Firm Dynamics with Frictional Product and Labor Markets*

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Abstract

This paper analyzes the joint dynamics of prices, output and employment across firms. We develop a dynamic equilibrium model of heterogeneous firms who compete for workers and customers in frictional labor and product markets. Idiosyncratic productivity and demand shocks have distinct implications for the firms' output and price adjustments. Using panel data on prices and output for German manufacturing firms, we calibrate the model to evaluate the quantitative contributions of productivity and demand for the labor market and the dispersions of prices and labor productivity. We further analyze the impact of shocks to the first and second moments of idiosyncratic risk on macroeconomic outcomes. An increase in demand uncertainty induces sizable declines in output and employment together with rising cross-sectional dispersion of price and output growth which are typical features of recessions in our data.

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1 Introduction

Firm heterogeneity matters for the labor market and for business-cycle dynamics. For instance, firms which differ in size, age or productivity create and destroy jobs at different rates and they respond to aggregate shocks in different ways, see e.g. Davis et al. (2006), Haltiwanger et al. (2013) and Moscarini and Postel-Vinay (2012). This motivates a large literature on the role of firms in macroeconomics, much of which builds on the seminal contributions of Hopenhayn (1992) and Hopenhayn and Rogerson (1993), sometimes augmented by richer labor market features.[1] In such models of firm dynamics, firms are hit by idiosyncratic transitory shocks to their revenue productivity which induces them to create or destroy jobs as they grow or contract over time. These shocks may reflect the price or the quantity components of revenue and hence could be induced by supply-side or demand-side disturbances. This seems a reasonable theoretical shortcut, given that most datasets have no separate information on firm-level prices and output. Yet one might expect that supply and demand affect the dynamics of firms in different ways and hence have distinct implications both for the cross-section as well as for aggregate dynamics.

Recent empirical findings highlight a prominent role of firm-specific demand for firm growth. Using U.S. data on narrowly defined industries that permit a distinction between price and quantity, Foster et al. (2008) examine the separate contributions of demand and productivity for firm performance, finding that demand variation is the dominant driver of firm growth and firm survival.[2] Hottman et al. (2016) use price and sales information from scanner data to infer the sources of firm heterogeneity on the basis of a structural model of monopolistic competition. They show that demand differences and demand variation (as reflected in time-varying "firm appeal" and "product appeal" parameters) are a more important source of cross-sectional variation of sales and firm growth than are markups or cost heterogeneity.[3]

This paper aims to understand the respective roles of demand and supply for firm dynamics and the labor market. To this end, we develop an equilibrium model with heterogeneous firms, frictions in product and labor markets, and separate roles of firm-specific demand and productivity. We calibrate the model to match features of price and output adjustments of

[1] Search and matching frictions in the labor market have been introduced into the Hopenhayn-Rogerson model framework by, e.g., Acemoglu and Hawkins (2014), Cooper et al. (2007), Elsby and Michaels (2013), Fujita and Nakajima (2016), Kaas and Kircher (2015) and Schaal (2017).

[2] The quinquennial manufacturing census data they use does not permit them to study the dynamics of firms over time. While there are no significant productivity differences across firms of different ages, younger firms charge lower prices than incumbents which suggests that these firms attempt to build a customer base (relationship capital). This idea motivates Foster et al. (2016) to build a structural econometric model of firm dynamics in which product demand stochastically adjusts slowly as firms actively expend resources to build a customer base.

[3] Argente et al. (2018) use similar data and show that most products are rather short-lived while firm appeal (i.e. demand) and product scope are the dominant factors of firm growth.

German manufacturing firms during the period 1995–2014 in order to quantify the importance of supply and demand for the variation of prices, output, employment and job flows, as well as the cross-sectional dispersion of prices and labor productivity. We further use the model to explore the effects of aggregate mean and uncertainty shocks to either demand or productivity and relate our findings to the business-cycle features in the data.

In Section 2, we build an equilibrium model in which heterogeneous firms compete for workers in a frictional labor market and simultaneously compete for buyers in a frictional product market. Demand and (physical) productivity are firm-specific state variables, and idiosyncratic shocks to either of these variables have distinct implications for the price, output and employment adjustments of a firm. Search frictions in labor and product markets imply that growing firms need to build a workforce as well as a customer base slowly over time. This sluggish adjustment gives rise to a dispersion of wages and prices across firms: firms with a higher desire to grow offer higher wages to recruit more workers and offer lower prices to attract new customers.

In Section 3, we calibrate this model to account for the joint dynamics of output and prices at the nine-digit product level in an administrative panel of manufacturing firms in Germany, which also contains information on employment, working hours and wage bill. The data shows that the overall dispersion of annual price growth is substantial, and yet substantially smaller than the dispersion of output growth across firms. Moreover, price growth correlates negatively with output growth across firms so that the majority (over 80 percent) of revenue growth dispersion is accounted for by output growth dispersion. We show that the negative correlation between price and output growth requires a substantial contribution of productivity (cost) shocks, while the dispersion of price growth necessitates a prominent role for demand shocks. Hence both supply and demand forces are key to the understanding of the joint microdynamics of prices and output. Yet when it comes to employment growth, job flows and unemployment, we show that demand shocks play a more dominant role. For instance, productivity shocks alone account for around 40 percent of job creation and almost no job destruction at continuing firms in our model. Furthermore, the unemployment rate would fall by almost two percentage points if demand shocks were absent. On the other hand, idiosyncratic productivity risk does not matter for the level of aggregate unemployment.

In line with our data, the quantitative model generates a negative cross-sectional correlation between firm-level prices and quantity labor productivity, which in turn implies that quantity labor productivity is more dispersed than revenue labor productivity. While demand and (physical) productivity shocks contribute about equally to the cross-firm dispersion of quantity labor productivity, almost all price dispersion in the model is induced by demand shocks. Compared to the data, however, the calibrated model generates too little overall dispersion of prices and productivity.

Finally, we examine the impact of different aggregate shocks on the economy. We compare declines in average productivity or demand, on the one hand, as well as increases in the uncertainty of productivity or demand, on the other hand. Shocks to the first moments of either productivity or demand cannot generate plausible responses as they fail to account for the co-movement of employment and output; they also cannot generate the counter-cyclical dispersion of firm growth that we document in the data. These model responses are induced by the flexibility of wages which are allowed to fall on impact whenever an adverse productivity or demand shock hits the economy. In contrast to first-moment shocks, an increase in demand uncertainty induces sizable declines in output and employment, together with a rise in output and price growth dispersion. Based on these findings, we argue that higher demand uncertainty is a plausible feature of recessionary episodes.

Related to our work are several recent contributions that introduce product market search frictions into macroeconomic models. Generally, search in product markets is meant to capture the observation that firms spend substantial time and resources for sales and marketing activities in order to attract customers.[4] In the presence of such frictions, Bai et al. (2017) and Michaillat and Saez (2015) argue that aggregate demand shocks play a more prominent role than aggregate technology shocks. Kaplan and Menzio (2016), Petrosky-Nadeau and Wasmer (2015) and Den Haan (2013) combine frictions in product and labor markets, introducing new amplification mechanisms for business-cycle dynamics. Albrecht et al. (2013), Paciello et al. (2018) and Shi (2016) examine price variability and sales policies in equilibrium models of product market search in which the customer base is a state variable. Unlike our paper, none of these contributions addresses firm heterogeneity and the role of firm-specific demand.

Firms in our model employ multiple workers and accumulate a customer base with multiple customers. In these respects, our model closely relates to Gourio and Rudanko (2014), who study customer acquisition as a costly and time-consuming process, as well as Kaas and Kircher (2015), who describe the hiring process under convex labor adjustment costs. Both papers use competitive search as in Moen (1997) so that firms use lower product prices to attract more customers (Gourio and Rudanko (2014)) or higher wages to attract more workers (Kaas and Kircher (2015)). Our paper combines these ingredients to develop a unified framework in which we can study the dynamics of firms in the presence of firm-specific demand and productivity variation.[5]

Our finding that uncertainty shocks help to generate plausible aggregate dynamics is in accordance with a recent literature on the role of uncertainty in macroeconomics. For instance, Bloom (2009), Bloom et al. (2018) and Bachmann and Bayer (2014) argue that time-varying uncertainty improves the fit of macroeconomic models with heterogeneous firms. Schaal (2017) considers a heterogeneous-firm model with labor market search, showing that uncertainty

[4] In the U.S., marketing expenditures are as high as 7.7% of GDP (Arkolakis, 2010).

[5] Relatedly, Roldan and Gilbukh (2018) introduce idiosyncratic productivity risk in a model similar to Gourio and Rudanko (2014) which they calibrate in order to match features of price and sales dispersion from scanner data, to study the response of markups to aggregate shocks, among others. Unlike us, they do not consider firm-specific demand shocks or labor market features.

shocks help to understand the volatility of aggregate unemployment. In these articles, uncertainty shocks are introduced as increases in the volatility of idiosyncratic (revenue) productivity. Our contribution is to distinguish between demand uncertainty and (physical) productivity uncertainty as separate influence factors for the overall uncertainty at the firm level. We show that demand uncertainty is a much more relevant feature of the business cycle than productivity uncertainty.[6]

Our work further relates to an empirical literature which investigates the dispersion of firm-level prices and productivity. While Abbott (1991) and Foster et al. (2008) document dispersion of producer prices in specific industries, Carlsson and Skans (2012) and Carlsson et al. (2017) use Swedish firm-level data for the manufacturing sector, finding that unit labor costs are transmitted less than one-to-one to output prices, and that much of the variation in output prices remains unexplained by productivity shocks. Furthermore, they find that employment responds negligibly to productivity shocks, while permanent demand shocks are the main driving force of employment adjustment.

2 The Model

In this section, we build a canonical model that describes the dynamics of firms in frictional product and labor markets. In the product market, firms compete for buyers via costly sales activities and by offering discounts on their products, which helps to accumulate a customer base. In the labor market, firms build up a workforce by spending resources on recruitment and by offering long-term contracts to new hires. Firms adjust their customer base and their employment stock in response to idiosyncratic demand and productivity shocks.

All firms are owned by a representative household which also encompasses worker and buyer members. A worker can be either employed at a firm or unemployed; likewise, a buyer can be either attached to a firm or unattached. Search in both markets is competitive: workers and buyers direct their search towards particular wage or price offers, trading off higher matching rates against lower match values.

We describe a stationary equilibrium in which search values of buyers and workers are constant over time, while individual firms' employment and output grow or shrink, depending on their idiosyncratic productivity and demand states. We then establish a welfare theorem which permits a tractable equilibrium characterization by a social planning problem.

[6] Different from this paper, Basu and Bundick (2017) and Leduc and Liu (2016) introduce demand uncertainty as time-varying volatility of the household's discount factor in New-Keynesian DSGE models.

2.1 The Environment

Goods and preferences. The representative household consumes the goods produced by firms as well as a separate numeraire good. Utility of the household is $\sum_t \beta^t[e_t + u(C_t)]$, where e_t is consumption of the numeraire good, u is a concave utility function and β is the household's discount factor. $C_t = \int y_t(f)c_t(f)d\mu_t(f)$ is a consumption aggregator which integrates over the measure μ_t of active firms f in period t at which the household buys $c_t(f)$ units of output. $y_t(f)$ is an idiosyncratic (firm-specific and time-varying) taste parameter which reflects, for instance, differences of local preferences or brand values between firms or over time.[7] This preference specification captures the idea that goods within an industry are close substitutes, so that C_t stands for the consumption of a narrowly defined industry's output.[8] In a stationary equilibrium, $C_t = C$ is a constant, and so is the marginal rate of substitution between the consumption aggregator and the numeraire, which is $u'(C)$. All prices, wages and costs defined below are expressed in units of the numeraire good.

Workers and customers. There is a constant stock $\bar{L}$ of workers who are members of the household. A worker can be either employed at a firm or unemployed. An unemployed worker produces b units of the numeraire good. The household also has a large number of potential buyers who are either attached to the customer base of a firm or who search for purchases elsewhere in the goods market.[9] Any active buyer (shopping or searching) imposes a cost c on the household; once matched to a firm, the buyer can buy up to one unit of the good produced by the firm per period. Attached customers or employed workers do not search for new matches.

Technology. A firm with L workers produces $xF(L)$ units of its output good. F is increasing, concave, and satisfies the Inada condition $F'(L) \to 0$ for $L \to \infty$. x is the firm's idiosyncratic productivity. With this reduced-form modeling of a firm's production technology, changes of x stand for any type of supply-side shocks such as technology changes or (unmodeled) price changes of factor inputs besides labor.

Demand. A firm with B customers can sell up to B units of output in the current period, given that each customer is constrained in the number of purchases. Every customer of the firm wishes to buy exactly one unit of the firm's good, as long as the unit price together with the shopping cost c is smaller than the marginal rate of substitution between the firm's good

[7] These taste parameters are the counterpart of the *firm appeal* and *product appeal* parameters of Hottman et al. (2016) in our framework.

[8] By focusing on the dynamics of firms, this framework deliberately abstracts from imperfect substitutability across industries or from industry-specific shocks. Such features could easily be introduced in a more general setting where u aggregates consumption from different industries. In fact, with symmetric Dixit-Stiglitz preferences and in the absence of industry-specific shocks, our framework is isomorphic to such a more general economy.

[9] A "buyer" should literally be interpreted as a unit of shopping time that can be used to either buy a good from a previously known seller or to search for a purchase elsewhere.

and the numeraire good, which is $u'(C)y$ where y is the firm-specific demand state. Because the good is non-storable, the firm is naturally constrained by $B \leq xF(L)$ in any period. If that inequality is strict, the firm wastes some of its output.[10]

Shocks. Both idiosyncratic states x and y follow a joint Markov process on a finite state space. We write $z = (x, y) \in Z$ and denote $\pi(z_+|z)$ the transition probability from z to z_+. For a firm of age a, we write $z^a = (z_0, \ldots, z_a)$ for the shock history from the entry period (firm age zero) up to the current period (firm age a), where $z_k = (x_k, y_k)$, $k = 0, \ldots, a$. $\pi^a(z^a)$ denotes the unconditional probability of that history event.

Recruitment and sales activities. For recruitment and sales, the firm spends $r(R, L)$ and $s(S, L)$ respectively, where R and S measure recruitment and sales effort and L is the size of the firm's workforce before it matches with new workers and customers. Recruitment and sales costs represent direct pecuniary costs as well as the opportunity costs of managers and staff assigned to these activities. Both r and s are increasing and convex in their respective first arguments. They are non-increasing in the size of the workforce L to possibly capture scale effects.

Labor market search. Search in the labor market is competitive. Recruiting firms offer long-term contracts to new hires. They are matched with unemployed workers in submarkets that differ by the offered contract values. In a given submarket, a firm hires $m(\lambda) \leq \lambda$ workers per unit of recruitment effort, where λ measures unemployed workers per unit of recruitment effort in the submarket, and m is a strictly increasing and concave function. Hence, $m(\lambda)/\lambda$ is the probability that an unemployed worker finds a job in this submarket, which decreases in λ. An employment contract specifies wage payments and separation probabilities contingent on realizations of firm-specific shocks. We write $\mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k \geq a}$ for the employment contract of a worker who is hired by a firm of age a. $w^a(z^k)$ is the wage that the worker earns when the firm has age $k \geq a$, conditional on the shock history z^k and conditional on staying employed at this firm. $\delta_w^a(z^{k+1})$ is the probability to separate from the firm in event history z^{k+1} with $k + 1 > a$.

Product market search. Search in the product market is also competitive, but firms cannot commit to long-term contracts.[11] Instead, firms that aim to expand the customer base offer discount prices p to new customers. In all subsequent periods, attached customers continue purchasing at this firm, but anticipate that the firm charges the reservation price p^R which makes the buyer exactly indifferent between buying at this firm and remaining inactive.

[10] In particular there are no inventories. It is a trivial extension to introduce inventories at the computational cost of adding another state variable to the firm's problem.

[11] The assumption that firms offer long-term contracts to workers though not to customers is intended to reflect realistic features of worker-firm and customer-firm relationships. Although long-term contracts with customers are common in some industries, they tend to be rather short. For German manufacturing firms, Stahl (2010) finds that although 50% of sales are undertaken in written contracts, the average contract duration is just nine months. With an annual calibration, the absence of price commitment seems a plausible abstraction. See Section 2.4 for an alternative pricing assumption with commitment.

Unattached buyers and selling firms are matched in submarkets which differ by the buyers' match values. Per unit of sales effort, the firm attracts $q(\varphi) \leq \varphi$ new customers, where φ is the measure of unattached buyers per unit of sales effort in the submarket, and q is an increasing and concave function. An unattached buyer searching for purchases is successful with probability $q(\varphi)/\varphi$, which is a decreasing function of φ.

Entry, separations and exit. New firms can enter the economy at cost $K > 0$ with zero workforce and zero customer base. They draw an initial productivity and demand state $z_0 = (x_0, y_0)$ from probability distribution π^0. Any existing firm, depending on its supply and demand shocks, separates from workers according to the contractual commitments. Separated workers can search for jobs in the same period. The firm may also decide not to serve some of its attached customers who then leave the firm's customer base. Workers quit the job into unemployment with exogenous probability $\bar{\delta}_w$, and buyers leave the customer pool of a firm with exogenous probability $\bar{\delta}_b$. This implies that the actual customer churn rate is bounded below by $\delta_b \geq \bar{\delta}_b$. Likewise, the contractual state-contingent worker separation rates are bounded below by $\delta_w \geq \bar{\delta}_w$. At the end of the period, any firm exits with probability δ in which case all its workers enter the unemployment pool and all its customers become unattached.

Timing. The timing within a period is as follows. First, firm-specific demand and productivity shocks are realized. Second, some workers and customers separate from firms. Third, firms search for new hires and customers. Fourth, production takes place, workers are paid and goods are sold. Fifth, firms exit with probability δ.

2.2 Competitive Search Equilibrium

We describe a stationary equilibrium in which search values of workers and customers, as well as the distributions of workers and customers across firm types, are constant over time. Any firm's policy only depends on the idiosyncratic shock history z^a where a is the firm's age. Hence we identify the different firm types with z^a.

2.2.1 Workers

Let U denote the value of an unemployed worker and let $W(\mathcal{C}^a, z^k)$ denote the value of an employed worker in contract $\mathcal{C}^a$ at firm z^k with $k \geq a$. These values represent the marginal contribution of the worker to the representative household's utility. Unemployed workers observe the set of contracts $\mathcal{C}^a$ at firm types z^a and the corresponding market tightness λ in the submarkets in which value-equivalent contracts are offered. An unemployed worker's Bellman equation is

$$U = \max_{(W(\mathcal{C}^a, z^a), \lambda)} \frac{m(\lambda)}{\lambda} W(\mathcal{C}^a, z^a) + \Big(1 - \frac{m(\lambda)}{\lambda}\Big)[b + \beta U] \,, \tag{1}$$

where maximization is over all submarkets $(W(\mathcal{C}^a, z^a), \lambda)$. With probability $m(\lambda)/\lambda$, the worker finds employment in which case the continuation value is $W(\mathcal{C}^a, z^a)$. Otherwise the worker earns unemployment income b and remains unemployed to the next period.
The employment value $W(\mathcal{C}^a, z^k)$ satisfies the Bellman equation

$$W(\mathcal{C}^a, z^k) = w^a(z^k) + \beta(1-\delta)\mathbb{E}_{z^k} W'(\mathcal{C}^a, z^{k+1}) + \beta\delta U \ . \tag{2}$$

This worker earns $w^a(z^k)$ in the current period. At the end of the period, the firm exits with probability δ in which case the worker becomes unemployed. Otherwise the worker stays employed to the next period which yields continuation value $W'(\mathcal{C}^a, z^{k+1})$ where the prime indicates the employment value before the firm separates from workers, i.e.

$$W'(\mathcal{C}^a, z^{k+1}) = [1 - \delta_w^a(z^{k+1})] W(\mathcal{C}^a, z^{k+1}) + \delta_w^a(z^{k+1}) U \ . \tag{3}$$

With contractual separation probability $\delta_w^a(z^{k+1})$, the worker leaves the firm and can search for employment in the same period (continuation utility U). Otherwise the worker stays employed with continuation utility $W(\mathcal{C}^a, z^{k+1})$.
It is convenient to define the option value of search in submarket (W, λ) by

$$\rho(W, \lambda) \equiv \frac{m(\lambda)}{\lambda} \Big(W - b - \beta U \Big) \ .$$

Then, the flow utility value of unemployment satisfies

$$(1-\beta)U = b + \rho^* \ , \tag{4}$$

where $\rho^* \equiv \max \rho(W(\mathcal{C}^a, z^a), \lambda)$ is the maximal search value over all submarkets. It follows that any contract that attracts unemployed workers (i.e., $\lambda > 0$) yields the same search value ρ^*.

2.2.2 Buyers

The household can send arbitrarily many buyers to the goods market at shopping cost c per buyer. Hence the marginal net contribution of any buyer to the household's utility must be zero. Searching buyers observe unit discount prices p offered by firms of different types. Buyers and firms are matched in submarkets that yield identical match values $u'(C)y - p$ to the buyer. Since the firm charges the reservation price for all attached customers in subsequent periods, the continuation value for the customer beyond the matching period is zero. Let φ denote buyer-to-sales-effort ratio in a generic submarket with matching probability $q(\varphi)/\varphi$. The expected gain from searching must equal the shopping cost:

$$c = \max_{(p,y,\varphi)} \frac{q(\varphi)}{\varphi} \Big[u'(C)y - p \Big] \ , \tag{5}$$

where maximization is over all submarkets (p, y, φ). Any discount price that attracts new customers (i.e., $\varphi > 0$) yields the same search value c. It follows that discount prices are linked to market tightness φ via

$$p = u'(C)y - \frac{c\varphi}{q(\varphi)} \ .$$

Reservation prices p^R charged on existing customers make the buyer indifferent between buying at this price after incurring the shopping cost c, or remaining inactive. Hence,

$$p^R = u'(C)y - c \ .$$

2.2.3 Firms

A firm of type z^a takes as given the workers hired in earlier periods, L^τ, $\tau = 0, \ldots, a-1$, together with their respective contracts $\mathcal{C}^\tau$.[12] It also takes as given the existing stock of the customer base B_-. Hence the firm's state vector is $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$. Let $J_a(\sigma)$ denote the value of the firm at the beginning of the period. The firm chooses recruitment policy $(\lambda, R, \mathcal{C}^a)$ and sales policy $(\delta_b, \varphi, S, p, p^R)$ to solve the problem

$$J_a(\sigma) = \max_{(\lambda,R,\mathcal{C}^a),(\delta_b,\varphi,S,p,p^R)} \Big\{ p^R B_-(1-\delta_b) + pq(\varphi)S - W - r(R, L_0) - s(S, L_0) + \beta(1-\delta)\mathbb{E}J_{a+1}(\sigma_+) \Big\} \tag{6}$$

subject to

$$\sigma_+ = [(L^{\tau+}, \mathcal{C}^\tau)_{\tau=0}^a, B, z^{a+1}] \ , \ \mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k\geq a} \ , \ \delta_w^a(.) \geq \bar{\delta}_w \ , \tag{7}$$

$$L^{\tau+} = (1 - \delta_w^\tau(z^a))L^\tau \ , \ \tau = 0, \ldots, a-1 \ , \ L^{a+} = m(\lambda)R \ , \ L_0 = \sum_{\tau=0}^{a-1} L^{\tau+} \ , \tag{8}$$

$$W = \sum_{\tau=0}^{a} w^\tau(z^a)L^{\tau+} \ , \tag{9}$$

$$B = B_-(1 - \delta_b) + q(\varphi)S \ , \ \delta_b \geq \bar{\delta}_b \ , \tag{10}$$

$$B \leq xF(L) \ , \ L = \sum_{\tau=0}^{a} L^{\tau+} \ , \tag{11}$$

$$\rho^* = \rho(W(\mathcal{C}^a, z^a), \lambda) \ \text{ if } \ \lambda > 0 \ , \tag{12}$$

$$p = u'(C)y_a - \frac{c\varphi}{q(\varphi)} \ \text{ if } \ \varphi > 0 \ , \ p^R = u'(C)y_a - c \ . \tag{13}$$

The firm's problem (6) is to maximize revenue from sales to existing and new customers minus expenditures for wages, sales and recruitment costs, plus the expected continuation profit. The firm is committed to separation rates $\delta_w^\tau(z^a)$ for workers hired in previous period $\tau < a$. For

[12] Without loss of generality, all workers hired by a firm of a given type are hired in the same contract, which is an optimal policy of the firm (see Kaas and Kircher (2015) for a formal argument).

workers hired in this period, the firm commits to future separation rates, $\delta_w^a(z^{k+1}) \geq \bar{\delta}_w$, $k \geq a$. Together with wages $w^a(.)$, they define the contract $\mathcal{C}^a$ offered to new hires. Equations (8) say how employment in different worker cohorts evolves over time. L_0 is the firm's workforce *before hiring* which affects recruitment and sales costs. Equation (9) states the wage bill of the firm. (10) says how the firm's customer stock evolves. Because the firm is not committed in the product market, it decides customer separations $\delta_b \geq \bar{\delta}_b$ (if required) freely. Condition (11) says that the firm cannot sell more than what it produces with its current workforce L. Regarding wage contract offers to new hires $\mathcal{C}^a$, as well as discount price offers p to new customers, the firm respects the search incentives of workers and customers, as expressed by constraints (12) and (13). That is, to attract more workers per recruitment effort (higher λ), the firm needs to offer a more attractive employment contract. Likewise, to attract more customers per sales effort (higher φ), the firm needs to offer a lower discount price. The last equation in (13) says that the firm optimally charges the reservation price p^R on existing customers.

2.2.4 Equilibrium

We can express all firm policy functions defined above to depend on the firm's history z^a, ignoring the dependence on pre-committed contracts and worker cohorts. This is feasible because such firm state variables evolve endogenously as functions of the firm's past shocks and policies. Hence, all firm policies (in stationary equilibrium) are functions of the idiosyncratic state history. For a firm of type z^a, we write $\lambda(z^a)$ and $R(z^a)$ for the recruitment policy, $\varphi(z^a)$ and $S(z^a)$ for the sales policy, and so on.[13] We also define

$$L(z^a) = \sum_{\tau=0}^{a} L^\tau(z^a) \ , \tag{14}$$

$$B(z^a) = B(z^{a-1})[1 - \delta_b(z^a)] + q(\varphi(z^a))S(z^a) \ , \tag{15}$$

for the stocks of workers and customers in firm history z^a, where $L^\tau(z^a) = L^\tau(z^{a-1})[1 - \delta_w^\tau(z^a)]$ if $a > \tau$, $L^a(z^a) = m(\lambda(z^a))R(z^a)$, and $B(z^{-1}) = 0$. Further, there are

$$N(z^a) = N_0(1 - \delta)^a \pi^a(z^a) \tag{16}$$

firms of type z^a when N_0 is the mass of entrant firms in any period. We are now ready to define the stationary equilibrium.

Definition: *A stationary competitive search equilibrium is a list of value functions U, W, W', J_a, firm policies λ, R, φ, S, δ_b, $\mathcal{C}^a = (w^a(.), \delta_w^a(.))$, $(L^\tau)_{\tau=0}^a$, L, B, p, p^R which are all functions of the firm type z^a, entrant firms N_0, aggregate consumption C and a search value ρ^* such that:*

[13] With abuse of notation, we do not index these functions by the firm's age.

(a) *Workers' value functions U, W, W' and the search value ρ^* describe optimal search behavior, equations (1)–(4).*

(b) *Buyers search optimally, equation (5), and aggregate consumption is given by*

$$C = \sum_{z^a} y_a N(z^a) B(z^a) \ . \tag{17}$$

(c) *Firms' value functions J_a and policy functions solve problem (6)–(13), and $L(.)$, $B(.)$ and $N(.)$ evolve according to (14), (15) and (16).*

(d) *Firm entry is optimal. That is, $N_0 > 0$ and*

$$K = \sum_{z^0} \pi^0(z^0) J_0(0, z^0) \ . \tag{18}$$

(e) *Aggregate resource feasibility:*

$$\bar{L} = \sum_{z^a} N(z^a) \Big\{ L(z^a) + [\lambda(z^a) - m(\lambda(z^a))] R(z^a) \Big\} \ . \tag{19}$$

Aggregate resource feasibility (e) requires that any worker either belongs to the workforce $L(z^a)$ at one of $N(z^a)$ firms of type z^a or that the worker is searching for a job in the same submarket as these firm types and does not find a job: precisely, $\lambda(z^a)R(z^a)$ workers are searching for employment per firm of type z^a, and share $1 - m(\lambda(z^a))/\lambda(z^a)$ of these workers are not successful and hence remain unemployed.

We can verify that the aggregate resource constraint for the numeraire good is satisfied in a stationary equilibrium. The budget constraint of the representative household in any period is

$$\begin{aligned} &\sum_{z^a} N(z^a) \Big[p^R(z^a) B(z^{a-1}) [1 - \delta_b(z^a)] + p(z^a) q(\varphi(z^a)) S(z^a) \Big] + e \\ = &\sum_{z^a} N(z^a) \Big[\pi(z^a) + \sum_{\tau \le a} L^\tau(z^a) w^\tau(z^a) \Big] + b \Big[\bar{L} - \sum_{z^a} N(z^a) L(z^a) \Big] \\ &-K N_0 - c \sum_{z^a} N(z^a) \Big\{ B(z^a) + [\varphi(z^a) - q(\varphi(z^a))] S(z^a) \Big\} \ . \end{aligned}$$

The left-hand side expresses the household's consumption expenditures for the different goods and for the numeraire e. The right-hand side gives the household's income which includes wage and profit income at all firm types z^a plus income from home production minus expenditures for the creation of new firms and for shopping. Shopping costs are paid both for searching and for attached buyers. Profit income of firm z^a is

$$\pi(z^a) = p^R(z^a)B(z^{a-1})[1-\delta_b(z^a)] + p(z^a)q(\varphi(z^a))S(z^a) - \sum_{\tau \le a} L^\tau(z^a)w^\tau(z^a)$$
$$- r(R(z^a), L_0(z^a)) - s(S(z^a), L_0(z^a)) \ .$$

Rearranging shows that the household's consumption of the numeraire good[14] is identical to the home production of the numeraire good net of the costs for recruitment, sales, firm entry, and shopping which are all paid in the numeraire good:

$$e = b\Big[\bar{L} - \sum_{z^a} N(z^a)L(z^a)\Big] - \sum_{z^a} N(z^a)\Big[r(R(z^a), L_0(z^a)) + s(S(z^a), L_0(z^a))\Big]$$
$$-KN_0 - c\sum_{z^a} N(z^a)\Big\{B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big\} \ .$$

2.3 Social Optimum and Firm Policies

A stationary competitive search equilibrium is identical to the solution of a social planning problem which maximizes the utility of the representative household, starting from the given initial distribution of customers and workers across firms. The social planner decides firms' recruitment and sales efforts and assigns workers and customers into submarkets which differ by the characteristics of the searching firms. In the Appendix, we formally define the planner's problem and show that it permits a rather simple recursive formulation at the level of individual firms. Let $\rho > 0$ denote the multiplier on the aggregate resource condition (19) which is a binding constraint for the planner. For a firm in a given period, the planner takes as given the firm's stocks of workers and customers, L_- and B_-, as well as the current productivity and demand state $z = (x, y)$. Write $G(L_-, B_-, z)$ for the social value of a firm, i.e., the contribution of the firm to the representative household's utility. It satisfies the recursive equation

$$G(L_-, B_-, z) = \max_{(\lambda, R, \delta_w), (\varphi, S, \delta_b)} \Big\{u'(C)yB - bL - r(R, L_-(1-\delta_w)) - s(S, L_-(1-\delta_w))$$
$$- \rho[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E}_z G(L, B, z_+)\Big\} \ , \quad (20)$$

subject to

$$L = L_-(1-\delta_w) + m(\lambda)R \ , \ B = B_-(1-\delta_b) + q(\varphi)S \ ,$$
$$B \le xF(L) \ , \ \delta_w \ge \bar{\delta}_w \ , \ \delta_b \ge \bar{\delta}_b \ .$$

[14] If $e < 0$, we assume that the household produces $-e$ units of the numeraire good which, together with unemployment income and net of shopping costs is identical to the firms' expenditures on entry, recruitment and sales.

The flow surplus of the firm includes the marginal utility value of output, $u'(C)yB$, net of the opportunity cost of employment, bL, recruitment and sales costs, $r(.)$ and $s(.)$, and net of shopping costs and social costs for the workers who are linked to this firm in the given period. Regarding the latter, there are L workers employed at the firm, and $(\lambda - m(\lambda))R$ unemployed workers who search for employment at this type of firm and do not find a job. Any of these workers can neither work nor search for jobs elsewhere in the economy and hence impose a social cost which is identical to the multiplier ρ on the aggregate resource constraint. Shopping costs are incurred by the B customers buying at this firm, but also by $(\varphi - q(\varphi))S$ unsuccessful customers who search for purchases in the same submarket as this firm. The planner neither needs to commit to separation rates, nor is there a need to discriminate between workers hired at different points in time (see the Appendix for details).

The Inada condition $F'(\infty) = 0$ and standard dynamic programming arguments imply that problem (20) has a solution $G : [0, L^{\max}] \times [0, B^{\max}] \times Z \to \mathbb{R}$ which is continuous in $(L_-, B_-) \in [0, L^{\max}] \times [0, B^{\max}]$ for some appropriately defined upper bounds $L^{\max}$ and $B^{\max}$.

Because $G(0, 0, z)$ denotes the social firm value upon entry, socially optimal entry requires that

$$K = \sum_z \pi^0(z) G(0, 0, z) \ . \tag{21}$$

We show that a joint solution of (20) and (21) together with aggregate consumption (17) and resource constraint (19) indeed give rise to a stationary planning solution. Moreover we prove a welfare theorem: the stationary planning solution corresponds to a stationary competitive search equilibrium where the social multiplier on the resource constraint coincides with the equilibrium search value of workers: $\rho = \rho^*$.[15]

Proposition 1 *Suppose that (ρ, G, N_0, C) solve the recursive social planning problem (20) together with (21), aggregate consumption (17) and the aggregate resource constraint (19), where $N(z^a)$ is defined by (16), and $L(z^a)$ and $B(z^a)$ are recursively defined by iterating over the policy functions of problem (20). Then:*

(a) The firm policies solve the sequential social planning problem which maximizes the discounted household's utility, starting from the initial distribution $(N(z^a), L(z^a), B(z^a))_{z^a}$.

(b) (ρ, G, N_0, C) corresponds to a stationary competitive search equilibrium with identical firm policies and search value $\rho^* = \rho$.

[15] Assuming alternatively that the household has a fixed stock of buyers (or shopping time) $\bar{B}$, rather than a large potential stock of buyers, does not change this characterization of a stationary equilibrium by a planning solution. The only difference is that c stands for the multiplier on a resource constraint for buyers, defined similarly as (19), and in a stationary equilibrium the buyers' search value c^* coincides with the social value c. This alternative formulation makes a difference, however, for comparative statics or for the analysis of aggregate shocks.

The welfare theorem (b) extends well-known efficiency results for competitive search economies (cf. Moen (1997)) to a setting with two-sided market frictions and firms with multiple workers and multiple buyers. Kaas and Kircher (2015) prove similar results for multi-worker firms in an environment without product market frictions (and without demand shocks). The main intuition for efficiency is that private search values of workers and customers in the competitive search equilibrium reflect their social values; by either posting long-term contracts (to workers) or discounts (to customers), firms fully internalize all congestion externalities of search. They also internalize the trade-off between costly search effort and higher matching rates. We elaborate on this trade-off in the next paragraph. Long-term contingent contracts further implement the socially efficient worker separation rates.

The socially optimal recruitment and sales policies permit a straightforward characterization which link the effort of search (recruitment and sales expenditures) to the matching rates, resembling earlier findings of Kaas and Kircher (2015) and Gourio and Rudanko (2014). Write γ for the multiplier on the constraint $B \leq xF(L)$. Then, the first-order conditions for R, λ, S, and φ, for positive recruitment and sales activities, are:

$$-r_1' - \rho\lambda + [\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma x F' - b]m = 0 \ , \tag{22}$$

$$-\rho R + [\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma x F' - b]m' R = 0 \ , \tag{23}$$

$$-s_1' - c\varphi + [\beta(1-\delta)\mathbb{E}G_2'(+) + (yu'(C) - \gamma)]q = 0 \ , \tag{24}$$

$$-cS + [\beta(1-\delta)\mathbb{E}G_2'(+) + (yu'(C) - \gamma)]q' S = 0 \ . \tag{25}$$

Here $\mathbb{E}G_i'(+)$, $i = 1, 2$, denote the derivatives of $\mathbb{E}G(L, B, z_+)$ with respect to the first and second arguments. Equations (22) and (23) can be combined to obtain

$$r_1'(R, L_-(1-\delta_w)) = \rho\Big[\frac{m(\lambda)}{m'(\lambda)} - \lambda\Big] \ . \tag{26}$$

Similarly, (24) and (25) yield a relation between φ and S:

$$s_1'(S, L_-(1-\delta_w)) = c\Big[\frac{q(\varphi)}{q'(\varphi)} - \varphi\Big] \ . \tag{27}$$

Condition (26) says that across firms (of a given size) recruitment effort and matching rates are positively related: the planner can achieve faster employment growth by spending more on recruitment (higher R) but also by assigning more workers to find employment at this type of firm. In the corresponding competitive search equilibrium, faster growing firms spend more on recruitment and offer higher salaries, thus attracting more workers (cf. Kaas and Kircher (2015)). Condition (27) expresses a similar relation in the product market: firms that spend more on sales also have lower discount prices (cf. Gourio and Rudanko (2014)).

Another straightforward insight of the first-order conditions of problem (20) is that the firm does not recruit workers and fire workers at the same time, i.e. $R > 0$ and $\delta_w > \bar{\delta}_w$ are mutually exclusive. To see this formally, it follows from (22) that $R > 0$ requires that

$$\rho < \frac{m(\lambda)}{\lambda}[\beta(1-\delta)\mathbb{E}G_1'(+) + \gamma x F' - b] \ .$$

For $\delta_w > \bar{\delta}_w$, the first-order condition is

$$\rho = \beta(1-\delta)\mathbb{E}G_1'(+) + \gamma x F' - b - (r_2' + s_2') \geq \beta(1-\delta)\mathbb{E}G_1'(+) + \gamma x F' - b \ ,$$

where the inequality follows since r and s are non-increasing in employment. Because of $m(\lambda)/\lambda \leq 1$, the two conditions are mutually exclusive. By a similar argument, the firm does not reject existing customers and attract new customers simultaneously. It may however be possible that the firm hires new workers and rejects customers at the same time (e.g., in response to a negative productivity shock). Conversely, it is conceivable that the firm fires workers and attracts new customers at the same time (e.g., in response to a positive productivity shock).

2.4 Revenue and Prices

Productivity and demand shocks impact the joint dynamics of the firms' revenue and output in distinct ways. While the firms' output dynamics $B(z^a) \leq xF(L(z^a))$ follows directly from the solution of the social planning problem, the dynamics of revenue requires the calculation of equilibrium prices in the decentralized competitive search equilibrium. Using (13), the revenue of firm z^a is

$$\begin{aligned} Re(z^a) &\equiv p^R(z^a)B(z^{a-1})(1-\delta_b(z^a)) + p(z^a)q(\varphi(z^a))S(z^a) \\ &= u'(C)y_a B(z^a) - c\Big[B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big] \ . \end{aligned}$$

Note that both prices p and p^R are increasing in firm-specific demand y. The firm's (average) price is $P(z^a) \equiv Re(z^a)/B(z^a)$ because $B(z^a)$ is the quantity of output units sold.
Other decentralizations without price discrimination (albeit with commitment) are also conceivable. Suppose for example that each firm charges the same price $p(z^a)$ for all its customers who also know that they are separated from firms with identical probability $\delta_b(z^a)$. Then optimal search requires that

$$c = \frac{q(\varphi(z^a))}{\varphi(z^a)} Q(z^a) \ ,$$

where the value of a customer $Q(z^a)$ buying from firm z^a satisfies the Bellman equation.

$$Q(z^a) = u'(C)y_a - p(z^a) + \beta(1-\delta)\mathbb{E}_{z^a}\Big([1-\delta_b(z^{a+1})][Q(z^{a+1}) - c]\Big) \ .$$

Given firm policies $\varphi(z^a)$ and $\delta_b(z^a)$, these two equations can be directly solved for non-discriminatory prices $p(z^a)$ and for the firms' revenue $Re(z^a) = p(z^a)B(z^a)$.[16]

[16] The model statistics on price dynamics that we report in Table 3 do not change much under this different pricing assumption. Details are available upon request.

We can also solve for wages in the competitive search equilibrium. In the quantitative experiments of the next section we consider flat wage contracts. Our results on wage variation change only little when we consider an alternative wage schedule satisfying an equal-pay constraint within the firm. See Appendix A for details on how to calculate wages under these two different scenarios.

3 Quantitative Analysis

In this section we calibrate the model to match certain statistics of the price and output adjustments of German manufacturing firms in order to examine the quantitative roles of demand and productivity for firm dynamics. We first describe the data and our calibration strategy. Then we quantify the separate contributions of demand and productivity for firm volatility, for the labor market and for productivity and price dispersion. Finally, we use the model to explore the impact of aggregate shocks on macroeconomic outcomes which we compared with the cyclical features in our data.

3.1 Data and Measurement

This section describes the data and how we treat them. Further details are contained in Appendix 4. We use administrative firm data for Germany (*Amtliche Firmendaten für Deutschland*, AFiD), which are provided by the Research Data Centers of the Federal and State Statistical Offices.[17] We work with the panel *Industriebetriebe* (manufacturing establishments) and the module *Produkte* for the years 1995–2014. The former is an annual panel which builds on monthly, quarterly and annual census statistics covering all establishments in manufacturing, mining and quarrying with 20 or more employees. These data contain annual information on employment, revenue and wage bill, while working hours are available for a subsample of establishments. The module *Produkte* builds on quarterly production statistics and has recordings on quantities and revenues for nine-digit products for these establishments. We merge the two panels in order to construct a matched establishment-product panel.

We drop establishments that do not operate throughout a given year or that report to employ fewer than 20 employees. We deflate nominal variables to 2010 euros using the GDP deflator. We further drop products that are measured in different units across establishments (less than one percent). We define a product-establishment-year observation to be valid if its quantity is not measured in euros (such as services without quantity information), if the product is produced in the current and in the previous year, and if it is not produced in contract work. We only consider those establishment-year observations where the valid products are representative

[17]For more information and how to access the data, see Malchin and Voshage (2009) and the website of the Research Data Centers http://www.forschungsdatenzentrum.de.

in the sense that they contribute at least 50 percent of the total revenue of the establishment (cf. Foster et al. (2008)).

Since we are interested in employment, output and price adjustments over time, we only consider establishment-year observations with positive employment and revenue information in the current and in the previous year. These establishments produce one or several products and have valid and consistent product information in both years, as defined above. Table 1 presents the distribution of establishments, employment and revenue by size class and by establishment type. More than half of revenue and over 42 percent of employment are concentrated in the largest five percent of establishments with more than 500 workers, while 40 percent of establishments employ less than 50 (though 20 or more) workers.[18]

Our data include single-establishment firms as well as establishments belonging to multi-establishment firms. The last row of Table 1 shows that over three quarters of establishments and over fifty percent of employment are in single-establishment firms. The statistics that we use in the following analysis do not change much when we restrict the panel to single-establishment firms. We thus refer to "firms" in the rest of this paper, keeping in mind that our statistics are calculated at the establishment level.

Table 1: Shares by Establishment Size and Type in Percent

	Establishments	Employment	Revenue
Establishment size			
20 – 49	40.4	8.7	5.3
50 – 249	46.8	32.1	26.6
250 – 499	7.8	16.9	15.9
500+	5.1	42.3	52.1
Single-unit firms	76.4	51.9	41.8

Source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

Measuring Firm Dynamics

We are interested in firm dynamics, i.e. year-to-year changes in revenue, output and prices. To obtain such measures, consider firm i observed in years $t-1$ and t. We track this firm's product

[18] The average establishment in our sample is quite large: the mean (median) size is about 150 (70) employees, which is due to the fact that our sample is truncated below at 20 employees and that it covers the manufacturing sector.

portfolio across the two years and consider its products j for which we have valid revenue $R_{ij\tau}$ and quantity information $Q_{ij\tau}$ in both years $\tau = t-1, t$. We compute a firm-product price as $P_{ij\tau} = R_{ij\tau}/Q_{ij\tau}$.[19] Then we measure the firm's (log) revenue, output and price growth rates as follows:

$$\hat{R}_{it} = \log\left(\frac{\sum_j R_{ijt}}{\sum_j R_{ij,t-1}}\right) \ , \ \hat{Q}_{it} = \log\left(\frac{\sum_j P_{ij,t-1}Q_{ijt}}{\sum_j R_{ij,t-1}}\right) \ , \ \hat{P}_{it} = \hat{R}_{it} - \hat{Q}_{it} \ .$$

That is, for multi-product firms, we use the previous year's prices to calculate value-weighted output growth. Equivalently, price growth is the log change of the firm-specific Paasche price index (cf. Carlsson and Skans (2012)). We further calculate log growth rates of employment $\hat{E}_{it}$, hours $\hat{H}_{it}$ and wage per hour $\hat{w}_{it}$ (whenever available). Our final sample includes 464,025 firm-year observations with valid growth rates for output, prices and employment over the period 1996–2014.[20] To reduce the impact of outliers and measurement error, we truncate the sample at 2% and 98% percentiles based on log price and output growth rates. To take out common industry or regional changes, we regress all log growth rates in a given year on region, industry, and region×industry dummies and take the residuals of these regressions as our empirical measures of firm growth rates.
Table 3 (first column) reports cross-sectional statistics of firm growth rates. Price growth is quite dispersed across firms, of a similar magnitude as the dispersion of employment growth, but much less dispersed than either output or revenue growth. Price growth correlates negatively with output growth: firms that expand production sell their products at lower prices on average. These results, which are conditional on industry and region, are similar for unconditional statistics as well as for statistics weighted by firm size (results available upon request).

3.2 Parameterization

We calibrate the model at annual frequency and set the discount factor to $\beta = 0.96$ to reflect a four-percent interest rate. The production function is Cobb-Douglas $F(L) = L^{\alpha}$ where $\alpha = 0.7$ gives rise to a labor income share of about 70 percent. We set the firm exit rate to $\delta = 0.02$ corresponding to the annual exit rate of German firms with 20 or more workers (see Fackler et al. (2013)), and the exogenous worker separation rate to $\bar{\delta}_w = 0.02$ so that the total separation rate in the stationary equilibrium is around 7 percent.[21] The exogenous customer separation rate is set to $\bar{\delta}_b = 0.43$; this number corresponds to the finding of Stahl (2010) that

[19] This price reflects a quality component of the product which may differ across firms producing the same nine-digit product in the same year, or across time in the same firm. Since we are interested in year-to-year changes of prices and quantities in the same firm, we presume that quality adjustments within the firm over the course of a year do not play an important role.

[20] 384,847 of these have valid measures of hours growth (and wage per hour growth).

[21] These targets are based on Fuchs and Weyh (2010) who measure plant-level job creation and destruction rates from the IAB Establishment History Panel for the period 2000–2006.

regular customers account for 57% of the annual revenue in German manufacturing firms. This parameter value implies rather high customer turnover so that no firm voluntarily decides to separate from customers, even when hit by rather adverse shocks (of the magnitudes calibrated below).

For recruitment and sales costs we adopt the constant-returns specifications $r(R, L) = \frac{r_0}{3}\left(\frac{R}{L}\right)^2 R$ and $s(S, L) = \frac{s_0}{3}\left(\frac{S}{L}\right)^2 S$; division by employment size makes sure that larger firms with proportionally higher recruitment and sales effort incur the same unit costs (cf. Merz and Yashiv (2007)). Convex recruitment costs give rise to sluggish employment adjustment with variation in job-filling rates and wages across firms. Similarly in the product market, convex sales costs are responsible for variation in discount prices across firms with different matching rates. The scale parameters r_0 and s_0 are set to match plausible shares of spending on recruitment and sales; specifically we target recruitment (sales) expenditures to be one (two) percent of GDP as in Christiano et al. (2016) (Arseneau and Chugh (2007)). For the labor market matching function, we choose the Cobb-Douglas form $m(\lambda) = m_0\lambda^{0.5}$. The elasticity of 0.5 is a standard value (e.g. Petrongolo and Pissarides (2001)) and the scale parameter m_0 is set to generate a stationary unemployment rate of 8.2 percent which is the data average of the OECD harmonized unemployment rate over the period 1995–2014. We follow Arseneau and Chugh (2007) and Mathä and Pierrard (2011) and adopt the same Cobb-Douglas functional form $q(\varphi) = q_0\varphi^{0.5}$ in the product market, and we set parameter q_0 such that the average matching probability of a shopper is 50 percent. While this choice is rather arbitrary, it is consistent with the observation that consumers visit on average 1.1–3 stores for a given purchase (cf. Lehmann and Van der Linden (2010)), suggesting that around half of all visits do not result in a match.[22]

The unemployment income parameter b is set to 60 percent of the average wage to represent the unemployment replacement rate in Germany (cf. Krebs and Scheffel (2013)). According to the time use survey of the German Statistical Office, the average person in Germany spends 45 minutes shopping per day (including transit time) and he/she spends five hours on market and non-market work (excluding shopping). Equating shopping costs to the opportunity cost of foregone earnings, we calibrate parameter c such that total shopping costs are equal to 15 percent of labor earnings.

The utility function has constant elasticity, $u(C) = \frac{u_0}{1-\sigma}C^{1-\sigma}$ with $\sigma \geq 0$. We set $\sigma = 2/3$ so that the elasticity of industry demand corresponds to the mean estimate for U.S. manufacturing industries of Chang et al. (2009). The marginal valuation of a good in the model equals $yu'(C)$ in units of the numeraire good. As the unit of measurement is arbitrary, we normalize the average value of $yu'(C)$ to unity by setting the mean value of the demand shock to $y = \bar{y} = 1$ and adjusting the scale parameter u_0 accordingly.

The entry cost parameter K (and thus the endogenous variable ρ) is set so that the average firm

[22]For both matching functions, we make sure that matching rates of workers and shoppers ($m(\lambda)/\lambda$ and $q(\varphi)/\varphi$ resp.) do not exceed one; that is we set $m(\lambda) = \min(\lambda, m_0\lambda^{0.5})$ and $q(\varphi) = \min(\varphi, q_0\varphi^{0.5})$.

employs 70 workers (the median firm size in our data), and we normalize mean productivity to unity, $x = \bar{x} = 1$.[23]

We now describe how the shock processes for firm-specific demand and productivity shocks are parameterized. Evidently, a firm's output and price policy differs in response to changes of either demand or productivity. This is illustrated in Figure 1 which shows the output and price responses to a permanent one-standard-deviation increase in either demand or productivity in period one. Higher demand allows the firm to increase its price on impact for both its existing customers and for new customers. Over time, the greater valuation of its product induces the firm to expand production and to attract more customers so that output increases. This is why the price (slightly) increases further after period one. A positive productivity shock allows the firm to produce more output on impact. To sell the additional output, the firm cuts the discount price to attract new customers. Because the price for existing customers does not change, the average price of that firm does not fall much in magnitude. Over time, the more productive firm hires additional workers and attracts more customers: output increases further and the discount price stays below the steady-state value from period two onward.

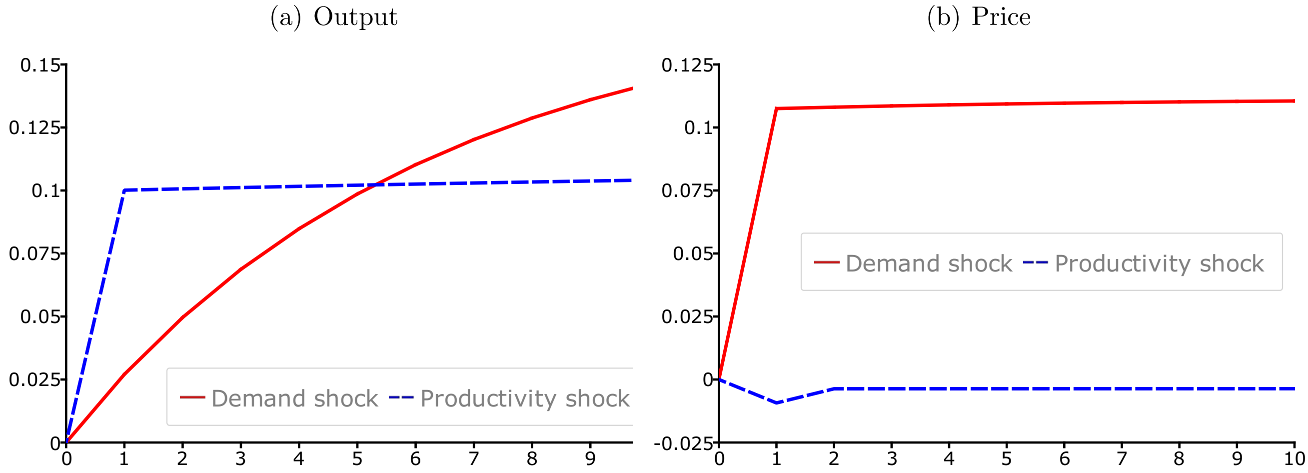

Figure 1: Output and Price Responses to Firm-Level Shocks.

Note: The graphs show the policy of a firm with average demand and productivity at its steady-state size in period zero which experiences a one-time permanent increase in productivity (blue, dashed) or demand (red, solid) by 10 percent in period one.

We assume that firm-specific productivity and demand parameters (x_t, y_t) follow AR(1) pro-

[23] Parameter K cannot be identified independently of the average values of firm productivity x because firm-level value functions are linearly homogeneous in the vector $(x, b, r_0, s_0^{-1/2}, \rho, K, B, S)$ (see problem (20), together with the assumed functional forms), so that all firm-level policies are independent of scaling transformations.

cesses

$$\log(x_{t+1}) = \rho^x \log(x_t) + \varepsilon^x_{t+1} \ ,$$
$$\log(y_{t+1}) = \rho^y \log(y_t) + \varepsilon^y_{t+1} \ ,$$

where the innovation terms ε^x and ε^y are jointly normally distributed with covariance matrix Σ_ε. The five parameters describing these processes (two autocorrelations and three parameters in symmetric matrix Σ_ε) are decisive for the joint dynamics of firm-level prices and output. While the covariance matrix controls the variability and co-movement of prices and output, persistence parameters are responsible for the frequency of price and output adjustments.
This leads us to choose the following five calibration targets: (i) the standard deviations of log price growth and log output growth (denoted $\sigma(\widehat{P})$ and $\sigma(\widehat{Q})$); (ii) the correlation coefficient of log price growth and log output growth $\rho(\hat{P}, \hat{Q})$; (iii) the shares of firms that adjust prices or output by less than two percent (upwards or downwards). The last two measures indicate the stickiness of prices and output; hence they are suitable calibration targets for the persistence parameters of the underlying productivity and demand shocks.
We find the five parameters for the shock processes via a simulated method of moments procedure where we minimize the unweighted squared percentage distance between the empirical and the model-implied moments. All our parameter choices are summarized in Table 2.

3.3 Firm Dynamics and Heterogeneity

The first five rows in Table 3 show that the model matches the calibration targets for price and output dynamics well. Output growth is more volatile than price growth, and prices are more sticky than output. Price growth and output growth are negatively correlated. Since a firm's log revenue growth is the sum of log price growth and log output growth, the cross-sectional standard deviation of revenue growth naturally splits into standard deviations of price and output growth. As implied by these numbers, output growth dispersion account for over 80 percent of the cross-sectional dispersion of revenue growth.[24]
The next rows in Table 3 demonstrate that the model performs reasonably well to capture the dispersions of revenue growth and employment growth. It also matches their respective correlations with output growth. Relative to the data, however, employment is somewhat more rigid in the model where 40.5 percent of firms do not adjust employment by more than two percent (compared to 25.7 percent in the data). This may be due to the non-convex labor adjustment costs in the presence of search frictions.
The cross-sectional standard deviation of average (hourly) wage growth is much larger in the data than in our model. One possible explanation for the gap are compositional changes of the

[24] In the data, the variances of revenue (price, output) growth are 0.0343 (0.0129, 0.0353) and the covariance of price and output growth is -0.0069. Hence the contribution of output growth to revenue growth is $(0.0353 - 0.0069)/0.0343 = 82.5$ percent (similar numbers for the model).

Table 2: Parameter Choices

Parameter	Value	Explanation/Target
β	0.96	Annual interest rate 4%
α	0.7	Labor income share
δ	0.02	Firm exit rate (Fackler et al. (2013))
$\bar{\delta}_w$	0.02	Worker separation rate 7%
$\bar{\delta}_b$	0.43	Customer retention rate 57%
r_0	0.334	Recruitment costs 1% of output
s_0	293.6	Sales costs 2% of output
m_0	0.460	Unemployment rate 8.2%
q_0	1.423	Customer matching rate 50%
b	0.111	Unemployment income 60% of average wage
c	0.070	Shopping costs 15% of earnings
K	67.98	Entry cost (average firm size $L = 70$)
Σ_ε^{xx}	0.00678	Standard deviation of output growth
Σ_ε^{yy}	0.00703	Standard deviation of price growth
Σ_ε^{xy}	-0.00502	Correlation price and output growth
ρ^x	-0.514	Log output growth in $[-0.02, 0.02]$
ρ^y	0.861	Log price growth in $[-0.02, 0.02]$

workforce which are not present in our model where all workers are homogeneous. Further, our model does not have an hours margin so we cannot compare hours adjustments. In the data, hours growth is more volatile than employment growth because firms use both the extensive and the intensive margins for labor adjustments.

To explore the separate roles of demand and productivity for firm dynamics, we report in the last two columns the model statistics if either demand shocks or productivity shocks are absent. Specifically, in these experiments we set the variance of either shock to zero while fixing the variance of the other shock. We leave all other model parameters unchanged and solve the model for a new steady state equilibrium.

In the absence of demand shocks ("x shocks only"), the model generates enough variation of output growth, but only little dispersion of price growth. Furthermore, output and price growth are too strongly negatively correlated. On the other hand, if productivity shocks are absent ("y shocks only"), the model generates substantial variation of price growth but too

Table 3: Firm Dynamics: Data versus Model

	Data	Model	x shocks only	y shocks only
$\sigma(\hat{P})$	0.113	0.115	0.025	0.106
$\sigma(\hat{Q})$	0.187	0.187	0.182	0.069
$\rho(\hat{P}, \hat{Q})$	-0.326	-0.324	-0.837	0.511
$\hat{P} \in [-2\%, +2\%]$	0.375	0.363	0.575	0.638
$\hat{Q} \in [-2\%, +2\%]$	0.123	0.131	0.187	0.513
$\sigma(\hat{R})$	0.185	0.185	0.161	0.153
$\sigma(\hat{E})$	0.116	0.097	0.062	0.098
$\rho(\hat{R}, \hat{Q})$	0.814	0.810	0.996	0.804
$\rho(\hat{E}, \hat{Q})$	0.236	0.342	0.253	1.000
$\hat{E} \in [-2\%, +2\%]$	0.257	0.405	0.811	0.410
$\sigma(\hat{w})$	0.111	0.010	0.012	0.010
$\sigma(\hat{H})$	0.143	–	–	–
JC rate (%)	2.9	3.4	1.4	3.4
JD rate (%)	3.0	2.0	0.0	2.0
u rate (%)	8.2	8.2	6.2	8.1

Note: Data statistics are employment-weighted averages of yearly cross-sectional statistics for residuals of firm growth rates, after controlling for industry and region. (Source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.) $\sigma(.)$ denotes a standard deviation, $\rho(.,.)$ is a correlation coefficient, and $\hat{X}$ is the log growth rate of variable X.

little variation of output growth, and price and output changes would be strongly positively correlated. This highlights that both productivity and demand shocks are necessary to capture the joint dynamics of prices and output across firms: productivity shocks (demand shocks) are the main factors behind output (price) variation, and their joint presence is required to generate the moderately negative correlation between output and price growth across firms.

Table 3 further shows that demand shocks are key to understanding employment dynamics. When only productivity shocks are present, the standard deviation of employment growth is only about half as large as in the data, and employment is much too sticky: in the model 81% of firms barely adjust employment when demand shocks are absent. In the data, in contrast, almost three quarters of firms adjust employment by more than two percent from one year to

the next.
On the other hand, demand shocks alone would generate more volatile employment growth, but they would predict that employment moves one-for-one with a firm's output, whereas in the data the correlation between output and employment growth is only moderately positive. In sum, both demand and productivity shocks are important factors to account for firms' employment adjustments, but demand shocks are the main driver of employment volatility.
The last three rows in Table 3 compare job flow rates and unemployment rates in the model and in the data.[25] With both shocks taken together, the model slightly overpredicts the magnitude of job creation, but it underpredicts the magnitude of job destruction for continuing firms. At the same time, job destruction at exiting firms (not reported here) is too large, which is due to our assumption of a size-independent exit rate. In the absence of demand shocks, the model accounts for only 41 percent of job creation, while job destruction drops to almost zero. In contrast, when productivity shocks are absent, job flow rates barely change compared to the full model with both shocks active. Hence, demand shocks are the major driving force for job creation and destruction at continuing firms. The last row shows that the model without demand shocks has a considerably lower unemployment rate (6.2 percent versus 8.2 percent in the data and in the model with both shocks active). In other words, idiosyncratic demand shocks account for a sizeable fraction of total unemployment in this model, whereas idiosyncratic productivity risk hardly matters for unemployment. The explanation is that there is basically no job destruction at continuing firms when demand shocks are absent.
It is well known that smaller firms are more volatile than larger firms. This is also true in our data, as the left graph in Figure 2 shows: the standard deviations of firm-level revenue growth, price growth, output growth and employment growth are all larger for smaller firms than for larger firms. The right graph shows that our model can broadly capture this fact, although it generates too much (too little) dispersion of employment growth at smaller (larger) firms.
Finally, our model generates some dispersion of prices and productivity across firms which we can relate to the data as follows. To distinguish between physical and revenue-based productivity, we calculate two measures. First, we measure *revenue labor productivity* (RLP) by dividing a firm's revenue by its labor hours. Second, we use average product prices to measure a firm's physical output. Dividing this number by hours yields a measure of *quantity labor productivity* (QLP). To minimize distortions generated by quality differences between firms, we use only products measured in physical units (i.e. length, area, volume or weight) and restrict the firm sample accordingly. The ratio $\tilde{P} \equiv \text{RLP/QLP}$ defines a firm-specific price index which exceeds one for those firms whose products are more expensive (in terms of a quantity-weighted average) than those produced at other firms (see Appendix B for details).

[25] Both in the data and in the model, we measure job creation and job destruction rates in the usual way: the job creation (destruction) rate is $jc_{it} = \frac{2\max(E_{it}-E_{i,t-1},0)}{E_{it}+E_{i,t-1}}$ ($jd_{it} = \frac{2\max(E_{i,t-1}-E_{i,t},0)}{E_{it}+E_{i,t-1}}$) where E_{it} ($E_{i,t-1}$) is year-t ($t-1$) employment at firm i. As in the data, the model sample is based on firms with 20 or more workers in both periods (hence it covers continuing firms only).

(a) Data (b) Model

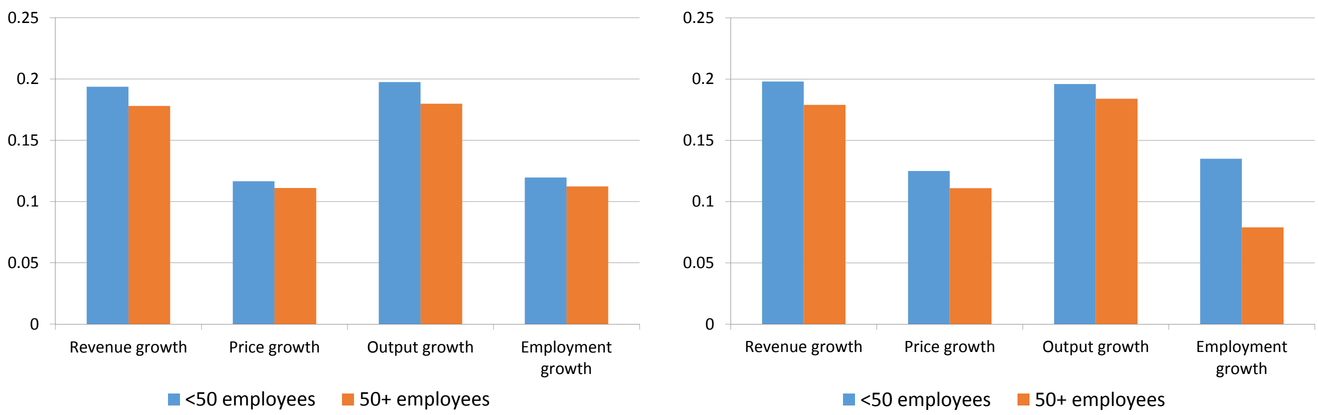

Figure 2: Firm Growth Dispersion for Smaller and Larger Firms.

Note: Standard deviations of log growth rates of firm-level revenue, price, output and employment. (Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.)

The first column in Table 4 shows the dispersion of revenue and quantity labor productivity and of the firm-specific price index in the data. Revenue labor productivity is considerably less dispersed than quantity labor productivity, which comes from a negative correlation between firms' prices and physical productivity (last row). This finding is in line with those of Foster et al. (2008) for physical and revenue total factor productivity (TFPQ and TFPR), indicating that more productive firms charge lower prices.

Our model can reproduce this last result: quantity labor productivity correlates negatively with prices so that revenue labor productivity is more dispersed than quantity labor productivity. However, our model generates only about one third of the dispersion of prices and about one fifth of the dispersion of productivity compared to the data. The unexplained dispersion in the data may reflect additional heterogeneity, e.g. permanent differences in firm-specific demand or productivity, that are not adequately captured in our quantitative model.

The last two columns of Table 4 show that both productivity and demand shocks contribute about equally to the dispersion of firm productivity, both in terms of revenue and quantity productivity. On the other hand, demand shocks are the main factor for price dispersion: differences in physical productivity alone would account for only seven percent of the price dispersion in our model.

Table 4: Price and Productivity Dispersion: Data versus Model

	Data	Model	x shocks only	y shocks only
σ(RLP)	0.674	0.144	0.104	0.137
σ(QLP)	0.973	0.161	0.118	0.124
$\sigma(\tilde{P})$	0.629	0.209	0.015	0.206
$\rho(\text{QLP}, \tilde{P})$	-0.726	-0.725	-0.924	-0.765

Note: Data statistics are averages of yearly cross-sectional statistics for residuals of RLP, QLP and $\tilde{P}$, after controlling for industry and region. (Source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.) $\sigma(.)$ denotes a standard deviation and $\rho(.,.)$ is a correlation coefficient.

3.4 Aggregate Dynamics

How does the model economy respond to aggregate shocks to the first or second moment of either productivity or demand risk? We are interested in the cyclical features of macroeconomic aggregates (i.e. output, employment and prices) as well as cross-sectional dynamics, in particular the dispersions of price and output growth across firms. To this end, we first look at the cyclicality of firm dispersion measures in our data. Then we analyze the impulse responses of different types of recessionary shocks on the model economy.

The literature documents counter-cyclical firm dispersion, based on the cross-sectional standard deviation of firms' revenue growth and other dispersion measures (e.g. Bloom et al. (2018)). Our data allow us not only to confirm these findings for the manufacturing sector in Germany but also to document the separate cyclicalities of price and output growth dispersion.[26] We find that both standard deviations of output growth and price growth are counter-cyclical. Since log revenue growth is the sum of log price growth and log output growth, both price and output growth dispersion contribute to the counter-cyclicality of revenue growth dispersion.

Figure 3 shows time series of the cross-sectional means and standard deviations of price growth, output growth and hours growth. Germany had two recessions in the sample period (2002/03 and 2009). In both recessions, unsurprisingly, the means of all three firm growth measures go down.[27] Moreover, during the 2002/03 recession and the subsequent recovery, output growth

[26] See also Bachmann and Bayer (2014) who document countercyclical dispersion of firm-level growth rates of employment, value added and factor productivity for Germany during the period 1973–1998. Berger and Vavra (2018) find countercyclical price growth dispersion in U.S. data.

[27] We use hours instead of employment here because the labor market reforms in Germany during the 2000s (Hartz I–IV) have decisively altered the employment dynamics in Germany. In particular, aggregate employ-

leads hours growth and price growth. As shown in the right panel of Figure 3, all three dispersion measures go up in both recessions, albeit by different magnitudes. The means and standard deviations of price growth are less volatile than the means and standard deviations of output growth or hours growth. Over the reported 19-year period, the means of the three growth measures are pro-cyclical while standard deviations are counter-cyclical.[28]

(a) Means (b) Standard deviations

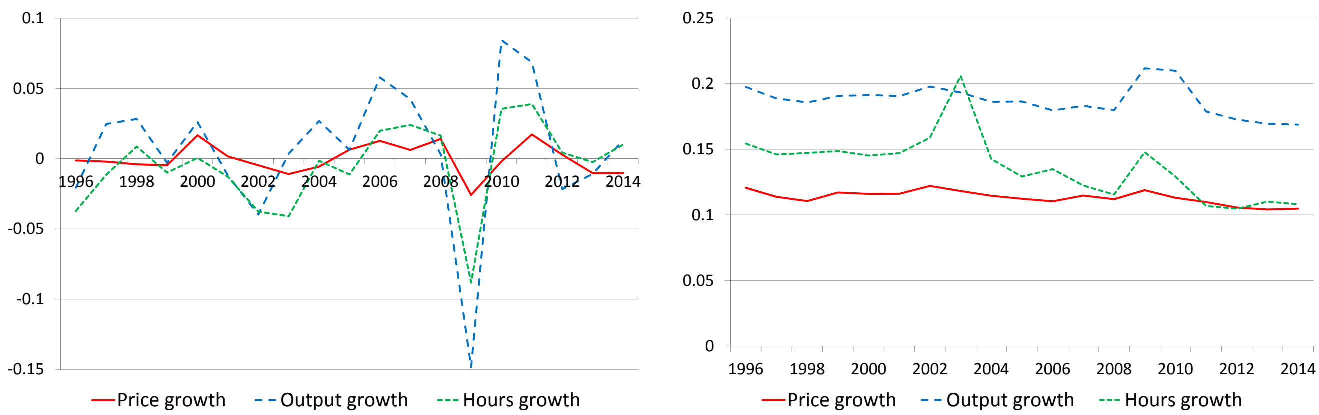

Figure 3: Means and Standard Deviations of Price, Output and Hours Growth Rates (1996–2014).

Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

To see how macroeconomic variables and firm-level dispersion measures react to aggregate shocks, we compare the impulse responses of our model economy to four types of aggregate events: (i) a decrease in either mean productivity $\bar{x}$ or mean demand $\bar{y}$ by five percent (mean productivity or demand shock); (ii) an increase in the standard deviation of shocks to firm productivity x or firm demand y by twenty percent (productivity or demand uncertainty shock).[29] In all four experiments, we consider the response to an unanticipated permanent shock together with the transition path. Despite the characterization by a social-planning

ment barely fell during the Great Recession (cf. Burda and Hunt (2011)).

[28]The correlations with (linearly) detrended value added in manufacturing are $(0.791, 0.759, 0.795)$ for the means of (price, output, hours) growth and $(-0.415, -0.643, -0.449)$ for the standard deviations of (price, output, hours) growth.

[29]Because AR(1) processes for the idiosyncratic states are expressed in the logs of x and y, we rescale the levels so that the means of x and y stay the same when the standard deviation of shocks increases. In both experiments we also leave the covariance term $\Sigma_{\varepsilon}^{xy}$ unchanged.

problem, aggregate dynamics has no block-recursive solution, in contrast to the directed search models of Menzio and Shi (2011) or Kaas and Kircher (2015). The reason is that marginal utility of consumption $u'(C)$ responds to changes in the distribution of firms which in turn feeds back into the firms' problem. Therefore we need to loop over the transition path of marginal utility $u'(C_t)$ together with workers' search values ρ_t to solve for the transition from one steady state to another.[30]

Figure 4 shows the economy's response to negative *mean* productivity and demand shocks. The top four graphs show the dynamics of aggregate output, employment, prices, and the number of firms. The negative productivity shock generates a five percent decline in output on impact while employment *increases* on impact, albeit by only a bit more than 0.1 percent. The countercyclical response of employment is a result of a substantial reduction of the workers' search value ρ_t in response to the shock which reduces wages and which makes it (slightly) more attractive for *incumbent* firms to hire. Potential entrants, in contrast, find it less profitable to enter so that the number of firms falls over time, resulting in a long-term decline in aggregate employment, which is again small, however (-0.2 percent). Panel (c) shows that firms pass on the higher labor costs to customers: prices increase on impact by more than four percent.

The response to a negative demand shock is quite different. Again, wages fall substantially in response to the decline in firm revenue which now leads to an even larger short-term increase in aggregate employment ($+0.35$ percent in the year after the shock), while the reduction of entry leads to a long-term decline in employment by -0.3 percent. Interestingly, output does not fall on impact (it even increases slightly) since incumbent firms simply cut prices to accommodate customers' lower valuations of their products. The long-term decline in output, induced by a smaller number of firms, is then rather modest (less than 1 percent).

The bottom two graphs in Figure 4 show the responses of price and output growth dispersion to the two shocks. The negative productivity shock raises the dispersion of output growth and reduces the dispersion of price growth, while the opposite is true in response to a negative demand shock. Intuitively, lower productivity raises the price level and reduces the output level (panels (a) and (c)), while the magnitude of idiosyncratic uncertainty stays the same. As growth rates are expressed in percentage terms, the dispersion of output growth rises while the dispersion of price growth falls. A negative demand shock has a smaller impact on the levels of prices and output, and because the price level falls, the standard deviation of price growth increases. In both cases, however, the magnitudes are rather small.

We conclude that aggregate shocks to the first moments of productivity or demand cannot generate the countercyclical dispersion of price and output growth that we observe in the

[30] Our model in Section 2 is described in steady state without aggregate risk. To incorporate the latter, we are assuming that the firms' wage contracts are contingent on the *aggregate* state of the economy. In response to aggregate events, contractual wages and separation rates adjust, which ensures that the response of the competitive-search equilibrium to these shocks is identical to the solution of the planning problem that we consider.

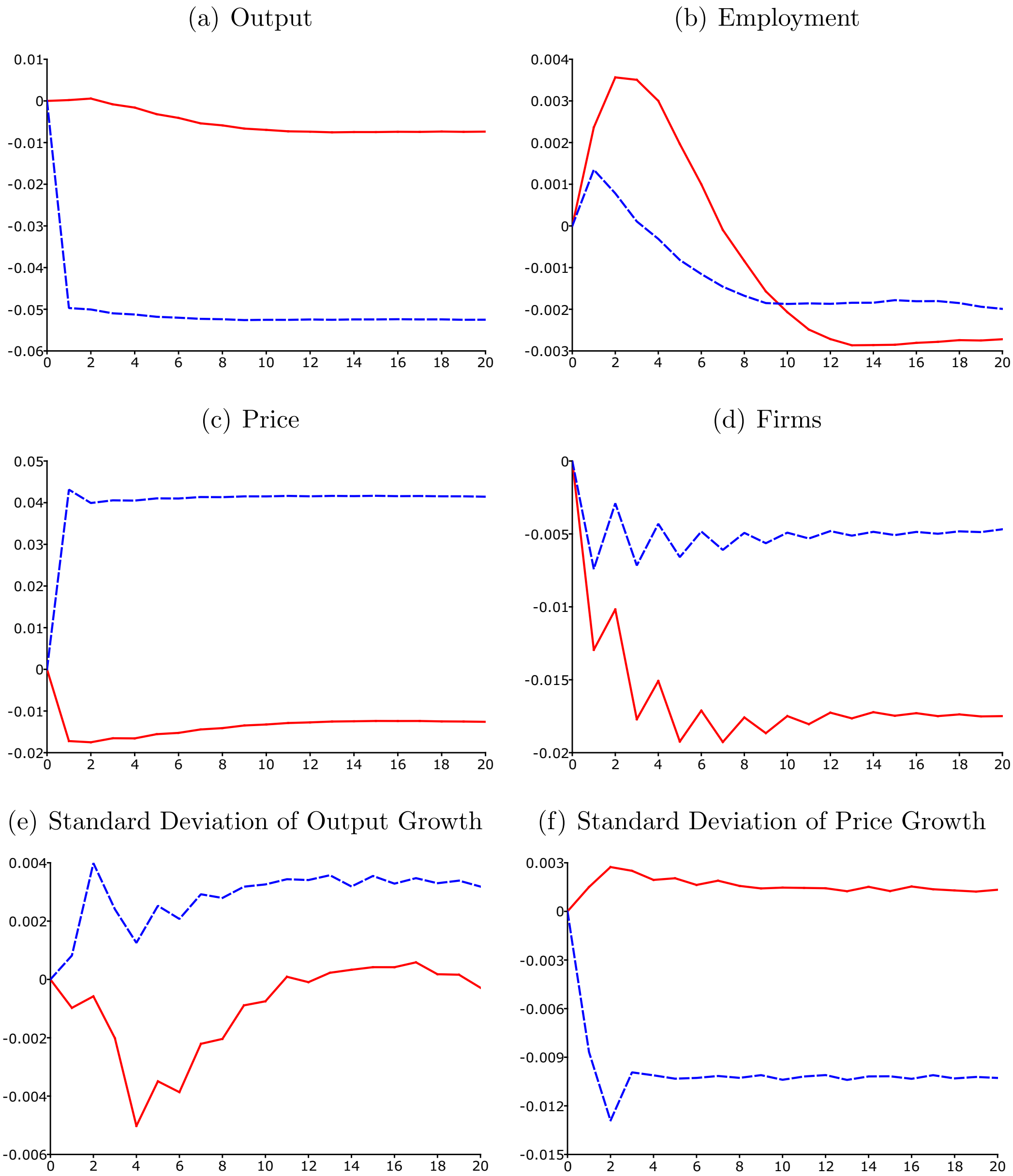

Figure 4: Responses to a Five Percent Decrease of Aggregate Productivity (dashed, blue) and Aggregate Demand (solid, red).

data. Moreover, both these shocks cannot generate a short-run comovement of employment and output which is due to the assumed flexibility of wages, as explained above.
Quite different is the reaction of the model economy to uncertainty shocks, as we illustrate in Figure 5. In particular, a demand uncertainty shock generates sizable declines in output and employment. Furthermore, the two reported measures of firm dispersion rise in response

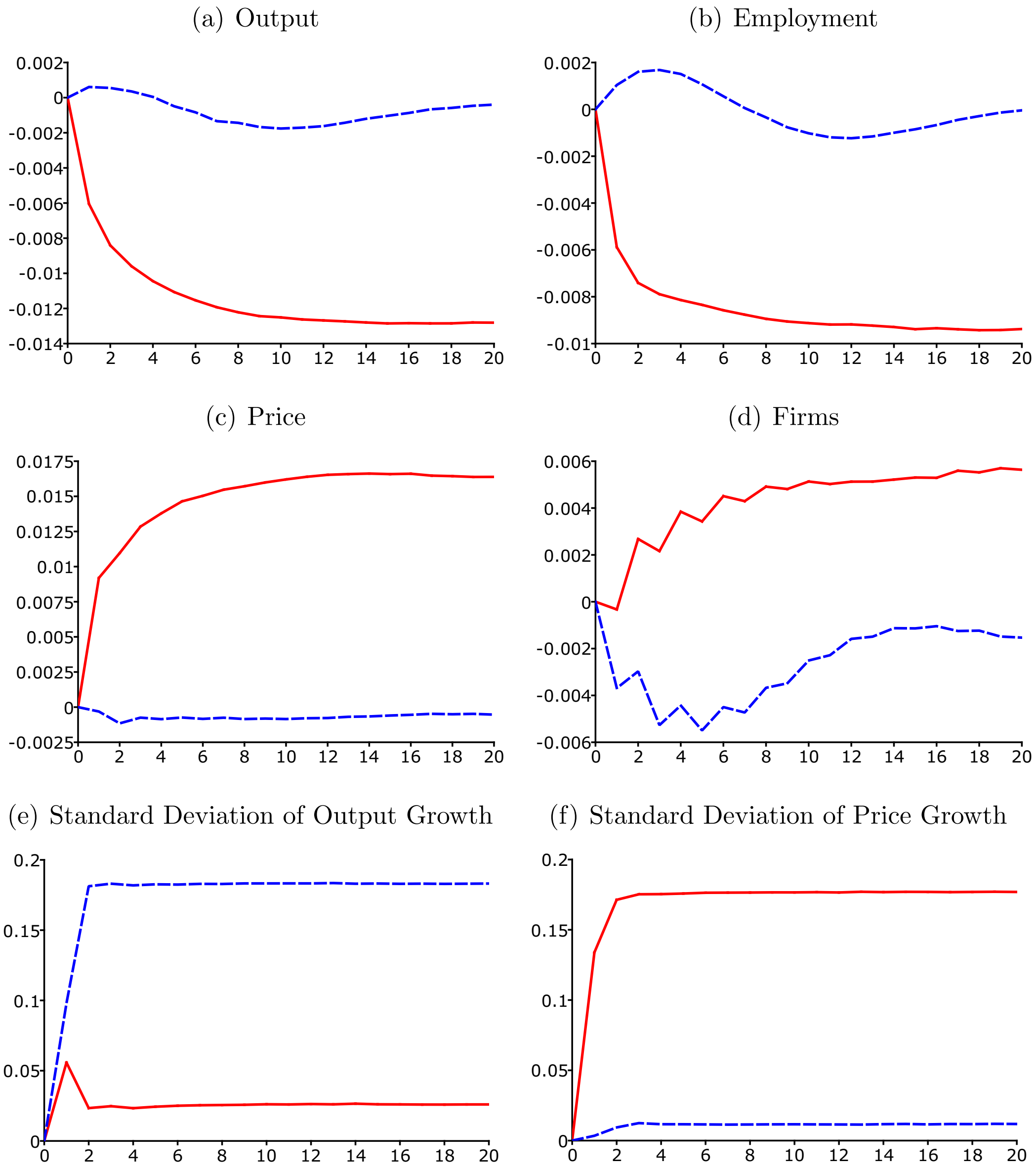

Figure 5: Responses to an Increase of Aggregate Productivity Uncertainty (dashed, blue) and Aggregate Demand Uncertainty (solid, red).

to greater demand uncertainty (panels (e) and (f)). Counterfactually, however, the aggregate price level goes up. The explanation is that the uncertainty shock benefits firms with high demand (which produce more and sell at higher prices) while it hurts firms with low demand (which produce less at lower prices). Since the aggregate price level is a quantity-weighted average of firm-level prices, the aggregate price index goes up due to the higher weight on larger firms.

An increase in productivity uncertainty, in contrast, generates much weaker (and positive) reactions of output and employment (on impact), and also the cross-firm dispersion of price growth changes only little. On the other hand, given that output growth dispersion is largely explained by idiosyncratic productivity shocks (see Table 3), greater productivity uncertainty generates a large increase in the standard deviation of output growth (panel (e)).
Based on these findings we conclude that higher demand uncertainty is a plausible feature of recessions: it generates declines in output and employment together with increasing firm dispersion. Although the price level goes up when the demand uncertainty shock is considered in isolation, it should be straightforward to induce a decline in the price level, together with plausible responses of the other variables, when the demand uncertainty shock is combined with a decline in mean demand, as suggested by the magnitudes of the responses in Figure 4 and in Figure 5. On the other hand, aggregate shocks to productivity, either to the first or to the second moment, do not deliver meaningful impulse responses in our model, at least when flexibility of wages is assumed, as we do in these exercises.

4 Conclusions

We consider heterogeneous firms who operate in frictional product and labor markets with convex sales and recruitment costs. Search frictions in the product market imply that firms are demand constrained, and hence must expend resources to spur demand. Likewise, frictions in the labor market make firms' adjustment to shocks sluggish, with consequences for the cross-sectional dynamics as well as for the aggregate economy. We distinguish between firm-level productivity and demand shocks which affect the firms' output and pricing policies in different ways. The parameters of these shock processes are calibrated in order to match salient features of price and output dynamics of a panel of German manufacturing firms.
While both demand and productivity shocks are necessary to describe the joint dynamics of prices and output in the data, demand variability is the dominant force to account for employment changes across firms. They are also the main driving force for job creation and job destruction, they are responsible for almost one quarter of total unemployment, and they are crucial for price dispersion among firms.
By means of several impulse response analyses, we highlight the importance of demand uncertainty for the business cycle. Considered in isolation, declines in aggregate demand or aggregate productivity cannot generate plausible recessions in the model economy with falling employment and output. By contrast, demand uncertainty shocks can qualitatively explain these patterns and additionally lead to increases in output and price growth dispersion in a recession which is in line with our data.
In sum, our work shows how product market conditions interact with labor market conditions to generate realistic firm dynamics in a fairly tractable model framework. Due to our assump-

tion of a representative household, some important product-labor market linkages that operate through the household sector, such as the different shopping behavior of unemployed workers (Krueger and Mueller (2010) or Kaplan and Menzio (2016)), are absent from our model. Introducing such features might have important implications for aggregate dynamics and should be an interesting avenue for further research.

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Appendix A: Proofs and Derivations

Proof of Proposition 1:

Part (a).

Consider first the sequential planning problem to maximize the discounted household utility for a given initial distribution of workers and customers among heterogeneous firms. For any time t and any firm's age a, write $z_{a,t} = (x_{a,t}, y_{a,t})$ for the firm's productivity and demand state, and write $z^{a,t} = (z_{0,t-a}, z_{1,t-a+1}, \ldots, z_{a,t})$ for the idiosyncratic state history. At time $t = 0$, the planner takes as given the initial firm distribution $(N(z^{a-1,-1}), L(z^{a-1,-1}), B(z^{a-1,-1}))_{a\geq 1, z^{a-1,-1}}$. The planner decides for all periods $t \geq 0$ and state-contingent firm histories $z^{a,t}$ the firm policies $\lambda(z^{a,t})$, $R(z^{a,t})$, $\varphi(z^{a,t})$, $S(z^{a,t})$, $\delta_w(z^{a,t})$, $\delta_b(z^{a,t})$, as well as entrant firms N_t so as to maximize discounted household utility

$$\sum_{t\geq 0} \beta^t \Bigg\{ u\Big(\sum_{z^{a,t}} N(z^{a,t}) y_{a,t} B(z^{a,t}) \Big) + b\bar{L} - K N_t$$
$$- \sum_{z^{a,t}} N(z^{a,t}) \Bigg[bL(z^{a,t}) + r(R(z^{a,t}), L_0(z^{a,t})) + s(S(z^{a,t}), L_0(z^{a,t}))$$
$$+ c\Big(B(z^{a,t}) + [\varphi(z^{a,t}) - q(\varphi(z^{a,t}))] S(z^{a,t}) \Big) \Bigg] \Bigg\} ,$$

subject to

$$\begin{aligned} L(z^{a,t}) &= L(z^{a-1,t-1})[1 - \delta_w(z^{a,t})] + m(\lambda(z^{a,t})) R(z^{a,t}) , \\ B(z^{a,t}) &= B(z^{a-1,t-1})[1 - \delta_b(z^{a,t})] + q(\varphi(z^{a,t})) S(z^{a,t}) , \\ L_0(z^{a,t}) &= L(z^{a-1,t-1})[1 - \delta_w(z^{a,t})] , \\ N(z^{a,t}) &= (1-\delta)\pi(z_{a,t}|z_{a-1,t-1}) N(z^{a-1,t-1}) , \end{aligned}$$

for $t \geq 0$ and $a \geq 1$,

$$\begin{aligned} &L(z^{0,t}) = m(\lambda(z^{0,t})) R(z^{0,t}) \ , \ B(z^{0,t}) = q(\varphi(z^{0,t})) S(z^{0,t}) , \\ &N(z^{0,t}) = \pi^0(z^{0,t}) N_t , \end{aligned}$$

for $t \geq 0$,

$$B(z^{a,t}) \leq x_{a,t} F(L(z^{a,t})) \ , \ \delta_w(z^{a,t}) \geq \bar{\delta}_w \ , \ \delta_b(z^{a,t}) \geq \bar{\delta}_b \ , \tag{28}$$

for $t \geq 0$ and $a \geq 0$, and subject to the resource constraint for all $t \geq 0$,

$$\bar{L} \geq \sum_{z^{a,t}} N(z^{a,t}) \Big[L(z^{a,t}) + \Big(\lambda(z^{a,t}) - m(\lambda(z^{a,t})) \Big) R(z^{a,t}) \Big] . \tag{29}$$

Write $\beta^t\rho_t$ for the multiplier on constraint (29). The Lagrange function of the planning problem (which is still subject to the constraints in (28)) is

$$\mathcal{L} = \sum_{t\geq 0}\beta^t\Bigg\{u\Big(\sum_{z^{a,t}} N(z^{a,t})y_{a,t}B(z^{a,t})\Big) - KN_t \tag{30}$$

$$-\sum_{z^{a,t}} N(z^{a,t})\Bigg[bL(z^{a,t}) + r(R(z^{a,t}), L_0(z^{a,t})) + s(S(z^{a,t}), L_0(z^{a,t}))$$

$$+c\Big(B(z^{a,t}) + [\varphi(z^{a,t}) - q(\varphi(z^{a,t}))]S(z^{a,t})\Big) + \rho_t\Big(L(z^{a,t}) + [\lambda(z^{a,t}) - m(\lambda(z^{a,t}))]R(z^{a,t})\Big)\Bigg]\Bigg\} .$$

The derivative of the Lagrangian with respect to firm $z^{a,t}$'s output $B(z^{a,t})$ is

$$\frac{d\mathcal{L}}{dB(z^{a,t})} = \beta^t N(z^{a,t})\Big[u'(C_t)y_{a,t} - c\Big] ,$$

with aggregate consumption $C_t \equiv \sum_{z^{a,t}} N(z^{a,t})y_{a,t}B(z^{a,t})$. By similar inspection of all other optimality conditions, the number of firms of type $z^{a,t}$, $N(z^{a,t})$, enters *linearly* all first-order conditions of the sequential planning problem with respect to firm-level policies, namely $B(.)$, $L(.)$, $\lambda(.)$, $\varphi(.)$, $S(.)$, $R(.)$. The number of firm types is thus irrelevant for the planner's firm-level policies which solve the firm-level problem, defined recursively for a given (bounded) sequence $(\rho_t, C_t)_{t\geq 0}$ by

$$G_t(L_-, B_-, z) = \max_{(\lambda,R,\delta_w),(\varphi,S,\delta_b)}\Big\{u'(C_t)yB - bL - r(R, L_-(1-\delta_w)) - s(S, L_-(1-\delta_w))$$

$$- \rho_t[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E}_z G_{t+1}(L, B, z_+)\Big\} , \tag{31}$$

subject to

$$L = L_-(1-\delta_w) + m(\lambda)R \ , \ B = B_-(1-\delta_b) + q(\varphi)S \ ,$$
$$B \leq xF(L) \ , \ \delta_w \geq \bar{\delta}_w \ , \ \delta_b \geq \bar{\delta}_b \ .$$

A solution $(G_t)_{t\geq 0}$ for this problem exists with functions $G_t : [0, L^{\max}] \times [0, B^{\max}] \times Z \to \mathbb{R}$ for appropriately define upper bounds $L^{\max}$ and $B^{\max}$. The proof of this assertion follows the same lines as in Lemma A.4, part (a), of Kaas and Kircher (2015).

To prove part (a) of Proposition 1, suppose that (ρ, G, N_0, C) solves the recursive social planning problem (20) together with (21), aggregate consumption (17) and the aggregate resource constraint (19) are satisfied. Then, for the constant sequences $\rho_t = \rho$ and $C_t = C$, value functions $G_t = G$ for all $t \geq 0$ also solve problem (31). If $N_t = N_0$ for all t, the resource constraint (29) is satisfied in all periods t because the distribution of firm types and the distribution of workers across firms is stationary: $N(z^{a,t}) = N(z^a)$, $L(z^{a,t}) = L(z^a)$, provided that $(N(z^a), L(z^a), B(z^a))$ is the initial firm distribution. Because individual firm policies solve problem (31), they also maximize the Lagrange function (30). On the other hand, the first-order condition of (30) with respect to N_t is

$$0 = -\beta^t K + \sum_{a\geq 0} \beta^{t+a}(1-\delta)^a \pi(z^a) \Big\{ u'(C) y_a B(z^a) - bL(z^a) - r(R(z^a), L_0(z^a)) - s(S(z^a), L_0(z^a))$$

$$- c\Big(B(z^a) + [\varphi(z^a) - q(\varphi(z^a))]S(z^a)\Big) - \rho\Big(L(z^a) + [\lambda(z^a) - m(\lambda(z^a))]R(z^a)\Big)\Big\}$$

$$= \beta^t \Big[- K + \sum_z \pi^0(z^0) G_t(0,0,z^0)\Big] \ ,$$

which is identical to condition (21) for $G_t = G$. Hence, $N_t = N_0$ for all $t \geq 0$ solve the sequential planning problem. Since aggregate resource feasibility is satisfied, these firm policies solve the sequential planning problem where the multiplier on (29) is $\beta^t \rho$. No other feasible allocation dominates the one defined by the individual firm's problem; the formal argument follows the same lines as the proof of Lemma A.4 in Kaas and Kircher (2015).

Part (b).

Consider (ρ, G, N_0, C) where G solves the recursive social planning problem (20) together with (21). Further, aggregate consumption is (17) and the aggregate resource constraint (19) is satisfied when $L(z^a)$, $B(z^a)$ etc. are defined by the policy functions of problem (20). The proof proceeds in three steps. First, we construct candidate employment contracts and firm policies that resemble the planning solution. Second, we show that the extended policies maximize the social firm value with commitment to previous contracts. Third, we show that the candidate policies also solve the private firm problems and hence correspond to a competitive search equilibrium.

First, define candidate equilibrium contracts $\mathcal{C}^{a*} = (w^{a*}(z^k), \delta_w^{a*}(z^{k+1})_{k\geq a}$ with separation rates $\delta_w^{a*}(z^k) \equiv \delta_w(z^k)$ from the policy functions of problem (20) (hence, separations are independent of the tenure in the firm). Candidate equilibrium wages $w(z^k)$ can be defined in different ways: for instance all workers may be paid flat wages over time, or all workers within the firm earn the same (equal treatment); see the corresponding equations at the end of this Appendix. We use equal-treatment wages for the remainder of this proof and hence specify candidate equilibrium wage contracts as $w^{a*}(z^k) = w(z^k)$, where $w(z^k)$ is defined as in (35). As in Section 2.2.3, define the generic state vector of the firm as $\sigma = [(L^\tau, \mathcal{C}^\tau)_{\tau=0}^{a-1}, B_-, z^a]$.

Second, let $G_a(\sigma)$ denote the *social* value of firm type z^a, assuming that the firm takes as given previous worker cohorts L^τ and the precommitted separation rates as specified in contracts $\mathcal{C}^\tau$, $\tau < a$. For the contracts $(\mathcal{C}^{\tau *})_{\tau=0}^{a-1}$ in the candidate equilibrium (and the corresponding worker cohorts $L^{\tau *}$) write σ^* for the firm's state vector. We show that these contracts, together with the other socially optimal firm policies, indeed solve the recursive social firm problem with commitment. The recursive problem to maximize social firm value is

$$G_a(\sigma) = \max_{(\lambda,R,\mathcal{C}^a),(\varphi,S,\delta_b)} \Big\{ u'(C) y_a B - bL - r(R, L_0) - s(S, L_0) \\ - \rho[L + (\lambda - m(\lambda))R] - c[B + (\varphi - q(\varphi))S] + \beta(1-\delta)\mathbb{E}G_{a+1}(\sigma_+) \Big\} , \qquad (32)$$

subject to (7), (8), (10) and (11). Wages in contracts $\mathcal{C}^\tau$ are obviously irrelevant for that problem. The same policies that solve problem (20), and in particular contracts $\mathcal{C}^{a*}$ for all $a \geq 0$, also solve problem (32). The only difference between these two problems is that the firm is precommitted to separation rates for existing workers in the latter but not in the former problem. But since policies for the latter problem are time consistent, both problems have the same solutions.

Third, it remains to show that these policies not only solve problem (32) but that they also maximize the *private* value of the firm, as specified in the recursive problem (6)–(13), provided that $\rho^* = \rho$. Substitution of (13) shows that

$$u'(C) y_a B - c[B + (\varphi - q(\varphi))S] = p^R B_-(1 - \delta_b) + pq(\varphi)S \ .$$

Hence, the left-hand side of that term in problem (32) can be replaced by the right-hand side together with constraint (13). Further, we can write the social labor costs

$$bL + \rho[L + (\lambda - m(\lambda))R] = (b + \rho)L_0 + [b + \rho\frac{\lambda}{m(\lambda)}]m(\lambda)R \ , \qquad (33)$$

with L_0 denoting employment of workers in previous cohorts (as defined in (8)). Given the precommitted contracts $\mathcal{C}^{\tau*}$, $\tau < a$, the first term can be written

$$\begin{aligned}(b+\rho)L_0 &= \sum_{\tau=0}^{a-1}[1 - \delta_w^{\tau*}(z^a)]L^\tau \cdot (b + \rho) \\ &= \sum_{\tau=0}^{a-1}[1 - \delta_w^{\tau*}(z^a)]L^\tau \Big[w^{\tau*}(z^a) - [W(\mathcal{C}^{\tau*}, z^a) - U] + \beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U]\Big] \\ &= -\sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U] + \sum_{\tau=0}^{a-1} L^{\tau+} w^{\tau*}(z^a) + \beta(1-\delta)\mathbb{E}\sum_{\tau=0}^{a-1} L^{\tau+}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U] \ .\end{aligned}$$

For any contract $\mathcal{C}^a = (w^a(z^k), \delta_w^a(z^{k+1}))_{k \geq a}$ offered to new hires $m(\lambda)R = L^{a+}$, the second term in (33) can be written

$$\begin{aligned}[b + \rho\frac{\lambda}{m(\lambda)}]m(\lambda)R &= [W(\mathcal{C}^a, z^a) - \beta U]m(\lambda)R \\ &= w^a(z^a)L^{a+} + \beta(1-\delta)\mathbb{E}[W'(\mathcal{C}^a, z^{a+1}) - U]L^{a+} \ .\end{aligned}$$

Substituting these expressions into (32) at $\sigma = \sigma^*$ shows

$$G_a(\sigma^*) = \max_{(\lambda,R,\mathcal{C}^a),(\varphi,S,p,p^R,\delta_b)} \Bigg\{ p^R B_-(1-\delta_b) + pq(\varphi)S - W + \sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U] - r(R, L_0) - s(S, L_0) + \beta(1-\delta)\mathbb{E}\Big\{ G_{a+1}(\sigma_+^*) - \sum_{\tau=0}^{a-1} L^{\tau+}[W'(\mathcal{C}^{\tau*}, z^{a+1}) - U] - L^{a+}[W'(\mathcal{C}^a, z^{a+1}) - U] \Big\} \Bigg\} , \tag{34}$$

where maximization is subject to (8)–(13) with $\sigma_+^* = [(L^\tau, \mathcal{C}^{\tau*})_{\tau=0}^{a-1}, (L^{a+}, \mathcal{C}^a), B, z^{a+1}]$. In this maximization problem, the term $\sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^{\tau*}, z^a) - U]$ is predetermined and thus not subject to the maximization. Therefore, we can define the private firm value

$$J_a(\sigma) \equiv G_a(\sigma) - \sum_{\tau=0}^{a-1} L^\tau[W'(\mathcal{C}^\tau, z^a) - U] ,$$

i.e. the difference between the social value of firm z^a and the surplus value of the existing workers. Then problem (34) (at given state vector σ^*) is equivalent to problem (6). In particular, the firm policies λ, R, φ, S, p and p^R and $\mathcal{C}^{a*}$ that solve (34) also solve (6). Moreover, because of $G(0, 0, z) = J_0(0, z)$, socially optimal entry (21) implies the equilibrium condition (18). Since resource constraints are satisfied, the stationary planning solution gives rise to a stationary competitive search equilibrium. □

Wages

We show how to obtain wage schedules in the competitive search equilibrium. We distinguish between two cases: (i) All workers are paid the same wage within a firm (equal treatment); (ii) all workers are paid flat wages over time. In both cases, as in the social planning problem specified in the text, separation rates for all workers in a firm are assumed identical: $\delta_w^\tau(z^a) = \delta_w(z^a)$ where a is the age of the firm and τ is the worker cohort (the age of the firm when the worker was hired). Further, firms are able to commit to wage contracts.[31]

Equal Treatment

We first consider the decentralization for which each firm pays the same wage to all its workers, i.e. $w^\tau(z^a) = w(z^a)$ for all $\tau \leq a$. In this case, worker values W and W' do not depend on the

[31]Rudanko (2018) considers a model in which multi-worker firms apply an equal-treatment wage policy in the absence of commitment. The competitive-search equilibrium in this case is not efficient and it gives rise to endogenous wage rigidity.

particular contract $\mathcal{C}^a$ and can therefore be written $W(z^a)$ and $W'(z^a)$, so that (1)–(4) become

$$\begin{aligned} U &= \frac{m(\lambda(z^a))}{\lambda(z^a)} W(z^a) + \Big(1 - \frac{m(\lambda(z^a))}{\lambda(z^a)}\Big)[b + \beta U] \ , \\ W(z^a) &= w(z^a) + \beta(1-\delta)\mathbb{E}_{z^a} W'(z^{a+1}) + \beta\delta U \ , \\ W'(z^a) &= [1 - \delta_w(z^a)] W(z^a) + \delta_w(z^a) U \ , \\ U &= b + \rho + \beta U \ . \end{aligned}$$

These equations can be solved for the worker surplus

$$W(z^a) - U = \rho \Big[\frac{\lambda(z^a) - m(\lambda(z^a))}{m(\lambda(z^a))} \Big] \equiv S^w(z^a) \ ,$$

and for wages:

$$w(z^a) = b + \rho + S^w(z^a) - \beta(1-\delta)\mathbb{E}_{z^a} \Big([1 - \delta_w(z^{a+1})] S^w(z^{a+1}) \Big) \ . \tag{35}$$

Flat Wages

Consider now the case (which we use in the numeric experiments) where every worker is paid a flat wage over time: w_τ is the constant wage of a worker hired in a firm at age τ, and $W(w_\tau, z^a)$ denotes the worker's value in this firm at history z^a, for $a \geq \tau$. We have the Bellman equations

$$\begin{aligned} W(w_\tau, z^a) &= w_\tau + \beta(1-\delta)\mathbb{E}_{z^a} W'(w_\tau, z^{a+1}) + \beta\delta U \ , \\ W'(w_\tau, z^a) &= [1 - \delta_w(z^a)] W(w_\tau, z^a) + \delta_w(z^a) U \ , \\ U &= b + \rho + \beta U \ , \end{aligned}$$

from which we obtain

$$W(w_\tau, z^a) - U = w_\tau - b - \rho + \beta(1-\delta)\mathbb{E}_{z^a}(1 - \delta_w(z^{a+1})) \Big[W(w_\tau, z^{a+1}) - U \Big] \ .$$

Hence, $W(w_\tau, z^a) - U = A(z^a)(w_\tau - b - \rho)$ where $A(z^a)$ satisfies

$$A(z^a) = 1 + \beta(1-\delta)\mathbb{E}_{z^a}(1 - \delta_w(z^{a+1})) A(z^{a+1}) \ .$$

To solve for wages, note that for any wage offer w_τ in a firm of type z^τ,

$$\begin{aligned} \rho = \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)} \Big[W(w_\tau, z^\tau) - b - \beta U \Big] &= \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)} \Big[W(w_\tau, z^\tau) - U + \rho \Big] \\ &= \frac{m(\lambda(z^\tau))}{\lambda(z^\tau)} \Big[A(z^\tau)(w_\tau - b - \rho) + \rho \Big] \ . \end{aligned}$$

This yields flat wages offered to new hires in a firm of type z^τ:

$$w_\tau = b + \rho + \frac{\rho}{A(z^\tau)} \frac{\lambda(z^\tau) - m(\lambda(z^\tau))}{m(\lambda(z^\tau))} \ .$$

In the numeric implementation where we solve firm policies using the recursive problem (20) with firm state vector (L_-, B_-, z), we find $A(L_-, B_-, z)$ by value function iteration over

$$A(L_-, B_-, z) = 1 + \beta(1-\delta)\mathbb{E}_z(1 - \delta_w(L, B, z_+))A(L, B, z_+) \ ,$$

where $L = L(L_-, B_-, z)$ and $B = B(L_-, B_-, z)$ are firm policies. Flat wages for new hires are

$$w(L_-, B_-, z) = b + \rho + \frac{\rho}{A(L_-, B_-, z)} \frac{\lambda(L_-, B_-, z) - m(\lambda(L_-, B_-, z))}{m(\lambda(L_-, B_-, z))} \ .$$

Appendix B: Data

4.1 Further Details

Over the period 1995–2014 covered by the data, several things change. First, the classification of industry and product codes was changed, so we use conversion codes from one standard to another.[32] We remove all products that split or merge going from one classification standard to another, and keep all products with the same measurement units, e.g., kilogram, meter etc, which neither split nor merge between standards.[33] Regarding industry classifications, the standards are WZ 93 covering 1995–2002, WZ 2003 covering 2003–2008, and WZ 2008 covering 2009–2014. Since we use two-digit industry codes as controls for our descriptive statistics, we convert standards at this level to the WZ 2003 classification. The titles or descriptions for standards WZ 93 and WZ 2003 are identical, allowing for a perfect conversion. When bringing WZ 2008 to WZ 2003, titles for four industries in WZ 2008 have no reasonable counterpart in WZ 2003; likewise five industries from WZ 2008 cannot be matched to WZ 2008. These industries are then left as they are. Further, we pool some two industries together and as a result of cleaning some industries are dropped.

Second, for years up to 2002 hours are reported only for blue-collar workers, while hours are reported for all employees in all subsequent years. To deal with this, we impute hours worked by blue-collar workers on the other employees. Third, until 2006 all firms report hours, while from 2007 only firms with 50 or more workers report hours.

Establishments in the data produce 2.46 products on average, with about 10 percent producing only one product and 55 percent producing more than five products. Table 5 shows the percentage distribution of establishments, employment and revenue across two-digit industries. Observe that there are considerable size differences, with the largest establishments in the production of motor vehicles and the smallest establishments in recycling, repair and installation. Table 6 shows the percentage distribution of establishments, employment and revenue across four German regions, each of which comprises several federal states. Establishments are largest in the South of Germany which hosts several car producers, and they are smallest in Eastern Germany.

[32] See https://www.klassifikationsserver.de for these conversion codes. For the mapping of product classification GP 95 into GP 2002, the relevant document was downloaded from the internet. All these documents can be shared upon request.

[33] This is done only for years 2001 and 2002, where the standard changes from GP 95 to GP 2002, years 2008 and 2009, where the standard changes from GP 2002 to GP 2009, and years 2011 and 2012 where the standard changes from GP 2009 to GP 2009, Version12. We also harmonize product codes for the state of Mecklenburg-Vorpommern where GP 2002 in the year 2001 was used instead of GP 95 by applying the standard GP 95.

Table 5: Distribution of Establishments, Employment and Revenue Across Industries for the Pooled Sample in Percent

Industry	Establishments	Employment	Revenue
Extraction of crude petroleum and natural gas	0.04	0.02	0.10
Basic metals	3.33	4.71	5.74
Chemicals and chemical products	4.20	6.88	9.27
Coke, refined petroleum products and nuclear fuel	0.17	0.23	2.19
Electrical machinery and apparatus n.e.c.	5.23	6.83	5.30
Fabricated metal products	16.00	10.42	6.69
Food products and beverages	16.25	10.91	11.96
Furniture	5.22	4.54	3.39
Machinery and equipment n.e.c.	13.53	16.66	13.68
Medical, precision and optical instruments	2.50	2.50	1.73
Motor vehicles, trailers and semi-trailers	1.86	11.60	20.18
Office machinery and computers	0.23	0.31	0.66
Other non-metallic mineral products	7.11	4.41	2.91
Other transport equipment	0.34	0.73	0.72
Pulp, paper and paper products	3.22	3.01	3.18
Radio, television and communication equipment	0.79	1.41	1.46
Rubber and plastic products	9.51	7.68	5.51
Textiles	2.36	1.63	1.03
Tobacco products	0.08	0.17	0.38
Wearing apparel; dressing and dyeing of fur	1.42	1.02	0.41
Wood and of products of wood and cork	3.17	1.65	1.35
Mining of coal and lignite; extraction of peat	0.09	0.21	0.13
Other manufacturing	0.63	0.62	0.43
Other mining and quarrying	1.20	0.40	0.23
Publishing, printing and reproduction of media	1.24	1.34	1.23
Recycling	0.28	0.10	0.12
Repair and installation of machinery/equipment	0.03	0.01	0.01

Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

4.2 Productivity and Price Dispersion

For cross-sectional price and productivity variation, quality differences between firms are a bigger concern than for our analysis of within-firm price and output adjustments. Therefore

Table 6: Distribution of Establishments, Employment and Revenue across Regions for the Pooled Sample in Percent

Region	Establishments	Employment	Revenue
East	16.14	10.83	10.61
West	35.64	36.53	35.64
North	12.07	12.90	15.16
South	36.16	39.74	38.59

Note: German federal states are grouped into "East" (Berlin, Brandenburg, Mecklenburg-Vorpommern, Saxony, Saxony-Anhalt), "West" (Hesse, North Rhine-Westfalia, Rhineland-Palatinate and Saarland), "North" (Bremen, Hamburg, Lower Saxony and Schleswig-Holstein) and "South" (Baden-Wuerttemberg and Bavaria). Data source: Research Data Centers of the Federal Statistical Office and Statistical Offices of the Länder, panel *Industriebetriebe* and module *Produkte*, survey years 1995–2014, own calculations.

we consider here a subsample of *homogeneous* products which are measured in physical units of weight, length, area, or volume, whereas we remove all products which are measured in other units such as "items" or "pairs". The underlying hypothesis is that products measured in physical units have a lower degree of processing, so that quality differences are less important.[34] We further remove all products which are produced by less than six establishments in order to be able to compute a meaningful average price for each product. As for our analysis of dynamics, we restrict the sample to those establishments whose valid products make at least 50% of the total revenue, as in Foster et al. (2008). We further follow their procedure and adjust proportionally the revenues (and quantities) of the establishment's products so that the sum of revenues in the relevant product sample equals the total revenue of the establishment. This adjustment is a valid modification of the data if the goods belonging to the homogeneous product sample are sufficiently representative for the set of all goods that this firm produces. Consider a given year and let I denote the set of establishments in this year and J the set of products after the above cleaning procedure. Let R_{ij} and Q_{ij} be the revenue and quantity values of product j in establishment i. Both these measured are possibly scaled up by the same factor to ensure that total sample revenue $\sum_j R_{ij}$ equals the total actual revenue of establishment i. As before define establishment i's price of product j by $P_{ij} = R_{ij}/Q_{ij}$. Then

[34] To give examples, this reduced sample includes products "1720 32 144: Fabric of synthetic fibers (with more than 85% synthetic) for curtains (measured in m^2)" and "2112 30 200: Cigarette paper, not in the form of booklets, husks, or rolls less than 5 cm broad (measured in t)", whereas it does not include "1740 24 300: Sleeping bags (measured in 'items')" and "2513 60 550: Gloves made of vulcanized rubber for housework usage (measured in 'pairs')" (numeric codes based on product classification 2002).

define the quantity-weighted average price of good j in the given year as

$$\bar{P}_j = \frac{\sum_i P_{it} Q_{ij}}{\sum_i Q_{ij}} \ .$$

Recall that summation if over at least six establishments for any valid product that we consider. Define the *firm-specific price index*:

$$\widetilde{P}_i = \frac{\sum_j P_{ij} Q_{ij}}{\sum \bar{P}_j Q_{ij}} \ . \tag{36}$$

This index expresses the firm's total revenue relative to the hypothetical revenue had the firm sold its products at the (quantity-weighted) average market prices.[35] It is a measure of the relative expensiveness of the firm in comparison to other firms producing the same products. Define revenue and quantity labor productivity of establishment i:

$$\mathrm{RLP}_i = \frac{\sum_j Q_{ij} P_{ij}}{H_i} \quad \text{and} \quad \mathrm{QLP}_i = \frac{\sum_j Q_{ij} \bar{P}_j}{H_i} \ ,$$

where H_i are working hours at establishment i. Then revenue labor productivity can be split into quantity productivity and the firm's price level:

$$\mathrm{RLP}_i = \mathrm{QLP}_i \cdot \widetilde{P}_i \ . \tag{37}$$

In other words, log revenue productivity is the sum of log quantity productivity and the logged firm-specific price index. We drop as outliers all log firm-specific price indices and log quantity labor productivity beyond the 2nd and 98th percentiles to arrive at $289,696$ establishment-year observations in the pooled sample.

[35] This concept is analogous to the construction of household-level price indices in Kaplan and Menzio (2015).