F\right)$ was nowhere near enough to lead to multiple equilibria.

It is interesting to note that all these concerns are entirely driven by the households' valuation of their children education. If their location decisions were only driven by labor market considerations, the slum-city divide is driven by the comparison of the utility costs of living in the slum, vs. housing cost of living in the city. Indeed, if households do not consider their children's human capital, e.g. $\beta = 0$, the threshold is generically unique and given by $p_t^N \xi_t = \tau_t\left(y_t^U\left(z_t^F\right) - \bar{c}^A\right)$.[38]

4.2 Determining the Rural-Urban Divide

We now determine the equilibrium divide between rural and urban areas, i.e. the split between rural μ_t^R and urban μ_t^U. Proceeding as before, we first derive a separation property. Second, we prove the existence of the equilibrium rural-urban threshold and discuss its uniqueness. Finally, we discuss how the determination of this threshold shapes up the emergence of slums and of low-skill urban jobs.

For any set of arbitrary prices it is easy to show that rural areas would be populated exclusively by the lower-tail of the skill distribution:

Lemma 4 ***Rural-Urban Separation:*** *Given the country's state variable $S_t = \left(X_t^A, X_t^N, \mu_t\right)$, some admissible prices $\left\{w_t^u, w_t^a,\ w_t^q, p_t^N, p_t^H\right\}$ and a regional partition of the country's skill distribution μ_t so that $Z_t^R \leq Z_t^F \leq Z_t^C$. Then, there exists a finite threshold $z_t^R > 0$ such that the rural location R is utility maximizing for all households with $z \in [0, z_t^R)$ and either urban locations F or C are optimal for all urban households $z \in [z_t^R, \infty)$.*

This result is even simpler to prove than the one in the previous section. Given prices and exposure to ideas, the value attained by any household in the rural areas is independent of their skills, i.e. $V_t^R = u\left[\theta\left(p_t^N\right)\left(w_t^u - \bar{c}^A\right)\right] + \beta G Z_t^R$ for all z. In contrast, the value to the household in in the urban area is $\max\left\{V_t^F(z), V_t^C(z)\right\}$ and therefore strictly increasing (when well defined, i.e. when $y_t^U(z) - \bar{c}^A > 0$.) The threshold is strictly positive because for z close to 0, the consumption would be non-positive.

[37] To see this, notice that for low values of z_t^F, the slum is small and the relative weight of the mass near z_t^F on the determination for Z_t^F is higher than it for Z_t^C. Exactly the opposite occurs for high levels of z_t^F, where the slum is large and Z_t^F will be relatively insensitive to z_t^F, while, in the limit, Z_t^C is largely determined by z_t^F.

[38] Uniqueness is only generic because the knife-edge possibility that the marginal slum dweller is a qualified worker. If so, the condition is $p_t^N \xi_t = \tau_t\left(w_t^q - \bar{c}^A\right)$ which can be satisfied by any threshold $z_t^F \in \left[z_{\min}, z_t^H\right]$.

To examine the equilibrium determination of the rural-urban threshold z_t^R, we write the value attained by the marginal household, with skills $z = z_t^R$, in each of the locations, once we have factored in the values of all relevant objects –prices, thresholds and ideas across regions– as set in general equilibrium. In particular, given z_t^R, the equilibrium determination of the urban labor markets threshold z_t^H and the implied urban wages w_t^a and w_t^q are from Proposition 1 and expression (25). The urban population, $\mu_t^U(\cdot) = \int_{[0,\, z_t^R]} \mu_t(\cdot)$, is divided between the city and the slum according to the threshold z_t^F characterized in Proposition 2. Writing both of them as $z_t^H\left(z_t^R\right)$ and $z_t^F\left(z_t^R\right)$, we compute all other relevant objects, $Y_t^A\left(z_t^R\right)$, $Y_t^N\left(z_t^R\right)$, $\sigma_t^C\left(z_t^R\right)$, $p_t^N\left(z_t^R\right)$, $p_t^H\left(z_t^R\right)$, $y_t^U\left(z_t^R\right)$, $Z_t^R\left(z_t^R\right)$, $Z_t^F\left(z_t^R\right)$, and $Z_t^C\left(z_t^R\right)$, solely in terms of the rural-urban threshold z_t^R. See the appendix for the details. Embedding all these conditional equilibrium functions, we can define the value for the marginal household in the three locations:

$$
\begin{aligned}
\mathbb{V}_t^R(z_t^R) &= u\left\{\theta\left[p_t^N\left(z_t^R\right)\right]\left(X_t^A - \bar{c}^A\right)\right\} + \beta G Z_t^R\left(z_t^R\right), \\
\mathbb{V}_t^F(z_t^R) &= u\left\{\theta\left[p_t^N\left(z_t^R\right)\right](1-\tau)\left(y_t^U\left(z_t^R\right) - \bar{c}^A\right)\right\} + \beta G Z_t^R\left(z_t^R\right), \\
\mathbb{V}_t^C(z_t^R) &= u\left\{\theta\left[p_t^N\left(z_t^R\right)\right]\left(y_t^U\left(z_t^R\right) - p_t^H\left(z_t^R\right) - \bar{c}^A\right)\right\} + \beta G Z_t^C\left(z_t^R\right),
\end{aligned}
$$

To compare with the value of rural areas, we only need the upper enveloped of the urban options,

$$
\mathbb{V}_t^U(z_t^R) = \max\left\{\mathbb{V}_t^F(z_t^R), \mathbb{V}_t^C(z_t^R)\right\}.
$$

The equilibrium condition boils down to finding a threshold z_t^R such that $\mathbb{V}_t^R(z_t^R) = \mathbb{V}_t^U(z_t^R)$.

Proposition 3 ***Existence of Equilibrium***: *Given the country's state $S_t = \left(X_t^A, X_t^N, \mu_t\right)$ and under Assumptions 1-3, there exists a competitive equilibrium as defined in Section 3.*

The equilibrium rural population is non–empty, i.e., $0 < z_t^R < \infty$. Pushing z_t^R to zero would lead the aggregate food production below the subsistence level $\bar{c}^A$. Therefore, the equilibrium z_t^R is always high enough to ensure $Y_t^A > \bar{c}^A$. Similarly, urban areas cannot be zero because this would lead to Y_t^N close to zero. Finally, cities cannot be empty either because otherwise, under Assumptions 1 and 2, there is always someone with skills high enough that they would opt for the city.

The key issue is whether the slums are empty, as we discuss below. An *equilibrium with slums* is one in which $\mathbb{V}_t^R(z_t^R) = \mathbb{V}_t^F(z_t^R) > \mathbb{V}_t^C(z_t^R)$, while an *equilibrium without slums* is one in which $\mathbb{V}_t^R(z_t^R) = \mathbb{V}_t^C(z_t^R) > \mathbb{V}_t^F(z_t^R)$, *and*, for all households, either the value of the countryside or the city strictly dominates home-schooling in the slum.[39]

Table 9 displays the different urban and employment configurations that can arise in equilibrium. The first is the case of a developed country urban area, where there is no slums and adaptable jobs are provided only by the high-end of the skill distribution. In this case, the marginal rural migrant to the city has skills that meet the qualification standard, $z_t^R \geq z_{\min}$, and the housing prices and/or the costs of living in a slum are such that this marginal migrant buys housing in the city. Abusing a bit the notation, we use $z_t^R = z_t^F$ to indicate the case when slums are empty. Counter-clockwise, in the second case, the urban slums are empty but there are low-skill workers providing adaptable jobs

[39] Formally, when the slum is empty, a household with skills z can attain

$$
\mathfrak{V}_t^F(z) = u\left[\theta\left[p_t^N\left(z_t^R\right)\right]\left(y_t^U(z) - \bar{c}^A\right)(1-\tau)\right] + \beta G z,
$$

by home-schooling their children while living in a slum. The condition is that, for all households, this option is dominated by either the city or the countryside, i.e.,

$$
\mathfrak{V}_t^F(z) \leq \max\left\{\mathbb{V}_t^R(z_t^R),\, u\left\{\theta\left[p_t^N\left(z_t^R\right)\right]\left(y_t^U(z) - p_t^H\left(z_t^R\right) - \bar{c}^A\right)\right\} + \beta G Z_t^C\left(z_t^R\right)\right\}.
$$

as indicated by $z_t^R = z_t^F < z_{\min}$. This case would represent some developed countries in the past with small and affordable cities, which have integrated the school system for all the inhabitants of the city. A third case is $z_t^F > z_t^R > z_{\min}$, when only the high-end of the skill distribution provides adaptable labor and slums are populated. This case could coincide with cities in which housing is so expensive that even some of the workers with enough qualification opt to live in slums. Finally, the fourth case is the one we view as more closely resembling Brazil. In this case, a mass of low-skill urban workers provides adaptable labor, and all or some of them live in slums.

Table 9: Urban Locations and Employment Configurations

	Urban Locations	
Urban Jobs	Cities Only	Cities and Slums
High-Skill Only	$z_t^R = z_t^F \geq z_{\min}$	$z_{\min} < z_t^R < z_t^F$
High- & Low- Skill	$z_t^R = z_t^F < z_{\min}$	$z_t^R < \min\left\{z_{\min}, z_t^F\right\}$

Closing up this section, it is worth mentioning that the issues of multiple equilibria are as pressing as before. The difference $\mathbb{V}_t^R(z_t^R) - \mathbb{V}_t^U(z_t^R)$ might be non-monotone in z_t^R, and, in particular, the value $\mathbb{V}_t^R(z_t^R)$ can be non-monotone in z_t^R because lower values of this threshold leads to higher relative prices for agricultural goods but lower school quality Z_t^R. Moreover, there are multiple effects via the housing prices and the possibly non-monotonic differences $Z_t^F\left(z_t^R\right) - Z_t^R\left(z_t^R\right)$ and $Z_t^C\left(z_t^R\right) - Z_t^F\left(z_t^R\right)$. Analytically, it is quite elusive to provide general conditions for uniqueness and to characterize the set of equilibria with any level of generality. However, for the set of functional forms we used to explore Brazil, we computationally ruled out the relevance of multiple equilibria issues. See Appendix D for special cases of the model that allow further analytical characterization.

5 Quantitative Analysis

In this section we use our model to explore the experience of Brazil from 1950 to 2010. We first describe the calibration of the model to Brazilian data, and then we use the calibrated model to explore a number of counterfactuals.

5.1 Calibrating the Model to Brazil

After explaining how we set the parameter values, initial conditions and exogenous variables, we proceed to discuss how well the model matches the key aspects of interest in the Brazilian data, including the emergence of slums and the locational distribution of human capital.

5.1.1 Parameter Values, Initial Conditions and Exogenous Variables

Our aim is on the broad, low-frequency urbanization and structural transformation patterns of Brazil since 1950. Because of the intergenerational focus of our model –and the sparsity in some of the available data– our view of the Brazilian experience is from a very high level. Our strategy is to set the initial conditions of the model to mimic the conditions of Brazil in 1950. Interpreting each period as roughly 30 years, we solve for the equilibrium of the model for two periods: The first period, $t = 1$, corresponds to the years 1951-1980, and the second period, $t = 2$, corresponds to the years 1981-2010. Following this strategy, we set the values for parameters (preferences, technology and skill formation), initial conditions (cross-section education levels), and the behavior of exogenous variables (sectoral productivities, housing costs and slum-dwelling costs.) Table 10, contains the functional forms and values for the calibrated model.

The preferences parameters are those of the utility function $u(\cdot)$, the valuation of the expected child's education β, and the Stone-Geary parameters, α and $\bar{c}^A$. For the utility function $u(\cdot)$ we experimented with a CRRA functional form, but the results hinged much more on the other preferences parameters. Thus, for simplicity, we opted for a linear $u(x) = x$, which allows us to use the explicit formulas in Appendix D. Doing so, we chose a value for β that would correspond to the discount factor in a fully altruistic model. Setting $\beta = 0.294$ $(= 0.96^{30})$ provides a natural relative weight between a household's current consumption and the child's formation of skills, given that one period in the model equals 30 years of calendar time. The weight $\alpha = 1\%$ on the consumption of agricultural goods follows the calibration in Herrendorf et al. (2014). Finally, the subsistence consumption level $\bar{c}^A$ is set aiming for the model to reproduce the employment shares in the agriculture sector in both periods, 1951-1980 and 1981-2010.

The parameters for the non-agricultural technology are the qualification threshold $z_{\min}$ and the output share η of qualified labor. As explained below, we interpret the distribution of z as "*effective*" labor market skills, which we do not observe. We anchor such a distribution with the actual distribution of observed years of schooling in the Brazilian labor force. Doing so, we set the qualification threshold as having completed 11 years of education, i.e. $z_{\min} = 11$. This value may be high for Brazil, especially during the first years, 1951-1980. To avoid overestimating the output share and equilibrium wages of adaptable labor, we set a high value for η. To this end, we proxy the share η as the share of all labor above the threshold $z_{\min} = 11$. Using the PNAD, we computed the share of all the non-agricultural income accrued by all workers with 11 or more years of education. This share ranges from 0.65 to 0.59, over the different years. We set $\eta = 0.6$. The results using nearby pairs $(z_{\min}, \eta)$ were fairly similar to those reported below.

The sectoral productivities, X_t^i, which we take as exogenous, are the primary forces driving structural transformation in the model. Using data from GGDC, we measure the productivity of the agricultural sector X_t^A as the ratio between the total value added and the total number of workers in agriculture. For the first period, $t = 1$ (i.e., 1951-980), we use the normalization $X_1^A = 1$, and set $X_2^A = \left(1 + \gamma^A\right) X_1^A$, where γ^A is the growth rate of the average productivity between the periods 1951-1980 and 1981-2010. The non-agricultural productivity levels for both periods, X_1^N and X_2^N are set to match the average production share of the non-agricultural sector in total output in both periods.

For the cost of housing in the city and the utility cost of living in slums we do not have clear data counterparts. On the one hand, the costs of living in slums in our model are in terms of utils, making them difficult to associate directly with any available measurement in the data, especially for the first period $t = 1$. On the other hand, housing in our model is highly stylized and difficult to associate with the myriad of housing and neighborhood options in such megacities as Rio and São Paulo. Therefore, we opted to set the values for the utility-tax of living in a slum, τ_t, and the housing costs of living in a city, ξ_t, so that the calibrated model matches the population shares in cities (for all Brazil) and slums (for Rio) during the sample period. The only departure is for the slum-dwelling costs, τ_2, for the second period, 1981-2010, which was set to $\tau_2 = 0.245$. We set this value following the information in the 1991 Census, that shows that average rent-to-income ratio for inhabitants of favelas in Rio de Janeiro was 24.5%. Notice that in our calibration, both slum-dwelling and city housing costs are increasing over time, i.e. $\tau_2 > \tau_1$ and $\xi_2 > \xi_1$, consistent with the notion that conglomeration have risen the costs of living in the cities and slums over time.

Table 10: Calibration of the Model to Brazil: 1950-2010

Parameters			**Exogenous Variables**		
Parameter	**Value**	**Source/Criterion**	**Variable**	**Value**	**Target/Criterion**
I. Preferences			III. City & Slum Dwelling Costs		
β	0.294	Model period = 30 years	τ_1	0.19	Slum & City Population
α	0.01	Herrendorf et al.(2014)	τ_2	0.245	Slum Rents, 1991.
$\bar{c}^A$	0.245	% Agric. Employment.	ξ_1	0.1	Slum & City Population
			ξ_2	0.6	Slum & City Population
II. Technology			IV. Sectoral Productivities		
η	0.6	% Output, HS+, 91.	X_1^A	1	Normalization
$z_{\min}$	11	High School Diploma.	X_2^A	2.5	Agric. Prod.,81-10
			X_1^N	10	% Non-Ag.Output, 50-80.
			X_2^N	11	% Non-Ag.Output, 81-10.
V. Human Capital Formation: $z' \sim \Gamma\left(Z_t^l, k\right)$					
k	2.4	Avg. years Schooling:50-10	Z_0^R	0.8	Schooling of old, 1950
ρ	1	Eliminate ancillary gains	Z_0^F	1	Schooling of old, 1950
			Z_0^C	2	Schooling of old, 1950

The parameters for human capital formation are crucial to evaluate whether slums are barriers or stepping stones for the country as a whole. We set those parameters as follows. The curvature parameter ρ in the location's exposure to ideas, Z_t^l, is set to $\rho = 1$, as our benchmark, with the sole purpose to eliminate any *ancillary* gains that would result from simply grouping the population across the different locations.[40] In the appendix we show that variations in ρ around this benchmark do not produce quantitatively significant changes. For the formation of human capital in each location, we parametrize the transition probability $Q\left(\cdot|Z_t^l\right)$ as a Gamma distribution with *scale* parameter Z_t^l (i.e., varying across locations and time) and common *shape* parameter k.

We set the initial values Z_0^l, those for 1950, according to the observed schooling attainment of the old population in Brazil at that time. According to the Barro-Lee database, in 1950 Brazilians with 25 years or more had an average of 1.96 years of schooling. For the older cohorts, the averages were much lower, 1.75 and 1.53 for those in the 50-54 and 60-64 age groups, respectively. To capture these lower numbers, we opted to set the initial conditions for our model at $Z_0^R = 0.8$ for the population in rural areas, a slightly higher level, $Z_0^F = 1$, for the population in urban slums, and a substantially higher level, $Z_0^C = 2$, for the population in the city. These gaps across locations are in line with Census data and with the fact that in 1950, 36% of the population were in urban areas, out of which around 7% were in urban slums, as shown by the data for Rio. For the *shape* parameter k in the Gamma distribution, notice that it is also equal to the growth factor G in the average skill formation in each region. We use this property and the fact that in Brazil education attainment has grown substantially over the years, especially for the younger cohorts, to set the value of k. According to the Barro-Lee dataset, the average schooling for Brazilians with 20-24 years moves from 2.37 in 1950, to 3.76 in 1980, and then to 9.94 in 2010. The gross growth in the average schooling years is 1.6 during the period $1950-1980$, and 2.6 during the period $1980-2010$. We set the parameter value $k = 2.4$ for all locations and periods. Doing so, we capture the overall increase in education in the country as a whole, without targeting the education distribution in the three locations.

[40] If $\rho \neq 1$, regrouprng the population across locations would lead to changes in the country's average skill accumulation. For instance, if $\rho \to -\infty$, then each Z_t^l would be equal to the respective lower bound in the support of each region. In such a case, partitioning an urban area between a city and a slum would lead to a much faster accumulation of human capital for the country as a whole. While these effects may be a relevant consequence of slums, at this stage we would rather abstract from them since we cannot cleanly discipline them in the data.

5.1.2 Results

Our calibrated model reproduces fairly closely the structural transformation in Brazil. As shown by Table 11, agriculture shares are initially underestimated, but the model's implied declines of these shares are fairly close to the ones observed. Likewise, the model reproduces the fall in the output share of agriculture in total output, although it underestimates this variable in both periods. The calibrated model does a better job matching the slum populations. Note that the population share living in the cities is underestimated by only 0.7 p.p. in 1980 and by 2.7 p.p. in 2010. The good fit of the model is not driven entirely by our calibration targets, since, for the second period, we pinned-down the utility-costs of living in a slum from the rents in the Census of 1991. Thus, it is remarkable that the model differs only by 0.14 percentage points with respect to the data.

Table 11: The Calibrated Model and Observed Data for Brazil

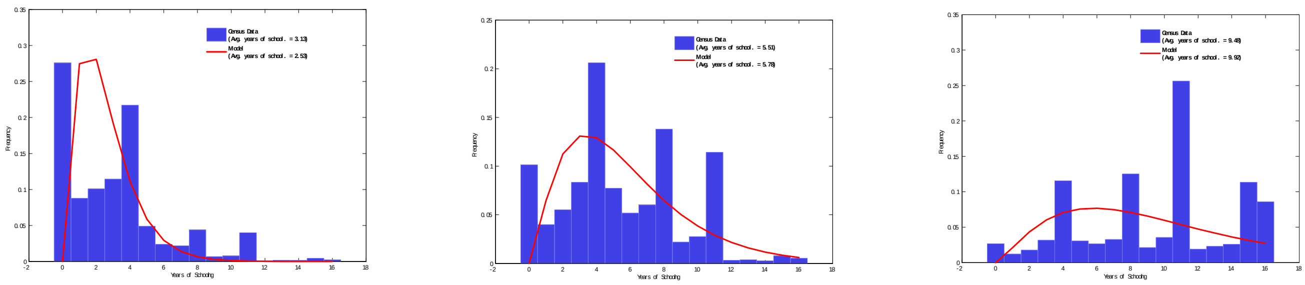
	1980		2010	
Variable	Data	Model	Data	Model
Population: (%)				
Slum Population: σ_t^F	10.34	10.96	18.70	18.84
City Population: σ_t^C	57.26	56.58	66.30	63.63
Agriculture: (%)				
Labor Share: L_t^A	38.15	32.46	16.70	17.53
Output Share: Y_t^A/Y	6.85	4.67	5.72	2.29
Average Schooling: (years)				
Rural Areas: $E\left(z'\|Z_t^R\right)$	1.46	2.20	3.13	2.53
Urban Slums: $E\left(z'\|Z_t^F\right)$	NA	4.07	5.51	5.78
Cities, Proper: $E\left(z'\|Z_t^C\right)$	NA	4.27	9.48	9.92

More interestingly, despite not being targets of our calibration, the model reproduces remarkably well the overall aspects of the distribution of human capital across rural areas, urban slums and cities proper for the final period, 1981-2010. Figure 6 shows these distributions from the data (histograms in bars) and the Gamma distributions (in lines) implied from the model, given the Z_1^l of the previous period. Obviously, there are significant differences, as the Gamma distribution cannot reproduce the multi-model aspects observed in the schooling attainment levels in the data. Yet, the distributions generated by the model captures important aspects of the data. For rural areas, most of the population is concentrated between zero and four years of schooling. For the slums, the model distribution captures that the peak in the data is around four years of schooling and that there is a sizable population with more than five years. For the city proper, as in the data, the model's distribution of schooling has a thick right tail, but misses, as expected, the spike at 11 years.

Finally, the model overestimates, to some extent, the urban-rural education differences for the years 1980-2010. On the one hand, the model underestimates the average education in the rural areas, 2.53 vs. 3.13 in the data. On the other hand, the model slightly overestimates the average schooling in urban slums and the city, 5.78 and 9.92 vs. 5.51 and 9.48 in the data, respectively.

Figure 6: Human Capital Distribution: Model and Data: 1980-2010.

a) Rural Area b) Slums c) City



All in all, we consider the calibrated model to be close enough to the key aspects of interest in the data, providing us with an useful tool for counterfactual experiments.

5.2 Counterfactual Experiments

We now use the calibrated model for counterfactual exercises. We first examine equilibria under two sets of alternative scenarios in terms of the costs of living in a slum or in the city. Second, we examine the implications of alternative degrees of integration between the schools of cities and the schools of the slums.

5.2.1 Cracking-down Slums

We start with the counterfactual experiment in which the government enacts measures to crack down on urban slums. Those are policies that governments in many developing countries have experimented with, including arresting slum dwellers and destroying their makeshift residences. In the model, these actions are captured by a higher utility costs τ_t of living in slums. Table 12 considers the implications of enacting these policies in the first period, raising τ_1, or in the second period, raising τ_2. For the former, we consider only a full crack-down on slums, i.e. $\tau_1 = 1$. For the latter, we consider two cases, a severe but partial crack-down, $\tau_2 = 0.5$ and a full crack down, $\tau_2 = 1$.

Table 12: Counterfactual Experiments: Cracking Down on Slums

Variable	Benchmark		Alternative Utility Costs of Living in a Slum					
			$\tau_1 \nearrow 1$		$\tau_2 \nearrow 0.5$		$\tau_2 \nearrow 1$	
	1980	2010	1980	2010	1980	2010	1980	2010
Population: (%)								
Urban Slums: σ_t^F	10.96	18.4	0.00	26.35	10.96	1.42	10.96	0.00
City proper: σ_t^C	56.58	63.63	17.34	57.98	56.58	79.61	56.58	29.99
Agriculture: (%)								
Labor Share: L_t^A	32.46	17.53	82.66	15.66	32.46	18.97	32.46	70.01
Output Share: Y_t^A/Y_t	4.67	2.29	1.43	2.75	4.67	2.04	4.67	2.26
Average Schooling (years)								
Rural Areas: $E\left(z' \vert Z_t^R\right)$	2.20	2.53	4.77	3.29	2.20	2.66	2.20	8.53
Urban Slums: $E\left(z' \vert Z_t^F\right)$	4.07	5.78	–	7.42	4.07	4.44	4.07	–
Cities, Proper: $E\left(z' \vert Z_t^C\right)$	4.27	9.92	7.33	10.21	4.27	8.44	4.27	14.88

Consider first the case, in columns 4-5 of Table 12, when in the first period, slums are eliminated by pushing τ_1 all the way to 1, making it impossible for households to subsist in an urban slum. By eliminating the option of living in urban slums, a much larger fraction of the population remains in rural areas, and the agricultural share of labor during the first period would have been 82.7%, much higher than the benchmark 32.5%. More interestingly, notice that: (a) despite the large share of employment in agriculture, the output share of that sector goes down substantially, from 4.7% to a very low 1.4%; and (b) the city proper becomes much smaller, just 17.3% of the population, from a much larger 56.6% in the benchmark. The reason behind these two responses lies in an implied higher relative price of non-agricultural goods which pushes up the housing costs of the city, $\xi_1 p_1^h$. The higher housing costs paired with a population of workers with very low skills as Brazil before 1980 explains the collapse of the city proper in the model. Under those circumstances, many households would have not been able to afford living in the city, and Brazil would have remained predominatly an agricultural country.

The equilibrium responses in terms of the regional differences in the formation of human capital are also worth noticing. Under the counterfactual experiment, not only the quantity but also the average skills of households living in rural areas would be higher. The implied expected skill formation in rural areas becomes much better than in the benchmark, 4.8 vs. 2.2 years, reinforcing the value for households to stay there. On the other hand, the city proper becomes much smaller and exclusive, with a substantially higher formation of skills, 7.3 vs. the benchmark's 4.27.

Even more interesting, are the implications for the second period, when the utility cost of living in a slum is kept at the benchmark level $\tau_2 = 0.245$. The model implies that the slums for this second period would be 40% *higher* than in the benchmark, accounting for 26.4% of the population vs. the benchmark (and data) 18.8%. The model suggests that for many dynasties in Brazil, living in urban slums during the years $1950 - 1980$ served as a stepping-stone to enter the cities proper in $1981 - 2010$.

Consider next the cases, in the remaining columns of Table 12, when the crack-down is in the second period. We consider two different degrees in which the anti-slum policies are hardened. In the first case, columns 6-7, τ_2 is only pushed to 0.5, which does not eliminate the slum, but makes it much smaller. Interestingly, in response to this change, the slum becomes much less selective, since the upper tail of those living there in the benchmark case would move to the city in the alternative scenario. Indeed, the city would have become substantially larger, 80% of the population compared to the benchmark 64%. It also becomes less selective, with an average formation of human capital of only 8.4 expected years of schooling, compared to 10 in the benchmark. Interestingly, the impact on the structural transformation of the country is relatively minor. This is partly explained by the higher level of skills in the population during the second period, 1981-2010.

Finally, consider the case when the crack-down in the second period is complete, $\tau_2 = 1$. As expected, the slums would disappear in that period. The impact on the structural transformation and urbanization are lower than in the experiment for the first period, but it is still quite large: The city would become smaller, 30% of the population, i.e. half the size it was in the benchmark, and the rural area would be much larger, 70% of the population. The general equilibrium response of the prices of non-agricultural goods and housing costs, as well as of the human capital formation in the countryside are similar as those in the first experiment.

5.2.2 Alternative Housing Costs in Cities

We now consider counterfactual experiments with alternative housing costs, which in our model are the endogenous barriers for a household to live in a city. Housing costs can be driven by conglomeration costs, i.e. construction becomes more costly as more urban lands are developed, and by regulations. Both forces can be captured in our model by variations in the units of non-agricultural goods ξ_t required to procure a house in the city. In Table 13, we report the results of our counterfactual experiments of variations in ξ_t. Columns 2 and 3 reproduce our benchmark.

Columns 4 and 5 explore the out first experiment, increasing the housing costs in the first period to $\xi_1 = 0.2$, i.e. twice as high as in the benchmark calibration. Columns 6 and 7 consider our second experiment, lowering housing costs in the second period to $\xi_2 = 0.3$, i.e., half of the benchmark value. Columns 8 and 9 consider our third experiment, raising the housing costs in the second period to $\xi_2 = 0.9$, fifty percent higher than in the benchmark calibration.

Table 13: Counterfactuals: Alternative Housing Costs in the City

Variable	Benchmark		Alternative Housing Costs in the City					
			$\xi_1 \nearrow 0.2$		$\xi_2 \searrow 0.3$		$\xi_2 \nearrow 0.9$	
	1980	2010	1980	2010	1980	2010	1980	2010
Population: (%)								
Urban Slums: σ_t^F	10.96	18.84	55.30	24.90	10.96	1.44	10.96	31.63
City proper: σ_t^C	56.58	63.63	12.24	57.50	56.58	81.03	56.58	50.84
Agriculture: (%)								
Labor Share: L_t^A	32.46	17.53	32.46	17.60	32.46	17.53	32.46	17.53
Output Share: Y_t^A/Y_t	4.67	2.29	4.73	2.28	4.67	2.34	4.67	2.26
Average Schooling (years)								
Rural Areas: $E\left(z'\vert Z_t^R\right)$	2.20	2.53	2.20	2.54	2.20	2.53	2.20	2.53
Urban Slums: $E\left(z'\vert Z_t^F\right)$	4.07	5.78	6.95	6.31	4.07	4.2	4.07	7.23
Cities, Proper: $E\left(z'\vert Z_t^C\right)$	4.27	9.92	8.14	10.63	4.27	8.32	4.27	11.44

As emphasized all along the paper, the costs of living in a city is a major determinant of urban slums. In our first counterfactual experiment, we see that a higher costs in the first period, precisely when the human capital distribution in Brazil is very low, would have led to a drastic reduction of the population in cities, to 12% from 57% in the benchmark and data, a massive increase in the slum population, to 55% from 11%, without much of an impact on the rural population or on the output share of agriculture. Expanding the slum population by contracting the city population makes *both* urban regions to improve their quality as places to rise children, because the low-end of the city distribution becomes the top-end of the slum distribution. In the new equilibrium, the average skill formation in slums and in the cities change from 4.1 and 4.3 years of school to 7 and 8.1, respectively.

In contrast, the second experiment shows that lowering housing costs in the presence of a higher distribution of human capital in the population can lead to a very substantial reduction of urban slums. By halving the second period housing costs from the benchmark calibration to $\xi_2 = 0.3$, the population in urban slums would have collapsed to less than 1.5%, and the city would expand in equal measure. Notice that with a more inclusive city, the average skill formation of the city also goes down to 8.32 years, but keep in mind that this covers a much larger population.

In the same vein, the third of these experiments shows that higher housing costs could have increased the slum population in the second period, but the increase is a far cry from the one obtained for the first experiment. As before, this is explained by the fact that in the second period, the population of Brazil had a much higher distribution of human capital. Even with such an increment, around 51% of the population would have remained in the cities. And, as in the first experiment, the average skill formation of both, cities and slums, would have been higher.

In sum, high housing costs are a major factor driver for the formation of urban slums. What these counterfactual experiments have added is showing, very clearly, that the impact of these costs depends crucially on the skill distribution of the country.

5.2.3 Urban School Integration

One of the most interesting findings in Section 2 was the asymmetric workings of a slum, in terms of granting access to adults to urban labor markets while keeping children segregated into schools of the slums. To be sure, school segregation in urban areas is a much more general phenomena, but, if anything, this reinforces our interest in exploring the implications of integrating the schools of cities and slums. To this end, we now consider a variation in the urban environment, in which the formation of skills for children growing up in urban areas depend on different weighted averages of the ideas circulating in the slum and the ideas circulating in the city. For children in slums,

$$z' \sim Q\left(\cdot|\hat{Z}_t^F\right), \text{ where } \hat{Z}_t^F = \iota_t Z_t^F + (1-\iota_t) Z_t^C,$$

while for children growing up in the city,

$$z' \sim Q\left(\cdot|\hat{Z}_t^C\right), \text{ where } \hat{Z}_t^C = \iota_t Z_t^C + (1-\iota_t) Z_t^F.$$

Here, $0.5 \leq \iota_t \leq 1$ is the weight that the children location of residence has on his skill formation. Our benchmark case is perfect segregation in both periods, i.e., $\iota_t = 1$ for both, $t = 1$ and $t = 2$. On the opposite extreme, $\iota_t = 0.5$, would mean perfect integration. Our counterfactual experiments are based on variations in ι_t. In the first experiment we consider a partial integration in the second period, setting $\iota_2 = 0.75$, keeping perfect segregation in the first period, $\iota_1 = 1$. In the experiment of case 2, we look at a partial degree of integration for both periods, i.e. $\iota_1 = \iota_2 = 0.75$. Finally, the third experiment, we look at the case of perfect integration for both periods, i.e. $\iota_1 = \iota_2 = 0.5$. The results are reported in Table 14, columns 4-5, 6-7 and 8-9, respectively.

Table 14: Counterfactual Experiments: Integrating Urban Schools

Variable	Benchmark		Alternative School Integration of Cities and Slums					
			ι_2= 0.75		ι_1= ι_2 = 0.75		ι_1= ι_2 = 0.5	
	1980	2010	1980	2010	1980	2010	1980	2010
Population: (%)								
Urban Slums: σ_t^F	10.96	18.84	10.96	23.23	46.32	14.60	46.32	12.94
City proper: σ_t^C	56.58	63.63	56.58	60.50	21.22	68.01	21.22	70.13
Agriculture: (%)								
Labor Share: L_t^A	32.46	17.53	32.46	16.27	32.46	17.39	32.46	16.92
Output Share: Y_t^A/Y_t	4.67	2.29	4.67	2.26	4.71	2.33	4.71	2.43
Average Schooling (years)								
Rural Areas: $E\left(z'\|Z_t^R\right)$	2.20	2.53	2.20	2.54	2.21	2.65	2.21	2.64
Urban Slums: $E\left(z'\|\hat{Z}_t^F\right)$	4.07	5.78	4.07	6.04	4.15	5.96	4.62	6.68
Cities, Proper: $E\left(z'\|\hat{Z}_t^C\right)$	4.27	9.92	4.26	7.67	4.21	8.08	4.62	6.68

In the first experiment, the partial access to schools from the city would motivate a substantial increment in the population living in slums. The city will shrink, partly because the schooling premium to live there has been reduced relative to the slums. The size of the overall urban population also increases marginally, from 82.5% to 83.7%. The effect on the relative price of non-agricultural goods, housing prices and the output share of the agricultural sector is also affected only marginally.

If the integration of urban schools is done in both periods, the results can be quite different. Under the second experiment, there would have been a surge in the slum population, to more than 46%, and a collapse of the city, to just above 21%, from the respective values of 11% and 57% in the benchmark. However, for the second period, the impact would have been reversed. The city population would have been higher than in the benchmark, 68% from 64%, and the slum population lower, below 15% from 19% in the benchmark. By integrating schools in the first period, the stepping-stone function of the slums becomes much stronger than in the benchmark case, and the mass of individuals with skills high enough to pay for a house in the city would have been higher in the second period.

The third experiment, perfect integration in both periods, shows the admitedly elusive case when city-slum school differences are completely dissipated. Under this experiment, the fraction of individuals living in slums is even lower in the second period. Interestingly, the population in the city proper would also be larger even if cities do not offer schooling advantages. This is the result of a higher distribution of skills for the population, and avoiding the utilty-costs of living in slums suffices to make living in the city the most desirable choice for more households.

While at the costs of reducing the quality of the city schools and initially inducing an even larger slum population, these experiments suggest that procuring slum-dwelling children with some access to the schools of the city, could eventually lead to larger cities with smaller urban slums.

6 Conclusions

On the surface, the urbanization and structural transformation patterns in Brazil (and other developing countries) seem to follow the same patterns observed in fast growing countries. Yet, beneath the massive rural-urban migration and employment reallocation to non-agricultural sectors, we uncovered a substantial growth in low-skill urban jobs and in urban slums. In this article, on the basis of salient micro evidence, we construct a simple dynamic model where the population distribution across locations and occupations, as well as the formation of human capital, are all endogenous. Following the evidence, there are substantial education advantages of cities over slums and of slums over rural areas. A key feature of the workings of slums is that they grant access to working-adults to the urban labor markets at large but restrict the human capital formation of children to inferior options. Despite those shortcomings, living in a slum is the preferred option for some low-skill households, in light of the high housing costs of the cities proper. We calibrate the model to the Brazilian data and examine different aspects of the formation and persistence of low-skill jobs and urban slums.

We argue that more than a barrier, slums are a stepping-stone for low-skill households and for the country as a whole. To be sure, slums are stepping-stones, not bridges, as they are associated with dire living conditions. But for low-skill households, they enhance the labor market and human capital formation opportunities once compared to the countryside. Indeed, we find that cracking down on slums –and secluding low-skill households to the rural areas– would slow down the acquisition of human capital in the low-end of the distribution, inducing even larger slums in the future. Yet, when compared to the city proper, slums are a barrier in terms of human capital formation opportunities. In fact, we show that procuring slum-dwelling children some access to schools in the city would foster their schooling attainment. On the aggregate, this would eventually lead to larger cities and smaller slums, precisely because the country's labor force would be less concentrated in low-skill workers.

To emphasize the essential general equilibrium aspects of rural-urban migration and structural transformation, our model is highly stylized, and we look at the data from a very high level. We see three well-defined avenues for future work. First, we should go beyond the simple peer-effects human capital formation model and explore more closely the factors driving the underwhelming schooling outcomes in the rural and poor urban areas. Accounting for location differences in peers, teachers, financing and other inputs, could suggest a wider scope for policy. Such an analysis should not be restricted to the micro aspects, since, as we have underlined in this paper, the macro implications of education policies can be as substantial as determining the skill composition of rural-urban migration and whether structural transformation is directed towards low- or high-skill urban occupations. For instance, high-quality schools in the countryside could prevent the formation of urban slums altogether. Second, it would be interesting to consider a multi-city or multi-neighborhood environment. As we have done here, such an analysis should incorporate urban slums as one of the residential options. For low-skilled households, such a richer setting could capture more finely the consumption/education trade-offs offered to low-skill households by the countryside, the different cities and the slums. For high-skilled households, it could capture more finely the multiple schooling options offered by cities. In the same vein, a third avenue is to extend our one-house/two-goods/three-occupations model, and consider richer models that can capture the impact of a fat tail on the income distribution on the demand of personal services and other low-skill urban jobs as a main driver for the formation of urban slums. All in all, those extensions deserve multiple papers on their own, and we view this paper as part of their foundation.

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A Data

The sample covers Brazil and the years 1950 to 2010. The series of real value-added and employment by sectors were taken from the Groningen Growth and Development Centre (GGDC) database[41], which provides long-run internationally comparable statistics for 42 countries (Africa, Asia, Latin America, Europe and United States). The data set includes series of value added, output deflators and persons employed for ten productive sectors.

Although the data covers ten broad sectors, here they were grouped into two major sectors: agriculture and non-agriculture, following the structural transformation literature. The sectors are defined by the ISIC Rev. 3[42] and were grouped as follows: agriculture consists of agriculture, forestry and fishing (01-05); and non-agriculture is composed by mining (10-14), manufacturing (15-37), public utilities (40-41), construction (45), wholesale and retail trade, hotels and restaurants (50-55), transport, storage and communication (60-64), finance, insurance and real estate (65-74), and community, social, personal and governments services (75-99). Following the literature, the productivity series were constructed as the ratio between the real value added and the persons employed by each sector for the period 1950 to 2010.

From the Brazilian Census, we explore interesting characteristcs and dynamics of the economy since 1950. The Census is a meticulous survey of all households in the country, conducted every ten years by the Brazilian Institute of Geography and Statistics (IBGE).[43] For the years 1991 and 2000, the Census provides an interesting variable, telling us if an household lives in a "subnormal

[41] See Timmer et al. (2014).

[42] International Standard Industrial Classification of All Economic Activities, Rev. 3.

[43] See *www.ibge.gov.br/english/.*

agglomerate" which defines as "a set of 51 or more housing units characterized by absence of a proper ownership title and at least one of the following aspects: (i) Irregular traffic routes or irregular size (shape) of land plot; (ii) Lack of essential public services such as garbage collection, sewage system, electricity and public lighting."

An alternative definition is the one used by the UN Habitat, which defines "a slum household" as "a group of individuals living under the same roof and lacking one or more of the following conditions: (i) Access to improved water; (ii) Access to improved sanitation; (iii) Sufficient-living area; (iv) Durability of housing; (v)Security of tenure.'Under this definition, the slum population in Brazil would be even larger, because it could include smaller groups of slum households scatter across the urban areas. For more details about the underestimation of the number of slum dwellers in Brazil, see Cavalcanti and Da Mata (2014). Finally, the description of a slum is almost equivalent to that of very poor settlements. Thus, following the Brazilian literature, we use here slums and "subnormal agglomerate" interchangeably.

We make intensive use of the Favela Census,[44] conducted by the state government of Rio de Janeiro in 2010. As explained in the text, this Census is a unique initiative of mapping and identifying the profile of residents who live in the three biggest slums in Rio: Alemão, Manguinhos and Rocinha.

The surveys for 1988 and 1996 of the PNAD (Pesquisa Nacional por Amostra de Domicílio)[45] have a special supplement which includes questions about parental education of the household head and the spouse. This database allowed us to compute the transition probability for entire country's rural and urban areas. We proxied slum households as those living in metropolitan areas with total income in the 35 percentile or lower.

B The Early History of Brazilian Favelas

The origin of slums can be traced back to the Golden Law (Lei Áurea) that in 1888 abolished slavery in Brazil. This law lacked any policy for inserting former slaves into the labor market or to provide basic services (like food, housing and health) and led to a large-scale migration of former plantation workers to the cities, in particular Rio de Janeiro, capital of Brazil during the years 1763 to 1960. These workers, unable to buy or rent formal housing, ended up living in tenements (cortiços), and illegal areas in hills, caves and swamps.

In 1889, a military coup overthrew the monarch Don Pedro II and established a republic backed by the landowning elites. During this time, strode Antonio Conselheiro, a peripatetic preacher, who wandered the Brazilian backlands and preached against slavery and the Brazilian Republic and against the separation of Church and State. Conselheiro settled in Canudos, in Bahia, and established the village of Belo Monte with its own social system and division of labor based on commom property and its own currency. Fearing the massive growth of the movement initiated in Belo Monte, in 1896, the republican government sent thousands of troops to the region, starting the War of Canudos (1896-1897), probably the biggest civil war in Brazil. Canudos was completed destroyed and the soldiers reclaimed the territory.

The victorious veterans retrurned to Rio de Janeiro to claim the land grants promised by the government. While waiting, they settled on a hillside alongside the former slaves and street vendors already camping there. The soldiers were never able to gain the lands promised by the government and gradually built their own shacks to replace their tents. The Canudos veterans named the hillside Morro da Favela, as the bushes there were reminiscent of the favela plant (Cnidoscolus quercifolius) found in Canudos. This first favela was later called Morro da Providência, and forever after the

[44] For more details see *www.emop.rj.gov.br/trabalho-tecnico-social/censos-comunitarios.*

[45] National Household Survey conducted every year in Brazil since 1976.

term favela has been used to refer to squatter settlements, shantytowns, and all types of irregular settlements, which in Brazil typically settle allong the hillsides (like Rocinha and Complexo de Alemao) and in the lowlands of Brazilian cities.

During the years 1902 and 1906, the major of Rio de Janeiro aggresively aimed the sanitation and planning of the city and to achieve his goals, favelas, cortiços, and shelters were destroyed. However, Morro da Favela remained untouchable. After that failed clean up policy, the number of squatter settlements increased from around 100 in 1906 to around 1,500 in 1933, with a population close to 10,000. Since the early twentieth century, we observe the first waves of rural-uban migration and the emergence of the first slums in Rio de Janeiro, but it was only after World War II that the process of urbanization and formation of slums became a national and widespread phenomena.

Although favela had been commonly used as a generic term for any squatter settlements, the first legal recognition of favelas was in the late of 1930, when the Building Code of 1937 prohibited the building of new favelas and banning the expansion of the existing ones, categorizing them as an urban aberration. See Pearlman (2010) and Pino (1997) for a richer account of the historical aspects of slums in Brazil and engaging details of the circumstances, decisions and outcomes of the households involved.

C Additional Information on Brazil

The massive rural-urban migration is reflected in the census data for Rio. In 1960, migrants accounted for 52.2 % of the slums' population in Rio; that share was 38.3% in the city of Rio proper. The following Table presents the information for Rio de Janeiro for the other years.

Table C1: Migrants, (% of total population) Rio de Janeiro

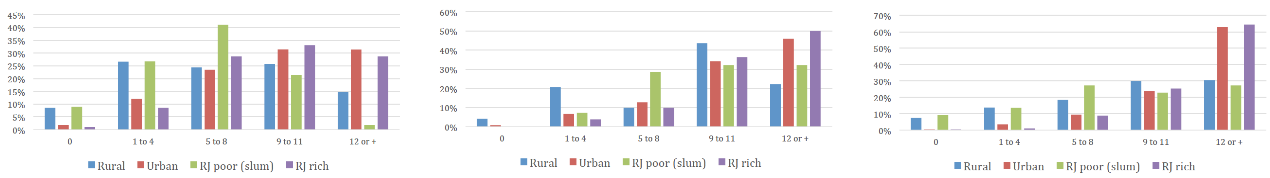
Year	Slums	City
1960	52.2	38.3
1991	29.8	27.7
2000	31.2	25.8

Source: Brazilian Census

As indicated above, the share of immigrants were higher in the 1950s and 1960s in Rio, as it was the capital and main city. Later on, the capital switched to Brasilia and the main city became Sao Paulo, which deviated the inflows of rural immigrants to these cities.

Figure C1 complements Figure 4 in the text, by showing the transition probabilities for children with a father with education attainment in the three remaining education levels. A nuance impact of location is evident for these higher education households, where the city clearly dominates slums, but, specially for the higher education parents, the rural areas dominate the slums. This would reinforce the observed pattern that highly educated households that move from the rural areas do so to the city and not to slums.

Figure C1: Brazil: Education Attainment Probabilities, Different Locations 1996



Fathers, Schooling 5-8 years   Fathers, schooling 9-11 years   Fathers, schooling 12+ years

D Analytical Details

D.1 Special Cases

Here, we consider two cases that allow further analytical characterization, as the equilibrium location choices as the roots of simple equations.

Myopic Economics: $\beta = 0$. Here, location decisions are entirely driven by labor market opportunities. An equilibrium without slums takes place when the marginal rural-urban migrant skips the slum, and pays for housing in the city. In this case, the price of a house is pinned-down by the equality of net-of-housing earnings between the countryside and the city,

$$p_t^H\left(z_t^R\right) = y_t^U\left(z_t^R\right) - X_t^A, \tag{26}$$

i.e., the consumption cost of the slum would be higher for the marginal migrant, i.e. $p_t^H\left(z_t^R\right) \leq \tau_t\left[y_t^U\left(z_t^R\right) - \overline{c}^A\right]$, a condition that trivially extends to all city dwellers. Since $p_t^H\left(z_t^R\right) = \xi_t p_t^N\left(z_t^R\right)$, we can use the expression (24) for $p_t^N\left(z_t^R\right)$ to put everything in terms of z_t^R. To this end, use $F_t\left(z_t^R\right)$ as the cdf associated to the measure μ_t, and with it, write the city size as $\sigma_t^C = 1 - F_t\left(z_t^R\right)$, agricultural output as $Y_t^A = X_t^A F_t\left(z_t^R\right)$. Also, as functions of z_t^R, write $Y_t^N\left(z_t^R\right)$ for the non-agricultural output resulting (25) and write the household urban earnings as $y_t^U\left(z_t^R\right) = p_t^N\left(z_t^R\right) X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^R, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^R\right)\right\}$, where $\chi\left(z_t^R\right)$ is the qualification indicator (a function that assumes a value of 1 if $z \geq z_{\min}$ or zero otherwise.) With all of those in (26), the rural-urban equilibrium threshold, z_t^R, is determined by the equality

$$\xi_t = X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^R, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^R\right)\right\} - \frac{X_t^A}{p_t^N\left(z_t^R\right)}, \tag{27}$$

where $p_t^N\left(z_t^R\right) = \frac{1-\alpha}{\alpha}\frac{X_t^A F_t\left(z_t^R\right) - \overline{c}^A}{Y_t^N\left(z_t^R\right) - \xi_t\left[1 - F_t\left(z_t^R\right)\right]}$ is the relative price of non-agricultural goods when there are not slums and the city is equal to the urban area. It is easy to see that under our assumptions there exist a unique threshold z_t^R that solves (27.)[46]

An equilibrium with slums arises when the marginal rural-urban migrant, $z = z_t^R$, has higher consumption in the slum than in the city, i.e. $X_t^A - \overline{c}^A = \left[y_t^U\left(z_t^R\right) - \overline{c}^A\right](1-\tau_t) > y_t^U\left(z_t^R\right) - p_t^H\left(z_t^R\right) - \overline{c}^A$. In such a case, the marginal entrant to the city proper, $z = z_t^F > z_t^R$, is determined by the condition $\xi_t p_t^N\left(z_t^R\right) = \tau_t\left[y_t^U\left(z_t^F\right) - \overline{c}^A\right]$. Following the steps of Section 4.1.2, for any given z_t^R, the equilibrium slum cut-off z_t^F solves

$$\xi_t = \tau_t X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^F, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^F\right)\right\} - \frac{\tau_t\overline{c}^A}{p_t^N\left(z_t^R, z_t^F\right)},$$

while the equilibrium z_t^R solves

$$\frac{X_t^A - \tau_t\overline{c}^A}{p_t^N\left(z_t^R, z_t^F\right)} = (1-\tau_t) X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^R, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^R\right)\right\}.$$

where $p_t^N\left(z_t^R, z_t^F\right) = \frac{1-\alpha}{\alpha}\frac{X_t^A F_t\left(z_t^R\right) - \overline{c}^A}{Y_t^N\left(z_t^R\right) - \xi_t\left[1 - F_t\left(z_t^F\right)\right]}$.

[46] There is always a unique crossing since the RHS, which is continuous and increasing in z_t^R, is negative for low values of z_t^R goes to $+\infty$ as z_t^R goes to $+\infty$.

Linear Utility: $\beta > 0$, $u(c) = c$. With linear utilities relative to overall consumption, we can also write down one or two conditions that define equilibria. An equilibrium without slums arises is given by the condition $V_t^R(z_t^R) = V_t^C(z_t^R)$, can be solved as

$$\xi_t = X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^R,\, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^R\right)\right\} - \frac{X_t^A}{p_t^N\left(z_t^R\right)} + \frac{\beta G\left[Z_t^C\left(z_t^R\right) - Z_t^R\left(z_t^R\right)\right]}{p_t^N\left(z_t^R\right)\theta\left[p_t^N\left(z_t^R\right)\right]},$$

where $p_t^N\left(z_t^R\right) = \frac{1-\alpha}{\alpha}\frac{X_t^A F_t\left(z_t^R\right) - \bar{c}^A}{Y_t^N\left(z_t^R\right) - \xi_t\left[1 - F_t\left(z_t^R\right)\right]}$ as in the static case. What differs from the case with $\beta = 0$, is that the price of houses in the city incorporate a premium for the better schooling opportunities, as denoted by the additional term $\frac{\beta G\left[Z_t^C\left(z_t^R\right) - Z_t^R\left(z_t^R\right)\right]}{p_t^N\left(z_t^R\right)\theta\left[p_t^N\left(z_t^R\right)\right]}$. Alternatively, this condition implies that, relative to the static case, the cutt-off z_t^R would be lower as households would be willing to sacrifice earnings so that their children access the higher quality urban schools.

An equilibrium with slums is defined by the conditions $V_t^R(z_t^R) = V_t^F(z_t^R) < V_t^C(z_t^R)$, and $V_t^F(z_t^F) = V_t^C(z_t^F)$ for $0 < z_t^R < z_t^F < \infty$. The second condition defines z_t^F and is given by

$$\xi_t = \tau_t X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^F,\, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^F\right)\right\} - \frac{\tau_t\bar{c}^A}{p_t^N\left(z_t^R, z_t^F\right)} + \frac{\beta G\left[Z_t^C\left(z_t^R, z_t^F\right) - Z_t^F\left(z_t^R, z_t^F\right)\right]}{p_t^N\left(z_t^R, z_t^F\right)\theta\left[p_t^N\left(z_t^R, z_t^F\right)\right]},$$

while the condition for the rural-urban cut-off z_t^R becomes,

$$\frac{X_t^A - \tau_t\bar{c}^A}{p_t^N\left(z_t^R, z_t^F\right)} - \frac{\beta G\left[Z_t^F\left(z_t^R, z_t^F\right) - Z_t^R\left(z_t^R\right)\right]}{\theta\left[p_t^N\left(z_t^R\right)\right]} = (1-\tau_t) X_t^N \max\left\{(1-\eta)\left(\frac{L_t^q}{L_t^a}\right)^{\eta} z_t^R,\, \eta\left(\frac{L_t^a}{L_t^q}\right)^{1-\eta}\chi\left(z_t^R\right)\right\}.$$

In both cases, the equilibrium conditions explicitly incorporate the value of schooling. The marginal rural-urban migrants are motivated to sacrifice some of their earnings so that their children have access to the better schooling prospects in the slums. Likewise, the marginal slum-city dweller is also willing to sacrifice some of their utility (net-of subsistance consumption) in order to switch the children from the slum schools to the city schools.

D.2 Proofs

We sketch the proofs for the simple analytical results in the text.

Proof of Lemma 1. Similar as in part (i) in Proposition 1, substituting μ_t^U for μ_t.■

Proof of Lemma 2. Recall that condition (24) is $p_t^N = \frac{1-\alpha}{\alpha}\frac{Y_t^A\left(\mu_t^U\right) - \bar{c}^A}{Y_t^N\left(\mu_t^U\right) - \xi_t\sigma_t^C}$, where μ_t^R, μ_t^F, μ_t^C are taken exogenously. Assumption 3 and the condition $Y_t^N\left(\mu_t^U\right) > \xi_t\sigma_t^C$ are both required for the price p_t^N to be positive. Under this circumstances, p_t^N is strictly increasing in the size of the city, σ_t^C since $Y_t^A\left(\mu_t^U\right)$ does not depend on the split between cities and slums and an increase in the size of the city σ_t^C would reduce the net non-agricultural output left for consumption, $Y_t^N\left(\mu_t^U\right) - \xi_t\sigma_t^C$, which is the denominator in (24). This proves part (a). Similarly, for part (b), notice that under the hypothesis of the lemma, given $Y_t^A\left(\mu_t^U\right)$ and σ_t^C, any increase in non-agricultural output $Y_t^N\left(\mu_t^U\right)$ would reduce the equilibrium price p_t^N.■

Proof of Proposition 1. Recall that the first order condition can be rearranged as

$$z^H = \frac{\eta}{1-\eta} * \frac{\left[\int_{z^R}^{\max\left\{z^R,\, z_{\min}\right\}} z\mu_t(dz) + \int_{z^H}^{\infty} z\mu_t(dz)\right]}{\left[F\left(z^H\right) - F\left(\max\left\{z^R,\, z_{\min}\right\}\right)\right]}.$$

As functions of z^H we have the following: Obviously, the left-hand-side is increasing and runs from 0 to $+\infty$. The right-hand-side, for any given z^R, is strictly decreasing and goes from $+\infty$ (when

z^H is close to $z_{\min}$) to $\frac{\eta}{1-\eta} * \frac{\left[\int_{z^R}^{z_{\min}} z\mu_t(dz)\right]}{[1-F(z_{\min})]} > 0$. Hence, there exist a single crossing. Moreover,if $z^R < z_{\min}$, then the right-hand-side boils down to

$$\frac{\eta}{1-\eta} * \frac{\left[\int_{z^R}^{z_{\min}} z\mu_t\left(dz\right) + \int_{z^H}^{\infty} z\mu_t\left(dz\right)\right]}{\left[F\left(z^H\right) - F\left(z_{\min}\right)\right]},$$

which is decreasing in z^R. Hence, its intersection with z^H is also decreasing. On the other hand, if $z^R > z_{\min}$, the right-hand side becomes

$$\frac{\eta}{1-\eta} * \frac{\left[\int_{z^H}^{\infty} z\mu_t\left(dz\right)\right]}{\left[F\left(z^H\right) - F\left(z^R\right)\right]},$$

which is strictly increasing in z^R, and so it will be the intersection with z^H.■

Proof Lemma 3 (Urban Separation) The proof of this result is straightforward. The utility gain of living in the city as opposed to the slum, $V_t^C(z) - V_t^F(z)$, is given by

$$u\left[\theta\left(p_t^N\right)\left(y_t^U\left(z\right) - p_t^h - \bar{c}^A\right)\right] - u\left[\theta\left(p_t^N\right)\left(y_t^U\left(z\right) - \bar{c}^A\right)\left(1-\tau\right)\right] + \beta\Delta_t^{C,F}, \tag{28}$$

where we have defined $\Delta_t^{C,\,F} \equiv E_t\left[z_{t+1}|Z_t^C\right] - E_t\left[z_{t+1}|Z_t^F\right] = G\left[Z_t^C - Z_t^F\right]$, which is positive by hypothesis. Given the wages w_t^q and w_t^a, the function $y_t^U\left(z\right)$ is weakly increasing in z, and strictly incresing for $z < z_{\min}$ or for $z > z_t^H$, and it is unbounded from above. There are minimum skill levels, $0 < z_F^l < z_C^l$, above which the consumption of households is positive for slum dwellers and city dwellers, i.e. $(1-\tau)\left(y_t^U\left(z\right) - \bar{c}^A\right) > 0$ and $y_t^U\left(z\right) - p_t^h - \bar{c}^A > 0$, respectively. The inequality $z_F^l < z_C^l$, indicates that for really low skilled population, living in a slum is be feasible while living in a city is not. In any event: (a) for all households with $z < z_F^l$, neither living in the slum or the city is feasible; so in equilibrium they will move back to the rural area; (b) for all households with $z_F^l < z < z_C^l$, the preferred choice is the slum, simply because the city is not feasible; (c) for households with $z \geq z_C^l$, the difference $D\left(z\right) \equiv u\left[\theta\left(p_t^N\right)\left(y_t^U\left(z\right) - p_t^h - \bar{c}^A\right)\right] - u\left[\theta\left(p_t^N\right)\left(y_t^U\left(z\right) - \bar{c}^A\right)\left(1-\tau\right)\right]$ is well-defined and increasing. Define z^{**} as the final threshold that equates the consumption costs of the slum with those of the city, i.e. the condition $y_t^U\left(z^{**}\right) = \left(p_t^h + \tau\bar{c}^A\right)/\tau$. Then, regardless of whether $u\left[\cdot\right]$ is unbounded or not, there is a level $z^* < z^{**}$ such that $D\left(z\right) + \Delta\left(Z_t^C, Z_t^F\right) > 0$, for all $z > z^*$.■

Proof of Proposition 2: (Existence of a Perfectly Sorted Urban Equilibrium) Increasing the threshold z_t^F makes the city more and more exclusive, i.e. $\lim_{z_t^F \nearrow \infty} Z_t^C = \infty$. Therefore, $\lim_{z_t^F \nearrow \infty} V_t^C(z_t^F) = \infty$ as long as $\lim_{z_t^F \nearrow \infty} u\left[\theta\left(p_t^N\right)\left(w_t^a z_t^F - p_t^h - \bar{c}^A\right)\right]$ remains bounded away from $-\infty$, which is easy to show. On the other hand, as we increase the threshold z_t^F, the population of the slums increases and, by assumption, the schooling prospects there remain bounded, $\lim_{z_t^F \nearrow \infty} Z_t^F < \infty$. So, even if $V_t^F(z_t^F)$ can grow without bound, it is always the case that $\lim_{z_t^F \nearrow \infty} u\left[\theta\left(p_t^N\right)\left(w_t^a z_t^F - p_t^h - \bar{c}^A\right)\right] - u\left[\theta\left(p_t^N\right)\left(w_t^a z_t^F - \bar{c}^A\right)\left(1-\tau\right)\right] > 0$. Therefore, $V_t^C(z_t^F) > V_t^F(z_t^F)$ for all z_t^F high enough. Since both $V_t^C(z_t^F)$ and $V_t^F(z_t^F)$ are continous, then, either: (a) crossing exists for $z_t^F \in (\frac{\bar{c}^A}{w_t^a}, \infty)$, and therefore an equilibrium with non-empty slums exist, or (b) a crossing does not exist, i.e., $V_t^C(z_t^F) > V_t^F(z_t^F)$ for all admissible $z_t^F \in (\frac{\bar{c}^A}{w_t^a}, \infty)$, in which case, the equilibrium exhibits empty slums and all the urban population is in the city proper.■

Proof of Lemma 4: (Rural-Urban Separation) The proof is virtually identical to that of Lemma 3.■

Proof of Proposition 3: (Existence of Equilibrium) The proof is virtually identical to that of Proposition 2.■