Money and Credit Revisited

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Abstract

Gu, Mattesini, and Wright (2016) show that if money is essential, then nomonetary credit (i.e., deferred payment to sellers) is irrelevant. We find that in an otherwise same model that also allows monetary credit (i.e., borrowing money from third parties), nonmonetary credit is relevant when money is essential. While nonmonetary credit can still be neutral locally, this result critically depends on the tightness of monetary credit. A tight monetary credit limit and loan market clearing restrict how real balances can respond to changes in nonmonetary credit. Money may serve as an accelerator or stabilizer, depending on the types of credit changes and overall credit condition. Our results suggest a subtle three-way relationship among money and the two types of credit.

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"The fine and complicated mechanism of the money and credit system is wrapped in obscurity."

Ludwig von Mises, The Theory of Money and Credit

1 Introduction

Credit is considered one of the key macroeconomic factors, especially after the 2008 financial crisis. Macroeconomists use three ways to study credit: to study it in cashless models, to study nonmonetary credit (i.e., deferred payment without using money) in monetary models, as in Lucas and Stokey (1987), and to study monetary credit (i.e., private borrowing and lending of money), as in Berentsen et al. (2007). The conventional wisdom says that nonmonetary credit matters in cashless models, and monetary credit matters in monetary models (e.g., Berentsen, Camera, and Waller, 2007; Ed and Andolfatto, 2009; Williamson, 2012; Li and Li, 2013; Araujo and Ferrarisz, 2020; and Berentsen, Martin, and Andolfatto, 2020). Gu, Mattesini, and Wright (2016, GMW thereafter), however, presents a striking result: if nonmonetary credit and money can be used in the same transactions, then nonmonetary credit is neutral when money is valued.[1] This result appears to contradict our understanding of the importance of credit. After all, the real world is a monetary economy, and there is little reason why money cannot be used with nonmonetary credit.

We revisit the issue of money and credit and reconcile the above inconsistency. Specifically, we show that if we also allow monetary credit in the GMW framework, nonmonetary credit matters even when money is valued. In other words, with monetary credit, the mechanism in GMW still works but no longer works perfectly. In addition, there is a fine and complicated relationship among money and the two

[1] Lucas and Stokey (1987) do not allow nonmonetary credit and money to be used in the same transactions.

types of credit. Our results also suggest that when studying credit conditions in the macro economy, it is important to incorporate money and to differentiate the two types of credit.

To understand these results, first shut down monetary credit so that the model becomes the same as that in GMW. When money is valued, tighter nonmonetary credit drives up money demand because the two are substitutes in transactions. The marginal cost of holding money is determined by the nominal interest rate set by the central bank, so that the marginal benefit of holding money must be constant as well, which is possible only if real balances rises to keep total liquidity constant. We call it the full teeterboard effect.

But things are different when monetary credit is also available. Now nonmonetary credit matters for whether monetary credit is binding, and if the interest rate on monetary loans is positive, then even marginal changes of nonmonetary credit can affect the allocation. We prove these results when debt limits are exogenous, or endogenous as in Kehoe and Levine (1993). We consider unsecured credit, and secured credit as in Kiyotaki and Moore (1997). The intuition is as follows. A tighter nonmonetary credit still drives up money demand. But the rise in real balances is now insufficient to keep the total liquidity constant. Market clearing in the (monetary) loan market connects the value of lenders' money (loan supply) and borrowers' monetary credit limit (loan demand). The teeterboard effect is constrained in this case, whereas it is unconstrained in GMW. Another interpretation is that, with monetary credit, the return of holding unspent money is determined by the interest rate on loans, which always zero in GMW but can be influenced by nonmontary credit in our model.

Our results suggests a fine and complicated relationship between money and credit when money is valued. For example, the two-way relationship between money

and nonmonetary credit critically depends on monetary credit. When unlimited, monetary credit does not matter, and the teeterboard effect works perfectly between money and nonmonetary credit. When monetary credit is extremely tight (i.e., with an excess supply of loans), money and the two types of credit are perfect substitutes, and the teeterboard effect works perfect between money and either type of credit. In these two cases, the teeterboard effect is unconstrained because loan market does not clear or loan demand is not constrained. When monetary credit is relatively tight, however, real balances decreases in nonmonetary credit (partial teetterboard effect, discussed in previous paragraph) but increases in monetary credit. In other words, money is now a complement to monetary credit is and an imperfect substitute to nonmonetary credit.

In terms of the literature of monetary theory, while many studies consider money and nonmonetary credit (e.g., Lucas and Stokey, 1987; and Townsend, 1989), or money and monetary credit (e.g., Berentsen, Camera, and Waller, 2011; Li and Li, 2013; He et al., 2015; and Araujo and Ferrarisz, 2020), this paper is the first to incorporate the three at the same time. Another important feature is that we follow GMW to consider the microfoundations that generates the role of money and to give money and credit equal chance in facilitating transactions, whereas many previous studies partition goods intro cash goods and credit goods (e.g., Lucas and Stokey, 1987), or assume costs or theft in favor of credit or money (He et al., 2005, 2008; Sanches and Williamson, 2010; Kahn et al., 2005; Kahn and Roberds, 2008; and Lotz and Zhang, 2015). Our results highlight the subtle three-way relationship between money and the two types of credit.

Building on GMW, our model also has two implications for the study of credit conditions in general, especially for models that do not characterize the role of money in transactions. First, while a contraction of nomonetary credit always decrease out-

put in cashless models, if we consider the transactional role of money, the same contraction still decrease output but only if the loan interest rate is above zero. Otherwise, money supports a floor of the economy with the perfect teeterboard effect. The same effect also implies that a marginal change of nonmonetary credit also has no effect on output when money is still valued but monetary credit is unconstrained.[2] Second, cashless models of credit are often not explicit about which type of credit they study, we show that this matters. When monetary credit is relatively tight, the endogenous real balance acts as a cushion to a tighter nonmonetary credit but as an accelerator to a tighter monetary credit. Therefore, when studying credit, it is important to incorporate money and to differentiate the two types of credit.[3]

The paper proceeds as follows. Section 2 describes the basic physical environment. Section 3 discuss property of the model with exogenous debt limit, including analysis of essentiality of money and whether changes in credit conditions affect the equilibrium. Section 4 shows that the main results are robust if the debt limit of monetary credit is endogenized by collateralized assets. This is useful because one might think the reason credit matters in Section 3 is because debt limit of monetary credit is exogenous, making the adjustment of the value of money difficult. Section 5 concludes.

2 Model

Time is discrete, and each period is divided into two subperiods: decentralized market (DM) and centralized market (CM). There are fixed buyers and sellers. In the DM, α fraction of buyers receive a preference shock and would like to consume. They are

[2]It is possible to have unconstrained monetary credit and constrained nonmonetary credit. Of course, if nonmonetary credit increases sufficiently, it can drive money out of circulation.

[3]Lagos and Zhang (2020) argue for modeling money in macroeconomics for a different reason. They show the cashless limit of a monetary economy is not the same as a cashless economy.

called active buyers, whereas the rest of buyers are called inactive buyers.[4] Let σ be the probability that an active buyer meets a seller. If sellers and active buyers trade in a competitive market, then $\sigma = 1$; if they trade bilaterally, then $\sigma < 1$.

The within-period utility functions of buyers and sellers are

$$\mathcal{U}^b = u(q) + U^b(x) - \ell \text{ and } \mathcal{U}^s = -c(q) + U^s(x) - \ell, \tag{1}$$

where q is the DM good, x is the CM good and ℓ is labor. Leisure is $1 - \ell$, and for now 1 unit of labor produces 1 units of x, so that the CM real wage, ω, is one. The constraints $x \geq 0$, $q \geq 0$ and $\ell \in [0, 1]$ are assumed not to bind, as can be guranteed in the usual ways. Also U^j, u and c are twice continuously differentiable and strictly increasing. Assume that $U'' \leq 0$, and that $u'' \leq 0 \leq c''$ with one equality strict, and $u(0) = c(0) = 0$.

There is discounting between the CM and DM according to $\beta = 1/(1+r)$, $r > 0$; any discounting between the DM and CM can be subsumed in the notation in (1). Goods q and x are nonstorable. There is an intrinsically worthless object called money that is storable; other storable assets are introduced below. The money supply per buyer M changes over time at rate π, so that $M_{+1} = (1+\pi) M$, where the subscript $+1$ (or -1) on a variable indicates its value next (or last) period. Changes in M are accompanined by lump sum transfers if $\pi > 0$ or taxes if $\pi < 0$. Money supply changes happen at the end of CM. We restrict attention to $\pi > \beta - 1$, or the limit $\pi \to \beta - 1$, which in this model is the Friedman rule; there is no monetary equilibrium with $\pi < \beta - 1$.

There are two types of credit: nonmonetary credit and monetary credit. Nonmonetary credit is like the credit studied in Gu, Mattesini, and Wright (2016). Buy-

[4] We can alternatively interpret α as the meeting probability in a ramdom matching model.

ers defer their payment to sellers but no money is involved when the transactions happen. Using monetary credit means buyers borrow money from a third party and use the borrowed money to pay sellers, as in Berentsen, Camera, and Waller (2007).

3 Exogenous Debt Limits

3.1 The CM Problem

The state of an agent in the CM is his wealth, $A = \phi m - d - bR\phi - T$, where ϕ is the value of his money m, in terms of numeraire x, d is (real) nonmonetary credit obtained from sellers, b is the nominal loan position (positive b means borrowing money and negative b means lending) from the financial market, R is the gross nominal interest rate on loans, and T is a lump-sum tax. The CM value function of agent j ($j =$ b or s) is as follows:

$$W_j(A) = \max_{x,\ell,\hat{m}} \left\{ U^j(x) - \ell + \beta V_j(\phi_{+1}\hat{m}) \right\} \ s.t.\ A + \omega\ell = x + \phi\hat{m}.$$

Using the budget equation to eliminate ℓ in the value function, it is easy to show that W_j is linear in A.

3.2 The DM Problem

For a buyer in the DM with real money balances ϕm, the value function is as follows

$$\begin{aligned} V_b(\phi m) =& W_b(\phi m - T) + (1-\alpha\sigma) m\phi (R-1) \\ &+ \alpha\sigma \max_{b,d,q} [u(q) - \phi m - d - b\phi R], \\ s.t.\ & v(q) = \phi m + d + b\phi, \qquad (2) \\ & d \leq D,\ b\phi R \leq B. \qquad (3) \end{aligned}$$

where we have used the fact W is linear in A, $\phi m + d + b\phi R$ is what the buyer gives up, and $v(q)$ is what the seller receives. The difference between the latter two is due to nominal interest rate the borrower pays to lenders. We have also assumed that inactive buyers and active buyers who do not meet a seller, with a total measure of $1-\alpha\sigma$, lend all of their money out. $v(q)$ is determined by a general trading protocal explained below. The two types of credit are limited by D and B, that is, $d \leq D$ and $b\phi R \leq B$.

The first-order conditions for d and b are

$$u'(q)/v'(q) - 1 \geq 0 \qquad (4)$$

$$u'(q)/v'(q) - R \geq 0, \qquad (5)$$

where inequality means contrained. Sellers do not hold money across periods. It is standard to use envelope conditions and first-order conditions to write the Euler equation for money demand for buyers

$$1 + i \geq \alpha\sigma \frac{u'(q)}{v'(q)} + (1-\alpha\sigma) R. \qquad (6)$$

4 Equilibrium

Since monetary equilibrium is a self-fulfilling prophacy, there is always a nonmonetary equilibrium. Let q^* be such that $u^{'}(q^*) = c^{'}(q^*)$. Define $q(D) = v^{-1}(D)$ as the q obtained in the nonmonetary equilibrium. Note that $q(D)$ is increasing when $D < D^* = v(q^*)$ and equals q^* when $D \geq D^*$.

4.1 Monetary Equilibrium

For moneytary equilibrium to exist, buyers must obtain a higher consumption than only using credit, $q(D)$. In a monetary equilibrium, active buyers must exhaust all of their nonmonetary credit, so $d = D$. Let q_i be the quantity traded in a monetary equilibrium. In general, q_i is a function, $q_i(B, D, i)$. Below we first study different cases in monetary equilibrium and then characterize the boundary conditions of these cases and the monetary equilibrium as a whole. The market clearing in the financial market (monetary credit) requires

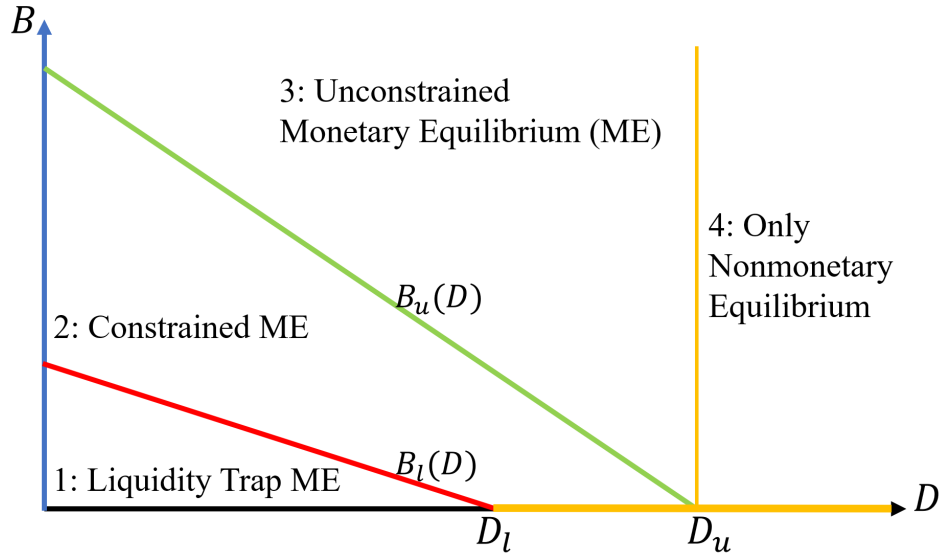
$$\alpha\sigma b \leq (1 - \alpha\sigma)\, m, \tag{7}$$

where the LHS and RHS represent the demand and supply of loans respectively, and the inequality means an excess supply of loans. Figure 1 plots the set of equilibrium in (D, B) space. **Below we describe different types of monetary equilibrium.**

4.1.1 Only Nonmonetary Credit

If $B = 0$, then there is no demand for monetary loans, so that the net return on monetary loans equals zero (i.e., $R = 1$). This establishes GMW as a special case of this paper. A marginal decrease of D does not affect q in the monetary equilibrium

Figure 1: Equilibrium Sets



Note: B and D are debt limits of monetary and nonmonetary credit respectively. Nonmonetary equilibrium always exists. If $B = 0$, then monetary equilibrium requires $D < D_\ell$. If $B > 0$, then monetay equilibrium requires $D < D_u$. In the Liquidity Trap monetary equilibrium, there is excess supply of loans and the nominal rate of monetary loans is zero.

since the Euler equation now becomes

$$i = \alpha\sigma \left[\frac{u'\left(\underline{q}_i\right)}{v'\left(\underline{q}_i\right)} - 1 \right]. \tag{8}$$

So q is entirely determined by i. We define $\underline{q}_i$ as the quantity of consumption in the monetary equilibrium with no monetary credit.

4.1.2 Liquidity Trap Equilibrium

If B is low enough, then we still have $R = 1$. According to the Euler equation (6), q is still given by (8). In this case, neither marginal changes in D or B affect q. From (2), since $b\phi R = B$ and $d = D$, we know that a lower D or B must be offset by an increase in real balances. Next, given i, let $B_\ell(D)$ be the threshold of B such that the economy is in the liquidity trap equilibrium if B is below $B_\ell(D)$. Using $b\phi R = B$,

$d = D$, and $R = 1$ the boundary $B_\ell(D)$ satisfies

$$v\left(\underline{q}_i\right) = m\phi + D + B_\ell(D) = \frac{\alpha\sigma}{1-\alpha\sigma} B_\ell(D) + D + B_\ell(D). \tag{9}$$

where $\underline{q}_i$ is defined in (8), and the second equality is due to the loan market clearing condition (7) with equality. Given i and D, (9) defines the function $B_\ell(D)$. If $B < B_\ell(D)$ then the supply of loans, $(1-\alpha\sigma)m$, can exceed the demand of loans, $b = B/\phi$. It is obvious that B_ℓ is decreasing in both i and D.

4.1.3 Unconstrained Monetary Equilibrium

Suppose B is large enough so that active buyers are unconstrained in choosing b. So (5) holds with equality. The Euler equation, (6), becomes $1 + i = R$. Therefore, (5) implies that

$$u'(\overline{q}_i)/v'(\overline{q}_i) = 1 + i, \tag{10}$$

which defines $\overline{q}_i$ as the quantity with unconstrained monetary credit. Given i, a marginal change of B or D would not affect $\overline{q}_i$.

Next consider boundary conditions. Given i, let $B_u(D)$ be the cutoff such that if $B \geq B_u(D)$, then agents are unconstrained in choosing b. Using $b\phi R = B_u(D)$, $R = 1 + i$, and $\alpha\sigma b = (1-\alpha\sigma)m$, the boundary $B_u(D)$ satisfies

$$v(\overline{q}_i) = D + \frac{B_u(D)}{(1-\alpha\sigma)(1+i)}, \tag{11}$$

where $\overline{q}_i$ is defined in (10). It is obvious that $B_u(D)$ is decreasing in D. The fact that $B_u(D)$ and $B_\ell(D)$ are both decreasing in D illustrates that the two types of credit are substitutes. We also know that the effect of i on $B_u(D)$ is ambiguous because it depends on the relative size of $(1+i)v'(q_i)\partial q_i/\partial i$ and $[v(q_i) - D]$.

The Euler equation (8) can be written as $u'\left(\underline{q}_i\right)/v'\left(\underline{q}_i\right) = 1 + \frac{i}{\alpha\sigma}$. Compare it with (10), we know $\underline{q}_i < \overline{q}_i$ unless $i = 0$. Therefore, $v\left(\underline{q}_i\right) < v\left(\overline{q}_i\right)$. Since we assume that $u'(q)/v'(q)$ is decreasing in q, we can compare (9) and (11) to conclude that $B_\ell(D) \leq B_u(D)$ with equality holding at $i = 0$. These results can be summarized as follows:

Lemma 1. *Given i, if $\alpha\sigma < 1$, then $\underline{q}_i \leq \overline{q}_i$ and $B_\ell(D) \leq B_u(D)$, with the equality holding when $i = 0$.*

4.1.4 Constrained monetary equilibrium

If $B \in (B_\ell, B_u)$, then the economy is in a constrained monetary equilibrium with binding borrowing constraint: $b\phi R = B$. The difference from liquidity trap equilibrium is now the monetary loan market clears so that $R > 1$.Since $d = D$, eliminate $m\phi$ in 2 using loan market clearing condition (7) with equality, we have the following DM goods demand equation:

$$v(q) = \frac{\alpha\sigma}{1-\alpha\sigma}B/R + D + B/R. \tag{12}$$

This equation and the Euler equation (6) form a two-equation system with two unknowns: q and R. The the Euler equation (6) plots an upward sloping curve in (R, q) space. The DM goods demand equation (12) plots a downward sloping curve in (R, q) space. A lower D or B would cause q and R both to decrease.

While it is not surprising that a change of B would affect the allocation as in Dai and He (2019), it is perhaps surprisingly that a change of D would also affect affect the allocation. This result contrasts that in GMW. In their setup, because buyers that do not need liquidity cannot lend their money out, so effectively R is forced to be 1 in the Euler equation (6). Therefore, a change of D does not matter in the monetary

equilibrium at all. Now we have monetary credit. Suppose D goes down. To make the q unchanged, we need R to be the same while $m\phi$ goes up to offset the decrease in D. This cannot be obtained because loan market clearing requires $m\phi = \frac{\alpha}{1-\alpha} b\phi = \frac{\alpha}{1-\alpha} B/R$. The intuition is that now the value of money cannot freely move to offset the decrease of D, the real money balances are restricted by the borrowing limit in the loan market as well.

4.1.5 Existence of Monetary Equilibrium

If $D \geq D^*$, agents can use credit to obtain q^* so that there is no need to use money. Monetary equilibrium does not exist. To check if a combinatioin of (B, D) can support an monetary equilibrium, we compare the consumption level if consumers only use nonmonetary credit, $q(D)$, and the consumption level if consumers hold money as well. If $B = 0$, the maximum consumption in a monetary equilibrium is $\underline{q}_i$, so monetary equilibrium does not exist if $D > v\left(\underline{q}_i\right)$. If $B > 0$, the maximum consumption in a monetary equilibrium is $\overline{q}_i$, so monetary equilibrium does not exist if $D > v(\overline{q}_i)$.

Proposition 1. *Given monetary policy i, there are three cases:*

1. Suppose $B > 0$. If $v(\overline{q}_i) \leq D$, then there is no monetary equilibrium and there is a nonmonetary equilibrium with $q \in [\overline{q}_i, q^*]$; if $D < v(\overline{q}_i)$, then there is a monetary equilibrium with $q \leq \overline{q}_i$ plus a nonmonetary equilibrium with $q(D) < \overline{q}_i$.

2. Suppose $B = 0$. If $v\left(\underline{q}_i\right) \leq D$, there is no monetary equilibrium and there is a nonmonetary equilibrium with $q \in [\underline{q}_i, q^*]$; if $D < v\left(\underline{q}_i\right)$, then there is a monetary equilibrium with $q = \underline{q}_i$ plus a nonmonetary equilibrium with $q(D) < \underline{q}_i$.

The novel results are summarized in the next proposition:

Proposition 2. *With exogenous policy and limits of direct and monetary credit (D and B), in*

(stationary) monetary equilibrium, changes in D or B is not irrelevant given i. Specifically, we have three cases:

1. If $D + B/(1-\alpha\sigma) < v\left(\underline{q}_i\right)$, then the nominal interest rate of monetary credit is zero, $q = \underline{q}_i$, and marginal changes in D or B are neutral given i;

2. If $D + B/(1-\alpha\sigma)(1+i) > v(\overline{q}_i)$, then the nominal interest rate of monetary credit equals i, $q = \overline{q}_i$, and marginal changes in D or B are neutral given i;

3. Otherwise, the nominal interest rate of monetary credit is less than i, $\underline{q}_i < q < \overline{q}_i$, and a marginal decrease in D or B would cause q and R both to decrease.

Proof. Assume $D + B/(1-\alpha\sigma) < v\left(\underline{q}_i\right)$, then in the financial market, money demand is less than the money supply. Hence the walrasian price of funds $R = 1$, then the Euler equation is reduced to $i = \alpha\sigma[\frac{u'(q)}{v'(q)} - 1]$, hence $q = \underline{q}_i$, therefore case 1 holds. Then assume $D + \frac{B}{(1-\alpha\sigma)(1+i)} > v(\overline{q}_i)$, then borrowers are not bounded in the financial market, by no arbitrage condition we have $R = 1 + i$, hence $\frac{u'(q)}{v'(q)} - 1 = i$, therefore $q = q_i$ and D and B are neutral, hence case 2 holds. Then assume $D + B/(1-\alpha\sigma) > v\left(\underline{q}_i\right)$ and $D + \frac{B}{(1-\alpha\sigma)(1+i)} < v(\overline{q}_i)$, the equilibrium (q, R) satisfy $1 + i = \alpha\sigma\frac{u'(q)}{v'(q)} + (1-\alpha\sigma)R$ and $v(q) = \frac{1}{1-\alpha\sigma}B/R + D$. Given the continuity of the two above equations, it is easy to show the existence of the equilibrium where $\underline{q}_i < q < \overline{q}_i$ and $1 < R < 1 + i$. And the first equation plots an upward sloping curve in (R, q) space while the second equation plots a downward sloping curve in (R, q) space, therefore, the equilibrium is unique and it is easy to show (q, R) increase with respect to both D and B. Hence, case 3 holds. □

Proposition 2 establishes the our result that nonmonetary credit, D, matters even when money is valued (i.e., in a monetary equilibrium). Specifically, nonmonetary credit matters for whether monetary credit is binding, and if the interest rate on monetary loans is positive (i.e., the constrained monetary equilibrium), then even marginal changes of nonmonetary credit can affect the allocation.

Before we turn to endogenous debt limits, it is useful to consider how the endogenous real balances respond to changes in exogenous credit limits. In Cases 1 and 2, marginal changes in D and B do not change the allocation because money serves as a cushion in the payment. For example, in Case 1 (liquidity-trap monetary equilibrium), $b\phi = B$, $d = D$, and $q = \underline{q}_i$, so we know from (2) that changes in B or D is completely offset by changes in $m\phi$. In this case, money and the two types of credit serve as perfect substitutes, so a reduction of credit raises the demand of money, whose value increase to offset the changes in credit. This is the intuition behind the results in Gu et al. (2016). The economics in Case 2 is similar but subtly different. Now the real value of monetary credit, $b\phi$, is connected to the value of money of lenders' due to loan market clearing (i.e., $b\phi = m\phi R\left(1-\alpha\sigma\right)/\alpha\sigma$). We also know that $q = q_i$, $R = \left(1+i\right)/\beta$, and $d = D$ so that, ceteris paribus, a marginal change in D is again completely offset by a change in ϕ.[5] However, while nonmonetary credit is still a substitute to money, money and monetary credit are no longer substitutes but rather complements: $b\phi$ depends positively on $m\phi$.

In case 3, the money borrowed is b, from (2), we know $(m+b)\phi = v\left(q\right) - D = \frac{1}{1-\alpha\sigma}B/R$. Given $\frac{\partial R}{\partial D} > 0$, then $\frac{\partial(m+b)\phi}{\partial D} = \frac{\partial[v(q)-D]}{\partial D} = -\frac{1}{1-\alpha\sigma}\frac{B}{R^2}\frac{\partial R}{\partial D} < 0$. Therefore, money is still a cushion of D in case 3, but not a perfect one. Then for $\frac{\partial(m+b)\phi}{\partial B}$, we have $\frac{\partial(m+b)\phi}{\partial B} = \frac{\partial[v(q)-D]}{\partial B} = \frac{1}{(1-\alpha\sigma)R} - \frac{1}{1-\alpha\sigma}\frac{B}{R^2}\frac{\partial R}{\partial B} = \frac{1-\varepsilon_{RB}}{(1-\alpha\sigma)R}$, where $\varepsilon_{RB} = \frac{B}{R}\frac{\partial R}{\partial B}$ is the elasticity of R with respect to B. For low elasticity ($\varepsilon_{RB} < \alpha\sigma$) and low inflation ($R$ is close to 1), we have $\frac{1-\varepsilon_{RB}}{(1-\alpha\sigma)R} > 1$ and hence money is an accelerator for monetary credit B. For high elasticity or high inflation, we have $\frac{1-\varepsilon_{RB}}{(1-\alpha\sigma)R} < 1$, hence money is a cushion for monetary credit B.

[5] A marginal change in B does not affect the allocation or the value of money because b is unconstrained.

5 Endogenous Debt Limits

Next, we show nonmonetary credit matters in the monetary equilibrium with endogenous secured credit as in Kiyotaki and Moore (1997), and when debt limits are endogenous as in Kehoe and Levine (1993).

5.1 Secured Monetary Credit with Asset

One might think the reason why nonmonetary credit matters is because the debt limit of monetary credit is exogenous, making it difficult for the value of money to adjust to the changing credit conditions. Now we consider endogenizing B with assets as collateral. Assets that can serve as collaterals are Lucas trees with dividends γ per unit of asset and a fixed supply E. Now the total asset of the a buyer in the CM is $A = \phi m - d - b\phi - T + (\psi + \gamma)\, a$, where ψ is the asset price, and a is the asset position. The value function in the CM is the same, whereas the value function in the DM now has two argument: $V^b\,(m, a)$. Another difference is that the borrowing constraint for monetary credit is now $b\phi R \leq B = \chi(\psi + \gamma)e$, where χ is the loan to value ratio in the loan market. We take χ as given, but the asset price ψ is endogenous, so whether the borrowing constraint is binding or not is still endogenous.

It is easy to show that in monetary equilibrium, $d = D$. The Euler equations for money is the same as in (6), while we have an Euler equation for the asset as follows:

$$\psi_{-1} = \beta(\psi + \gamma)\left[1 + \alpha\sigma\chi\left(\frac{u'\,(q)}{v'\,(q)}\frac{1}{R} - 1\right)\right]. \tag{13}$$

Next, suppose agents face binding monetary credit. In equilibrium $a = E$. Eliminate $m\phi$ using market clearing (7) with equality and eliminate ψ using (13), we can

rewrite the DM goods demand equation (12) as follows:

$$v(q) = \frac{\alpha\sigma}{1-\alpha\sigma}\frac{1+r}{r-\alpha\sigma\chi(\frac{u'(q)}{v'(q)}\frac{1}{R}-1)}\chi\gamma E/R + D + \frac{1+r}{r-\alpha\sigma\chi(\frac{u'(q)}{v'(q)}\frac{1}{R}-1)}\chi\gamma E/R.$$

Combine this equation with the Euler equation for money demand, we have a two-equation system with two unknowns: q and R. The Euler equation (6) plots an upward sloping curve in (R, q) space. It is easy to show by contridiction that the above equation plots a downward sloping curve in (R, q) space. a lower ρ and χ would cause q and R both to decrease as well. But most importantly, similar to the benchmark case, a lower D would cause q and R both to decrease. This is because while the borrowing limit, B, is endogneous as it is supported by an endogenous asset value, it still restricts how real balance can respond to changes in nonmonetary credit.[6] Therefore, the rise in real balance is not enough to keep the total liquidity constant. The following proposition summerizes these results and characterizes the monetary equilibrium:

Proposition 3. *With exogenous policy and limits of direct and monetary credit ($D < D_\ell$ and ρ), in (stationary) monetary equilibrium, changes in D or ρ or χ is not irrelevant given i. Specifically, we have three cases:*

1. *If $D + \frac{1+r}{r-i\chi}\chi\gamma A/(1-\alpha\sigma) < v\left(\underline{q}_i\right)$, then the nominal interest rate of monetary credit is zero, $q = \underline{q}_i$, and marginal changes in D or B are neutral given i;*
2. *If $D + \frac{\chi(1+1/r)\gamma A}{(1-\alpha\sigma)(1+i)} > v(\overline{q}_i)$, then the nominal interest rate of monetary credit is i, $q = \overline{q}_i$, and marginal changes in D or B are neutral given i;*

[6] The market clearing in the loan market requires $m\phi = \frac{\alpha\sigma}{1-\alpha\sigma}b\phi = \frac{\alpha\sigma}{1-\alpha\sigma}B/R$, where the borrowing limit is determined as follows:

$$B = (1+r)\chi\rho E/\left[r - \alpha\sigma\chi(\frac{u'(q)}{g'(q)}\frac{1}{R} - 1)\right].$$

It cannot freely adjust as in GMW.

3. Otherwise, the nominal interest rate of monetary credit is less than i, $\underline{q}_i < q < \overline{q}_i$, and a marginal decrease in D or ρ or χ would cause q and R both to decrease.

5.2 Secured Monetary Credit with Reproducible Capital

Now suppose monetary credit is secured by capital, with ρ and δ the rental and depreciation rates. The constant return to scale production funciont is $f(N, K)$, where N is total employment and K is the total capital stock. Profit maximization implies $\omega = f_1(N, K)$ and $\rho = f_2(N, K)$. Let k be the holding of capital stock of an individual at the beginning of a period. We focus on the monetary equilibria and assume $D < D_\ell$. Now the total asset of the a buyer in the CM is $A = \phi m - d - b\phi - T + (1 - \delta + \rho)k$. The value function in the CM is the same, whereas the value function in the DM now has two argument: $V^b(m, k)$. Another difference is that the borrowing constraint for monetary credit is now $b\phi R \leq B = \chi(1 - \delta + \rho)k$, where χ is the loan to value ratio in the loan market.

It is easy to show $d = D$. The money and capital distributions are again degenerate across buyers because the marginal costs of bringing more money and capital are linear and does not depend on trading history or asset positions. The Euler equations for money and capital are:

$$(1+i)\frac{U'(x_{-1})}{U'(x)} = 1 + (1 - \alpha\sigma)(R - 1) + \alpha\sigma\left[\frac{u'(q) - v'(q)U'(x)}{v'(q)U'(x)}\right], \tag{14}$$

$$\frac{1}{\beta}\frac{U'(x_{-1})}{U'(x)} = (1 - \delta + \rho)[1 + \alpha\sigma\chi\frac{u'(q) - v'(q)U'(x)}{v'(q)U'(x)}]. \tag{15}$$

It is easy to show that sellers would not carry money or capital at the end of CM in most cases, and if they do that would be because they are indifferent between carrying any amount. So we assume sellers do not carry money or capital. The

first-order condition for x and the market clearing for x are as follows:

$$U'(x)f_1(N,K) = 1,$$
$$f(N,K) + (1-\delta)K - K_{+1} = 2x, \tag{16}$$

where we have used the following facts: the total measure of agents is two, only buyers hold capital, and sellers also consumes the same amount of x.

Now suppose the monetary credit borrowing constraint is binding while the loan market clears. We have a four-equation dynamic system: (14), (15), (16) and (12) with $B = \chi(1-\delta+\rho)k$. As the economy accumulates more capital over time, the borrowing constraint on monetary credit will be gradually relaxed. Capital provides liquidity because it supports monetary credit. However, it is clear from (12) with $B = \chi(1-\delta+\rho)k$ that when D changes, the value of the liquidity capital provides is affected. While real balance, supported by capital accumulation, will respond to changes in nonmonetary credit, it is again not unresctricted. Therefore, nonmonetary credit still matters in this monetary equilibrium and will also determine whether we are in such a constrained monetary equilibrium.

5.3 Reputation

Next, consider endogenous debt limits as in Kehoe and Levine (1993). We allow agents to renege on direct and indirect debt. An advantage of this setup is that we are able to endogensly determine both two types of credit limits jointly. There is some monitoring, to be specific, we check whether buyers honor their nonmonetary and monetary debt with probabilities μ_D and μ_B. We assume that buyers honor their public taxes. In terms of timing, buyers simultaneously choose one of the following options: pay both d and b; pay only d; or pay only b; or pay neither.

Then they are randomly monitored by the seller and the monetary credit market (e.g., banking sector). If a buyer reneges (removed "one either one,") but is not caught will choose $(x, \ell, \hat{m})$ in the CM as before. But buyers caught reneging on d or b is banished to autarky and can not participate in DM. The autarky payoff is $\underline{W}(\phi m) = \phi m + \frac{u(x^*)-x^*+T}{1-\beta}$=$\phi m + U_0(1+r)/r$, where $U_0 = u(x^*) - x^* + T$.

To ensure that buyers pay their debts, we impose the following incentive constraints:

$$W_b(\phi m - b - d) \geq (1-\mu_D)W_b(\phi m - b) + \mu_D \underline{W}(\phi m),$$
$$W_b(\phi m - b - d) \geq (1-\mu_B)W_b(\phi m - d) + \mu_B \underline{W}(\phi m),$$
$$W_b(\phi m - b - d) \geq (1-\mu_D)(1-\mu_B)W_b(\phi m) + (\mu_D + \mu_B - \mu_D\mu_B)\underline{W}(\phi m).$$

If the incentive constraints on repaying d or b both hold, then the incentive constraints on repaying both are abundant. Then reshape the first two incentive constraints, we have

$$d \leq \mu_D[W_b(\phi m - b) - \underline{W}(\phi m)],$$
$$b \leq \mu_B[W_b(\phi m - d) - \underline{W}(\phi m)].$$

We know $W_b(0) = U_0(1+r)/r + \frac{1}{r}\alpha\sigma[u(q) - v(q) + b/R - b] - \phi m$. Insert $W_b(\phi m - b)$,$W_b(\phi m - d)$ and $\underline{W}(\phi m)$ into the above two constraints and using $v(q) = \phi m + d + b/R$, we have

$$d \leq \mu_D\{\tfrac{1}{r}\alpha\sigma[u(q) - v(q) + b/R - b] - \phi m - b\},$$
$$b \leq \mu_B\{\tfrac{1}{r}\alpha\sigma[u(q) - v(q) + b/R - b] - \phi m - d\}.$$

From the above equations, we know for the fixed point, we have $(\frac{1}{\mu_D} - 1)D = (\frac{1}{\mu_B} - 1)B$ so that D and B are porportional to each other. We now endogenize D and

B by adapting the method in Alvarez and Jerman (2000) or Gu et al. (2016). First, pick up arbitrary D and B, Then the RHS of the above two equations depends on D and B. We put down the RHS as:

$$\Phi_D(D,B) = \begin{cases} \frac{\mu_D \alpha\sigma}{r}[u(q^*) - q^*], & \text{if } D \geq v(q^*) \\ \frac{\mu_D \alpha\sigma}{r}\left\{u\left[v^{-1}(D)\right] - D\right\}, & \text{if } v(\overline{q}_i) \leq D < v(q^*) \\ \frac{\mu_D \alpha\sigma}{r}\Gamma(q) - \mu_D \frac{i}{1+i}\left[v(q) - D\right]\Upsilon + D, & \text{if } v(\overline{q}_i) - B_i \leq D < v(\overline{q}_i) \\ \frac{\mu_D \alpha\sigma}{r}\Gamma(q) + D, & \text{if } D < v(\underline{q}_i) - B/(1-\alpha\sigma) \\ \frac{\mu_D \alpha\sigma}{r}\Gamma(q) + \mu_D \frac{\alpha\sigma + r}{r}\frac{1-R}{R}B + D, & \textit{otherwise}. \end{cases}$$

$$\Phi_B(D,B) = \frac{\frac{1}{\mu_D} - 1}{\frac{1}{\mu_B} - 1}\Phi_D(D,B).$$

where $B_i = B/(1-\alpha\sigma)(1+i)$, $\Gamma(q) = u(q) - \frac{\alpha\sigma + r}{\alpha\sigma}v(q)$, and $\Upsilon = (\alpha\sigma + r)(1-\alpha\sigma)/r$.

We can show the existence and uniqueness of D by adapting the method in Gu et al. (2016). The next step is show that changes in nonmonetary credit condition (i.e., μ_D) will affect q in the last case in $\Phi_D(D,B)$. The proof should be similiar with the benchmark case. The following proposition characterize different equilibrium under endogenous debt limits.

Proposition 4. *Given a feasible policy i, $\exists(\widehat{D},\widehat{B}) = (\Phi_D(D,B), \Phi_B(D,B)) > (0,0)$, There are five cases:*

1. If $\widehat{D} + \widehat{B}/(1-\alpha\sigma) \leq v\left(\underline{q}_i\right)$, then the nominal interest rate of monetary credit is zero, $q = \underline{q}_i$.

2. If $\widehat{D} + \widehat{B}/(1-\alpha\sigma) > v\left(\underline{q}_i\right)$ and $\widehat{D} + \frac{\widehat{B}}{(1-\alpha\sigma)(1+i)} < v(\overline{q}_i)$, then the nominal interest rate of monetary credit is less than i, $\underline{q}_i < q < \overline{q}_i$.

3. If $\widehat{D} + \frac{\widehat{B}}{(1-\alpha\sigma)(1+i)} \geq v(\overline{q}_i)$ and $\widehat{D} < v(\overline{q}_i)$, then the nominal interest rate of monetary credit is i, $q = \overline{q}_i$.

4. If $\widehat{D} \geq v(\overline{q}_i)$ and $\widehat{D} < v(q^*)$, there is no monetary equilibrium and there is a non-monetary equilibrium with $q \in (\overline{q}_i, q^*)$.

5. If $\widehat{D} \geq v(q^*)$, there is no monetary equilibrium and there is a non-monetary equilibrium with $q = q^*$.

The following proposition establishes how changes in nonmonetary credit condition affect the allocation:

Proposition 5. *With endogenous monetary and nonmonetary credit limits, in (stationary) monetary equilibrium, changes in D are not neutral if B is tight and loan markets clear. Otherwise D is neutral.*

Proof. In monetary equilibrium, we have $\widehat{D} < v(q_i)$. Then we look into three subcases. 1) $\widehat{D} + \widehat{B}/(1-\alpha\sigma) \geq v\left(\underline{q}_i\right)$ and $\widehat{D} + \frac{\widehat{B}}{(1-\alpha\sigma)(1+i)} \leq v(q_i)$, 2) $\widehat{D} + \widehat{B}/(1-\alpha\sigma) < v\left(\underline{q}_i\right)$, and 3) $\widehat{D} + \frac{\widehat{B}}{(1-\alpha\sigma)(1+i)} > v(q_i)$ and $\widehat{D} < v(q_i)$. For case 1, the equilibrium (q, R) satisfy $1 + i = \alpha\sigma\frac{u'(q)}{v'(q)} + (1-\alpha\sigma)R$ and $v(q) = \frac{1}{1-\alpha\sigma}\widehat{B}/R + \widehat{D}$. The first equation plots an upward sloping curve in (R, q) space while the second equation plots a downward sloping curve in (R, q) space. Given the continuity of the two above equations, it is easy to show the existence and uniqueness of the equilibrium where $\underline{q}_i < q < \overline{q}_i$ and $1 < R < 1 + i$. And we know (q, R) increase with respect to both $\widehat{D}$ and $\widehat{B}$, hence changes in D are not neutral. For case 2, loan demand is less than supply, therefore, $R = 1$ and $i = \alpha\sigma[\frac{u'(q)}{v'(q)} - 1]$, hence $q = \underline{q}_i$ and changes in D are neutral. For case 3, borrowers are not bounded, by no arbitrage condition we have $R = 1 + i$ and $\frac{u'(q)}{v'(q)} - 1 = i$. Therefore, changes in D are neutral. □

6 Concluding Remarks

This paper shows that allowing agents to borrow money from third parties to pay sellers makes a crucial change in the GMW framework: now nonmonetary credit

condition matters when money is valued. While nonmonetary credit can still be neutral locally, this result critically depends on the tightness of monetary credit. A tight monetary credit limit and loan market clearing restrict how real balances can respond to changes in nonmonetary credit. Money may serve as an accelerator or stabilizer, depending on the types of credit and overall credit condition. Our results highlight the fine and complicated relationship between money and credit and that between the two types of credit. If we compare cashless models and in monetary models, GMW shows the importance of incorporate money in the study of credit. We show that in monetary models, some qualitative properties of credit can still be similar to those in cashless models. However, there are also big differences both qualitatively and quantitatively. It is important to not only incorporate money but also differentiate the two types of credit.

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