Treating Symmetric Buyers Asymmetrically*

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Abstract

We investigate a finite-horizon dynamic pricing problem of a seller where he cannot pre-commit to any future price-path. Even when the buyers are *ex-ante symmetric* (though non-anonymous) to the seller, the seller can charge *different* prices to different buyers. We show that this asymmetric treatment of symmetric buyers *generates higher revenue* than the optimal symmetric mechanism. We change the random tie-breaking allocation rule, used for symmetric mechanisms, to generate higher revenue for the seller. We show that the result holds even in static environment, though the marginal benefit of price discrimination increases with the time horizon of the game.

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1 Introduction

From the standard microeconomic theory we know that when buyers are *ex-ante* symmetric to a seller, he treats them symmetrically, e.g. single monopoly price charged by a monopolist. On the other hand, asymmetric treatment by the seller usually occurs when there is some form of *ex-ante* heterogeneity among the buyers, e.g. third degree price discrimination, asymmetric auctions (Maskin and Riley, 2000) etc. If the seller has some information on the heterogeneity of the buyers' willingness to pay, he exploits that extra information to treat them differently in order to generate higher revenue. In contrast to this standard theory, there have been several instances where a firm charges different prices to different buyers, but *no observable heterogeneity* among the buyers could be identified as the underlying cause of price discrimination.

The website Staples.com has been found to charge different prices to different observationally similar buyers for exactly the same product. According to Mikians *et al* (2012), they found that the same Swingline stapler ordered through the website from two nearby zip codes had prices $14.29 and $15.79 respectively. They further examined prices for 29 random products, each under 200 random zip codes, but did not find any significant observable attribute of these online orders as a causality for price discrimination. A further Wall Street Journal investigation on the pricing strategy of Staples.com by Valentino-De-Vries *et al* (2012) found that the people who got discounted prices had in fact *higher* average income than those getting higher prices, thus denouncing the logic of asymmetric willingness-to-pay as an explanation for price discrimination[1].

Another example of such price discrimination is by Amazon in 2000. They offered different prices to different consumers for a same DVD. When a lawsuit was filed against Amazon, they admitted that they were 'arbitrarily' charging different prices to different consumers. To quote the Wall Street Journal article, "offering different prices to different people is legal, with a few exceptions for race-based discrimination and other sensitive situations." So far, the standard theories of price discrimination have failed to provide explanations for these price dispersions.

In this paper, to investigate this issue we construct an asymmetric dynamic price-posting mechanism for a seller when the buyers are *ex-ante* symmetric. Our paper shows that if we allow the seller to use asymmetric mechanisms, under a mild and payoff irrelevant assumption that the buyers are non-anonymous to the seller, it *strictly increases* the seller's

[1] Valentino-DeVries, Jennifer, J. Singer-Vine and A. Soltani .2012. "Websites Vary Prices, Deals Based on Users' Information". *Wall Street Journal*, December 24.

revenue compared to the benchmark case of optimal symmetric mechanism.

We restrict our model within the realm of *price-posting mechanisms.* Of course in our symmetric setting, the most optimal mechanism for the seller would be to run a first or second price auction with appropriate reserve prices (Myerson, '81). But our model applies only to those situations where other types of mechanisms, like running an auction, are either not feasible, or not preferred by the seller. This may be due to various reasons, but we do not go into the details. [2]Later this section we describe an additional technical reason why some such auctions are not possible for the seller.

Instead, our model follows closely to that of Hörner and Samuelson (2011) which, under the feasible set of posted prices, derives the revenue-maximizing price path for symmetric buyers in the context of revenue-management industries, like airlines, packaged tours, television industries etc. Their mechanism is a symmetric mechanism, posting a single common price in each period to all the buyers. But we show that this *symmetric treatment of symmetric buyers,* as proposed by Hörner and Samuelson (2011), acts as a *binding constraint* to the seller. If we relax this constraint and allow the seller to *asymmetrically* treat the buyers, it strictly increases the seller's revenue.

We additionally make a payoff-irrelevant assumption of *non-anonymity* apart from *ex-ante* symmetry. We assume that the seller can exactly identify the buyers' identities. Although payoff-irrelevant, the non-anonymity assumption has an important impact on the choice of mechanisms of the seller. When we symmetrically treat symmetric buyers in general, we implicitly assume *anonymity*, which can potentially be disentangled from *symmetry.* When we relax this assumption, the choice of mechanism and hence the revenue of the seller differ.

Treating symmetric buyers equally and asymmetric buyers unequally thus implies that the only source of non-anonymity (i.e. knowing the buyer-identities) is some *payoff-relevant* property of the buyers, for e.g. the value distributions. In different revenue-management industries, the buyers can be non-anonymous to the seller, yet can have *ex-ante* same value distributions. An example can be television and radio industries. When a television channel sells advertising time slots to different companies, the number of buyers are not sufficiently large and hence they are not completely anonymous. The television company knows which companies have actually bid for that slot. Also, compared to the traditional purchasing habit by visiting to the shop, under online shopping consumers are actually non-anonymous to the seller, as the seller can almost exactly track the buyers by locating the IP addresses. The latter case refers to our original example of Staples.com and Amazon.[3]

[2]Einav *et al* (2013) uses data from eBay.com to show that online sellers are increasingly preferring posted-prices than auctions. A number of other papers like Hammond (2010), Hammond (2013) etc. also show similar trends of favoring posted-prices over auctions.

[3]We use the term 'non-anonymity' in a strict sense so that we abstract away from any other ex-ante

Since we show our result in the context of revenue-management industries, let us elaborate some distinct features of revenue-management pricing as the following: a) there is only a fixed quantity of good for sale, b) there is a fixed deadline within which the seller has to sell the good. Property (b) implies that the good becomes obsolete after the deadline. When an airline company prices tickets for passenger seats, it has to sell the tickets before the actual date of flight. Property (a) implies that in general there is excess demand in the market. So there is always an inherent competition among the buyers to acquire the good.

In our model we consider a seller who faces a fixed and known number of buyers. There is a deadline within which the good needs to be sold. The buyers are *ex-ante* homogeneous in the sense that they draw their private values from an identical and known distribution. The seller can set prices in each of the finite instants of time which the buyers can either accept and end the game, or can reject in which case the game moves to the next period for possible price revisions. The seller cannot *ex-ante* pre-commit to any fixed price paths, so in our model each price has to be *sequentially rational* and the equilibrium that we focus on is the *perfect Bayesian equilibrium*. Regarding the allocation rule of the seller we depart from the standard literature. The standard allocation rule in such mechanisms entails a basic norm of *equal treatment of equals and unequal treatment of unequals*. For example the optimal allocation rule for strategic buyers in Hörner and Samuelson (2011) (since the basic set up of our model is closest to theirs) is to post a single price in each period and allocate the good if any one buyer accepts in that period. If none of the buyers accept in a particular period the game moves to the next period for price revisions. If there is a tie in a particular period, the seller *randomly* allocates the good to the accepting buyers. Our mechanism differs in this tie-breaking allocation rule. We show that under non-anonymity instead of randomly allocating the good in case of a tie, if he allocates the good *deterministically* to any one of the accepting buyers, then the seller is better off.

Unlike the standard theories of price discrimination which rely on *ex-ante* heterogeneity among the buyers (for e.g. third degree price discrimination) in this paper price discrimination arises as a consequence of diversified choice of options. By asking for a high price from one buyer, the seller takes a high risk high return gamble. In order to compensate for that he sets a low price for the other in case the high risk option does not pay off. In fact when there are two buyers, we show that one of the prices is above and another below the price that is set in the single price mechanism.[4]

We compare our dynamic model results with the corresponding one-period case and show

heterogeneity of the buyers apart from the fact that the seller only knows the identities of the buyers with no other additional information.

[4] We can also explain the intuition in our model similar to that in the standard price discrimination literature. Our seller also makes use of some additional information which comes from the non-anonymity assumption. This additional information (by which he can distinguish among the buyers), although payoff-irrelevant, affects the choice of mechanism by the seller.

that even in *static scenario*, *i.e.* without the effect of any dynamics, the non-anonymous mechanism revenue-dominates the anonymous mechanism, which further strengthens our result. We show that even in a one period model charging the static monopoly price is not optimal when the buyers are non-anonymous. Instead of a single monopoly price it is better for the monopolist to charge two different prices in a static framework.

The dynamic scenario, on the other hand, helps to determine the inter-temporal trade-off in differential pricing. Comparing with the one-period case we show that in case of dynamics as we approach the deadline, price dispersion increases. We also show that the non-anonymous mechanism not only increases revenue both in static and dynamic cases, but as the time horizon of the game increases, the difference in revenue between the mechanisms also increases. Thus the non-anonymous mechanism becomes more effective, the larger the time horizon of the game.

A recent study (Kotowski, 2017) uses a similar intuition to treat symmetric bidders asymmetrically in the case of first-price auctions. An auctioneer can set different reserve prices to different bidders instead of a common reserve price. They show that this differential treatment can sometimes raise the revenue for the seller. The necessary condition for revenue increment in their paper is that the bidders' valuations should come either from irregular distributions, or regular distributions with some discontinuity. However we show that for posted-price mechanisms, even for continuous and regular distributions satisfying monotone hazard rate condition, asymmetric treatment can generate higher revenue than a symmetric mechanism.

We have already mentioned that our set-up restricts the feasible set of mechanisms to only posted-price mechanisms. The posted-price mechanism resembles somewhat to a Dutch auction and we know that a *Dutch auction* with a *positive reserve price* can implement the most general revenue-maximizing optimal mechanism when we include all possible mechanisms under the feasible set (Myerson, 1981). A Dutch auction has two broad features: fine discrimination of the buyer types, as well as a positive terminal price. In our scenario, the seller faces a trade-off between setting positive reserve prices and fine price discrimination. If he sets positive reserve prices he has to have non-negligible buyer valuation range whom he charges the same price. This is due to lack of commitment. He cannot commit to a Dutch auction-like mechanism with a positive reserve price. Our paper shows that in situations where a seller cannot run his favorite mechanism to maximize his revenue, (and remains within the realm of price-posting mechanisms), by asymmetrically treating the buyers, he can at least increase his revenue compared to a symmetric price-posting mechanism.

For full characterization of the equilibrium we take an additional assumption that the valuations of the buyers are drawn from an uniform distribution. This is to ensure that the equilibrium that we get is unique. It can be noted that the buyers' game is a game of strategic complementarity- the more likely the other buyers are to wait, the more incentive a buyer has for waiting. In general a game of strategic complementarity allows for multiple equilibria. So we take uniformly distributed buyer valuations in our model and show that the equilibrium is unique and interior.

Literature on Dynamic Mechanism Design

Our paper contributes to the growing literature on revenue maximizing dynamic mechanism design. This literature can be broadly classified into two types: *with commitment* and *without commitment*. Mechanisms with commitment literature differs in the different ways in which dynamics has been defined. The first strand of literature is where the population changes over time. Pai and Vohra (2009) presents a model where the population of potential buyers arrive and depart over the course of a finite time horizon. The time at which each agent arrives and departs from the market is his private information. Gershkov and Moldovanu (2010) examine a similar model with dynamic population of randomly arriving buyers. Said (2008, 2009), Gallien (2006) and Board and Skrzypacz (2010) are papers along the similar strand. There is another set of papers where the population is fixed but the information is dynamically changing. (For e.g. see Courty and Li, 2000, Eso and Szentes, 2000, Pavan, Segal and Toikka, 2009a and Battaglini, 2005).

However all the papers mentioned above assume that the seller can *pre-commit* to the entire path of mechanisms. The literature on dynamic mechanisms *without commitment* is relatively new and fewer works have been done so far in this literature. (For e.g. see Hörner and Samuelson, 2011, Skreta, 2015 etc.). Our paper adds to this literature on dynamic mechanisms without commitment. Although Skreta (2015) considers a sequence of optimal mechanisms where the seller can choose any potential selling mechanism, we restrict ourselves to only those situations where price posting is the only feasible mechanism. In this way we take the indirect mechanisms approach in order to portray a specific type of interaction between buyer and seller. We closely follow the model of Hörner and Samuelson (2011).

Secondly, in our setting, since a seller having no commitment power is tempted to lower down the price in subsequent periods in order to tempt the buyers to buy the good, this gives a similarity of the setting with the literature on durable-goods monopoly and Coase conjecture (Ausubel and Deneckere, 1989 and Gul, Sonnenschein and Wilson, 1986), but the durable goods literature differs from the revenue management literature in its infinite horizon setting. The fixed deadline gives the seller enough commitment power. Another difference is that in the literature on durable goods there are enough goods compared to the number of buyers. In our model the scarcity of goods induces inherent competition among the buyers for buying the goods. This competition among the buyers incentivizes them to accept the good earlier. Thus the monopolist in our model, even if he is in a Coasian dynamics, will eventually violate the Coase conjecture because the inherent competition among the buyers will not allow the price to go down to marginal cost even if we allow the time interval between price revisions to be close to zero.

The rest of the paper is arranged as follows. In section 2.1 and 2.2 we set up the model for the anonymous and the non-anonymous buyers respectively. In sections 2.3 and 2.4 we illustrate our main results in the cases of one and two period versions of the model respectively. In section 2.5 we derive our main result of defining and characterizing the equilibrium for a general $T(>2)$ period model. Section 2.6 deals with revenue comparison of the two mechanisms and section 3 concludes.

2 The Model

2.1 Anonymous Price Posting Mechanisms

We consider a general T-period dynamic game where a seller posts take-it-or-leave-it prices to sell an indivisible good to n buyers, where $n \geq 2$. The good is to be consumed at the end of the T periods and after that it becomes valueless. Thus the seller has to sell the good within the T periods. We denote the T periods as $\{\Delta, 2\Delta, \ldots, T\Delta \equiv 1)$, where we assume that $\frac{1}{\Delta}$ is an integer equal to T. Thus time period t denotes the number of periods remaining in the game, and hence the first period is denoted by T and likewise, $t = T-1$ denotes the next period while $t = 1$ is the last period.

The **timeline** for the game is as follows: In each period t, the seller announces a price $p_t \in \mathbb{R}$, and the buyers upon observing the price simultaneously decide whether to accept or to reject the price. If only one of the buyers accepts the price, the game ends and the good is sold to the accepting buyer at price p_t. If more than one buyer accept, then the good is randomly allocated to one of the accepting buyers at the announced price. If no one accepts the good, the game moves to the next period $t-1$.

Each buyer draws his private valuation v independently and identically from a known distribution $F : [0,1] \to [0,1]$ such that F is strictly increasing and continuously differentiable. A buyer with valuation v who obtains the good at price p derives a payoff of $(v-p)$. The seller having no intrinsic valuation over the good has a payoff equal to the price p at which the good is sold.[5]

We denote this game as Γ^T. A non-trivial **history** $h_t \in H_t$ is the history at period t where the game does not end effectively. The history h^t at period $t \in \{1, \ldots, T\}$, consists of prices till period $(t-1)$: $\{p^1, \ldots, p^{t-1}\}$. The set of all possible histories at period t is H^t, and we assume $H^1 \equiv \emptyset$. A behavior strategy of the seller $\{\sigma_S^t\}_{t=1}^T$ is a sequence of prices p_t which maps from the history to a probability distribution of prices. A behavior strategy of

[5]Without loss of generality, we assume that all parties discount future payoffs using the same discount rate of 1.

a buyer i, $\{\sigma_i^t\}_{t=1}^T$, is a collection of maps from his type, history of prices, and current price to a probability of acceptance, i.e.,

$$\sigma_i^t : [0,1] \times H_t \times \mathbb{R} \to \{0,1\}.$$

The solution concept we adopt in the paper is *perfect Bayesian equilibrium*.[6] We assume that the seller does not have any commitment power and each price has to be sequentially rational given the previous history and the belief about the optimal continuation payoff. Although in real world we do find cases where the seller uses different commitment devices, but in the present scenario, a seller without commitment will always do better than a seller without commitment, for at least two buyers. In this section, we shall focus on an anonymous price posting mechanism where the seller posts a single price in each period to all buyers and the buyers use symmetric strategies, $\sigma_i^t = \sigma_j^t$ for $i \neq j$.That is, we assume that the buyers of same type base their strategies on the same conditional distribution. The strategy of a buyer depends only on his valuation but not on his identity. In Subsection 2.2, we shall consider a non-anonymous price posting mechanism where the seller offers different prices to different buyers and accordingly, the buyers adopt different strategies, i.e., $\sigma_i^t \neq \sigma_j^t$ for $i \neq j$.

In an anonymous price-posting mechanism, the seller posts a single price to all the buyers in each period. Each individual buyer chooses a particular time period (if any) to accept the corresponding prevailing price and ends the game. The buyers face a non-trivial competition problem in each period. In particular, a buyer with higher valuations are more anxious to accept earlier as it is possible that the other buyers may "snatch" the good earlier, leaving him with zero payoff. In particular, the buyers' problem is an optimal stopping problem, where an individual buyer chooses an optimal price in the price path which he can accept, taking his opponent's strategy as given. Consequently, the buyers' game is one with strategic complementarity. The marginal gain from waiting one extra period increases for a buyer, the more likely he believes that his opponents will also wait. In general, in a game of strategic complementarity, there is a possibility of multiple equilibria.[7] To avoid this issue of multiple equilibria, we shall take a specific case of uniform distribution of buyer valuations while solving the model, in which case we can find a unique equilibrium to the problem. For the rest of this section, we shall follow Hörner and Samuelson (2011) closely in describing the buyers' and the seller's problems explicitly.

Given our focus on symmetric perfect Bayesian equilibrium, the buyers who accept at time period t are those whose valuations exceed a critical threshold v_t. Our next lemma, taken directly from Hörner and Samuelson (2011) illustrates the seller's posterior beliefs after

[6]Existence of such an equilibrium in our setting is similar to that in Horner and Samuelson (2011), and follows from standard arguments (see Chen (2012)).

[7]For a particular example of a case where multiple equilibria can arise, see Hörner and Samuelson (2011).

a history of no sales up to a particular time period.

Lemma 1. (Hörner and Samuelson (2011)) *Let $n \geq 2$. Fix an equilibrium, and suppose period t has been reached without a price having been accepted. Then the seller's posterior belief is that the buyers' valuations are identically and independently drawn from the distribution $F(v)/F(v_{t+1})$, with support $[0, v_{t+1}]$, for some $v_{t+1} \in (0,1]$.*

In the last period a buyer accepts a price if it is below or equal to his valuation. In the earlier periods each buyer faces a trade-off whether to accept at the posted price, or to wait till the next period . If he waits till the next period, he may get the good at a lower price, but the probability of getting the good decreases. If he accepts, he may get the good at a higher price, compared to waiting till next period, but the probability that he gets the good becomes higher.

Consider an arbitrary time period t and a buyer i with valuation v. Given a critical threshold v_t, buyer i's expected payoff from accepting the price p_t is :

$$\begin{aligned} &F(v_t)^{n-1} \sum_{j=0}^{n-1} \tfrac{1}{j+1} \tbinom{n-1}{j} \left(1 - \tfrac{F(v_t)}{F(v_{t+1})}\right)^j \left(\tfrac{F(v_t)}{F(v_{t+1})}\right)^{n-1-j} (v - p_t) \\ &= F(v_t)^{n-1} \tfrac{1-(F(v_t)/F(v_{t+1}))^n}{1-F(v_t)/F(v_{t+1})} \tfrac{(v-p_t)}{n}. \end{aligned} \tag{1}$$

Notice that here $F(v_t)^{n-1}$ is the probability that no buyer with higher valuations accepts a higher price. The term $1/(j+1)$ is the probability that buyer i receives the good when j other buyers accept the posted price p_t. The binomial expression after $1/(j+1)$ is the probability that exactly j other buyers accept the price p_t : since the valuations of the opponents are drawn identically and independently from the distribution F, the term $\frac{F(v_t)}{F(v_{t+1})}$ is the conditional probability that an opponent's valuation is less than v_t, given that the opponent's valuation is below v_{t+1} (recall that v_t and v_{t+1} are the critical threshold valuations above which a buyer accepts the price p_t). The probability $\left(1 - \frac{F(v_t)}{F(v_{t+1})}\right)$ is hence the corresponding conditional probability that an opponent who has not accepted in period $(t+1)$ accepts the price p_t in period t. The monetary gain of $(v - p_t)$ is buyer i's ex post payoff when i is awarded the good at price p_t.

If the critical threshold v_t is interior, then a buyer with valuation exactly equal to v_t should be indifferent between accepting the current price and waiting for another period to accept. To be explicit, in period t, if buyer i with valuation v_t accepts p_t, his expected payoff can be written as (similar to (1)):

$$\frac{1 - (F(v_t)/F(v_{t+1}))^n}{1 - F(v_t)/F(v_{t+1})} \frac{(v_t - p_t)}{n}. \tag{2}$$

On the other hand, if buyer i with type v_t waits for another period to accept price p_{t-1}, his expected payoff is

$$\begin{aligned} & \left(\frac{F(v_t)}{F(v_{t+1})}\right)^{n-1} \sum_{j=0}^{n-1} \frac{1}{j+1} \binom{n-1}{j} \left(1 - \frac{F(v_{t-1})}{F(v_t)}\right)^j \left(\frac{F(v_{t-1})}{F(v_t)}\right)^{n-1-j} (v_t - p_{t-1}) \\ = & \left(\frac{F(v_t)}{F(v_{t+1})}\right)^{n-1} \frac{1 - (F(v_{t-1})/F(v_t))^n}{1 - F(v_{t-1})/F(v_t)} \frac{(v_t - p_{t-1})}{n}. \end{aligned} \tag{3}$$

In (3), the first term $\left(\frac{F(v_t)}{F(v_{t+1})}\right)^{n-1}$ is the probability that the good is still available for sale in the next period, i.e., the probability that none of his opponents has already bought the good at the start of next period. As before, the term $\frac{F(v_{t-1})}{F(v_t)}$ is the conditional probability that an opponent's valuation is less than v_{t-1}, given that the opponent's valuation is below v_t. All other terms are analogous to the expression in (1). As mentioned previously, if this critical threshold v_t is interior, then a v_t-type buyer is indifferent between accepting at price p_t in this period and waiting for the next period to accept p_{t-1}. In other words, we have

$$\frac{1 - (F(v_t)/F(v_{t+1}))^n}{1 - F(v_t)/F(v_{t+1})} \frac{(v_t - p_t)}{n} = \left(\frac{F(v_t)}{F(v_{t+1})}\right)^{n-1} \frac{1 - (F(v_{t-1})/F(v_t))^n}{1 - F(v_{t-1})/F(v_t)} \frac{(v_t - p_{t-1})}{n}. \tag{4}$$

Thus the above equation recursively defines a set of thresholds v_t such that for a buyer with valuation v if the optimal time period to accept is t, then $v \in [v_t, v_{t+1})$.

The seller's optimization problem is to choose a sequence of prices $\{p_t\}_{t=T}^1$ so as to maximize his expected payoff:

$$\max_{\{p_t\}_t} \pi_T(v_T) = \max_{\{p_t\}_t} \sum_{t=1}^{T} [F(v_{t+1})^n - F(v_t)^n] p_t,$$

where $[F(v_{t+1})^n - F(v_t)^n]$ is the probability that the highest valuation of the buyers lies in the interval $[v_t, v_{t+1})$ and the good is sold at price p_t (recall that the seller attaches value 0 to the good).

To solve the problem using a procedure like backward induction, it is convenient to write the seller's expected payoff in t recursively as follows:

$$\pi_t(v_{t+1}) = \left(1 - \left(\frac{F(v_t)}{F(v_{t+1})}\right)^n\right) p_t + \left(\frac{F(v_t)}{F(v_{t+1})}\right)^n \pi_{t-1}(v_t),$$

where as before, $(1 - (F(v_t)/F(v_{t+1}))^n)$ is the probability that a buyer accepts price p_t in period t and $\pi_{t-1}(v_t)$ is the continuation expected payoff.

While conceptually it is straightforward to apply a backward induction process to solve for the seller's optimal sequence of prices, the problem is complicated by the possibility of multiple equilibria in the continuation game, i.e., multiple sequences of critical thresholds $\{v_t\}$ can be consistent with a sequence of equilibrium prices. In addition, which equilibrium prevails in the continuation game can depend arbitrarily on the price p_t offered by the seller in period t and on the entire history of the prices offered by the seller, complicating the issue further.

2.2 Non-Anonymous Price Posting Mechanisms

We now introduce non-anonymous price posting mechanisms in our current framework and we start with formally describing the buyers' game and the seller's maximization problem as we did in Section 3.1. The key difference between an anonymous price posting mechanism and a non-anonymous price posting mechanism is that in a non-anonymous price posting mechanism, the price offered to each buyer in each period can be identity dependent, i.e., different buyers can face different take-it-or-leave-it price offers in each period. Offering such identity dependent posted prices typically requires that the seller can identify different buyers throughout the game. While this is not a particularly strong assumption (i.e., the seller can simply assign each buyer a particular number that will be fixed throughout the T periods), such mechanisms will be typically feasible in settings where the number of the buyers is not too large.

Notice that while the identities of the buyers seemingly provide extra information of the buyers to the seller, such identities can be completely arbitrary and is hence completely payoff-irrelevant from the ex ante point of view. As our main objective is to compare the revenue performance of the anonymous price posting mechanism studied in Hörner and Samuelson (2011) with that of a non-anonymous price posting mechanism, we assume that apart from the identities of the buyers, the buyers are otherwise *ex-ante* symmetric in their valuations. There is no other asymmetry, or payoff relevant information from the buyers. Thus, although the seller can clearly identify the buyers, he does not have any information of the buyer types, just like the previous case.

Notice that in a non-anonymous price posting mechanism, the seller adopts a strategy of unequal treatment of equals even when the buyers types are *ex-ante* identical. From the buyers' perspectives, if different buyers are treated differently, the strategies adopted by the different buyers are necessarily different. As a result, the equilibrium we shall focus on in a non-anonymous price posting mechanism is an asymmetric perfect Bayesian equilibrium where the buyers use asymmetric strategies, $\sigma_i^t \neq \sigma_j^t$, for $i \neq j$. In such an equilibrium, the strategy of a buyer depends not only on the type of the buyer but also on the buyer's identity.

One important issue we have to deal with in a non-anonymous price posting mechanism is the tie-breaking rule when multiple buyers accept the offers from the seller. In the anonymous price posting mechanism, if more than one buyers accept the good at a given period, the good can be randomly allocated to the accepting buyers without affecting the seller's payoff in that given period. In a non-anonymous price posting mechanism, a tie-breaking rule can significantly affect the seller's payoff since different buyers are facing different prices In particular, the seller can modify this random allocation rule to a deterministic allocation rule in order to achieve a higher expected payoff. For example he can specify that among all the buyers accepting the seller's offers, the buyer with the highest price offer is allocated the good with probability 1. It should be noted however that there are many other tie-breaking rules the seller can adopt and the above tie-breaking rule (allocating to the accepting buyer with the highest offered price) is not necessarily the revenue-maximizing rule in the entire dynamic game. We shall however restrict our attention to such an intuitive tie-breaking rule and we shall show that such a rule suffices for the non-anonymous price posting mechanism to generate strictly higher expected payoff for the seller than an anonymous price posting mechanism.

Our above discussion eventually leads to a mechanism where the seller sets different prices to different buyers, and if there is a tie, he allocates the good to the accepting buyer with the highest price in each period. As a result, in a non-anonymous price posting mechanism, we shall have n different price paths, each designed for a particular buyer, for the seller instead of a single one as in the case of anonymous buyers.

To simplify issues, we shall only consider that there are 2 buyers. Qualitatively the analysis will remain the same, but for an n buyers case, the corresponding analysis would become much more cumbersome and difficult to handle. Suppose at each period t the seller sets two different prices p_t (buyer 1) and q_t (buyer 2) to the two different buyers, and without loss of generality, we assume that $p_t > q_t$. As before, the critical valuation thresholds are important, and we denote u_t (buyer 1) and v_t (buyer 2) to be the threshold valuations at time t for the two buyers respectively.

In a non-anonymous price posting mechanism, the buyers are treated differently in equilibrium. As a result, the indifference conditions that pin down the corresponding threshold types will be different for the two buyers. Thus the threshold types in a non-anonymous mechanism will also be different for the two buyers in each period.

In time period t, the incentives for a u_t-type of buyer 1 is given by the indifference condition

$$(u_t - p_t) = \frac{F(v_t)}{F(v_{t+1})}(u_t - p_{t-1}). \tag{5}$$

Notice that in period t, buyer 1 can get the good with certainty if he accepts the offer. On the other hand, if he rejects the offer, the game goes to the next period $(t-1)$ only in the event that buyer 2 has also rejected his own price offer in period t.

Similarly, in time period t, the incentives for a v_t-type of buyer 2 is given by the indifference condition

$$\frac{F(u_t)}{F(u_{t+1})}(v_t - q_t) = \frac{F(u_t)}{F(u_{t+1})}\frac{F(u_{t-1})}{F(u_t)}(v_t - q_{t-1})$$
$$\Rightarrow (v_t - q_t) = \frac{F(u_{t-1})}{F(u_t)}(v_t - q_{t-1}). \tag{6}$$

Recall that given the tie-breaking rule, buyer 2 can only get the good if buyer 1 rejects the offer. So in period t, he can get the good only with probability $F(u_t)/F(u_{t+1})$. If buyer 2 rejects the offer at period t, and if in the event that the game goes to the next period, again he can win the good with probability $F(u_{t-1})/F(u_t)$, i.e., only if buyer 1 again rejects the offer.

The seller's optimization problem in each time period t is to choose p_t and q_t to maximize $\pi_t(u_t, v_t)$ given the continuation payoff $\pi_{t-1}(v_{t-1})$

$$\max_{p_t,q_t} \pi_t(u_t, v_t) =$$
$$\left[\left(1 - \frac{F(u_t)}{F(u_{t+1})}\right) p_t + \frac{F(u_t)}{F(u_{t+1})}\left(1 - \frac{F(v_t)}{F(v_{t+1})}\right) q_t + \frac{F(v_t)}{F(v_{t+1})}\frac{F(u_t)}{F(u_{t+1})}\pi_{t-1}(v_{t-1})\right]$$

Suppose the seller charges p_t and q_t to buyers 1 and 2 respectively. The seller gets the lower price q_t if buyer 1 rejects the offer in period t, i.e., only for the event that buyers 1's valuation is lower than his own threshold level, while buyer 2's valuation is above his threshold level in period t. Also, the seller gets the higher price p_t if buyer 1's valuation is higher than his own threshold level no matter what the valuation of buyer 2 is. If both have their valuations below their own threshold levels, the game moves on to the next period.

As mentioned previously, throughout this paper we are concerned with situations where the seller cannot commit to future prices. In the anonymous price posting mechanism, we have assumed that each price chosen by the seller has to be sequentially rational. There is no pre-committed price path that the seller announces beforehand. As we shall see in the next section, if the seller is allowed to treat different buyers differently, the seller might be tempted to do so to increase his expected payoff. So treating them equally can act as a commitment, i.e., an allocation rule of offering the same price to all the buyers and distributing the good with equal probability to any accepting buyer is a commitment on the part of the seller.

In the next subsection we shall use a simple motivating example to illustrate how the seller can increase his expected payoff by treating different buyers differently. To ease exposition, we shall consider a model with two buyers and two periods. We will explicitly solve the two-period model to derive the price paths for anonymous and non-anonymous price posting mechanisms respectively. To do so, we shall assume that the valuations of the buyers are drawn from uniform distribution over $[0, 1]$. This is done not only to avoid computational complexity but also to abstract away from the issues of multiple equilibria in the present buyers game of strategic complementarity. We will show that when the seller has the option of treating different buyers differently, his expected payoff can be actually strictly better if he opts for a non-anonymous price posting mechanism.

2.3 Static Case (T=1)

We first show the effect of price discrimination on revenue in the simplest one period model. In the one period model, the seller posts a price p and the buyer can either accept or reject the price. If the buyer accepts the price p, then he gets the object and realizes a payoff of $v - p$ and the seller realizes a payoff of p. If the buyer rejects the price p, the the seller keeps the object and realizes a payoff of 0.

Clearly, it is a dominant strategy for the buyer to accept the price p if $v > p$ and reject it if $v < p$. Given this, under the anonymous mechanism the seller gets a payoff of $(1 - v^2)p$, by posting a price p. If p^* is an optimal solution then

$$p^* = \frac{1}{\sqrt{3}}$$

and the seller gets a revenue of 0.38

Under non-anonymous mechanism, if the seller can post different prices p and q to buyers 1 and 2 respectively, he gets a payoff of $(1 - u)p + u(1 - v)q$. From first order conditions, the optimal prices are

$$\begin{aligned} p^* &= \frac{5}{8} \\ q^* &= \frac{1}{2} \end{aligned}$$

and the seller's optimal revenue is 0.39. Thus the seller's revenue increases by 0.01 under the non-anonymous mechanism.

A final important issue that still needs to be clarified is exactly what factor is driving the difference in the performance of the two mechanisms. If the distributions of valuations of the buyers were different, it would be intuitive that the seller should adopt a non-anonymous price posting mechanism if such a "horizontal price discrimination" is also allowed on top of the intertemporal price discrimination. This is because the asymmetry of the distributions would give the seller additional payoff-relevant information on the buyers which the seller would want to make use of. So it would had been natural for him to treat different buyers differently. In our dynamic framework, when the seller treats the buyers differently in the first period and the game moves on to the next period, the buyers will be ex ante different in the next period due to their differential treatments in the first period although they started with symmetry. Hence the asymmetry between the buyers in the last period comes purely from their different treatments in the first period. The additional information in the second period generated by the asymmetric treatments can to some degree drive the difference between the two mechanisms.

We argue, however, that the main driving force of the performance difference of the two mechanisms comes from the fact that different treatments of the buyers intensify the competition of the buyers. This is more clearly seen by comparing the two mechanisms in the static version of the model. Even without any dynamic considerations, a non-anonymous price posting mechanism performs better than an anonymous price posting mechanism. As discussed, posting different prices to different buyers intensifies the competition between the buyers, which in turn drives up the seller's expected revenue.

2.4 Analysis of the Mechanism for T=2

Consider a two-period example. The valuation of each buyer is drawn independently from a uniform distribution over [0,1]. We first analyze the optimal sequence of prices when the seller chooses an anonymous price posting mechanism, i.e., he sets a single price in each period to both buyers. We solve the model using backward induction, starting from the last period, i.e., $t = 1$.

In $t = 1$, the seller maximizes his expected payoff:

$$\begin{aligned} \max_{v_1} \pi_1(v_1) &= \left(1 - \left(\frac{v_1}{v_2}\right)^2\right) p_1 \\ \text{s.t.} &: \ p_1 \leq v_1. \end{aligned}$$

In the maximization problem, v_1 and v_2 are the equilibrium critical valuation thresholds in the two periods. The constraint implies that a v_1-type buyer accepts the price in the last period only if his valuation is at least as high as the price. In the objective function, $\left[1 - (\frac{v_1}{v_2})^2\right]$ is the probability that at least one of the buyers have a valuation greater than v_1, conditional on that they both had valuations less than v_2, which comes from the fact that the good remained unsold after the first period. So, this is the probability that the good is sold in the last period. Since this is the last period, the constraint is binding. The seller finds no reason to charge a price less than v_1 in the last period. He then chooses the optimal v_1-type buyer whom he wants to target so that the objective function is maximized. The above discussion implies that the seller faces the following problem:

$$\max_{v_1} \pi_1(v_1) = \left(1 - \left(\frac{v_1}{v_2}\right)^2\right) v_1.$$

The corresponding first-order condition is

$$\frac{\partial \pi_1(v_1)}{\partial v_1} = 0 \Rightarrow v_1^* = p_1^* = \frac{v_2}{\sqrt{3}}, \tag{7}$$

implying that the optimal continuation payoff is

$$\pi_1(v_1^*) = \frac{2v_2}{3\sqrt{3}}. \tag{8}$$

In the first period, i.e., $t = 2$, denote the seller's price to be p_2. First consider the buyers' problem. The incentive constraint for the buyers is (i.e., the indifference condition for a type-v_2 buyer):

$$\frac{1 - v_2^2}{1 - v_2}(v_2 - p_2) = v_2 \frac{1 - (v_1/v_2)^2}{1 - v_1/v_2}(v_2 - v_1). \tag{9}$$

Given our discussion in Section 2.1, we know that the probability that a buyer accepts the price and obtains the good in the first period can be obtained via a binomial expression $\sum_{j=0}^{1} \frac{1}{j+1}(1 - v_2)^j (v_2)^{1-j} = \frac{1}{2}\frac{1-v_2^2}{1-v_2}$. So, the left hand side of the constraint (9) is the expected payoff of a v_2-type buyer in the first period when he accepts the offered price. The right hand side of the constraint (9) is the expected payoff to the buyer if he waits till the last period to buy the good. Since v_2 is the indifferent type buyer, the left hand and right hand sides of (9) should be equal to each other.

We now consider the seller's problem in the last period. The seller maximizes:

$$\begin{aligned} \max_{v_2} \pi_2(v_2) &= [(1 - v_2^2)p_2 + v_2^2 \pi_1(v_1)] \\ \text{s.t. } \frac{1 - v_2^2}{1 - v_2}(v_2 - p_2) &= v_2 \frac{1 - (v_1/v_2)^2}{1 - v_1/v_2}(v_2 - v_1). \end{aligned}$$

The seller chooses the optimal v_2 threshold to maximize his expected payoff. If any of the buyers accept the price (this happens with probability $(1 - v_2^2)$), he gets p_2, otherwise the game proceeds to the last period, in which case he gets $\pi_1(v_1)$.

Using our results in (9), we can solve for the optimal prices explicitly:

$$p_2^* = 0.58 \text{ and } p_1^* = 0.479.$$

In particular, notice that the optimal prices are decreasing over time. The optimal prices together with (9) imply that the seller's optimal expected revenue is $\pi_2(v_2^*) = 0.4$.

Observe that the optimal price in the last period $p_1^* = v_1^* = v_2^*/\sqrt{3} > 0$, i.e., the optimal price in the last period is above the marginal cost. While such a result is similar to the standard result of a durable-goods monopolist's pricing strategy in a similar two-period model, the price path obtained in our setting is intrinsically different. Similar to the Coase Conjecture, which states that when a monopolist does not have any commitment power in setting prices in a dynamic framework, the prices chosen by the seller should gradually decrease over time (and towards the marginal cost, which is zero here, in an infinite horizon model),. the optimal price path $\{p_1^*, p_2^*\}$ is also driven by the fact that the good is limited relative to the demand, i.e., there is excess demand in the market and the buyers compete with each other to acquire the good: Intuitively, a buyer may wait for one extra period for the price to fall, but at the same time he fears that the good might be snatched by his opponent in the current period, in which case he gets nothing. This provides him an incentive to buy the good earlier. This inherent competition among the buyers drives the optimal price path to be different from that in a Coasian framework.

We now consider the case where the seller adopts a non-anonymous price posting mechanism. We denote the prices offered by the seller as p_t (buyer 1) and q_t (buyer 2) with $p_t > q_t$, and the critical valuation thresholds as u_t (buyer 1) and v_t (buyer 2) in period $t = 1, 2$. As described in Section 3.2, the incentive constraint for buyer 1 is:

$$(u_2 - p_2) = v_2(u_2 - u_1), \tag{10}$$

while the incentive constraint for buyer 2 is:

$$u_2(v_2 - q_2) = u_1(v_2 - v_1). \tag{11}$$

Next, we consider the seller's maximization problem in the last period. The objective function of the seller is:

$$\pi_2(v_2) = [(1 - u_2)v_2p_2 + (1 - u_2)(1 - v_2)p_2 + u_2(1 - v_2)q_2 + u_2v_2\pi_1(v_1)]$$

Thus the seller gets the lower price q_2 only in the event that buyer 1 rejects the offer while buyer 2 accepts his offer. Similarly, the seller gets the higher price p_2 when buyer 1 accepts the offer regardless of the decisions of buyer 2, and when nobody accepts in the first period, the price offering game moves on to the last period.

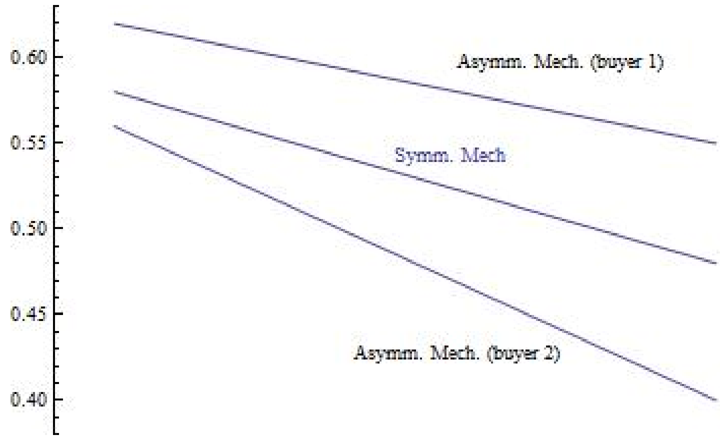

Figure 1: Single Price Path for Symmetric Mechanism and Two Price Paths for Asymmetric Mechanism for buyers 1 and 2 respectively

Hence the seller's optimization problem can be written as:

$$\begin{aligned} \max_{u_2,v_2} \pi_2(v_2) &= [(1-u_2)v_2p_2 + (1-u_2)(1-v_2)p_2 + u_2(1-v_2)q_2 + u_2v_2\pi_1(v_1)] \\ \text{s.t.} &: (u_2 - p_2) = v_2(u_2 - u_1) \text{ and } u_2(v_2 - q_2) = u_1(v_2 - v_1). \end{aligned}$$

Using a similar approach as that of the anonymous price posting mechanism, we find that the optimal prices chosen by the seller in the two periods are

$$p_2^* = 0.62, q_2^* = 0.56, \text{ and } p_1^* = 0.55, q_1^* = 0.402.$$

The optimal price paths lead to an optimal expected revenue of $\pi_2(v_2^*) = 0.404$. There are some interesting observations to be noted here. It is easy to see that the expected revenues are such that $\pi_2^A = 0.404 > \pi_2^N = 0.4$. In other words, the possibility of unequal treatment of equals strictly increases the payoff of the seller. The following table compares the performance of the optimal anonymous mechanism with the optimal non-anonymous mechanism of the two-period model more explicitly.

	Anonymous Mechanism	Non-Anonymous Mechanism
Price in period 2	0.58 (p_2)	0.62 (p_2^1), 0.56 (p_2^2)
Price in period 1	0.48 (p_1)	0.55 (p_1^1), 0.40 (p_1^2)
Expected revenue	0.4	0.404

Table 1: Comparison of prices and revenues in the non-commitment case

Denote p_t as the optimal price in period t under the anonymous mechanism, $t \in \{1, 2\}$

and denote p_t^i as the optimal price in period t for buyer $i \in \{1,2\}$, $t \in \{1,2\}$ under the non-anonymous mechanism. A first observation from the table is that the two prices in each period under the non-anonymous mechanism are a "spread" from the corresponding price under the anonymous mechanism, i.e., $p_t^1 > p_t > p_t^2$ for each t. Hence buyer 1 is charged with a price higher than the anonymous mechanism price while buyer 2 is charged a price lower than the anonymous mechanism price. One possible explanation is that we can view this as risk sharing motive for the seller: The seller charges a higher price to buyer 1 to take a high risk, high return gamble, while at the same time, the seller charges a lower price to buyer 2 as a fallback option in case the high price gamble does not work out.

A second useful observation is that $|p_t^i - p_t|$ is decreasing in t for each $t = 1,2$ and $i = 1,2$. In other words, in the earlier period the spread of the prices is less than that in the final period. In the second period which is the final period to sell the good, the seller tends to diversify even more (i.e., reducing the "risk") so that it is more likely that at least one of the buyers accepts the good in the final period. To be more explicit, let's consider the price variations for the buyers in the two mechanisms in detail. It can be shown that the line of the price path for buyer 2 in the non-anonymous mechanism is steeper than that of buyer 1 in the non-anonymous mechanism, while the slope of the line of the price path for the anonymous mechanism lies in the mid-way. In addition, we can see that the price difference (between the two mechanisms) for buyer 1 is relatively higher in the first period than that of buyer 2, i.e., $|p_2^1 - p_2| > |p_2^2 - p_2|$, while in the final period the price difference for buyer 2 is higher, i.e. $|p_1^1 - p_1| < |p_1^2 - p_1|$.

Comparing the two period model with the one period case, one can also get the effect of dynamic environment on revenue increment. Although the static case result shows the strength of the non-anonymous mechanism itself in increasing the revenue without the effect of any dynamics, its comparison with the two period model helps to understand how this changes with the change in the time horizon.

Let $D_T = |\pi_{NA} - \pi_A|$ denote the absolute difference between the two mechanisms in a T period game, for $T = 1,2$. We see that $D_2 = 0.04 > D_1 = 0.01$.Thus with our two period model, we show that the revenue increment for the seller under non-anonymous mechanism increases with the time horizon. By treating the symmetric buyers asymmetrically in the first period, the buyers actually become 'asymmetric' from the seller's point of view. Thus the asymmetric treatment becomes more effective in the last period. These are formally stated in the following proposition.

Proposition 1: *Let $D_T = |\pi_{NA} - \pi_A|$ for a T-period game, $T = 1,2$. Then, $D_2 > D_1$.*

Next we check which buyer gains from this differential pricing. Let v^1 and v^2 be the private valuations of buyers 1 and 2 respectively. Ex-post expected payoff of buyer 1 under non-anonymous mechanism is

$$(1 - u_2)(v^1 - p_2) + u_2 v_2 (1 - \frac{u_1}{u_2})(v^1 - p_1)$$

Ex-post expected payoff of buyer 2 is

$$u_2(1 - v_2)(v^2 - q_2) + u_2 v_2 \frac{u_1}{u_2}(1 - \frac{v_1}{v_2})(v^2 - q_1)$$

Under the anonymous mechanism the ex-post expected payoff of buyer i is

$$(1-v_2)v_2(v^i - p_2) + (1-v_2)(1-v_2)\frac{(v^i - p_2)}{2} + v_2^2((1-\frac{v_1}{v_2})\frac{v_1}{v_2}(v^i - p_1) + (1-\frac{v_1}{v_2})(1-\frac{v_1}{v_2})\frac{(v^i - p_1)}{2})$$

Putting in the equilibrium values of prices and threshold valuations, we get the following proposition:

Proposition 2: *For a two-period model, under non-anonymous mechanism, and compared to the anonymous mechanism, expected payoff of buyer 1 falls, and the expected payoff of buyer 2 rises if* $v^2 > \frac{1}{2}$.

Buyer 1 is disadvantaged by the higher price charged under non-anonymous mechanism. For buyer 2, although the allocation rule is not in favor of him, that gets offset by the benefit of lower price if his valuation is sufficiently high.

Thus the implication that we get from these exercises is that asymmetric equilibrium exists even in an *ex-ante* symmetric setting, and moreover the asymmetric mechanism is the optimal one under posted price domain. Another interesting implication that our static version of the example gives is that under the assumption of non-anonymity, in an otherwise *ex-ante* symmetric framework, the setting of the single static monopoly price is not the optimal price mechanism for the seller. It is quite common in the standard monopoly pricing literature that the monopolist price discriminates to extract the maximum producer surplus. The horizontal discrimination happens when the buyers come from different segments of population which have different demand structure and the monopolist has some information over the respective demands or the valuations, *i.e.* when there is an asymmetry in the distribution of valuations. Our example shows that even in a symmetric setting horizontal discrimination is the optimal one should the monopolist know about the identities of the buyers.

2.5 Analysis for any Finite T>2

2.5.1 Non-anonymity

This subsection characterizes the equilibrium price path of the seller in a general T period case in the case for non-anonymous buyers, and then in the next subsection we compare the equilibrium with that in the case of anonymous buyers. We assume that the valuations of the buyers are drawn independently from an identical distribution $F[0,1]$.

The characterization of the equilibrium comes from its recursive formulation. Suppose period t is reached without the good being sold in the earlier periods. We can recall that the seller's t^{th} period continuation payoff is

$$\pi_t(u_{t+1}, v_{t+1}) = (1 - \frac{F(u_t)}{F(u_{t+1})})p_t + \frac{F(u_t)}{F(u_{t+1})}(1 - \frac{F(v_t)}{F(v_{t+1})})q_t + \frac{F(v_t)}{F(v_{t+1})}\frac{F(u_t)}{F(u_{t+1})}\pi_{t-1}(u_{t-1}, v_{t-1}).$$

Suppose that for any time period t, and for every set of valuations (u_t, v_t), there is an unique and interior equilibrium for the continuation game with $t-1$ periods remaining and the buyers' valuations being drawn from $[0, u_t]$ and $[0, v_t]$. The threshold valuation buyers in each period of the continuation game (u_{t-k}, v_{t-k}) for any $k \in [1, t-1]$, are indifferent between accepting the current prices and waiting for the next period, rendering the interior solution of the game. In each period t, the seller maximizes $\pi_t(u_{t+1}, v_{t+1})$ given his continuation payoff. The buyers' incentive constraints fix (u_t, v_t) in period t, $\pi_{t-1}(u_{t-1}, v_{t-1})$, (u_{t-1}, v_{t-1}), and (p_{t-1}, q_{t-1}) are fixed by the continuation payoff, and the seller then maximizes his current payoff by choosing (p_t, q_t). The entire model can then be solved recursively by backward induction.

In the final period, *i.e.* with 1 period to go, the problem is a static problem and the optimal prices for the monopolist are the two static monopoly prices instead of a single monopoly price as he discriminates among the non-anonymous buyers taking u_2 and v_2 as given. Then, given the payoff in the last period, we can backwardly solve for the prices in all the previous periods, and thus the entire price paths of the monopolist can be traced. There will be two prices in each period, one higher than the other, thus generating two price paths over the period.

Another interesting feature of the problem that needs to be discussed is how it differs from a standard optimal auction design in the case of posted prices. It is well-known that under the case of posted prices, the optimal mechanism for the seller is a Dutch auction with a positive reserve price when the virtual valuations are increasing. (For the detailed discussion, see Myerson '81). But this would not be an optimal mechanism under the present scenario. A Dutch auction results in a fine discrimination among the buyers' valuation types while the positive reserve price excludes the lower valuation buyers from being allocated.

In a discriminatory mechanism, it would extend to two parallel Dutch auctions along with two optimal reserve prices. But in our case, we show that this will not be the case as the positive terminal prices would not allow a fine discrimination of the buyer types as there will always be two non-negligible buyer valuation ranges (for two buyers respectively) for which the same prices would be charged in each period.

To illustrate the idea, we consider the last period where the monopolist sets the discriminatory static prices to determine the final period threshold valuations. These threshold valuations are non-zero if they are lower than their respective upper bound of their posterior distributions, which are nothing but the threshold valuations of the previous period. Thus, for each buyer, there is a non-negligible gap between the two threshold valuations. The range of valuations within this gap was charged the same price in the previous period. With an induction logic we can claim that in every period under positive terminal prices, there would be two respective ranges of buyers' types who would be charged the same prices. This is stated formally below.

Proposition 3: *Suppose that the distribution function F has no atoms. If* $\lim_{\Delta\to 0} u_{\Delta 1} > 0$ *and* $\lim_{\Delta\to 0} v_{\Delta 1} > 0$*, then for all* k*,*

$$\lim_{\Delta\to 0} u_{\Delta k+1} > \lim_{\Delta\to 0} u_{\Delta k}$$

$$\lim_{\Delta\to 0} v_{\Delta k+1} > \lim_{\Delta\to 0} v_{\Delta k}$$

where $u_{\Delta k}$ *and* $v_{\Delta k}$ *are the threshold buyers' valuation types who are indifferent between accepting and rejecting the period-k price.*

Proof: Suppose that $u_{\Delta 1} > 0$ and $v_{\Delta 1} > 0$. Thus given $u_{\Delta 2}$ and $v_{\Delta 2}$,

$$(u_{\Delta 1}, v_{\Delta 1}) = \arg\max(1 - F_2(u))u + F_2(u)(1 - F_2(v))v,$$

where $F_2(u)$ and $F_2(v)$ are the posterior distributions such that $(u_{\Delta 1}, v_{\Delta 1})$ is contained in $(u_{\Delta 2}, v_{\Delta 2})$. Now if $F_2(u)$ and $F_2(v)$ have strictly positive density, then

$$u_{\Delta 2} > u_{\Delta 1}$$

$$v_{\Delta 2} > v_{\Delta 1}.$$

Since $\lim_{\Delta\to 0} u_{\Delta 1} > 0$ and $\lim_{\Delta\to 0} v_{\Delta 1} > 0$, thus we can get $\lim_{\Delta\to 0} u_{\Delta 2} > \lim_{\Delta\to 0} u_{\Delta 1}$ and $\lim_{\Delta\to 0} v_{\Delta 2} > \lim_{\Delta\to 0} v_{\Delta 1}$. Again, by an argument of induction we can establish this inequality for any earlier period k.

Thus a fine discrimination of buyer types by running a Dutch auction as well as setting positive terminal prices is not possible for the seller.

Uniform Distribution : We now assume the distribution of buyers' valuation to be uniformly distributed on $[0,1]$. The specification of uniform distribution helps to find the unique solution to the problem and would allow us to find an explicit characterization of the equilibrium. We can pin down the unique equilibrium from the buyers' indifference conditions. Buyer 2's indifference condition gives

$$\begin{aligned} v_t - q_t &= \frac{u_{t-1}}{u_t}(v_t - q_{t-1}) \\ &= \beta_{t-1}(v_t - q_{t-1}) \\ &= \beta_{t-1}(v_t - v_{t-1}) + \beta_{t-1}(v_{t-1} - q_{t-1}) \\ &= \beta_{t-1}(1-\gamma_t)v_t + \beta_t(v_{t-1} - q_{t-1}), \text{ where } \beta_t = \frac{u_t}{u_{t+1}} \text{ and } \gamma_t = \frac{v_t}{v_{t+1}}. \end{aligned} \tag{12}$$

Proceeding recursively, we can write the indifference condition as

$$v_t - q_t = \sum_{\tau=1}^{t-1}(1-\gamma_\tau)(\Pi_{l=\tau}^{t-1}\beta_l)v_{\tau+1}. \tag{13}$$

Again, writing $v_t - q_t$ as $v_t(1 - \frac{q_t}{v_{t+1}}\frac{1}{\gamma_t})$, we can rewrite the above equation as

$$(1 - \frac{q_t}{v_{t+1}}\frac{1}{\gamma_t}) = \beta_{t-1} + \sum_{\tau=1}^{t}\beta_{t-\tau}\Pi_{\tau=1}^{t}\gamma_{t-\tau}^2 - \sum_{\tau=2}^{t}\beta_{t-\tau}\gamma_{t-\tau}\Pi_{\tau=2}^{t}\gamma_{t-\tau+1}^2 - \beta_{t-1}\gamma_{t-1}. \tag{14}$$

The left hand side of the equation is monotonic in γ_t while the right hand side is independent of γ_t. Thus the equation can pin down γ_t as a function of $\frac{q_t}{v_{t+1}}$. Thus in the continuation game with t periods to go, given the price offered by the monopolist, there can be only one threshold type of Buyer 2 who is indifferent between accepting the price and waiting for the next period.

Similarly we can write down buyer 1's indifference condition and substitute recursively as

$$\begin{aligned} u_t - p_t &= \frac{v_t}{v_{t+1}}(u_t - p_{t-1}) \\ &= \gamma_t \sum_{\tau=1}^{t-1}(1-\beta_\tau)(\Pi_{l=\tau}^{t-1}\gamma_l)u_{\tau+1}. \end{aligned} \tag{15}$$

Similarly writing $u_t - p_t$ as $u_t(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t})$, the above equation can be rewritten as

$$\frac{1}{\gamma_t}(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t}) = \sum_{\tau=1}^{t}\Pi_{\tau=1}^{t}\gamma_{t-\tau}\beta_{t-\tau} - \sum_{\tau=1}^{t}\Pi_{\tau=1}^{t}\beta_{t-\tau}\gamma_{t-\tau+1}\gamma_{t-\tau} - \beta_{t-1}. \tag{16}$$

The left side of the equation is a function of β_t and γ_t, and since γ_t is pinned down from buyer 2's indifferent condition, thus the left side becomes monotonic in only β_t, while the right hand side is independent of it, thus pinning down β_t.Thus we can claim that the equilibrium of the monopolist's problem is unique.

The monopolist's problem is then to maximize his expected payoff

$$\pi_t(u_{t+1}, v_{t+1}) = (1 - \frac{u_t}{u_{t+1}})p_t + \frac{u_t}{u_{t+1}}(1 - \frac{v_t}{v_{t+1}})q_t + \frac{u_t}{u_{t+1}}\frac{v_t}{v_{t+1}}\pi_{t-1}(u_t, v_t).$$

This along with the indifference conditions of the buyers gives $\pi_t(u_{t+1}, v_{t+1})$ as a linear function of u_{t+1} and v_{t+1}. This again suggests that the solution is unique. This can be stated formally in the following lemma:

Lemma 2: *In the continuation game with t periods remaining, the prices for the two buyers p_t and q_t, and the monopolist's payoff function are linear functions of u_{t+1} and v_{t+1} for every t.*

Thus we can see that in this equilibrium the prices that the monopolist sets at any period t and the t^{th} period revenue of the monopolist are linear functions of u_{t+1} and v_{t+1}. From the buyers' problem we can ensure that the solution to this problem is unique in the sense that in each period we get two unique threshold valuations for the two buyers respectively, and thus the two prices that the monopolist sets for the two buyers respectively in each period are unique. A detailed proof of it is shown in the Appendix. The idea is to start from the last period. In the last period it is straightforward to show that the solution is unique. Then we apply the logic of induction on the number of periods and show that this is the case for any general t^{th} period. In any period the solution is unique given the continuation game.

The seller's problem on the other hand shows that the solution is indeed interior. The first order conditions from the seller's maximization problem characterize the price path of the monopolist in any general t^{th} period, and the second order condition shows that the solution is interior. The interior solution implies that in each period there exist some buyer valuations that do accept the prices in that period. The following set of first order conditions define the price paths of the monopolist and show the very existence of asymmetric equilibria in our otherwise symmetric setting. The second order condition along with Proposition 1 would show that the solution is interior, while the buyers' problem pins down the solution to be unique. The corresponding t^{th} period first order conditions that define the price paths are

$$\beta_t : -2(1-\beta_t)\beta_t[\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} - \gamma_t(1-\gamma_t)\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}$$

$$+ [\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} + (1-2\beta_t)u_{t+1} + (1-\gamma_\tau)\gamma_t v_{t+1} + \gamma_t\pi_{t-1} = 0 \qquad (17)$$

and

$$\gamma_t : -\beta_t(1-2\gamma_t)[\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}] + \beta_t(1-2\gamma_t)v_{t+1} + \beta_t\pi_{t-1} = 0 \quad (18)$$

The monopolist thus sets prices in each period according to the threshold cut-off rules such that the corresponding cut-off types are indifferent between accepting the price and waiting for the next period. The buyers on the other hand follow the strategy in any period to accept the price if their valuations(or types) are strictly greater than the respective cutoff valuations in that period, otherwise they wait for the next period. This gives the unique perfect Bayesian equilibrium of the continuation game, which is stated in the following proposition.

Proposition 4: *When the buyers are non-anonymous, at any period t, if the monopolist's posterior beliefs are* $[0, u_{t+1}]$ *and* $[0, v_{t+1}]$*, then in the unique perfect Bayesian equilibrium, the* t^{th} *period prices are given by*

$$p_t = u_t - \gamma_t \sum_{\tau=1}^{t-1}(1-\beta_\tau)(\Pi_{l=\tau}^{t-1}\gamma_l)u_{\tau+1}$$

and

$$q_t = v_t - \sum_{\tau=1}^{t-1}(1-\gamma_\tau)(\Pi_{l=\tau}^{t-1}\beta_l)v_{\tau+1},$$

and given prices $\widetilde{p}_t$ *and* $\widetilde{q}_t$*, buyers 1 and 2 with their respective valuations* $u > u_t(\widetilde{p}_t, u_{t+1})$ *and* $v > v_t(\widetilde{p}_t, v_{t+1})$*, the threshold types at time period t, accept their prices, and buyers 1 and 2 with their respective valuations* $u < u_t(\widetilde{p}_t, u_{t+1})$ *and* $v < v_t(\widetilde{p}_t, v_{t+1})$ *reject the prices, where* $u_t(\widetilde{p}_t, u_{t+1})$ *and* $v_t(\widetilde{p}_t, v_{t+1})$ *are given by*

$$(1-\frac{\widetilde{q}_t}{v}) = \beta_{t-1} + \sum_{\tau=1}^{t}\beta_{t-\tau}\Pi_{\tau=1}^{t}\gamma_{t-\tau}^2 - \sum_{\tau=2}^{t}\beta_{t-\tau}\gamma_{t-\tau}\Pi_{\tau=2}^{t}\gamma_{t-\tau+1}^2 - \beta_{t-1}\gamma_{t-1}.$$

and

$$\frac{1}{\gamma_t}(1-\frac{\widetilde{p}_t}{u}) = \sum_{\tau=1}^{t}\Pi_{\tau=1}^{t}\gamma_{t-\tau}\beta_{t-\tau} - \sum_{\tau=1}^{t}\Pi_{\tau=1}^{t}\beta_{t-\tau}\gamma_{t-\tau+1}\gamma_{t-\tau} - \beta_{t-1}.$$

Proof: See the Appendix.

2.5.2 Anonymity

This subsection deals with the benchmark case of anonymity of the buyers to the monopolist. The monopolist cannot distinguish among the buyers so he treats the buyers symmetrically.

In each period he posts a single price. If one of the buyers accept the price he gives the good to that buyer. If none of them accepts, the game moves on to the next period. In the event that ore than one buyer accept the good in a given period he randomly allocates the good to all the accepting buyers. If there are 2 buyers and the buyers' valuations are drawn from the distribution $F(.)$, the monopolist's t^{th} period maximization problem is:

$$Max_{p_t}\pi_t(v_{t+1}) = Max_{p_t}[(1-(\frac{F(v_t)}{F(v_{t+1})})^2)p_t + (\frac{F(v_t)}{F(v_{t+1})})^2\pi_{t-1}(v_{t-1})],$$

where v_t is the threshold valuation of the buyers in period t and p_t is the price in period t. Similar to the previous case in each period t the seller maximizes $\pi_t(v_{t+1})$ given his continuation payoff. The buyers' incentive constraints fix v_t in period t, $\pi_{t-1}(v_{t-1})$, v_{t-1}, and p_{t-1}, are fixed by the continuation payoff, and the seller then maximizes his current payoff by choosing p_t. The entire model can again be solved recursively by backward induction where the last period price is the static monopoly price. Thus the entire price path can be traced.

We can directly switch to the assumption of uniform distribution of the buyers' valuations and uniquely pin down the equilibrium solution to the benchmark problem. The buyers' indifference condition is given by the following equation

$$\frac{1-\gamma_t^2}{1-\gamma_t}(v_t - p_t) = \gamma_t\frac{1-\gamma_{t-1}^2}{1-\gamma_{t-1}}(v_t - p_{t-1}) \tag{19}$$

By recursive substitution, the equation can be rewritten as

$$\frac{1-\gamma_t^2}{1-\gamma_t}(1-\frac{p_t}{v_t}) = \gamma_t(1-\Pi_{\tau=1}^{t-1}\gamma_\tau^2) \tag{20}$$

Again, writing $\frac{p_t}{v_t}$ as $\frac{p_t}{v_{t+1}}\frac{1}{\gamma_t}$, we can rewrite the above equation as

$$\frac{1-\gamma_t^2}{1-\gamma_t}(1-\frac{p_t}{v_{t+1}}\frac{1}{\gamma_t}) = \gamma_t(1-\Pi_{\tau=1}^{t-1}\gamma_\tau^2) \tag{21}$$

Dividing both sides by γ_t the left hand side is monotonic in γ_t while the right hand side is independent of γ_t.Thus the equation pins down γ_t as a function of $\frac{p_t}{v_{t+1}}$ and the solution is unique. The monopolist's problem is then to maximize his expected payoff subject to the buyers' indifference condition.

$$Max_{p_t}\pi_t(v_{t+1}) = Max_{p_t}[(1-(\frac{v_t}{v_{t+1}})^2)p_t + (\frac{v_t}{v_{t+1}})^2\pi_{t-1}(v_{t-1})].$$

This along with the indifference condition on the buyers again gives $\pi_t(v_{t+1})$ as a linear function of v_{t+1} which suggests that the solution is unique. This is stated formally in the following lemma which is the corresponding lemma to Lemma 2.

Lemma 3: *In the continuation game with t periods remaining, the price for the two buyers p_t and the monopolist's payoff function are linear functions of v_{t+1} for every t.*

The seller's problem shows that the solution is interior, i.e. there exists some buyer valuations in each period who accept the good. The following proposition formally defines the perfect Bayesian equilibrium in the anonymous case. The difference with the non-anonymous buyers case is that there is only one price in each period. We define the equilibrium formally in the Proposition below.

Proposition 5: *When the buyers are anonymous, at any period t, if the monopolist's posterior belief is $[0, v_{t+1}]$, then in the unique perfect Bayesian equilibrium, the t^{th} period price is given by*

$$p_t = 1 - \frac{1-\gamma_t}{1-{\gamma_t}^2}\gamma_t(1 - \sum_{\tau=1}^{t} \Pi_{\tau=1}^{t} \gamma_{t-\tau}^2)$$

and given price $\widetilde{p_t}$, buyers 1 and 2 with their valuations $v > v_t(\widetilde{p_t}, v_{t+1})$, the threshold type at time period t, accept their prices, and buyers 1 and 2 with their valuation $v < v_t(\widetilde{p_t}, v_{t+1})$ reject the price, where $v_t(\widetilde{p_t}, v_{t+1})$ is given by

$$1 - \frac{\widetilde{p_t}}{v} = \frac{1-\gamma_t}{1-{\gamma_t}^2}\gamma_t(1 - \sum_{\tau=1}^{t-1} \Pi_{l=\tau}^{t-1} \gamma_l^n)$$

Proof: See Hörner and Samuelson (2011).

2.6 Revenue Comparison

In this subsection we will see the intuition behind the higher revenue that a non-anonymous mechanism generates. Since the difference between the two mechanisms lies in the allocation rules, let us first write down the allocation rules for both the mechanisms. Let v^1 and v^2 be the valuations of buyers 1 and 2 respectively. For the **anonymous mechanism**, the probability of allocating the good at any period t to buyer i is

$$\phi_{it}^A = \begin{cases} 1 \text{ if } v^i > v_t \text{ and } v^j < v_t \\ 0 \text{ if } v^i < v_t \text{ and } v^j > v_t \\ \frac{1}{2} \text{ if } v^i > v_t \text{ and } v^j > v_t \end{cases}$$

For the **non-anonymous mechanism**, the probability of allocating the good at any period t to buyer 1 is

$$\phi_{1t}^{NA} = \begin{cases} 1 \text{ if } v^1 > u_t \\ 0 \text{ if } v^i < v_t \end{cases}$$

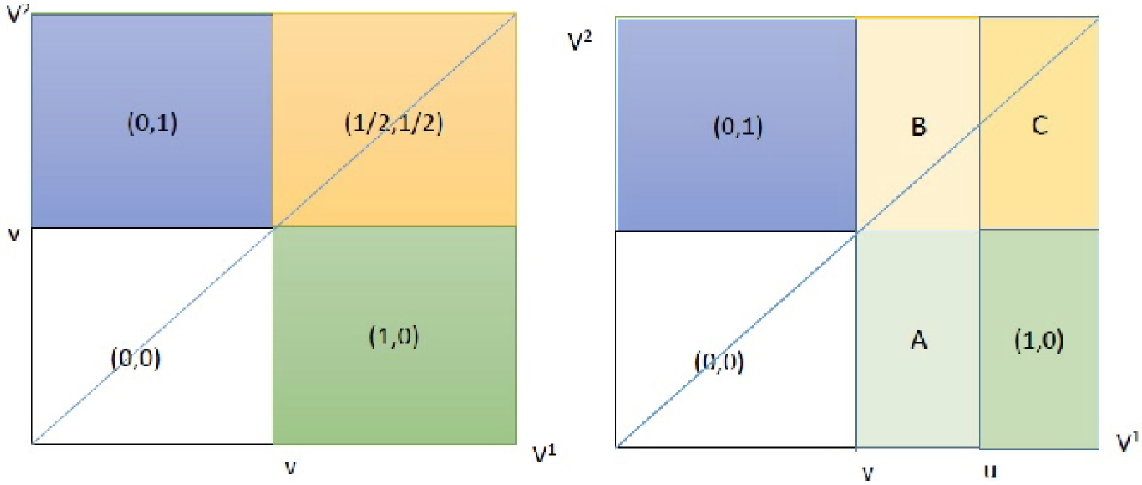

Figure 2: Allocation Rules for Anonymous (left) and Non-anonymous (right) Mechanisms

Similarly, the probability of allocating the good at any period t to buyer 2 is

$$\phi_{2t}^{NA} = \begin{cases} 1 \text{ if } v^1 < u_t \\ \text{and } v^2 > v_t \\ 0 \text{ if } v^2 < v_t \end{cases}$$

We will argue about the higher revenue through a local perturbation method. First let us consider the static case. Let v be the equilibrium cutoff valuation (and also price) under the anonymous mechanism for both the buyers. Let us assume without loss of generality that for buyer 1, this cutoff valuation is perturbed locally to u, where $u = v + k$, $k > 0$ is the amount of perturbation. Figure 2 shows the different allocation rules in different regions under this perturbation.

Following Figure 2, the regions A, B and C are the regions where the allocation rule changes. In Region A, the perturbation implies no-trade compared to the anonymous case where buyer 1 was allocated the good. Thus in Region A, the non-anonymous mechanism decreases revenue for the seller. In Regions B and C, for the anonymous mechanism, each buyer is allocated the good with probability $\frac{1}{2}$. Under perturbation, the allocation probability shifts in favor of buyers 2 and 1 respectively for Regions B and C. Table 2 shows the allocation rules and the seller's revenues in these regions.

Region	Anonymous Mechanism	Non-Anonymous Mechanism
A	$(1,0)$; $(1-v)v^2$	$(0,0)$; 0
B	$(\frac{1}{2},\frac{1}{2})$; $(1-v)^2v$	$(0,1)$; $(v+k)(1-v)v$
C	$(\frac{1}{2},\frac{1}{2})$; $(1-v)^2v$	$(1,0)$; $(1-(v+k))(1-v)(v+k)$

Table 2: Allocation Rules and Seller's Revenue in Different Regions

We need to make sure that the potential gain in revenue by shifting the allocation probabilities to either buyer in Regions B and C dominates the certain revenue loss under Region A. Thus the non-anonymous mechanism generates higher revenue if the following condition holds:

$$((1-(v+k))+v)(1-v)(v+k) > 2(1-v)^2v + (1-v)v^2 \tag{22}$$

We can conclude that there exists $\underline{k}, \overline{k} > 0$, such that for all perturbations $k \in (\underline{k}, \overline{k})$, the expected revenue of the non-anonymous mechanism dominates that of the anonymous mechanism. Here there is a trade-off with two opposing forces working together. A higher cutoff value for buyer 1 with perturbation implies a higher price for the seller in case he sells to buyer 1, but there is also a higher probability that the good remains unsold. The idea is that the positive effect should outweigh the negative one. If k is too high, the price the seller gets is greater, but the negative impact of no-trade probability outweighs the positive impact. If k is too low, the no-trade probability is of course low, but it is still higher than that in the anonymous mechanism. Thus the lower positive impact it gets with lower increase in price (as the perturbation k is low) cannot outweigh the negative impact. This rationalizes the range of values for perturbation amount.

The logic can be applied to the dynamic $T-$period game as well. In each period t, we perturb the cut-off value of buyer 1 by an amount $k_t > 0$. Of course, for a dynamic game, the trade-off is more complicated. We show that *for each continuation game*, if we perturb the cutoff valuations of buyer 1 in each period from the last period upto that period, the expected revenue of the seller at each continuation game strictly increases with perturbation. This is formalized in the following Proposition.

Proposition 6: *Let v_t be the revenue maximizing cutoff valuation in period t for anonymous mechanism. Suppose under non-anonymous mechanism, the cutoff valuation for buyer 1 (chosen arbitrarily) in period t is increased to $u_t = v_t + k_t$. Then for each t, there exists $\overline{k_t}$, $\underline{k_t} > 0$, such that for all perturbations $k_t \in (\underline{k_t}, \overline{k_t})$, the non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism.*

Proof: See the Appendix.

Revenue for Non-Uniform Distributions

We have shown how the non-anonymous mechanism increases revenue for the seller for an uniform distribution. For non-uniform distributions, we will shed light on some distribution properties for generation of higher revenues. We will also restrict ourselves to the static case comparison.

Monotone hazard rate (MHR) condition implies that for a random variable x following a probability distribution $F(x)$, the following term

$$Q(x) = \frac{1 - F(x)}{f(x)}.$$

is monotone in x. If $Q(x)$ is non-decreasing in x, we call it **decreasing hazard rate (DHR)** condition. If $Q(x)$ is non-increasing in x, we call it **increasing hazard rate (IHR)** condition.[8]

Let v be the equilibrium cutoff value for the anonymous mechanism. We perturb the cutoff value for buyer 1 by an amount k. Referring to the the allocation rules in Figure 2, the following table shows the revenues of the seller in different regions.

Region	Anonymous Mechanism	Non-Anonymous Mechanism
A	$(1, 0)$; $Q(v)F(v)f(v)v$	$(0, 0)$; 0
B	$(\frac{1}{2}, \frac{1}{2})$; $Q(v)(1 - F(v))f(v)v$	$(0, 1)$; $Q(v)f(v)v(F(u) + F(v) - 1)$
C	$(\frac{1}{2}, \frac{1}{2})$; $Q(v)(1 - F(v))f(v)v$	$(1, 0)$; $Q(u)Q(v)(1 - F(v))f(v)f(u)u$

Table 3: Allocation Rules and Seller's Revenue in Different Regions

The non-anonymous mechanism generates a higher revenue than the anonymous mechanism if the following condition holds:

$$I = Q(v)f(v)v(Q(u)f(u)u - 2Q(v)f(v) + F(u) - \frac{F(v)}{f(v)}) > 0. \tag{23}$$

We can verify that $Q(v) > 0$, and $u = v + k, k > 0$. If $F(u) > \frac{F(v)}{f(v)}$ and $Q(u) >> Q(v)$, I can be positive. This means if the distribution of valuations follows a sharp **DHR** condition, it becomes sufficient to generate higher revenue by even a small perturbation k. One example of a distribution with DHR condition is a Pareto distribution.

For a distribution that follows **IHR** condition, $Q(u) < Q(v)$. For such distributions, for the condition $I > 0$ to hold, the gap between u and v should be sufficiently high. Thus for a high enough perturbation k, the non-anonymous mechanism can generate higher revenue.

[8] Actually the inverse of $Q(x)$ is defined as the Hazard Rate.

However, there should also be an upper bound on k such that $Q(u)$ does not become too high compared to $Q(v)$. Thus for a given closed and bounded set of perturbation values k, the non-anonymous mechanism generates higher revenue when the distribution follows the IHR condition. Earlier we have already shown that for Uniform distribution, which follows IHR condition, the non-anonymous mechanism creates higher revenue for the seller. This is depicted in the following Proposition.

Proposition 7: *Let v be the revenue maximizing cutoff valuation for anonymous mechanism. Suppose under non-anonymous mechanism, the cutoff valuation for buyer 1 (chosen arbitrarily) is increased to $u = v + k$.*

*i) When $F(.)$ follows **DHR** condition: Let $Q(x) = \frac{1-F(x)}{f(x)}$.* Let $Q(u) >> Q(v)$. *Then for even very small perturbation k, the non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism.*

*ii) When $F(.)$ follows **IHR** condition: There exists $\overline{k}, \underline{k} > 0$, such that for all perturbations $k \in (\overline{k}, \underline{k})$, the non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism.*

3 Conclusion

We have constructed a non-anonymous price mechanism in the ex-ante symmetric environment such that our mechanism revenue-dominates the symmetric mechanism in Hörner and Samuelson (2011). Our mechanism gives a prescription that in situations where running auctions are not feasible, and posted prices are the only feasible options, then treating buyers asymmetrically can at least increase the revenue compared to treating them symmetrically.

It would be interesting to further look into the mechanism to identify the most 'optimal' or the revenue-maximizing mechanism under the current feasible set of posted-price mechanism, when we relax the constraint of 'treating equals equally'. We acknowledge that very much like the random tie-breaking rule, our deterministic tie-breaking rule is also a commitment on the part of the seller. In complete absence of commitment, one should ideally relax all the constraints and allow the seller to optimize on the tie-breaking rule sequentially rationally in every period to maximize his revenue. This should be an interesting extension for a future research.

Our model is a dynamic model with discrete and finite time periods. It would be interesting to see an infinite horizon version of our current setting to see if the differential pricing from the seller persists in the limit as time become infinite.

Appendix

Proof of Proposition 4:

We have assumed without loss of generality that the two buyers face t^{th} period prices p_t and q_t respectively with $p_t \geq q_t$. We start from the last period, *i.e.* $t = 1$. In the last period, the buyers accept the price if and only if their valuations are at least the prices they face in that period *i.e.* $u_1 \geq p_1$ and $v_1 \geq q_1$ respectively for buyers 1 and 2. The seller updates his posterior belief that the buyers' valuations are drawn from uniform distributions in the range $[0, u_2]$ and $[0, v_2]$ respectively.

The seller sets $u_1 = p_1$ and $v_1 = q_1$. The objective function of the seller is:

$$\begin{aligned} & (1 - \frac{u_1}{u_2})u_1 + \frac{u_1}{u_2}(1 - \frac{v_1}{v_2})v_1 \\ = & (1 - \beta_1)\beta_1 u_2 + \beta_1(1 - \gamma_1)\gamma_1 v_2 \end{aligned}$$

where $\beta_1 = \frac{u_1}{u_2}$ and $\gamma_1 = \frac{v_1}{v_2}$.

From the first order conditions we get:

$$\beta_1 \; : \; (1 - 2\beta_1)u_2 + ((1 - \gamma_1)\gamma_1)v_2 = 0 \qquad (24)$$

$$\gamma_1 \; : \; (1 - 2\gamma_1)\beta_1 v_2 = 0 \qquad (25)$$

Solving the first order conditions,

$$\begin{aligned} u_1 &= \frac{4u_2 + v_2}{8} \\ v_1 &= \frac{v_2}{2} \end{aligned}$$

As we can see, in the last period, u_1 and v_1 can be expressed as linear functions of u_2 and v_2.

The value of the problem is

$$\pi_1 = \mu_1 u_2 + \upsilon_1 v_2$$

where $\mu_1 = (1 - \beta_1)\beta_1$ and $\upsilon_1 = \frac{\beta_1}{4}$. Thus in the last period the solution is linear in u_2 and v_2.

Now we use the logic of induction on the number of time periods to show that the solution is unique for any general t^{th} period problem. Let us first fix t and assume that for all periods upto $t - 1$, the solution is unique and is characterized by μ_{t-1}, β_{t-1} and γ_{t-1}. Now let us consider the t^{th} period problem where the posterior beliefs are that the valuations of the two buyers are drawn from uniform distributions in $[0, u_{t+1}]$ and $[0, v_{t+1}]$ respectively.

The indifference conditions of the two buyers in the t^{th} period are:

$$u_t - p_t = \frac{v_t}{v_{t+1}}(u_t - p_{t-1}) \tag{26}$$

and

$$v_t - q_t = \frac{u_{t-1}}{u_t}(v_t - q_{t-1}). \tag{27}$$

Writing $\frac{v_t}{v_{t+1}} = \gamma_t$ and $\frac{u_t}{u_{t+1}} = \beta_t$, we can write for buyer 1,

$$\begin{aligned} u_t - p_t &= \gamma_t(u_t - p_{t-1}) \\ &= \gamma_t(u_t - u_{t-1}) + \gamma_t(u_{t-1} - p_{t-1}) \\ &= \gamma_t(1 - \beta_{t-1})u_t + \gamma_t(\gamma_{t-1}(1 - \beta_{t-2})u_{t-1} + \gamma_{t-1}(u_{t-2} - p_{t-2})) \\ &= \sum_{\tau=1}^{t-1}(1 - \beta_\tau)(\Pi_{l=\tau+1}^{t}\gamma_l)u_{\tau+1}. \end{aligned} \tag{28}$$

Similarly, for buyer 2,

$$\begin{aligned} v_t - q_t &= \gamma_{t-1}(v_t - q_{t-1}) \\ &= \gamma_{t-1}(v_t - v_{t-1}) + \gamma_{t-1}(v_{t-1} - q_{t-1}) \\ &= \gamma_{t-1}(1 - \gamma_{t-1})v_t + \gamma_{t-1}(\gamma_{t-2}(1 - \gamma_{t-2})v_{t-1} + \gamma_{t-2}(v_{t-2} - q_{t-2})) \\ &= \sum_{\tau=1}^{t-1}(1 - \gamma_\tau)(\Pi_{l=\tau}^{t-1}\gamma_l)v_{\tau+1}. \end{aligned} \tag{29}$$

Now again let us consider buyer 1. Buyer 1's indifference condition can also be written as:

$$(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t}) = (\sum_{\tau=1}^{t-1}\Pi_{l=t-1}^{t-\tau+1}\gamma_l^2(1 - \beta_{t-\tau}) \tag{30}$$

We can write a similar expression for buyer 2 as well. Thus we have characterized the buyers' behavior completely and uniquely. Given the sequences $\{\beta_t\}_{t=t-1}^{T}$ and $\{\gamma_t\}_{t=t-1}^{T}$ in each period t, we can pin down β_t and γ_t uniquely as functions of $\frac{p_t}{u_{t+1}}$ and $\frac{p_t}{v_{t+1}}$.

In the above equation, the left hand side is monotonic in β_t while the right hand side is independent of it. Thus β_t can be pinned down uniquely given u_{t+1} and the values in the continuation game.

Next we come to the seller's problem. The seller's expected payoff is:

$$\pi_t(u_{t+1}, v_{t+1}) = (1 - \frac{u_t}{u_{t+1}})p_t + \frac{u_t}{u_{t+1}}(1 - \frac{v_t}{v_{t+1}})q_t + \frac{u_t}{u_{t+1}}\frac{v_t}{v_{t+1}}\pi_{t-1}(u_t, v_t).$$

The seller maximizes the objective function subject to the indifference conditions of the buyers.

The first order conditions from the seller's maximization problem characterize the price path of the monopolist in the t^{th} period.

$$\beta_t : -2(1-\beta_t)\beta_t[\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} - \gamma_t(1-\gamma_t)\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1} \quad (31)$$

$$+ [\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} + (1-2\beta_t)u_{t+1} + (1-\gamma_\tau)\gamma_t v_{t+1} + \gamma_t \pi_{t-1} = 0$$

and

$$\gamma_t : -\beta_t(1-2\gamma_t)[\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}] + \beta_t(1-2\gamma_t)v_{t+1} + \beta_t\pi_{t-1} = 0 \quad (32)$$

From the second order condition it can be shown that the solution is also interior. Thus the solution to the t^{th} period problem is unique and interior given the continuation game.

Proof of Proposition 6

Let v_t be the equilibrium cutoff valuation for each buyer under the anonymous mechanism. We perturb the cutoff valuation for buyer 1 in each period t by an amount k_t. We apply backward induction and start from the last period. Let us first assume that only the last period cutoff valuation for buyer 1 is perturbed by an amount k_1. In the last period, the perturbed cutoff value for buyer 1 is

$$u_1 = v_1 + k_1.$$

Therefore if $\kappa_1 = \frac{k_1}{v_2}$ and $\alpha_2 = \frac{v_2}{u_2}$, then

$$\beta_1 = (\gamma_1 + \kappa_1)\alpha_2$$

Corresponding to Figure 2, the anonymous mechanism revenue in regions A, B and C together is

$$\mu_1^A = (1-\gamma_1)\gamma_1^2 + 2(1-\gamma_1)^2\gamma_1$$

The corresponding non-anonymous mechanism revenue for the three regions is

$$\begin{aligned}\mu_1^{NA} &= (\gamma_1+\kappa_1)(1-\gamma_1)\gamma_1\alpha_2 + (1-(\gamma_1+\kappa_1)\alpha_2)(1-\gamma_1)(\gamma_1+\kappa_1)\alpha_2 \\ &= \alpha_2(1-\gamma_1)(\kappa_1+\gamma_1)(\gamma_1-\kappa_1\alpha_2-\alpha_2\gamma_1+1)\end{aligned}$$

The difference between the two revenues is

$$\Delta_1 = \mu_1^{NA} - \mu_1^A = (1-\gamma_1)\left(-\kappa_1^2\alpha_2^2 - 2\kappa_1\alpha_2^2\gamma_1 + \kappa_1\alpha_2\gamma_1 + \kappa_1\alpha_2 - \alpha_2^2\gamma_1^2 + \alpha_2\gamma_1^2 + \alpha_2\gamma_1 + \gamma_1^2 - 2\gamma_1\right) \tag{33}$$

From the inequality in (32) there must exist $\overline{\kappa_1}$, $\underline{\kappa_1} > 0$, such that $\kappa_1 \in (\underline{\kappa_1}, \overline{\kappa_1})$, so that $\Delta_1 > 0$. Thus we show that if only the last period cutoff value for buyer 1 is perturbed, given the equilibrium history, the revenue increases.

Next we perturb buyer 1's cutoff values from the last period till period $(t-1)$. We assume that for the continuation game starting from period $(t-1)$, $\Delta_{t-1} > 0$, where $\Delta_{t-1} = \mu_{t-1}^{NA} - \mu_{t-1}^{A}$ is the difference in revenues for the continuation game starting from period $t-1$. Also, let Λ_{t-1} be the difference in total flow of revenue (as a sum of current revenue and the continuation revenue). We also assume that $\Lambda_{t-1} > 0$. Then we are required to show that $\Lambda_t > 0$.

We can now determine how the price in each period changes with the perturbations. Let $\rho_t = \frac{p_t}{v_{t+1}}$. From equations (28) and (29), we can pin down perturbed model $\rho_t^P(\gamma_t + \kappa_t) = \rho_t(\gamma_t) + \theta_t$, for $\theta_t > 0, \rho_t'(\gamma_t) > 0, \rho_t^{P\prime}(\gamma_t + \kappa_t) > 0$. So the t^{th} period expected revenues are the following:

$$\mu_t^A = (1-\gamma_t)\gamma_t\rho_t(\gamma_t) + 2(1-\gamma_t)^2\rho_t(\gamma_t)$$

$$\mu_t^{NA} = (\gamma_t + \kappa_t)(1-\gamma_t)\rho_t(\gamma_t)\alpha_{t+1} + (1-(\gamma_t+\kappa_t)\alpha_{t+1})(1-\gamma_t)\rho_t^P(\gamma_t+\kappa_t)\alpha_{t+1}$$

The difference between the two revenues is

$$\begin{aligned}\Delta_t \;&=\; \mu_t^{NA} - \mu_t^A = (1-\gamma_t)((\gamma_t+\kappa_t)\rho_t(\gamma_t)\alpha_{t+1} + (1-(\gamma_t+\kappa_t)\alpha_{t+1})\rho_t^P(\gamma_t+\kappa_t)\alpha_{t+1} \\ &\quad -\gamma_t\rho_t(\gamma_t) - 2(1-\gamma_t)\rho_t(\gamma_t))\end{aligned} \tag{34}$$

Analytical solution to the range of κ_t is non-trivial. But since $\rho_t'(\gamma_t) > 0, \rho_t^{P\prime}(\gamma_t+\kappa_t) > 0$, we can apply the same logic for the last period revenue. The value of κ_t should be sufficiently high so that the positive impact of higher price outweighs the negative impact higher probability of no-trade event. Also, there is an upper bound of κ_t beyond which the negative impact probability of no-trade becomes too high to outweigh the positive effect. Thus there should exist, in each period t, $\overline{\kappa_t}$, $\underline{\kappa_t} > 0$, such that $\kappa_t \in (\underline{\kappa_t}, \overline{\kappa_t})$, so that $\Delta_t > 0$.

Δ_t is the difference in revenues for only for the combined Regions A, B and C. The total difference in revenues is

$$\Lambda_t = \Delta_t + (\gamma_t + \kappa_t)\alpha_{t+1}\Lambda_{t-1} \tag{35}$$

Since $\Lambda_{t-1} > 0$ and $\Delta_t > 0$, we have $\Lambda_t > 0$. Then we can apply the logic of induction to claim that for any Γ_T, $T > 2$, we have $\Lambda_T > 0$. This completes the proof.

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