Bank Market Power and Central Bank Digital Currency: A Case of Imperfect Substitution

Xun Gong
School of Economics, Zhejiang University

Han Han
School of Economics, Peking University

Yu Zhu
School of Economics, Renmin University of China

January 2023

Abstract

This paper studies the effect of a central bank digital currency (CBDC) on bank intermediation when the CBDC is an imperfect substitute for bank deposits and the deposit market is imperfectly competitive. We show that a CBDC can promote bank intermediation and output if the CBDC rate is in some intermediate range. A CBDC that is a better substitute has larger positive effects. It can raise lending and output by more. It starts to increase lending and output at a lower interest rate and it promotes intermediation and output for a wider range of interest rates. Therefore, to better reap the benefit of a CBDC, it should be designed as a close substitute to bank deposits.

JEL Codes: E50, E58.

Keywords: Central bank digital currency; Banking; Market power; Disintermediation; Design; Imperfect substitution

1 Introduction

Recently, there is a growing interest among the central banks in issuing central bank digital currencies (CBDCs), a digital form of central bank money for retail payments. The Bank for International Settlements surveyed 65 central banks in 2020 and finds that 86% are engaging in work regarding a CBDC; 60% have started experiments or proofs-of-concept for a CBDC; and 14% have moved forward to development and pilot arrangements (see Boar and Wehrli 2021).

Despite the wide interest, a common concern of issuing a CBDC is that it competes with deposits and raise commercial banks' funding costs. As a result, it can reduce lending, leading to bank disintermediation. There are multiple policy papers that discuss this concern. The early ones include Mancini-Griffoli et al. (2018) and the 2018 report by the Committee on Payments and Market Infrastructures of the Bank for International Settlements.

These policy discussions triggered a line of academic research studying the effects of a CBDC on banking. Keister and Sanches (2022) show that a CBDC disintermediates banks if the banking sector is perfectly competitive. However, it can still be welfare improving. Andolfatto (2021) argues that if there is a monopoly commercial bank, the CBDC can increase the deposit quantity and promote financial inclusion without affecting the loan quantity. Chiu, Davoodalhosseini, Jiang and Zhu (2022), henceforth CDJZ (2022), show that a CBDC can increase both deposits and loans if the deposit market is not perfectly competitive. They show that the result is robust under various model assumption and quantify the effect of a CBDC on the US economy.

While these works provide important insights, they focus on the case that the CBDC is either a perfect substitute to deposits (deposit-like) or a perfect substitute to cash (cash-like). The argument of the key results relies on that the CBDC sets an interest rate floor for deposits, which occurs only if the CBDC is a perfect substitute to deposits. A natural question is what happens if the CBDC is an imperfect substitute to deposits. This is an important question for two reasons. First, the commercial banks have been serving retail customers for a long time. Therefore, they may have advantages so that it is difficult for the CBDC to be a perfect substitute to deposits. Second, it is an important design question as to how close the CBDC should be to deposits.

This paper aims to provide insights into this question. We build on the general equilibrium model of CDJZ (2022). In our model, banks create deposits and issue loans to entrepreneurs. They have market power in deposit creation. Entrepreneurs use the loans, in the form of deposits, to buy investment good produced by households. Households later on use the deposits to trade consumption goods. Besides deposits, households have access to a CBDC, which bears an interest and can also be used to trade consumption goods.[1] We allow the CBDC to be an imperfect substitute to bank deposits: It can be used in only a fraction of the transactions where bank deposits can be used (deposit transactions). If this fraction is higher, the CBDC is a better substitute to the bank deposits. We then study how the effect of the CBDC changes with this fraction.

Qualitatively, we show that a CBDC can increase deposits and loans even if it is an imperfect substitute to deposits. Therefore, the main result of CDJZ (2022) remains valid. Intuitively, when banks have market power in deposit creation, they restrain the deposit supply to keep the deposit rate below the level under perfect competition. A CBDC forces commercial banks to raise the deposit rate, which leads to higher demand for the deposits. This can have two consequences. First, if the CBDC rate is in some intermediate range, banks are willing to satisfy all the demand for deposits, leading to more deposits. Because banks get more funding by issuing deposits, they lend out more. Therefore, the CBDC increases deposits and loans. Second, if the CBDC rate is too high, banks raise the loan rate to compensate for the increase in the deposit rate, leading to a decrease in loan demand. As a result, banks do not need as much funding. Therefore, they reduce deposit supply. Then

[1]The interest on the CBDC can also be interpreted as reduction in holding costs such as account fees.

a CBDC reduces deposits and loans, leading to bank disintermediation. A CBDC that is a better substitute to deposits forces banks to increase deposit rate more. Therefore, it starts to promote bank intermediation at a lower CBDC rate and can lead to more increase in deposits and loans. But it also starts to disintermediate banks at a lower CBDC rate. These qualitative results are robust under different market structures of the loan market.

To assess the quantitative effect, we calibrate the model to the US economy and then introduce a CBDC with different designs. The baseline calibration has a perfectly competitive loan market. If the CBDC serves 95% of deposit transactions, it increases lending when its interest rate is in the interval $(0.57\%, 1.69\%)$, and it increases output when its rate is in $(0.57\%, 1.82\%)$. These intervals change to $(3.50\%, 3.92\%)$ and $(3.50\%, 4.00\%)$ if the CBDC serves only 40% of deposit transactions. The lengths of the intervals are shorter than their counterparts when the CBDC serves 95% of deposit transactions. Therefore, a CBDC promotes bank intermediation and output for a larger set of interest rates if it is a better substitute to bank deposits. At the maximum, the CBDC can increase increase lending and output by 1.52% and 0.30% respectively, if it serves 95% of deposit transactions. These numbers reduces to 0.66% and 0.13%, if the CBDC serves only 40% of deposit transactions.

If the loan market is imperfectly competitive, the effect of a CBDC is qualitatively the same but quantitatively smaller. If the CBDC serves 95% of deposit transactions, it promotes bank lending and output when its rate is in $(0.57\%, 1.08\%)$ and $(0.57\%, 1.35\%)$, respectively. The maximum increases in bank lending and output are 0.7% and 0.13%. If the CBDC serves 40% of deposit transactions, it promotes bank lending and output when its rate is in $(3.50\%, 3.69\%)$ and $(3.50\%, 3.73\%)$, respectively. The maximum increases in lending and output are 0.30% and 0.06%.

Research on the effect of a CBDC is emerging rapidly. The early works have relatively simple substitution pattern between the CBDC and bank deposits. Recently, more papers start to consider the case where the CBDC and bank deposits are imperfect substitutes, including Agur et al. (2021), Assenmacher et al. (2021), and Perazzi and Elena (2022). The mechanism we focus on is not present in these papers. Agur et al. (2021) have a perfectly competitive banking sector, while Assenmacher et al. (2021), and Perazzi and Elena (2022) both predict a more attractive CBDC reduces bank intermediation. There are also works adopting the empirical industry organization approach, which models payment products as horizontally differentiated products. These work includes Li (2022); Li, Usher and Zhu (2023); Whited, Wu and Xiao (2022). This approach can incorporate various observed characteristics of different payment methods and is useful to study the design of a CBDC. Different from these work, we adopt a macroeconomic approach, which allows us to study the general equilibrium effects. Also,these works either model the substitution pattern among payment products through a CES aggregator or a logit demand system.[2] We instead micro-found the substitution pattern by acceptance decision, which is arguably easier to interpret.

There are other lines of research on the CBDC that is complementary to our studies. Brunnermeier and Niepelt (2019) and Niepelt (2020) derive conditions under which introducing a CBDC has no effects on macroeconomic outcomes, including bank intermediation. A number of studies focus on the role of CBDCs as a monetary policy tool. This includes Barrdear and Kumhof (2021), Davoodalhosseini (2021), Dong and Xiao (2021), and Jiang and Zhu (2021). Another line of research studies the financial stability implications of a CBDC such as the risk-taking behavior of banks and bank runs. Recent works by Chiu et al. (2020), Fernández-Villaverde et al. (2020), Schilling et al. (2020), Keister and Monnet (2020), Monnet et al. (2020), and Williamson (2020*b*) have made some important progress. For research related to the design of a CBDC, see Agur et al. (2020), Wang (2020), Garratt and van Oordt (2021), Kahn et al. (2021) and Kahn and van Oordt (2022). For policy discussions on CBDCs, see Fung and Halaburda (2016); Engert and Fung (2017); Mancini-Griffoli et al. (2018); Chapman and Wilkins (2019); Davoodalhosseini and Rivadenyra (2020); Davoodalhosseini

[2]Assenmacher et al. (2021) do so indirectly through incorporating a CES aggregator of capital in the production function.

et al. (2020); and Kahn et al. (2020).[3]

The rest of the paper is organized as follows. Section 2 study a baseline model with imperfect competition in the loan market and perfect competition in the deposit market. Section 3 extends the baseline model to allow for imperfectly competitive loan market. Section 4 calibrates the model and assesses quantitatively how the effect of a CBDC depends on its subsitution pattern with deposits. Section 5 discusses the implications of the results on the design of a CBDC and conclude. All the proofs are collected in the appendix.

2 Baseline Model

Our model is a variant of CDJZ (2022), which is based on Lagos and Wright (2005). Time is discrete and continues forever. There are four types of agents: a continuum of households with measure 2, a continuum of entrepreneurs with measure 1, a finite number of N banks owned by the households, and the government. The discount factor between two periods is $\beta \in (0, 1)$. Each period t is divided into two subperiods. In the first sub-period, agents interact in a frictional decentralized market (DM). In the second subperiod, agents interact in a Walrasian centralized market (CM). There are two perishable goods: y in the DM and x in the CM.

A household can be a buyer or a seller with equal probability and the type is permanent. In the DM, a buyer wants to consume y and his utility is $u(y)$ with $u'(0) = \infty$, $u' > 0$, and $u'' < 0$. A seller can produce y with cost y on the spot. A buyer randomly meets a seller. Upon a meeting, the buyer and the seller decide terms of trade. The efficient consumption is y^* that satisfies $u'(y^*) = 1$. Buyers lack commitment and hence no credit is viable in the DM. As a result, buyers must use a means of payment to exchange for y, which we will discuss later. The terms of trade are determined by buyers making take-it-or-leave-it offers. In the

[3] Our paper is also related to the literature on private digital currencies and currency competition; see Chiu and Koeppl (2019); Fernández-Villaverde and Sanches (2019); Schilling and Uhlig (2019); Zhu and Hendry (2019); Benigno et al. (2020); Choi and Rocheteau (2020); and Zhou (2020). For a complete introduction to the issues in digital currencies, see Schar and Berentsen (2020).

CM, households work and consume x. Labor h is transformed into x one-for-one. The utility from consuming x is $U(x)$ with $U'(0) = \infty$, $U'(x) > 0$, and $U''(x) < 0$. Buyers' and sellers' period utilities, denoted by $\boldsymbol{U}^B$ and $\boldsymbol{U}^S$, are

$$\begin{aligned}
\boldsymbol{U}^B (x, y, h) &= u(y) + U(x) - h, \\
\boldsymbol{U}^S (x, y, h) &= -y + U(x) - h.
\end{aligned}$$

Entrepreneurs live for two periods. They are born in the current CM and die in the next CM. Entrepreneurs cannot work and consume only when old. When they are young, they have access to an investment opportunity that transforms x current CM goods to $f(x)$ CM goods in the next period, where $f'(0) = \infty$, $f'(\infty) = 0$, $f' > 0$, and $f'' < 0$. Entrepreneurs would like to borrow from households to invest. However, entrepreneurs and households lack commitment and cannot enforce debt repayment, so no credit arrangement among them is feasible.

Bank are infinitely lived and owned by households. They can commit to repay their liabilities and enforce debt repayment from entrepreneurs. Therefore, they can act as intermediaries between households and entrepreneurs to finance investment projects. A bank can finance its loans by issuing two liabilities, liquid checkable deposits and illiquid term deposits, and by internal financing, retained profits.[4] Checkable deposits can be used as a medium of exchange to facilitate trades between buyers and sellers in the DM. Banks are subject to a reserve requirement that a bank's reserve holding must cover at least a fraction $\chi \geq 0$ of its checkable deposits. Banks maximize the discounted sum of dividend paid to households. For now, we assume that they engage in Cournot competition in the deposit market and perfect competition in the loan market.

The government is a combination of monetary and fiscal authorities. The monetary authority, or the central bank, issues two forms of liabilities: central bank reserves, and a

[4]Without loss of generality, we assume banks do not raise equity.

CBDC. The reserves are whole-sale electronic balances that pay a net nominal interest rate $i_r \geq 0$. They can be held only by banks. The CBDC is a digital token or electronic entry that can be used for retail payments. It pays a net nominal interest i_e. To simplify presentation, we abstract from physical currency (cash) in the theoretical analysis.[5] But we will introduce it in the quantitative analysis. We focus on stationary monetary policies, where the total liabilities of the central bank (CBDC, and reserves) grow at a constant gross rate $\mu > \beta$ and the central bank stands ready to exchange the CBDC and reserves at par in the CM. The government collects revenues from the issuance of new liabilities to pay interest on the CBDC and reserves, and the difference finances lump-sum transfers (T) to households (a negative T represents lump-sum taxes).

In the DM, Buyers use the CBDC and checkable deposits to purchase y. These payment methods are distinguished by the type of transactions they facilitate (Lester et al. 2012 and Zhu and Hendry 2019). With probability α, a buyer meets a seller. He can always use checkable deposits to purchase y. But the CBDC, which is an imperfect substitute to checkable deposits, can be used only with probability τ. If $\tau < 1$, the CBDC is less useful than checkable deposits. If $\tau = 1$, the CBDC and checkable deposits are perfect substitutes.

In the rest of this section, we first study the household problem, which leads to the demand function for checkable deposits. Given this demand function, we then study the Cournot game among banks, which leads to the aggregate loan supply. Lastly, we combine it with the aggregate loan demand from entrepreneurs to obtain the equilibrium. We focus on the stationary equilibrium where real variables are constant over time.

2.1 Households

Define $\overrightarrow{a} = (b, d, e)$ to be vector of the real value of term deposits, checkable deposits and the CBDC including the interest payment and let $\overrightarrow{i} = (i_b, i_d, i_e)$ be the vector of corresponding

[5]Introducing physical currency does not change the theoretical results but makes presentation more cumbersome.

net nominal returns. Then $\overrightarrow{R} = (1 + \overrightarrow{i})/\mu$ is the vector of gross real returns and $\overrightarrow{\phi} = 1/\overrightarrow{R}$ is the price vector of one unit real purchasing power next period.

In the CM, a buyer chooses consumption x, labor h, and the asset portfolio brought into the next period $\overrightarrow{a}' = (b', d', e')$. His value function is:

$$W^B(\overrightarrow{a}) = \max_{x,h,\overrightarrow{a}'} \left\{U(x) - h + \beta V^B(\overrightarrow{a}')\right\}, \text{s.t. } x + \overrightarrow{\phi} \cdot \overrightarrow{a}' = T + h + \overrightarrow{1} \cdot \overrightarrow{a},$$

where $\overrightarrow{1} = (1, 1, 1)$ and "$\cdot$" denotes the inner product of two vectors. The first-order condition with respect to the asset portfolio $\overrightarrow{a}'$ is

$$\beta \frac{\partial}{\partial a} V^B(\overrightarrow{a}') \leq \phi_a, \text{ with equality if } a' > 0 \text{ for } a = b, d, e. \tag{1}$$

Clearly, buyers choose the same portfolio $\overrightarrow{a}'$ and $\frac{\partial}{\partial a} W^B(\overrightarrow{a}) = 1$ for $a = b, d, e$, which, together with (1), implies $b' > 0$ only if $\phi_b = \beta$. Because term deposits cannot be used as means of payment, the return must compensate the delay in consumption for them to be held. The buyer's DM value function is

$$V^B(\overrightarrow{a}) = \alpha\tau[u(Y(d+e)) - P(d+e)] + \alpha(1-\tau)[u(Y(d)) - P(d)] + W^B(\overrightarrow{a}), \tag{2}$$

where the consumption and payment of the buyer, Y and P, are two functions that will be discussed later. Their arguments are the real payment balances usable in the meeting. With probability $\alpha\tau$, both the CBDC and checkable deposits can be used. Therefore, the usable payment balances are $d + e$. With probability $\alpha(1-\tau)$, only checkable deposits can be used. Therefore, the usable balances are d.

Without loss of generality, assume that a seller does not bring any asset into the DM. Then a type j seller's CM value function is

$$W_j^S(\overrightarrow{a}) = \max_{x,h} \left\{U(x) - h + \beta V_j^S(\overrightarrow{0})\right\}, \text{s.t. } x = T + h + \overrightarrow{1} \cdot \overrightarrow{a},$$

where $j = 1$ if the seller accepts the CBDC and 0 if otherwise. The DM value function is

$$V_j^S(\overrightarrow{0}) = \alpha[-Y(\widetilde{L}_j) + P(\widetilde{L}_j)] + W_j^S(\overrightarrow{0})$$

where $\widetilde{L}_0 = \tilde{d}$ and $\widetilde{L}_1 = \tilde{d} + \tilde{e}$ are the usable payment balances held by the seller's counterparty.

The terms of trade in the DM are determined by buyers making take-it-or-leave-it offers and solve

$$\max_{y,p} \{u(y) - p\}, \text{s.t. } p \geq y \text{ and } p \leq L,$$

The solution is

$$Y(L) = P(L) = \min(y^*, L).$$

We can combine it with (1) and (2) to obtain the Euler equations:

$$\phi_d \geq \alpha\tau\beta\lambda(e+d) + \alpha(1-\tau)\beta\lambda(d) + \beta \text{ with equality iff } d > 0, \tag{3}$$

$$\phi_e \geq \alpha\tau\beta\lambda(e+d) + \beta \text{ with equality iff } e > 0, \tag{4}$$

where $\lambda(L) = \max\{u'(L) - 1, 0\}$ is the liquidity premium and with a little abuse of notations, d and e denote real value of checkable deposits and the CBDC next period. Equation (3) is the Euler equation for checkable deposits. The left hand side is the marginal cost of holding checkable deposits: the buyer needs to give up ϕ_d current consumption. The right hand side is the marginal benefit. It has two components. First, the buyer can redeem the deposits and get one unit of CM consumption. Because of discounting, this is captured by β. Second, the buyer gets extra surplus because checkable deposits allow him to consume in the DM. This is captured by the first two terms on the right hand side of (3). The liquidity premium, λ, captures the marginal value of usable payment balances. It is the Lagrangian multiplier on the constraint that the buyer cannot pay more than his usable payment balances. If the marginal cost exceeds the marginal benefit, $d = 0$. Otherwise, the buyer holds enough d to equate the marginal cost and the marginal benefit. Equation (4) is the Euler equation for e and has the same interpretation. It is different from (3) because only a τ fraction of sellers accept the CBDC.

For a given $\phi_e \geq \beta$, there exist two cut-offs:

$$\underline{\phi}_d^\tau = \frac{\phi_e + (\tau - 1)\beta}{\tau} \geq \phi_e, \tag{5}$$

$$\bar{\phi}_d^\tau = \phi_e + \beta\alpha(1-\tau)\lambda(0). \tag{6}$$

If $\phi_d > \bar{\phi}_d^\tau$, the buyer does not hold checkable deposits. Then $d = 0$ and e solves

$$\phi_e = \alpha\tau\beta\lambda(e) + \beta.$$

If $\phi_d < \underline{\phi}_d^\tau$, the buyer does not hold the CBDC. Then $e = 0$ and d solves

$$\phi_d = \alpha\beta\lambda(d) + \beta.$$

If $\phi_d \in (\underline{\phi}_d^\tau, \bar{\phi}_d^\tau)$, the buyer holds both checkable deposits and the CBDC. Then, d and e solve

$$\begin{aligned}\phi_d &= \alpha\tau\beta\lambda(d+e) + \alpha(1-\tau)\beta\lambda(d) + \beta, \\ \phi_e &= \alpha\tau\beta\lambda(d+e) + \beta.\end{aligned}$$

The above analysis defines an inverse demand function of checkable deposits next period: $\mathbf{\Phi}_{d,\tau}(d)$. For reference, if there is no CBDC, d solves

$$\phi_d = \alpha\beta\lambda(d) + \beta.$$

This defines the inverse demand of checkable deposits without the CBDC: $\hat{\mathbf{\Phi}}_d(d)$.

Figure 1 shows the inverse deposit demand without a CBDC (blue) and with a CBDC under $\tau < 1$ (black). The two curves overlap if $\phi_d < \underline{\phi}_d^\tau$, i.e. $\hat{\mathbf{\Phi}}_d(d)$ and $\mathbf{\Phi}_{d,\tau}(d)$ are identical. In this region, the CBDC does not affect the demand for checkable deposits because the price (the interest rate) of the latter is sufficiently low (high). Because $\tau < 1$ and $\lambda(0) = \infty$, $\bar{\phi}_d^\tau = \infty$. Households always hold checkable deposits because they can facilitate transactions that the CBDC cannot. For reference, we also plot the deposit demand under $\tau = 1$ (red), which was the focus of CDJZ (2022). It overlaps with the blue curve only if $\phi_d < \phi_e$: because

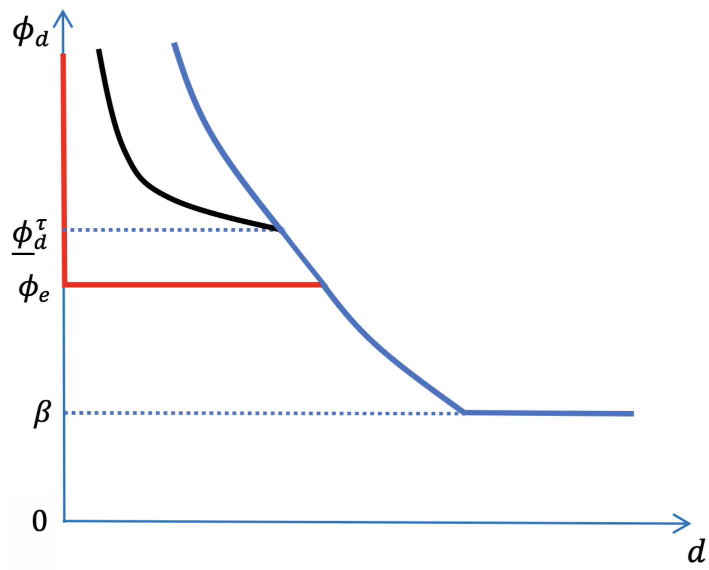
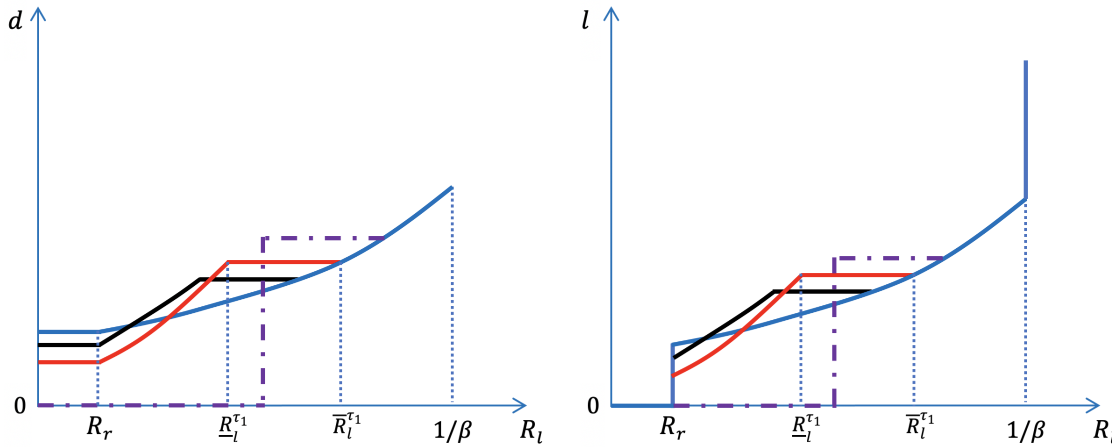


Figure 1: Demand for checkable deposits under $\tau < 1$ (black), $\tau = 1$ (red) and without a CBDC (blue).

the CBDC is more useful under $\tau = 1$, a lower price (a higher interest rate) on checkable deposits is needed to make the CBDC irrelevant. Moreover, the demand for checkable deposits drops to 0 as ϕ_d moves above ϕ_e, i.e. $1/\phi_e$ is the floor of the gross real rate on checkable deposits as discussed in CDJZ (2022). Because the CBDC and checkable deposits are perfect substitutes, households hold only the one with a higher rate of return. If $\tau < 1$, the demand for checkable deposits is always positive and the CBDC no longer sets the interest floor for checkable deposits.

2.2 Banks

Banks are infinitely-lived and maximize the discounted sum of dividend paid to their owners. Banks engage in Cournot competition in the deposit market and perfect competition in the loan market. Each period, bank j chooses checkable deposits (d_j), term deposits (b_j), loan quantity l_j, reserve quantity r_j and dividend paid to owners δ_j, taking as given the gross real rates for reserves (R_r) and loans (R_l), the inverse demand function for deposits ($\mathbf{\Phi}_{d,\tau}(d)$), and other banks' checkable deposits ($D_{-j} = \sum_{i \neq j} d_i$). To be consistent with notations used



(a) Deposit Supply (b) Loan Supply

Figure 2: Supply of checkable deposits and loans under $\tau = \tau_1$ (red), $\tau = \tau_2$ (black). $\tau = 1$ (dash-dot purple) and without a CBDC (blue), where $1 > \tau_1 > \tau_2$.

for household demand, d_j and b_j are the real value of checkable deposits and term deposits next period including interest payment. Then, the bank's value function is:

$$\Omega(\Pi) = \max_{b_j, d_j, l_j, r_j, \delta_j} \delta_j + \beta \Omega(\Pi')$$

$$\text{st } \delta_j + l_j + r_j = \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j + \beta b_j + \Pi$$

$$\Pi' = R_l l_j + R_r r_j - d_j - b_j$$

$$\delta_j \leq \Pi, \; r_j > \chi \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j$$

The first constraint is the resource constraint. It says that funding from the real profit (Π) and deposits covers dividend payment, investment in loans and reserves. Banks are allowed to use profit for investment. Notice that funding available for investment depends on the value of deposits before interest: $\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j$ and βb_j. The second constraint specifies how profit depend on deposits and investment. The last two inequalities impose that the dividend cannot exceed the profit and the reserve requirement is satisfied at the end of this period. We also implicitly impose that all the control variables are non-negative. The bank is indifferent between using retained profit and term deposits for investment. Therefore, without loss of generality, we can assume that a bank does not retain profits for investment purposes, i.e., $\delta = \Pi$. Then the bank does not save and its problem is effectively static, which can be rewritten as

$$\begin{aligned} &\max_{b_j, d_j, l_j, r_j} \{R_l l_j + R_r r_j - b_j - d_j\} \\ &\text{s.t. } l_j + r_j = \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j + \beta b_j, \ r_j \geq \chi \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j. \end{aligned} \tag{7}$$

We now solve the Cournot game among banks given R_l, which results in the deposit and loan supply as functions of R_l. We focus on the case where $R_r < R_e < 1/\beta$ and $R_l \leq 1/\beta$.[6] Define $\xi = \max\{(1-\chi)R_l + \chi R_r, R_r\}$ to be the return of checkable deposits. Intuitively, if the loan rate is higher than the reserve rate, a bank invests in loans until the reserve requirement is binding and the return of checkable deposits is $(1-\chi)R_l + \chi R_r$. If the reserve rate is higher than the loan rate, a bank invest all funds in reserves and the return of checkable deposits is R_r. Then one can rewrite (7) as

$$\max_{d_j} \xi \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j) d_j + R_l \beta b_j - b_j - d_j. \tag{8}$$

Because $\mathbf{\Phi}_{d,\tau}$ has kinks, in a symmetric equilibrium, d satisfies

$$\Delta_\tau^-(d) = \nabla^- \mathbf{\Phi}_{d,\tau}(Nd) d + \mathbf{\Phi}_{d,\tau}(Nd) \geq 1/\xi, \tag{9}$$

$$\Delta_\tau^+(d) = \nabla^+ \mathbf{\Phi}_{d,\tau}(Nd) d + \mathbf{\Phi}_{d,\tau}(Nd) \leq 1/\xi, \tag{10}$$

where ∇^- and ∇^+ denote the left and right derivatives, respectively. Moreover, $b_j > 0$ only if $R_l = 1/\beta$.

We maintain the following assumption throughout the paper.

Assumption 1 *The inverse demand function without a CBDC, $\hat{\mathbf{\Phi}}_d$, satisfies the following: (1) For any $D_{-j} < y^*$ and $\xi > 0$, either there exists $\bar{d} \in (0, y^* - D_{-j})$ such that $\hat{\mathbf{\Phi}}'_d(D_{-j} + d)d + \hat{\mathbf{\Phi}}_d(D_{-j} + d) \lesseqgtr 1/\xi$ iff $d \gtreqless \bar{d}$ or $\hat{\mathbf{\Phi}}'_d(D_{-j} + d)d + \hat{\mathbf{\Phi}}_d(D_{-j} + d) < 1/\xi$ for all $d < y^* - D_{-j}$. (2) $\hat{\Delta}(d) = \hat{\mathbf{\Phi}}'_d(Nd)d + \hat{\mathbf{\Phi}}_d(Nd)$ is monotonically decreasing on $[0, y^*/N)$ and is bigger than $1/R_r$ for d sufficiently small. (3) $\hat{\Phi}_d(D)D$ is increasing in D.*

[6] Notice that $R_l > 1/\beta$ cannot occur in the general equilibrium because it leads to unbounded supply of loans.

Assumption 1(1) and (2) guarantee there is a generically unique symmetric equilibrium in the Cournot game. Assumption 1(3) implies the loan quantity is increasing in D, which guarantees uniqueness of the general equilibrium. It is satisfied, for example, if $-yu''(y)/u'(y) < 1$. Also notice that Assumption 1 puts restrictions only on the inverse deposit demand without a CBDC, $\hat{\mathbf{\Phi}}_d$. But it is sufficient for analyzing the equilibrium with a CBDC.

We now charaterize $\mathbf{d}_\tau(R_l)$, a bank's supply of checkable deposits in the Cournot equilibrium with a CBDC, where τ indicates the dependence on the substitution pattern between the CBDC and bank deposits. First, let $\hat{\mathbf{d}}(R_l)$ be the supply of checkable deposits in the Cournot equilibrium without a CBDC given R_l, which solves

$$\hat{\mathbf{\Phi}}'_d(Nd)d + \hat{\mathbf{\Phi}}_d(Nd) = 1/\xi$$

as a function in d. Let $\tilde{\phi}_e^\tau = \tau/R_r + (1-\tau)\beta$. In the main text, we focus on the case where $\tilde{\phi}_e^\tau > \phi_e > \hat{\Phi}_d[N\hat{\mathbf{d}}(1/\beta)]$ ($\underset{\sim}{R}_e^\tau < R_e < 1/\hat{\Phi}_d[N\hat{\mathbf{d}}(1/\beta)]$ where $\underset{\sim}{R}_e^\tau = 1/\tilde{\phi}_e^\tau$).[7] Other cases are studied in the appendix. From this point on, we will base most of our discussion on the gross real rates, which are the inverse of the prices. However, we will still use the notation ϕ_e, the price of the CBDC, because it is sometime more convenient.

We focus on the case with $\tau < 1$, i.e. the CBDC and checkable deposits are imperfect substitutes. For the case with $\tau = 1$, please refer to CDJZ (2022). Define $\bar{D}_\tau = \hat{\mathbf{\Phi}}_d^{-1}(\underline{\phi}_d^\tau)$, which is the smallest amount of checkable deposits needed to drive the CBDC out of circulation. By (5), $\underline{\phi}_d^\tau$ is increasing in ϕ_e or decreasing in R_e. Because $\hat{\mathbf{\Phi}}_d(d)$ is decreasing in d, $\bar{D}_\tau$ increases with R_e. Then

$$\mathbf{d}_\tau(R_l) = \begin{cases} \tilde{\mathbf{d}}_\tau(R_l) & \text{if} \quad R_l < \underline{R}_l^\tau, \\ \bar{D}_\tau/N & \text{if} \quad \underline{R}_l^\tau < R_l < \bar{R}_l^\tau, \\ \hat{\mathbf{d}}(R_l) & \text{if} \quad \bar{R}_l^\tau < R_l \leq 1/\beta. \end{cases} \tag{11}$$

[7] If this condition does not hold, some of the branches in (11) disappear. The condition $\phi_e > \hat{\Phi}_d[N\hat{\mathbf{d}}(1/\beta)]$ implies introducing the CBDC does not change the deposit supply if R_l is sufficiently close to $1/\beta$, which implies that the last branch in (11) exists. The condition $\tilde{\phi}_e^\tau > \phi_e$ implies that the first branch in (11) exists. It also implies that the supply of checkable deposits with a CBDC is lower than the supply without a CBDC if $R_l \leq R_r$.

where $\underline{R}_l^\tau$ and $\bar{R}_l^\tau$ solve $\Delta_\tau^-(\bar{D}_\tau) = 1/\xi$ and $\Delta_\tau^+(\bar{D}_\tau) = 1/\xi$ as equations in R_l, respectively. Notice that ξ also depends on R_l. Let $\tilde{\mathbf{d}}_\tau(R_l)$ solve

$$\phi_e + \alpha\beta(1-\tau)\lambda(Nd) + \alpha\beta(1-\tau)\lambda'(Nd)d = 1/\xi \tag{12}$$

as an equation in d. If $d < \bar{D}_\tau$, (12) is the same as $\mathbf{\Phi}'_{d,\tau}(Nd)d + \mathbf{\Phi}_{d,\tau}(Nd) = 1/\xi$.

The red curve in Figure 2(a) illustrates the supply of checkable deposits. The blue curve is the supply of checkable deposits if there is no CBDC. If $R_l < R_r$, banks invest all funding in reserves and the marginal return from checkable deposits is independent of R_l. Therefore, the supply of checkable deposits is constant. It is increasing with R_l on $[R_r, 1/\beta]$. A higher loan rate makes issuing checkable deposits more profitable, leading to a higher supply.

The red curve is the supply of checkable deposits with a CBDC if $\tau = \tau_1 < 1$. If $R_l > \bar{R}_l^{\tau_1}$, banks are willing to pay a sufficiently high interest on checkable deposits even if there is no CBDC. As a result, introducing the CBDC does not affect the economy and the blue curve and the red curve overlap. If R_l is between $\underline{R}_l^{\tau_1}$ and $\bar{R}_l^{\tau_1}$, the red curve is constant at $\bar{D}_\tau/N$, which is above the blue curve. This occurs because there is a kink in the demand of checkable deposits as shown in Figure 1. This leads to a downward jump in the marginal profit of checkable deposits at $d = \bar{D}/N$.Therefore, for a range of R_l, the marginal profit is positive if $d < \bar{D}_\tau/N$ and negative if $d > \bar{D}_\tau/N$. In this region, banks find it optimal to drive the CBDC out of circulation but the CBDC is an effective outside option that disciplines banks' market power. If R_l is between R_r and $\underline{R}_l^{\tau_1}$, the red curve is strictly increasing. It crosses the blue curve from below at $R_l = \tau_1/[\phi_e - (1-\tau_1)\beta]$. Therefore, a CBDC with $\tau = \tau_1$ reduces the supply of checkable deposits if $R_l < \tau_1/[\phi_e - (1-\tau_1)\beta]$ and raises the supply if $\tau_1/[\phi_e - (1-\tau_1)\beta] < R_l < \bar{R}_l^{\tau_1}$.

The black curve shows the supply of checkable deposits with a CBDC under $\tau = \tau_2 < \tau_1$. It has a similar shape as the red curve but it lies above the red curve if R_l is low and lies below the red curve if R_l is in some intermediate range. Therefore, a higher τ may increase or decrease the supply of checkable deposits.

For reference, the dash-dot purple curve shows the supply of checkable deposits if they are perfect substitutes to the CBDC, i.e. $\tau = 1$. Different from the case with $\tau < 1$, the dashed-dot curve stays at 0 if R_l is not sufficiently high. This reflects the fact that the CBDC rate is an floor for the interest rate of checkable deposits. If R_l is not sufficiently high, banks cannot afford to match the CBDC rate and banks shut down. This is not the case if $\tau < 1$ because the CBDC no longer sets the interest floor for checkable deposits.

Proposition 1 *The Cournot game has a unique equilibrium for a generic $R_l < 1/\beta$.*[8] *The resulting supply of checkable deposits satisfies:*

(1) $\mathbf{d}_\tau(R_l) > \hat{\mathbf{d}}(R_l)$ *if* $R_l < \tau/[\phi_e - (1-\tau)\beta]$ *and* $\mathbf{d}_\tau(R_l) < \hat{\mathbf{d}}(R_l)$ *if* $\tau/[\phi_e - (1-\tau)\beta] < R_l < \bar{R}_l^\tau$.

(2) If $1 > \tau_1 > \tau_2$ *and* $\tilde{\phi}_e^{\tau_2} > \phi_e$, *there exists a cut-off* R_l^c *such that* $\mathbf{d}_{\tau_1}(R_l) < \mathbf{d}_{\tau_2}(R_l)$ *for all* $R_l < R_l^c$ *and* $\mathbf{d}_{\tau_1}(R_l) > \mathbf{d}_{\tau_2}(R_l)$ *for all* $R_l^c < R_l < \bar{R}_l^{\tau_1}$.

Proposition 1 has two messages: (1) introducing or decreasing the supply of after-interest checkable deposits depending on R_l; (2) the supply of after-interest checkable deposits may increase or decrease with τ depending on R_l. Because Assumption 1 implies $\mathbf{\Phi}_{d,\tau}(D)D$ is increasing in D, a CBDC that is a closer substitute to deposit does not necessarily lead to fewer pre-interest checkable deposits, which is an important funding source of banks.

Given the deposit supply, the loan supply function of a bank can be written as

$$\mathbf{l}_\tau(R_l) = \begin{cases} 0 & \text{if} \quad R_l < R_r, \\ [0, (1-\chi)\mathbf{\Phi}_{d,\tau}\left(N\mathbf{d}_\tau(R_l)\right)\mathbf{d}_\tau(R_l)] & \text{if} \quad R_l = R_r, \\ (1-\chi)\mathbf{\Phi}_{d,\tau}\left(N\mathbf{d}_\tau(R_l)\right)\mathbf{d}_\tau(R_l) & \text{if} \quad R_r < R_l < 1/\beta, \\ [(1-\chi)\mathbf{\Phi}_{d,\tau}\left(N\mathbf{d}_\tau(1/\beta)\right)\mathbf{d}_\tau(1/\beta), \infty] & \text{if} \quad R_l = 1/\beta. \end{cases} \tag{13}$$

If the loan rate is lower than the reserve rate, a bank invest all funding in reserves. If the loan rate is higher than the reserve rate, it invest all funding in loans until the reserve requirement

[8] If $\chi = 0$, there can be a Cournot equilibrium with a positive bank profit and a continuum of equilibria with a 0 profit, in which case we select the equilibrium with positive bank profit.

is binding. If the loan rate is the same as the reserve rate, the bank is indifferent between any loan quantity as long as the reserve requirement is satisfied. If $R_l = 1/\beta$, the bank issues term deposits and is indifferent between any loan quantity above the amount that can be financed by the checkable deposits. To obtain a bank's loan supply without a CBDC, we simply replace $\mathbf{d}_\tau$ and $\mathbf{\Phi}_{d,\tau}$ in (13) by $\hat{\mathbf{d}}$ and $\hat{\mathbf{\Phi}}$.

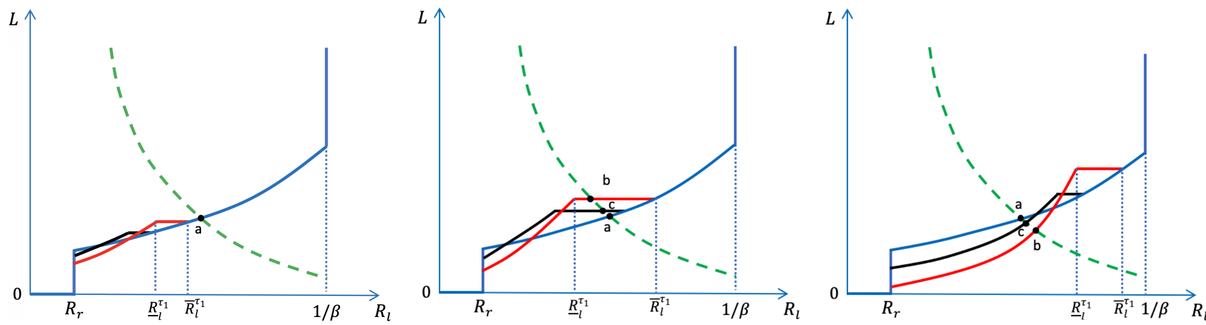
The loan supply function of a bank is shown in Figure 2(b), where the red and black curves are again under $\tau = \tau_1$ and τ_2, respectively. They overlap with the blue curve, the loan supply without a CBDC, if $R_l < R_r$ or if R_l is sufficiently large. The red curve, for example, is above the blue curve for a range of R_l, i.e. a CBDC can still raise a bank's loan supply even if it is not a perfect substitute to checkable deposits. It is constant at $(1-\chi)\hat{\Phi}_d(\bar{D}_{\tau_1})\bar{D}_{\tau_1}/N$ on $(\underline{R}_l^{\tau_1}, \bar{R}_l^{\tau_1})$, reflecting that the supply of checkable deposits is constant in the this region. We can sum the loan supply of every bank to obtain the aggregate loan supply: $\mathbf{L}^s_\tau(R_l) = N\mathbf{l}_\tau(R_l)$. Similarly, we can get the aggregate loan supply without a CBDC denoted by $\hat{\mathbf{L}}^s$.

2.3 Entrepreneur Problem and Equilibrium

Entrepreneurs take the loan rate as given and choose the loan quantity to maximize profit:

$$\max_l f(l) - R_l l.$$

The first order condition is $f'(l) = R_l$. Because there is a continuum of entrepreneurs with measure 1, the aggregate loan demand function is $\mathbf{L}^d(R_l) = f'^{-1}(R_l)$. The equilibrium R_l^* equates the aggregate loan supply and the aggregate loan demand, i.e., $\mathbf{L}^d(R_l^*) = \mathbf{L}^s_\tau(R_l^*)$. Because the loan supply curve is increasing under Assumption 1 and the loan demand curve is decreasing, R_l^* is unique and other equilibrium objects are also uniquely determined. For example, the equilibrium quantity of checkable deposits is $D_\tau^* = N\mathbf{d}_\tau(R_l^*)$ and the loan quantity is $L_\tau^* = \mathbf{L}^d(R_l^*)$. For reference, define the equilibrium loan rate without a CBDC as $\hat{R}_l^*$, which satisfies $\mathbf{L}^d(\hat{R}_l^*) = \hat{\mathbf{L}}^s(\hat{R}_l^*)$.



(a) Low R_e (b) Intermediate R_e (c) High R_e

Figure 3: Equilibrium under Different τ. The blue curve is the loan supply without the CBDC. The solid red is the loan supply with τ_1 and the solid black is the loan supply with τ_2 where $\tau_1 > \tau_2$. The dashed green is the loan demand curve.

Proposition 2 *There exists a unique monetary equilibrium.*

We can analyze the properties of the equilibrium using the aggregate loan supply and the aggregate loan demand. Figure 3 shows the equilibrium under three different R_e. In all panels, the blue curve is the aggregate loan supply without a CBDC. The red curve and black curves are the loan supplies under τ_1 and τ_2, respectively, where $1 > \tau_1 > \tau_2$. By definition, the aggregate loan supply curves have the same shape as the loan supply curves of an individual bank, which are shown in 2(b). The green dashed curve is the aggregate loan demand. Its intersection with the loan supply curves correspond to the equilibrium under different conditions: point a is the equilibrium without a CBDC, point b the equilibrium with a CBDC under $\tau = \tau_1$ and point c the equilibrium under $\tau = \tau_2$.

We start with investigating the equilibrium under $\tau = \tau_1$. The aggregate loan supply is the red curve. An increase in R_e (a decrease in ϕ_e) raises $\underline{R}_l^{\tau_1}$, $\bar{R}_l^{\tau_1}$ and $\tau_1/[\phi_e - (1-\tau_1)\beta]$, which is the loan rate at which the red curve intersects the blue curve. It also shifts the increasing segment of the red curve to the right and moves up the horizontal segment by increasing $\bar{D}_{\tau_1}$.

If R_e is below $R_{1,e}^{\tau_1}$, which solves $\Delta_\tau^+(\bar{D}_\tau) = 1/\xi$ with $\xi = \max((1-\chi)\hat{R}_l^* + \chi R_r, R_r)$, point a is to the right of $\bar{R}_l^{\tau_1}$, as shown in Figure 3(a). The equilibrium with a CBDC coincides with that without CBDC coincide. If $R^{\tau_1}_{1,e} < R_e < R^{\tau_1}_{2,e}$, where $R^{\tau_1}_{2,e} = 1/[\tau_1/\hat{R}^*_l + (1-\tau_1)\beta]$, point a falls between $\tau_1/[\phi_e - (1-\tau_1)\beta]$ and $\bar{R}^{\tau_1}_l$, as shown in the Figure 3(b). The green curve intersects the red curve when it is above the blue curve. Therefore, point b is higher than point a, i.e. the CBDC with $\tau = \tau_1$ raises bank lending. Notice that in this case, b is on the horizontal segement of the red curve. Therefore, the CBDC is not used in the equilibrium. It is purely an outside option to discipline banks market power.[9] If R_e is slightly higher, point b will still be higher than point a, but will lie on the increasing segment of the red curve. Then the CBDC is used in the equilibrium. If $R_e > R^{\tau_1}_{2,e}$, point a is to the left of $\tau_1/[\phi_e - (1-\tau_1)\beta]$, as shown in Figure 3(c). Point b is then below point a, i.e. the CBDC with $\tau = \tau_1$ disintermediates banks .

One can show that the loan quantity is maximized at $\bar{R}^{\tau_1}_e$ that solves

$$\alpha(1-\tau_1)\beta\lambda(\bar{D}_{\tau_1}) + 1/R_e + \alpha(1-\tau_1)\beta\lambda'(\bar{D}_{\tau_1})\bar{D}_{\tau_1}/N = \frac{1}{(1-\chi)f'((1-\chi)\underline{\phi}^{\tau_1}_d\bar{D}_{\tau_1}) + \chi R_r} \tag{14}$$

as an equation in R_e, where $\bar{D}_{\tau_1}$ and $\underline{\phi}^{\tau_1}_d$ both depend on R_e. Graphically, at $\bar{R}^{\tau_1}_e$, the green curve intersects the red curve at the left end of its horizontal segment and the equilibrium loan rate R^*_l is equal to $\underline{R}^{\tau_1}_l$. If $R^{\tau_1}_{1,e} < R_e < \bar{R}^{\tau_1}_e$, point b lies between $\underline{R}^{\tau_1}_l$ and $\bar{R}^{\tau_1}_l$. Then, the loan quantity is $(1-\chi)\hat{\mathbf{\Phi}}_d(\bar{D}_{\tau_1})\bar{D}_{\tau_1}$, which is increasing in R_e. If $R_e > \bar{R}^{\tau_1}_e$, point b lies on the increasing segment of the red curve, to the left of $\underline{R}^{\tau_1}_l$. A higher R_e shifts this increasing segment to the right. As a result, point b moves down along the loan demand curve, leading to a lower loan quantity.

The above discussion does not depend on the value of τ_1. Therefore, the following proposition holds.

Proposition 3 *If $\tau > 0$, the following holds:*

(1) Compared to the equilibrium without a CBDC, the CBDC increases bank lending iff $R_e \in (R^{\tau}_{1,e}, R^{\tau}_{2,e})$. It reduces bank lending iff $R_e > R^{\tau}_{2,e}$.

[9] This is related to Lagos and Zhang (2022), who study the disciplining role of money in the cashless limit.

(2) The loan quantity is increasing if $R_e \in (R^{\tau}_{1,e}, \bar{R}^{\tau}_e)$ and decreasing if $R_e > \bar{R}^{\tau}_e$, with the maximum achieved at $R_e = \bar{R}^{\tau}_e$.

This proposition describes how introducing a CBDC changes the equilibrium. It is analogous to Proposition 3 in CDJZ (2022) but is more general because it allows the CBDC to be an imperfect substitute to checkable deposits. The CBDC can promote bank lending even if it is inferior to checkable deposits in terms of payment functions. Intuitively, banks have incentives to keep the supply of checkable deposits below the level under perfect competition, which reduces interest payment on checkable deposits. The CBDC can weaken this incentive by making the demand for checkable deposits more elastic, leading to more checkable deposits in the equilibrium.[10] Because banks have more funding, they lend out more.

We next analyze how the effect of a CBDC depends on τ. The black curve in Figure 3 shows the loan supply under $\tau_2 < \tau_1$. It has a shape similar to the red curve but with some differences. First, it joins the blue curve at a lower R_l, i.e. $\bar{R}^{\tau_2}_l < \bar{R}^{\tau_1}_l$. Therefore, there exists a range of R_e at which $\hat{R}^*_l$ is between $\bar{R}^{\tau_2}_l$ and $\bar{R}^{\tau_1}_l$. Then the CBDC promotes bank intermediation if $\tau = \tau_1$ but does not affect bank lending if $\tau = \tau_2$. Second, the black curve is above the red curve at R_l close to R_r and below the red curve at R_l close to $\bar{R}^{\tau_1}_l$. If R_e is not too big, as shown in Figure 3(b), the equilibrium loan quantity under τ_1 is higher than that under τ_2. If R_e is sufficiently big, as in Figure 3(c), the equilibrium loan quantity under τ_1 is lower than that under τ_2. One can show that a higher τ leads to higher maximum quantities of checkable deposits and loans, denoted as $\bar{D}^*_{\tau}$ and $\bar{L}^*_{\tau}$, which are achieved at $R_e = \bar{R}^{\tau}_e$.

Proposition 4 *If $1 > \tau_1 > \tau_2$, the following holds.*

(1) There exists $R_{3,e}$ such that $L^*_{\tau_1} > L^*_{\tau_2}$ if $R_e \in (R^{\tau_1}_{1,e}, R_{3,e})$ and $L^*_{\tau_1} < L^*_{\tau_2}$ if $R_e > R_{3,e}$.

(2) $\bar{D}^*_{\tau_1} > \bar{D}^*_{\tau_2}$ and $\bar{L}^*_{\tau_1} > \bar{L}^*_{\tau_2}$.

[10] If $\tau = 1$, the CBDC is a perfect substitute to checkable deposits. The demand for checkable deposits is perfectly elastic at $R_d = R_e$. Therefore, the CBDC rate serves as a floor for the interest rate on checkable deposits.

This result has two messages. First, if τ is higher, the CBDC promotes bank intermediation more if R_e is not too high but disintermediate banks more if R_e is sufficiently high. Second, a higher τ always leads to a larger maximum positive effect on bank intermediation. Intuitively, the magnitude of the positive effect depends on the ability of the CBDC to promote competition in the deposit market. This ability is higher if the CBDC is a closer substitute to checkable deposits. Quantitative analysis in Section 4 also shows that a CBDC with a higher τ leads to a larger maximum output, promotes bank intermediation and output for a wider range of R_e. Therefore, if the central bank can choose R_e properly, it is always useful to design the CBDC as a close substitute to checkable deposits.

3 Imperfect Loan Market Competition

We have so far considered a perfectly competitive loan market. Now we show that our results hold if the loan market is imperfectly competitive à la Cournot. The household problem and the entrepreneur problem stay unchanged. Banks internalize the effect of their lending decisions on the loan rate. Then bank j solves

$$\begin{aligned}
\Omega(\Pi) = & \max_{\delta_j, b_j, d_j, l_j, r_j} \delta_j + \beta\Omega(\Pi') \qquad (15) \\
& \text{st } \delta_j + l_j + r_j = \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \Pi + \beta b_j \\
& \quad \Pi' = \mathbf{R}_l(L_{-j} + l_j)l_j + R_r r_j - d_j - b_j \\
& \quad \delta_j \leq \Pi,\ r_j > \chi\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j,\ b_j \geq 0
\end{aligned}$$

where $\mathbf{R}_l(L_{-j} + l_j) = f'(L_{-j} + l_j)$ is the inverse demand function for loans and $L_{-j} = \sum_{i \neq j} l_i$ is the total loan supply of banks other than j.

In the appendix, we show that the equilibrium can be solved by finding the shadow value of checkable deposits, $R_I \in [R_r, 1/\beta]$, which depends on the Lagrangian multiplies on the constraints that relate the loan quantity to the checkable deposit quantity. Given R_I, the aggregate checkable deposit quantity D in the symmetric equilibrium satisfies

$$1 \leq \left[\nabla^{-}\mathbf{\Phi}_{d,\tau}(D)\frac{D}{N} + 1 \geq \mathbf{\Phi}_{d,\tau}(D)\right]\left[(1-\chi)R_I + \chi R_r\right] \tag{16}$$

$$1 \geq \left[\nabla^{+}\mathbf{\Phi}_{d,\tau}(D)\frac{D}{N} + \mathbf{\Phi}_{d,\tau}(D)\right]\left[(1-\chi)R_I + \chi R_r\right]. \tag{17}$$

The aggregate loan supply depends on D in the same way as in the previous section. If $R_I = R_r$, the reserve requirement is not binding. Therefore, the aggregate loan quantity can be any value between 0 and $(1-\chi)D\mathbf{\Phi}_{d,\tau}(D)$. If $R_I > R_r$, the reserve requirement is binding and the aggregate loan supply is $(1-\chi)D\mathbf{\Phi}_{d,\tau}(D)$. If $R_I = 1/\beta$, banks are willing to use term deposits to finance loans. Therefore, the loan supply can be any value in $[(1-\chi)D\mathbf{\Phi}_{d,\tau}(D), \infty]$. Therefore, the aggregate loan supply is

$$\mathbf{L}_{\tau}(R_I) = \begin{cases} [0, (1-\chi)\mathbf{\Phi}_{d,\tau}(D)\, D] & \text{if} \quad R_l = R_r, \\ (1-\chi)\mathbf{\Phi}_{d,\tau}(D)\, D & \text{if} \quad R_r < R_l < 1/\beta, \\ [(1-\chi)\mathbf{\Phi}_{d,\tau}(D)\, D, \infty] & \text{if} \quad R_l = 1/\beta. \end{cases}$$

It is identical to the aggregate loan supply in the baseline model except it depends on R_I instead of R_l. Given R_I, the aggregate loan quantity L in the Cournot game satisfies

$$R_I = \mathbf{R}_l(L) + \mathbf{R}_l'(L)\frac{L}{N}. \tag{18}$$

This gives the aggregate loan demand function $\mathbf{L}^d(R_I)$. It differs from the loan demand in the baseline model in two ways. First, it depends on R_I instead of R_l. Second, it has the additional term $\mathbf{R}_l'(L)L/N$, which comes from the Cournot competition in the loan market: banks internalize their impact on the loan rate. To proceed, we impose the following assumption.

Assumption 2 *The production function f satisfies $2f''(L) + f'''(L)L < 0$ for all $L > 0$.*

Assumption 2, together with Assumption 1 implies that there is a unique symmetric equilibrium in the Cournot game among banks, which guarantees the uniqueness of the general equilibrium. Under this assumption, $\mathbf{L}^d(R_I)$ is decreasing. We can analyze the equilibrium by plotting the loan demand and loan supply as functions of R_I. The diagram is the same as Figure 3, except the x-axis is R_I instead of R_l. Because the loan supply function is unchanged except it now depends on R_I, we can obtain a result identical to Proposition 1. We can then follow the analysis in Section 2 and show all other results hold.

Proposition 5 *Propositions 2-4 hold if banks engage in Cournot competition in both the deposit and the loan markets.*

4 Quantitative Analysis

We now quantify the effect of a CBDC with different designs on the US economy. The design parameter of particular interest is τ. If τ is close to 1, the CBDC is good substitute to checkable deposits in payment.

To map the model to data, we extend the baseline model in several dimensions. First, we introduce cash and another two types of meetings. Assume that with probability α_1, a buyer gets into a meeting (type 1 meeting) where only cash can be used. With α_2, a buyer gets into a meeting (type 2 meeting) where cash cannot be used but checkable deposits can be used. This is the type of meetings we consider in the previous sections. With probability α_3, a buyer gets into a meeting (type 3 meeting) where both cash and checkable deposits can be used. Then the probability of meeting a seller $\alpha = \alpha_1 + \alpha_2 + \alpha_3$. The CBDC is an imperfect substitute to checkable deposits. It can use used in a τ fraction of type 2 and type 3 meetings. Second, we introduce a proportional cost for banks to handle deposits, denoted as c. Third, we allow sellers to have some market power in the DM by assuming that the terms of trade are determined by Kalai bargaining with bargaining power θ to the buyer. If $\theta < 1$, the seller has market power. We parameterize $U(x) = B\log(x)$, $f(x) = Ax^{\eta}$, $u(y) = \left[(y+\varepsilon)^{1-\sigma} - \varepsilon^{1-\sigma}\right]/(1-\sigma)$ and define $\omega = \alpha_i/\alpha$ for $i = 1, 2, 3$. Notice that α captures the size of the DM and ω_i is the fraction of type i meetings in DM.

We calibrate the extended model to the US economy between 2014 and 2019. We use the following data for this exercise: (1) data from the Survey of Consumer Payment Choice (SCPC) and the Diary of Consumer Payment Choice (DCPC) from the Federal Reserve Bank of Atlanta; (2) call report data from the Federal Financial Institutions Examination Council; (3) new M1 series from Lucas and Nicolini (2015); and (4) several time series on macro variables and reserves from Federal Reserve Economic Data (FRED).

The calibration strategy is similar to CDJZ (2022) with some differences. The ωs, which capture the acceptance decisions of sellers, are obtained from the SCPC (Greene and Stavins 2018) and the DCPC (Premo 2018). We use the same time period as in CDJZ (2022) and set $\omega_1 = 4.58\%$, $\omega_2 = 26.44\%$, and $\omega_3 = 68.98\%$. We pick (σ, B) and the bargaining power (θ) jointly to match the money demand curve and a 20% retail markup. Different from CDJZ (2022), the DM size α is chosen to match the ratio of DM output to total output in the model with the ratio of retail sector value-added to GDP in data. The idea is to interpret the DM as the retail market.[11] We use data from 1987 to 2008 following CDJZ (2022). Data on the value-added of the retail sector is available only since 2005. Therefore, we use the average ratio of retail sector value-added to GDP between 2005 and 2008.

We set η to match the elasticity of commercial loans with respect to the loan rate. Choose χ to match the average ratio of required reserves to the total balance in transaction accounts in 2014–2019. Set $i_r = 1.02\%$ to match the average interest rate on required reserves. To calibrate A, c, and N, we use several statistics on the banking sector in 2014-2019 calculated from the call report data. We choose c to match average non-interest expenditures, excluding expenditures on premises or rent, per dollar of assets. Pick A and N jointly to match the average interest rate on transaction accounts and the spread between the loan rate and the

[11] We also tried to follow CDJZ (2022) and choose (σ, B, α) to match the money demand curve. The results are similar, but the fit of the money demand curve does not change much with α, which leads to the concern of weak identification. Therefore, we report the calibration that uses the retail sector value-added to GDP ratio as a target. The potential weak identification can arise partly because we calculate GDP differently from CDJZ (2022). In their model, banks are owned by bankers who consumes profits. In our model, banks are owned by households and all profits are paid as dividend to households.

Parameters	Notation	Value	Calibration Targets
Calibrated externally			
Discount factor	β	0.96	Standard in literature
Curvature of production	η	0.66	Elasticity of commercial loans
Reserve requirement	χ	5.60%	2014–19 avg. required reserves/trans. balances
Interest rate on reserves	i_r	1.02%	2014–19 avg. IORR
Cost of handling deposits	c	0.02	Avg. operating cost per dollar asset 2.02%
Gross money growth rate	μ	1.0152	2014–19 avg. annual inflation 1.52%
Frac. of type 1 trades	ω_1	4.58%	SCPC 2016
Frac. of type 2 trades	ω_2	26.44%	SCPC 2016
Frac. of type 3 trades	ω_3	68.98%	SCPC 2016
Calibrated internally			
Prob. of DM trading	α	0.4	Value-Added of Retail Sector to GDP 6.1%
Coeff. on CM consumption	B	2.65	Money demand 1987-2008
Curv. of DM consumption	σ	1.59	Money demand 1987-2008
Total factor productivity	A	1.50	Rate on transaction accounts 0.3049%
Number of banks	N	25	Spread b/w transaction accounts and loans 3.39%
Buyer's bargaining power	θ	0.9982	Retailer markup 20%

Table 1: Calibration Results: Baseline Model

rate on transaction accounts. We consider two versions of the model. One with a perfectly competitive loan market and the other with a Cournot loan market.

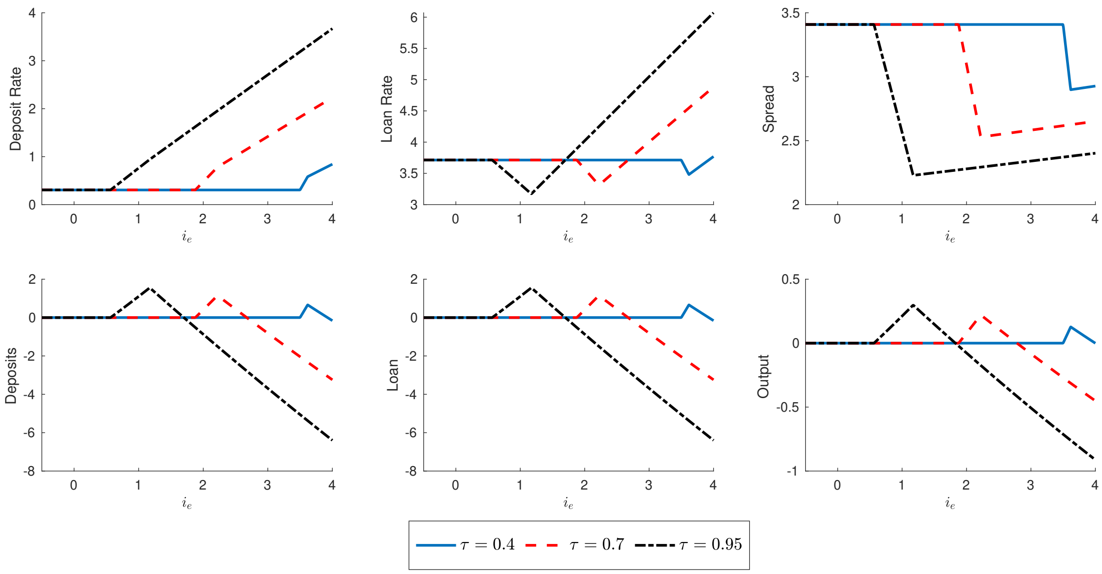
4.1 Perfectly Competitive Loan Market

Table 1 shows the calibration results. We investigate how the effect of a CBDC depends on its interest rate i_e and τ. Figure 4 shows the results.

The first row shows the effects on the nominal deposit rate, the norminal loan rate and the spread, i.e., the difference between the deposit rate and the loan rate. The second row shows the deposit quantity, the loan quantity and output. The horizontal axis is the nominal interest rate on the CBDC, i_e. The rates are in percentage points and the quantities are the percentage deviation from the equilibrium without a CBDC. The solid blue curve has $\tau = 0.4$, the dashed red curve has $\tau = 0.7$ and the dash-dot black has $\tau = 0.95$.

First, notice that the CBDC increases the deposit rate, although it no longer sets the floor for the deposit rate as in CDJZ(2022). This is because that the CBDC is not a perfect

Figure 4: Effects of a CBDC under Different i_e and τ



substitute to deposits. Therefore, the demand for deposits is positive even if the rate is lower than the CBDC rate. If τ is higher, i.e., the CBDC is a better substitute to deposits, the CBDC needs to pay a lower rate to become effective. If $\tau = 0.4$, the CBDC has an effect on the economy when i_e is above 3.50%. If $\tau = 0.7$ and 0.95, this number decreases to 1.88% and 0.57%, respectively. Because the CBDC is more attractive under a higher τ, a lower interest rate is needed to make it a good alternative to deposits. Moreover, when the CBDC is effective, the pass-through from the CBDC rate to the deposit rate is stronger if τ is higher, i.e., the dash-dot black line is steeper than the solid blue and the dashed red.

As the CBDC rate increases, the banks first find it optimal to compete aggressively. They issue enough deposits to satisfy all the demand for electronic payment balances (deposits and the CBDC). Because the deposit rate is elevated, the demand for deposits is higher, resulting in more deposits. Because banks have more funding, they lend out more, leading to a decrease in loan rate and an increase in loan quantity. At the maximum, the CBDC can increase deposits and lending by 0.66% if $\tau = 0.4$, 1.14% if $\tau = 0.7$ and 1.54% if $\tau = 0.95$. Consistent with our theoretical results, the maximum positive impact of the CBDC on deposits and loans increases as τ increases. The maximum drop in the loan rate also increases with τ.

Notice that before the maximal loan quantity is reached, the CBDC is not used and serves merely as an outside option to discipline banks' market power. Because the CBDC is not attractive enough, banks' optimal decision is to drive it out of circulation. This is related to Zhu and Hendry (2019), who study a policy game of the central bank and a private digital currency issuer and establish a similar result. They also show that if the private digital currency is strictly more useful than the central bank money, the optimal strategy of the central bank is to make its money as an outside option.

When the CBDC rate becomes too high, it is no longer optimal for banks to compete aggressively with the CBDC. If banks drive the CBDC out of circulation, they have too much funding, which drives down the lending rate and leads to losses. Instead, the banks reduces deposit supply and allow the CBDC to circulate in the economy. At the same time, they lend less and get a higher loan rate. Therefore, a higher CBDC rate reduces deposits and loans, and raises the loan rate. If the CBDC rate is too high, deposits and loans fall below the level without a CBDC. This occurs at $i_e > 3.92\%$ if $\tau = 0.4$, at $i_e > 2.67\%$ if $\tau = 0.7$ and at $i_e > 1.69\%$ if $\tau = 0.95$. Therefore, if the CBDC is a better substitute to bank deposits, it disintermediates banks at a lower rate. However, the CBDC with a higher τ promotes bank intermediation for a longer interval of interest rates: if $\tau = 0.95$, the length of the interval is 1.12%; if $\tau = 0.7$, the length is 0.79%; and if $\tau = 0.4$, the length is 0.42%.

Output follows a pattern similar to deposits and loans. It first increases and then decreases as the CBDC rate increases. At the maximum, the CBDC increases output by 0.13% under $\tau = 0.4$, by 0.22% under $\tau = 0.7$ and 0.30% under $\tau = 0.95$. The CBDC increases output compared to the case without a CBDC if $i_e \in (3.50\%, 4.0\%)$ under $\tau = 0.4$, if $i_e \in (1.88\%, 2.78\%)$ under $\tau = 0.7$, and if $i_e \in (0.57\%, 1.82\%)$ under $\tau = 0.95$. Notice that the length of the range also increases as τ increases, i.e., if τ is higher, it is easier to find i_e such that introducing a CBDC increases output.

Lastly, the spread between the loan rate and the deposit rate first decreases and then increases because banks first competes aggressively, which decreases the loan rate and increases the deposit rate. But if the CBDC rate is too high, banks have to cover the higher deposit rate by raising the loan rate.

It is worth pointing out that unlike in CDJZ (2022), banks never behave as if the banking sector is perfectly competitive. This is because the CBDC is not a perfect substitute to bank deposits. Therefore, banks always have market power in the segment of market not served by the CBDC, which enables them to make a profit.

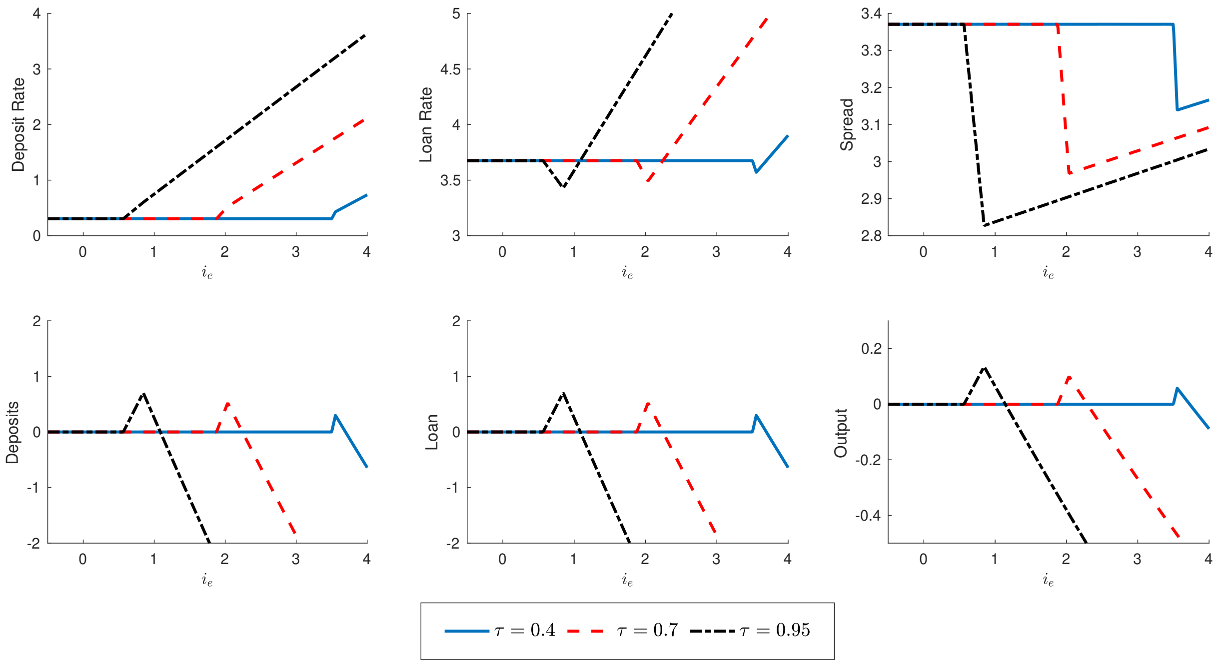
4.2 Imperfectly Competitive Loan Market

To assess the robustness of the findings, we consider the version where the loan market is imperfectly competitive à la Cournot. To do so, we recalibrate the model using the same method. Notice that only A and N change in the new calibration because other parameters are calibrated independent of the loan market structure. It turns out the change in A is small but N increases dramatically. Intuitively, banks now have market power in both the deposit and the loan markets. To generate the same spread, more banks are needed.

Figure 5 shows the effect of i_e under $\tau = 0.4, 0.7, 0.95$. The graphs are qualitatively similar to those in the previous section. The CBDC can promote bank intermediation and increase output. If τ is larger, the CBDC becomes effective at a lower i_e and can increase output and lending by more and for a wider range of i_e. But it starts to disintermediate banks and decrease output also at a lower i_e.

Quantitatively, the effect of a CBDC is much smaller. If $\tau = 0.4$, the CBDC promotes bank lending and output when its rate is in $(3.50\%, 3.69\%)$ and $(3.50\%, 3.73\%)$, respectively. It increases bank lending and output by 0.30% and 0.06%, respectively. Similarly, if $\tau = 0.7$, the CBDC increases lending if $i_e \in (1.88\%, 2.24\%)$ and increases output if $i_e \in (1.88\%, 2.29\%)$. The maximum increases in lending and output are 0.52% and 0.1%,

Figure 5: Effects of a CBDC under Different i_e and τ



respectively. If $\tau = 0.95$, the CBDC promotes bank lending and output when its rate is in $(0.57\%, 1.08\%)$ and $(0.57\%, 1.35\%)$, respectively. At the maximum, it increases loans and output by 0.7% and 0.13%. Both the lengths of the intervals and the maximum effects are about half of their values in the baseline calibration. Intuitively, the CBDC affects the economy by disciplining the market power of banks in the deposit market. In the baseline model, the loan market is perfectly competitive. A large market power in the deposit market is need to match the spread in the data. If the loan market is imperfectly competitive, part of the spread is attributed to the loan market. Therefore, the deposit market is more competitive and the CBDC has a smaller effect.

5 Discussion and Conclusion

We study the effect of a CBDC on bank intermediation and output when it is an inferior substitute to checkable deposits in the sense that it can only be used in a fraction of transactions served by checkable deposits. This may be the realistic scenario because commercial banks are more experienced in retail payment than the central bank. Therefore, it can be difficult for the central bank to create a product that is superior to bank deposits. Even in this situation, the CBDC can increase bank lending and output. Therefore, the result of CDJZ (2022) does not depend on that the CBDC is a perfect substitute to checkable deposits in retail payment.

It can be desirable to design the CBDC as a good substitute to checkable deposits. A CBDC that is a better substitute to checkable deposits has a positive effect on the economy at a lower interest rate and for a wider range of interest rates; it leads to a stronger the pass-though from the CBDC rate to the deposit rate; it can increase lending and output by more. The downside is that it also disintermediates banks at a lower rate. However, according to our baseline calibration, even if the CBDC serves 95% of deposit transactions, the CBDC rate needs to be around 1.82% for disintermediation to occur when the reserve rate is 1.02%. If the CBDC does not pay interest, this means that the CBDC has to have a big advantage in account/transaction fees to disintermediate banks. This implies that the disintermediation concern may not be important. A bigger concern is that if the CBDC is not a good substitute to bank deposits, it would not affect the economy even if the rate is high. For instance, in the baseline calibration if the CBDC serves only 40% of deposit transactions, it does not affect the economy unless its interest rate is above 3.5%. Therefore, to better reap the benefit of the CBDC, it should be designed as a close substitute to bank deposits in payment.

An important question is how to design the CBDC such that it serves a high fraction of deposit transactions. This question is beyond the scope of this paper and achieve this goal would involve a combination of technological and business model designs. The former guarantees the CBDC has sufficient functionality and the latter provides economic incentives for agents to adopt the CBDC. Recent studies such Li (2022) and Huyhn et al. (2021) made some important progress on this front. More future research is needed.

A Supply of Checkable Deposits

In the main text, we consider the case where $\tilde{\phi}_e^\tau > \phi_e > \hat{\Phi}_d[N\hat{d}(1/\beta)]$. This section characterizes the supply of checkable deposits in other cases under $\tau < 1$. For the case with $\tau = 1$, please refer to CDJZ (2022).

Define $\bar{\phi}_e^\tau = \alpha\beta\tau\lambda(N\hat{\mathbf{d}}(R_r)) + \beta$ and $\hat{\phi}_e^\tau$ solves

$$\alpha\beta\lambda(\bar{D}_\tau) + \alpha\beta(1-\tau)\lambda'(\bar{D}_\tau)\bar{D}_\tau/N = 1/R_r$$

as an equation in ϕ_e, where $\bar{D}_\tau$ is decreasing in ϕ_e by definition. One can check that $\bar{\phi}_e^\tau > \hat{\phi}_e > \tilde{\phi}_e^\tau$. If $\phi_e > \bar{\phi}_e^\tau$, the supply of checkable deposits and loans are the same as those without a CBDC. If $\bar{\phi}_e^\tau > \phi_e > \hat{\phi}_e^\tau$, $\underline{R}_l^\tau$ does not exist but $\bar{R}_l^\tau$ exists. Therefore,

$$\mathbf{d}_\tau(R_l) = \begin{cases} \bar{D}_\tau/N & \text{if} \quad R_l < \bar{R}_l^\tau, \\ \hat{\mathbf{d}}(R_l) & \text{if} \quad < \bar{R}_l^\tau < R_l \leq 1/\beta. \end{cases}$$

If $\hat{\phi}_e^\tau > \phi_e > \hat{\Phi}_d[N\hat{d}(1/\beta)]$, we obtain (11). Notice that if $\bar{\phi}_e^\tau > \phi_e > \tilde{\phi}_e^\tau$, $\mathbf{d}_\tau(R_l) > \hat{\mathbf{d}}(R_l)$ for all $R_l \leq R_r$.

Next, suppose $\hat{\Phi}_d[N\hat{d}(1/\beta)] > \phi_e > \underline{\phi}_e^\tau$, where $\underline{\phi}_e^\tau = \alpha\beta\tau\lambda(D) + \beta$ and D satisfies

$$\alpha\beta\lambda(D) + \beta + (1-\tau)\alpha\beta\lambda'(D)D/N = \frac{1}{(1-\chi)/\beta + \chi R_r}.$$

Notice that $\hat{\Phi}_d[N\hat{d}(1/\beta)] > \phi_e$ implies that introducing a CBDC changes the supply of checkable deposits even if $R_l = 1/\beta$. Therefore, $\bar{R}_l^\tau$ does not exist. Moreover, $\phi_e > \underline{\phi}_e^\tau$ implies that the supply of checkable deposits reaches $\bar{D}_\tau$ before R_l reaches $1/\beta$. Therefore, $R_r < \underline{R}_l^\tau < 1/\beta$ and

$$\mathbf{d}_\tau(R_l) = \begin{cases} \tilde{\mathbf{d}}_\tau(R_l) & \text{if} \quad R_l < \underline{R}_l^\tau, \\ \bar{D}_\tau/N & \text{if} \quad \underline{R}_l^\tau < R_l \leq 1/\beta, \end{cases}$$

If $\underline{\phi}_e^\tau > \phi_e > \beta$, both $\underline{R}_l^\tau$ and $\bar{R}_l^\tau$ do not exist. Then if $R_l \leq 1/\beta$, $\mathbf{d}_\tau(R_l) = \tilde{\mathbf{d}}_\tau(R_l)$.

B Proofs

Unless stated otherwise, all proofs are under $\underline{R}_e < R_e < 1/\hat{\Phi}_d[N\hat{d}(1/\beta)]$. We first establish two useful lemmas.

Lemma 1 *Let $\tilde{R}_l$ satisfy $(1-\chi)\tilde{R}_l + \chi R_r = 1/\phi_e$. Then $\tilde{\mathbf{d}}_\tau(R_l)$ is decreasing in τ if $R_l < \tilde{R}_l$ and increasing in τ if $R_l > \tilde{R}_l$.*

Proof. Rewrite (12) to get

$$\alpha\beta\lambda(Nd) + \alpha\beta\lambda'(Nd)d = (1/\xi - \phi_e)/(1-\tau). \tag{19}$$

If $R_l < \tilde{R}_l$, $1/\xi - \phi_e > 0$. The right hand side is positive and increasing in τ. If $R_l > \tilde{R}_l$, $1/\xi - \phi_e < 0$. The right hand side is negative and decreasing in τ. By Assumption 1, the left hand side is decreasing in d. Therefore, $\tilde{\mathbf{d}}_\tau(R_l)$ is decreasing (increasing) in τ if $R_l < \tilde{R}_l$ ($R_l > \tilde{R}_l$). Moreover, if $R_l = \tilde{R}_l$, $\tilde{\mathbf{d}}_\tau(R_l) = \tilde{d}$ and is independent of τ. ■

Lemma 2 *If $\tau > 0$ and $\phi_e < \tilde{\phi}_e^\tau$, $\tilde{\mathbf{d}}_\tau(R_l) = \hat{\mathbf{d}}(R_l)$ implies $\tilde{\mathbf{d}}'_\tau(R_l) > \hat{\mathbf{d}}'(R_l)$*

Proof. Recall that $\hat{\mathbf{d}}(R_l)$ solves

$$\alpha\beta\lambda(Nd) + \alpha\beta\lambda'(Nd)d = 1/\xi - \beta. \tag{20}$$

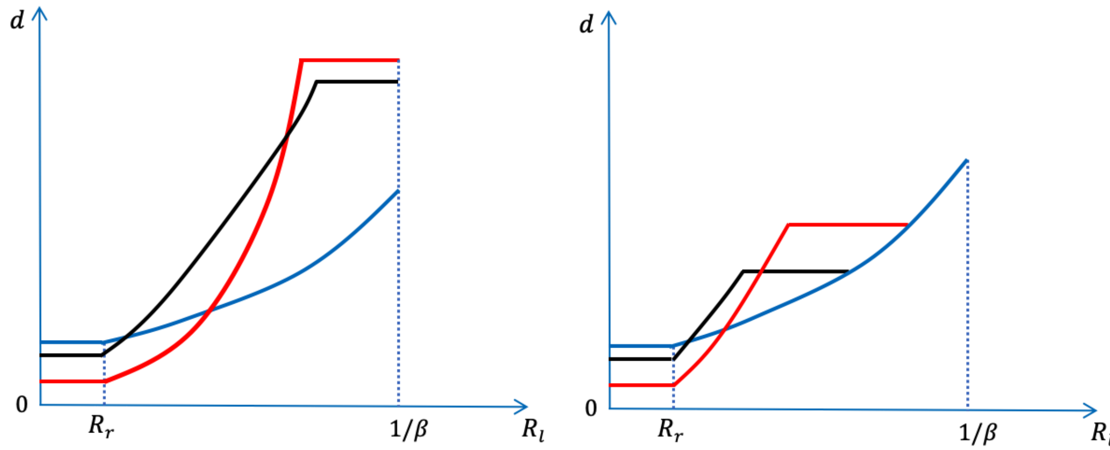
Because $\phi_e < \tilde{\phi}_e$, the right hand side of (19) is bigger than that of (20) if $R_l < R_r$. Therefore, $\tilde{\mathbf{d}}_\tau(R_l) < \hat{\mathbf{d}}(R_l)$ for all $R_l < R_r$. Therefore, $\tilde{\mathbf{d}}_\tau(R_l) = \hat{\mathbf{d}}(R_l)$ can hold only if $R_l > R_r$. Notice also that $\alpha\beta\lambda(Nd)+\alpha\beta\lambda'(Nd)d = \hat{\Delta}(d)-\beta$. Then (19) and (20) imply that if $\tilde{\mathbf{d}}_\tau(R_l) = \hat{\mathbf{d}}(R_l)$

$$\hat{\mathbf{d}}'(R_l) = -\frac{1/\xi^2(1-\chi)}{\hat{\Delta}'(\hat{\mathbf{d}}(R_l))} < -\frac{1/\xi^2(1-\chi)}{\hat{\Delta}'(\tilde{\mathbf{d}}'_\tau(R_l))(1-\tau)} = \tilde{\mathbf{d}}'_\tau(R_l), \tag{21}$$

where the inequality because Assumption 1 implies that $\hat{\Delta}'(d) < 0$ and $\tau > 0$. This proves the lemma. ■

Proof of Proposition 1. To show the uniqueness, we need to prove that there is a unique d that satisfies (9) and (10). Let $\Delta_\tau(d) = \mathbf{\Phi}'_{d,\tau}(Nd)d + \mathbf{\Phi}_{d,\tau}(d)$. We first show that Δ_τ is decreasing on $[0, \bar{D}_\tau/N)$ and on $(\bar{D}_\tau/N, y^*/N)$. If $d \in [0, \bar{D}_\tau/N)$,

$$\begin{aligned}\Delta_\tau(d) &= \alpha(1-\tau)\beta\lambda(Nd) + \phi_e + \alpha(1-\tau)\beta\lambda'(Nd)d \\ &= (1-\tau)\hat{\Delta}(d) + \phi_e - (1-\tau)\beta.\end{aligned}$$



(a) Case 1 (b) Case 2

Figure 6: Checkable Deposit Supply of a Bank. The red curve and the black curve are has $1 > \tau_1 > \tau_2$, respectively. The blue curve is the supply without a CBDC.

By Assumption 1, $\hat{\Delta}(d)$ is decreasing in d, so is $\Delta_\tau(d)$. If $d \in (\bar{D}_\tau/N, y^*/N)$, $\Delta_\tau(d) = \hat{\Delta}(d)$, which is decreasing. If $d = \bar{D}_\tau/N$, $\phi_e = \alpha\tau\beta\lambda(Nd) + \beta$ and

$$\Delta_\tau^-(d) = \Delta_\tau(d) = \hat{\Delta}(d) - \alpha(1-\tau)\beta\lambda'(Nd)d > \hat{\Delta}(d) = \Delta_\tau^+(d).$$

This means $\Delta_\tau(d_1) > \Delta_\tau(d_2)$ if $d_1 \in [0, \bar{D}_\tau/N)$ and $d_2 \in (\bar{D}_\tau/N, y^*/N)$. By Assumption 1, $\Delta_\tau(d) > 1/\xi$ for d sufficiently small and $\Delta_\tau(d) < 0$ if d sufficiently close to y^*/N. Moreover, $\Delta_\tau^-(d) = \Delta_\tau^+(d) = \Delta_\tau(d)$ is continuous on $[0, \bar{D}_\tau/N)$ and $(\bar{D}_\tau/N, y^*/N)$. Then (9) and (10) hold for a unique $d \in [0, y^*/N)$, which proves the existence and uniqueness of the equilibrium.

Proposition 1(1) is proved by the discussion before the proposition. We only need to prove Proposition 1(2). We distinguish two cases:

Case 1: $\tilde{R}_l < \underline{R}_l^\tau$ for $\tau = \tau_1, \tau_2$. This case occurs only if $R_e > 1/\hat{\Phi}_d[N\hat{d}(1/\beta)]$. For completeness, we still discuss it here. Figure 6(a) shows this case. The red curve is the supply of checkable deposit under τ_1 and the black curve is under τ_2. The blue curve is the supply of checkable deposits without a CBDC.[12] If $R_l < \tilde{R}_l$, $\mathbf{d}_\tau(R_l) = \tilde{\mathbf{d}}_\tau(R_l)$. By Lemma 1, $\mathbf{d}_{\tau_1}(R_l) < \mathbf{d}_{\tau_2}(R_l)$. If $1/\beta > R_l > \tilde{R}_l$, either $\mathbf{d}_\tau(R_l) = \tilde{\mathbf{d}}_\tau(R_l)$ or $\mathbf{d}_\tau(R_l) = \bar{D}_\tau/N$. Because

[12] Notice that $\tilde{d} > \hat{\mathbf{d}}(1/\beta)$ if $\chi > 0$.

both are increasing in τ, the second part of this proposition holds with $R_l^c = \tilde{R}_l$.

Case 2: $\tilde{R}_l > \underline{R}_l^\tau$ for at least one τ. Notice that $R_e < 1/\hat{\Phi}_d[N\hat{d}(1/\beta)]$ implies $\tilde{R}_l > \underline{R}_l^\tau$ for τ_1 and τ_2. Figure 6(b) shows this case. We start with showing $\underline{R}_l^{\tau_1} > \underline{R}_l^{\tau_2}$. To see this, suppose towards contradiction, $\underline{R}_l^{\tau_1} < \underline{R}_l^{\tau_2}$. There are two possibilities. The first possibility is $\underline{R}_l^{\tau_1} < \underline{R}_l^{\tau_2} < \tilde{R}_l$. By definition, $\tilde{\mathbf{d}}_\tau(\underline{R}_l^\tau) = \bar{D}_\tau/N$. Because, $\tilde{\mathbf{d}}_\tau(R_l)$ is decreasing in τ if $R_l < \tilde{R}_l$ and $\bar{D}_\tau$ is increasing in τ, $\tilde{\mathbf{d}}_{\tau_1}(\underline{R}_l^{\tau_2}) < \tilde{\mathbf{d}}_{\tau_2}(\underline{R}_l^{\tau_2}) = \bar{D}_{\tau_2}/N < \bar{D}_{\tau_1}/N$. This implies $\underline{R}_l^{\tau_1} > \underline{R}_l^{\tau_2}$, which is a contradiction. The second possibility is $\underline{R}_l^{\tau_1} < \tilde{R}_l < \underline{R}_l^{\tau_2}$. We can arrive a contradiction by using a similar argument. Therefore, it must be $\underline{R}_l^{\tau_1} > \underline{R}_l^{\tau_2}$. Because $\underline{R}_l^{\tau_1} > \underline{R}_l^{\tau_2}$ and $\tilde{R}_l > \underline{R}_l^\tau$ for at least one τ, $\tilde{R}_l > \underline{R}_l^{\tau_2}$. By Lemma 1, $\mathbf{d}_{\tau_1}(R_l) < \mathbf{d}_{\tau_2}(R_l)$ if $R_l < \underline{R}_l^{\tau_2}$. If $R_l \in (\underline{R}_l^{\tau_1}, \bar{R}_l^{\tau_1})$, $\mathbf{d}_{\tau_1}(R_l) = \bar{D}_{\tau_1} > \mathbf{d}_{\tau_2}(R_l)$. Therefore, $\mathbf{d}_{\tau_1}(R_l)$ crosses $\mathbf{d}_{\tau_2}(R_l)$ at least once on $(\underline{R}_l^{\tau_2}, \underline{R}_l^{\tau_1})$. We only need to show that the crossing can happen only once. Suppose towards contradiction, there are more than one crossing. There must exist $R_l \in (\underline{R}_l^{\tau_2}, \underline{R}_l^{\tau_1})$ such that $\mathbf{d}_{\tau_1}(R_l) = \mathbf{d}_{\tau_2}(R_l)$ and $\mathbf{d}'_{\tau_1}(R_l) < \mathbf{d}'_{\tau_2}(R_l)$. Such R_l can be either smaller than $\bar{R}_l^{\tau_2}$ or bigger than $\bar{R}_l^{\tau_2}$. If it is smaller than $\bar{R}_l^{\tau_2}$, $\mathbf{d}_{\tau_2}(R_l)$ is constant and $\mathbf{d}'_{\tau_2}(R_l) = 0$. Because $\mathbf{d}_{\tau_1}(R_l)$ is strictly increasing on $(\underline{R}_l^{\tau_2}, \underline{R}_l^{\tau_1})$, $\mathbf{d}'_{\tau_1}(R_l) > \mathbf{d}'_{\tau_2}(R_l)$. If this R_l is bigger than $\bar{R}_l^{\tau_2}$, then $\mathbf{d}_{\tau_2}(R_l) = \hat{\mathbf{d}}(R_l)$. Because $R_l < \underline{R}_l^{\tau_1}$, $\mathbf{d}_{\tau_1}(R_l) = \tilde{\mathbf{d}}_{\tau_1}(R_l)$. Then by Lemma 2, $\mathbf{d}'_{\tau_1}(R_l) > \mathbf{d}'_{\tau_2}(R_l)$. Therefore, we arrive at a contradiction and such R_l does not exist. Then proposition holds with R_l^c being the crossing point of $\mathbf{d}_{\tau_1}(R_l)$ and $\tilde{\mathbf{d}}_{\tau_1}(R_l)$ in $(\underline{R}_l^{\tau_2}, \underline{R}_l^{\tau_1})$. ■

Proof of Proposition 4.

Proof of the first claim: By the proof of Proposition 1, $\mathbf{d}_{\tau_1}(R_l) < \mathbf{d}_{\tau_2}(R_l)$ if $R_r < R_l < R_l^c$ and $\mathbf{d}_{\tau_1}(R_l) > \mathbf{d}_{\tau_2}(R_l)$ if $R_l^c < R_l < \bar{R}_l^{\tau_1}$. Because $\Phi(D)D$ is increasing in D, the same relationship holds for the aggregate loan supply, i.e. $\mathbf{L}^s_{\tau_1}(R_l) < \mathbf{L}^s_{\tau_2}(R_l)$ if $R_r < R_l < R_l^c$ and $\mathbf{L}^s_{\tau_1}(R_l) > \mathbf{L}^s_{\tau_2}(R_l)$ if $R_l^c < R_l < \bar{R}_l^{\tau_1}$. Notice that as R_e increases, R_l^c shifts to the right. Define $R_{3,e}$ be the solution to $R_l^* = R_l^c$ where both R_l^* and R_l^c depend on R_e, i.e. the loan demand intersects the loan supply curves at R_l^c. If $R_{1,e} < R_e < R_{3,e}$, the loan demand curve intersects both curves to the right of R_l^c, implying that $L_{\tau_1}^* > L_{\tau_2}^*$ and if $R_e > R_{3,e}$, the loan demand curve intersects both curves to the left of R_l^c, implying that $L_{\tau_1}^* < L_{\tau_2}^*$. This proves the first part of this proposition.

Proof of the second claim: We start with proving that for a given τ, lending is maximized at $\bar{R}_e^\tau$. Notice that $\bar{R}_e^\tau$ is uniquely defined because the left hand side of (14) is decreasing in R_e while the right hand side is increasing in R_e. We show that the loan quantity is lower if R_e is bigger or smaller than $\bar{R}_e^\tau$.

If $R_e > \bar{R}_e^\tau$, then $\bar{D}_\tau > \bar{D}_\tau^*$ and $(1-\chi)\underline{\phi}_d^\tau \bar{D}_\tau > \bar{L}_\tau^*$. Then the right hand side of (14) is bigger than its left hand side. This suggests that the equilibrium D_τ^* cannot be bigger than $\bar{D}_\tau$. Therefore, D_τ^* solves

$$\alpha(1-\tau)\beta\lambda(D) + 1/R_e + \alpha(1-\tau)\beta\lambda'(D)D/N = \frac{1}{(1-\chi)f'((1-\chi)\mathbf{\Phi}_{d,\tau}(D)D) + \chi R_r}. \quad (22)$$

Suppose towards contradiction the equilibrium loan quantity, $L_\tau^* = (1-\chi)\mathbf{\Phi}_{d,\tau}(D_\tau^*)D_\tau^*$, is higher than $\bar{L}_\tau^*$. Then $f'(L_\tau^*) < f'(\bar{L}_\tau^*)$. For (22) to hold, D_τ^* has to be smaller than $\bar{D}_\tau^*$. Because $\mathbf{\Phi}_{d,\tau}(D_\tau^*) < \underline{\phi}_d^\tau$, $(1-\chi)\mathbf{\Phi}_{d,\tau}(D_\tau^*)D_\tau^* < \bar{L}_\tau^*$, which is a contradiction. Therefore, the equilibrium loan quantity must be smaller if $R_e > \bar{R}_e^\tau$.

If $R_e < \bar{R}_e^\tau$, the equilibrium loan quantity is either $(1-\chi)\underline{\phi}_d^\tau \bar{D}_\tau$ or $\hat{L}^*$. The former is smaller than $\bar{L}_\tau^*$ becasue it is increasing in R_e and the latter is smaller because of banks' market power. Then we have established that the maximum loan quantity is achieved at $R_e = \bar{R}_e^\tau$.

We are now ready to prove this claim. By the previous discussion, $\bar{D}_{\tau_2}^*$ satisfies

$$\frac{1/\xi - \phi_e}{1-\tau_2} = \alpha\beta\lambda(\bar{D}_{\tau_2}^*) + \alpha\beta\lambda'(\bar{D}_{\tau_2}^*)\bar{D}_{\tau_2}^*/N, \quad (23)$$

$$\frac{\phi_e - \beta}{\tau_2} = \alpha\beta\lambda(\bar{D}_{\tau_2}^*), \quad (24)$$

$$R_l = f'\left((1-\chi)\hat{\mathbf{\Phi}}_d(\bar{D}_{\tau_2}^*)\bar{D}_{\tau_2}^*\right), \quad (25)$$

for some ϕ_e, where ξ depends on R_l. If $\tau = \tau_1$, we can find ϕ'_e such that

$$\frac{\phi'_e - \beta}{\tau_1} = \alpha\beta\lambda(\bar{D}^*_{\tau_2})$$

We now show under ϕ'_e, $D^*_{\tau_1} \geq \bar{D}^*_{\tau_2}$. If $D^*_{\tau_1} = \bar{D}^*_{\tau_2}$, then R_l and ξ are unchanged. Because $D^*_{\tau_1}$ satisfies (23) and (24) with τ_2 replaced by τ_1 and ϕ_e replaced by ϕ'_e,

$$\begin{aligned}\frac{1/\xi - \phi'_e}{1-\tau_1} &= \frac{1/\xi - \alpha\beta\lambda(\bar{D}^*_{\tau_2}) - \beta}{1-\tau_1} + \alpha\beta\lambda(\bar{D}^*_{\tau_2}) \\ &< \frac{1/\xi - \alpha\beta\lambda(\bar{D}^*_{\tau_2}) - \beta}{1-\tau_2} + \alpha\beta\lambda(\bar{D}^*_{\tau_2}) = \frac{1/\xi - \phi_2}{1-\tau_2}.\end{aligned}$$

The inequality holds because $\tau_1 > \tau_2$ and by (23) and (24),

$$1/\xi - \alpha\beta\lambda(\bar{D}^*_{\tau_2}) - \beta = (1-\tau_2)\alpha\beta\lambda'(\bar{D}^*_{\tau_2})\bar{D}^*_{\tau_2}/N < 0.$$

This implies that if $R_l = f'\left((1-\chi)\hat{\mathbf{\Phi}}_d(\bar{D}^*_{\tau_2})\bar{D}^*_{\tau_2}\right)$

$$\frac{1/\xi - \phi'_e}{1-\tau_1} < \alpha\beta\lambda(\bar{D}^*_{\tau_2}) + \alpha\beta\lambda'(\bar{D}^*_{\tau_2})\bar{D}^*_{\tau_2}/N.$$

According to our equilibrium condition, we either have $D^*_{\tau_1} = \bar{D}^*_{\tau_2}$, which occurs if

$$1/\xi - \beta > \alpha\beta\lambda(\bar{D}^*_{\tau_2}) + \alpha\beta\lambda'(\bar{D}^*_{\tau_2})\bar{D}^*_{\tau_2}/N,$$

or $D^*_{\tau_1} > \bar{D}^*_{\tau_2}$. Moreover, if we reduce ϕ_e slightly from ϕ'_e, one can use a similar argument as the above and exploit continuity to show that $D^*_{\tau_1} \geq \bar{D}_{\tau_1} > \bar{D}^*_{\tau_2}$. Therefore, $L^*_{\tau_1} > \bar{L}^*_{\tau_2}$ under this ϕ_e, which establishes the second part of this proposition. ■

Proof of Proposition 5. We only prove existence and uniqueness of the equilibrium. Other parts can be proved following the same strategy for Propositions 3 and 4.

First, we solve the bank's problem (15). Eliminate δ_j from the objective function using the budget constraint, i.e., the first constraint. Then combine the budget constraint with the constraint $\delta_j \leq \Pi$ to obtain

$$\begin{aligned}\Omega(\Pi) =& \Pi + \max_{b_j,d_j,l_j,r_j} \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \beta b_j - l_j - r_j + \beta\Omega(\Pi')\\ &\text{st } l_j + r_j - \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j - \beta b_j \geq 0\\ &\quad \Pi' = \mathbf{R}_l(L_{-j} + l_j)l_j + R_r r_j - d_j\\ &\quad r_j > \chi\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j,\ b_j \geq 0.\end{aligned}$$

This shows that $\Omega'(\Pi) = 1$. Therefore, we can use the expression for Π' to obtain

$$\begin{aligned}\Omega(\Pi) = &\max_{b_j,d_j,l_j,r_j} \{\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \beta b_j - l_j - r_j + \beta\left[\mathbf{R}_l(L_{-j} + l_j)l_j + R_r r_j - d_j - b_j\right]\}\\ &+ \Pi + \beta\Omega(0)\\ \text{st } &l_j + r_j - \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j - \beta b_j \geq 0\\ &r_j > \chi\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j,\ b_j \geq 0.\end{aligned}$$

Assumptions 1 and 2 imply that the objective function is concave on the relevant range and the constraints define a convex restriction set. Therefore, the optimizer is unique and the FOCs are sufficient for optimality. Let γ_1, γ_2 and γ_3 be the Lagrangian multipliers on the first, second and third constraint of the above maximization problem. Then we can write the Lagrangian as

$$\begin{aligned}\max_{b_j,d_j,l_j,r_j} &\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \beta b_j - l_j - r_j + \beta\left[\mathbf{R}_l(L_{-j} + l_j)l_j + R_r r_j - d_j - b_j\right]\\ &+ \gamma_1\left[l_j + r_j - \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j - \beta b_j\right] + \gamma_2\left[r_j - \chi\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j\right] + \gamma_3 b_j\end{aligned}$$

where $\gamma_1, \gamma_2, \gamma_3 \geq 0$ are strictly positive if the constraints are binding. The FOCs are

$$d_j : \left[\nabla^{-}\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)\right](1 - \gamma_1 - \gamma_2\chi) - \beta \geq 0 \tag{26}$$

$$d_j : \left[\nabla^{+}\mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)d_j + \mathbf{\Phi}_{d,\tau}(D_{-j} + d_j)\right](1 - \gamma_1 - \gamma_2\chi) - \beta \leq 0 \tag{27}$$

$$l_j : 1 - \gamma_1 - \beta\left[\mathbf{R}_l(L_{-j} + l_j) + \mathbf{R}_l'(L_{-j} + l_j)l_j\right] = 0 \tag{28}$$

$$r_j : 1 - \gamma_1 - \gamma_2 = \beta R_r \tag{29}$$

$$b_j : \gamma_1\beta - \gamma_3 = 0 \tag{30}$$

In the symmetric equilibrium, d_j, l_j, r_j and b_j are the same across banks. Therefore, the Lagrangian multipliers are also the same across banks.

To obtain the equilibrium, define $R_I = (1-\gamma_1)/\beta$. Imposing symmetry, we can rewrite (26) to (28) as (16)-(18) in the main text. Under Assumptions 1 and 2 there is a unique (D, L) that solve (16)-(18) given R_I.

If the reserve requirement is not binding, $\gamma_2 = 0$ and (29) implies $R_I = R_r$. Because $R_r < 1/\beta$, $\gamma_1 > 0$ and $\gamma_3 > 0$. Therefore, $b_j = 0$ and $\delta_j = \Pi$ for all j, which implies $(1-\chi)\mathbf{\Phi}_{d,\tau}(D)D > L$.

If the reserve requirement is binding, $\gamma_2 > 0$, then $R_I > R_r$ by (29). There are two cases.

If $R_I < 1/\beta$, $\gamma_1 > 0$ and (30) implies $\gamma_3 > 0$. Therefore, $\Pi = \delta_j$ and $b_j = 0$ for all j, which implies $(1-\chi)\mathbf{\Phi}_{d,\tau}(D)D = L$.

If $R_I = 1/\beta$, $\gamma_1 = \gamma_3 = 0$. Banks are willing to retain profit or issue term deposits to finance loans. Without loss of generality, we assume they do not retain profit for lending but rely on term deposits. Then $L \geq (1-\chi)\mathbf{\Phi}_{d,\tau}(D)D$.

This analysis leads to the loan supply function $\mathbf{L}^s_\tau(R_I)$ described in the main text. It is increasing in R_I. And the loan demand function $\mathbf{L}^d(R_I)$ is determined by (18), which is downward sloping and positive for any R_I. These two curves have a unique intersection, which establishes the existence and uniqueness of the equilibrium. ■

References

[1] I. Agur, A. Ari, and G. Dell'Ariccia (2022) "Designing Central Bank Digital Currencies." *Journal of Monetary Economics* 125, 62-79.

[2] D. Andolfatto (2021) "Assessing the Impact of Central Bank Digital Currency on Private Banks." *The Economic Journal* 31(634), 525-540.

[3] O. C. Ashenfelter, H. Farber, and M. R Ransom (2010) "Labor Market Monopsony." *Journal of Labor Economics* 28(2), 203-210.

[4] J. Barrdear and M. Kumhof (2021) "The Macroeconomics of Central Bank Issued Digital Currencies." *Journal of Economic Dynamics and Control*, 104148.

[5] P. Benigno, L. Schilling, and H. Uhlig (2022) "Cryptocurrencies, Currency Competition and the Impossible Trinity." *Journal of International Economics* 136, 103601.

[6] A. Berentsen, G. Camera, and C. Waller (2007) "Money, Credit and Banking." *Journal of Economic Theory*, 135(1), 171-195.

[7] C. Boar and A. Wehrli (2021) "Ready, Steady, Go? Results of the Third BIS Survey on Central Bank Digital Currency."Bank for International Settlements Working Paper, NO. 114.

[8] J. Boyd and G. De Nicolo (2005) "The theory of bank risk taking and competition revisited." *The Journal of Finance* 60(3), 1329-1343.

[9] M. Brunnermeier and D. Niepelt (2019) "On the Equivalence between Private and Public Money." *Journal of Monetary Economics* 106(C), 27-41.

[10] K. Burdett and K. Judd (1983) "Equilibrium Price Dispersion." *Econometrica* 51(4), 955-969.

[11] K. Burdett and D. Mortensen (1998) "Wage Differentials, Employer Size, and Unemployment." *International Economic Review* 39(2), 257-273.

[12] J. Chapman and C. Wilkins (2019) "Crypto "Money": Perspective of a Couple of Canadian Central Bankers." Bank of Canada Staff Discussion Paper 2019-1.

[13] J. Chiu and T. Koeppl (2019) "The Economics of Cryptocurrencies – Bitcoin and Beyond." Bank of Canada Staff Working Paper 2019-40.

[14] J. Chiu, M. Davoodalhosseini, J. Jiang and Y. Zhu (2020) "Safe Payments." Bank of Canada Staff Working Paper 2020-53.

[15] J. Chiu, M. Davoodalhosseini, J. Jiang and Y. Zhu (2022) "Bank Market Power and Central Bank Digital Currency: Theory and Quantitative Assessment." Journal of Political Economy,forthcoming.

[16] M. Choi and G. Rocheteau (2021) "Money Mining and Price Dynamics." *American Economic Journal: Macroeconomics* 13(4), 246-94.

[17] Committee on Payments and Market Infrastructures (2018) "Central Bank Digital Currencies." Bank of International Settlements.

[18] D. Corbae and P. D'Erasmo (2020) "Capital Buffers in a Quantitative Model of Banking Industry Dynamics. " Mimeo.

[19] M. Davoodalhosseini (2022) "Central Bank Digital Currency and Monetary Policy," *Journal of Economic Dynamics and Control* 142, 104150.

[20] M. Davoodalhosseini and F. Rivadeneyra (2020) "A Policy Framework for E-Money." *Canadian Public Policy* 46(1), 94-106.

[21] M. Davoodalhosseini, F. Rivadeneyra, and Y. Zhu (2020) "CBDC and Monetary Policy." Bank of Canada Staff Analytical Note 2020-4.

[22] M. Dong, S. Huangfu, H. Sun, and C. Zhou (2021). "A Macroeconomic Theory of Banking Oligopoly." *European Economic Review* 138, 103864.

[23] M. Dong and S. Xiao (2021). "Central Bank Digital Currency: A Corporate Finance Perspective." Mimeo.

[24] I. Dreschler, A. Savov, and P. Schnabl (2017) "The Deposit Channel of Monetary Policy." *Quarterly Journal of Economics* 132, 1819-1976.

[25] W. Engert and B. Fung (2017) "Central Bank Digital Currency: Motivations and Implications." Bank of Canada Staff Discussion Paper 2017-16.

[26] J. Fernández-Villaverde and D. Sanches (2019) "Can Currency Competition Work?" *Journal of Monetary Economics* 106(C), 1-15.

[27] J. Fernández-Villaverde, D. Sanches, L. Schilling, and H. Uhlig (2021) "Central Bank Digital Currency: Central Banking for All?" *Review of Economic Dynamics* 41, 225-242.

[28] C.J. Flinn (2006) "Minimum Wage Effects on Labor Market Outcomes under Search, Matching, and Endogenous Contact Rates."*Econometrica* 74, 1013-1062.

[29] B. Fung and H. Halaburda (2016) "Central Bank Digital Currencies: A Framework for Assessing Why and How." Bank of Canada Staff Discussion Paper 2016-22.

[30] R. Garratt and M. van Oordt (2021) "Privacy as a Public Good: A Case for Electronic Cash."*Journal of Political Economy* 129(7), 2157-2180.

[31] C. Greene and J. Stavins (2018) "The 2016 and 2017 Surveys of Consumer Payment Choice: Summary Results." Federal Reserve Bank of Atlanta Discussion Paper No. 18-3.

[32] C. Gu, C. Monnet, E. Nosal, and R. Wright (2019) "On the Instability of Banking and Financial Intermediation." Working Papers 1901, Department of Economics, University of Missouri.

[33] A. Head, L. Q. Liu, G. Menzio and R. Wright (2012) "Sticky Prices: A New Monetarist Approach." *Journal of the European Economic Association*, 10: 939-973.

[34] B. Holmström, and J. Tirole (1998) "Private and public supply of liquidity." *Journal of political Economy*, 106.1: 1-40.

[35] J. Jiang and Y. Zhu (2021) "Monetary Policy Pass-Through with Central Bank Digital Currency." Bank of Canada Staff Working Paper.

[36] C. M. Kahn and M. van Oordt (2022) "The Demand for Programmable Payments. "Tinbergen Institute Discussion Paper, Vol. 2022-076.

[37] C. M. Kahn, M. van Oordt and Y. Zhu (2021) "Best before? Expiring central bank digital currency and loss recovery . "Bank of Canada Staff Working Paper , 2021-67.

[38] C. M. Kahn, F. Rivadeneyra, and R. Wong (2020) "Should the Central Bank Issue E-Money." *Journal of Financial Market Infrastructures*, 8(4), 1-22.

[39] T. Keister and C. Monnet (2022) "Central Bank Digital Currency: Stability and Information." *Journal of Economic Dynamics and Control* 142, 104501.

[40] T. Keister and D. Sanches (2022) "Should Central Banks Issue Digital Currency?" *Review of Economic Studies*, 89(2).

[41] M. Kumhof and C. Noone (2018) "Central Bank Digital Currencies—Design Principles and Balance Sheet Implications." Bank of England Staff Working Paper, No. 725.

[42] P. Kurlat (2019) "Deposit Spreads and the Welfare Cost of Inflation." *Journal of Monetary Economics* (106), 78-93.

[43] R. Lagos and R. Wright (2005) "A Unified Framework of Monetary Theory and Policy Analysis." *Journal of Political Economy* 113, 463-484.

[44] R. Lagos and S. Zhang (2019) "The Limits of *onetary* Economics: On Money as a Medium of Exchange in Near-Cashless Credit Economies." NBER Working Paper No. w25803.

[45] R. Lagos and S. Zhang (2022) "The Limits of *onetary* Economics: On Money as a Constraint on Market Power." *Econometrica* 90(3), 1177-1204.

[46] B. Lester, A. Postlewaite and R. Wright (2012) "Information, Liquidity, Asset Price and Monetary Policy." *Review of Economic Studies* 79, 1209-1238.

[47] K. Liu (2019) "The Impact of the Dodd-Frank Act on Small U.S. Banks. " Mimeo.

[48] R.E. Lucas and J.P. Nicolini (2015) "On the stability of money demand. " *Journal of Monetary Economics* 73(C), 48-65.

[49] T. Mancini-Griffoli, M. S. Martinez, I. A. Peria, A. Ari, J. Kiff, A. Popescu, and C. Rochon (2018) "Casting Light on Central Bank Digital Currency." International Monetary Fund Staff Discussion Notes No. 18-08.

[50] J. Meaning, B. Dyson, J. Barker, and E. Clayton (2018) "Broadening Narrow Money: Monetary Policy with a Central Bank Digital Currency." Bank of England Staff Working Paper No. 724.

[51] C. Monnet, A. Petursdottir, and M. Rojas-Breu (2020). "Central Bank Account for All: Efficiency and Stability." Mimeo.

[52] D. Niepelt (2020) "Monetary Policy with Reserves and CBDC: Optimality, Equivalence, and Politics." Available at SSRN: https://ssrn.com/abstract=3740324.

[53] Payment Canada (2020) "COVID-19 Pandemic Dramatically Shifts Canadians' Spending Habits."Available at https://www.payments.ca/about-us/news/covid-19-pandemic-dramatically-shifts-canadians%E2%80%99-spending-habits.

[54] J. Premo (2018) "Guide to the 2016 Diary of Consumer Payment Choice."Federal Reserve Bank of Boston: Consumer Payments Research Center.

[55] G. Rocheteau, R. Wright and C. Zhang (2018) "Corporate Finance and Monetary Policy."*American Economic Review* 108, 1147-1186.

[56] F. Schar and A. Berentsen (2020) "Bitcoin, Blockchain, and Cryptoassets: A Comprehensive Introduction." MIT Press.

[57] D. Scharfstein and A. Sunderam (2016) "Market Power in Mortgage Lending and the Transmission of Monetary Policy." Unpublished working paper. Harvard University.

[58] L. Schilling, J. Fernández-Villaverde, and H. Uhlig (2020) "Central Bank Digital Currency: When Price and Bank Stability Collide." Available at SSRN: https://ssrn.com/abstract=3606226.

[59] L. Schilling and H. Uhlig (2019) "Some Simple Bitcoin Economics." *Journal of Monetary Economics* 106(C), 16-26.

[60] Y. Wang, T. Whited, Y. Wu, and K. Xiao (2022) "Bank Market Power and Monetary Policy Transmission: Evidence from a Structural Estimation." *The Journal of Finance* 77(4), 2093-2141.

[61] Z. Wang (2020) "Tax Compliance, Payment Choice, and Central Bank Digital Currency." Available at SSRN: https://ssrn.com/abstract=3755573.

[62] S. Williamson (2022) "Central Bank Digital Currency: Welfare and Policy Implications. " *Journal of Political Economy* 130(11), 2829-2861.

[63] S. Williamson (2022) "Central Bank Digital Currency and Flight to Safety. " *Journal of Economic Dynamics and Control* 142, 104146.

[64] S. Zhou (2020) "Anonymity, Secondary Demand, and the Velocity of Cryptocurrency." Mimeo.

[65] Y. Zhu and S. Hendry (2019) "A Framework for Analyzing Monetary Policy in an Economy with E-Money." Bank of Canada Staff Working Paper. 2019-1.