Normalizing the Central Bank's Balance Sheet: Implications for Inflation and Debt Dynamics*

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Abstract

We explore the effects of reducing the overall size of the central bank's balance sheet and lowering its maturity structure. To do so, we consider an environment where fiscal policy is passive and the central bank follows the Taylor principle. Moreover, the monetary authority has size and compositional targets of its balance sheet. When short-term public debt exhibits premia, changes in the central bank's balance sheet have implications for long-run inflation, real allocations and for the uniqueness of equilibria. To ensure a unique and a locally stable steady state, the central bank should target a low enough maturity composition of its balance sheet. In our numerical exercise, calibrated to the United States, we find that long-term debt holdings by the central bank should be less than 1.8 times of their short-term positions. Moreover, the process of balance sheet normalization should be slow enough. Compared to a traditional Taylor rule, a modified rule, that takes into account the premium, increases the prevalence of multiplicity of steady states and delivers lower welfare. Thus, we argue that the traditional Taylor rule is appropriate for managing interest rates in the presence of premia.

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1 Introduction

After reducing policy interest rates close to zero, in 2009 the Federal Reserve started to implement unconventional monetary policies. These involved purchases of assets that were not traditionally present in its portfolio. These operations dramatically increased the size of the central bank's balance sheet and expanded its longer-term public debt position.[1] However, as soon as the economy of the United States started to recover, the discussion among policy makers turned to *monetary policy normalization*. This process would require the implementation of policies so that, in the future, the central bank would operate as it previously did.[2] During the transition period, the new conduct of monetary policy would not only involve interest rate management, but also a new set of policies aimed at drastically changing the central bank's balance sheet. In 2014 the Federal Open Market Committee (FOMC), through the Policy Normalization Principles and Plans, outlined three key actions in this process: (i) begin increases in short-term interest rates,[3] (ii) reduce the size of the central bank's balance sheet, and (iii) maturity reduction in order to have a composition in the balance sheet similar to the one prior to the Great Recession.[4] So far the literature has not fully explored the macroeconomic consequences of such *normalization* process. This is main purpose of this paper.

Rather than analyzing the quantitative easing policies that lead to the substantial expansion of the central bank's balance sheet and significant change in its composition, this paper studies various aspects of the *monetary policy normalization* process. In particular, we take as given the post Great Recession characteristics of the central bank's balance sheet and provide answers to the following questions. What are the consequences for economic activity, inflation and debt dynamics of having a central bank reduce the size of its balance sheet and its maturity structure? How will changes in the size of the central bank's balance sheet and its maturity structure alter the inherent links between monetary and fiscal policy? What type of rules should the central bank use when *normalization* takes place?

[1] In December 2007, the Federal Reserve securities were entirely Treasury securities, and of these 32.1 percent were Treasury bills (short-term government debt). In December 2014, the Federal Reserve held no short-term Treasury securities; 41 percent of its security holdings were long-maturity mortgage-backed securities and 58.1 percent of its security holdings were long-maturity Treasury notes and bonds.

[2] As early as June 2013, Federal Open Market Committee minutes already include some discussion on policy normalization and the long-run composition of the balance sheet.

[3] The Federal Open Market Committee (FOMC) took this step in December 2015.

[4] For more information we refer the reader to the "Policy Normalization Principles and Plans" (Board of Governors, 2014).

To answer these questions, we consider a cashless flexible price environment where agents face stochastic trading opportunities and limited commitment in some markets.[5] Agents have access to nominal public debt, of different maturities, to smooth consumption. However, not all assets are equally pledgeable.[6] As a result, short-term public debt carry a liquidity premium.[7] Other than private agents trading with each other, there is also a government with two different authorities: fiscal and monetary. Both of these trade with the private sector in a multilateral, frictionless and competitive financial and goods markets. The fiscal authority needs to finance an exogenous stream of expenditures through taxes and the issuance of short and long-term nominal bonds. The fiscal authority also decides the long-run quantity of debt to be issued as well as its composition. Moreover, the operating procedures for fiscal policy consist of a tax rule that takes into account public debt and a rule for its target maturity composition. The monetary authority, on the other hand, manages interest rates through a Taylor rule. To highlight the role of premia on short-term public debt, we analyze two specifications. The *traditional* Taylor rule, whereby the central bank only responds to deviations of inflation from its target. We also consider a *modified* Taylor rule, a long the lines of Cúrdia and Woodford (2010), that takes into account bond premia. Finally, in contrast to most of the literature, we consider two additional monetary rules that specify the evolution of the size and composition of the central bank's balance sheet. These operating procedures for monetary policy help us capture some of the features set out in the Policy Normalization Principles and Plans by the Federal Reserve in 2014. It also helps operationalize the monetary policy normalization process by explicitly considering the central bank's balance sheet size and composition targets. In addition, this approach allows us to conduct

[5] Given that we are studying monetary policy normalization, considering an environment without reserves is rather innocuous. This is the case as the normalization process would be one where we have asset run-offs and the central bank would trade long for short-term bonds. For more on this approach to monetary policy normalization, we refer the reader to Rickets et al. (2014) and Bullard (2017).

[6] Pledgeability refers to the amount of the asset that is accepted as collateral. In particular, we assume that short-term government bonds are acceptable as collateral, while long-term bonds are not.
The collateral nature of short-term government debt has been largely overlooked by the literature and we argue is key when thinking about the conduct of monetary policy. This is especially salient in today's current environment of low interest rates and large central bank balance sheets.

[7] The asymmetric treatment of the collateral properties among assets is one way, among other alternatives, to generate a liquidity premium. We refer the reader to Venkateswaran and Wright (2014), Berentsen and Waller (2018), Andolfatto and Martin (2018), Canzeroni et al. (2016), among others, for examples where bonds serves as collateral, yielding then a liquidity premium. Other authors such as Herrenbrueck and Geromichalos (2016), Lee et al. (2016) and Domínguez and Gomis-Porqueras (2019), among others, consider secondary over the counter markets to generate such a premium, whiel Berentsen and Waller (2011) consider a multilateral and competitive market.

a variety of monetary policy normalization counterfactuals.

Our main results are as follows. When public debt does not exhibit premia, the stationary equilibria is unique and the prescriptions for determinacy of equilibria are similar to those found in Leeper (1991). For long-run public debt and its dynamics, what is crucial is the total real debt net of the debt held by the central bank. However, how much public debt is held by the private sector does not affect inflation dynamics. We also find that changes to the composition of the balance sheet (either in terms of desired maturity structure target or the specific response to deviations from its compositional target) does not impact inflation nor debt dynamics. This is not surprising as the economy is Ricardian and all public debt is priced fundamentally.

When short-term bond premia exists, changes in the central bank's balance sheet can alter real allocations. In particular, by changing the composition of assets in its balance sheet, the central bank can influence households' portfolio decisions. This is the case as agents face imperfect asset substitutability when trading in frictional and decentralized markets. We also find that the pledgeability of short-term bonds has first-order consequences for long-run inflation as well as for the uniqueness and local stability of steady states. For a given balance sheet size target, when following active monetary policies and to ensure a unique and locally stable steady state inflation rate, it is crucial that the central bank targets a low enough maturity composition of government bonds. When calibrated to the United States, we find that when the central bank holdings of long-term debt is less than 1.8 times than that of their short-term bond holdings, the resulting stationary equilibria is unique and locally determinate. These results are robust to whether interest rate management policies take short-term public debt premium into account or not.[8] Moreover, the process of balance sheet normalization should be slow enough and depend on current economic conditions. This ensures a long-run inflation rate that is close to its target (when the monetary authority does not consider the premium) and higher welfare independently of whether the monetary authority takes the premium into account or not. The specifics determining the speed and economic conditions critically depend on the central bank's balance size target and the fiscal authority's debt and compositional targets.

Under a modified Taylor rule that takes into account of the premium of short-term debt, we find that the long-run inflation is equal to the central bank's target. However, such interest rate

[8] With a Taylor rule that takes into account the premium, steady state inflation is unique, but steady state bond holdings may not be.

policy increases the prevalence of multiple steady states. Thus, the compositional target becomes even more important to deliver desirable equilibria. In addition, these stationary equilibria result in lower welfare. Thus, given that a traditional Taylor rule is less likely to deliver real indeterminacies and yields higher welfare, this policy should be the preferred one when managing interest rates. Finally, our findings also show that an adequate balance sheet normalization process critically depends on the size and maturity composition targets of the fiscal authority. These findings highlight that further coordination between fiscal and monetary authorities is needed when monetary normalization begins.

This paper is organized as follows. Section 2 follows with a review of the recent literature. Section 3 presents the economic environment. Section 4 characterizes the resulting dynamic equilibrium. Section 5 provides a numerical example. Finally, Section 6 concludes.

2 Literature Review

This paper connects with two strands of literature. One that studies unconventional monetary policies when the economic environment has short and long-term government bonds. The other literature that we relate to is the one on monetary policy normalization.

Within the context of the Great Recession, there is now a voluminous empirical literature that has tried to evaluate the effectiveness of unconventional monetary policies enacted by the Federal Reserve.[9] However, fewer theoretical studies have analyzed the inherent trade-offs of a central bank trading public debt of different maturities to implement such policies. Notable exceptions are that of Harrison (2011), Chen et al. (2012), among others. In particular, Harrison (2011) considers the New Keynesian segmented financial market framework of Andres et al. (2004), where agents have different preferences over public debt of different maturities.[10] Harrison (2011) shows that to the extent that asset purchase programs reduce long-term interest rates, aggregate demand can be stimulated.[11] Within a similar framework, Chen et al. (2012) consider various nominal and real rigidities and estimate the effects of large scale asset purchase programs. Relative to no intervention, the authors find that US GDP growth increases by less than a third

[9] We refer to Dell'Ariccia et al. (2018) for an excellent survey.

[10] In this setting, trading debt of different maturities exert an additional effect, through imperfect asset substitutability, on long-term yields and aggregate demand. This latter demand effect is separate from the traditional effect on the expected path of short-term rates.

[11] This leads to higher inflation through a conventional New Keynesian Phillips curve.

of a percentage point and inflation barely changes.[12] Within a flexible price environment with fiat money, government debt of different maturities, credit and banking, Williamson (2016) shows that when private banks face scarcity of collateralizable wealth, the economy displays an upward-sloping nominal yield curve. Under such scenario, the author finds that central bank purchases of long maturity government debt are always a good idea. Within a similar framework, Williamson (2019) compares two policies in a two sector general equilibrium banking model where the central bank runs a floor system.[13] The author shows that an increase in the central bank's balance sheet can have redistributive effects and reduce welfare. In contrast, a reverse repo facility at the central bank puts a floor under the interbank interest rate, always improving welfare. When the central bank holds an explicit portfolio of short and long-term public debt, Reis (2017) studies unconventional monetary policies in a simplified sticky price environment where agents have access to one and two-period nominal government bonds. The author shows that the power of quantitative easing policies to alter real allocations is due to the interest payment on reserves.[14] Within the same spirit, Arce et al. (2019) evaluates quantitative easing policies in a New Keynesian model with heterogeneous banks. The authors show that a lean-balance-sheet regime with temporary and prompt quantitative easing achieves similar stabilization and welfare outcomes than that of a large-balance-sheet regime where managing interest rates is the primary adjustment margin.[15]

After signs of economic recovery, practitioners discussed three main approaches to undoing unconventional monetary policies enacted during the financial crisis.[16] One policy option is to leave excess reserves unchanged and pay interest on reserves. Another strategy is to absorb reserves through reverse repos, central bank bills, or term deposits. Finally, another option is to sell assets purchased under quantitative easing programs. When compared to the one on unconventional monetary policies, the literature examining the consequences of such exit strategies is rather small. An important contribution is the work of Armenter and Lester (2017). These authors provide a model that captures the institutional details of the US money market

[12]Key for their results is the role of transaction costs and segmented markets in effectively compressing term premia.

[13]Only retail banks can hold reserves, but these banks are subject to capital requirements.

[14]Reis (2017) notes that reserves are a special public asset that is neither substitutable by currency nor by government debt.

[15]A key features that helps deliver their main findings is that banks trade funds in an interbank market characterized by matching frictions.

[16]These are typically referred as *exit strategies*.

and study the central bank's ability to increase the policy rate in an environment with large excess reserves. The authors find that a cap on volume at the overnight reverse repurchase agreements facility poses a risk to successful monetary policy implementation. This risk increases as the target range rises (holding the spread between the interest on overnight excess reserves and the overnight reverse repurchase agreements rates fixed) but falls as the spread widens. A long similar lines, Berentsen et al. (2018) study the consequences of paying interest on reserves. The authors find that it is optimal if the central bank has full fiscal support. If the central bank has no fiscal support, reducing reserves is optimal. This can be achieved by reserve absorbing operations, which hold the size of the balance sheet constant, or by selling assets, which reduces the size of the balance sheet. When calibrated to the Swiss franc repo market, the authors find that absorbing reserves using term deposits is equivalent to selling assets and thus unwinding quantitative easing.[17]

We depart from the important contributions in these two different literatures by studying monetary the consequences of reducing central bank's assets. Moreover, we provide explicit balance sheet rules, consistent with the Policy Normalization Principles and Plans, that take into account the desired size and composition of their public debt holdings.

3 The Environment

Time is discrete and there is a continuum of infinitively-lived agents of measure one that discount the future at a rate $\beta \in (0, 1)$. As in Lagos and Wright (2005) and Rocheteau and Wright (2005), agents face stochastic trading opportunities and trade in sequential markets. Each period has two sub-periods. The first one corresponds to a decentralized and frictional specialized goods market (DM). The other sub-period is a frictionless and competitive centralized market (CM). In DM agents receive a preference shock that determines whether they are consumers or producers in this market. After preference shocks are realized, DM consumers are randomly and bilaterally matched with DM producers. When trading in DM, other than search frictions, agents also face limited commitment. As a result, producers do not provide unsecured credit. To obtain DM credit, consumers are required to post collateral.[18] However, not all assets have

[17]Note that the central bank's balance sheet size remains unchanged with term deposits whereas selling assets reduces the central bank's balance sheet.

[18]We refer to Kiyotaki and Moore (1997) for more on the need to collaterize loans.

the same pledgeability properties. In particular, short-term bonds can be used as collateral, while long-term debt can not. In the second sub-period, agents enter a centralized, frictionless and competitive market. In CM all agents can produce and consume a general perishable good, re-adjust their portfolio of short and long-term government bonds, settle their private debt and pay their tax liabilities. Nominal government bonds are the only durable objects in the economy that can help buyers and sellers smooth their consumption.

Preferences. Agents have preferences over the consumption of CM goods (x_t), effort to produce CM goods (h_t), consumption of specialized DM goods (q_t) and effort to produce DM goods (e_t). The expected utility of the i-agent is then given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln(x_t) - h_t + \chi_{i,t} \frac{q_t^{1-\xi}}{1-\xi} + \frac{\chi_{i,t} - \chi}{\chi} e_t \right], \tag{1}$$

where $\xi \in (0,1)$ is the inverse of the intertemporal elasticity of substitution of DM consumption and $\chi_{i,t} = \{0, \chi\}$ is an idiosyncratic time-varying shock. This preference shock is such that, when $\chi_{i,t} = 0$, then the i-agent is a DM producer, and when $\chi_{i,t} = \chi > 0$, the i-agent is a DM consumer. This shock is independently distributed across all agents. Finally, E_0 denotes the linear expectation operator with respect to an equilibrium distribution of idiosyncratic agent types.

Technologies. All DM and CM goods in the economy are produced with a linear technology where labor is the only input. The production function is such that one unit of labor yields one unit of output.

Assets. Agents in this economy have access to one period government nominal debt, which we denote by B_t^S. From now we refer to this type of debt as short-term bonds. Agents also have access to a more general portfolio of public nominal debt, which we denote by B_t^L. Following Woodford (2001), this latter debt instrument has a nominal payment structure equal to $\rho^{T-(t+1)}$, where $T > t$ and $0 < \rho < 1$. This asset can be interpreted as a portfolio of infinitely many nominal bonds, with weights along the maturity structure given by $\rho^{T-(t+1)}$.[19] From now on we refer these as long-term bonds and their price is denoted by Q_t. On the other hand, nominal

[19] For example, one-period debt corresponds to $\rho = 0$, while a consol bond is consistent with $\rho = 1$.

interest rate corresponding to a short-term public debt purchased at time t is represented by R_t.

Frictions and Trades. In the first sub-period, agents face stochastic trading opportunities. In particular, DM consumers (buyers) and DM producers (sellers) are bilaterally matched with probability $\sigma \in [0,1]$. After matching takes place, these agents also face limited commitment when trading in DM. Thus, to consume q in DM, buyers can promise the DM seller a payment of q in the next CM. However, due to limited commitment, the DM buyer can renege on his future payment. This possibility allows assets to have a role as collateral. The usual interpretation of such arrangement is that if a borrower reneges on his promise, his assets are seized. This contingency dissuades opportunistic default. Note, however, that, as in Rocheteau et al. (2018), one can also describe DM trade as a repurchase agreement, where a buyer getting q gives assets to a seller, who gives them back at prearranged terms in the next CM.[20] We assume that not all assets are equally pledgeable when trading in DM as in Rocheteau et al. (2018) and Dong et al. (2019). In particular, we assume that short-term are acceptable as collateral, while long-term bonds are not.[21]

Government

As in Del Negro and Sims (2015), among others, we distinguish between the central bank's balance sheet and the rest of the government budget constraint. Below we describe how these two different institutions implement policy.

Fiscal Authority

This government agency needs to finance an exogenous and constant stream of expenditures, which we denote by G, and outstanding debt interest payments. To finance them, the fiscal authority has access to lump-sum CM taxes, τ_t^{CM}, and the issuance of short and long-term nominal bonds. The corresponding budget constraint for the fiscal authority is then given by

$$\tau_t^{CM} + \phi_t B_t^S + Q_t \phi_t B_t^L + T_t^C = G + R_{t-1} \phi_t B_{t-1}^S + (1 + \rho Q_t)\, \phi_t B_{t-1}^L, \tag{2}$$

[20] As in Rocheteau et al. (2018), we do not propose a deep theory of repurchase agreements (repos). We refer to Antinolfi et al. (2015) or Gottardi et al. (2015) for more on repos.

[21] This is consistent with the United States repo market where the majority of trades involve three months or less. We refer the reader to the Repurchase Agreement Operational Details for additional information, which can be found at Federal Reserve Bank of New York.

where $\phi_t \equiv \frac{1}{P_t}$ is the real price of the CM good and T_t^C is the transfer given by the central bank. The real value of all bond issuance is $\phi_t B_t = \phi_t \left(B_t^S + Q_t B_t^L\right)$. We assume that it is bounded above by a sufficiently large constant to avoid Ponzi schemes.

To describe the specific operating procedures for fiscal policy, we consider the debt sustainability framework commonly used for policy analysis by the IMF.[22] In particular, we consider a fiscal rule that links taxes with the quantity of real government debt outstanding. More specifically, we have that

$$\tau_t^{CM} = \gamma_0 + \gamma^S \left(\phi_{t-1} B_{t-1}^S - b^{S*}\right) + \gamma^L \left(\phi_{t-1} Q_{t-1} B_{t-1}^L - b^{L*}\right), \tag{3}$$

where γ_0 determines how taxes are set regardless of the economy's debt structure and γ^S $\left(\gamma^L\right)$ captures how taxes respond to the level of short-term (long-term) bonds. Finally, a variable with an asterisk denotes its corresponding target. Thus, b^{S*} and b^{L*} represent the real target levels for short and long-term public debt, respectively. From now on, we define total real debt, $b_t = \phi_t B_t$, as follows

$$b_t = b_t^S + Q_t b_t^L,$$

where we have that $b_t^S = \phi_t B_t^S$ and $b_t^L = \phi_t B_t^L$. Taking these into account, the fiscal rule can then be written as follows

$$\tau_t^{CM} = \gamma_0 + \gamma_1 \left(b_{t-1} - b^*\right). \tag{4}$$

Implicit in this rule is that the fiscal authority also specifies a constant composition of the new issuance of long and short-term bonds so that $\Omega \equiv \frac{Q_t B_t^L}{B_t^S}$, where the balance sheet composition target is such that $\Omega^* = \frac{Q^* b^{*L}}{b^{*S}}$. In particular, we have that with $\gamma_1 = \left(\frac{\gamma^S + \gamma^L \Omega}{1+\Omega}\right)$ and the total debt target is $b^* = b^{S*} + Q^* b^{L*}$.

Monetary Authority

In contrast to most of the literature, we consider additional monetary rules. More precisely, the central bank specifies the targets regarding the size and composition of its balance sheet. To capture some features of the *Policy Normalization Principles and Plans* outlined by the FOMC in 2014, we consider the following rules

$$b_t^M = \gamma_{-1}^M b_{t-1}^M + \gamma_0^M + \gamma_1^M \left(b_t - b^*\right), \tag{5}$$

[22] We refer to Article IV in the country reports as well as Ghosh et al. (2013) and the references therein for more on debt sustainability frameworks.

$$\frac{1}{(1+\Omega_t^M)} b_t^M = \eta_0^M + \eta_1^M \left(\frac{1}{(1+\Omega)} b_t - \frac{1}{(1+\Omega^*)} b^* \right), \tag{6}$$

where $b_t^M = \theta_t^S b_t^S + Q_t \theta_t^L b_t^L$ represents the monetary authority's total bond holdings and θ_t^S (θ_t^L) is the fraction of all outstanding short-term (long-term) public debt held by the central bank. We denote the composition of public debt in the hands of the monetary authority by $\Omega_{t-1}^M = \frac{Q_{t-1}\theta_{t-1}^L B_{t-1}^L}{\theta_{t-1}^S B_{t-1}^S}$. Taking the operating procedures for fiscal policy into account, the asset composition of the central bank is then given by $\Omega_{t-1}^M = \frac{\theta_{t-1}^L}{\theta_{t-1}^S}\Omega$. Finally, we have that $\gamma_{-1}^M = \frac{1}{\beta}$, $\gamma_0^M = (1-\frac{1}{\beta})b^{M*}$ and $\eta_0^M = \frac{1}{(1+\Omega^{M*})} b^{M*}$, where b^{M*} represents the target level of total public debt held by the central bank and Ω^{M*} is the compositional target of these bonds.

In addition to deciding the composition and size of the balance sheet, the central bank also manages interest rates through a Taylor rule. In particular, we have that

$$R_t = \alpha_0 + \alpha_1 (\Pi_t - \Pi^*) - \alpha_2 s_t, \tag{7}$$

where s_t represents the premium on short-term public debt, which will be endogenously determined, $\Pi_{t+1} = \frac{\phi_t}{\phi_{t+1}}$ denotes the gross inflation rate, α_0 is a constant that determines how interest rates are set regardless of the economy's inflation rate, while α_1 captures how interest rates respond to inflation rate departures from its target Π^*.[23] Finally, α_2 measures the central bank's response to observed debt premia. With $\alpha_2 = 0$, we recover a *traditional* Taylor rule that does not take into account premia. With $\alpha_2 = 1$, we capture a *modified* Taylor rule a long the lines of Cúrdia and Woodford (2010), which explicitly takes into account bond premia.[24]

The corresponding budget constraint for the monetary authority is then given by

$$T_t^C + \theta_t^S \phi_t B_t^S + \theta_t^L Q_t \phi_t B_t^L = R_{t-1}\theta_{t-1}^S \phi_t B_{t-1}^S + \theta_{t-1}^L (1+\rho Q_t) \phi_t B_{t-1}^L. \tag{8}$$

3.1 Agent's Problem

Given the sequential nature of the environment, we solve the representative agent's problem backwards. Thus, we first solve the CM and then the DM problems.

[23]This corresponds to the long-run inflation rate of an economy with no bond premia.

[24]According to McCulley and Toloui (2008) and Taylor (2008), spread adjustments to interest rate policy settings should be equal to the size of the increase in spreads. This situation corresponds to $\alpha_2 = 1$. In a sticky price model, Cúrdia and Woodford (2010) analyze intermediate responses where $\alpha_2 \in (0, 1)$.

CM Problem

In this market all agents can produce and consume the CM good, while trading in a frictionless and competitive market. Agents can settle their CM trades with any assets, CM goods or CM labor.

Given a portfolio of nominal government bonds ($\tilde{B}^S_{t-1}$,$\tilde{B}^L_{t-1}$) at the beginning of CM, the problem of a representative agent is as follows

$$W\left(\tilde{B}^S_{t-1},\tilde{B}^L_{t-1},\tilde{L}_{t-1}\right)=\max_{x_t,h_t,\tilde{B}^S_t,\tilde{B}^L_t}\left\{\ln(x_t)-h_t+\ \beta\,V^{DM}\left(\tilde{B}^S_t,\tilde{B}^L_t\right)\right\}\quad \text{s.t.} \tag{9}$$

$$x_t+\phi_t\tilde{B}^S_t+Q_t\phi_t\tilde{B}^L_t+\phi_t\tilde{L}_{t-1}=h_t-\tau^{CM}_t+\phi_t\left(1+\rho Q_t\right)\tilde{B}^L_{t-1}+\phi_t R_{t-1}\tilde{B}^S_{t-1},$$

where V^{DM} is the agent's expected DM value function, $\tilde{B}^S_t$ ($\tilde{B}^L_t$) denotes the agent's short-term (long-term) bond holdings in period t and $\tilde{L}_{t-1}$ represents the nominal payment of an agent that was granted a loan in DM. Note that while the initial portfolio of nominal government bonds ($\tilde{B}^S_{t-1}$, $\tilde{B}^L_{t-1}$) is the same across agents, the secured loan, $\tilde{L}_{t-1}$, may be different. This is the case as when agents trade in DM, buyers obtain the good through a secured loan and no bonds change hands between buyers and sellers.

The corresponding first-order conditions are given by

$$\frac{1}{x_t}-1=0, \tag{10}$$

$$-\phi_t+\beta\,\frac{\partial V^{DM}\left(\tilde{B}^S_t,\tilde{B}^L_t\right)}{\partial\hat{B}^S_t}=0, \tag{11}$$

$$-\phi_t Q_t+\beta\,\frac{\partial V^{DM}\left(\tilde{B}^S_t,\tilde{B}^L_t\right)}{\partial\hat{B}^L_t}=0. \tag{12}$$

The associated envelope conditions are $\frac{\partial W_t}{\partial\tilde{B}^S_{t-1}}=\phi_t R_{t-1}$, $\frac{\partial W_t}{\partial\tilde{B}^L_{t-1}}=\phi_t\left(1+\rho Q_t\right)$ and $\frac{\partial W_t}{\partial\tilde{L}_{t-1}}=-\phi_t$.

It is important to note that households hold fractions $(1-\theta^S_t)$ and $(1-\theta^L_t)$ of all short and long-term bonds issued at time t, respectively. Because of quasi-linearity, all agents enter DM with the same portfolio of bond holdings, as they do not depend on past holdings. This implies then that all agents after CM hold $\tilde{B}^S_t=\left(1-\theta^S_t\right)B^S_t$ and $\tilde{B}^L_t=\left(1-\theta^L_t\right)B^L_t$. The rest of the public debt is held by the central bank.

DM Problem

At the beginning of each period, agents experience a preference shock. With probability $\frac{1}{2}$ $\left(\frac{1}{2}\right)$ an agent becomes a consumer (producer) in the ensuing DM. Thus, before the shocks are realized, the corresponding value function of an agent with a portfolio of short and long-term bonds $(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})$ is given by

$$V_b^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) = \frac{1}{2}\left[V_b^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) + V_s^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})\right],$$

where V_j^{DM} is the expected DM utility of an agent of type $j = \{b, s\}$, where subscript b denotes a DM consumer (buyer) and subscript s represents a DM producer (seller).

After the preference shock is realized, consumers and producers are randomly and bilaterally matched. Other than search frictions, agents in this market also face limited commitment. The value of DM consumer with a beginning of period portfolio $(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1})$ is given by

$$V_b^{DM}(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}) = \sigma\left(\chi\frac{q_t^{1-\xi}}{1-\xi} + W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, \tilde{L}_{t-1}\right)\right) + (1-\sigma)W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, 0\right),$$

where q_t denotes the quantity of DM goods purchased and σ is the matching probability. Because of limited commitment (agents can renege on their future payments), in order to trade, DM consumers use their short-term bonds as collateral.[25] This is the case as they are the only assets that are pledgeable. Thus, the amount of credit extended in DM is such that $\tilde{L}_{t-1} \leq \eta\hat{B}^S_{t-1}$, where η is the pledgeablility parameter that determines the fraction of short-term bonds that can be used as collateral. This fraction captures the severity of the limited commitment problem in DM.

Similarly, the expected utility of a DM producer is given by

$$V_s^{DM}(\hat{B}^S_{t-1}, \hat{B}^L_{t-1}) = \sigma\left[-q_t + W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, -\tilde{L}_{t-1}\right)\right] + (1-\sigma)W\left(\tilde{B}^S_{t-1}, \tilde{B}^L_{t-1}, 0\right).$$

The trading protocol in this frictional market is determined ex-post by a DM consumer take-it-or-leave-it offer with threat point of no trade. In order to induce trade, a DM consumer needs to propose terms of trade that satisfy the DM producer's participation constraint and be consistent

[25] As in Kiyotaki and Moore (1997), Bernanke et al. (1999), Iacoviello (2005), Andolfatto and Martín (2018) and Berentsen and Waller (2018), among others, because of limited commitment, the loan extended to DM consumers has to be collaterized.

with his borrowing constraint. Formally, the terms of trade solve the following problem

$$\max_{q_t,\tilde{L}_{t-1}} \left\{ \chi \frac{q_t^{1-\xi}}{1-\xi} + W\left(\tilde{B}^S_{b,t-1}, \tilde{B}^L_{b,t-1}, \tilde{L}_{t-1}\right) - W\left(\tilde{B}^S_{b,t-1}, \tilde{B}^L_{b,t-1}, 0\right) \right\} \text{ s.t.}$$

$$\tilde{L}_{t-1} \leq \eta \tilde{B}^S_{b,t-1},$$

$$-q_t + W\left(\tilde{B}^S_{s,t-1}, \tilde{B}^L_{s,t-1}, -\tilde{L}_{t-1}\right) \geq W\left(\tilde{B}^S_{s,t-1}, \tilde{B}^L_{s,t-1}, 0\right),$$

where $\tilde{B}^S_{b,t-1}(\tilde{B}^S_{s,t-1})$ and $\tilde{B}^L_{b,t-1}(\tilde{B}^L_{s,t-1})$ represent the consumer's (producer's) short and long-term nominal bond holdings, respectively. As in Kiyotaki and Moore (1997), the collateral used by DM consumers do not change hands in DM as they are used in case of default, which never occurs in equilibrium.

Since DM consumers make a take-it-or-leave-it offer, they extract all the surplus from the match. This leaves the DM producer indifferent between producing or not trading. After substituting the various CM value functions, we have that $q_t = \phi_t \tilde{L}_{t-1}$. Thus, the resulting optimal terms of trade are then given by

$$\chi q_t^{-\xi} = 1 + \lambda_t,$$

$$\lambda_t \left(\eta \tilde{B}^S_{b,t-1} - \tilde{L}_{t-1}\right) = 0,$$

where λ_t is the Lagrange multiplier associated with the loan payment constraint arising from the limited commitment problem. These imply the following DM consumer's envelope conditions

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}^S_{b,t-1}} = \sigma \left[\frac{\chi}{q_t{}^\xi} \frac{\partial q_t}{\partial \tilde{L}_{t-1}} \frac{\partial \tilde{L}_{t-1}}{\partial \tilde{B}^S_{b,t-1}} - \phi_t \frac{\partial \tilde{L}_{t-1}}{\partial \tilde{B}^S_{b,t-1}} + \phi_t R_{t-1} \right] + (1-\sigma)\phi_t R_{t-1},$$

$$\frac{\partial V_b^{DM}}{\partial \tilde{B}^L_{b,t-1}} = \phi_t \left(1 + \rho Q_t\right).$$

For the DM producer, we have instead

$$\frac{\partial V_s^{DM}}{\partial \tilde{B}^S_{s,t-1}} = \phi_t R_{t-1}, \quad \frac{\partial V_s^{DM}}{\partial \tilde{B}^L_{s,t-1}} = \phi_t \left(1 + \rho Q_t\right).$$

Throughout the rest of the paper, we focus on equilibria where the borrowing constraint binds. Thus, we have that $q_t = \phi_t \tilde{L}_{t-1} = \phi_t \eta \tilde{B}^S_{b,t-1}$. As a result, the marginal effects of bringing additional units of short and long-term bonds into DM imply the following intertemporal Euler equations

$$\phi_t = \beta\phi_{t+1}\left\{R_t + s_{t+1}\right\}, \tag{13}$$

$$\phi_t Q_t = \beta\phi_{t+1}\left(1 + \rho Q_{t+1}\right). \tag{14}$$

In addition to receiving an interest rate R_t, holding short-term bonds also includes a liquidity premium. This is the case as it can help expand the consumption possibilities in DM. Such additional value is captured by the present value of the Lagrangian multipliers of the collateral constraints it relaxes. From equation (13), the endogenous value of the liquidity premium on short-term debt is captured by s_{t+1}, which is given by $s_{t+1} = \frac{\sigma}{2}\left(\frac{\chi}{q_{t+1}{}^{\xi}} - 1\right)\eta$.

4 Dynamic Equilibrium

Having characterized the representative agent's optimal decisions, we can now define the corresponding dynamic equilibrium for this economy.

Definition 1 *Given the operating procedures for monetary policy (equations (5), (6) and (7)) and fiscal policy (equation (4) and* $B_t^L = B_t^S\Omega$*), public spending* $\{G\}_{t=0}^{\infty}$ *and initial conditions* $(B_{-1}^S, B_{-1}^L, \theta_{-1}^S, \theta_{-1}^L)$*, a dynamic equilibrium is a sequence of consumptions* $\{x_t, q_t\}_{t=0}^{\infty}$*, assets and prices* $\left\{B_t^S, B_t^L, Q_t, R_t, \phi_t\right\}_{t=0}^{\infty}$ *satisfying the agents' problems and clearing all markets.*

After imposing the agents' optimal decisions, market clearing and substituting $b_t^S = \frac{1}{(1+\Omega)}b_t$, $Q_t b_t^L = \frac{\Omega}{(1+\Omega)}b_t$, $\theta_t^S b_t^S = \frac{1}{\left(1+\Omega_t^M\right)}b_t^M$ and $Q_t\theta_t^L b_t^L = \frac{\Omega_t^M}{\left(1+\Omega_t^M\right)}b_t^M$, it is easy to show that the dynamic equilibrium is characterized by the following system of dynamic equations

$$x_t = 1, \tag{15}$$

$$q_{t+1} = \eta\left(\frac{b_t}{(1+\Omega)} - \frac{b_t^M}{(1+\Omega_t^M)}\right)\frac{1}{\Pi_{t+1}}, \tag{16}$$

$$\Pi_{t+1} = \beta\left(R_t + s_{t+1}\right), \tag{17}$$

$$\Pi_{t+1}Q_t = \beta\left(1 + \rho Q_{t+1}\right), \tag{18}$$

$$T_t^C + b_t^M = \frac{1}{\beta}b_{t-1}^M - \frac{s_t}{\Pi_t}\frac{b_{t-1}^M}{1+\Omega_{t-1}^M}, \tag{19}$$

$$\tau_t^{CM} + b_t + T_t^C = G + \frac{1}{\beta}b_{t-1} - \frac{s_t}{\Pi_t}\frac{1}{1+\Omega}b_{t-1}, \tag{20}$$

where R_t is determined by the Taylor rule (given by equation (7)), τ_t^{CM} is specified by the fiscal rule (given by equation (3)) and normalization monetary policies, b_t^M and Ω_t^M are given by equations (5) and (6), respectively.

As we can see, consumption in CM, x_t, is efficient. However, efficiency in DM consumption, q_{t+1}, depends on the size of the short-term bond liquidity premium. After substituting the monetary and fiscal rules, we can rewrite the dynamic equilibrium as a sequence $\{\Pi_{t+1}, b_t, b_t^M, \Omega_t^M\}$ that satisfies the following non-linear equations

$$\Pi_{t+1} = \beta \left(\alpha_0 + \alpha_1 \left(\Pi_t - \Pi^*\right) - \alpha_2 s_t + s_{t+1}\right), \tag{21}$$

$$b_t - b_t^M = G - \gamma_0 - \gamma_1 \left(b_{t-1} - b^*\right) + \frac{1}{\beta}\left(b_{t-1} - b_{t-1}^M\right) - \frac{s_t}{\Pi_t}\left(\frac{b_{t-1}}{1+\Omega} - \frac{b_{t-1}^M}{1+\Omega_{t-1}^M}\right), \tag{22}$$

$$b_t^M = \frac{1}{\beta} b_{t-1}^M + \gamma_0^M + \gamma_1^M \left(b_t - b^*\right), \tag{23}$$

$$\frac{b_t^M}{(1+\Omega_t^M)} = \frac{b^{M*}}{(1+\Omega^{M*})} + \eta_1^M \left(\frac{b_t}{(1+\Omega)} - \frac{b^*}{(1+\Omega^*)}\right). \tag{24}$$

It is important to highlight that the characterization of the dynamic equilibria depends on whether a liquidity premium exits or not. Next we examine these two possibilities.

Case 0: No Liquidity Premium

In this scenario, the dynamic equilibria is described by

$$\Pi_{t+1} = \beta \left[\alpha_0 + \alpha_1(\Pi_t - \Pi^*)\right],$$

$$b_t = G - (\gamma_0 - \gamma_0^M) + \frac{1}{\beta} b_{t-1} - \left(\gamma_1 - \gamma_1^M\right)\left(b_{t-1} - b^*\right).$$

As in environments that do not include central bank balance sheet policies, the evolution of inflation is independent of total real debt in the economy. It is easy to show that the corresponding steady state is unique and given by

$$\Pi = \Pi^* = \beta\alpha_0, \;\; \& \;\; b = b^* = \left(\frac{\beta}{1-\beta}\right)\left[(\gamma_0 - \gamma_0^M) - G\right],$$

which implies that the buyer consumes the first best in DM, $q = 1$, and that the price of the long-term nominal bonds is given by $Q = \frac{\beta}{\Pi - \beta\rho}$. This implies that short and long-term government bonds are priced fundamentally.

The local dynamics associated with this stationary equilibria are characterized by the following Jacobian

$$J = \begin{pmatrix} \beta\alpha_1 & 0 \\ \gamma_2^M & \frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) \end{pmatrix},$$

which results in a monetary and a fiscal eigenvalue that are independent of each other. As a result, the normative prescriptions for determinacy of equilibria are similar to those found in Leeper (1991). Nevertheless, it is worth highlighting that the real debt that is relevant for the fiscal eigenvalue is the one held by the public. More precisely, what is relevant for local stability properties is the total real debt net of the debt held by the central bank. Thus the speed at which the central bank adjusts its size relative to its target affects the evolution of real debt. Moreover, it also affects the corresponding local stability properties.[26] However, it does not affect inflation dynamics. On the other hand, changes to the composition of the balance sheet (either in terms of desired maturity structure target or the specific response to deviations from target) does not impact inflation nor debt dynamics. This is not surprising as the economy is Ricardian and all public debt is priced fundamentally.[27]

For the remainder of the paper and following Leeper (1991), we refer to active (passive) monetary policy to one that satisfies $\beta\alpha_1 > 1$ ($\beta\alpha_1 < 1$) and to passive (active) fiscal policy to one that satisfies $\frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) < 1$ ($\frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) > 1$).

Case 1: Liquidity Premium

When holding additional short-term bonds expand agents' DM consumption possibilities, the dynamic equilibria can be summarized by the evolution of inflation and total government debt

$$\Pi_{t+1} = \Pi^* + \beta\alpha_1(\Pi_t - \Pi^*) + \beta\left(s_{t+1} - \alpha_2 s_t\right),$$

$$b_t = G - (\gamma_0 - \gamma_0^M) + \frac{b_{t-1}}{\beta} - \left(\gamma_1 - \gamma_1^M\right)(b_{t-1} - b^*) - \frac{s_t}{\Pi_t}\left(\frac{b_{t-1}}{1+\Omega} - \frac{b^{M*}}{(1+\Omega^{M*})}\right) + \\ + \frac{s_t}{\Pi_t}\eta_1^M\left(\frac{b_{t-1}}{(1+\Omega)} - \frac{b^*}{(1+\Omega^*)}\right),$$

[26]In Leeper (1991), Woodford (1998), among others, the central bank does not hold any public debt. This situation can be thought as one where $\gamma_1^M = 0$.

[27] As pointed out by Wallace (1981) and Lucas (1984), when an economy is Ricardian the maturity structure of government debt is totally irrelevant.

where $s_{t+1} = \frac{\sigma}{2}\left(\frac{\chi}{q_{t+1}{}^{\xi}} - 1\right)\eta$, and $q_{t+1} = \eta\left(\frac{b_t}{(1+\Omega)} - \frac{b^{M*}}{(1+\Omega^{M*})} - \eta_1^M\left(\frac{b_t}{(1+\Omega)} - \frac{b^*}{(1+\Omega^*)}\right)\right)\frac{1}{\Pi_{t+1}}$. The corresponding targets $\Pi^* = \beta\alpha_0$, and $b^* = \frac{\beta}{1-\beta}\left(\left[(\gamma_0 - \gamma_0^M) - G\right] + \alpha_2\frac{s^*}{\eta}q^*\right)$, where we have that $s^* = \frac{\sigma}{2}\left(\frac{\chi}{q^{*\xi}} - 1\right)\eta$ and $q^* = \eta\left(\frac{b^*}{(1+\Omega^*)} - \frac{b^{M*}}{1+\Omega^{M*}}\right)\frac{1}{\Pi^*}$.

In contrast to the environment without a premium on short-term debt, the evolution of inflation and total real debt are not independent of each other. Moreover, operating procedures for monetary and fiscal policy both affect the price level. This is the case even when the monetary authority follows a Taylor principle and fiscal policy is passive. This is not surprising as the spread on short-term bonds depends on both fiscal and monetary policies.

After imposing that the economy is in steady state, it is easy to show that the stationary equilibria is characterized by the following two implicit equations

$$\Pi = \Pi^* + \frac{\beta(1-\alpha_2)}{(1-\beta\alpha_1)}\, s(\Pi, b),$$

$$\left(1 - \frac{1}{\beta} + \left(\gamma_1 - \gamma_1^M\right)\right) b = G - (\gamma_0 - \gamma_0^M) + \left(\gamma_1 - \gamma_1^M\right) b^* -$$
$$-\frac{s(\Pi, b)}{\Pi}\left(\left(1 - \eta_1^M\right)\frac{b}{1+\Omega} - \frac{b^{*M}}{(1+\Omega^{*M})} + \eta_1^M\frac{b^*}{(1+\Omega^*)}\right).$$

where $s(\Pi, b) \geq 0$ is the long-run bond premium associated with short-term public debt. Such spread depends on the long-run inflation and total debt circulating in the economy.

As we can see, when the premium is not taken into account by the monetary authority when managing interest rates, $\alpha_2 = 0$, the liquidity premium leads to deviations of steady state inflation from the central bank's target. This deviation increases with the premium $s(\Pi, b)$. As in Domínguez and Gomis-Porqueras (2019), it is also easy to see that an active (passive) monetary policy, $\beta\alpha_1 > 1$ ($\beta\alpha_1 < 1$), induces steady state inflation rates below (above) the target level Π^*. In contrast, when the spread adjustments to interest rate policy settings are equal to the size of the increase in spreads, $\alpha_2 = 1$, we find that steady state inflation is unique and equal to the target level $\Pi = \Pi^* = \beta\alpha_0$. Note that interest rate management policies corresponding to $\alpha_2 = 0$ (traditional Taylor rule) or $\alpha_2 = 1$ (modified Taylor rule that reacts one for one to spreads) lead to the same target inflation. In contrast, the government debt target critically depends on whether the monetary authority takes into account the premium ($\alpha_2 = 1$) or not ($\alpha_2 = 0$). Finally, it is important to highlight that generically multiple steady states can not be ruled out. This is the case as the implicit functions describing the long-run inflation and total real debt are highly non-linear. Note that even when $\alpha_2 = 1$, the implicit equation characterizing the long-run real debt in the economy is highly non-linear, allowing the possibility for multiple steady states to exist.

Now both central bank's balance sheet targets (size and composition) impact the long-run equilibria for inflation and total real debt. This is the case as household's demand for short-term public debt is quite different from that of the central bank. When households acquire an additional unit of short-term debt, they are able to expand their consumption possibilities in DM. Instead, the central bank does obtain any additional payoff from such increase in its portfolio. Thus, the willingness to pay for such asset is quite different. This is a situation not observed when the economy is satiated with short-term debt as in Case 0. Furthermore, balance sheet targets can also potentially affect the existence of multiple stationary equilibria.

In a neighborhood of a steady state, local dynamics are described by the following Jacobian

$$J = \begin{pmatrix} \frac{\beta\alpha_1 - \alpha_2(1-\omega_1)}{\omega_1} + \beta\omega_4\omega_2 & \beta\omega_4\left(\frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) - \omega_3 - \alpha_2\right) \\ \omega_2 & \frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) - \omega_3 \end{pmatrix},$$

where $\omega_1 = 1 - \beta\ \frac{\partial s}{\partial \Pi}$, $\omega_2 = -\left(\frac{\partial s}{\partial \Pi}\Pi - s\right)\frac{Z_b}{\Pi^2}$, $\omega_3 = \frac{\partial s}{\partial b}\frac{Z_b}{\Pi} + \frac{s}{\Pi}\frac{1}{1+\Omega}\left(1 - \eta_1^M\right)$ and $\omega_4 = \frac{\frac{\partial s}{\partial b}}{\omega_1}$, where $Z_b = \left(\frac{b}{1+\Omega} - \frac{b^{M*}}{(1+\Omega^{M*})} - \eta_1^M\left(\frac{b}{(1+\Omega)} - \frac{b^*}{(1+\Omega^*)}\right)\right)$.

As we can see, generically, the corresponding monetary and fiscal eigenvalues depend on all monetary and fiscal policy rules. In particular, both eigenvalues are affected by the various central bank's balance sheet targets and speed of adjustments relative to the target gaps. Thus, the specifics of the normalization process are going to be key for inflation and debt dynamics.

To further illustrate the importance of balance sheet policies, let us consider an extreme situation where the economy is such that the underlying parameters deliver $\omega_2 \approx 0$. This corresponds to a situation where the evolution of inflation is independent from that of total real debt. Then the resulting eigenvalues for this economy are given by

$$\lambda^M \approx \frac{\beta\alpha_1 - \alpha_2\left(1 - \omega_1\right)}{\omega_1},$$

$$\lambda^F \approx \frac{1}{\beta} - \left(\gamma_1 - \gamma_1^M\right) - \frac{\partial s}{\partial b}\frac{Z_b}{\Pi} - \frac{s}{\Pi}\frac{1}{1+\Omega}\left(1 - \eta_1^M\right).$$

In an economy with premia and with $\alpha_2 = 0$, the monetary eigenvalue is larger than in an economy without premia since $\frac{\partial s}{\partial \Pi} > 0$. However, such result can not be established once the monetary policy rule takes into account the premium $\alpha_2 = 1$. Thus, taking spreads into account, when setting interest rate policies, dampens monetary policy effects. However, such interest rate policy setting delivers ambiguous effects for the fiscal eigenvalue. Nevertheless, the size of the fiscal eigenvalue is affected by the target level of bond holdings by the central bank (b^{M*}), its maturity target (Ω^{M*}) and how maturity responds to current economic conditions (η_1^M).

5 A Numerical Exploration

To gain further insights on the consequences for macroeconomic aggregates and local stability of having different normalization policies, we resort to numerical analysis. To discipline the underlying parameters describing the economy, our calibration strategy is as follows. The period we consider covers from 1985 to 2014. To be closer to the literature that examines monetary and fiscal policy interactions in non-search environments, we set $\sigma = 1$ so that agents can always find a counter-party in DM. To pin down the pledgeability parameter, we follow Gorton et al. (2010) and consider haircuts associated with short-term bonds. This yields $\eta = 1$.[28] To discipline the rest of the parameters, we use historical US data at an annual frequency. In particular, to pin down preference parameters we rely on interest rate data, inflation and GDP obtained from the Federal Research of St. Louis Economic Data (*FRED*). Information on short-term bonds holdings is collected from various issues of the *Treasury Bulletin* of the United States. Finally, we assume that policy parameters are consistent with active monetary and passive fiscal policies of an economy that has no short-term bond premia.

In our analysis, we define short-term bonds as the marketable interest-bearing public debt maturing within one year by the end of the fiscal period. We calculate the amount of short-term bonds held by the domestic private sector by assuming that the proportion of foreign and international held bonds is evenly distributed across the different maturities.[29] The 3-Month Treasury constant maturity rate is represented by R_t. We then define the premium in our model, s_{t+1}, as the difference between the 10-Year and the 3-Month Treasury constant maturity. The data analog for the inflation rate in our model, Π_t, is the Consumer Price Index (all items, 2015 = 100). Finally, nominal quantities of short-term bonds privately and domestically held

[28] We later conduct sensitivity analysis for σ and η.

[29] The proportion of privately-held bonds in the hands of foreigners and international investors has been steadily growing over time. This proportion was 17 per cent in 1985 while over 40 per cent in 1999.

as well as output are converted to real and detrended by the average real growth rate of GDP during that period, 2.2 per cent.[30]

Using the corresponding data on inflation, interest rates and short-term government bond premia, we use equation (17) to derive an implied annual discount factor. We then set β as the average for that period, which is equal to 0.97. In our model, the ratio of short-term bonds held by households to output equals $\frac{q_t \frac{\Pi_t}{\eta}}{1+\frac{1}{2}\sigma q_t}.$ Given the values for η and σ, we use data on privately and domestically held short-term bonds and GDP to construct an implied time series for q_t. We further assume that the highest value of q_t, over this 30-year period, coincides with the efficient one, which is given by $q = \chi^{\frac{1}{\xi}}$. We then consider a grid for ξ. For a given value of ξ we compute a value for χ. Then for each pair (χ, ξ) we calculate the implied short-term bond premium s_{t+1} over time. We then choose the pair (χ, ξ) that minimizes the mean square error between the implied short-term bond premia and the historical US data.[31] This procedure yields $\chi = 0.74$ and $\xi = 0.14$.

For the policy parameters, we make the following assumptions. We set $G = 0.23$, consistent with a federal government spending to GDP ratio of 22 per cent as of 2014 Q4. We set b^* to 0.75, which is consistent with a target of public debt to GDP ratio of 72 per cent. The maturity of the public debt is fixed at $\Omega = \Omega^* = 2.35$, to represent the long-term to short-term public debt ratio. For the fiscal rule, given by (4), we fix the intercept, γ_0, to match a public debt (domestically held) to output ratio of 72 per cent. Note that, in order to match it, γ_0 adjusts with γ_0^M. The slope of the fiscal rule is set so that fiscal policy is passive in an economy without the premium. In particular, we consider $\gamma_1 - \gamma_1^M = \frac{1}{\beta} - 1 + 0.0025$, where γ_1 also adjusts with different values of γ_1^M.

To discipline the management of interest rates by the central bank, given by equation (7), we fix the intercept to match an annual inflation target of 2 per cent.[32] This implies $\alpha_0 = 1.05$. For the slope that describes the sensitivity to deviations from the inflation target, we assume $\alpha_1 = 1.5$, which is consistent with an active monetary policy (in an economy without premium) that satisfies the Taylor Principle. For the rest of the policy parameters, we consider a range of

[30] In our model, output equals $1 + \frac{1}{2}\sigma q_t$, while in the data is measured by the Gross Domestic Product (GDP).
[31] We exclude two observations with negative premia, corresponding to the years 2000 and 2006.
[32] On 25 January 2012, U.S. Federal Reserve Chairman Ben Bernanke set a 2% target inflation rate. Until then, the Federal Open Market Committee (FOMC), did not have an explicit inflation target but regularly announced a desired target range for inflation (usually between 1.7% and 2%).

values consistent with various choices of targets and responses to current economic conditions that characterize the balance sheet normalization process.

The resulting parameters and calibration targets are summarized in Table 1.

Table 1: Calibration Targets

Parameter	Target
$\beta = 0.9735$	Implied by (17) with data on Π, R and s in 1985-2014
$\chi = 0.7399$ $\xi = 0.1357$	Short-term bond premia defined by the difference between the 10-Year and the 3-Month Treasury in 1985-2014
$G = 0.2281$	Federal government expenditure of 21.97 % of GDP as of 2014 Q4
$b^* = 0.7457$	Public debt (domestically held) of 71.84 % of GDP as of 2014 Q4
$\Omega^* = 2.3527$	Long-term to short-term public debt (domestically held) as of 2014 Q4
γ_0	Adjusts with γ_0^M to match a public debt (dom. held) of 71.84 % of GDP as of 2014 Q4
γ_1	Adjusts with γ_1^M to ensure passive fiscal policy (without a premium)
$\alpha_0 = 1.0478$	Inflation target of 2 %
$\alpha_1 = 1.5000$	Active monetary policy (without a premium)

5.1 Quantitative Results

To evaluate the effect of the normalization process we consider the following experiment. In 2014 the Federal Reserve had a balance sheet of 4.5 trillion, which corresponds to 45 per cent of GDP. The model counterpart is b^M equal to 0.47. According to FOMC statements, the explicit target size of the balance sheet has been set to be between 2.5 trillion and 3 trillion. In our benchmark, we study the consequences of a reduction to 3 trillion, which implies a target of $b^{M*} = 0.31$. Then, given the size target, we consider different targets for the maturity composition, Ω^{M*}, and responsiveness to deviations from compositional targets, η_1^M, and compute the resulting equilibria. To discipline the range of compositional targets, it is important to note that in 2007 the Federal Reserve had a maturity of long-term bonds that corresponded to $\Omega^{M*} = 2.125$.[33] For robustness, we also explore the properties of equilibria of having a target of 25 per cent of GDP (corresponding $b^{M*} = 0.26$), which would be consistent with a reduction of the balance sheet to 2.5 trillion.

When computing the various experiments, we consider two different interest rate management settings. One is the traditional Taylor rule and the other one is a modified Taylor rule that explicitly takes into account bond premia.

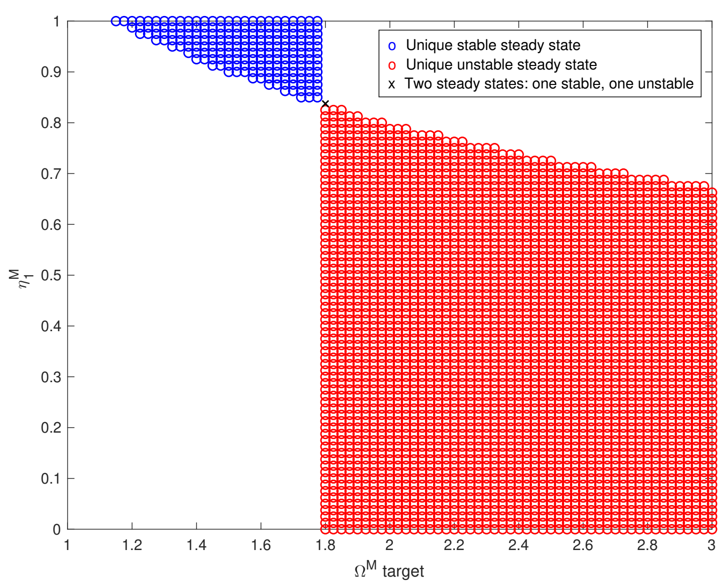
[33]By end of 2014, the Federal Reserve held no short-term treasuries, implying a substantially larger value for Ω^{M*}.

Traditional Taylor Rule

Given the parameters in Table 1, we find that the targets and speed of adjustment in the balance sheet normalization process are important in determining uniqueness and local stability stability properties. More specifically, given the target size of the balance sheet, b^{M*}, then both the maturity compositional target, Ω^{M*}, and the degree of responsiveness to deviations from the compositional target, η_1^M, matter for uniqueness of stationary equilibria and its associated local stability. We find, however, that the degree of responsiveness to target size deviations, γ_1^M, is unimportant. This might be the case as we re-adjust γ_1 so that $\gamma_1 - \gamma_1^M$ remains constant. We do so to ensure that the implied eigenvalues in an economy without a premium remain the same across all our simulations. This allows for meaningful comparisons across the various equilibria. Thus, from now on we fix $\gamma_1^M = 0.50$.

The results of normalizing the central bank's balance sheet when the management of interest rates does not take into account the premium, $\alpha_2 = 0$, are illustrated in Figure 1.

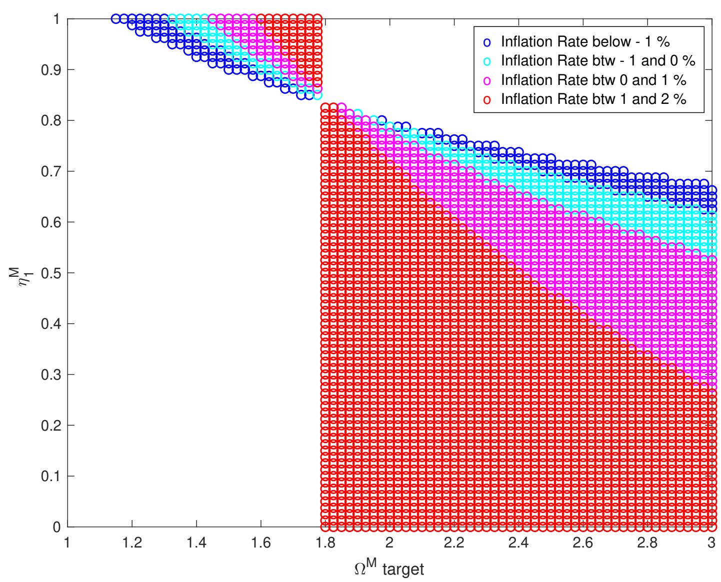
Figure 1: Uniqueness and Stability of Steady States



As we can see from Figure 1, the policy parameter space consistent with multiple steady states is very small. To ensure unique and local determinate equilibria, the central bank needs to target a composition of the balance sheet such that Ω^{M*} <1.8. Notice that this value is substantially lower than the 2007 levels, which was around 2.1. In addition, normalization needs to be sufficiently slow. More precisely, η_1^M needs to be larger than 0.8.[34] We argue that for higher levels of the compositional target, Ω^{M*}, the balance sheet of the central bank increases liquidity in the market, inducing the economy to move to the other side of the liquidity Laffer curve. As a result, the steady state becomes unstable.

Next, we explore the effect of the balance sheet normalization process on long-run inflation. As we previously showed, when the monetary authority does not take into account the premium, $\alpha_2 = 0$, the long-run inflation does not reach its target.[35] In particular, given an active monetary policy, the steady state levels of inflation are below the target level of 2 per cent. But how low are they? Figure 2 reports the net long-run inflation rate of economies with a unique stationary equilibria.

Figure 2: Steady State Inflation Rates



As shown in Figure 2, for a compositional target Ω^{M*} below 1.8, further reductions deliver lower long-run inflation rates. In particular, for low enough maturity targets, the net inflation rate becomes negative. Moreover, for a given composition target, Ω^{M*}, when the responsiveness to deviations from the compositional target, η_1^M, is larger, the resulting inflation rate increases. As a result, it brings the long-run inflation rate closer to the central bank's target. However, for maturity targets Ω^{M*} above 1.8, the effects are reversed. This asymmetric response is, once again,

[34]Note that for a given maturity target Ω^{M*}, a low η_1^M implies a high speed of adjustment. Higher speeds of adjustment can also lead to local instability.

[35]When the monetary authority does not take into account the premium, $\alpha_2 = 0$, inflation is miscalculated and so is bond holdings and maturity composition. When the target of inflation is hit, all other targets are also hit.

a reflection of the equilibrium moving to the other side of the liquidity Laffer curve. On that side of the curve, lowering the maturity composition target increases the steady state inflation. Moreover, for a given Ω^{M*}, the higher is the responsiveness η_1^M, the lower the inflation.

We find qualitatively similar results when the central bank's debt target is lower and set to $b^{M*} = 0.26$.[36] In this new size setting, to ensure unique and local determinate equilibria, the central bank needs a lower compositional target; i.e, Ω^{M*} <1.3. To achieve a similar level of liquidity in the hands of the public, a lower target size of the central bank's balance sheet requires a lower compositional target. We refer the reader to the Appendix for more details.

In addition we also explore the effect of having different responses to deviations from the inflation target. We find such changes to the management of interest rates hardly affect the threshold composition target. Nevertheless, a more aggressive response to inflation leads to a wider range of parameter values for which an equilibrium with premium exists and also allows for a larger parameter space consistent of multiplicity of steady states. Finally, for additional robustness, we also perform additional sensitivity analysis with respect to changes to the degree of pledgeability of short-term bonds η and the extent of search frictions σ in the economy. Overall, we find that a lower pledgeability (lower η) and higher search frictions (lower σ) have minimal changes to the threshold level of Ω^{M*} needed to ensure uniqueness and stability.[37] We refer the reader to the Appendix for more details on these various robustness checks.

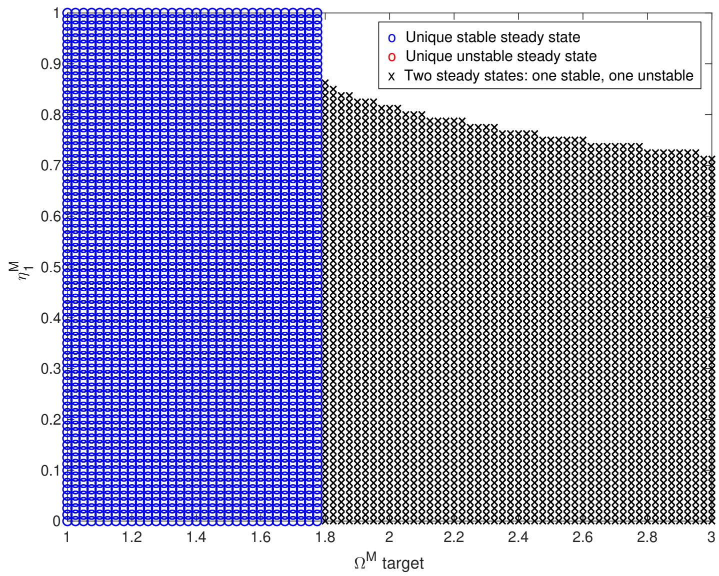
Summarizing, when the central bank follows a traditional Taylor rule, the process of balance sheet normalization should entail a reduction of the holdings of long-term debt in order to ensure uniqueness and stability. Such reduction should be adequate but not severe and slow enough if the central bank wants to achieve an inflation rate close to its target.

Modified Taylor Rule

As previously shown, when the monetary authority sets interest rates according to a modified Taylor rule, $\alpha_2 = 1$, the central bank can achieve a unique steady state inflation that equals its target. However, is the steady state government debt unique? Are those steady states locally stable? These answers can be found in Figure 3 below.

[36]This corresponds to 25 % of GDP or equivalent to 2.5 trillion.
[37]This could be due to the calibration strategy.

Figure 3: Uniqueness and Stability of Steady States



As shown in Figure 3, as long as the central bank's maturity composition target is low enough, $\Omega^{M*} < 1.8$, the resulting stationary equilibria is unique and locally stable. Above this compositional threshold, we find many normalization policy configurations that deliver multiple steady states. Moreover, we find that the response to deviations from the maturity composition, η_1^M, does not affect the number nor the stability properties of stationary equilibria. The maturity composition target, Ω^{M*}, is now even more important. This is the case as it can not only deliver locally determinate equilibria, but also rule out real indeterminacies. In particular, not having a large proportion of long-term debt allows the central bank to eliminate self-fulfilling prophecies consistent with an economy with two equilibria on both sides of the liquidity Laffer curve that have different locally determinacy properties. As is the case when the central bank follows a traditional Taylor rule, a lower size target value of bond holdings by the central bank, b^{M*}, yields similar qualitative results. Such size reduction implies a lower compositional target that delivers unique and locally determinate equilibria. We refer the reader to the Appendix for more details.

In terms of achieving the target inflation, Π^*, the *modified* Taylor rule that takes into account the effect of the premium ($\alpha_2 = 1$) outperforms the *traditional* Taylor rule ($\alpha_2 = 0$). Is that also true in terms of welfare? We find that is not the case. Figure 4 reports the DM consumption-equivalent welfare associated with every steady state. We then compare it with the welfare of an economy where the efficient level of DM consumption is attained. For multiple steady states, the left (right) figures show the welfare costs of the better (worse) steady state.

Figure 4a: Welfare Costs, $\alpha_2 = 0$

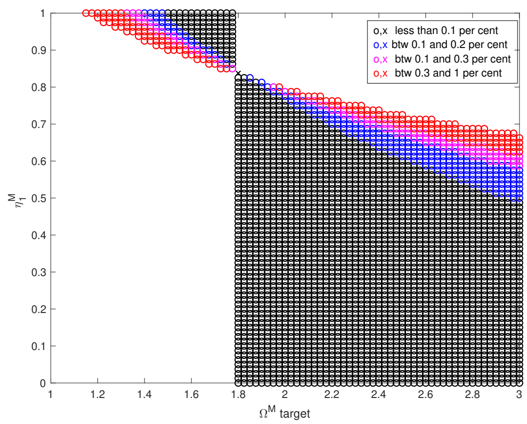


Figure 4b: Welfare Costs, $\alpha_2 = 0$

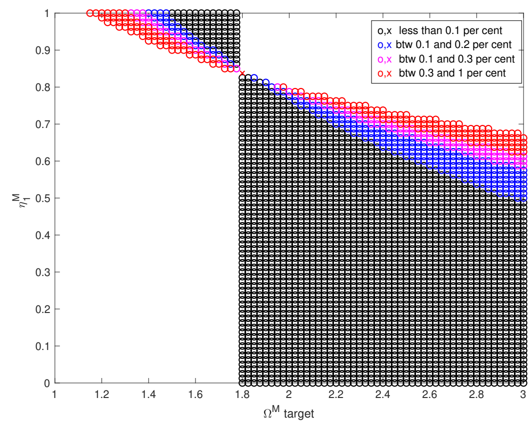


Figure 4c: Welfare Costs, $\alpha_2 = 1$

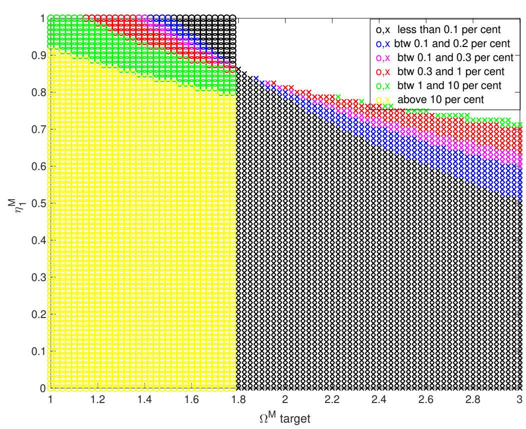
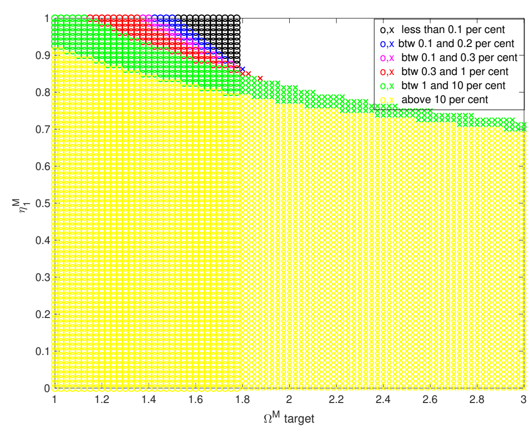


Figure 4d: Welfare Costs, $\alpha_2 = 1$



These figures show that the modified Taylor rule leads to a larger parameter space consistent with existence of equilibria. However, the prevalence for multiple equilibria is also much higher. Overall, we find that unique and stable equilibria tend to deliver lower welfare costs when the responsiveness of the maturity composition to current conditions, η_1^M is higher, which implies a slower speed of adjustment. When multiple steady states exist, one of the steady states has agents facing much lower welfare. Moreover, our numerical findings also suggest that the welfare associated with a *traditional* Taylor rule always outperforms that of the *modified* Taylor rule.

Summarizing, while achieving better inflation targets, the *modified* Taylor leads to many more situations where real indeterminacies are possible. Such multiplicity of steady states allows for increased volatility as one can always construct sunspot equilibria between the various steady states.[38] Moreover, the *modified* Taylor rule delivers lower welfare. In light of these findings, we

[38] See Azariadis (1981), among others, for detailed discussion on sunspot equilibria.

conclude that following a traditional Taylor rule is a more desirable policy for managing interest rates.

Debt Management by the Fiscal Authority

So far when analyzing balance sheet normalization policies, we have not discussed how policies regarding debt issuance by the fiscal authority change the nature of the equilibria. Taking into account current increased fiscal pressures, we explore the consequences of considering an alternative larger target for total debt. In particular we consider an increase to 75% of GDP (which corresponds to $b^* = 0.78$) relative to the benchmark of 72% (which corresponds to $b^* = 0.7457$). We also analyze a situation where the issuance of long-term bonds by the fiscal authority decreases to $\Omega^* = 2.00$. Figure 5 illustrates our findings for an economy where the central bank follows a traditional Taylor rule; i.e, $\alpha_2 = 0$.[39]

Figure 5a: Stability & Uniqueness, $b^* = 0.78$

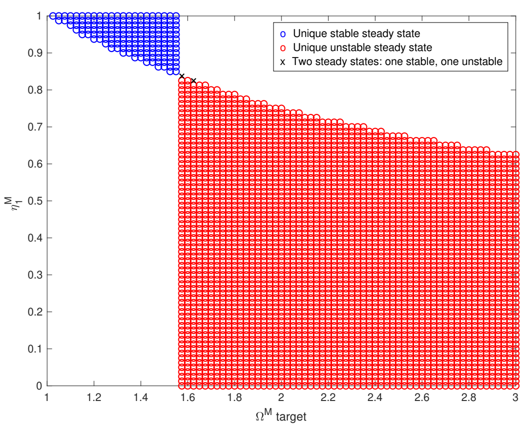


Figure 5b: Inflation Rates, $b^* = 0.78$

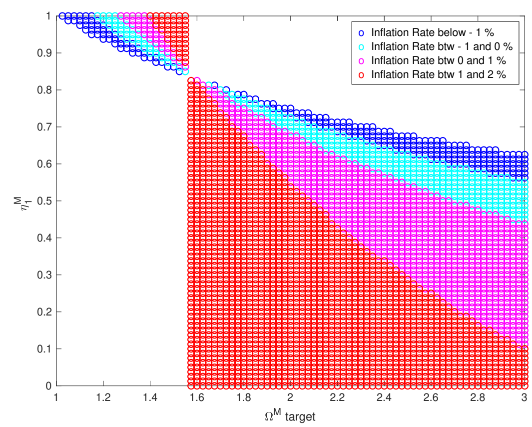


Figure 5c: Stability & Uniqueness, $\Omega^* = 2.00$

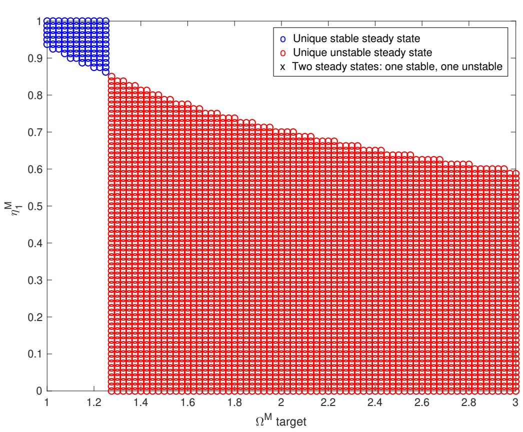
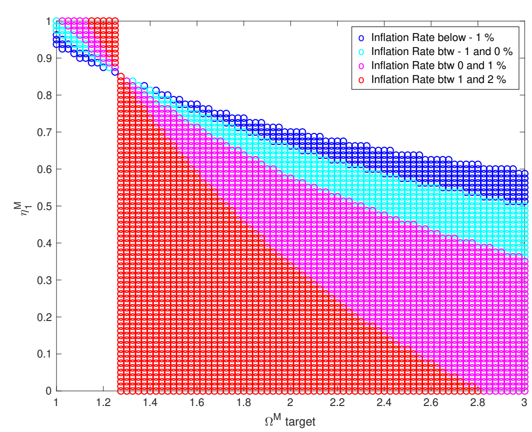


Figure 5d: Inflation Rates, $\Omega^* = 2.00$



[39] The results for $\alpha_2 = 1$ are similar as before, but with a lower composition threshold as those with $\alpha_2 = 0$.

As we can see from Figure 5, the central bank's response to the new fiscal realities would require a lower maturity threshold target and a similar adjustment speed. We again interpret such findings as suggesting that what is relevant is the liquidity held by the public. Moreover, these findings also highlight the importance of having even further coordination between monetary and fiscal authorities to achieve desirable equilibria, when the normalization process begins.

6 Conclusions

This paper has explored the effects of reducing the overall size of the central bank's balance sheet and changing its composition towards short-term public debt. When public debt does not exhibit premia, the stationary equilibria is unique and the prescriptions for determinacy of equilibria are similar to those found in Leeper (1991). Changes to the composition of the balance sheet (either in terms of desired maturity structure target or the specific response to deviations from its compositional target) does not impact inflation nor debt dynamics. However, once the economy exhibits bond premia, we find that changes in the central bank's balance sheet have important implications for long-run inflation, real allocations, government debt as well as for uniqueness and local stability of stationary equilibria. In order to ensure a unique and stable steady state, the central bank should target a low enough maturity composition in its portfolio. In our numerical exercise, we find that long-term debt holdings by the central bank should be less than 1.8 their holdings of short-term bonds. Moreover, the process of balance sheet normalization should be slow enough and depend on current economic conditions in order to guarantee stability and deliver a long-run inflation rate close to its target. While hitting the inflation target, the modified Taylor rule, that explicitly takes into account spreads, makes the existence of multiple equilibria more likely and delivers lower welfare. Given these findings, the traditional Taylor rules is the preferred policy for managing interest rates when agents in the economy face a liquidity premia. Finally, our findings also show that an adequate balance sheet normalization process critically depends on the size and maturity composition targets of the fiscal authority. These findings highlight that further coordination between fiscal and monetary authorities is needed when monetary normalization begins.

In this paper, we have focused on the implications of the central bank's balance sheet within a closed economy. With an international perspective, Taylor (2019) suggests that the recent increases in the central banks' balance sheet took place with the aim of affecting exchange rates and that have resulted in an international balance sheet contagion and excessive exchange rate volatility. These are important aspects that we leave for future research.

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Appendix: Robustness Check

Figure 6: Sensitivity Analysis for Traditional Taylor, $\alpha_2 = 0$

Figure 6a: Stability & Uniqueness, $b^{M*} = 0.26$

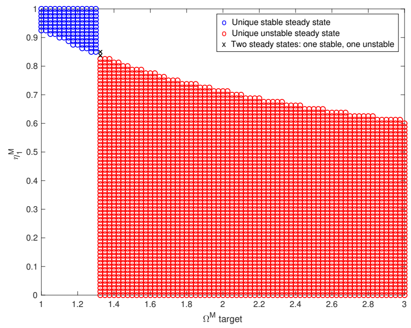

Figure 6b: Inflation Rates, $b^{M*} = 0.26$

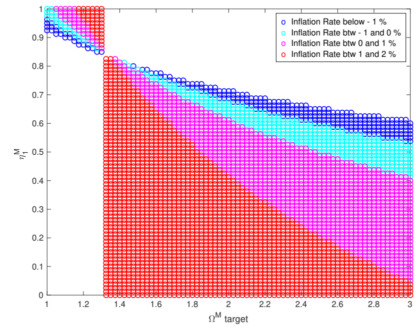

Figure 6c: Stability & Uniqueness, $\alpha_1 = 2.00$

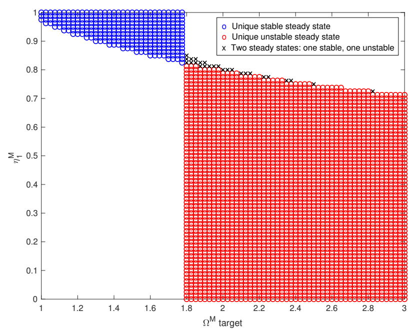

Figure 6d: Inflation Rates, $\alpha_1 = 2.00$

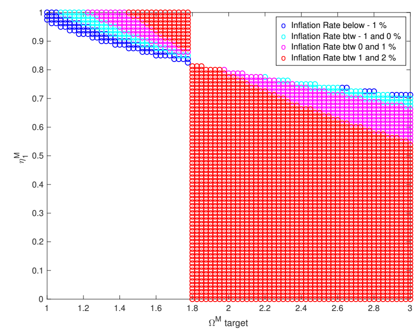

Figure 6e: Stability & Uniqueness, $\eta = 0.50$

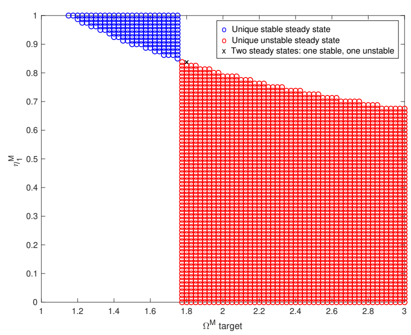

Figure 6f: Inflation Rates, $\eta = 0.50$

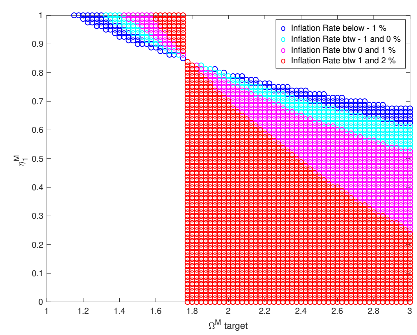

Figure 6g: Stability & Uniqueness, $\sigma = 0.50$

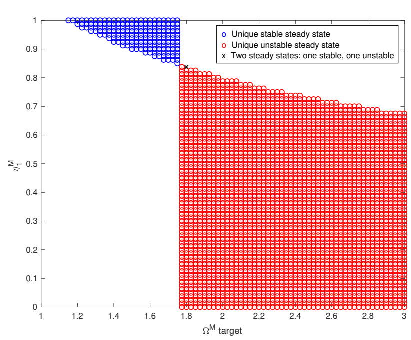

Figure 6h: Inflation Rates, $\sigma = 0.50$

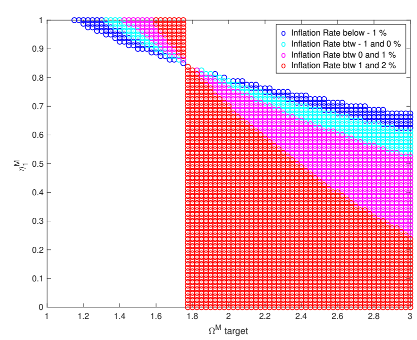

Figure 7: Sensitivity Analysis for Modified Taylor, $\alpha_2 = 1$

Figure 7a: Stability & Uniqueness, $b^{M*} = 0.26$

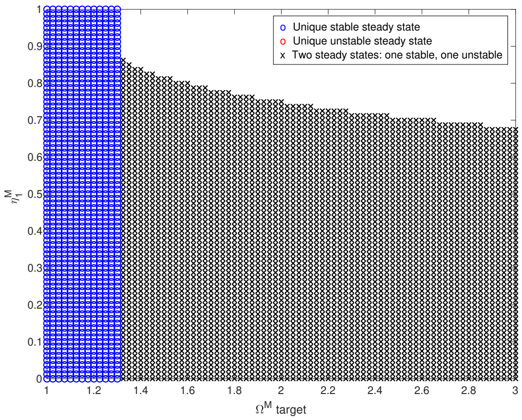

Figure 7b: Stability & Uniqueness, $\alpha_1 = 2.00$

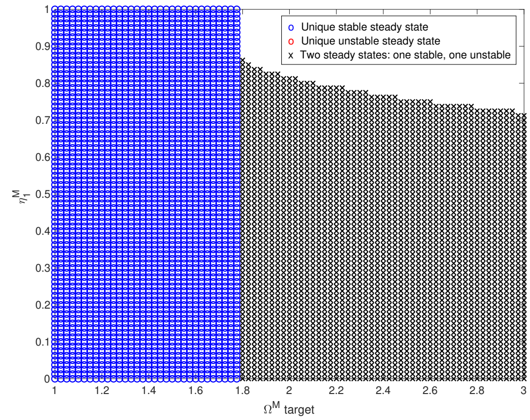

Figure 7c: Stability & Uniqueness, $\eta = 0.50$

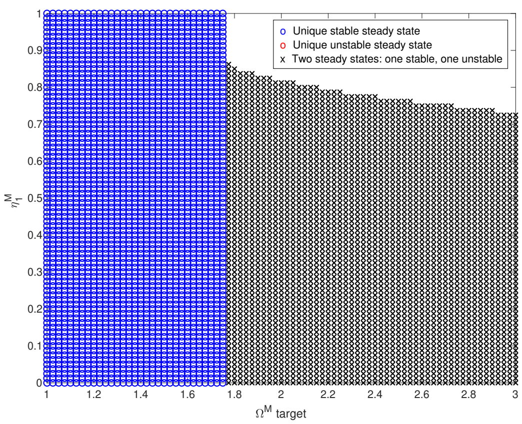

Figure 7d: Stability & Uniqueness, $\sigma = 0.50$