Does Level- k Behavior Imply Level- k Thinking?

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Abstract

The level-k literature classifies subjects into different Lk types by their observed levels of reasoning in games. However, it remains unclear whether the observed level-k behavior is determined by belief or reasoning ability. This study proposes a strategy to identify the ability-bounded Lk subjects, who could not reason more than k steps of iterated best responses and thus have reached the upper bound of their reasoning capacity. The identification utilizes a combination of simultaneous and sequential ring games. In the sequential games, it requires an extra reasoning step to best respond as Lk , and thus the ability-bounded ones would fail the task. Results show that more than half of the L2 and L3 subjects are ability-bounded. Additionally, subjects' CRT scores, a measure of their cognitive ability, support the separation of the these types. The findings suggest that both belief and reasoning ability could determine the observed levels, and thus one must be cautious when trying to infer belief or ability from the existing level-k data.

Keywords: behavioral game theory, level- k , high-order belief, bounded rationality JEL Codes: C70, C91

1 Introduction

The level-k theory is proposed to model player's systematic deviation from Nash equilibrium by allowing for inconsistent beliefs (Nagel, 1995; Stahl and Wilson, 1994, 1995; Ho et al., 1998). The model starts with the irrational or non-strategic L0 type, who is usually assumed to choose randomly or use certain salient strategies. L1 acts as if all the rest of the players are L0 and each higher Lk best responds to $L(k-1)$. The level-k model has been used extensively to explain non-equilibrium behavior from labs and fields (See Crawford et al. (2013) for a detailed review).

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Experimental literature on the level-k model mostly focuses on the identification of Lk types by looking at the number of reasoning levels used by the subjects. Most subjects in these experiments appear to be using no more than two or three levels of reasoning when playing a game for the first time. The natural question then is, why do they stop at these low levels? What can be inferred about people's reasoning process and reasoning ability in general?

To tackle this question, we need to understand what determines the observed levels. The original level-k model assumes that a subject using k levels of reasoning is responding to his belief that the opponent reasons $k-1$ levels. Since only belief levels enter the story, it is implicitly assumed that subjects' ability is never binding. In reality, however, we cannot rule out the possibility that the observed Lk behavior might result from bounded reasoning ability. In this alternative story, even if the subject might be aware that the opponents are more sophisticated than $k-1$ levels, his limited ability prevents him from developing too many thinking steps. In the latter case, observed Lk behavior might not reflect the subject's assessment of the opponent's rationality, but, instead, reflects his own cognitive limit.

Understanding ability bounds is relevant to a lot of research questions. For example, researchers are interested in using people's reasoning ability in one game to predict their performance in other games (Georganas et al., 2015) or the economic outcomes in the real world. Currently, these studies use observed levels as proxies of reasoning abilities, but more ideal analysis would be based on the identified ability bounds (perhaps together with identified beliefs). In addition, there are larger heterogeneities among the players from the real world than in the laboratories. Data on people's belief and ability bounds could help us predict their behavior when faced with more seasoned opponents, a situation always encountered in reality.

Evidence on belief and ability bounds could also add to our understanding of how people form beliefs. Do they form belief based on their own abilities? Or, are belief levels far below the subjects' ability bounds? In fact, previous studies that explore how levels are endogenously determined (Alaoui and Panta, 2016) and the correlation of levels between different games (Georganas et al., 2015) both apply models that distinguish between belief and reasoning ability. Therefore, separation of the two decisive factors could shed light on a more predictive theory stemming from the level-k model.

Nevertheless, since belief and ability are not directly observable from choice data, existing literature provides little evidence on cleanly identified ability bounds. This study attempts to identify the *ability-bounded* Lk subjects: those who behave as Lk and are not able to reason beyond k steps of best responses. The identification utilizes a combination of simultaneous and sequential ring games. The ring games are first studied in the innovative work of Kneeland (2015). She uses a set of simultaneous ring games to separate different orders of rationality. Here I provide an illustration of the separation of ability-bounded L2 subjects with 3-player ring games.

Let us first consider a 3-player simultaneous ring game G (Figure 1). Player 1's (P1) payoff depends on his and Player 2's (P2) actions. Player 2's payoff depends on his and Player 3's (P3)

actions. Player 3's payoff depends on his and Player 1's actions. In particular, Player 3 has a dominant strategy. Players 1 and 2's iterated dominant strategies could be derived from Player 3's best response, and thus the unique surviving profile $(b1, a2, b3)$ could be solved through three rounds of iterated dominance.

Note: In G , the three players move simultaneously. In G' , players 1 and 3 move in the first stage. Player 2 moves in the second stage after observing players 1's and 3's choices.

Figure 1: G and G' : 3-player ring games

Consider a subject Ann who consistently chooses the (iterated) dominant strategies as Players 2 and 3, but not as Player 1. She would be classified as L2. However, it remains unclear whether her failure as Player 1 is due to her inability to reason iteratively to Charlie's problem, or due to her lack of confidence that Bob would respond to Charlie's optimal choice. The two stories have exactly the same behavioral implication in this game, which is also the case in most games in the existing literature. Therefore, it is often hard to tell whether ability is binding for a subject.

Suppose we could somehow manipulate Ann's belief, assuring her that Bob would respond to Charlie's rationality. With this belief, if she still does not take Charlie's move into consideration, we could conclude that her ability is binding. In the existing literature, this idea is mostly implemented by telling Ann that Bob is an experienced player, or that Bob is strongly incentivized. However, Ann's interpretation of such messages might vary across individuals and thus cannot be cleanly controlled.

This study proposes a different approach. Instead of Bob's sophistication, Ann is informed that Bob would first observe everybody else's action before making his own move. By making this into a two-stage sequential game G' , an $L2$ Ann should believe that Bob would best respond to whatever he observes from the first stage. What remains is whether Ann has the ability to understand the strategic situation in this game, and solve the 3 (iterated) best response problems through the same reasoning process as in the simultaneous game. If she still fails to best respond as Player 1 in the sequential game, then it could not be because of her belief: she could be identified as ability-bounded L2.

If the L2 Ann best responds as Player 1 in the sequential game, it suggests that she is able to understand the strategic relationship between Bob and Charlie, but does believe that Bob is 2nd-order rational in the simultaneous game. Therefore, she is classified as the *belief-driven* L2.

Note that cautions should be taken when interpreting the ability-bound defined here and hence the separation of the two types. As described in the level-k theory, to develop one additional level of reasoning, a player needs to both understand the strategic relationship among the players, and to have the ability to conceive of "what others think", or to develop a Theory of Mind (ToM). The ability bound identified in this study concerns the understanding of the strategic situation and the search of best responses, but not the maximum levels of ToM. For instance, it is possible that Ann understands how Bob responds to Charlie when she knows that Bob would observe Charlie's action, but has difficulty in conceiving of how Bob thinks about Charlie when all of them are playing simultaneously. The latter requires one more layer of ToM about Bob's theory of Charlie. But, of course, the ability of ToM in a particular game is related to the understanding of strategic situation. If Ann does not even understand that she needs to find out Bob's response to Charlie, she certainly would not form that level of ToM. So the ability bound here actually provides an upper bound of the subject's ToM-ability-bound (discussed in more detail in ??). In this sense, this study provides a lower bound of the proportion of ToM-ability-bounded subjects.

The actual experiment uses 4-player games, which identifies the ability-bounded types from L2 and L3 subjects. A total of 184 subjects participated in the experiment, and enough observations were collected from 179 of them to perform the analysis. There are 50 and 39 subjects classified as L2 or L3 respectively in the simultaneous games. More than half of these subjects failed to perform the extra step in the sequential game, suggesting that a considerable number of subjects are bounded by their ability. I then perform a subject-by-subject type classification (Figure 2). Out of the 50 $L2$ subjects, 21 are identified to be ability-bounded $L2$, and 21 are belief-driven $L2$ (the remaining 8 subjects are classified as $L1$, $L3$ or unidentified). Out of the 39 L3 subjects, 20 are classified as ability-bounded L3 and 15 as belief-driven L3 (the remaining 4 subjects are classified as L0 or unidentified). The results show that the subjects' rationality levels and ability bounds are mostly consistent throughout the experiment, and that over half of the observed L2 and L3 subjects have reached their upper boundaries of reasoning.

To help better understand the correlation between cognitive ability and observed levels, the Cognitive Reflection Test (CRT) scores were collected from the participants after the main part of the experiment. The CRT score is a measure of cognitive ability in decision making. I find that the CRT scores increase with levels among the lower types, and then flatten out. Especially, I find that among the L2 subjects, the belief-driven ones performed much better in CRT than the ability-bounded ones. But among the players who behave as L3 or higher, the differences in CRT scores are not significant.

Overall, I find the existence of both the high ability subjects responding to lower-order belief and the low ability subjects who have reached their cognitive limit when behaving as $L2$ or $L3$, which demonstrates large heterogeneity in subjects' reasoning ability. The results suggest that

Figure 2: Decomposition of L2 and L3 subjects

both subjects' belief and their reasoning ability could be the decisive factors of their exhibited levels. The CRT scores support the separation of the ability-bounded and belief-driven subjects.

The remainder of the paper proceeds as follows. The next section summarizes related literature. The game, experimental design, and identification strategies are presented in ?? and ??. Section 5 is the experimental procedure. Section 6 reports the experimental results. Section 7 concludes.

2 Related Literature

The recent two decades saw the emergence of a vast literature on the level- k model. This section summarizes only the studies that are closest to this paper. The simultaneous ring games used in this study are similar to the ones proposed by Kneeland (2015). Her methodology and other studies on ring games will be discussed in detail in the following sections, and hence are not included here. There is also a class of literature assuming that Lk players believe that the opponents are drawn from a distribution of all the lower types, e.g. the Cognitive Hierarchy model from Camerer et al. (2004). Since the games in this study do not distinguish between the level-k model and the Cognitive Hierarchy model, players in this study are assumed here to have degenerate belief, as in the original level-k model.

There are a few studies looking at subjects' reaction to the information about the opponents' types. Given the information that the opponents are higher types, the high ability subjects should be able to respond by raising their own levels, while the ability-bounded ones are not able to adjust their behavior. Some studies find positive results at aggregate level, suggesting that at least a large proportion of the subjects' ability limits are not binding (For examples, see Palacios-Huerta and Volji, 2009; Agranov et al., 2012; Slonim, 2005; Alaoui and Panta, 2016).

If the heterogeneity is explored in more detail, however, the results seem to be mixed. In Gill and Prowse (2015), who use repeated beauty contest games, the higher cognitive ability subjects respond positively to the cognitive abilities of their opponents, while the lower cognitive ability subjects do not. In addition, Georganas et al. (2015) find similar effects in their undercutting games, but not in the two-player guessing games. Alaoui and Panta (2017) also find the evidence of ability-bounded subjects.

A more direct way to identify whether subjects are best responding to their belief is to elicit their belief. Belief elicitation in strategic games has been studied in numerous papers. However, evidence is mixed on whether beliefs could be successfully elicited without altering behavior and whether subjects do act according to their stated beliefs. Some studies find that subjects' actual play in the games is not affected after belief elicitation (Nyarko and Schotter (2002), Costa-Gomes and Weizsacker (2008), Manski and Neri (2013)), while both Ruststrom and Wilcox (2009) and Gachter and Renner (2010) find that incentivized elicitation alters choices. In both Nyarko and Schotter (2002) and Manski and Neri (2013), most subjects appear to be best responding to their first-order belief. However, in Bhatt and Camerer (2005), only 66% of the choices match with the stated first-order belief. Costa-Gomes and Weizsacker (2008) also find that most choices are one step below the stated first-order belief. One possible explanation of the occasional failure of belief elicitation could be the hedging behavior of risk-averse subjects, as discussed in Kadane and Winkler (1988), Blanco et al. (2010) and Armantier and Treich (2013).

Additional strategies are developed to infer beliefs from observed levels. Arad and Rubinstein (2012) use a very simple undercutting game, but still find that subjects mostly use no more than three levels of reasoning. They conclude that this could not be due to obstacles in thinking, and thus must be due to non-equilibrium beliefs. Other methods include tracking players' information search (Costa-Gomes et al. (2001), Costa-Gomes and Crawford (2006) and Brocas et al. (2014)), translating their beliefs from communication records between players (Burchardi and Penczynski, 2014), or providing tutorials to reduce the reasoning cost (the *Tutorial Method* in Alaoui and Panta (2017)). Bhatt and Camerer (2005) use brain image to search for the connection between the brain activities of making choices and stating belief.

The mixed findings in the previous studies, most of which are on aggregate levels, suggest heterogeneity in the population. Thus, this study seeks to identify players' belief and cognitive ability bounds at the individual level. However, the nature of some existing techniques, such as belief manipulation, makes it very hard to implement a within-subjects design, while beliefelicitation might affect the reasoning levels. This paper employs a novel identification strategy that only requires choice data, which circumvents the challenge from belief manipulation or belief elicitation and could easily achieve a within-subjects design.

Friedenberg et al. (2018) also explore the relationship between belief levels and reasoning ability in a ring game setting, but they adopt a different definition of reasoning ability. Friedenberg et al.'s (2018) "strategic bound" is defined as the levels a player forms theory of, but the theories might not be rational, and their "ability bound" is the maximum levels a player is able to conceive of, but does not necessarily best respond. For example, a player could believe that the opponent is using an irrational decision rule and thus best respond to it. They look into the difference between reasoning about rationality and strategic reasoning, and find that 47% of their subjects have strictly lower rationality bounds than strategic bounds.

Besides which, this paper finds positive correlation between CRT scores and observed levels, which extends the literature that explores the relationship between cognitive ability and strategic thinking. One of the closely related studies is Gill and Prowse (2015), who use the Raven's test as a measure of cognitive ability and find that the high ability subjects converge to equilibrium more quickly and earn more. Another related study, Georganas et al. (2015), uses four tests, including the CRT. They find that none of the test scores is a good prediction of observed levels, but CRT scores could predict earnings. In addition, Rydval et al. (2009) find that higher working memory, need for cognition, and premeditation are associated with a higher likelihood of obeying dominance. Works on the Beauty Contest Games also show that the subjects who score high in the CRT, or other tests of cognitive ability, play the strategies closer to equilibrium (Burnham et al., 2009; Schnusenberg and Gallo, 2011; Branas-Garza et al., 2012).

3 The Ring Games

The main part of the experiment consists of three sets of 4-player ring games, which are named as SIMUL (the simultaneous games), SEQ-P2 and SEQ-P3 (the sequential games, see Figure 3). The SIMUL set contains two ring games, G1 and G2, with simultaneous moves. The two sets of sequential games are labeled by second-movers' positions. In the games of SEQ-P2, G3 and G4, Player 2 is the second-mover. Players 1, 3 and 4 move simultaneously in the first stage and Player 2 moves in the second stage after observing their actions. In the SEQ-P3 games, G5 and G6, Player 3 moves in the second stage. The sequence of moves and the information structure are common knowledge among all the players.

Each player has three actions to choose from, and the payoffs are represented by 3×3 matrices. Player 1's (P1) payoff is determined jointly by his own action and Player 2's (P2) action. Player 2's payoff is determined by his and Player 3's (P3) actions. Player 3's payoff depends on his and Player 4's (P4) actions. Player 4's payoff depends on his and Player 1's actions.

It would be useful to define the opponent system in the ring games as follows. In a simultaneous or sequential ring game, define the *first direct opponent* of a player to be the opponent whose action would affect his payoff. Therefore, the first direct opponent of Player i is Player $(i + 1)$ for $t = 1, 2, 3$ and is Player 1 for $i = 4$. Then the kth direct opponent of a player could be defined to be the opponent whose action would affect his $(k-1)$ th direct opponent's payoff. In the ring games, the set of kth direct opponent of a player is a singleton for all k .

In each of these ring games, Player 4 always has a strictly dominant strategy. Therefore

Figure 3: The ring games

in each game there is a unique strategy profile that survives Iterated Elimination of Strictly Dominated Strategies (IESDS) in simultaneous games, or Iterated Conditional Dominance (ICD) in sequential games.

In addition, in each pair of rings (G1 and G2, G3 and G4, G5 and G6), the payoff matrices of the two Players 1, 2 and 3 positions are identical. The two Player 4 positions have the same three strategies, but the strategies are labeled in different orders. This feature, combined with the dominant strategies at Player 4 positions, is designed specifically for Lk type classification, which will be discussed in the subsequent subsections.

4 Cognitive Types and Identification Strategy

In this subsection, I first propose a model in which a player's type could be characterized by both his cognitive ability and his belief about the opponents' types. Similar assumptions could be found in Alaoui and Panta (2016), Choi (2012), Strzalecki (2014) and Georganas et al. (2015). Then the identification strategy will be discussed.

4.1 Reasoning Steps and Ability Bound

A player using one step of reasoning considers only his own situation, and lacks the strategic thinking that other players are also reasoning and best responding. In the ring games, such a player focuses on his own payoff matrix. He will be able to identify the dominated strategies if he has one, but will not pay attention to the dominated strategies in others' matrices.

To perform one more step of reasoning, the player needs to realize that he should try to figure out the action of his first direct opponent. The next step would be to to understand how the first direct opponent's action depends on the 2nd direct opponent's action, and so on. Each extra step of reasoning corresponds to the inspection of an additional payoff matrix and the calculation of a best response.

Players are assumed to be cognitively constrained. In a certain game, the maximum number of reasoning steps player i could perform, or his ability bound, is denoted by a_i . a_i reflects the player's cognitive ability. As suggested by Alaoui and Panta (2016) , a_i might depend on the incentives of the game. But since the incentive level is not varied in this experiment, I do not consider cost-benefit analysis, and use a_i alone to capture the player's cognitive cost.

Relationship with Levels of Theory of Mind

The ability bound here is defined as the maximum number of best responses the player is able to reason through along the ring. In simultaneous games, number of reasoning steps coincides with the levels of Theory of Mind (ToM) one needs to develop, but the number of reason steps and the levels of ToM are different in sequential games. To see this, consider game G' in the illustrative example. As Player 1, Ann needs to understand and compute 3 steps of best responses, but only needs to form 2 levels of ToM. Compared with the simultaneous game, she doesn't need to conceive of what Bob thinks about Charlie, though she still needs to think how Bob responds to Charlie's action. Therefore, the ability bound studied in this paper is about the ability to understand a strategic situation and search for the best responses, but not about the upper limit of ToM ability.

That said, I am not suggesting that the computation of best responses in this game is the major challenge for the subjects. In fact, over 95% of the subjects successfully identify the dominant strategies and respond correctly as second movers, so it does not appear to be a hard task. The cognitive load lies in their attempt to understand how an opponent's action depends on other players and to make connection of their rules of play.

Define the ToM-ability-bound as the maximum levels of iterated best responses one is able to conceive of in the simultaneous game. Then having a ToM-ability-bound of k implies that the player is able to form k levels of ToM and reason through k best responses along the ring¹. So a player's ability-bound a_i is weakly higher than his ToM-ability-bound. In this sense, this study provides a lower bound of the proportion of ToM ability-bounded subjects.

Relationship with Rounds of Iterated Conditional Dominance

The relationship between the reasoning steps in sequential games and sequential rationality, or rounds of iterated conditional dominance (ICD) (Battigalli and Siniscalchi, 2002; Shimoji and Watson, 1998), is worth mentioning. The steps of reasoning defined here do not coincide with the rounds of ICD in the sequential ring games. For example, in G' , Ann (as Player 1) will be able to identify the iterated dominated strategies in her own payoff matrix by performing only two rounds of ICD². However, this implies that she understands that Bob is best responding, and that Charlie is also acting optimally, which requires that she reasons through three best responses.

4.2 Identifying Lk Behavior in Simultaneous Ring Games

Lk Types in Simultaneous Ring Games

In a simultaneous ring game, Player i's cognitive type $t_i \in T_i$ is characterized by (a_i, b_i) , where $a_i \in \{0, 1, 2, ...\}$ is the ability bound, and $b_i \in \Delta(T_{-i})$ represents the player's belief about his opponents' types. For simplicity, a player is assumed to hold the same belief about all 3 opponents.

¹The ToM-ability-bound is specified as the ability to form ToM and best respond, which is regarded as one "level" in the level-k literature. It is emphasized here because, as pointed out by Friedenberg et al. (2018), a player might form one level of ToM that is not a best response. Alaoui and Panta (2017) provide more detailed discussion.

²In the first round of ICD, Ann would eliminate the (conditionally) dominated strategies in all the information sets of the game, which means that she identifies Charlie's dominated strategies and Bob's (second-mover) dominated strategies in the three information sets where Bob is active. In the second round, with the reduced game, Ann would identify the iterated dominated strategies in her own matrix.

This study only considers degenerate beliefs, which restricts that $b_i \in T_{-i}$.

In this study, the cognitively-constrained types are defined as the ones who has ability bound a_i , and holds the belief that the opponents are $(k-1)$ th-order rational, but not ability-bounded. This assumption simplifies the notations, for players' high-order beliefs are only defined on the opponents' beliefs, but not ability bounds. It also follows closely the level-k literature, as the players are assumed to have the "level-k" types of beliefs³. A test of the validity of the assumption in the 4-player ring games will be presented in the results section.

Let's first define the cognitively-unbounded types with k th-order rationality, R_i^k , and then the observed Lk types with cognitive constraints could be defined based on the R_i^k types.

The irrational type is characterized by the set

$$
R_i^0 = \{(a_i, b_i) | a_i = 0, b_i = \emptyset\}.
$$

This type does not engage in any optimization and serves mainly as the anchoring point of analysis.

For any $k = 1, 2, 3, 4$, define the types whose behaviors depend only on their orders of rationality as

$$
R_i^k = \{(a_i, b_i) | a_i > 4, b_i \in R_{-i}^{k-1}\}.
$$

The player's cognitive limit is assumed to be greater than 4, which is the maximum reasoning steps required to understand and solve the whole game. Therefore, cognitive limit never plays a role in determining the R_i^k type's behavior.

For a player with an ability bound that might be binding, exhibiting Lk behavior in the simultaneous games implies having kth-order rationality, and being able to perform k steps of reasoning. Therefore, the observed Lk types could be defined as

$$
Lk_i = \{(a_i, b_i)|a_i \ge k, b_i \in R_{-i}^{k-1}\}.
$$

Identification of Lk Behavior

Subjects' choices in the simultaneous ring games could be used to infer their reasoning steps. That is, each subject could be put in to one of the Lk_i sets, where $k = 0, 1, 2, 3, 4$ represents the steps of reasoning.

An Lk player who uses k steps of thinking reasons through his own problem, his first direct opponent's problem, his second direct opponent's problem, and so on. The kth step would be the $(k-1)$ th direct opponent's decision problem. He stops here and will not examine the kth direct opponent's strategies. This leads to the first identifying assumption:

³The distinction between beliefs in rationality and beliefs in ability might be nontrivial in more complicated games and might worth further investigation. For example, Alaoui and Panta (2017) find that subjects respond to the change in the opponents' incentives.

A1: A player using k steps of reasoning does not respond to the changes in mth direct opponent's payoff, for all $m \geq k$.

A1 is developed from the **exclusion restriction** (ER) in Kneeland (2015) and is modified so that it could be applied to both simultaneous and sequential ring games⁴. Al implies that a player's choice will not be affected by the changes in a distant opponent's decision situation, if he is not reasoning all the way to that opponent.

The identification strategy works as follows. In G1 and G2, the payoff matrices are identical in the Players 1, 2 and 3 positions respectively. The only difference is in the payoff matrices of Player 4. Hence A1 implies that an L1 type, who reasons one step in the simultaneous games, chooses the dominant strategies as Player 4 but makes the same choice in both rings as Players 1, 2 and 3. The behavior pattern of other Lk types could also be predicted. An $L2$ subject makes the same choices as Players 1 and 2 in both G1 and G2, and chooses the (iterated) dominant strategies as Players 3 and 4. An L3 subject makes the same choice at the two Player 1 positions and chooses the (iterated) dominant strategies as Players 2, 3 and 4. L4 or higher types choose the (iterated) dominant strategies at all player positions.

It should be noted that both $A1$ and ER from Kneeland (2015) imply that a player always chooses the same action when presented with the same strategic decision problem. If this assumption does not hold, it could cause large noise in type classification. See Lim and Xiong (2016) and Jin (2016) for detailed discussions.

Another identifying assumption commonly used in the literature is that L0 chooses all the possible actions with equal probability, and solves higher types' best responses recursively. The predicted action profiles of this assumption are special cases of that of A1. The type classification using the uniformly randomizing L0 assumption appears to be less robust among lower types, but the pattern in the main result holds for higher types. The more detailed analysis and results will be reported in Appendix C.2.

4.3 Identifying Ability-bounded Subjects with Sequential Games

For any subject identified as Lk , his behavior in the sequential games could be used to identify whether he is ability-bounded. That is, whether this subject has $a_i = k$ or $a_i > k$.

Cognitive Types in Sequential Ring Games

In the sequential ring games, a player *i*'s type is characterized by $t'_{i} = (a'_{i1}, a'_{i2}, b'_{i1}, b'_{i2}) \in T'_{i}$, where $a'_{i1} \in \{0, 1, 2, ...\}$ and $a'_{i2} \in \{0, 1\}$ are the player's the ability bounds as first-movers and secondmovers respectively, and $b'_{i1}, b'_{i2} \in T'^k_{-i}$ represents the player's belief about his opponents' types

⁴The assumption ER in Kneeland (2015) says that a player with k th- but not higher-order belief in rationality does not respond to the changes in $(k+1)$ th- or higher-order opponents' payoffs.

as first-movers and as second-movers⁵. Though I do not use the terms of sequential rationality, such as kth-order strong belief in rationality, b'_{i1} and b'_{i2} are capturing player i's belief about the opponents who are active in different information sets.

Again, let's first define the types, $R_i^{\prime k}$, whose ability bounds never bind and thus their behavior depends only on the orders of rationality:

$$
R_i'^0 = \{ (a'_{i1}, a'_{i2}, b'_{i1}, b'_{i2}) | a'_{i1} = a'_{i2} = 0, b'_{i1}, b'_{i2} = \emptyset \},\,
$$

and for $k = 1, 2, 3, ...,$

$$
R_i^{\prime k} = \{ (a'_{i1}, a'_{i2}, b'_{i1}, b'_{i2}) | a'_{i1} > 4, a'_{i2} = 1, b'_{i1} \in R_{-i}^{\prime k-1}, b'_{i2} \in R_{-i}^{\prime \min\{k-1, 1\}} \}.
$$

Then, for $k = 1, 2, 3, 4$, define the ability-bounded Lk types as

$$
Lka_i = \{ (a'_{i1}, a'_{i2}, b'_{i1}, b'_{i2}) | a'_{i1} = k, a'_{i2} = 1, b'_{i1} \in R'^{k-1}_{-i}, b'_{i2} \in R'^{\min\{k-1,1\}}_{-i} \},
$$

and the belief-driven Lk types as

$$
Lkb_i = \{ (a'_{i1}, a'_{i2}, b'_{i1}, b'_{i2}) | a'_{i1} > k, a'_{i2} = 1, b'_{i1} \in R'^{k-1}_{-i}, b'_{i2} \in R'^{\min\{k-1,1\}}_{-i} \}.
$$

Note that an Lka player is allowed to hold the belief that requires more reasoning steps than his ability bound a'_{i1} to respond. To see this, consider G' in the illustrative example, and assume that Ann has $a'_{i1} = 2$, and belief $b'_{i1}, b'_{i2} \in R'^{1}_{-i}$. Even though she believes that both Bob and Charlie are rational, she is not able to finish all the reasoning steps to best respond as Player 1. In this case, the player is not able to form a concrete idea of the actions the opponents might take, due to his cognitive limit. Rather, he has an abstract belief of the sophistication level of the opponents. Although such situations occur frequently in everyday lives, there are few theories on people's strategy against obviously stronger opponents. It is assumed that when a player faces a more sophisticated opponent, he would perform the maximum reasoning steps as he could. That is, he would reason up to a'_{i1} steps. A similar assumption is used in Alaoui and Panta (2016).

Assumptions on the Similarity of Simultaneous and Sequential Games

The sequential ring games are framed in such a way that the path of reasoning stays the same as in the simultaneous games. More specifically, no matter whether there is a second-mover, the reasoning path always naturally follows as the players' own problem, the first direct opponent's problem, the second direct opponent's problem, and the third direct opponent's problem. The

⁵Note that even Player *i* is a first-mover position, an a'_{i2} is still assigned as a character to this player, and vice versa for the second-mover position. Therefore such a type characterization could be used on all player positions, which makes it more convenient to define higher-order beliefs.

identification strategy of this study relies on this similarity, so that meaningful comparisons could be made between subjects' behavior in simultaneous and sequential games.

To make it clearer, I state three assumptions on the relationship between a subject's types in the simultaneous and sequential ring games.

A2: For each subject, $a_i = a'_{i1}$, and $a_i \ge 1$ implies $a'_{i2} = 1$, $\forall i$. That is, a subject's ability bound stays the same in simultaneous and sequential ring games.

As a first-mover, one needs to go through the same thought process to search for best responses in both the simultaneous and sequential games. It is assumed in A2 that a subject's ability bound should not change as a first-mover. As for the second-mover positions, it is assumed that if the subject behaves as at least L1 in the simultaneous games, he will be able to best respond as a second-mover in the sequential games.

Due to the same argument, a subject's belief in other first-movers' rationality should not change either.

A3: $b_i \in R_i^k$ if and only if $b'_{i1} \in R_i^k$. That is, a subject's belief hierarchy about their opponents' rationality as first-movers stays the same in simultaneous and sequential ring games.

As for the second-movers in sequential games, although it also takes one step of reasoning to best respond, the task complexity is not exactly the same as the one step reasoning in the simultaneous game. Thus, I only assume that the task for the second-mover is easier, if not the same, than the task for one reasoning step in the simultaneous games.

A4: If $b_i \in R_i^k$ and $k \ge 1$, then it must be that $b'_{i2} \in R_i'^1$.

Combining A3 and A4, it implies that if a subject's kth-order belief says that the opponents is at least 1st-order rational in the simultaneous games, then in his kth-order belief the opponents are capable of best responding as the second-movers in the sequential games.

There are some concerns that reasoning in sequential games is easier than in simultaneous games. For example, if one is facing the second-mover as his first direct opponent, it is more likely to induce him to start examining his opponent's situation. In this regard, the ability bound might be higher in sequential games for some subjects. If this is true, then it implies that this study is identifying an lower bound of the proportion of the ability-bounded subjects. The tests on whether ability bounds stay the same across different games will be presented in the results part.

Identifying Ability-bounded Subjects

With these three assumptions, it could be predicted that each subject identified as Lk in the simultaneous games could behave as either as Lka or Lkb in the sequential games. The behavioral

patterns of all the Lka and Lkb types could be predicted, as shown in Table 3. In particular, if a player does not reason enough steps so that he could identify the iterated dominant strategy at a certain position, by assumption A1, he chooses the same action at this position in that pair of games (the SEQ-P2 pair or the SEQ-P3 pair).

		Ρ1	P ₂	P ₃	P ₄			P ₁	P ₂	P3	P ₄
L1b	SEQ-P2	\times	2nd	\times	$\sqrt{ }$	L1a	SEQ-P2	\times	2nd	\times	$\sqrt{ }$
	SEQ-P3	\times	\times	2nd	$\sqrt{ }$		SEQ-P3	\times	\times	2nd	$\sqrt{ }$
L2b	SEQ-P2	\times	2nd		$\sqrt{ }$	L2a	SEQ-P2	\times	2nd		$\sqrt{ }$
	SEQ-P3	\times	$\sqrt{ }$	2nd	$\sqrt{ }$		SEQ-P3	\times	\times	2nd	$\sqrt{ }$
L3b	SEQ-P2	$\sqrt{ }$	2nd			L3a	SEQ-P2	\times	2nd		
	$\rm SEQ\text{-}P3$		$\sqrt{ }$	2nd	$\sqrt{ }$		SEQ-P3	\times	$\sqrt{ }$	2nd	$\sqrt{ }$
L4b	SEQ-P2		2nd		$\sqrt{ }$	L4a	SEQ-P2	$\sqrt{ }$	2nd		
	SEQ-P3			2nd			SEQ-P3			2nd	$\sqrt{ }$

Note: [√] denotes choosing the (iterated) dominant strategy at this position, × otherwise. If a player does not identify the (iterated) dominant strategy at a certain position (e.g. P1 of SEQ-P2), then according to assumption "A1", he chooses the same action at this position in the two games (e.g. he chooses the same action at P1 in the two games of SEQ-P2).

"2nd" denotes second-movers. All the types but L0 should play the dominant strategies in the subgame as secondmovers.

(*): Belief-driven L3 plays the iterated dominant strategy only if he believes that the opponents are 2nd-order rational and not ability-bounded.

Table 1: Predicted action profiles of each Lka and Lkb type

 $L2a$ and $L2b$ could be separated using the Player 2 positions of SEQ-P3. An $L2b$ subject has $a_i = a'_{i1} > 2$, and holds the belief that the first-mover opponents are rational. At the Player 2 position, he would believe that Player 4 chooses the dominant strategy. In addition, given A4, he should also believe that Player 3 best responds after observing Player 4's action. Hence, an $L2b$ subject will be able to solve for his own best response through 3 steps of reasoning. An $L2a$ subject, on the other hand, is only able to think two steps and is thus not able to best respond as P2 of SEQ-P3.

Similarly, L3a and L3b could be separated using the Player 1 position of the sequential games. An L3b subject holds the belief that the first-mover opponents are 2nd-order rational, and that the second-mover best responds. Since L3b could reason at least 4 steps, he is able to reason all the way back to his own problem and best respond in both $SEQ-P3$ and $SEQ-P2$. An $L3a$ subject has $a_i = a'_{i1} = 3$. Therefore, his ability bound restricts him from using more than 3 steps of reasoning, and thus will not be able to best respond as Player 1.

In addition, by comparing L3 subjects' behavior in SEQ-P3 and SEQ-P2, it could be tested whether L3b subjects believe that their opponents are 2nd-order rational (and not ability-bounded) or the ability-bounded L2a who could think only 2 steps. In SEQ-P3, Player 1 needs to believe that Player 2 is 2nd-order rational and reason 3 steps to play the best response. Therefore, if there exists any belief-driven L3 subject who believes that the opponents are the ability-bounded $L2a$, he will best respond as P1 in SEQ-P2, but not as P1 in SEQ-P3.

With this design, $L1b$ and $L1a$ cannot be separated. $L4b$ and $L4a$ could be separated with 5-player ring games, which are not used in this experiment.

What if A2 does not hold well, and some subjects could reason more steps in sequential games, as mentioned in the earlier subsection? In this case, some Lka subjects might be classified as Lkb because they demonstrate higher reasoning ability in the sequential games. But those who are identified as Lka are guaranteed to be Lka . If so, as discussed before, this study provides an lower bound of the proportion of ability-bounded subjects.

5 Experimental Procedures

The experiment was conducted at the Experimental Social Science Laboratory (XLab) at UC Berkeley. A total of 184 subjects, who were recruited from Berkeley undergraduate classes, participated in 6 different sessions, with between 20 to 36 subjects in each session. Each experimental session lasted about one and a half hours. The average earnings were \$25, plus a \$5 participation fee, which were paid in private after each session.

The data was collected through an online interface. Subjects were not allowed to interact directly, and their identities were kept confidential. After subjects read the instructions, the instructions were read aloud by an experimenter. Then the subjects were given a short quiz to test their understanding of the game structure, followed by 4 unpaid practice games to help them get familiar with the interface.

In the main part of the experiment, the subjects played at the 24 positions of the games in a random order. In each game they were matched with a new group. Subjects received no feedback until all 24 games were finished. Though the subjects were not allowed to write during the experiment, the online interface allowed them to mark any cell in the payoff matrices by a click of the mouse. In this way they were able to easily track the best responses across matrices. The subjects were not allowed to make changes once they had confirmed their choices.

There was a time limit of 60 seconds for the subjects to complete each game. If they failed to choose in one game, the earnings for this game would be zero, and the system would randomly pick from the three choices for them to calculate the payoffs of their opponents. The secondmovers were given an additional 30 seconds after the first-movers had submitted their choices⁶.

⁶There are 12 subjects who failed to choose in one game (out of 184×24 games in total). For 7 out of the 12 subjects, missing one choice does not affect their type identification. Five subjects (406, 412, 610, 814, 1017) are

The decision time of each game was recorded for each subject.

After the subjects had completed all 24 games, they took the Cognitive Reflection Test. The CRT is composed of three short questions and is designed to measure subjects' cognitive ability (Frederick, 2005).

At the end of each session, subjects finished a survey of their demographic characters. One of the 24 games was randomly chosen for payment. One out of the three questions in the CRT were chosen for payoff and the subjects got \$0.25 if their answers were correct.

6 Experimental Results

6.1 Data Description

The percentage of subjects who comply with iterated dominance at each of the 24 positions is reported in Table 4^7 . For the second-movers, I look at whether they choose the dominant strategies in the subgames.

		Player 1	Player $2^{(A)}$	Player $3^{(C)}$	Player 4
SIMUL	G1	34.2%	56.0%	77.7%	97.3%
	G ₂	46.7%	54.3%	79.3%	97.8%
		Player 1	Player 2 (2nd)	Player 3	Player 4
SEQ-P2	G ₃	47.3%	95.1%	80.4%	96.7%
	G ₄	42.9%	93.5%	77.7%	94.6%
		Player 1	Player $2^{(B)}$	Player 3 (2nd)	Player 4
SEQ-P3	G5	49.5%	71.2%	96.2%	98.4%
	G ₆	51.6%	65.2%	96.2%	97.3%

Note: $N = 179(2nd)$ denotes second stage movers.

Table 2: Compliance rates of (iterative) dominating strategies

The compliance rates are quite high at all of the Player 4 positions and the second-mover positions (Player 2 of SEQ-P2 and Player 3 of SEQ-P3). Over 95% of the subjects choose the dominant strategies at these positions, which suggests that the majority of the participants understand the payoff structure and are capable of identifying strict dominance. The compliance

affected and are excluded from the type distribution presented in the following section.

⁷Out of the 184 participants, 172 finished all the choices within the time limit; 7 failed to choose in one game but the missing choice does not affect their type classification. The following analysis is based on the 179 subjects. The 5 subjects are excluded because they failed to choose in one game and the missing choice affects their type classification.

rates decrease as the required belief levels and reasoning steps go up, which is consistent with the prediction of the level- k model: fewer players achieve higher levels of reasoning.

A trace of the treatment effects of the sequential moves could be found in the differences in compliance rates. For example, while the P2 positions of both SIMUL and SEQ-P3 require three steps of reasoning, it requires one to be 3rd-order rational in SIMUL (marked as "A"), but only 2nd-order in SEQ-P3 (marked as "B"). If subjects' behavior is solely determined by their beliefs, we should expect higher compliance rates at P2 of SEQ-P3 $(B > A)$. Such a pattern could indeed be found in the data. However, the compliance rates at P2 of SEQ-P3 are lower than P3 of SIMUL (marked as " C ", $C > B$), which also requires the player to be 2nd-order rational but only two reasoning steps. It might imply that not every subject responding to 2nd-order rational belief in the simultaneous games is able to proceed to the third step. A similar pattern can be found among the L3 subjects.

6.2 Observed Lk Behavior in Simultaneous Games

Up to L4 behavior could be identified from subjects' choices in the simultaneous games. The type classification method is due to Kneeland (2015). It is assumed that each subject's behavior is determined by a single type, which remains constant throughout the experiment. A subject i deviates from his Lk type's choice profile with probability ϵ_{ik} , which is i.i.d. across games. When a subject deviates from his own type, it is assumed that he chooses the other two strategies with equal probability. The likelihood of a player i being type k given his action profile can be defined as

$$
d_{ik}(\epsilon_{ik}, x_{ik}) = (1 - \epsilon_{ik})^{G - x_{ik}} \left(\frac{\epsilon_{ik}}{2}\right)^{x_{ik}},\tag{1}
$$

where G is the number of games and x_{ik} is the number of observations that do not match the predicted profile of type k.

A subject is assigned to the type k with the highest likelihood d_{ik} , which is equivalent to finding the lowest number of deviations x_{ik} . If a subject's action profile matches exactly with a type's predicted profile, he will be assigned to this type. However, if $\min_k(x_{ik}) > 0$, there might be more than one minimum x_{ik} . Following Kneeland (2015), this subject will be assigned to the lowest type that has the minimum number of deviations.

If an action profile deviates too much from the predicted profiles of L_1-L_4 , the subject would be labeled as the irrational L0 or the *unidentified*. The unidentified ones are defined as deviating from Lk predictions, but picking the dominant strategies as Player 4 in both two rings. Hence, they are at least capable of avoiding dominated strategies and should be distinguished from the irrational, unpredictable L0 type. The unidentified subjects might be using some rules of their own that cannot be captured by the level-k model.

Therefore a cutoff point is needed so that those subjects with $\min_k(x_{ik})$ larger than the cutoff will be assigned to L0 or the unidentified group. Table 5 reports the type assignment results with

the cutoffs being 0, 1 or 2 deviations. In the first row, when a subject cannot be matched exactly to an Lk type, he is assigned to $L0$ or unidentifiable. This seems to be too strict as there are over 60% of the subjects left unidentified. When allowing for 1 deviation, the share of the unidentified subjects drops down to 15% , with more subjects being assigned to one of the four Lk types. If the 2-deviation cutoff is used, there is a further drop in the number of the unidentified subjects and an increase in $L1$ and $L2$. The numbers of higher types do not change.

Level	L ₀	L1	L ₂	L ₃	L4	UI
0 deviation	8	5	17	17	19	113
	4.5 $%$	2.8 \%	9.5%	9.5%	10.6%	63.1 $%$
random choice	89.0 %	0.4%	0.1%	0.1%	0.0%	10.4%
1 deviation	7	26	50	39	30	27
	3.9%	14.5%	27.9 %	21.8 \%	16.8%	15.1%
random choice	86.4 %	4.4 %	1.0%	0.3%	0.2%	7.7%
2 deviations	5	40	56	39	30	9
	2.8%	22.3%	31.3%	21.8 %	16.8 %	5.0%
random choice	72.5%	18.9%	4.2%	1.5%	0.7%	2.2%

Note: $N = 179$. Each subject is classified as $L_1 - L_4$ with no more than 0, 1 or 2 deviations from the predicted action profiles. Otherwise they are assigned to $L0$ or the unidentified group. The subjects classified as unidentified are able to choose dominant strategies as Player 4 but do not match any of the predicted patterns. They are at least rational, which makes them different from L0. The random choices are simulated with 10,000 randomly choosing subjects.

Table 3: Type assignment from SIMUL games

To determine which cutoff is the most appropriate in this study, a sample of 10,000 random choosing subjects is simulated and analyzed through the type assignment process. This analysis focuses on how many of these subjects could be correctly assigned to L0 and how many are wrongly assigned to one of the Lk types. When allowing for 1 deviation, over 86% of them are classified as L_0 , and around 5% go to the Lk types. It does not differ too much from the 0-deviation cutoff. However, when allowing for 2 deviations, over 25% of the random choosing subjects are assigned as Lk , which is too high to be acceptable. Therefore, the 1-deviation cutoff is chosen in the main analysis, for it gives reliable results and provides enough observations of the Lk types for the following analysis. The results using the 0-deviation and 2-deviation cutoffs, from which a similar pattern could be found as in the main results, are reported in Appendix C.1.

The type classification in Kneeland (2015) also uses the 1-deviation cutoff. The 1-deviation cutoff type distribution in my data is very close to the distribution found in Kneeland (2015). If the unidentified subjects are excluded, the Fisher's exact test comparing these two categor-

ical distributions yields a p-value of 0.926, suggesting that they are statistically not different⁸. However, the proportion of unidentified subjects, 15.1%, is much larger compared to the 1.2% in Kneeland's (2015) data⁹, which is due to larger deviation rates of my subjects.

Just as the one in Kneeland (2015), this type distribution is relatively higher than the findings in the literature (for example, Stahl and Wilson (1995) and Costa-Gomes et al. (2001)), which might be attributed to the special features of ring games. There are 27.9% and 21.8% of the subjects classified as L2 and L3 respectively with the 1-deviation cutoff, which serves as the starting point of the following analysis on the separation of Lka and Lkb.

6.3 Identification of Ability-bounded Subjects

In order to separate the ability-bounded and belief-driven types, I classify subjects using the 20 first-mover choices from SIMUL and the two sets of sequential games (excluding the 4 choices as second-movers) with a likelihood function similar to (1) . Six types $(L1, L2a, L2b, L3a, L3b)$ are included in the type assignment. The predicted actions of these types could be found in Section 4.3.

Subjects are assigned to a type with no more than 3 deviations from the type's predicted action profile¹⁰. When a subject deviates too much from a predicted profile, he is assigned to the unidentified category if he chooses 5 out of the 6 dominating strategies at Player 4 positions correctly, otherwise he is classified as L_0 . A randomly choosing subject has only 0.2% of a chance to correctly choose 5 dominant strategies.

Result 1 About half of the L2- and L3-behaving subjects in the simultaneous games are ability-bounded.

Table 6 shows how the identified types change from using the 8 positions in the simultaneous games to using the 20 first-mover positions in the simultaneous plus sequential games. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications. Most of the $L1$ and $L4$ subjects stay at the same levels when more observations are included, and all of the L0 subjects in the simultaneous games are still identified as L0. Overall, only approximately 15% of the subjects fall into a different category, and the majority of the subjects appear to be quite consistent across the three sets of games.

 $8E$ ven with the 2-deviation cutoff and thus only 5% unidentified subjects, Fisher's exact test still does not reject that the distribution in this study is different from Kneeland's (2015), though with a lower p-value of 0.853.

⁹Kneeland (2015) does not include a category of unidentified subjects, since there is only one such subject in her main treatment. This subject is assigned to L0 in her original paper.

 10 The 3-deviation results are used in the main analysis, as the probabilities of subjects making 3 errors or fewer out of 20 decision tasks are roughly comparable to the probabilities of them making no or one error out of 8 decisions, if their error rates are between 10% and 20%. The results using 2-deviation or 4-deviation cutoffs are reported in Appendix A.

		$Baseline + Sequential Types$								
		L0	L1	L2a	L2b	L3a	L3b	> L4	UI	sum
	L ₀	6	θ	θ	θ	θ	θ	0	1	7
	L1	$\overline{0}$	22	1	1	θ	$\mathbf{1}$	$\overline{0}$	1	26
	L2	θ	3	21	21	$\boldsymbol{0}$	1	$\overline{0}$	4	50
Baseline Types	L3	1	θ	$\overline{0}$	θ	20	15	$\overline{0}$	3	39
	L4	θ	θ	$\overline{0}$	1	1	θ	26	2	30
	UI	1	1	1	1	θ	1	θ	22	27
	sum	8	26	23	24	21	18	26	33	179

Note: $N = 179$. Subjects are assigned to a type with no more than 3 deviations. Otherwise they are classified as L0 or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 4: Type assignment according to the observations from SIMUL, SEQ-P2 and SEQ-P3

21 out of the 50 subjects who exhibit L2 behavior in the simultaneous games are identified as the ability-bounded $L2a$, who could do, at most, two steps of reasoning, compared with the other 21 L2b, who have higher ability and could proceed to three reasoning steps and still respond to their 2nd-order rational belief. Of the 39 L3 subjects in the simultaneous games, more than half are identified as $L3a$, and 15 are identified as the higher ability $L3b$. The results suggest that around half of the L2 and L3 behavior observed in the simultaneous games is due to lack of ability to think further.

As previously mentioned, there are some concerns that subjects' belief bounds and ability bounds might not remain the same in simultaneous and sequential games, especially since sequential reasoning might be considered easier. How would such inconsistency affect the observed type distribution?

First, consider a subject who has higher belief levels in the sequential ring games. Such a subject would behave as different Lk types. For example, there is one subject who is classified as L1 in SIMUL, but L2b if we look at all the games. His belief bound must have changed. Fortunately, such inconsistency would only be reflected in the switched levels, but would not affect the ratio of Lka and Lkb subjects. Therefore, if we exclude all the subjects who receive different type classifications, and focus only on the numbers in bold in Table 6, inconsistency in belief bounds should not affect the conclusion that half of the $L2$ and $L3$ subjects are ability bounded.

If some subjects have the same belief levels, but higher ability bounds in the sequential games,

the number of Lkb subjects might be overestimated. For example, a subject might be 2nd-order rational, and an ability bound of 2 in simultaneous games and 3 in sequential games. He is actually ability-bounded in the simultaneous games by definition, but would be assigned to the belief-driven L2b. In this case, the results here provide a lower bound of the proportion of the ability-bounded subjects.

I next look into the type switch patterns in more detail to analyze subjects' consistencies across the three sets of games.

Result 2 Subjects' behavioral patterns in the sequential games are close to the theoretical predictions assuming that their rationality levels and ability bounds are consistent across simultaneous and sequential games. Most subjects fall into the predicted categories.

Figure 4 gives the actual type switch patterns of the identified Lk type from SIMUL. In each diagram, the solid bar describes the behavior of these Lk types in $SEQ-P2$ or $SEQ-P3$. According to Table 3, subjects' behavior could be sorted into three categories based on their choices as firstmovers in the sequential games. In SEQ-P2, the subjects could be playing the (iterated) dominant strategies only at P4 position, at P4 and P3 positions, or at all of the P4, P3 and P1 positions. In SEQ-P3, the three categories are playing the (iterated) dominant strategies at P4 position, at P4 and P2 positions, or at P4, P2 and P1 positions. The three categories correspond to the subjects best responding to 1st-, 2nd-, and 3rd-order rational belief in the sequential games. Each subject could be assigned to one of the categories by the method in Section 6.2 and the 1-deviation cutof¹¹.

In theory, these subjects' behavior should follow Table 3 if their belief levels and ability levels remain the same in both simultaneous and sequential games. However, one could be misidentified to a different type if he deviates too much from his own type. In order to determine how many misidentifications from the predicted categories could occur, for each of the $L1-L4$ types, I simulate the choices of 10,000 pseudo-subjects in sequential games, assuming that all of them are either Lka or Lkb. The average deviation rate of each type used in the simulation is from the type classification results of the simultaneous games in Section 6.2. In Figure 4, the patterned bars of each category give the predicted behavior pattern of Lka or Lkb obtained from simulation. According to the simulation, around 85% of the subjects should fall into the predicted categories if their belief and ability bound do not change.

¹¹ According to A1, a subject best responding to 1st-order rational belief in SEQ-P2 plays dominant strategies as Player 4, but chooses the same actions as Players 1 and 3. This gives the predicted action profile for the "P4" category in SEQ-P2, which includes L1b and L1a. Similarly, a subject responding to 2nd-order rational belief in SEQ-P2 should play the (iterated) dominant strategies as Players 3 and 4 and choose the same action as Player 1, which gives the action profile for the "P4P3" category in SEQ-P2, including L2a, L2b and L3a. A subject responding to 3rd-order rational belief in SEQ-P2 should play the (iterated) dominant strategies as Players 1, 3 and 4, which is the "P4P3P1", including L3b, L4a and L4b. The actions of the three categories in SEQ-P3 could be predicted in the same way.

L1 (N = 26) in SEQ-P3 25 20 15 10 5 $\pmb{0}$ $P₄$ **P4P2** P4P2P1 $LO + UI$ actual data ⊠ predicted-L1b & L1a

P4P2P1

 $LO + UI$

Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 1-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

Figure 4: Behavior pattern of each Lk type in sequential games

Let's first take a look at the L1 and L4 types, and L2 in SEQ-P2. The predictions of Lka or Lkb behavior are the same for these subjects. Their actual choices in SEQ-P2 and SEQ-P3 share similar patterns with the simulated distributions, though less concentrated on the theoretically predicted categories, possibly due to the presence of inconsistent subjects. According to Table 7, exact tests of goodness-of-fit show that in three of these five cases (L1 in SEQ-P3, L2 in SEQ-P2 and L4 in SEQ-P3) the actual type distributions are not different statistically from the predicted ones at the significance level of 0.05 (Table 7). In the case of $L4$ subjects in SEQ-P2, the difference is at the 0.05 level but not the 0.01 level.

Since there might exist both $L2a$ and $L2b$ in SEQ-P3, and both $L3a$ and $L3b$ in SEQ-P2 and SEQ-P3, the type distributions of the L2 and L3 subjects could be different from the simulated ones. However, the difference should only be reflected in the predicted categories. Specifically, for L2's behavior in SEQ-P3, the difference from the simulated distribution should only be found in the best response rates at Player 2 position. Therefore small p -values should only occur in the "P4" and "P4P2" categories and the proportions of the other categories should not differ too much from the simulated ones. Similarly, for L3's behavior in SEQ-P2 and SEQ-P3, the difference should only be reflected in the best response rates at Player 1 position (reflected in Categories (2) and (3)). The follow-up test of each of the four categories confirms that very few subjects fall outside of the predicted categories.

Thus it could be concluded that the simulated type distributions are good predictions of the aggregate pattern, which provides strong evidence that most subjects are consistent in belief and ability bounds.

Result 3 No difference has been found in L3's behavior in SEQ-P2 and SEQ-P3, suggesting that most L3b subjects does not believe that their opponents ability-bounded.

In the theoretical framework, a simplifying assumption was made that a player holds belief only in the opponents' rationality but not in their ability bounds. This assumption only affects the behavioral implications for L3 subjects. It remains to check the validity of this assumption. In SEQ-P2, in order to play the iterated dominant strategy as Player 1, L3b needs to believe that the opponents reason two steps, while in SEQ-P3 L3b needs to believe that the opponents reason three steps. If all L3b subjects hold the belief that their opponents are 2nd-order rational and not ability-bounded, then SEQ-P3 and SEQ-P2 should make no difference for them, which serves as the null hypothesis in this test.

Under the null hypothesis, there should be the same proportion of L3 subjects identified as $L3b$ in SEQ-P2 and SEQ-P3. Otherwise, if a subject believes that the opponents are $L2a$, he will not comply with iterated dominance as Player 1 in SEQ-P3. In this case, fewer L3 subjects would be identified as L3b in SEQ-P3 than in SEQ-P2.

In the actual data, no difference could be found statistically in the type distributions from the two sets of sequential games (see Figure 4). A closer look at the behavior patterns of the L3

Note: the numbers in bold fonts represents the 0.05 significance level test result, with the null hypothesis that the actual data and simulated data are drawn from the same distribution. The first column shows the original tests including all four categories. Columns (2)-(5) show the follow-up tests of each category vs. the sum of all the other categories. For the four follow-up tests, the significance level is corrected by $0.05/4 = 0.0125$.

Table 5: p-values of the exact tests of goodness-of-fit: actual vs. predicted distributions

subjects (Table 8) has confirmed that most of these subjects behave consistently as $L3b$ or $L3a$ in both SEQ-P3 and SEQ-P2. No evidence is found to reject the hypothesis that L3b believes that their opponents are always best responding to the 2th-order rational belief¹².

6.4 CRT Scores and k-Levels

This subsection explores the correlation between subjects' identified reasoning ability from the dominance-solvable games and a measure of their cognitive ability. The Cognitive Reflection Test is used as a quick measure of cognitive ability in this experiment. If the $L2b (L3b)$ subjects are more capable of reasoning than the $L2a (L3a)$ subjects as predicted in the theoretical model, the difference might also be reflected in their CRT scores.

 12 It should be noted that with a sample size of 39 L3 subjects, to get a power higher than 0.8, it requires at least 25% of them responding to the belief that the opponents are bounded by two steps of thinking (Faul et al., 2007). So there may very well exist a small number of such subjects, who could not be detected in this experiment due to lack of power.

		behavior in SEQ-P2						
		P4P3 P4P3P1	UI					
	P4	3	2					
	P ₄ P ₂		9					
in SEQ-P3	P ₄ P ₂ P ₁		5	10				

Table 6: Behavioral patterns of the 39 L3 subjects in the two sequential games

The Cognitive Reflection Test is designed by Frederick (2005) to test people's cognitive ability in decision making. It is composed of three short questions as follows:

- (a) A bat and a ball cost 1.10 in total. The bat costs a dollar more than the ball. How much does the ball cost? \ldots cents.
- (b) If it takes 5 machines 5 min to make 5 widgets, how long would it take 100 machines to make 100 widgets? __ min.
- (c) In a lake, there is a patch of lily pads. Every day, the patch doubles in size. If it takes 48 days for the patch to cover the entire lake, how long would it take for the patch to cover half of the lake? $__$ days.

All three questions have intuitive but erroneous answers, and one needs to reflect on them for a brief moment to get them right.

Researches have found correlations of over 0.4 between CRT scores and other cognitive ability measures, such as the SAT, the Wonderlic Personnel Test, and the Vocabulary and Matrix Reasoning subtests (Frederick, 2005; Obrecht et al., 2009; Toplak et al., 2011). The CRT is also reported to relate to some important decision making characteristics. The subjects who score high in the CRT are more patient, and are more willing to take risks (Frederick, 2005). In addition, because the CRT tasks separate impulsive and reflective thinkers, it suggests that the ones who score higher in the test tend to do more rational thinking and are less likely to succumb to heuristics and bias (Oechssler et al., 2009).

The participants in this experiment obtained a mean CRT score of 1.64, which is comparable to the results in the literature. In Figure 5 the mean CRT scores are displayed by types. Among the lower types (L0-L2b), subjects' CRT scores increase with their identified levels. Two-sided t-tests show that the differences are significant between these groups, except for $L1$ and $L2a$. The most striking result is that the L2a subjects, who are identified as having lower ability bound, did much worse in CRT than the L2b subjects, although both types exhibited the same behavior

in the simultaneous games. The results of CRT and type identification corroborate each other, suggesting that cognitive ability plays an important role in strategic reasoning.

Note: Standard errors are reported in the error bars.

Figure 5: Average CRT scores by types

The differences between high types $(L2b-L4)$ are not that clear. T-tests show that the means are not different for these four groups. All of these subjects are able to do at least three steps of reasoning in the ring games, and their average scores are pretty high¹³. Therefore, one possible explanation of why higher types are not separable by CRT scores could be that the CRT results do not distinguish between the subjects whose reasoning abilities have reached a certain high level.

Another interesting observation is the difference between L0 and unidentified subjects. The average CRT score of L0 subjects is close to 0. Actually, 7 out of 8 subjects in this group scored 0 in the test. Contrastingly, the unidentified group performs much better and their average score is not statistically different from the mean of the whole sample. It supports the separation of these unidentified subjects from the irrational L0. The unidentified subjects could be of high cognitive ability, but do not follow the prediction of the level-k model. Or it could be that they are inconsistent and act as different types across different games.

¹³To serve as a comparison, Fredrick (2005) conducted the CRT on 3,428 subjects, primarily college students, and found a mean score of 1.24. The mean scores from top universities like Princeton and Harvard are around 1.4-1.6. Only MIT students get a high mean of over 2.

6.5 Robustness Check: Learning Effect

An essential assumption in the above analysis is that each subject's belief bound and ability bound remain constant throughout the experiment, with a few deviations due to random preference shift or trembling hand. However, if certain learning effects prevailed, the subjects became more proficient in identifying iterated dominant strategies as the experiment proceeded. Then the observed pattern could be driven by the order of games in the experiment. For example, the subjects who played the more difficult positions, such as Player 1 or Player 2 positions, in the later periods would have a larger chance of best responding. In this case, the higher types might not be the group with higher ability, but the group with more opportunity to learn.

A detailed analysis of learning effects will be reported in Appendix B. In summary, I find mild learning effects at a couple of player positions, but the type classification results do not appear to be affected by learning. This subsection compares the type distributions of the whole sample and the groups who might have an advantage in learning, showing that learning has almost no effect in shaping the type distribution.

Given the special structure of the ring games, it might be advantageous to play the Player 4 positions first, for it could help the subjects to figure out early in the experiment that the games could be solved by iterated dominance. In addition, playing as second-movers might also help, for it motivates the subject to look into the dependency relationships between the opponents and himself. If this is true, then the subjects who played more Player 4 positions or second-movers in the earlier stage are more likely to behave as higher types. To address this concern, I check whether the performances of these subjects differ from the whole sample.

In each session, subjects played the 24 games in different random orders. Table 9 gives the number of subjects who played more Player 4 positions and second-mover positions in the earlier 12 games. The numbers of Player 4 positions are denoted by $n(P4)$, and the number of second-mover positions by $n(PSM)$. There are 44 subjects who played at more than four Player 4 positions in the earlier 12 games, and 53 subjects who played at more than six Player 4 or second-mover positions. These subjects are regarded as having advantages in learning. The following results still hold if other cutoffs are used to determine advantages.

n(P4)		$\geq 1 \quad \geq 2 \quad \geq 3 \quad \geq 4 \quad \geq 5 \quad \geq 6$	
- N		$\begin{array}{ c c c c c c c c c } \hline 179 & 155 & 124 & \textbf{44} & 31 & 0 \\ \hline \end{array}$	
$n(P4) + n(PSM)$ $ \geq 3 \geq 4 \geq 5 \geq 6 \geq 7 \geq 8$			
- N		179 154 108 53 36	

Note: $n(P4)$ denotes the number of Player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second-mover positions played by the subject in the earlier 12 games. The bold fonts represent the cutoffs used in the following analysis.

Table 7: Number of subjects who played more advantageous positions in earlier 12 games

As shown in Figure 6, no evidence has been found that they performed better than the rest of the subjects. The distributions of these groups are not shifted toward higher types. The goodness-of-fit tests show that the type distribution of the subjects with $n(P4) + n(PSM) \ge 6$ is not statistically different from the distribution of the whole sample, and that, for the group with $n(P4) \geq 4$, the differences only occur in the L3b and L4 categories. Therefore it is safe to say that the identified patterns of the main results are not affected by subjects' learning of iterated dominance.

Note: $n(P4)$ denotes the number of player 4 positions played by the subject in the earlier 12 games. $n(PSM)$ denotes the number of second-mover positions played by the subject in the earlier 12 games.

Figure 6: Type distributions of the subjects who have an advantage in learning

7 Conclusion

Existing literature on the level-k model focuses on identifying people's reasoning levels in strategic situations. This study seeks to identify whether the observed level-k behaviors are driven by bounded cognitive ability. It is found that out of the 50 and 39 subjects classified as L2 and L3 from their choices in the simultaneous games, around half have reached their upper boundaries of reasoning, and are identified as the ability-bounded L2a and L3a. The rest of them demonstrated higher reasoning ability than their originally identified levels, and are classified as the belief driven $L2b$ and $L3b$. In addition, the CRT scores are significantly higher for the high ability $L2b$ than the low ability $L2a$, which supports the separation of the two types. But higher types $(L2b, L3a)$, L3b and L4) are not distinguishable using CRT scores.

The findings suggest large heterogeneity in subjects' abilities to best respond to even low

order beliefs. Thus, the observed low levels in the previous studies could be explained by both the presence of low ability types and the low-order beliefs of high ability types. Although the high types have incorrect beliefs, their low-order beliefs are not entirely unfounded, given the large proportion of the cognitively bounded subjects.

The existing literature has demonstrated the descriptive power of the level-k model. To make it also an explanatory and predictive model, it requires a better understanding of why people behave at certain levels, or where people get their belief from. For example, the existence of ability-bounded subjects in this study shows that a lot of people might not start with a clear idea of the opponents' levels. Rather, their belief could be formed through the anchoring and adjusting process suggested by Brandenburger and Li (2015), and this process would stop when they have reached their cognitive boundaries. Therefore, one direction of future works could be to explore the relationship between players' ability bounds and belief formation.

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APPENDIX

A Type Classification of Lkb and Lka with Alternative Cutoffs

Note: $N = 179$. Subjects are assigned to a type with, at most, 2 deviations. Otherwise they are classified as L0 or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 8: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (2 deviations)

Note: $N = 179$. Subjects are assigned to a type with, at most, 4 deviations. Otherwise they are classified as L0 or unidentified. The subjects classified as unidentified are able to choose 5 out of 6 dominant strategies as player 4 but do not match any of the predicted patterns. The numbers in bold fonts represent the subjects who remain in the same Lk category in the two type classifications.

Table 9: Type assignment according to the observations from SIMUL, SEQ-P3 and SEQ-P2 (4 deviations)

B Additional Analysis on Learning Effects

A thorough analysis on learning effects is reported in this section. I start by checking whether there exist systematic type shifts in the data. That is, whether subjects are more likely to shift to a higher (lower) type in the later (earlier) period of the experiment. Since the choices in all 20 first-mover positions are needed to pin down a subject's type, it is not possible to estimate one's type in the early and late parts of the experiment separately. Instead, I look at the deviations from the choice pattern of one's assigned type. The deviation observed in the type assignment process could be sorted into one of three cases: (1) a non-equilibrium strategy is chosen at a position where the type should have chosen an equilibrium strategy. (2) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and at least one of the two strategies chosen is an equilibrium strategy. (3) two different strategies are chosen at the same position of the two paired rings where the type should be on off-equilibrium path and choose the same strategy, and neither of the two strategies chosen is an equilibrium strategy.

Case (1) implies that the subject might shift to a lower level when playing that game and Case (2) corresponds to the shift to a higher level. If learning affects subjects' behavior, they should be more likely to deviate to a higher level in the later half of the experiment and more likely to a lower level in the earlier half. If, however, the growth of fatigue plays a more important role, it should be opposite. Of course it could not be ruled out that some deviations in Case (1) and (2) are caused by preference shifts or mistakes. But if the occurrences of preference shifts or mistakes are assumed to be time-invariant, then they could be canceled out when only the differences of the earlier and later halves are examined.

		8 Baseline Games		20 first-mover Games			
		Case (1) Case (2) Case (3) \vert Case (1) Case (2) Case (3)					
Shifts	downward upward			downward upward			
Earlier	12	26	25	42		53	
Later	ച	27		24	79		

Note: the first two cases are counted in the earlier and later 12 games respectively. L0 and unidentifiable subjects are excluded.

Table 10: Deviations sorted into three cases

Table 12 reports the deviations of the classified subjects, from both the 8 SIMUL games and the 20 first-mover games type assignments. L0 and unidentifiable subjects are excluded from this analysis, because according to the definition L0s could be using any combination of strategies, and since I could not identify the decision rules of the unidentifiable it would be hard to determine which choices are deviations from their rules. The 24 games were played in a random order and

the orders of play were different for each subject. Cases (1) and (2) could therefore be put into two categories, that is whether these deviations occur in the first or later 12 games. In Case (3) it could only be observed that subjects are choosing two different strategies at the same position, but it is impossible to tell which one is the deviation (or both could be deviations). So only the total numbers of deviations in Case (3) are reported.

In Case (1), players deviate to a lower type. This kind of deviation is more likely to happen in the first half of the experiment, suggesting some sort of learning effects. However, the occurrences of Case (2) deviations, which imply a shift to a higher level, are quite close between earlier and later periods of the experiment.

I next run a probit regression to determine the learning effect specifically at each position.

$$
Probability(Y_i) = \alpha + \beta_1 L12_i + \beta_2 POS_i + \beta_{3j}POS_{ij} \times L12_i + \epsilon_i,
$$
\n(2)

where $Y_i = 1$ when an equilibrium strategy is chosen, and $Y_i = 0$ otherwise; $L12_i = 1$ if that choice is made in the later 12 games, and $L12_i = 0$ otherwise; POS_{ij} denotes the position dummy at position j. The session fixed effects are also controlled.

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. (2nd) denotes second stage movers. Player 4 of G5 is dropped due to collinearity.

Table 11: Experience effect at each position $(\beta_1 + \beta_{3j})$

Table 13 reports the coefficients $\beta_1 + \beta_{3j}$ of each position j. One position dummy, player 4 of G5, is dropped because of collinearity. Significant positive learning effects are found at only 3 of the 23 positions.

Since the identification uses at least a pair of ring games, the learning effects at three positions is unlikely to affect the type distribution. It is reported in Section 6.5 that the performance of the subjects who could have better opportunities to learn is not different from the whole sample. Here I further show with multilogit regressions that playing more player 4 positions or secondmover positions in the earlier periods does not affect the probability of being assigned to a high type (Table 14 and Table 15).

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 12: Multilogit regression of types on Learning Effects (1)

Note: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Standard errors are reported in the parentheses. Each row shows a regression with a different type being the base outcome. Session fixed effects are included in the multilogit regressions. Unidentifiable subjects are excluded.

Table 13: Multilogit regression of types on Learning Effects (2)

C Other Results

C.1 Type Switch Pattern with Alternative Cutoffs

Table 5 has shown what the type distribution looks like allowing for 0 or 2 deviations. If no deviation is allowed for during the type assignment process, there will be a large fraction of subjects (113 out of 184) who could not be put into any of the L0-L4 categories. With the 0-deviation cutoff, since few subjects' behavior profile could be matched to a certain type with 0 deviation in all three sets of games, it is hard to determine a clear pattern of each type's behavior in the sequential games. As shown in Figure 7, although most $L4$ subjects behave as predicted in both sequential games, half of the L1, L2 and L3 subjects fall into the unidentified category. So it is less clear what proportion of the subjects are best responding to their belief and what proportion are bounded by reasoning ability in the sequential games.

When allowing for 2 instead of 1 deviation in the matching process, the risk of misidentifying the randomly choosing subjects to an Lk type increases. But it is still worth looking at the results with the 2-deviation cutoff, in which almost all the subjects could be classified.

Most previously unidentified subjects in the simultaneous games are moved to one of the lower types $(L1 \text{ and } L2)$ when allowing for 2 deviations, and there is no change in the number of L3 and L4 subjects. The behavioral pattern of each type in the sequential games appears to be similar as in the main analysis (Figure 8). But it should be noted that since it identifies more lower types from the previously unidentifiable pool by using the 2-deviation cutoff, the use of this cutoff picks up slightly high proportion of the ability-bounded L2a and L3a subjects.

Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterated) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterated) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 0-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

Figure 7: Behavioral patterns in sequential games (0 deviation)

 $LO + UI$

P4P3P1

 $LO + UI$

20

10

 $\pmb{0}$

P₄

P4P3

Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with ER and 2-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

Figure 8: Behavioral patterns in sequential games (2 deviations)

C.2 Type Assignment with the Assumption of Uniform Randomizing L0

The assumption used to predict Lk behavior in the ring games is that an Lk player does not respond to the changes in $(k + 1)$ - or higher direct opponents' payoffs $(A1)$. An alternative assumption, which is widely used in Lk experiments, is that the irrational $L0$ type uniformly randomize on all the possible actions (UP: uniform prior on L0).

		L ₀	L1	L2	L ₃	L4	UI
SIMUL	ER	$\overline{7}$	26	50	39	30	27
		3.9%	14.5%	27.9%	21.8%	16.8%	15.1\%
	UP	6	10	43	39	53	28
		3.4%	5.6%	24.0%	21.8%	29.6%	15.6%
		L ₀	P4	P4P3	P ₄ P ₃ P ₁		UI
SEQ-P2	ER	$\overline{7}$	43	69	46		14
		3.9%	24.0%	38.5%	25.7%		7.8%
	UP	7	$32\,$	60	78		$\sqrt{2}$
		3.9%	17.9%	33.5%	43.6%		1.1%
		L ₀	P ₄	P ₄ P ₂	P ₄ P ₂ P ₁		UI
SEQ-P3	ER	5	68	46	45		15
		2.8%	38.0%	25.7%	25.1%		8.4%
	UP	5	22	46	94		12
		2.8%		12.3\% 25.7\%	52.5%		6.7%

Note: $N = 179$. Subjects are assigned to a type with no more than 1 deviation when using A1, with no more than 2 deviations when using UP. Otherwise they are assigned to L0 or the unidentified group. The subjects classified as unidentified play dominant strategies as Player 4 but do not match any of the predicted patterns.

Table 14: Type assignment using the two assumptions ER and UP

The distribution of assigned types using the predicted Lk behavior of UP is given in Table 16. The 2-deviation cutoff is used here, so a random choosing subject only has a probability of less than 5% of being assigned to one of the Lk types, which is comparable to the main results. The distributions shift to the right in all three sets of games. In the simultaneous games, the number of L_4 subjects almost doubles, and the number of L_1 drops by more than a half. The shift is even larger in SEQ-P3. The number of subjects assigned to "P4" category under UP is less than a third of the number under A1. And the number in "P4P2P1" category increases by more than 100%. A closer look at the type changes confirms that everyone's level rises or at least stays the same. Nobody goes to a lower level under UP . Nevertheless, the numbers of L_0 and the unidentified are quite close. There are 6 subjects classified as L0 and 14 classified as unidentified under both assumptions.

I try to perform the same analysis of each type's behavioral pattern in the sequential games (Figure 9). However, the overestimation of lower types in the two sequential games makes the treatment effects less clear. A large number of L1- and L2-behaving subjects in SIMUL are classified as "P4P2P1" in the sequential games, which is inconsistent with the theoretical prediction. It is impossible to tell how much subjects are Lka with such large inconsistency across the three sets of games. The overestimation is less severe among higher types, and a similar pattern could be observed on L3 and L4 subjects as in the main analysis.

Type classification with UP tends to overestimate the lower types. This is because the predicted action profile of each type on the off-equilibrium path using UP is a special case of that using A1. Since UP puts stronger restrictions on the off-equilibrium path, any subject has a smaller chance to be assigned to a low type. The overestimation would be more severe if fewer subjects follow the prediction of UP , as observed in SEQ-P3. Hence I use A1 in the main analysis, which I believe provides more robust and reliable results.

Note: In SEQ-P2, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P3 ("P4P3"), and at P4, P3 and P1 ("P4P3P1"). In SEQ-P3, subjects are sorted into three categories: choosing the (iterative) dominant strategies at P4 ("P4"), at P4 and P2 ("P4P2"), and at P4, P2 and P1 ("P4P2P1"). Each subject is assigned to a category with UP and 2-deviation cutoff. The solid bars give the actual type distribution. The checked and stripe bars give the simulated type distribution assuming that these subjects are Lkb or Lka. In certain cases the behaviors of Lkb and Lka are not distinguishable, and they are represented by the same checked bar.

L1 ($N = 10$) in SEQ-P3 10 8 6 $\overline{4}$ $\overline{2}$ $\mathbf 0$ $P₄$ P4P2 P4P2P1 $LO + UI$ actual data ⊠ predicted-L1b & L1a

 $LO + UI$

Figure 9: Behavioral patterns with the uniform prior assumption (UP)

D Instructions

You are about to participate in an experiment in decision making. The experiment includes 24 decision problems and 9 short questions. You will get separate instructions for the short questions after the decision making part. If you follow the instructions and make good decisions, you may earn a considerable amount of money. Besides that, you will also get a \$5 participation fee. All the money that you earn will be paid to you in private after today's session.

You may not write during the experiment. It is also important to remain silent throughout the experiment and not to look at other people's work. If you have any questions or need assistance of any kind, please raise your hand, and the experimenter will come to you. Otherwise, if you write, laugh, talk with other participants, exclaim out loud, etc., YOU WILL BE ASKED TO LEAVE. Thank you.

Your earnings will be determined by your decisions and the decisions of other participants in the experiment. Before making your decisions, you will be able to gather information about how your earnings and other participants' earnings depend on your and their decisions. Then you need to take an understanding test and have the opportunity to practice on 4 problems. You will NOT be paid for the practice rounds.

The experiment has 24 rounds. In each round, you will be matched with a RANDOMLY selected group of the other participants. A new group will be formed and matched with you in each round. You will not know which ones of the other participants you are matched with, and your identity and the identities of the other participants will never be revealed.

Each round concerns a DECISION SITUATION in which you and the other participants in your group separately and independently make decisions called CHOICES. Together, your and their choices determine your and their earnings in this round, which may be different for each member in the group.

Neither your nor their choices in a round will affect how you or the other participants are matched or the decision situations you and they face in the rest of the experiment.

In each round, you will choose among three alternatives. The other members in your group will also have three alternatives to choose from. Your earnings will depend on the combination of your choice and one of the other group member's choice. These earnings possibilities will be represented in a table like the one below. Your choices will determine the row of the table. The choice of one of the other members in your group, whom we call Participant X, will determine the column of the table. You may choose from "a, b, c" and Participant X may choose from "d, e, f". The cell corresponding to this combination of choices will determine your earnings.

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them

Participant Z is the observer in this group.

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices.

Participant Y's earnings depend on Participant Y's and Participant Z's choices.

Participant Z's earnings depend on Participant Z's and your choices

Please choose: ea eb ec

You will not be able to change your choice after you have confirmed it.

CONFIRM

For example, given the above earnings tables, if you choose b and Participant X chooses e, then you will earn 8 dollars for this round. If instead Participant X chooses f, you will earn 14 dollars.

Your earnings will always be listed in the FIRST table. Participants X, Y and Z's earnings are listed in the other three tables. X's earnings depend upon his/her choice and Y's choice. Y's earnings depend upon his/her choice and Z's choice. Z's earnings depend upon his/her choice and your choice.

For example, if you choose a, Participant X chooses e, Participant Y chooses h, Participant Z chooses l, then you will earn 8 dollars, X will earn 2 dollars, Y will earn 10 dollars and Z will earn 18 dollars.

The earnings tables will be given to every member of the group. After you have made the decision in that round, you need to click the button to confirm you choice and wait to proceed to the next problem after all the other group members have finished their choices. The earnings tables may change from round to round, so you should always look at the earnings carefully at the beginning of each round.

Besides, you may highlight any cell by clicking on it to help you making the decisions (see screenshot below).

There is a time limit of 60 seconds for you to make choice in each round. There will be a counting down clock showing how much time is left for this round. The clock is located on the upper right corner, as shown in the screenshot below. If you fail to make a decision within the time limit, you will earn \$0 in this round. If any group member fails to make a choice within the time limit, the system will randomly pick a choice out of his three alternatives when calculating the earnings of other group members. For example, given the earnings tables below, if you choose b, X chooses f, Y fails to choose, Z chooses l, and the system chooses g for Y, then you will earn \$8, X will earn \$8, Y will earn \$0 and Z will earn \$10.

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them

Participant X is the observer in this group.

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices. Participant Y's earnings depend on Participant Y's and Participant Z's choices. Participant Z's earnings depend on Participant Z's and your choices. Please choose: ea eb ec You will not be able to change your choice after you have confirmed it.

CONFIRM

In certain rounds, one of the group members will be RANDOMLY selected to be the OBSERVER. An observer will be able to look at the other group members' choices before making his/her choice. At the beginning of each round, We will inform every member in the group which member has been assigned to be the observer. (But, of course, his/her true identify will always remain anonymous.)

For example, if Participant X is selected to be the observer, you and Participants Y and Z will see this sentence on the screen:

"Participant X is the observer."

And participant X will see this on the screen:

"You are the observer."

Below is a screenshot of the interface for an observer. After the other participants have made their choices, these choices will appear in the box.

Your earnings are given by the blue numbers. You can highlight any cell by clicking on them.

You are the observer in this group. Your group mates' choices will appear in the box below.

Your earnings depend on your and Participant X's choices.

Participant X's earnings depend on Participant X's and Participant Y's choices.

Participant Y's earnings depend on Participant Y's and Participant Z's choices.

Participant Z's earnings depend on Participant Z's and your choices.

Please choose: o a o b o c

You will not be able to change your choice after you have confirmed it.

CONFIRM

The observer will be given an extra of 30 seconds to make his/her choice, which means the observer will have 90 seconds in that round. If you are the observer, please wait patiently until your group-mates' choices show up in the boxes.

Whether you or the other members will be selected as the observer does NOT depend on your or their previous choices nor the group formation. You will be randomly matched with a new group in each round and the identities of all the group members will always remain anonymous.

Once you have confirmed your choice for a round, you will not be able to change your choice for that round. After you have made your choices for all of the 24 rounds, one round will be randomly selected for payment at the end of the experiment. Every participant in this room will be paid based on his/her choices and the choices of his/her randomly selected group-mates in this round. Any of the rounds could be selected. So you should treat each round like it will be the one determining your payment.

You will be informed of your total earnings, the round chosen for payment, what choice you made in that round and the action of your randomly matched group-mates only at the end of the experiment. You will not learn any other information about the actions of other participants' in the experiment. You final payment will be the earnings calculated by the system plus the \$5 participation fee. All the money that you earn will be paid to you in private after today's session.