A Multisector Perspective on Wage Stagnation*

L. Rachel Ngai[†] Orhun Sevinc [‡]

June 2021

Abstract

Low-skill workers are concentrated in sectors that experience fast productivity growth and yet their wages have been stagnating. We document evidence from U.S. states showing that a multisector perspective is crucial to understanding this divergence and stagnation. Key to our mechanism is the fall in the relative price of the low-skill sector caused by faster productivity growth. When outputs are complements across sectors, this leads to a reallocation of low-skill workers into the high-skill sector where their marginal product of labor is stagnant. We show this mechanism is quantitatively important for low-skill wage stagnation, its divergence from aggregate labor productivity, and the rise in wage inequality from 1980 to 2010.

Keywords: Wage stagnation, Wage-productivity divergence, Wage inequality, Multisector model

JEL Classification: E24, J23, J31

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*We thank Mun Ho and Akos Valentinyi for the help with the US KLEMS data, Zsofia Barany, Lukasz Drozd, Alan Manning, Joseba Martinez, Marti Mestieri, Ben Moll, Chris Pissarides, Richard Rogerson and John Van Reenen for helpful discussion, and participants at Copenhagen Business School, London Business School, LSE, London Macro workshop at UCL, Labor Market Adjustment Workshop at ECB, National University of Singapore, Philadelphia Macro Workshop, XIII Koc University Winter Workshop in Economics, Richmond Fed, RIDGE Workshop, TOBB University, VIII Workshop on Structural Transformation and Macro Dynamics at Cagliari, CEPR Macro and Growth Meeting, Tilburg, Uppsala, University of Groningen and York University for comments. Hospitality from London Business School is gratefully acknowledged.

[†]London School of Economics, CFM, and CEPR. Email: L.Ngai@lse.ac.uk

[‡]Central Bank of the Republic of Turkey, and CFM. Email: Orhun.Sevinc@tcmb.gov.tr

1 Introduction

Low-skill workers have experienced very little wage growth, despite working mostly in sectors with fast productivity growth. In the U.S., the real wage of non-college workers increased by 20% between 1980-2010, which is less than half the increase in aggregate labor productivity.[1] The stagnation persists even after controlling for age, race, gender, education, and occupation, so it is not due to compositional changes in low-skill employment.[2] Hours worked by these workers represent two-thirds of overall hours worked, so their wage experiences are important in our understanding of aggregate wage dynamics.

There is a large literature on the fall in the relative wage of low-skill workers. But the underlying forces driving relative wages do not necessarily contribute to the low-skill wage stagnation. Understanding the reasons for the low-skill wage stagnation, in combination with the rising skill premium, is especially important. Taken together they reject the view that a rising tide lifts all boats, apparently some boats are left behind in absolute term.

Our main objective is to understand the stagnation in the low-skill real wage and its divergence from aggregate labor productivity, and the growing wage inequality between low-skill and high-skill workers. We offer a novel multisector perspective for understanding the low-skill wage stagnation through changing relative prices driven by uneven productivity growth across sectors. We show that this mechanism is quantitatively important in accounting for the three facts simultaneously.

The multisector perspective that we propose is that the marginal product of

[1]The precise increase in the non-college real wage ranges from 15% to 25% depending on the choice of price deflators, composition adjustment, the inclusion of non-wage compensation and self-employed, and whether it is only for nonfarm business sectors. See Appendix A1.3. However, regardless of these choices, the findings that the non-college real wage is stagnant and lags behind aggregate labor productivity growth are robust.

[2]As documented in Acemoglu and Autor (2011), low-skill wage stagnation co-exists with occupational polarization according to which the wages of low-wage occupations have been growing faster than the wages of middle-wage occupations. The low-skill wage stagnation is about a group of workers with given education qualifications whereas polarization is defined over given occupational groups irrespective of who is employed there. Sevinc (2019) documents the role of skill heterogeneity within occupations in understanding these two patterns.

low-skill workers grew differently across sectors and low-skill workers have been moving from faster growth sectors to slower growth ones. This caused a slowdown in the aggregate marginal product of low-skill workers, which contributed to the aggregate low-skill wage stagnation. It is important to emphasize that the sectoral real wage, which is what workers care about, does not measure the sectoral marginal product of labor. In a competitive labor market, the marginal product of labor is measured by the "product wage", which equals the nominal wage deflated by the sectoral value-added price. Two facts emerge from the data: (1) low-skill real wages grew similarly but product wages grew differently across sectors because of the changes in relative prices across sectors, and (2) the share of low-skill hours increased in sectors with slower growth in product wages. These facts highlight that sectoral reallocation *alone* is not the reason for low-skill wage stagnation; we also require that the reallocation is into sectors with slower growth in product wages due to rising relative prices.

Following on from this argument, we build our explanation based on a model of uneven productivity growth across sectors, which was also the starting point of Baumol (1967) in his seminal paper on growth stagnation. Baumol (1967) derived his result from the labor reallocation into the stagnant sector. For our results on the stagnation of low-skill wages, its divergence from aggregate productivity and the growing wage inequality, we need in addition capital, heterogeneous labor and different skill intensities across sectors to interact with the uneven productivity growth.

To motivate our argument, we group the U.S. economy into high-skill and low-skill sectors according to the importance of high-skill labor, the data implies that the high-skill sector experienced slower productivity growth and rising relative price during 1980-2010.[3] The growth rate of low-skill wages were almost identical across the two sectors. The rise in the relative price of the high-skill sector translated into a stagnant low-skill product wage in the high-skill sector; but the

[3]The high-skill sector includes: finance, insurance, government, health, and education services. The low-skill sector includes all remaining industries. See Section 2.2 and Data Appendix A1 for details.

low-skill product wage was growing in the low-skill sector because of its falling relative price.

The basic mechanism can be understood in a two-sector and two-input model, with both sectors using high-skill and low-skill workers. The crucial assumptions are outputs of the two sectors are gross complements and productivity growth is faster in the low-skill sector. Together they imply a rise in the relative price of the high-skill sector and a labor reallocation towards the high-skill sector. Given that the expanding sector has a faster price growth, this reallocation process reflects a shift of low-skill workers into the sector with a slower-growing product wage, which contributes to the low-skill wage stagnation.[4] This between-sector mechanism implies a shift towards the sector that uses high-skill workers more intensively so it acts like a skill-biased shift, which increases the relative wage of the high-skill workers.

Our between-sector mechanism relies on uneven productivity growth across sectors. As an alternative explanation, one might invoke imperfect labour markets and a fall in low-skill workers' bargaining power, which spread unevenly across sectors. Evidence for this is provided by Stansbury and Summers (2020), who found that worker bargaining power fell more in low-skill industries than in high-skill ones. However, uneven productivity growth is still needed to generate the observed changes in relative prices, as this simple equation shows. Express the low-skill wage as $w_{lj} = p_j MPL_j \pi_j$, where MPL_j is low-skill marginal product and π_j captures the worker's bargaining power in an imperfect market. We have argued that low-skill wages increased at similar rates across sectors while the prices of high-skill industries are growing faster. For these facts to be consistent with the finding of Stansbury and Summers, the marginal product of low-skill labor in high-skill intensive industries must be falling relative to that in low-skill intensive industries. Rather than act as an alternative, changes in bargaining

[4] In other words, specializing in sectors with faster productivity growth works against the low-skill workers, as the output they produce is getting cheaper over time. This has a similar flavor, but the mechanism is different, to the early trade literature on immiserizing growth, where faster productivity growth results in a country being worse off because of deteriorating terms of trade (Bhagwati, 1958).

power reinforce our explanation.[5]

The basic model described above explains the divergence of the low-skill wage and the aggregate labor productivity by predicting a rise in wage inequality. Using an accounting identity, which expresses the total value-added of the economy as the sum of total factor payments, we show that there are two other drivers for the divergence. They are the falling labor income share and the rising relative cost of living, measured by the ratio of the consumption deflator and the output deflator. To have a full account of the divergence, we need to introduce capital to the basic model.

The model is calibrated to match key features of the U.S. labor market from 1980 to 2010. In addition to the between-sector mechanism through uneven productivity growth, the quantitative model allows for four other labor market changes: a fall in the relative price of capital, an increase in the relative supply of high-skill workers and changing production weights on low-skill workers and high-skill workers. The between-sector mechanism alone can contribute up to 85% of the divergence by predicting 68% of the rise in wage inequality and all the rise in the relative cost of living. Though other changes can contribute to the rise in wage inequality and the divergence, only the between-sector mechanism and the falling production weights of low-skill workers (for example, due to a low-skill replacing technical change) can generate the low-skill wage stagnation. The key difference between the two is that the between-sector mechanism is needed for generating the observed sector-specific trends in the low-skill product wages.

These results highlight one key difference between the one-sector and multi-sector perspective. Slowing down the low-skill replacing technical change can improve low-skill wage growth in a one-sector economy but at the cost of slowing down aggregate productivity growth, i.e. there is a trade-off between aggregate productivity growth and low-skill wage growth. The mutlisector perspective avoids this trade-off. This is because higher low-skill wage growth can be achieved by

[5] Income effects, i.e. high-skill goods have higher income elasticity, can drive sectoral reallocation but they do not have direct effect on relative prices, which is needed to generate the sector-specific product wages. They can affect relative prices indirectly through other equilibrium variables, see Section 3.5.1.

improving total factor productivity growth of high-skill sectors, which also boost the aggregate productivity growth.

The role of different price deflators and falling labor income share have been empirically documented as the sources of the decoupling of the average wage and productivity (e.g. Lawrence and Slaughter, 1993; Stansbury and Summers, 2017). This paper shows that a majority of labor force, i.e. the low-skill workers, suffer from an even larger divergence due to the growing skill premium. Since the seminal work of Katz and Murphy (1992), there has been a large literature studying the effects of the skill-biased technical change on the skill premium (see Goldin and Katz, 2009, for a review).

The skill-biased technical change that simply improves the relative productivity of high-skill workers, however, cannot explain wage stagnation for low-skill workers (Johnson, 1997; Acemoglu and Autor, 2011). This has partly contributed to a growing literature on the effect of automation (see recent examples, Zeira, 1998; Acemoglu and Restrepo, 2018; Martinez, 2019; Moll et al., 2019; Caselli and Manning, 2019; Hémous and Olsen, 2020, among others).[6] There are other potential explanations for the low-skill wage stagnation, such as de-unionization and decline in the minimum wage (Lee, 1999; Dustmann et al., 2009), the increasing monospony power (Manning, 2003), increasing imports (Autor et al., 2013)[7], and the decline in urban premium for non-college workers due to region-specific occupational changes (Autor, 2019).[8] Our contribution to this literature is to show the importance of uneven productivity growth across the low-skill and the high-skill intensive sectors.

Our mechanisms for relative wages across different types of workers are related

[6]This is accompanied by a parallel growing empirical literature on the effect of automation on employment, wages and labor income shares (see e.g., Autor and Salomons, 2018; Graetz and Michaels, 2018; Acemoglu and Restrepo, 2019; Bonfiglioli et al., 2020; Kapetaniou and Pissarides, 2020; Chen et al., 2021, among others).

[7]The decline in manufacturing is an important part of our mechanism and it is modeled as a result of uneven productivity growth. Both Autor et al. (2013) and Kehoe et al. (2018) find that trade accounts for a quarter or less for the decline in U.S. manufacturing, and Kehoe et al. (2018) specifically shows that most of the decline is due to uneven productivity growth.

[8]To the extent that most of the expansion in high-skill services happens in urban areas, our mechanism is consistent with the finding of Autor (2019) on the decline of urban premium for the non-college workers.

to Krusell et al. (2000), Ngai and Petrongolo (2017) and Buera et al. (2020).[9] But unlike them, our objective is to understand low-skill wage stagnation and its divergence from aggregate labor productivity. Our finding that falling relative price of capital and capital-skill complementarity contribute to the rise in the skill premium is consistent with the one-sector model of Krusell et al. (2000), but we show that it also boosts the low-skill wage. In other words, factors that imply a rise in wage inequality do not always lead to low-skill stagnation. Our two-sector model of uneven productivity growth and consumption complementarity is closer to the models of Ngai and Petrongolo (2017) and Buera et al. (2020), but as we show, the presence of capital is important in our explanation of the growth of group-specific wages and the wage-productivity divergence. In the absence of capital, these models would predict that the average wage grows at the same rate as the aggregate labor productivity, which would over-predict the growth in the average wage. It follows that their models could not match the rate of group-specific wage growth and the groups' relative wage simultaneously.

Section 2 presents motivating facts on low-skill wage stagnation. Section 3 uses a two-sector and two-input model to show the basic mechanism of how uneven productivity growth can lead to low-skill wage stagnation. Section 4 presents the full model with capital. The quantitative importance of the mechanism is presented in Section 5 when the model is calibrated to match key features of the U.S. labor market.

2 Motivation

This section first present data from U.S. states to motivate the importance of multisector perspective for understanding the low-skill wage stagnation. It then presents trends in a two-sector version of the U.S. economy to motivate the mechanism we propose through uneven productivity growth across sectors.

[9] See also studies of relative wages across occupations in multisector models, (see e.g., Autor and Dorn, 2013; Bárány and Siegel, 2018, among others).

2.1 Motivating Facts from U.S. States

We use GDP by state from the BEA's Regional Economic Accounts, which provide nominal and real GDP by state at the industry level. We restrict our focus from 1980 to 2010. We compute nominal and real GDP at the 11 one-digit sectors and obtain price indexes as the ratio of nominal to real GDP. Wages are from IPUMS Census extracts for 1980, 1990, and 2000, and the American Community Survey (ACS) for 2010 in order to achieve sufficient number of observations at the state-sector level. We calculate composition adjusted wages of low-skill workers at the year-state-sector level using 216 demographic groups based on six age, two sex, two race, three education, and three occupation categories. Due to the lack of historical consumer price indexes at the state-level, we deflate state-level nominal wages by the national level PCE price index to obtain real wages by state.[10] We calculate low-skill product wages at the state-sector level as nominal wage divided by state industry price. See Data Appendix A1 for details.

To begin with, we conduct a shift-share analysis and find that the within-state component accounts for all the changes in the low-skill wage growth at the national level. Hence, we focus on the within-state facts in this section.

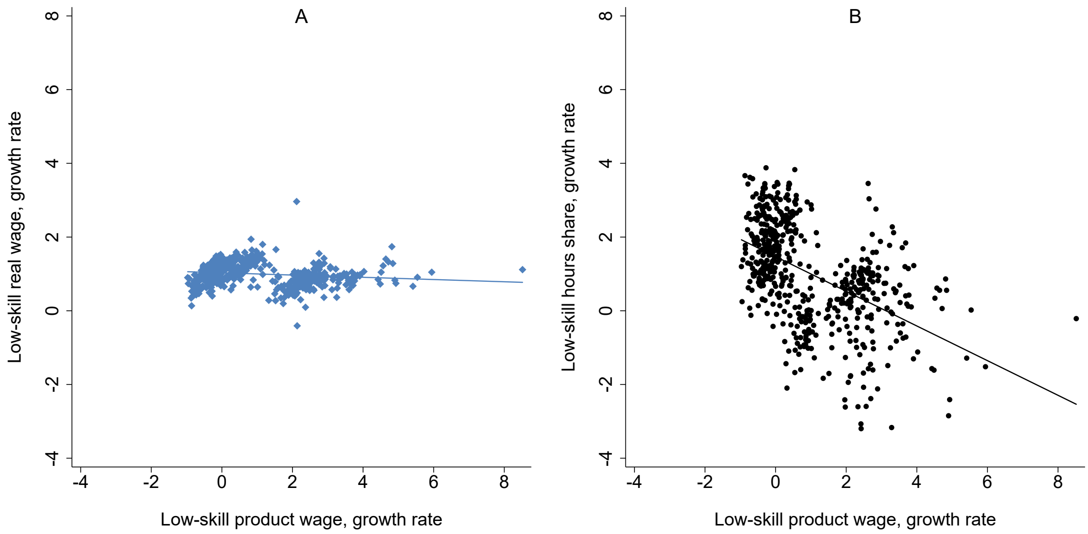
Observation 1: Low-skill labor reallocates from sectors with faster growing low-skill product wage into slower ones.

The importance of changing relative prices across sectors is motivated by the difference between the real wage and the"product wage". The real wage is the nominal wage deflated by an aggregate consumption price index while the product wage is the nominal wage deflated by the sectoral value-added price. Figure 1A plots the growth in the low-skill real wage against the low-skill product wage growth.[11] The substantial changes in relative prices across sectors imply large variations in the growth of low-skill product wages, compared to small variations

[10] As an alternative, we reproduce the evidence presented below using state-level output price as the deflator for real wages. Our observations are robust to using this alternative deflator and available upon request.

[11] All growth rates are adjusted for state fixed effects to ensure that the data pattern is not driven by variation across states. Specifically, we regress the growth rates at the state-sector level on state fixed effects and use the residuals scaled up by the average national growth rate.

Figure 1: Growth in Product Wage, Real Wage, and Hours Shares by U.S. States



Notes: The annual growth of sectoral real wages on the left panel and the growth of hour shares on the right panel are plotted against the growth of product wages of the low-skill workers for the same period. Real wage is calculated as nominal wage divided by the PCE price index. Sectoral product wage is calculated as nominal wage divided by sectoral value-added output price. Growth rates between 1980 and 2010 of 11 sectors are annualized. The figure shows the pooled observations for 51 states where each variable's growth rate is adjusted for state fixed effects. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix A1 for the construction of variables and sectors.
Source: BEA Regional Economic Accounts, Census, and ACS.

in the growth of low-skill real wages. Figure 1B shows that low-skill workers are reallocating into sectors with slower growth in product wage.

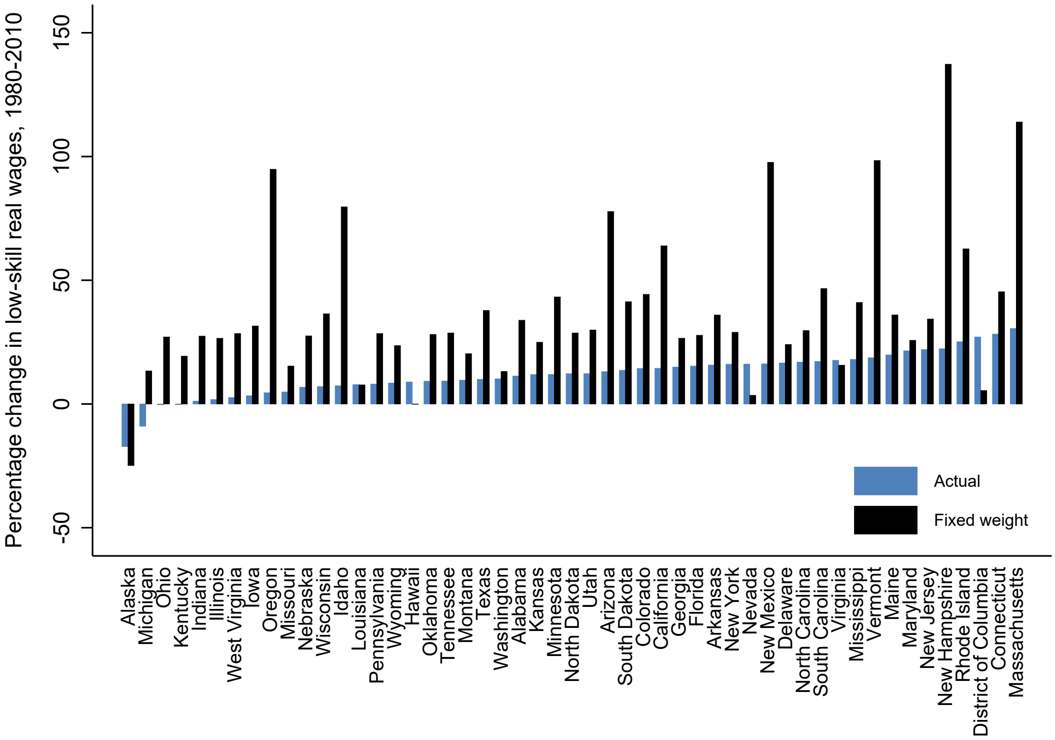
Observation 2: The multisector perspective is quantitatively important in accounting for low-skill wage stagnation.

To understand the impact of sector-specific trends in product wages and labor reallocation documented in Observation 1, we express aggregate low-skill real wage as a weighted sum of the sectoral low-skill product wages:

$$\frac{w_l}{P_C} = \sum_j \frac{w_{lj}}{p_j}\alpha_j; \quad \alpha_j \equiv \frac{p_j}{P_C}\frac{L_j}{L}; \tag{1}$$

where w_l is the aggregate low-skill nominal wage, P_C is the aggregate consumption price index, w_{lj} and p_j are the low-skill nominal wage and value-added price in sector j, and the weight α_j is the product of the relative price p_j/P_C and the share of low-skill labor L_j/L in sector j.

Figure 2: The Importance of Multisector Perspective by U.S. States



Notes: The figure shows the percentage change in real wages by state from 1980 to 2010. Wages are deflated by PCE. The black bar shows the wage growth when the weights on the sectoral product wages (α_j in equation (1)) are fixed. Low-skill is defined as education less than university degree. Composition adjusted wages are calculated as the weighted mean of 216 cells. See Data Appendix A1 for the construction of variables. States are sorted by actual real wage growth.
Source: BEA Regional Economic Accounts, Census, and ACS.

Observation 1 implies that the weight α_j are rising for sectors with slower-growing product wages because of their rising relative prices and hour shares. To see the importance of these changes, Figure 2 reports the percentage changes in the low-skill real wage by state when the weight α_j is fixed (as if in a one-sector economy) against the actual changes. The median ratio of the percentage increase in wage relative to the actual is 2.5.[12]

Observation 3: The growth of sectoral low-skill hours shares, sectoral prices, and sectoral low-skill product wages are skill-biased.

Observation 2 reveals that the sector-specific changes in prices and low-skill hour shares are important for understanding the low-skill wage stagnation. Using

[12]The change is larger than the actual change in all except five states.

Table 1: Sectoral Growth and Skill Intensity

	Share of Low-skill Hours		Sectoral Price		Low-skill Product Wage	
	(1)	(2)	(3)	(4)	(5)	(6)
Skill Intensity						
Hours	2.24		4.98		-3.86	
	(0.39)		(0.44)		(0.39)	
Compensation		1.47		3.32		-2.49
		(0.31)		(0.29)		(0.31)

Notes: Table shows the coefficients of the skill intensity variables estimated from equation (2). The dependent variable is the annualized growth rate of sectoral low-skill hours share in (1)-(2), sectoral value-added price in (3)-(4), product wage in (5)-(6) in each decade from 1980 to 2010 by state. Sectoral product wage is calculated as nominal wage divided by sectoral value-added price. High-skill is defined as education equal to or greater than university degree. Skill intensity in hours is calculated as the sample mean of sectoral hours of high-skill divided by total hours in the sector. Skill intensity in labor compensation is calculated as the sample mean of sectoral compensation of high-skill divided by total compensation in the sector. Composition adjusted wages are calculated as the fixed-weighted mean of 216 cells. See Data Appendix A1 for the construction of variables and sectors. All specifications include state and decade fixed effects. The number of observations is 1683. Robust standard errors are in parentheses. All reported coefficients are significant at the 1 percent level.

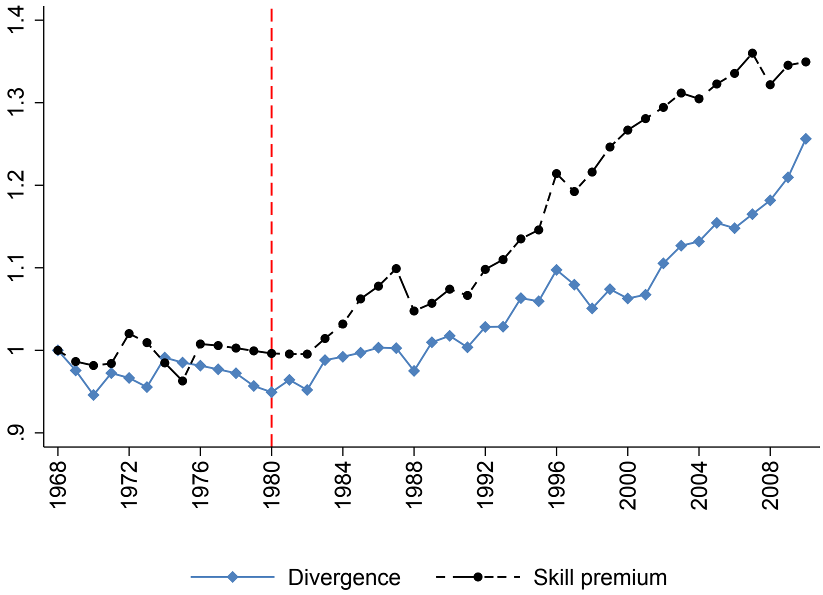
the following simple regression, we show that these changes are skill-biased:

$$g_{njt} = \theta s_{nj} + \gamma_n + \gamma_t + \epsilon_{njt}, \tag{2}$$

where g_{njt} is the growth rate of low-skill hours share, price or low-skill product wage of sector j in state n and decade t; s_{nj} is the long-run skill-intensity of sector j in state n, γ_n and γ_t are state and decade fixed effects that control for state- and decade-specific elements affecting the economy-wide growth rates and ϵ_{njt} is the disturbance term. The slope term θ indicates the strength of conditional correlation between the growth rates and skill intensity.

Table 1 reports the estimated θ from equation (2), where the three left-hand side growth variables are regressed on two alternative skill intensity measures based on hours and labor compensation. Columns (1)-(4) show that the growth in both the share of low-skill hours and value-added price are positively correlated with skill intensity. The product wage growth, on the other hand, is negatively correlated with skill intensity measures in columns (5)-(6). In other words, low-skill workers are reallocating into sectors with higher skill intensity with slower growth in low-skill product wages and rising relative prices.

Figure 3: The divergence and the rise in skill premium



Notes: Divergence is the ratio of aggregate labor productivity relative to the aggregate low-skill real wage. Skill premium is the ratio of the high-skill wage relative to the low-skill wage. Both ratios are normalized to 1 in 1968. Low-skill is defined as education less than university degree. Composition adjusted wages control for age, sex, race and education within the high-skill and the low-skill. See Data Appendix A1 for the construction of variables and sectors.
Source: WORLD KLEMS and CPS.

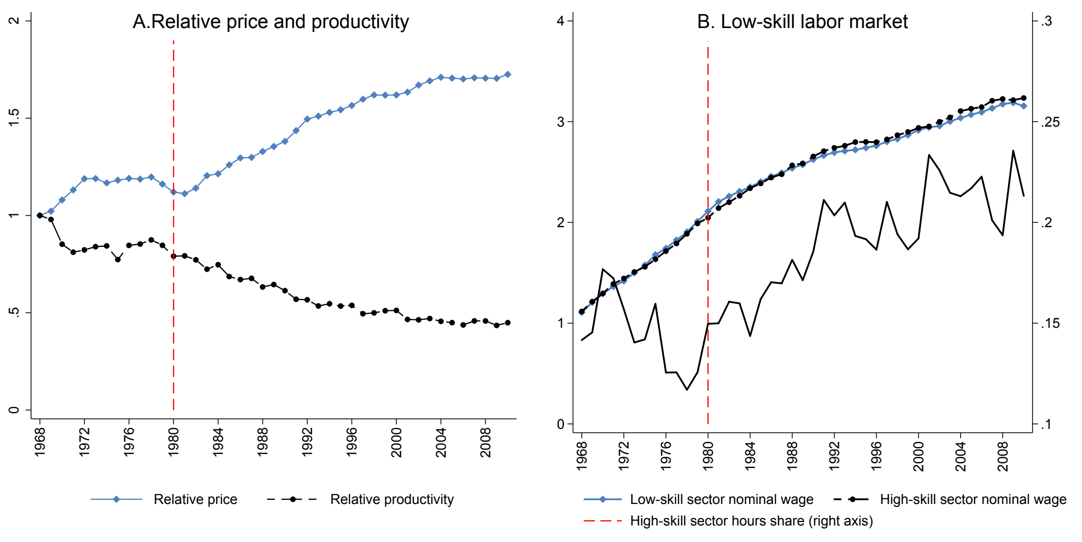
2.2 Two-sector Trends in the U.S.

The objective of the paper is to understand low-skill wage stagnation, its divergence from aggregate labor productivity and the rising skill premium since 1980. As shown in Figure 3, these patterns were not present prior to 1980 as the low-skill real wage is growing at about the same rate as the high-skill wage and the aggregate labour productivity.[13] Motivated by Observation 1-3, we aggregate sectors into two sectors based on the level of skill intensity to examine the potential of our mechanism in understanding Figure 3.

We use the WORLD KLEMS to compute value-added shares, prices, and labor income shares and group sectors into a low-skill sector and a high-skill sector according to the importance of high-skill workers in each sector. The high-skill sector includes: finance, insurance, government, health and education services,

[13]Among other factors, one reason could be the faster growth in relative supply of high-skill workers during this period at 4.1% compared to the 2.0% growth post-1980.

Figure 4: Relative prices, Relative Productivity and Labor reallocation



Notes: Panel A shows the value-added price and real labor productivity of the high-skill sector relative to the low-skill sector, normalized to one in 1968. Panel B shows the composition adjusted log nominal wages of the low-skill workers in the high-skill and the low-skill sectors, and the share of low-skill hours in the high-skill sector. See Data Appendix A1 for the construction of variables and sectors.
Source: WORLD KLEMS and CPS.

and the low-skill sector includes the remaining industries.

To construct a consistent measure for the divergence between labor productivity growth and the low-skill real wage, we compute the aggregate wages by merging the KLEMS data on total compensation and hours with the distribution of demographic subgroups in the CPS. The labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. Given the distribution of demographic subgroups is taken from the CPS, the implied relative wage is the same as the CPS. [14] The Data Appendix A1 provides all the remaining details about our approach to the data.

The mechanism we propose builds on the assumption that the low-skill sector

[14]Similar to Section 2.1, wages are composition adjusted (age, sex, race and education within high-skill and low-skill). We do not control for occupation for the rest of the analysis because unlike other controls, occupation is a choice variable for the worker. However, the evolution of wages are similar when controls for occupations are included.

has faster productivity growth, which implies a rise in the relative price (a slower growth of the low-skill product wages) in the high-skill sector and a reallocation of low-skill workers into the high-skill sector. Figure 4 shows that our mechanism is consistent with the timing of the divergence reported in Figure 3.

Figure 4A shows that the relative price and the relative productivity of the high-skill sector were broadly constant prior to 1980. Since then, the relative price of the high-skill sector was rising, mirroring the decline in its relative productivity.[15] Figure 4B shows that the low-skill nominal wages are similar across the two sectors, supporting the view of an integrated low-skill labor market. What is hidden behind the similarity of sectoral low-skill wages is that the sectoral low-skill product wage is growing much slower than in the high-skill sector because of the rise in the relative price of the high-skill sector. These sector-specific trends in the low-skill product wages contribute to the low-skill wage stagnation because of a reallocation low-skill workers into the high-skill sector shown in Figure 4B.[16]

3 The Basic Mechanism

3.1 The Basic Model Setup

There is a measure H of high-skill household and a measure $L = 1 - H$ of low-skill households. Each household is endowed with one unit of time which they supply to the market inelastically. Household i maximizes utility defined over consumption of the output from the two sectors c_{ij}, $j = h, l$:

$$U_i = \ln c_i; \quad c_i = \left[\psi c_{il}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\psi)\, c_{ih}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

[15] The rise in the relative price of the high-skill sector is also documented as a key feature of "skill-biased structure change", together with the rise in the value-added share and the share of labor compensation in the high-skill sector, by Buera et al. (2020) using a panel of countries over the years 1970-2005. For the U.S., we show that a substantial part of the rise in the share of labor compensation in the high-skill sector is due to the rising skill premium, while the remaining part is due to the reallocation of the low-skill workers documented in Figure 4B.

[16] In contrast, Appendix A1.4 shows that high-skill workers did not experience the same reallocation and the income share of high-skill workers is rising faster in the low-skill sector since 1980, see Appendix Figure A2.

subject to the budget constraint :

$$p_h c_{ih} + p_l c_{il} = w_i, \tag{4}$$

where w_i is the wage of household i.

The economy consists of two sectors: the high-skill sector and the low-skill sector. The representative firm in sector $j = h, l$ uses low-skill labor and high-skill labor as input with a CES production function:

$$Y_j = A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{5}$$

where parameter ξ_j captures the importance of low-skill labor in sector j where $\xi_l > \xi_h$. H_j and L_j are the high-skill and low-skill labor used in sector j.

The goods market clearing and labor market conditions are:

$$Y_j = C_j; \quad j = h, l \tag{6}$$

$$H_h + H_l = H; \quad L_h + L_l = L, \tag{7}$$

3.2 Household' Optimization

Household $i = h, l$ maximizes utility taking prices p_h and p_l as given. The optimal decision of household i implies the marginal rate of substitution across the two goods equal to their relative prices:

$$\frac{c_{ih}}{c_{il}} = \left[\frac{p_l}{p_h} \left(\frac{1-\psi}{\psi} \right) \right]^{\varepsilon}, \tag{8}$$

thus relative expenditure is given by

$$x \equiv \frac{p_h c_{ih}}{p_l c_{il}} = \left(\frac{p_h}{p_l} \right)^{1-\varepsilon} \left(\frac{1-\psi}{\psi} \right)^{\varepsilon}. \tag{9}$$

Using the budget constraint to derive individual's demand:

$$p_l c_{il} = x_l w_i; \quad p_h c_{ih} = x_h w_i; \quad x_l \equiv \frac{1}{1+x}, x_h \equiv \frac{x}{1+x}, \tag{10}$$

where x_j is the expenditure share of good j. These expenditure shares are identical across all household because of the homothetic preference. Aggregating across households, the aggregate demand for good j is :

$$p_j C_j = x_j (H w_h + L w_l) \tag{11}$$

so the aggregate relative demand relative expenditure are the same as the individual's:

$$\frac{C_h}{C_l} = \left[\frac{p_l}{p_h}\left(\frac{1-\psi}{\psi}\right)\right]^{\varepsilon}; \qquad \frac{p_h C_h}{p_l C_l} = x, \tag{12}$$

Using the equilibrium condition from the household's optimization, Appendix A2.1 shows that the price index for consumption basket is the same across all household and derive the aggregate consumption price index as:

$$P_C = \left[\psi^{\varepsilon} p_l^{1-\varepsilon} + (1-\psi)^{\varepsilon} p_h^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{13}$$

3.3 Firm's Optimization

All sectors are perfectly competitive and the representative firm in each sector takes wages of high-skill and low-skill labor as given and maximizes profit. The optimal decision of the firm implies the marginal rate of technical substitution across high-skill and low-skill labor is equal to their relative wages:

$$\frac{H_j}{L_j} = \sigma_j^{\eta} q^{-\eta}; \quad q \equiv \frac{w_h}{w_l} \quad \sigma_j \equiv \frac{1-\xi_j}{\xi_j}; \tag{14}$$

where q is the wage of high-skill labor relative to the low-skill labor. Given $\xi_l > \xi_h$ implies that $\sigma_h > \sigma_l$, so the high-skill sector has a higher skill-intensity.

The income share of the low-skill in sector j is

$$J_j(q) \equiv \frac{w_l L_j}{w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\left(\frac{1-\xi_j}{\xi_j}\right)^{\eta}\right]^{-1} \tag{15}$$

and the relative income share across sectors is:

$$\frac{J_h(q)}{J_l(q)} = 1 - \frac{1}{1+\sigma_h^{-\eta} q^{\eta-1}}\left(1 - \left(\frac{\sigma_l}{\sigma_h}\right)^{\eta}\right) \tag{16}$$

Given $\sigma_h > \sigma_l$, the low-skill income share is lower in the high-skill sector.

3.4 Equilibrium Prices and Allocation

The equilibrium wages are:

$$w_l = p_j \frac{\partial Y_j}{\partial L_j}; \qquad \frac{\partial Y_j}{\partial L_j} = A_j \left[J_j(q)\xi_j^{-\eta}\right]^{\frac{1}{1-\eta}}, \tag{17}$$

$$w_h = q w_l = p_j A_j q \left[J_j(q)\xi_j^{-\eta}\right]^{\frac{1}{1-\eta}}. \tag{18}$$

The expression of low-skill income share $J_j(q)$ in (15) implies that $[J_j(q)]^{\frac{1}{1-\eta}}$ is decreasing while $q[J_j(q)]^{\frac{1}{1-\eta}}$ is increasing in q. Thus, an increase in the relative wage q implies a fall in the marginal product of low-skill and a rise in the marginal product of high skill. Intuitively, this is because a higher relative wage q implies a lower high-skill to low-skill labor ratio.

The free mobility of labor implies the relative price of the high-skill sector:

$$\frac{p_h}{p_l} = \left(\frac{A_l}{A_h}\right)\left(\frac{\xi_l}{\xi_h}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_h(q)}{J_l(q)}\right)^{\frac{1}{\eta-1}}. \tag{19}$$

It shows that an increase in the relative productivity of the low-skill sector contributes to a rise in the relative price of the high-skill sector. An increase in the relative wage of the high-skill also increases the relative price of the high-skill sector given (16) implies $[J_h(q)/J_l(q)]^{\frac{1}{\eta-1}}$ is increasing in q.

Appendix A2.4 shows that the equilibrium of the model can be summarized as solving for relative wage q and the share of low-skill labor in the high-skill sector

$l_h \equiv L_h/L$ using two conditions:

$$l_h = S\left(q;\zeta\right) = \frac{\zeta\sigma_l^{-\eta}q^{\eta}-1}{\left(\sigma_h/\sigma_l\right)^{\eta}-1}; \quad \zeta \equiv \frac{H}{L} \tag{20}$$

$$l_h = D\left(q;\hat{A}_{lh}\right) = \left(1+\frac{J_l\left(q\right)}{J_h\left(q\right)x\left(q;\hat{A}_{lh}\right)}\right)^{-1}, \tag{21}$$

where ζ is the relative supply of high-skill labor and the relative expenditure x is derived from (9) and (19) as:

$$x\left(q;\hat{A}_{lh}\right) = \hat{A}_{lh}^{1-\varepsilon}\left(\frac{J_h\left(q\right)}{J_l\left(q\right)}\left(\frac{\xi_l}{\xi_h}\right)^{\eta}\right)^{\frac{1-\varepsilon}{\eta-1}}; \quad \hat{A}_{lh} \equiv \frac{A_l}{A_h}\left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}} \tag{22}$$

In a nutshell, the condition $S\left(q;\zeta\right)$ is derived using the labor market clearing conditions in equation (7) and the firm's optimal input usage in (14). It is increasing in q given $\sigma_h > \sigma_l$. In other words, given the low-skill sector uses the low-skill workers more intensively, the reallocation of low-skill labor from the low-skill sector to the high-skill sector is associated with higher relative wage q. The condition $D\left(q;\hat{A}_{lh}\right)$ is derived using the goods market clearing conditions in equation (6) and the household's optimal consumption in (12). The relative expenditure share $x\left(q;\hat{A}_{lh}\right)$ summarizes the effect of relative productivity on demand through its effect on relative prices. A rise in $\hat{A}_{lh}$ increases the relative price of the high-skill sector, which increases the relative expenditure x, resulting in higher l_h for any given q.

Given the equilibrium (q, l_h), the allocation of high-skill labor follows from (14), the relative price p_h/p_l follows from (19), and the relative expenditure x follow from (22).

3.5 Low-Skill Wage Stagnation

Using (17), the low-skill real wage can be expressed as:

$$\frac{w_l}{P_C} = \left(\frac{w_l}{p_l}\right)\left(\frac{p_l}{P_C}\right) = A_l\left[J_l(q)\xi_l^{-\eta}\right]^{\frac{1}{1-\eta}}\left(\frac{p_l}{P_C}\right), \tag{23}$$

where (p_l/P_C) can be obtained from the consumption price index in (13). It is important to note that productivity growth itself has a positive effect on the level of low-skill real wage. This can be seen by substituting the P_C in (13) and the relative price in (19) into (23):

$$\frac{w_l}{P_C} = \left[\hat{A}_l^{\varepsilon-1}\left(\xi_l^{-\eta}J_l\right)^{\frac{1-\varepsilon}{\eta-1}} + \hat{A}_h^{\varepsilon-1}\left(\xi_h^{-\eta}J_h\right)^{\frac{1-\varepsilon}{\eta-1}}\right]^{\frac{1}{\varepsilon-1}}; \tag{24}$$

$$\hat{A}_l \equiv \psi^{\frac{\varepsilon}{\varepsilon-1}}A_l \quad \hat{A}_h \equiv (1-\psi)^{\frac{\varepsilon}{\varepsilon-1}}A_h \tag{25}$$

which is increasing in productivity parameters A_l and A_h. Clearly the low-skill real wage will be stagnant if A_h and A_l are stagnant. But the main issue in the data is that low-skill real wage is lagging behind productivity.

Proposition 1 spells out the basic mechanism of the paper: low-skill workers are concentrated in sectors with faster productivity growth but they do not benefit as much since the output they produce is getting cheaper and is complementary to the high-skill labor.

Proposition 1 *When the output of the two sectors are complements ($\varepsilon < 1$), a rise in the relative productivity of the low-skill sector contributes negatively to the change in the low-skill real wage, reducing the direct positive effect from the rise in productivity.*

Proof. Suppose A_h is fixed and there is an increase in A_l. Higher $\hat{A}_{lh}$ implies higher p_h/p_l in (19), resulting in higher x in (22) given $\varepsilon < 1$, thus shifts up $D\left(q; \hat{A}_{lh}\right)$ in (21). Given $\xi_l > \xi_h$, $S(q;\zeta)$ in (20) is increasing in q, the increase in $D\left(q; \hat{A}_{lh}\right)$ results in higher q and higher l_h. Higher q further increases relative price p_h/p_l in (19) given $[J_h(q)/J_l(q)]^{\frac{1}{\eta-1}}$ is increasing in q from (16). The rise in p_h/p_l and q imply two negative effects on low-skill real wage in (23): a fall in p_l/P_C from (13), and a lower w_l/p_l given $[J_l(q)]^{\frac{1}{1-\eta}}$ is decreasing in q from (15). It follows that the change in low-skill real wage is smaller than the increase in A_l.

■

Proposition 1 highlights the importance of faster productivity growth in the low-skill sector and consumption complementarity.[17] Suppose the productivity growth was the same across sectors, i.e. $\hat{A}_{lh}$ does not change, then there will be no change in relative prices and there will be no shift in $D\left(q; \hat{A}_{lh}\right)$ thus no change in relative wage. Productivity growth will benefit all workers equally and the growth of real wages will be the same as productivity growth. On the other hand, in the absence of consumption complementarity, faster productivity growth in the low-skill sector will imply either no change (when $\varepsilon = 1$) or a fall (when $\varepsilon > 1$) in the relative expenditure on high-skill sector (see (22)). It follows from (21) that it will imply either no change or a downward shift in $D\left(q; \hat{A}_{lh}\right)$, resulting in no change or a fall in the relative wage, removing the negative effect on the marginal product of low-skill labor in (23).

It is worth noting that if both sectors use inputs with the same weights ($\xi_l = \xi_h$), then the two sectors only differ in terms of their productivity. The Baumol's cost disease is present due to uneven productivity growth and consumption complementarity, but it will apply to all workers equally. In this world, the relative wage is independent of relative productivity. This is because equation (14) implies that the factor intensity is identical across sectors and equal to the relative supply, so the relative wage is determined by relative supply.

3.5.1 Demand shifts towards high-skill sector

In addition to uneven productivity growth, a demand shift towards the high-skill sector can also lead to labor reallocation. This demand shift can be induced by a rising income if the high-skill goods have a higher income elasticity than low-skill goods. As shown by Comin et al. (2021), a fall in the preference parameter ψ in the homothetic CES utility function (3) is a reduced form way of capturing income effects in a more general non-homothetic CES utility function.[18] Thus, by examining the effect of a fall in ψ, we can learn about the effect of a demand shift

[17] Note that Proposition 1 relies on the elasticity of substitution across high-skill and low-skill goods to be below one ($\epsilon < 1$). It does not put restriction on the elasticity of substitution across high-skill and low-skill labor (η).

[18] This can be seen explicitly from comparing the relative expenditure derived in (12) with the relative expenditure derived from a non-homothetic CES utility function in Comin et al. (2021).

towards the high-skill sector on low-skill wage.

Using (22), a fall in ψ implies an increase in $\hat{A}_{lh}$ and a rise in the relative expenditure, thus it has a similar effect on the relative wage and low-skill labor allocation as an increase in the relative productivity A_l/A_h. But it will not have a direct effect on relative prices of the high-skill sector as shown in equation (19).[19] Thus it cannot generate a large dispersion in the growth rates of product wage across sectors, as shown in Figure 1 and it will not contribute much to the low-skill wage stagnation given changes in relative prices are necessary to generate the dispersion.

4 Low-Skill Wage and Productivity Divergence

The basic model delivers the key mechanism on how uneven productivity growth can contribute to low-skill wage stagnation. It generates a divergence in the low-skill wage and aggregate productivity by predicting a rise in wage inequality. This section first shows that there are two other potential drivers behind the divergence in the data that are missing from the basic model. It then presents a full model to incorporate all three drivers. Finally, it shows factors that imply a rise in wage inequality always contribute to the divergence but do not necessarily contribute to low-skill wage stagnation.

4.1 Accounting Identity

An accounting relationship between low-skill wage and aggregate labor productivity exists given the sum of value-added must equal to sum of factor payment:

$$\beta \sum_j p_j Y_j = \sum_i w_i M_i, \tag{26}$$

[19] It will have an equilibrium effect on the relative prices through the rise in q by changing J_h/J_l but the effect is small as it depends on the differences in the parameters ξ_h and ξ_l as shown in (16).

where p_j and Y_j is the price and real value-added of sector j, w_i and M_i are the wage and market hours by labor input i, and β is the labor income share. Let P_Y be the aggregate output price index and M be the total market hours, the identity implies

$$\beta y = w, \quad y \equiv \frac{\sum_j p_j Y_j}{M}, \quad w \equiv \frac{\sum_i w_i M_i}{M}, \tag{27}$$

where y is the nominal aggregate labor productivity and w is the average nominal wage in the economy. So the ratio of real productivity relative to low-skill real wage is:

$$\underbrace{\frac{y/P_Y}{w_l/P_C}}_{\textit{Real}} = \underbrace{\left(\frac{y}{w_l}\right)}_{\textit{Nominal}} \underbrace{\left(\frac{P_C}{P_Y}\right)}_{\textit{Deflator}}, \qquad \frac{y}{w_l} = \underbrace{\left(\frac{w}{w_l}\right)}_{\textit{Wage Inequality}} \underbrace{\left(\frac{1}{\beta}\right)}_{\textit{Labor Share}} \tag{28}$$

It shows that the real divergence in the low-skill wage and productivity can be due to growth in the relative cost of living and a nominal divergence in the low-skill wage and productivity. The nominal divergence itself can be driven by the growth in wage inequality (the ratio of average wage relative to low-skill wage) and a fall in labor income share.

Two of the drivers for the divergence, ratio of deflators and labor income share, are missing from the basic model. In the basic model, given both sectors only produce consumption goods, the value-added shares of the economy are the same as the expenditure shares. Thus it implies the consumption price deflator and the output price deflator are the same. Second, in the absence of capital, the labor income share is equal to 1 in the basic model. The remaining parts of this section present a full model that incorporates all three drivers of the divergence.

4.2 The Model Economy

This section extends the basic model to include capital. To keep the framework simple, we assume the output of the low-skill sector can be converted into capital and there is full depreciation of capital. In the quantitative exercise, the objective is to compare the labor market changes from 1980 to 2010 instead of studying the time path.

4.2.1 The model setup

The household problem is the same as the basic model but the firm's problem is different. The representative firm in sector $j = l, h$ uses low-skill labor, high-skill labor, and capital as input with the following production function:

$$Y_j = A_j F_j \left(G_j \left(H_j, K_j\right), L_j\right) \tag{29}$$

$$F_j \left(G_j \left(H_j, K_j\right), L_j\right) = \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + \left(1-\xi_j\right) \left[G_j \left(H_j, K_j\right)\right]^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{30}$$

$$G_j \left(H_j, K_j\right) = \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + \left(1-\kappa_j\right) H_j^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{31}$$

where parameter κ_j measures the importance of capital within the capital-skill composite. The new assumption is that there is capital-skill complementarity, $\rho < 1$. Together with $\eta > 1$, the nested CES structure implies that the elasticity of substitution across low-skill and capital are larger than the substitution across high-skill and capital.

The market clearing condition for the high-skill sector and the labor market clearing conditions are the same as before. The output of the low-skill sector can be used as consumption goods or converted into $1/\phi$ unit of capital, where ϕ can be interpreted as the price of capital relative to the low-skill goods.[20] As in Greenwood et al. (1997), an investment-specific technical change can be implemented as a fall in ϕ. The low-skill goods and the capital market clearing conditions are:

$$Y_l = C_l + \phi K, \tag{32}$$

$$K = K_h + K_l. \tag{33}$$

[20]We show in the Appendix A2.3 that this two-sector model can be mapped into a three-sector model where the low-skill sector is an aggregation of a consumption goods sector and a capital goods sector under the assumption that they have identical production functions except with a sector-specific TFP index. In this environment, the relative price of capital is equal to ϕ which is the inverse of their relative TFP.

4.2.2 Firm's optimal decision

The representative firm in each sector takes the price of capital q_k, high-skill labor w_h and low-skill labor w_l as given to maximize profit. The optimal decision of the firms implies the marginal rate of technical substitution across any two inputs is equal to its relative price. Across high-skill and capital input:

$$\frac{H_j}{K_j} = (\chi\delta_j)^{-\rho}; \qquad \delta_j \equiv \frac{\kappa_j}{1-\kappa_j}, \chi \equiv \frac{w_h}{q_k}. \tag{34}$$

Define $\tilde{I}_j$ as the high-skill income relative to total income that goes to high-skill and capital:

$$\tilde{I}_j \equiv \frac{w_h H_j}{q_k K_j + w_k H_j} = \frac{1}{1+\chi^{\rho-1}\delta_j{}^{\rho}}, \tag{35}$$

where the last equality follows from the condition (34). Appendix A2.2.1 shows that the relative skill-intensity in each sector is:

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta}\,(1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}}\,\tilde{I}_j{}^{\frac{\eta-\rho}{1-\rho}}, \tag{36}$$

Thus, the income share of low-skill in sector j is:

$$J_j \equiv \frac{w_l L_j}{q_k K_j + w_h H_j + w_l L_j} = \left[1 + q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\,(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1}, \tag{37}$$

The income share of high-skill in sector j is:

$$I_j \equiv \frac{w_h H_j}{q_k K_j + w_h H_j + w_l L_j} = (1-J_j)\,\tilde{I}_j. \tag{38}$$

Finally, the total labor income share in sector j is derived in Appendix A2.2.2 as

$$\beta_j = I_j + J_j = J_j\left[q^{1-\eta}\sigma_j^{\eta}\left[\tilde{I}_j\,(1-\kappa_j)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right]. \tag{39}$$

4.2.3 Equilibrium prices and allocation

Appendix A2.2.3 shows that the equilibrium low-skill wage has the same expression as (17) with the income share J_j derived in (37). Thus labor mobility implies the relative price of the high-skill sector has the same expression as in (19).

The equilibrium conditions on input prices and output prices imply an equilibrium condition across the relative prices χ and q. It is shown in Appendix A2.3 that the two-sector model can be mapped into a three-sector model where ϕ is the price of capital relative to low-skill goods, so $\phi = q_k/p_l$. Using the firm's optimal conditions, the equilibrium price of capital implies:

$$\chi = q\frac{A_l}{\phi}\left(J_l\xi_l{}^{-\eta}\right)^{\frac{1}{1-\eta}}. \tag{40}$$

Substituting J_l in (37), Appendix A2.4.1 derives q as a function of χ:

$$q = \chi\left[\left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}, \tag{41}$$

which is increasing in χ. Given q is a function of χ, it follows that I_j, J_j and $\tilde{I}_j$ are also functions of χ. Appendix A2.4 derives the new equilibrium conditions as:

$$l_h = S\left(\chi;\zeta,\frac{\phi}{A_l}\right) \equiv \frac{\zeta q\left(\chi;\frac{\phi}{A_l}\right)^{\eta}\sigma_l^{-\eta}(1-\kappa_l)^{\frac{\rho(\eta-1)}{1-\rho}}\tilde{I}_l(\chi)^{\frac{\eta-\rho}{\rho-1}}-1}{(\sigma_h/\sigma_l)^{\eta}\left(\frac{1-\kappa_l}{1-\kappa_h}\right)^{\frac{\rho(\eta-1)}{1-\rho}}\left(\frac{\tilde{I}_l(\chi)}{\tilde{I}_h(\chi)}\right)^{\frac{\eta-\rho}{\rho-1}}-1}. \tag{42}$$

$$l_h = D\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right) \equiv \left[1+\frac{J_l\left(\chi;\frac{\phi}{A_l}\right)}{J_h\left(\chi;\frac{\phi}{A_l}\right)}\left(\frac{1}{x\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right)\beta_l(\chi)}+\frac{1-\beta_h(\chi)}{\beta_l(\chi)}\right)\right]^{-1}, \tag{43}$$

where the relative expenditure share $x\left(\chi;\hat{A}_{lh},\frac{\phi}{A_l}\right)$ has the same expression as (22). Note that when $\kappa_j \to 0$, $\beta_j \to 1$, the two equilibrium conditions are the same as (20) and (21). These two conditions together solve for (χ, l_h) and the relative wage q is obtained from (41). The value-added share of the high-skill sector is derived in the Appendix A2.5 as:

$$v_h \equiv \frac{p_jY_j}{\sum_j p_jY_j} = \left[1+\left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1}, \tag{44}$$

4.3 Low-Skill Wage stagnation and Wage Inequality

This subsection uses the full model to show that factors that imply a rise in wage inequality do not always contribute to low-skill wage stagnation. Using the optimal capital-skill ratio in (34), the production function can be expressed as a function of high-skill and low-skill labor:

$$Y_j = \tilde{A}_j \left[\lambda_j H_j^{\frac{\eta-1}{\eta}} + (1-\lambda_j) L_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{45}$$

$$\tilde{A}_j \equiv A_j \left(\xi_j + (1-\xi_j) \left(\frac{1-\kappa_j}{\tilde{I}_j} \right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)} \right)^{\frac{\eta}{\eta-1}} \; ; \; \lambda_j \equiv \frac{(1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}{\xi_j + (1-\xi_j)\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\left(\frac{\rho}{\rho-1}\right)\left(\frac{\eta-1}{\eta}\right)}}, \tag{46}$$

which takes a similar form as the aggregate production used in the literature (see Heathcote et al., 2010), where the aggregate skill-biased shift is captured by an increase in the λ of an aggregate production function. Our model provides two endogenous sources of the aggregate skill-baised shift. First, falling relative price of low-skill goods (driven by uneven productivity growth) induces a labor reallocation towards the high-skill sector due to consumption complementarity. This implies an increase in aggregate λ when $\lambda_h > \lambda_l$, contributing to a between-sector skill-biased shift, which is shown as an important source for the increase in aggregate skill intensity in the data (see Appendix (??)). Second, falling relative price of capital (driven by uneven productivity and investment specific technical change) implies an increase in $\tilde{I}_j$ due to capital-skill complementarity. This implies an increase in λ_j, contributing to a within-sector skill-biased shift.

Both sources of endogenous skill-biased shifts imply a rise in wage inequality but they have different effects on the level of low-skill wage growth. The between-sector shift induces a shift from the low-skill sector with high $(1-\lambda_l)$ to the service sector with low $(1-\lambda_h)$, so it reduces the aggregate $(1-\lambda)$ contributing to a slow growth in low-skill wage. The within-sector shift, through rising $\tilde{I}_j$, reduces $(1-\lambda_j)$ in both sectors but this effect is offset by the implied rise in the effective productivity $\tilde{A}_j$ due to the capital-skill complementarity (i.e. $\rho < 1$, see

46). Thus the within-sector shift can contribute to a rise in wage inequality but it does not necessarily contribute to the low-skill wage stagnation.

There are other sources of skill-biased shifts not captured by the model and can be interpreted as exogenous changes in κ_j and ξ_j which increase λ_j. For instance, as a result of automation some tasks performed by low-skill are replaced by machines (Acemoglu and Autor, 2011), or skill-biased organizational change that increases the importance of human capital (Caroli and Van Reenen, 2001).

Using the nested CES production function specified in (29), the skill-biased shifts discussed above can be put into perspective using the three classes of technical changes in Johnson (1997). The fall in κ_j is an *intensive skill-biased technical change* which raises the marginal product of high-skill workers without affecting those of low-skill labor directly, thus it contributes to wage inequality but does not necessarily imply low-skill wage stagnation. The fall in ξ_j is an *extensive skill-biased technical change* which increases the marginal product of high-skill workers and lowers the marginal product of low-skill workers, thus contributing to both wage inequality and low-skill wage stagnation. What is interesting is the rise in A_h and A_l, which are *skill-neutral technical change* at the sectoral level but becomes skill-biased at the aggregate level because of different factor intensities across sectors, contributing to both rising wage inequality and low-skill wage stagnation.

4.4 The Decoupling of Wage and Productivity in the Model

The accounting identity in Section 4.1 shows that the divergence of low-skill wage from the aggregate labor productivity can be due to rising wage inequality, falling labor income shares and rising relative cost of living. We now study them through the lens of the model.

As shown in equation (28), the divergence in real terms is a product of divergence in nominal terms and the price deflators P_C/P_Y. For the model with two labor inputs, the nominal divergence is equal to:

$$\frac{y}{w_l} = \frac{(w_h/w_l - 1)\,\mu_H + 1}{\beta}; \quad \beta = \beta_l v_l + \beta_h v_h, \quad \mu_H = \frac{M_h}{M_l + M_h} \tag{47}$$

Given the share of high-skill market hours μ_H, the model implies a rise in the relative wage due to the two sources of endogenous skill-biased shifts. It predicts a rise in the high-skill income share and a fall in the low-skill income share in both sectors, and a shift towards the high-skill sector, thus has an ambiguous prediction on labor income share β.

The growth of the relative price indexes P_C/P_Y is obtained from the difference in the growth of the two deflators. The growth of both price indexes are weighted averages of the sectoral prices: the expenditure share x_j is the weight used for P_C (see Appendix A2.1) and the value-added share v_j is the weight used for P_Y (the Tornqvist formula). Given the expenditure share of the high-skill sector exceeds its value-added share, the model predicts a rise in the relative cost of living P_C/P_Y by predicting a rise in the relative price of the high-skill sector.

5 Quantitative Results

We calibrate the model to match key features of the U.S. from 1980 to 2010. The forces that drive the mechanism of the model are $(A_{lT}/A_{l0}, A_{hT}/A_{h0}, \phi_T/\phi_0)$. They are calibrated to match the rise in the relative price of the high-skill sector, the fall in the relative price of capital, and the aggregate labor productivity growth. The production weight of each input in the production function $\{\xi_{lt}, \xi_{ht}, \kappa_{lt}, \kappa_{ht}\}_{t=0,T}$ are set to match the sectoral income share while the relative supply of high-skill labor (ζ_0, ζ_T) are set to match the aggregate income shares of high-skill and low-skill labor.[21] In sum, the prediction on wages and wage-productivity divergence

[21] Our calibration procedures share some features with Buera et al. (2020) but the crucial difference is that their model abstracts from capital. As discussed earlier, the average wage grows at the same rate as the aggregate labor productivity in a model without capital. So by matching aggregate labor productivity, their model over-predicts growth in the average wage, thus cannot match the growth of low-skill wage and high-skill wage simultaneously while matching the skill premium.

are driven by changes in five set of parameters: $\hat{A}_{lh}$ in equation (22), the relative price of capital ϕ, the production weights $\{\xi_l, \xi_h, \kappa_l, \kappa_h\}$ and the relative supply of high-skill labor ζ.[22]

5.1 Data Targets

The data targets reported in Table 2 are constructed using the two-sector data described in Section 2.2.[23] Data from the five-year average 1978-1982 is used for 1980 and 2006-2010 for 2008. As shown in Table 2 the high-skill income share (I_j) increases while the low-skill income shares (J_j) fall in both sectors. The total labor income share ($I_j + J_j$) falls in the low-skill sector, rise in the high-skill sector, and fall for the overall economy. The price of high-skill sector relative to the price of low-skill sector grows at 1.4% and the annual growth of the aggregate labor productivity deflated by the price of the low-skill sector was 2.1% during this period. Using the ratio of P_K/P_Y from the BEA and the ratio P_Y/P_l from the KLEMS, the implied price of capital relative to low-skill sector ϕ declines at 0.5% per year.[24]

5.2 Calibration

The elasticity of substitution across high-skill and low-skill labor $\eta = 1.4$ is taken from Katz and Murphy (1992) and the elasticity of substitution across capital and high-skill labor $\rho = 0.67$ is taken from Krusell et al. (2000). There is no direct

[22]Given the definition of $\hat{A}_{lh}$ in equation (22) and $\hat{A}_l$ in equation (25), we do not need to separate the preference parameter ψ from A_l and A_h to solve for the model.

[23]To compute (w_h, w_l), we allow the efficiency unit of labor to be different within subgroups (gender, age, education, and race) of a skill-type, e.g. one hour of a high-school graduate is not equal to that of high school dropout in efficiency units. The relative efficiency unit of an average high-skill relative to an average low-skill is assumed to be one, where the average worker in each skill-type is defined by long-run hours shares of subgroups. Instead of choosing the average worker as the reference group, we could make alternative assumptions such as assuming the relative efficiency for a particular subgroup, e.g. an 18-25 years old white male, then compute the relative efficiency for an average high-skill relative to an average low-skill. As long as the relative efficiency does not change substantially over time, the quantitative result on low-skill wage stagnation is robust to this alternative assumption given we match the initial w_l in the data.

[24]It is worth noting that the growth of P_Y in KLEMS is growing at 2.94% which is almost identical to that of BEA at 2.86%.

Table 2: Calibration Data Summary

	Level							Growth (% p.a.)		
	J	J_h	J_l	I	I_h	I_l	q	$\frac{y}{p_l}$	ϕ	$\frac{p_h}{p_l}$
1980	0.41	0.23	0.46	0.17	0.33	0.12	1.44	-	-	-
2008	0.28	0.21	0.31	0.28	0.44	0.21	1.94	2.1	-0.5	1.4

estimate of the elasticity of substitution across high-skill and low-skill goods ϵ. The literature on the structural transformation finds that the elasticity of substitution across agriculture, manufacturing, and services is close to zero (Herrendorf et al., 2013). Given we re-group these three sectors into two sectors, this is likely to imply a higher degree of substitution. The equilibrium condition (8), on the other hand, implies that the own-price elasticity of the two goods is $-\varepsilon$. Ngai and Pissarides (2008) report a range of estimates for the price elasticity of services ranging from -0.3 to 0, this is informative but not an exact estimate for $-\varepsilon$ which is the price elasticity of the high-skill sector in our model. Based on these estimates, we use $\varepsilon = 0.2$ as our baseline value for the elasticity of substitution across high- and low-skill sectors. We conduct sensitivity analysis in Appendix A3.3.

The relative wage q and incomes shares reported in Table 2 are used to determine the relative supply of high-skill efficiency labor ζ and the input weights $(\xi_l, \xi_h, \kappa_l, \kappa_h)$ in the two periods. In the aggregate economy, the income share of high-skill relative to the low-skill is:

$$\frac{I_t}{J_t} = \frac{w_{ht}H_t}{w_{lt}L_t} = q_t\zeta_t, \tag{48}$$

which implies a value for the relative supply of high-skill efficiency labor ζ_t given data on (q_t, I_t, J_t).[25]

Given a value for ϕ/A_l, equation (40) can be used together with the equations on income shares to set the input weights to match sectoral income shares in the data. To simplify the explanation, denote 1980 as period 0 and 2008 as period

[25]Note that the H_j and L_j are not the raw market hours by the high-skill and low-skill workers in the data. The composition adjusted high-skill hours H_j in sector j is computed as high-skill income in sector j divided by the composition adjusted high-skill wage, similarly for L_j.

T. We normalized $\phi_0/A_{l0} = 1$, this pins down all input weights in period 0 (see Appendix A3.1 for details). Using these parameters condition (42) implies a value of l_{h0}. The value of $\hat{A}_{lh0}$ is then set to match the relative wage q_0 using condition (43).

For a given level of A_{lT}/A_{l0}, data on the fall in ϕ_t implies a value for ϕ_T/A_{lT}, which pins down all inputs weights in period T. We then set the change in the relative productivity A_{lhT}/A_{lh0} to match the increase in the relative price of the high-skill sector in the data. Finally, we adjust A_{lT}/A_{l0} so that the predicted changes in the aggregate labor productivity deflated by the price of the low-skill sector, y/p_l, matches the data. It is important to note that the model is not calibrated to match the relative wage in period T.[26]

Table 3 reports the calibrated parameters. The implied annual growth of ϕ, A_{lh}, A_l, ζ and input weights (κ_j, ξ_j) are reported in Panel B of Table 3.[27] Matching the rise in the relative price of the high-skill sector implies faster productivity growth in the low-skill sector.[28] Matching the aggregate income shares of the high-skill and low-skill labor implies a rise in the relative supply of high-skill efficiency labor. Matching the sectoral income shares, on the other hand, requires changes in the input weights reflecting other sources of skill-biased shifts that are exogenous to our model. Changes in these parameters drive the quantitative results of the model.

Using the calibrated parameters the model delivers predictions on wages, allocation of labor, relative prices, and labor productivity for each sector. As shown in Appendix Table A3, the model accounts for 96% of the rise in relative wage q,

[26]The relative wage in period T is only used together with the income shares to calibrate $\{\xi_{lT}, \xi_{hT}, \kappa_{lT}, \kappa_{hT}\}$. As an alternative, we could choose growth in relative productivity, A_{lhT}/A_{lh0}, to match the rise in the relative wage using equation (43) for period T. However, given the objective of the quantitative exercise is to examine the proposed mechanism in accounting for stagnant low-skill wage and its divergence, and the mechanism is governed by the changing relative prices, we choose to match the changes in relative prices instead of relative wage.

[27]Note that negative growth in κ_j does not necessarily mean a decrease in the usage of capital. It only implies a fall in the input weight of capital in the capital-skill composite.

[28]The growth in A_j of the model is not the same as the TFP growth in the data. The ranking is consistent with the data that the low-skill sector has faster TFP growth than the high-skill sector.

Table 3: Parameters of Calibration

A. Parameters from the literature				
Parameters	Values			Source
ε	0.2			Benchmark value, see main text
ρ	0.67			Krusell et al. (2000)
η	1.4			Katz and Murphy (1992)
B. Calibrated parameters				
Parameters	1980	2010	Growth (% p.a.)	Target
ϕ			-0.50	Price of capital relative to the low-skill sector
A_l			1.10	Labor productivity deflated by price of the low-skill sector
A_{lh}			1.82	Relative price of the high-skill sector
ξ_l	0.33	0.25	-0.93	Sectoral income share. See Appendix A3.1
ξ_h	0.20	0.19	-0.13	Sectoral income share. See Appendix A3.1
κ_l	0.74	0.69	-0.21	Sectoral income share. See Appendix A3.1
κ_h	0.41	0.33	-0.79	Sectoral income share. See Appendix A3.1
ζ	0.29	0.50	1.92	Relative aggregate labor income shares

86% of the rise in the share of low-skill labor in the high-skill sector l_h, and the constant share for the high-skill labor observed in the data. Consistent with the data, it predicts a fall in labor income share in the low-skill sector and a rise in labor income share in the high-skill sector, and a decline in aggregate labor income share. The role of each of the five parameters $(A_l/A_h, \phi, \xi_j, \kappa_j, \zeta)$ can be found in Appendix Table A3. All parameters are important for the rise in the relative wage. The between-sector mechanism through the rise in A_l/A_h is crucial for the low-skill labor reallocation and the rise in the value-added share of the high-skill sector. The fall in ξ_j, on the other hand, is needed for the lack of high-skill labor reallocation and the fall in the labor income share in the low-skill sector.

The sectoral real labor productivity growth in the model is

$$\frac{y_j}{p_j} \equiv \frac{Y_j}{L_j + H_j} = A_j \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}} \left(\frac{1}{1 + H_j/L_j}\right), \tag{49}$$

which shows that in addition to A_j, other factors also contribute to the sectoral labor productivity growth. The calibrated model predicts the sectoral labor productivity growth is 2.2% for the low-skill sector and -0.2% for the high-skill sector, which match the 2.3% and 0% observed in the data almost perfectly.[29]

[29] The calibration implies that the A_h is falling. Aggregating individual sectors using the Tornqvist indexes, we compute the TFP growth to be 0.82% for the low-skill and -0.37% for

5.3 Predictions on Wage-Productivity Divergence

Table 4 reports the percentage change in the real divergence, decomposed into changes in the relative cost of living, wage inequality, and the aggregate labor income share. Since KLEMS data does not contain information on consumption, we simply take P_C/P_Y as the ratio of PCE and GDP implicit deflators from the BEA.[30]

The data (row 1) provides an empirical decomposition for the accounting identity in equation (28). During this 30-year period, the negative forces imposed by the rising relative cost of living, growing wage inequality, and falling aggregate labor income share largely offset the impact of rising productivity on low-skill real wage. The rise in the relative cost of living contributes to 10%(=2.8/27) of the real divergence, the increase in the wage inequality contributes to 70% (=19/27) and the fall in the aggregate labor income share accounts for the remaining 20% of the real divergence.[31] The baseline (row 2) can account for all the real and the nominal divergence. The remaining rows of Table 4 examine each of the five forces that drives these changes.

Row 3 and 4 of Table 4 show that both sources of endogenous skill-biased shifts contribute to the real divergence by predicting a rise in the wage inequality and a rise in the relative cost of living. Among the two sources, the falling relative price of capital (Row 4) contributes more through the wage inequality, while the uneven productivity growth (Row 3) contributes through both channels. It shows that the uneven productivity growth alone can account for 85% (=23/27) of the

the high-skill sector. The decline in A_h is also in line with the findings of Aum et al. (2018) and Bárány and Siegel (2021). The former finds negative growth for high-skill occupations (Professional and Management) while the latter finds negative growth for abstract occupation. We do not model occupations, but their findings could be the sources of the falling A_h given these occupations are concentrated in the high-skill sector.

[30] This implies P_C/P_Y increased by 2.8% as reported in Table 4. If we were to use CPI which grows faster than PCE, the increase in P_C/P_Y would be at 11.5%. This alternative value will imply a larger real divergence and slower real wage growth in the data row in Table 4 and 5, but does not affect the predictions of the model. Due to the concerns that CPI tends to bias the increase in the cost of living (Boskin et al., 1998), we follow the literature in using the PCE deflator.

[31] The literature on the average wage and productivity divergence often uses the nonfarm business sector. In Appendix A1.3 we conduct the empirical decomposition for the accounting identity in equation (28) using similar data.

Table 4: Real and Nominal Divergence, Cumulative Percentage Change, 1980-2008

		Real	Nominal			Deflator	
		$(y/w_l)(P_C/P_Y)$	y/w_l	w/w_l	β	P_C/P_Y	p_h/p_l
(1)	Data	27	24	19	-3.4	2.8	49
(2)	Model	34	23	19	-3.7	8.3	matched
		Counterfactual (keeping all else constant at 1980)					
(3)	$A_l/A_h \uparrow$	23	9.3	13	3.5	12	79
(4)	$\phi \downarrow$	11	9.7	14	3.7	1.5	8.8
(5)	$\xi_j \downarrow$	28	29	19	-7.5	-0.6	-3.3
(6)	$\kappa_j \downarrow$	6.3	5.9	14	7.6	0.3	2.1
(7)	$\zeta \uparrow$	-4.6	-3.4	-7.1	-3.8	-1.2	-6.6

Note: the combined effects are not the sum of the individual effects because the model is not linear.

real divergence by predicting 68% (=13/19) of the rise in wage inequality and all the rise in the relative cost of living. Row 5 and 6 of Table 4 show that changes in production weights contribute to the real divergence by predicting a rise in wage inequality but only the fall in ξ_j (Row 5) can generate a fall in the labor income share. Finally, the increase in the relative supply of high-skill labor contributes negatively to the divergence as it reduces wage inequality but it contributes to a fall in the labor income share (Row 7). This is because higher relative supply of high-skill induces an increase in the capital income share due to capital-skill complementarity.

5.4 Predictions on Wage Stagnation

Table 4 shows that all sources of skill-biased shifts are important in accounting for the divergence of the low-skill wage from aggregate productivity. We next turn to their effects on the growth of the low-skill real wage. As highlighted in equation (23), their effects depend crucially on how they affect the growth of the low-skill product wage in each sector (w_l/p_j).

Table 5 shows that among them, only the uneven productivity growth (Row 3)

Table 5: Productivity and Wages, Cumulative Percentage Change, 1980-2008

		y/P_Y	w_l/P_C	y/p_l	w_l/p_l	w_l/p_h
(1)	Data	60	26	78	44	-3.4
(2)	Model	61	20	matched	44	-3.2

Counterfactual (keeping all else constant at 1980)

		y/P_Y	w_l/P_C	y/p_l	w_l/p_l	w_l/p_h
(3)	$\boldsymbol{A_l/A_h} \uparrow$	**43**	**17**	**68**	**54**	**-14**
(4)	$\phi \downarrow$	81	63	85	69	55
(5)	$\boldsymbol{\xi_j} \downarrow$	**53**	**19**	**51**	**18**	**22**
(6)	$\kappa_j \downarrow$	61	51	62	53	50
(7)	$\zeta \uparrow$	80	89	77	83	96

and the falling production weights of low-skill workers (Row 5) can contribute to the low-skill wage stagnation, but they deliver different patterns for the low-skill product wages. In the data (Row 1) the low-skill product wage in the low-skill sector rose by 44% and fell in the high-skill sector due to the rise in the relative price of the high-skill sector. Consistent with the data, the uneven productivity growth (Row 3) implies a 54% rise in the low-skill sector and a fall in the high-skill sector, by predicting a rise in the relative price. The uneven productivity growth implies a reallocation from the low-skill sector with high ξ_l to the high-skill sector with low ξ_h, contributing to a decline in the average ξ in the economy. The falling production weights of low-skill workers (Row 5), however, predicts slow growth in low-skill product wages in both sectors, which misses the sector-specific trends observed in the data. Finally, as discussed in Section 4.3, both the falling relative price of capital (Row 4) and lower κ_j (Row 6) boost the growth in low-skill real wage by increasing the growth of the low-skill product wages in both sectors.

6 Conclusion

Despite working mostly in sectors with fast productivity growth, the average real wage for low-skill workers is stagnant because of the divergence in low-skill wage and productivity driven by rising wage inequality, falling labor share and rising relative cost of living. This paper shows that uneven productivity growth across sectors can produce the low-skill wage stagnation, growing wage inequality, and the wage-productivity divergence simultaneously. Quantitatively, the model does a good job in accounting for these three facts.

A key message of the multisector perspective is that there does not need to be a trade-off between the low-skill real wage and aggregate labor productivity. By improving the total factor productivity growth of the high-skill intensive sectors, we can slowdown the growth of their relative prices and boost the growth of the low-skill wage and the aggregate productivity.

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Appendix

A1 Data Appendix

A1.1 Industry Data

A1.1.1 Nation-level data

The main dataset at the industry level is the March 2017 Release of the Unites States data from the WORLD KLEMS database (Jorgenson et al., 2017), which reports industry value-added, price indexes, labor compensation, and capital compensation. The data are reported using the North American Industry Classification System (NAICS), which is the standard used by Federal statistical agencies in classifying business establishments in the U.S. This provides the data needed to compute value-added share, relative prices, and labor income shares for all industries that are consistent with the official statistics.

To classify sectors into high-skill and low-skill sectors, we use April 2013 Release of the U.S. data from the WORLD KLEMS (Jorgenson et al., 2013) which provides a labor input file that allows the computation of low- and high-skill workers' share in labor compensation and value-added. High-skill is defined as education greater than or equal to college degree. Table A1 reports the long-run (1980-2010) average high-skill share in total value-added and total labor income for 15 one-digit industries. For a sector to be classified as high-skill we require that the long-run high-skill labor income shares out of total labor income and total value-added to be jointly above the total economy average. The high-skill service sector includes finance, insurance, government, health and education services (code J ,L, M, N), and the remaining industries are grouped into the low-skill sector.

Using this classification we map the 65 NAICS industries of the KLEMS 2017 Release and the three-digit *ind1990* codes of the CPS into the two broad sectors for the quantitative analysis. Value-added and labor compensation for each broad sector are obtained by summing over industries in each broad sector. Sectoral

Table A1: High-Skill Income Shares by Industry, 1980-2010 average

Industry	Code	High-skill share in Value-added	High-skill share in Labor income
Agriculture, Hunting, Forestry and Fishing	AtB	10	19
Mining and Quarrying	C	11	32
Total Manufacturing	D	20	31
Electricity, Gas and Water Supply	E	9	30
Construction	F	14	16
Wholesale and Retail Trade	G	22	30
Hotels and Restaurants	H	14	18
Transport and Storage and Communication	I	16	25
Financial Intermediation	**J**	**33**	**55**
Real Estate, Renting and Business Activity	K	21	55
Public Admin	**L**	**29**	**40**
Education	**M**	**58**	**77**
Health and Social Work	**N**	**39**	**49**
Other Community, Social and Personal Services	O	23	31
Private Households with Employed Persons	P	16	16
All Industries	TOT	25	40

Notes: The table reports the share of high-skill workers in total value-added and labor income by industry. High-skill is defined as education greater than or equal to college degree. Labor income reflects total labor costs and includes compensation of employees, compensation of self-employed, and taxes on labor.
Source: April 2013 Release of the WORLD KLEMS for the U.S.

value-added prices are calculated as Tornqvist indexes, where value-added shares are used as weights. For the ratio of aggregate consumption price deflator and output price deflator, we use the BEA's implicit price deflators of GDP and Personal Consumption Expenditures, respectively. The price of capital is calculated as the investment in total fixed assets divided by the chain-type quantity index for investment in total fixed assets (Tables 1.5 and 1.6 of the BEA's Fixed Assets Accounts).

Industries in Figures 1 and 2 are the one-digit industries reported in Table A1 with some regrouping. Due to the low number of observations in CPS we merge agriculture (AtB) with mining (C) and other services (O) with private households (P). We also regroup public administration (L), education (M), and health and social work (N) as a single industry to ensure consistency in industry definitions.[32] Our mapping across KLEMS 2013, KLEMS 2017, and CPS industries is provided

[32]For instance, public education is included in the general government industry in KLEMS 2017, while it is part of education in KLEMS 2013.

in Table A2.

A1.1.2 State-level data

We use GDP by state from the BEA's Regional Economic Accounts for value-added sector prices at the state-level. BEA reports nominal and real GDP (chained at constant dollars) by industry for 51 states by SIC between 1963-1997, and by NAICS between 1997-2010. In order to calculate sectoral prices, we first aggregate the industry data in 11 consistent sectors according to Table A2. Next, using the common year of observation 1997, we carry forward the SIC-based series by the growth rates of the NAICS-based series. Finally, we calculate sectoral price indexes as the ratio of nominal to real GDP. Our bridging strategy produces national sectoral growth rates similar to those reported in the KLEMS data. In particular, the correlation coefficients between the long-run U.S.-level sectoral growth rates from both sources are 0.97, 0.91, and 0.90 for nominal value-added, real value-added, and prices, respectively.

A1.2 Wages, Efficiency Hours, and Productivity

We use March Current Population Survey Annual Social and Economic Supplement (ASEC) data from 1978 to 2012 (Ruggles et al., 2017). Our sample includes wage and salary workers with a job aged 16-64, who are not student, retired, or in the military. Hourly wage is calculated as annual wage income divided by annual hours worked, where the latter is the product of weeks worked in the year preceding the survey and hours worked in the week prior to the survey. Top coded components of annual wage income are multiplied by 1.5. Workers with weekly wages below $67 in 1982 dollars (based on PCE price index) are dropped.

Our treatment of Census for years 1980, 1990, 2000, and ACS for 2010 in Section 2 follows the same steps with the above paragraph except that wages lower than the first percentile are set to the value of the first percentile following Autor and Dorn (2013).

The composition adjusted mean wages of low-skill workers for each of the sectors, used in Figures 1 and 2, are computed using the CPS data as follows. Within each sector, we calculate mean wages weighted by survey weights for each of 216 subgroups composed of two sexes, white and non-white categories, three education categories (high school dropout, high school graduate, some college), six age categories (16-24, 25-29, 30-39, 40-49, 50-59, 60-64 years), and three occupation categories (high-wage occupations including professionals, managers, technicians, and finance jobs, middle-wage occupations including clerical, sales, production, craft, and repair jobs, operators, fabricators, and laborers, and low-wage occupations including service jobs). Sector-level means by skill are calculated using the long-run average hours share of each subgroup in the labor market as weights. This way we obtain a measure of industry wage that only compares growth differences of subgroups across industries. However, applying long-run hours share by subgroup can still affect industry means through composition when for some subgroups there are missing observations in some of the industries. Cells containing missing wages are imputed for each year of the dataset using a regression of the log of hourly wages on industry dummies and dummies including the full set of interactions of subgroups. We assign predictions from this regression to the missing wage observations while keeping the observed wages. The growth rate of sector wages with and without imputation are very close. Finally, we deflate nominal wages by the PCE price index for real wages and by the value-added price index for product wages.

The composition adjusted wages used in Section 2 are constructed using Census and ACS data due to the need for sufficient number of observations at the state level. The steps of the composition adjusted wage calculation are identical to what is explained above with one exception. There is an additional layer of states so that the composition adjustment is performed within each of the 51 states. We deflate nominal wages by the national PCE price index due to the absence of consistent state-level consumption prices in our period.

For the quantitative analysis, used in Table 4 and 5, the aggregate wage has to be consistent with the measure of aggregate productivity, so we use the aggregate labor compensation and aggregate hour from the KLEMS. More specifically, to compute the composition-adjusted wage for the average high-skill and average low-skill workers, we merge KLEMS 2013 data on total labor compensation and hours with the distribution of demographic subgroups in the CPS. We form 120 subgroups based on two sex, two race, five education, six age categories. Low-skill includes high school dropout, high school graduate, and some college; high-skill includes college graduates and post-college degree categories. Compensation for each subgroup is calculated as compensation share (from CPS) times total compensation (from KLEMS). The hours worked of each subgroup is calculated in a similar way. The wage for each subgroup is then calculated as total compensation divided by total hours. The aggregate wage for low-skill and high-skill are calculated as the average wage of the relevant subgroups using their long-run (1980-2010) hour shares as weights. It is important to note that the labor compensation variable of KLEMS includes both wage and non-wage components (supplements to wages and salaries) of labor input costs as well as reflecting the compensation of the self-employed, and hours variable in KLEMS are adjusted for the self-employed. Thus KLEMS provides a more reliable source of aggregate compensation and aggregate hours in the economy. This procedure is equivalent to rescale the CPS total hours and total wage income to sum up to KLEMS total.

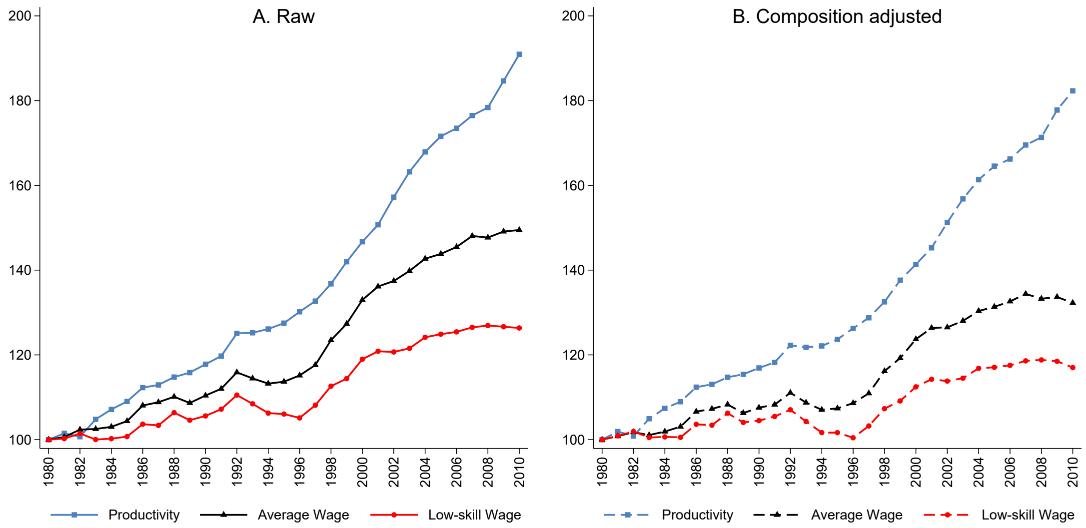
Efficiency hours, corresponding to (H, L) in the model, are computed as the labor compensation divided by composition-adjusted wage for high-skill and low-skill workers respectively. Total efficiency hours are the sum of low- and high-skill efficiency hours. We calculate real labor productivity as total value-added divided by total efficiency hours and deflate with the output price index.

Table A2: Industry Mapping

NACE (KLEMS 2013)	NAICS (KLEMS 2017)	IND1990 (CPS)
AtB & C	Farms, Forestry, Fishing, and Related Activities, Oil and Gas Extraction, Mining, Except Oil and Gas, Support Activities for Mining	Agriculture, Forestry, and Fisheries, Mining
D	Wood Products, Nonmetallic Mineral Products, Primary Metals, Fabricated Metal Products, Machinery, Computer and Electronic Products, Electrical Equipment, Appliances, and Components, Motor Vehicles, Bodies and Trailers, and Parts, Other Transportation Equipment, Furniture and Related Products, Miscellaneous Manufacturing, Food and Beverage and Tobacco Products, Textile Mills and Textile Product Mills, Apparel and Leather and Allied Products, Paper Products, Printing and Related Support Activities, Petroleum and Coal Products, Chemical Products, Plastics and Rubber Products	Manufacturing
E	Utilities	Utilities
F	Construction	Construction
G	Wholesale Trade, Retail Trade	Wholesale Trade, Retail Trade
H	Accodomation, Food Services and Drinking Places	Hotels and Lodging Places, Eating and Drinking Places
I	Air Transportation, Rail Transportation, Water Transportation, Truck Transportation, Transit and Ground Passenger Transportation, Pipeline Transportation, Other Transportation and Support Activities, Warehousing and Storage, Publishing Industries, Except Internet (Includes Software), Motion Picture and Sound Recording Industries, Broadcasting and Telecommunications, Data Processing, Internet Publishing, and Other Information Services	Transportation, Communications
J	Federal Reserve Banks, Credit Intermediation, and Related Activities, Securities, Commodity Contracts, and Investments, Insurance Carriers and Related Activities, Funds, Trusts, and Other Financial Vehicles	Finance, Insurance
K	Real Estate, Rental and Leasing Services and Lessors of Intangible Assets, Legal Services, Computer Systems Design and Related Services, Miscellaneous Professional, Scientific, and Technical Services, Management of Companies and Enterprises, Administrative and Support Services, Waste Management and Remediation Services	Real Estate, Business Services, Professional Services*
L & M & N	Educational Services, Ambulatory Health Care Services, Hospitals and Nursing and Residential Care Facilities, Social Assistance, Federal General Government, Federal Government Enterprises, State and Local General Government, State and Local Government Enterprises	Public Administration, Education*, Health and Social Services*
O & P	Performing Arts, Spectator Sports, Museums, and Related Activities, Amusements, Gambling, and Recreation Industries, Other Services, Except Government	Sanitary and Personal Services, Private Households, Entertainment and Recreation Services, Museums, Art Galleries, and Zoos, Labor Unions, Religious Organizations, Membership Organizations, n.e.c.

Notes: The table shows the mapping of KLEMS 2013 industries to KLEMS 2017 and CPS industries. The description of KLEMS 2013 industries is provided in Table A1. Industries marked with * do not have separate sections in CPS industry classification. They are constructed as follows. Professional Services: Engineering, architectural, and surveying services, Accounting, auditing, and bookkeeping services, Research, development, and testing services, Management and public relations services, Miscellaneous professional and related services, Legal services. Education: Elementary and secondary schools, Colleges and universities, Vocational schools, Educational services, n.e.c. Health and Social Services Offices and clinics of physicians, Offices and clinics of dentists, Offices and clinics of chiropractors, Offices and clinics of optometrists, Offices and clinics of health practitioners, n.e.c., Hospitals, Nursing and personal care facilities, Health services, n.e.c., Job training and vocational rehabilitation services, Child day care services, Family child care homes, Residential care facilities, without nursing, Social services, n.e.c.

Figure A1: Divergence in the BLS Nonfarm Business Sector Data



Notes: The figure plots low-skill and average hourly real wage and average hourly real labor productivity in the U.S. economy, all normalized to 100 in 1980. Raw (composition adjusted) wage and hours are used in Panel A (B). Real labor productivity is from the Bureau of Labor Statistics (BLS). Real hourly wages are calculated by merging hours and income shares in the Current Population Survey (CPS) with the total hours and labor income in BLS. Productivity is deflated by the output price index. Wages are deflated by Personal Consumption Expenditure (PCE) price index. Low-skill is defined as education less than a college degree. Composition adjusted wages are calculated as the fixed-weighted mean of 120 demographic groups, where the fixed weights are groups' long-run employment shares. See the appendix subsection for the construction of variables.
Source: BLS nonfarm business sector multifactor productivity statistics, CPS, and authors' calculations.

A1.3 Divergence in the BLS Nonfarm Business Data

This subsection compares the wage growth and the decomposition of low-skill wage and productivity divergence by KLEMS, on which results in the main text are based, with Bureau of Labor Statistics (BLS) nonfarm business productivity data. BLS nonfarm business data is typically used by the papers on U.S. wage-productivity divergence (e.g. Lawrence and Slaughter, 1993; Lawrence, 2016; Stansbury and Summers, 2017), and its labor share is a widely cited headline measure (Elsby et al., 2013).

In order to compute wages at skill-level that are consistent with the BLS productivity series' hourly compensation growth, the share of annual wage income and total hours of 120 demographic groups from March CPS are used. Demographic groups are based on six age, two gender, two race, and five education categories. Compensation (hours) for each subgroup is calculated as compensation (hours) share times BLS total compensation (hours). BLS-consistent wages for each subgroup is calculated as total compensation divided by total hours. Average and low-skill wages are then calculated as the mean hourly wages of relevant subgroups weighted by their hours share. In the composition adjusted wages, long-run hours shares are used as weights. This is the same procedure as we followed for the quantitative analysis with two exceptions. First, we further exclude agriculture, private households, and public administration sectors to comply with nonfarm business sector. Second, aggregate labor income and hours are rescaled to those of nonfarm business sector. For real wages Personal Consumption Expenditure price index (PCE) is used as the wage deflator.

Real labor productivity is U.S. nonfarm business nominal output divided by nonfarm total composition adjusted hours and deflated by the output price deflator from BLS. The average wage for all workers is calculated as total compensation divided by total composition adjusted hours of the nonfarm business sector. Composition adjusted or efficiency hours are calculated for each skill as the total compensation divided by composition adjusted wages.

Figure A1 plots the raw and composition adjusted low-skill real wage, average real wage, and real labor productivity. From 1980 to 2010, the low-skill wage growth is around 25 percent which shrinks just below 20 percent when adjusted for compositional changes. These figures are slightly lower from those suggested by KLEMS (Table 5) and somewhat higher than those calculated directly from CPS. The former difference stems from the industry coverage that particularly affects growth rates in labor income, which is lower in the nonfarm business sector. Hours grow at the same rate in both. On the contrary, the latter difference, i.e. slower wage growth in CPS, is driven by the stronger growth in CPS hours compared to those in the macro sources, despite a bit higher growth in CPS wage income.[33]

As shown in Figure A1, low-skill real wage growth is less than a quarter of the labor productivity growth, suggesting a higher real divergence than what is calculated from KLEMS. The reason for a higher divergence is partly greater

[33] See Stewart and Frazis (2019) for an up-to-date discussion on the hours estimated by CPS and other BLS measures. Although total annual hours estimated from CPS is seen as problematic, authors recommend the use of CPS for comparing hours across demographic groups, which is consistent with our data approach.

decline in labor share of nonfarm business (7 percent as opposed to 3.4 in KLEMS), which is already hinted by the discussion above regarding the stronger labor income growth in KLEMS. A second but more important reason is the large growth in the BLS nonfarm business output deflator compared to the BEA's output deflator. Accordingly, the relative cost of living increases by 13 percent compared to 2.8 in KLEMS. Not surprisingly, inequality growth is the same in the two sources given that they both employ hours and income distribution of CPS. Recall Table 4 implies increasing inequality, declining labor share, and rising relative cost of living accounts for 70, 20 and 10 percent of the real divergence respectively. The corresponding decomposition based on nonfarm business sector are 48, 19 and 33 percent, implying a larger role for the rising relative cost of living for the real divergence, and a larger role of labor share relative to wage inequality for the nominal divergence.

A1.4 Hours and income share of high-skill workers

In contrast to the reallocation of low-skill hours, Figure A2A shows that the allocation of high-skill hours across the two sectors are rather constant since 1980. Figure A2B shows that the high-skill income share are rising in both sectors, where the common driver is the increase in the relative supply of high-skill workers. The reallocation of low-skill hours into the high-skill sector contributed to a faster rise in the high-skill income share in the low-skill sector since 1980.

References

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Elsby, M. W., B. Hobijn, and A. Sahin (2013). The decline of the US labor share. Brookings Papers on Economic Activity 2013 (2), 1-63.

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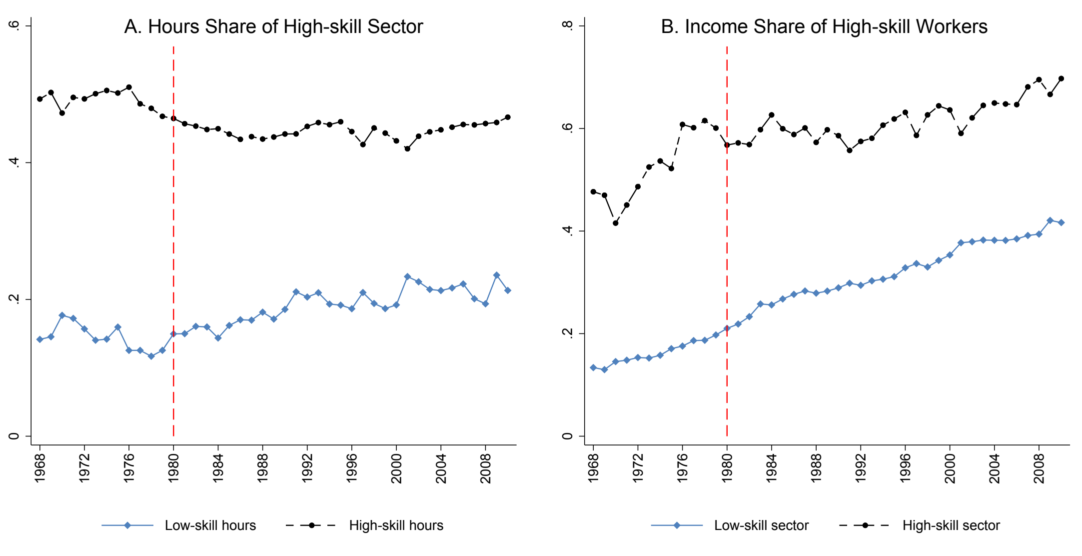
Figure A2: Trends in Hours and Labor Income



Notes: Panel A shows the share of low-skill hours and the share high-skill hours in the high-skill sector. Panel B shows the income share of high-skill workers in the low-skill and the high-skill sector. See Data Appendix A1 for the construction of variables and sectors.
Source: WORLD KLEMS and CPS.

Challenges for Public Policy.

Ruggles, S., K. Genadek, R. Goeken, J. Grover, and M. Sobek (2017). Integrated public use microdata series: Version 7.0 [dataset]. https://doi.org/10.18128/D010.V7.0.

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A2 Theory Appendix

The proof here is for the general case. It can be applied to the basic model with no capital by setting $\kappa_j = 0$.

A2.1 Deriving Consumption Price Index

Define p_{ci} as household i' price index for the consumption basket:

$$p_{ci}c_i = p_l c_{il} + p_h c_{ih} = c_{il} p_l \left(1 + x\right).$$

From the utility function,

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi} \right) \left(\frac{c_{ih}}{c_{il}} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

substituting the optimal condition (8),

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi} \right) \left(\frac{p_l}{p_h} \left(\frac{1-\psi}{\psi} \right) \right)^{\varepsilon-1} \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

simplify to

$$\frac{c_i}{c_{il}} = \psi^{\frac{\varepsilon}{\varepsilon-1}} \left[1 + \left(\frac{1-\psi}{\psi} \right)^{\varepsilon} \left(\frac{p_l}{p_h} \right)^{\varepsilon-1} \right]^{\frac{\varepsilon}{\varepsilon-1}} = \left[\psi \left(1+x \right) \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{A1}$$

thus using the expression for c_i/c_{il} in (A1), the consumption price index becomes

$$p_{ci} = \left(\psi \left(1+x \right) \right)^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1+x \right),$$

which is identical across households due to the assumption of a homothetic preference with identical weight, so it is also the same as the aggregate price index for consumption P_C. Using the expression for x in (8),

$$P_C = p_{ci} = \psi^{\frac{\varepsilon}{1-\varepsilon}} p_l \left(1 + \left(\frac{p_h}{p_l} \right)^{1-\varepsilon} \left(\frac{1-\psi}{\psi} \right)^{\varepsilon} \right)^{\frac{1}{1-\varepsilon}},$$

which simplifies to

$$P_C = \left[\psi^{\varepsilon} p_l^{1-\varepsilon} + \left(1-\psi \right)^{\varepsilon} p_h^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{A2}$$

Thus

$$\begin{aligned}
\frac{P_{Ct}}{P_{Ct-1}} &= \left[\frac{\psi^{\varepsilon}p_{lt}^{1-\varepsilon}+(1-\psi)^{\varepsilon}p_{ht}^{1-\varepsilon}}{\psi^{\varepsilon}p_{lt-1}^{1-\varepsilon}+(1-\psi)^{\varepsilon}p_{ht-1}^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[\frac{\psi^{\varepsilon}p_{lt-1}^{1-\varepsilon}}{\psi^{\varepsilon}p_{lt-1}^{1-\varepsilon}+(1-\psi)^{\varepsilon}p_{ht-1}^{1-\varepsilon}}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}+\frac{(1-\psi)^{\varepsilon}p_{ht-1}^{1-\varepsilon}}{\psi^{\varepsilon}p_{lt-1}^{1-\varepsilon}+(1-\psi)^{\varepsilon}p_{ht-1}^{1-\varepsilon}}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \\
&= \left[x_{lt}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}+x_{ht}\left(\frac{p_{ht}}{p_{ht-1}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.
\end{aligned}$$

A2.2 Equilibrium Prices

A2.2.1 Deriving the ratio H_j/L_j

Equating MRTS across high-skill and low-skill labor to relative wages:

$$q=\frac{1-\xi_j}{\xi_j}\left(\frac{L_j}{\tilde{H}_j}\right)^{\frac{1}{\eta}}(1-\kappa_j)\left(\frac{G_j(H_j,K_j)}{H_j}\right)^{\frac{1}{\rho}},$$

which can be re-written as

$$q=\sigma_j(1-\kappa_j)\left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}}\left(\frac{G_j(H_j,K_j)}{H_j}\right)^{\frac{\eta-\rho}{\rho\eta}};\qquad \sigma_j\equiv\frac{1-\xi_j}{\xi_j}$$

where using equation (34), we can derive:

$$\begin{aligned}
\frac{G_j(H_j,K_j)}{H_j} &= \left[\kappa_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}}+(1-\kappa_j)\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}}\left[\delta_j\left(\frac{K_j}{H_j}\right)^{\frac{\rho-1}{\rho}}+1\right]^{\frac{\rho}{\rho-1}} \\
&= (1-\kappa_j)^{\frac{\rho}{\rho-1}}\left(\delta_j^{\rho}\chi^{\rho-1}+1\right)^{\frac{\rho}{\rho-1}},
\end{aligned}$$

thus we have

$$\frac{G_j(H_j,K_j)}{H_j}=\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}}. \tag{A3}$$

Substituting (A3) into the MRTS condition across high-skill and low-skill:

$$q = \sigma_j (1 - \kappa_j) \left(\frac{L_j}{H_j}\right)^{\frac{1}{\eta}} \left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\eta-\rho}{(\rho-1)\eta}},$$

which implies

$$\frac{H_j}{L_j} = (\sigma_j/q)^{\eta} (1-\kappa_j)^{\frac{\rho(\eta-1)}{(\rho-1)}} \tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}.$$

A2.2.2 Labor income shares

The high-skill income share is

$$I_j = [1 - J_j] \tilde{I}_j, \tag{A4}$$

using (35) and (37),

$$I_j = \frac{\tilde{I}_j}{1 + q^{\eta-1}\sigma_l^{-\eta} \left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-1}{\rho-1}}} \tag{A5}$$

The total labor income shares is

$$\begin{aligned} \beta_j &= I_j + J_j = (1 - J_j) \tilde{I}_j + J_j \\ &= J_j \left[\frac{1-J_j}{J_j} \tilde{I}_j + 1\right], \end{aligned}$$

substitute (35) and (37),

$$\beta_j = J_j \left[q^{1-\eta}\sigma_j^{\eta} \left[\tilde{I}_j (1-\kappa_j)^{-\rho}\right]^{\frac{\eta-\rho}{1-\rho}} + 1\right].$$

A2.2.3 Equilibrium low-skill wage $\mathbf{w_l}$

The price for low-skill efficiency labor equals to the value of its marginal product:

$$w_l = \xi_j p_j A_j \left(\frac{F_j (G (H_j, K_j), L_j)}{L_j}\right)^{\frac{1}{\eta}}$$

where using the production function

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \left[\left(1-\xi_j\right)\left[\frac{G_j\left(H_j,K_j\right)}{L_j}\right]^{\frac{\eta-1}{\eta}}+\xi_j\right]^{\frac{\eta}{\eta-1}}$$
$$= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left[\frac{G_j\left(H_j,K_j\right)}{H_j}\right]^{\frac{\eta-1}{\eta}}\left(\frac{H_j}{L_j}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}},$$

substitute (A3) and (36) to obtain

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j\left(\frac{1-\kappa_j}{\tilde{I}_j}\right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)}\left(q^{-\eta}\sigma_j^{\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-\rho}{1-\rho}}\right)^{\frac{\eta-1}{\eta}}+1\right]^{\frac{\eta}{\eta-1}}$$
$$= \xi_j^{\frac{\eta}{\eta-1}}\left[\sigma_j^{\eta}q^{1-\eta}\left(1-\kappa_j\right)^{\frac{\rho(\eta-1)}{(\rho-1)}}\tilde{I}_j^{\frac{\eta-1}{1-\rho}}+1\right]^{\frac{\eta}{\eta-1}}.$$

Using the income shares (37)

$$\frac{F_j\left(G\left(H_j,K_j\right),L_j\right)}{L_j} = \left(\frac{\xi_j}{J_j}\right)^{\frac{\eta}{\eta-1}}, \tag{A6}$$

and low-skill wage is

$$w_l = \xi_j^{\frac{\eta}{\eta-1}}p_jA_j\left[J_j\right]^{\frac{1}{1-\eta}}.$$

A2.3 Mapping the Two-Sector Model into a Three-Sector Setting

Consider a three sector-economy where the service sector is as before, but in addition to the low-skill sector, there is a capital sector with the same production function as the low-skill sector in the baseline model. Assume the production function of the low-skill sector and the capital sector are identical except for their TFP index, equating the MRTS across the three inputs of production implies that the following two Lemmas.

Lemma A1 *Given the production functions for the low-skill sector and capital sector are identical except the TFP A_j, the relative inputs used in the low-skill* *sector is the same as that of the capital sector:*

$$\frac{H_l}{K_l} = \frac{H_k}{K_k}, \frac{H_l}{L_l} = \frac{H_k}{H_l}, \tag{A7}$$

and the relative price of the two sectors is the inverse of their TFP:

$$\frac{p_l}{q_k} = \frac{A_k}{A_l}. \tag{A8}$$

Proof. Given $\kappa_l = \kappa_k$, it follows from (34) that $\frac{H_l}{K_l} = \frac{H_k}{K_k}$, thus (35) implies $\tilde{I}_l = \tilde{I}_k$, and together with $\xi_l = \xi_k$, optimal condition (36) implies $\frac{H_l}{L_l} = \frac{H_k}{L_k}$. It also follows from (37) and (A5) that $J_j = J_k$ and $I_j = I_k$, thus mobility of low-skill labor across the low-skill and capital sector implies the relative price is the inverse of the TFP from (19). ■

Lemma A2 *Given the production functions for the low-skill sector and capital sector are identical except their TFP, the low-skill sector and capital sectors can be aggregate into one sector with the following constraint:*

$$Y_l + \frac{q_k}{p_l} Y_k = A_l F_l \left(G_l \left(H_l + H_k, K_l + K_k \right), L_l + L_k \right) \tag{A9}$$

Proof. Given the production function is homogenous of degree 1,

$$\begin{aligned} & p_l Y_l + q_k Y_k \\ = \; & p_l A_l F_l \left(G_l \left(H_l, K_l \right), L_l \right) + q_k A_l F_l \left(G_l \left(H_k, K_k \right), L_k \right) \\ = \; & p_l A_l H_l F_l \left(G_l \left(1, \frac{K_l}{H_l} \right), \frac{L_l}{H_l} \right) + q_k A_l H_k F_l \left(G_l \left(1, \frac{K_k}{H_k} \right), \frac{L_k}{H_k} \right) \end{aligned}$$

Lemma A1 implies that

$$\frac{K_l + K_k}{H_l + H_k} = \frac{K_l}{H_l}; \qquad \frac{L_l + L_k}{H_l + H_k} = \frac{L_l}{L_k}$$

together with the result on relative price equation (A8),

$$p_l Y_l + q_k Y_k = p_l F_l \left(G_l \left(H_l + H_k, K_l + K_k\right), L_l + L_k\right),$$

thus result follows. ■

Lemma A2 implies that we can work with a two-sector economy where the final goods from the low-skill sector can be transformed into one unit of consumption goods and $1/\phi \equiv p_l/q_k = A_k/A_l$ unit of capital goods.

A2.4 Allocation of High-Skill Efficiency Labor

A2.4.1 Expressing q as function of χ

Using (17), the equilibrium condition for price of capital is:

$$q_k = \frac{q}{\chi} p_l A_l \left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}}$$

Given $\phi = q_k/p_l$,

$$\chi = q\frac{A_l}{\phi}\left[J_l \xi_l^{-\eta}\right]^{\frac{1}{1-\eta}}.$$

Using the definition of income share $J_l(\chi, q)$ in (37),

$$\begin{aligned} \chi &= q\xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[1 + q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}} \\ &= \xi_l^{\frac{\eta}{\eta-1}}\frac{A_l}{\phi}\left[q^{\eta-1} + \sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}} \end{aligned}$$

rearranging

$$q^{\eta-1} + \sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}} = \left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{\frac{\eta}{1-\eta}}$$

so

$$q = \left[\left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\tilde{I}_l(\chi)\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{\frac{1}{\eta-1}},$$

Given the expression for $\tilde{I}_l$ in (35),

$$\begin{aligned} q &= \left[\left(\frac{\phi\chi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(1+\chi^{\rho-1}\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}} \\ &= \chi\left[\left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} - \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta-1}}, \end{aligned}$$

so $q > 0$ requires

$$\begin{aligned} \left(\frac{\phi}{A_l}\right)^{\eta-1}\xi_l^{-\eta} &> \sigma_l^{\eta}\left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{1-\eta}{1-\rho}} \\ \left[\left(\chi^{1-\rho}+\delta_l^{\rho}\right)(1-\kappa_l)^{\rho}\right]^{\frac{\eta-1}{1-\rho}} &> \left(\frac{\phi}{A_l}\right)^{1-\eta}(1-\xi_l)^{\eta} \end{aligned}$$

which requires

$$\chi > \chi_{\min} \equiv \left[\left(\frac{A_l}{\phi}\right)^{1-\rho}(1-\xi_l)^{\frac{\eta(1-\rho)}{\eta-1}}(1-\kappa_l)^{-\rho} - \delta_l^{\rho}\right]^{\frac{1}{1-\rho}}.$$

Deriving equation for $S\left(\chi;\zeta,\frac{\phi}{A_l}\right)$: The labor market clearing condition for high-skill worker implies:

$$\frac{H_l+H_k}{L_l+L_k}(L_l+L_k) + \frac{H_h}{L_h}L_h = H,$$

using Lemma 2 and high-skill labor market,

$$\frac{H_l}{L_l}(L-L_h) + \frac{H_h}{L_h}L_h = H,$$

thus the share of low-skill efficiency labor in the high-skill sector is:

$$l_h \equiv \frac{L_h}{L} = \frac{H/L - H_l/L_l}{H_h/L_h - H_l/L_l}, \tag{A10}$$

simplify to

$$l_h = \frac{\zeta/(H_l/L_l) - 1}{(H_h/L_h)/(H_l/L_l) - 1},$$

substitute MRTS condition (36)

$$l_h = \frac{\zeta \sigma_l^{-\eta} q^\eta (1-\kappa_l)^{\frac{\rho(\eta-1)}{1-\rho}} \tilde{I}_l^{\frac{\eta-\rho}{\rho-1}} - 1}{(\sigma_h/\sigma_l)^\eta \left(\frac{1-\kappa_h}{1-\kappa_l}\right)^{\frac{\rho(\eta-1)}{\rho-1}} \left(\frac{\tilde{I}_h}{\tilde{I}_l}\right)^{\frac{\eta-\rho}{1-\rho}} - 1}$$

.For the special case $\kappa_j \to 0, \tilde{I}_l \to 1$

$$l_h = \frac{\zeta \sigma_l^{-\eta} q^\eta - 1}{(\sigma_h/\sigma_l)^\eta - 1}$$

Deriving equation for $D\left(\chi; \hat{A}_{lh}, \frac{\phi}{A_l}\right)$: The goods market clearing conditions and the relative demand implies:

$$x = \frac{p_h C_h}{p_l C_l} = \frac{P_h Y_h}{P_l (Y_l - \phi K)}$$

which can be written as:

$$\frac{p_h Y_h}{p_l Y_l} = x \left(1 - \frac{\phi K}{Y_l}\right), \tag{A11}$$

where using relative price (19), x is derived as

$$x = \hat{A}_{lh}^{1-\varepsilon} \left(\frac{\xi_h^{-\eta} J_h}{\xi_l^{-\eta} J_l}\right)^{\frac{1-\varepsilon}{\eta-1}}; \hat{A}_{lh} \equiv \frac{A_l}{A_h} \left(\frac{1-\psi}{\psi}\right)^{\frac{\varepsilon}{1-\varepsilon}}$$

and using the capital market clearing condition, K is derived as:

$$K = K_h + K_l = \frac{K_h}{L_h} L_h + \frac{K_l}{L_l} (L - L_h)$$

so the relative demand equation (A11) can be written as

$$\frac{p_h Y_h}{x p_l Y_l} = 1 - \frac{\phi}{Y_l} \left[\frac{K_h}{L_h} L_h + \frac{K_l}{L_l} (L - L_h)\right],$$

given $\phi \equiv q_k/p_l$, rewrite it in terms of low-skill income share J_j :

$$\begin{aligned}
\frac{J_l}{xJ_h}\left(\frac{L_h}{L_l}\right) &= 1-\frac{q_kJ_l}{q_lL_l}\left[\frac{K_h}{L_h}L_h+\frac{K_l}{L_l}\left(L-L_h\right)\right] \\
&= 1-\frac{J_l}{L_l}\left[\frac{q_kK_h}{q_lL_h}L_h+\frac{q_kK_l}{q_lL_l}\left(L-L_h\right)\right] \\
&= 1-\frac{J_l}{L_l}\left[\frac{1-\beta_h}{J_h}L_h+\frac{1-\beta_l}{J_l}\left(L-L_h\right)\right],
\end{aligned}$$

where the last equality follows from the definition of β_j. Finally:

$$\frac{J_l}{xJ_h}\left(\frac{l_h}{1-l_h}\right)=1-\frac{J_l}{1-l_h}\left[\frac{1-\beta_h}{J_h}l_h+\frac{1-\beta_l}{J_l}\left(1-l_h\right)\right],$$

thus the demand for l_h is:

$$l_h=\frac{\beta_l}{\beta_l+\frac{J_l}{J_h}\left(\frac{1}{x}+1-\beta_h\right)}.$$

For the special case of no capital, $\beta_j \to 1$:

$$l_h=\left(1+\frac{J_l}{xJ_h}\right)^{-1}.$$

A2.5 Value-Added Shares

The value-added shares of the high-skill sector is:

$$v_h=\left[1+\frac{p_lY_l}{p_hY_h}\right]^{-1}=\left[1+\frac{p_lA_lF_l/L_l}{p_hF_h/L_h}\frac{L_l}{L_h}\right]^{-1}$$

Using relative prices (19) and (A6),

$$v_h=\left[1+\left(\frac{1-\lambda_h}{1-\lambda_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_l}{J_h}\right)^{\frac{1}{\eta-1}}\left(\frac{1-\lambda_l}{J_l}\right)^{\frac{\eta}{\eta-1}}\left(\frac{J_h}{1-\lambda_h}\right)^{\frac{\eta}{\eta-1}}\left(\frac{L_l}{L_h}\right)\right]^{-1}$$

simplify to

$$v_h=\left[1+\left(\frac{J_h}{J_l}\right)\left(\frac{1-l_h}{l_h}\right)\right]^{-1},$$

given l_h, v_h is determined.

A2.5.1 Endogenous skill-biased shift

The production function is

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j K_j^{\frac{\rho-1}{\rho}} + (1-\kappa_j) H_j^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\frac{K_j}{H_j} \right)^{\frac{\rho-1}{\rho}} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \end{aligned}$$

Using the MRTS condition (34),

$$\begin{aligned} Y_j &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\kappa_j \left(\chi \frac{\kappa_j}{1-\kappa_j} \right)^{\rho-1} + (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left[\left(\chi^{\rho-1} \left(\frac{\kappa_j}{1-\kappa_j} \right)^{\rho} + 1 \right) (1-\kappa_j) \right]^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\ &= A_j \left[\xi_j L_j^{\frac{\eta-1}{\eta}} + (1-\xi_j) \left(\frac{1-\kappa_j}{\tilde{I}_j} \right)^{\frac{\rho}{\rho-1}\left(\frac{\eta-1}{\eta}\right)} H_j^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \end{aligned}$$

A3 Quantitative Results

A3.1 Calibration

This section explains how the weights of each input are calibrated to match the sectoral income shares of high-skill and low-skill for period 0 and period T.

A3.1.1 Normalization of $\phi/\mathbf{A_l}$

The initial $\frac{\phi}{A_l}$ can be normalized to 1. By definition of $\tilde{I}_j$

$$\tilde{I}_j = \left[1 + \frac{K_j}{\chi H_j} \right]^{-1} \implies \frac{K_j}{\chi H_j} = \frac{1-\tilde{I}_j}{\tilde{I}_j},$$

which is independent of ϕ/A_l. Also by definition of J

$$J_j^{-1} = \left[1 + \frac{K_j}{\chi H_j}\right] q\frac{H_j}{L_j} + 1$$

so $\frac{H_j}{L_j}$ is independent of ϕ/A_l as well. Therefore it follows from (A10) that l_h is independent of ϕ/A_l. So the allocation of low-skill labor is independent of ϕ/A_l. Given H_j/L_j and K_j/H_j are independent of ϕ/A_1, so the allocation of all inputs are independent of ϕ/A_1. This shows that we can normalize $\phi/A_{l0} = 1$ as it does not affect input allocations across the three sectors. The value of ϕ_T/A_{lT} is then determined by the growth in the relative price of capital ϕ_T/ϕ_0 and the growth in low-skill productivity A_{lT}/A_{l0}.

A3.1.2 Calibration of κ_1, ξ_1

Given ϕ/A_l, equation (40) express χ as a function of ξ_l given data on q and J_l :

$$\chi = qA_k\left[J_l\xi_l^{-\eta}\right]^{\frac{1}{1-\eta}} = qA_kJ_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}.$$

Substitute this into $\tilde{I}_l$ in (35) to solve out δ_l explicitly:

$$\delta_l = \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}}$$

which implies a value of $\kappa_l = \frac{\delta_l}{1+\delta_l}$ for any given level of ξ_l. Thus the income share equation (37) provides an implicit function to solve for ξ_l:

$$J_l = \left[1 + q^{1-\eta}\sigma_l^{\eta}\left[\tilde{I}_l\left(1-\kappa_l\right)^{-\rho}\right]^{\frac{\eta-1}{1-\rho}}\right]^{-1},$$

which can be used to solve for ξ_l given data on $\left(\tilde{I}_l, J_l\right)$. This procedure pins down χ, ξ_l and κ_l. Note that

$$\left(1-\kappa_l\right)^{-1} = 1 + \delta_l = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\chi^{1-\rho}\right]^{\frac{1}{\rho}} = 1 + \left[\frac{1-\tilde{I}_l}{\tilde{I}_l}\left(\frac{q\phi}{A_l}J_l^{\frac{1}{1-\eta}}\xi_l^{\frac{\eta}{1-\eta}}\right)^{1-\rho}\right]^{\frac{1}{\rho}}$$

so

$$\sigma_l^\eta \left[(1-\kappa_l)^{-1}\right]^{\frac{\rho(\eta-1)}{1-\rho}} = \sigma_l^\eta \left[1 + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}} \left(qA_k J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}} \xi_l^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}$$

The implicit function is

$$f(\xi_l) = \left[1 + q^{1-\eta}\left[\left(\frac{1-\xi_l}{\xi_l}\right)^{\frac{\eta(1-\rho)}{\rho(\eta-1)}} + \left(\frac{1-\tilde{I}_l}{\tilde{I}_l}\right)^{\frac{1}{\rho}} \left(\frac{q\phi}{A_l} J_l^{\frac{1}{1-\eta}}\right)^{\frac{1-\rho}{\rho}} (1-\xi_l)^{\frac{\eta(1-\rho)}{(\eta-1)\rho}}\right]^{\frac{\rho(\eta-1)}{1-\rho}}\right]^{-1} - J_l,$$

thus we have

$$\begin{aligned} f'(\xi_l) &> 0 \\ \lim_{\xi_l \to 1} f(\xi_l) &= 1 - J_l > 0 \\ \lim_{\xi_l \to 0} f(\xi_l) &= -J_l < 0 \end{aligned}$$

so there is an unique solution for $\xi_l \in (0,1)$ for any given ϕ/A_l.

A3.1.3 Calibration of κ_h, ξ_h

Using income shares $\tilde{I}_h$ in ((35)):

$$\delta_h = \left[\frac{1-\tilde{I}_h}{\tilde{I}_h}\chi^{1-\rho}\right]^{\frac{1}{\rho}} \Longrightarrow \kappa_h = \frac{\delta_h}{1+\delta_h}$$

given $\tilde{I}_h$ and χ, κ_h is obtained. Using J_h in (37):

$$\sigma_h = \left[\frac{1-J_h}{J_h} q^{\eta-1}\left[\tilde{I}_h (1-\kappa_h)^{-\rho}\right]^{\frac{1-\eta}{1-\rho}}\right]^{\frac{1}{\eta}},$$

given $\kappa_h, \tilde{I}_h, J_h$ and q, so ξ_h is obtained.

A3.2 Results for Other Variables

The performance of the model on other key variables is summarized in Table A3.

Table A3: Actual and Predicted Values for Key Variables

		q	l_h	h_h	v_h	β_l	β_h	β
	Data 1980	1.44	0.14	0.46	0.24	0.59	0.56	0.58
(1)	Data 2008	1.94	0.21	0.46	0.29	0.53	0.65	0.56
	Model 1980	matched	matched	matched	matched	matched	matched	matched
(2)	Model 2008	1.92	0.20	0.45	0.28	0.52	0.65	0.56
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_l/A_h \uparrow$	2.08	0.20	0.54	0.32	0.60	0.61	0.60
(4)	$\phi \downarrow$	2.11	0.16	0.46	0.26	0.59	0.62	0.60
(5)	$\xi_j \downarrow$	2.37	0.16	0.37	0.22	0.52	0.60	0.54
(6)	$\kappa_j \downarrow$	2.12	0.16	0.47	0.27	0.61	0.65	0.63
(7)	$\zeta \uparrow$	1.07	0.13	0.42	0.23	0.56	0.56	0.56

Table A4: Data and Model Predictions, $\varepsilon = 0.5$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	33	61	21	23	m	45	-2.7
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_l/A_h \uparrow$	21	44	19	8.1	68	55	-13
(5)	$\xi_j \downarrow$	28	52	19	29	51	17	21

A3.3 Alternative Elasticity Parameters

The elasticity parameters in the baseline are set to the values used in the related literature. This section considers alternative values for these elasticities. Given the calibration procedures, changing the elasticity parameters will change the values for other parameters. In the interest of space, we do not report those values. These parameter values are available upon request.

A3.3.1 Elasticity of substitution across high-skill and low-skill goods

As discussed in the main text, there is no direct estimate for ε in our model but there is evidence suggesting that it is small. We now consider a higher value of $\varepsilon = 0.5$. An increase in ε implies that the model requires a higher growth in A_{lh} to match the observed growth in relative prices, as a result other parameters are also affected.

As shown in Table A4 the baseline results (2) are not affected given the calibration procedures. The more important question is whether it will affect the role played by the between-sector mechanism, i.e. a rise A_{lh}. As shown in row (3), the between-sector mechanism remains important for the stagnation in low-skill real wage and it continues to account for a significant fraction of real divergence and wage inequality. Compared to the baseline results in Table 4 and Table 5, it predicts a slightly faster rise in the low-skill real wage and a slightly smaller fraction of the real divergence. Compared to the role played by falling ξ in row (5), its advantage remains in predicting a rise in the relative price of the high-skill sector, which is needed for the sector-specific trends in low-skill product wages and a rise in the relative cost of living.

A3.3.2 Elasticity of substitution across capital and high-skill labor

The estimate of $\rho = 0.67$ in Krusell et al. (2000) is for the aggregate economy using data for 1963-1992. We can also infer the elasticity of substitution across capital and high-skill labor ρ using the equilibrium condition (34) and data on income shares and relative input prices, Using the equilibrium condition (34), the response in relative income shares to changes in relative prices of high-skill and capital input is

$$\ln\left(\frac{I_{jT}/\left(1-\beta_{jT}\right)}{I_{j0}/\left(1-\beta_{j0}\right)}\right) = (1-\rho)\ln\left(\frac{\chi_T}{\chi_0}\right), \tag{A12}$$

where by definition, $\chi = w_h/q_k = \phi(w_h/p_l)$, so its growth can be obtained from data on the relative price of capital and the high-skill wage deflated by price of low-skill sector. Given the data in 2, equation (A12) implies ρ is 0.39 using income shares from the low-skill sector and 0.59 using income shares from the high-skill sector, which give an average of 0.49. If we were to use the aggregate income shares instead, equation (A12) implies $\rho = 0.48$. Thus we report the results for $\rho = 0.5$ in Table A5. It shows that the results for the full model (row 2) is almost identical to those in Table 4 and Table 5. The contribution of the between-sector mechanism (row 3) to the real divergence and low-skill wage stagnation is also similar.

Table A5: Data and Model Predictions, $\rho = 0.5$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	61	20	23	m	45	-3.0
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_l/A_h \uparrow$	21	38	14	8.5	63	50	-16
(5)	$\xi_j \downarrow$	27	47	15	28	46	14	16

A3.3.3 Elasticity of substituion across low-skill and high-skill labor

The estimate of $\eta = 1.4$ in Katz and Murphy (1992) is for the aggregate economy using data for 1963-1987. For a similar period, 1963-1992, Krusell et al. (2000) finds $\eta = 1.67$ and $\rho = 0.67$ for the nested aggregate production function including capital. Using more recent data, abstracting from capital, Acemoglu and Autor (2012) find values within the range 1.6–1.8. Higher η implies a smaller exogenous decline in ξ_l is needed to account for the decline in labor income shares in the low-skill sector. Table A6 reports the results for $\eta = 2.0$. It shows the between-sector mechanism (row 3) has a more important role in accounting for the divergence as the required decline in ξ_l reduced to -0.46% compared to -0.93% in the baseline. As in the baseline, the between-sector mechanism is important for generating the sector-specific trends in low-skill product wages.

Table A6: Data and Model Predictions, $\eta = 2.0$, 1980-2008 % Change

		$(y/w_l)(P_C/P_Y)$	y/P_Y	w_l/P_C	y/w_l	y/p_l	w_l/p_l	w_l/p_h
(1)	data	27	60	26	24	78	44	-3.4
(2)	model	34	60	20	24	m	44	-3.4
	Counterfactual (keeping all else constant at 1980)							
(3)	$A_l/A_h \uparrow$	23	31	6.7	11	52	36	-19
(5)	$\xi_j \downarrow$	19	45	22	19	45	22	22