Large shocks travel fast*

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Abstract

We document a sizeable increase in the frequency of price adjustments following the large energy shocks of 2022. We use a tractable New Keynesian model, calibrated to the pre-shock data, to interpret such a pattern. The calibration highlights the state-dependence of firms' decisions: prices are adjusted rapidly when markups are misaligned. In the model, a large cost shock triggers a swift increase in the frequency of price adjustments, causing a rapid pass-through from costs to prices. Time-dependent models, as the Calvo model, miss this frequency response, failing to capture the sudden inflation surge after a large shock.

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1 Introduction

The frequency of price adjustment determines the speed with which shocks transmit through the economy in New Keynesian models. For example, Gopinath and Itskhoki (2010) document that a higher frequency of price changes is associated with a higher pass-through of exchange rate shocks, attributing the cross-sectional differences to primitives affecting the curvature of the profit function (see also Devereux and Yetman (2010)). Many empirical studies use micro data to document a large cross-sectional heterogeneity in the frequency of price adjustments, but there have been few attempts to study what causes changes in this frequency over time.¹ In this paper we focus on the *size* of the aggregate shock as a key determinant of the frequency of price changes. We document that large shocks increase the frequency of price adjustments, leading to a faster pass-through from costs to prices.

Our main finding is that large cost shocks induce firms to react faster than in normal times. For instance, after years of overall stationary behavior, the frequency of price changes increased in 2022 and 2023, more than doubling across all sectors and countries, following a two-digit increase of the energy costs for these firms. This is pictured in Figure 1, which uses data on the frequency of price changes in the food and beverages sectors of several countries, and in the manufacturing and services sector of France. We explain the increase of the repricing activity with a simple model where a firm is more likely to reprice if its markup is further away from its desired level. Since a large cost shock reduces the profit margin markedly, many firms react by changing prices. This simple mechanism, at the core of State-Dependent (SD) models, is absent in the Time-Dependent (TD) models that are commonly used by Central Banks and popular in the New Keynesian literature, such as Taylor (1980) and Calvo (1983).² Thus, while TD and SD models may behave similarly in the presence of small aggregate shocks, as argued by Gertler and Leahy (2008), Alvarez,

¹Cavallo (2019) found that online competition led to an increase in the frequency of price changes for US multi-channel retailers between 2008 and 2017. See Gautier et al. (2022) for recent extensive evidence on the frequency of price changes across the industries of European countries.

²The extensive margin is also emphasized by Gagnon et al. (2012). Relatedly, Hall (2023) argues that larger idiosyncratic shocks will make prices more flexible.

Lippi, and Passadore (2016), and Auclert, Rigato, Rognlie, and Straub (2023), they differ significantly in the presence of large shocks.³ This difference carries policy implications because prices increase much faster in a SD than in a TD model after a large shock.⁴

We base the analysis on a rich granular dataset on price setting behavior in the food and beverages industry of several countries. We interpret the data using a generalized statedependent model, following the seminal work of Caballero and Engel (1993, 1999). We develop a new tractable model of state-dependent pricing that can be solved analytically. This model nests a broad class of SD models, such as Golosov and Lucas (2007) or Nakamura and Steinsson (2010), as well as TD models, such as the well-known Calvo (1983) model. The key features of the model are determined by three "deep parameters". We prove that these parameters are identified by three cross-sectional moments of price setting behavior: the frequency of price changes, the standard deviation and the kurtosis of the size of price changes. In particular, we show analytically that the state-dependence of the firm's pricing policy is identified by the kurtosis of the price changes. We measure these moments before 2022, a period over which aggregate shocks were arguably small, and use them to calibrate the model. The calibration reveals a sizeable amount of state dependence in firms' behavior: the probability that a price is adjusted increases as the gap between the actual and the desired price grows. This result confirms the seminal findings of Eichenbaum, Jaimovich, and Rebelo (2011) and the recent ones by Gautier et al. (2022); Karadi et al. (2023); Dedola et al. (2023), highlighting the empirical relevance of state-dependent pricing.⁵

Our analysis has a simple structure. We calibrate the model using data on price setting behavior observed before the energy cost shocks of 2022. We interpret this calibration as a representation of the firms' behavior in a "steady state", i.e. a situation where aggregate

 $^{^{3}}$ The size of the shock is measured relative to the standard deviation of price changes. Small cost shocks, such as those considered in VAR analyses, change marginal costs by less than 1%, or less than 1/10 of the standard deviation of price changes.

⁴Ascari and Haber (2021) document the nonlinear responses of aggregate prices and output to large monetary policy shocks using US data from 1969 to 2007.

⁵Gagnon (2009) and Nakamura et al. (2018) document that the frequency of price adjustments increases when inflation is high, a feature captured by state-dependent models.

shocks are small. We then use the policy function of the calibrated model to analyze how the economy responds to a large cost shock. The mechanism explaining the burst of repricing activity in our analysis is that a large shock shrinks the firm's profit margin, creating an incentive to change prices. This exercise is developed using the firm's steady state decision rule. Considering the general equilibrium feedback on the firm's decisions has only minor consequences for our findings (see Appendix D).

Other papers have shown rapid firm adjustments to large shocks: Bonadio, Fischer, and Sauré (2019) and Auer, Burstein, and Lein (2021) offer empirical evidence of quick currency invoicing changes after the large 2015 Swiss appreciation. Karadi and Reiff (2019) analyze Hungary's substantial VAT increase in 2004 and 2006 using a menu cost model, showing a prompt aggregate price level response. Our work is closely related to Alvarez and Neumeyer (2020), who examine inflation in Argentina following major utility cost increases in 2014 and 2016, and a sharp exchange rate devaluation in 2018. They find that such shocks lead to a behavior akin to an economy with flexible prices, using a canonical menu cost model with high-inflation. Our study differs in the data (we focus on low-inflation countries) and in the modeling. Concerning the latter, they consider a Golosov-Lucas model, which has the highest possible degree of state dependence. Our formulation lets the data speak about the exact degree of state dependence, a key feature to determine the response of the economy to a large shock.

The paper is organized as follows. The next section outlines the model. Section 3 presents the calibration of the model. Section 4 uses the model to analyze the propagation of a large energy shock, discussing the asymmetric response to cost increases vs cost decreases. Section 5 concludes.

2 A tractable sticky-price model

This section presents a New Keynesian model inspired by the seminal ideas of Caballero and Engel (1993, 1999). We present a new tractable model based on a simple microfoundation of the costly price adjustment: each firm controls the *probability* to change its price. At each moment the firm pays a cost to affect this probability (this is similar to Woodford (2009); Alvarez et al. (2022)). The model predicts that the probability of adjusting the price increases in the "price gap", the distance between the firm's current price and the profitmaximizing price. The elasticity of this probability to the price gap identifies the degree of "state dependence" of the model. We will discuss how to identify this important parameter in Section 3.

Households. The household preferences are given by

$$\int_0^\infty e^{-\rho t} U\left(C(t), H(t), M(t)\right) dt \quad \text{where} \quad U(C, H, M) \equiv \frac{C^{1-\epsilon}}{1-\epsilon} - \alpha H + \log\left(\frac{M}{P}\right), \quad (1)$$

where ρ is the discount rate, ϵ the relative risk aversion, α the disutility of labor, C(t) is a consumption composite, H(t) the labor supply and M(t)/P(t) real money balances at time t. Let $c_i(t)$ denote the consumption of good i. The composite C(t) is

$$C(t) \equiv \left[\int_{0}^{1} \left(A_{i}(t)c_{i}(t) \right)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$$
(2)

where $A_i(t)$ denote preference shocks and η is a parameter determining the price elasticity of demand. Labor supply equals the aggregate labor hired by firms, $H(t) = \int_0^1 h_i(t) di$. Households maximize utility subject to the budget constraint, choosing consumption for each good *i*, labor, and real balances for each *t* (see Appendix A). **Firms.** The economy is populated by a unit mass of firms, indexed by $i \in [0, 1]$. Each firm produces one good, $c_i(t)$, using labor (h_i) and energy (m_i) as inputs, according to

$$c_i(t) = \left(\frac{h_i(t)}{Z_i(t)}\right)^{1-\zeta} m_i(t)^{\zeta} \tag{3}$$

where $\zeta \in (0,1)$ is the energy share in production. The marginal cost of production of firm *i* is: $\mathrm{mc}_i(t) = KE(t)^{\zeta} (W(t)Z_i(t))^{1-\zeta}$ where W(t) is the nominal wage, E(t) is the price of the energy input, $Z_i(t)$ is a firm-specific productivity level, and $K \equiv \zeta^{-\zeta}(1-\zeta)^{\zeta-1}$ is a constant. We assume that $Z_i(t) = \exp(\sigma z_i(t))$, where $\{z_i\}$ are standard Brownian motions, independent across *i*, with standard deviation parameter σ . We assume that $A_i(t) = Z_i(t)^{1-\zeta}$ so the (log of) marginal cost and the preference shock are perfectly correlated.⁶

The price that maximizes current profits is given by the marginal cost times a constant markup:

$$p_i^*(t) = \frac{\eta}{\eta - 1} \mathrm{mc}_i(t) \; .$$

If prices are sticky, the state of firm *i* is summarized by its price gap $x_i(t)$, namely the wedge between the actual price $p_i(t)$ and the profit-maximizing price $p_i^*(t)$:

$$x_i(t) \equiv \log p_i(t) - \log p_i^*(t) . \tag{4}$$

Absent pricing frictions the gap is identically zero, i.e. each firm charges the profit-maximizing price $p_i(t) = p_i^*(t)$. If the price is not adjusted, the price gap changes due to trend inflation (increasing nominal costs) and idiosyncratic productivity shocks, as well as energy cost shocks. As long as the nominal price is kept constant the evolution of the price gap obeys

$$dx_i(t) = -\mu \, dt + \sigma \, dz_i(t),\tag{5}$$

⁶This assumption, also used in Woodford (2009); Bonomo et al. (2010); Midrigan (2011); Alvarez and Lippi (2022), allows the problem to be described by a scalar stationary state variable, the price gap x. This is used to write the dynamic programming problem of the firm as well as to keep the expenditure shares stationary across goods in the presence of permanent idiosyncratic shocks.

where z_i is a standard Brownian motion describing productivity shocks and μ is the growth rate of nominal wages per unit of time, i.e. the inflation rate.⁷

The price-setting problem. The firm's value function v(x) solves the following problem:

$$\rho v(x) = F(x) - \mu v'(x) + \frac{\sigma^2}{2} v''(x) + \min_{\ell \ge 0} \{\ell \cdot (v(x^*) - v(x)) + (\kappa \ell)^{\gamma}\}$$
(6)

where the flow cost function $F(x) = 1 - \eta \left(e^x - \frac{\eta - 1}{\eta}\right) e^{-\eta x}$ represents forgone profits measured as a fraction of maximum (static) profits (see Appendix A). The variable x^* is the optimal price gap that is chosen upon adjustment, satisfying $v'(x^*) = 0$. In every moment the firm chooses an effort rate ℓ for price resetting, so that with probability ℓ per unit of time the price gap is reset to x^* . The optimal choice for ℓ balances the benefit of a price reset, given by $v(x^*) - v(x)$, with the cost of effort, given by the convex power function $(\kappa \ell)^{\gamma}$, with $\gamma > 1$ and $\kappa > 0$.

We note that the flow cost F(x) is asymmetric around x = 0, because of the asymmetry of the profit function. Such a feature arises in several models since having a "high price" is preferable to having a "low price" (in the latter case the firm sells many units with a low, possibly negative, margin). The asymmetry implies that a firm's incentive to adjust prices is larger when its price is "low", a negative x, than when it is "high". We will use this feature to discuss the firm's asymmetric response to cost increases versus cost decreases.

The price-setting behavior implied by equation (6) is summarized by a generalized hazard function, $\Lambda : \mathbb{R} \to \mathbb{R}_+$, as postulated in Caballero and Engel (1993). The function gives the probability (per unit of time) that a firm with price gap x will change its price. The first order condition for ℓ gives the generalized hazard function $\Lambda(x) = \ell^*$:

$$\Lambda(x) = \frac{1}{\kappa} \left(\frac{v(x) - v(x^*)}{\kappa \gamma} \right)^{\frac{1}{\gamma - 1}}, \quad \text{for each } x \in (-\infty, \infty) \quad .$$
(7)

⁷This is rationalized by assuming the money supply grows at an exogenous rate μ .

This policy rule implies that price changes occur probabilistically, with an intensity that depends on the benefits of adjustment, $v(x) - v(x^*)$, and on the parameters of the cost function, $\{\gamma, \kappa\}$. Hence the probability of a price adjustment depends on x, the magnitude of the firm's desired adjustment.

The value function $v(\cdot)$ and the generalized hazard function $\Lambda(\cdot)$ have a minimum at x^* and are increasing in $|x - x^*|$. Intuitively, the hazard is increasing in the distance between xand the optimal reset gap x^* : a larger distance increases the incentives to adjust, as shown in the first panel of Figure 2 for three calibrations designed to fit Germany, the UK and the US (described below). A v-shaped hazard emerges for all calibrations and is the hallmark of state dependence (as opposed to a flat one).⁸ This finding complements previous evidence on the relevance of state-dependent hazards, see e.g. Eichenbaum et al. (2011); Gautier et al. (2022); Karadi et al. (2023); Dedola et al. (2023).

Building on the generalized hazard function models analyzed in Alvarez, Lippi, and Oskolkov (2022), our model introduces a specific analytical form for the effort cost, $(\kappa \ell)^{\gamma}$, allowing for analytical tractability. A large number of models are nested by this framework, including the canonical Calvo model with a constant hazard $\Lambda(x) = 1/\kappa$ as $\gamma \uparrow \infty$, and the canonical Ss model as $\gamma \downarrow 1$, where the hazard is flat at zero for $|x| < \bar{x}$ and diverges for $|x| > \bar{x}$.⁹ Intermediate cases cover the so called Calvo-Plus model by Nakamura and Steinsson (2010), and the random menu cost problem of Dotsey and Wolman (2020). Hence, the parameter γ is critical to determine the *shape* of the hazard function, i.e. to determine the degree of state dependence. A small γ implies a steep v-shaped profile, revealing a high state-dependence of pricing decisions. As γ increases state-dependence is dampened, and eventually vanishes (flat hazard) as $\gamma \uparrow \infty$.¹⁰

We note that the hazard function displays a higher slope to the left because, as discussed

⁸It also appears in models with "costly control" of the timing and magnitude of adjustments, see Costain and Nakov (2019); Costain et al. (2022).

⁹As $\gamma \downarrow 1$, equation (7) shows that \bar{x} is defined by the value matching condition $v(\bar{x}) = v(x^*) + \kappa$.

¹⁰Some analysis of equation (7) shows the hazard function satisfies $\Lambda'(x^*) = 0$ for $\gamma \in (1,3)$, so the function is U shaped. Instead, for $\gamma > 3$, the hazard function has an Υ shape, namely $\lim_{x \downarrow x^*} \Lambda'(x) \to \infty$, as is the case of Spain in Figure 2.

above, negative price gaps lead to larger forgone profits. This makes positive price changes more likely. Such a behavior has been documented empirically by Karadi, Schoenle, and Wursten (2021) and Luo and Villar (2021).¹¹

Aggregation. The hazard $\Lambda(x)$ and the law of motion of x determine the steady-state distribution of the firms' price gaps, with density function $m : \mathbb{R} \to \mathbb{R}_+$ solving the Kolmogorov forward equation:

$$\Lambda(x) \cdot m(x) = \mu m'(x) + \frac{\sigma^2}{2} m''(x), \quad \text{for each } x \neq x^*.$$
(8)

with boundary conditions $\lim_{x\downarrow x^*} m(x) = \lim_{x\uparrow x^*} m(x), 1 = \int_{-\infty}^{\infty} m(x) dx$, and $\lim_{x\to\pm\infty} m(x) = 0$. The steady-state density function m is pictured in the second panel of Figure 2.

3 Mapping the model to the "steady state" data

In this section, we derive the model's prediction for the frequency and size distribution of price changes in the absence of aggregate shocks (the "steady state"). We calibrate the model using pre-2022 data on price setting behavior in the food and beverages industries of several countries. The model captures key patterns about the frequency, scale and shape of the distribution of price changes observed in the data.

The frequency of price adjustments N is given by

$$N = \int_{-\infty}^{\infty} \Lambda(x)m(x) \, dx \tag{9}$$

The equation counts the total number of price adjustments per period by weighting the rate at which firms adjust at each x, $\Lambda(x)$, with the density of firms m(x).

Recall that upon any price change the firm resets its gap from x to the optimally chosen

¹¹The asymmetric flow cost also affects the optimal reset price-gap x^* . The first panel of Figure 2 shows that $x^* > 0$, to hedge against negative gaps. Fernandez-Villaverde et al. (2015) discuss the asymmetry of the profit function and its business cycle implications.

return point x^* , hence the size of the adjustment is $\Delta x = x^* - x$. This occurs with probability $\Lambda(x)$ per unit of time for a density m(x) of firms with price gap x. Thus, the distribution of the size of price changes has the following density:

$$q(\Delta x) \equiv \frac{\Lambda(x)m(x)}{N}.$$
(10)

From the distribution q, many cross-sectional moments can be computed as the variance and kurtosis of the size of price changes, $Var(\Delta x)$ and $Kurt(\Delta x)$. The next proposition establishes properties that are useful for the identification of the model's fundamental parameters, and for simple comparisons across economies. See Appendix B for the proof.

PROPOSITION 1. (*Identification and Scaling*) Consider an economy with zero inflation, fundamental parameters $\{\eta, \rho, \sigma^2, \kappa, \gamma\}$, and $\{v(x), \Lambda(x), m(x), N, q(\Delta x)\}$ solving equations (6)-(10) above. The following holds:

$$N \cdot \operatorname{Var}(\Delta x) = \sigma^2 . \tag{11}$$

Consider the second-order approximation of the flow cost $F(x) \approx Bx^2$, with $B \equiv \frac{\eta(\eta-1)}{2}$, and another economy denoted by tildes such that for two parameters a > 0 and s > 0, the observables satisfy

$$\widetilde{\operatorname{Var}}(\Delta x) = a \operatorname{Var}(\Delta x), \qquad \widetilde{N} = \frac{s}{a} N_{s}$$

then $\tilde{\sigma}^2 = s \, \sigma^2$. Additionally, if $\tilde{\gamma} = \gamma$ then $\tilde{v}(x) = v\left(\frac{x}{\sqrt{a}}\right) \frac{a^2}{s} \left(\frac{\tilde{B}}{B}\right)$, $\tilde{m}(x) = m\left(\frac{x}{\sqrt{a}}\right) \frac{1}{\sqrt{a}}$ and

$$\tilde{\Lambda}(x) = \frac{s}{a} \Lambda\left(\frac{x}{\sqrt{a}}\right) \quad , \quad \tilde{q}(x) = q\left(\frac{x}{\sqrt{a}}\right) \frac{1}{\sqrt{a}} \quad , \tag{12}$$

provided that $\tilde{\rho} = \frac{s}{a} \rho$ and $\tilde{\kappa} = \kappa \frac{1}{s} a^{\frac{\gamma+1}{\gamma}} \left(\frac{\tilde{B}}{B}\right)^{\frac{1}{\gamma}}$.

The three key takeaways of the proposition are: a method to calibrate σ^2 , a one-toone map between the parameter γ and the kurtosis of price changes, and a scaling of the parameters ρ and κ for economies with different frequencies and variances of price changes. We describe each of these results next.

First, equation (11) offers a simple method to calibrate σ^2 from observables. It also reveals the fundamental trade-off in sticky price models: given the size of the idiosyncratic shocks σ^2 , the firm's pricing policy dictates the balance between the costs of adjusting prices (N) versus the costs of price misalignments (Var(Δx)).

Second, equation (12) indicates that if two economies have the same γ parameter then they will have the same size-distribution of price changes, q and \tilde{q} , up to a scale transformation. Recall that γ is key in determining the curvature of the hazard function, as discussed after equation (7). This finding implies that there is a direct relationship between the parameter γ and the kurtosis of price changes (a *shape* parameter), regardless of the frequency or variance of price changes in those economies.¹²

Third, the proposition shows how to re-calibrate the parameter $\tilde{\kappa}$ for the tilde economy accounting for variations in the frequency and variance of price changes, while holding the γ parameter constant. For instance, two economies that differ only in the frequency of price changes $\tilde{N} > N$ can be modeled by a s > 1 and a = 1, namely a smaller effort cost parameter $\tilde{\kappa} = \kappa \frac{1}{s}$.

We note that the proposition assumes no trend inflation and a second-order approximation of the flow costs. These assumptions provide an accurate approximation to our calibration, featuring a small inflation and the exact profit function.

The dataset. We base our analysis on granular data on price setting behavior provided by PriceStats and related to The Billion Prices Project (Cavallo and Rigobon, 2016). This granular data includes daily frequencies and size of price changes for numerous firms. The

$$\widetilde{\operatorname{Kurt}}(\Delta x) \equiv \frac{\int_{-\infty}^{\infty} \tilde{q}(x) x^4 dx}{\left(\widetilde{\operatorname{Var}}(\Delta x)\right)^2} = \frac{a^2 \int_{-\infty}^{\infty} q\left(\frac{x}{\sqrt{a}}\right) \frac{1}{\sqrt{a}} \left(\frac{x}{\sqrt{a}}\right)^4 dx}{a^2 \left(\operatorname{Var}(\Delta x)\right)^2} = \operatorname{Kurt}(\Delta x)$$

 $^{^{12}}$ To see this, notice

data, gathered via web scraping, comprises daily, product-level details from various retailers' websites, including product id, price, category, and sale status. The data is detailed, covering the entire lifespan of all products sold by these retailers, and provides prices that are similar to those obtained in offline stores (Cavallo, 2017). The data is comparable across countries, collected using identical techniques for similar categories of goods over the same time period. This data set is also known to reduce the measurement error present in other sources, for example, due to time aggregation using the average revenue (see Cavallo (2018)). We use a subset of data from several European countries and the US, from January 1st 2019 to July 22nd 2023. We apply a v-shape algorithm to filter sales and focus on regular prices, as is standard in the literature (see Appendix C for more details on the data-cleaning process).

We focus on the "Food and Beverages" category which has experienced one of the highest rates of inflation during this period and has the largest weight in the goods CPI basket for most countries. We chose this sector because of the high quality and the comparability of the data across countries. Other sectors, such as industry and services, record a similar increase in the frequency of price revisions, as depicted in the bottom panel of Figure 1.¹³ We emphasize that the higher steady-state frequency of price changes in the "Food and Beverages" category, as compared to other sectors, does not tilt our results towards estimating larger degree of state dependence (state dependence is related to the parameter γ , which is independent of the frequency of price changes, see Proposition 1).

In Table 1 we present summary statistics of price setting behavior for several countries. The standard deviation of price changes varies from 15% to 31% across countries. This highlights the significant role of idiosyncratic productivity shocks. The kurtosis measure is very similar across countries and ranges between 2 and 3.

Accurately estimating kurtosis is crucial because it informs us about the curvature of the hazard function Λ . Kurtosis is a scale-free statistic that can be largely affected by the mixing of distributions with different scales. For instance mixing normal distributions with

¹³Appendix E contains similar evidence for Spain and Italy based on CPI data. See Montag and Villar (2023) for evidence from the US CPI data.

different variances can yield a kurtosis much higher than 3 in spite of the fact that each of the underlying distributions is normal. To measure kurtosis in Table 1 we follow Alvarez, Lippi, and Oskolkov (2022) in accounting for unobserved heterogeneity. In particular, we assume (i) that each product *i* within a category has a product-specific scale b_i , such that $\Delta x_i = b_i \Delta x$, where Δx is a random variable common across products and (ii) that price changes Δx_t , Δx_s in different dates $t \neq s$ are independent. The kurtosis of Δx can be computed from the distributions { Δx_i } accounting for unobserved heterogeneity using

$$\operatorname{Kurt}(\Delta x) = \frac{\mathbb{E}[(\Delta x_{it})^4]}{\mathbb{E}[(\Delta x_{it})^2 (\Delta x_{is})^2]},\tag{13}$$

where \mathbb{E} indicates expectation across products and time of price changes. The empirical counterparts are estimated with

$$\widehat{\mathbb{E}}[(\Delta x_{it})^4] = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{T(i)} \sum_{t=1}^{T(i)} (\Delta x_{it})^4,$$
$$\widehat{\mathbb{E}}[(\Delta x_{it})^2 (\Delta x_{is})^2] = \frac{1}{I} \sum_{i=1}^{I} \frac{1}{T(i)(T(i)-1)} \sum_{t=1}^{T(i)} \sum_{s=1, s \neq t}^{T(i)} (\Delta x_{it})^2 (\Delta x_{is})^2,$$

where I is the number of products and T(i) is the number of price changes for product i.

Calibration. For the food and beverages sectors of several countries we apply Proposition 1 to identify $\{\sigma, \kappa, \gamma\}$, using three cross-sectional moments, measured before 2022: the frequency of price changes, the standard deviation and kurtosis of the size of price changes. We use standard values for the other parameters: $\eta = 6$ for the price elasticity of demand (implying a markup of 20%), a time discount $\rho = 0.05$ and an inflation rate of $\mu = 2\%$ consistent with inflation at steady state. The calibrated parameters are shown in the last three columns of Table 1. We find a sizeable degree of state dependence, represented by (a small value of) the parameter γ , identified by a relatively small kurtosis of price changes. This finding is robust across countries. The first panel of Figure 2 depicts this fact: the calibrated shape of the hazard function is clearly v-shaped in all countries, displaying a comparable curvature (the parameter γ) and somewhat differing levels needed to match the different mean frequency of price changes (the parameter κ).

The calibration produces the price change distributions $q(\Delta x)$ shown in Figure 2. These feature a small mass of tiny price changes, a greater mass of positive changes, and the distribution's bimodality (evident in most countries except the US).

4 The propagation of an aggregate cost shock

In Figure 1 we observed that the frequency of price adjustments rose quickly after a large energy shock. In this section we use the calibrated model to rationalize this observation. We consider an economy at steady state and study the dynamics that follow an aggregate shock to the firms' marginal cost through a permanent increase in energy prices. To be conservative, we consider a 20% shock to marginal costs. It is worth noting that the cost shocks recorded by food production are even larger of the magnitude we are considering. For example, in the United States, the Department of Agriculture estimated that in 2022, the operating costs per acre for producing eleven farm crops and meats surged by 36%, with fuel, lubricants, and electricity costs per acre registering an even more pronounced increase of over 43%. Similarly, in 27 European countries, Eurostat estimated that in 2022, the costs of inputs used in agricultural production increased by 31% on average, with energy and lubricant costs surging by 53%.¹⁴

We illustrate the propagation of large and small shocks using the model calibration for the average of the food and beverages sectors described in Table 1. We hit the steady state economy with an unexpected once-and-for-all shock to energy prices that increases the firms' marginal cost. This displaces the distribution of price gaps by $\delta = 20\%$ percentage points

¹⁴The www.ers.usda.gov/data-products/ the US data for can be found atcommodity-costs-and-returns/. The products include hogs, cow-calf, milk, cotton, sovbean, peanut, sorghum, oats, corn, barley, and wheat. The data for Europe can be found at https://doi.org/10.2908/APRI_PI15_INA

"to the left", as shown by the dashed distribution in the first panel of Figure 3.¹⁵ The firms would then like to increase their prices to bring the markup back to the optimal level. This incentive shapes the dynamic response of the price level and the frequency.

For simplicity we develop the analysis assuming that firms use the steady state decision rule, $\Lambda(x)$.¹⁶ We let $\hat{m}(x,t) : \mathbb{R} \times [0,\infty) \to \mathbb{R}_+$ be the distribution of price gaps t periods after the shock. Intuitively, the distribution depends on the time elapsed because the initial shock places the distribution away from its steady state. The distribution $\hat{m}(x,t)$ solves a Kolmogorov forward equation with initial condition (see Alvarez and Lippi (2022))

$$\hat{m}(x,0) = m(x+\delta) \tag{14}$$

The shock $\delta > 0$ thus shifts the whole distribution m(x) to the left, as an increase in energy prices reduces the price gap of all firms.

Panel (a) of Figure 3 depicts the distribution of price gaps m, which peaks around the vertical line $x = x^*$, and the distribution of firms' price gaps after the shock $\hat{m}(x, 0)$. Recall that $x^* > 0$ and is approximately 10% due to the asymmetry of the profit function, discussed above. Trend inflation is small, at 2%, and it does not significantly affect the magnitude of x^* : a trend inflation of 2% leads to $x^* \approx 0$ when a symmetric flow cost, such as the approximation $F(x) \approx Bx^2$, is used. A large shock shifts the distribution m(x) to the left, so that most firms' prices are further away from their desired prices (at around -20% price gap). In this region the probability of adjustment, $\Lambda(x)$, is larger for most firms. This is the mechanism by which a large shock triggers an increase number of price adjustments. Intuitively, the costs of not adjusting the price are much larger after the shock.

Using $\hat{m}(x,t)$ we can describe the path for the aggregate price level and the frequency of

¹⁵A shock $e^{\delta/\zeta}$ to energy prices affects the marginal cost by e^{δ} , as implied by equation (3).

¹⁶This assumption is often used in the literature, see Baley and Blanco (2021). In Appendix D we solve the general equilibrium problem, with feedback from aggregates to the individual firm's decision, and show that results are similar.

price changes after an aggregate shock, as

$$P(t) \approx P(0) + \int_{-\infty}^{\infty} x \cdot \hat{m}(x,t) \, dx \, - \, (X_{ss} - \delta), \tag{15}$$

$$N(t) = \int_{-\infty}^{\infty} \Lambda(x) \cdot \hat{m}(x,t) \, dx \tag{16}$$

where $X_{ss} = \int_{\mathbb{R}} x \cdot m(x) dx$. The right hand side of equation (15) can be written more succinctly as $P(0) + X(t) - (X_{ss} - \delta)$, where X(t) denotes the mean price gap at time t.

Panel (c) of Figure 3 illustrates a marked rise in frequency following a large shock, of a magnitude that is akin to the one observed across all countries in our sample during 2022-2023, see Figure 1. As mentioned, the aggregate cost shock locates many firms in a region where the hazard rate is high, $\Lambda(x)$. This leads to a sustained increase in the frequency of price changes. Evidently this mechanism is not at work if the hazard function is flat, as in time-dependent models. These results are robust in our sample of countries because the calibrated hazard functions are v-shaped in all countries.

We stress that the marked response of the frequency does not depend on the relatively high frequency of price changes in the food and beverages industry. What is necessary for the result to obtain is that the hazard function is v-shaped. As shown by Proposition 1, the extensive margin response depends on the value of γ , hence on the kurtosis of price changes. It does not depend on the steady-state frequency of price changes, which is driven by κ . To see this formally note that the impact response of the frequency to the shock is $N_0 \equiv \int_{-\infty}^{\infty} \Lambda(x)m(x+\delta)dx$. Consider another economy differing only in the steady state frequency $\tilde{N} = sN$. A straightforward application of equation (12) yields

$$\frac{\tilde{N}_0}{\tilde{N}} = \frac{\int_{-\infty}^{\infty} \tilde{\Lambda}(x)\tilde{m}(x+\delta)dx}{\tilde{N}} = \frac{s\int_{-\infty}^{\infty} \Lambda(x)m(x+\delta)dx}{sN} = \frac{N_0}{N}$$

proving that the percentage increase of the frequency after the shock is identical in the two economies. To obtain a different response it is necessary to consider economies that differ in their degree of state dependence, i.e. with very different levels of kurtosis. **Inflation dynamics.** Panel (b) of Figure 3 shows that the propagation of a large shock features a faster pass-through than one predicted by a Calvo model matching the same frequency, and size, of price changes. This result mirrors the observation about frequency. Upon a large shock the calibrated model is characterized by more firms adjusting prices upwards resulting in faster passthrough. Inspecting the slope of the price path, the figure illustrates that failing to account for the large increase in the frequency of price revisions, as would occur in a time-dependent model, leads to a substantial initial underestimation of inflation followed by a subsequent overestimation. A feature much discussed in the recent inflationary episode.

We note that one feature not captured by our simple model is the half-life of the shock, which is faster than observed in the data. Enriching the model to have strategic complementarities in price setting would slow down the propagation of the shock, while retaining the state-dependence of the firms' decision and thus preserve a sizeable response along the extensive margin, as shown in Alvarez, Lippi, and Souganidis (2023). Such an extension comes at the cost of a more involved model, which we leave for future work.

In summary, the state-dependent model features a dynamic response to a large shock that resembles the data on the frequency of adjustments observed after the energy shocks of 2022. Furthermore the forecast of the frequency of price changes and the path of inflation vary significantly with the chosen model. The implications of a purely time-dependent model are counterfactual because the model misses the extensive margin response documented in Figure 1.

Starting with the last quarter of 2022, there has been a rapid decrease in energy prices, prompting the question of how swiftly firms will respond by adjusting their prices downward. Different features of the economic environment may induce an asymmetric response to positive vs negative cost shocks, such as a sizeable trend inflation or a variable markup.¹⁷ Indeed an asymmetric response to positive vs negative cost shocks is present in our model. Panel

 $^{^{17}}$ See Peltzman (2000) and Tappata (2009) for more evidence and interpretations of the "rockets and feathers" phenomenon, i.e., that prices rise faster than they fall.

(d) of Figure 3 depicts the asymmetric response of the frequency to a large positive versus a large negative shock (both of the same size). This behavior is a direct consequence of the asymmetric shape of the hazard function, $\Lambda(x)$, and in particular of the fact that the hazard has a higher slope when the firm markup is low ($x < x^*$) than when it is high (stemming from the asymmetry of the profit function discussed above). Unlike a positive shock (dotted line), a negative shock (thick line) leads to a less pronounced rise in the frequency of price changes due to lower probabilities of adjustments for high price gaps ($x > x^*$).¹⁸ Although the pass-through of a negative shock is relatively slower than its positive counterpart, it continues to be faster than the passthrough predicted by a purely time-dependent model. In short, our model with asymmetries shows that prices are more flexible upon a cost increase than upon a cost decrease.

In this section we examined the propagation of an aggregate costs shock of $\delta = 20\%$, in a model calibrated to the food and beverages data. We note that our findings are robust to the magnitude of the shock chosen. As depicted in Figure 4, shocks of 10% and 15% also lead to a significant increase in frequency, with factors of 1.5 and 2, respectively. It is apparent in Figure 4 that shocks larger than 5% introduce non-linearities, a result that aligns with those in Auclert et al. (2023).¹⁹

5 Conclusions

We use a tractable New Keynesian model, calibrated using granular data from the food and beverages industry of several countries, to study the propagation of a large supply shock. The model matches some key features of the data, including the (pre-shock) distribution of the size of price changes and the significant increase in the frequency of price changes following a large shock. The findings underscore the state-dependent nature of price setting decisions, where the probability of a price adjustment increases as the firm's price deviates from its

¹⁸Such an asymmetry cannot be caused by a high trend inflation in this model because the flow cost is convex in the size of price gaps x. Thus price gaps $x > x^*$ are even more costly.

¹⁹Blanco et al. (2024) find similar nonlinear responses in an SD model with pricing complementarities.

desired level.

The main implication of our research is that large shocks lead to a rapid pass-through from costs to prices causing a temporary surge in inflation, i.e., we show that large shocks travel fast. Understanding the different firm-level reactions to large and small shocks is critical to predict inflation dynamics following a large shock. Our findings highlight the limitations of time-dependent models, commonly used by central banks. By failing to account for the increased frequency of price adjustments, the time-dependent models are unable to capture the fast inflation run up after the shock, as well as the subsequent quick slowdown.

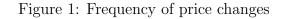
Future research should extend the analysis to more categories of goods, and explore the role of strategic complementarities in shaping the speed of pass-through of large shocks.

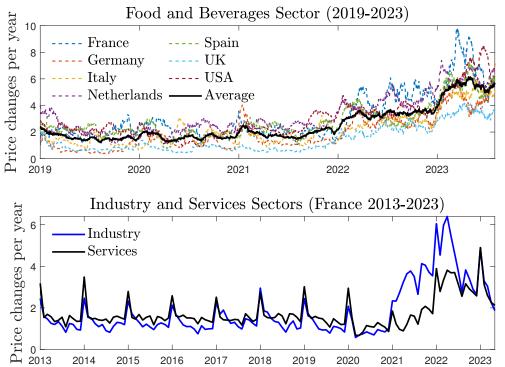
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Note: The top panel uses data from PriceStats (see Section 3). A simple average is plotted on a thick line. The data source for the bottom panel is the Banque de France Monthly Business Survey (see Dedola et al. (2023)); we map the original data (share of firms changing prices per month) into number of price changes per year.

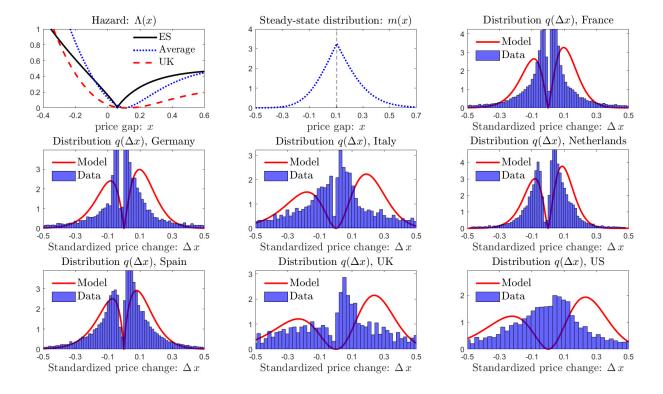
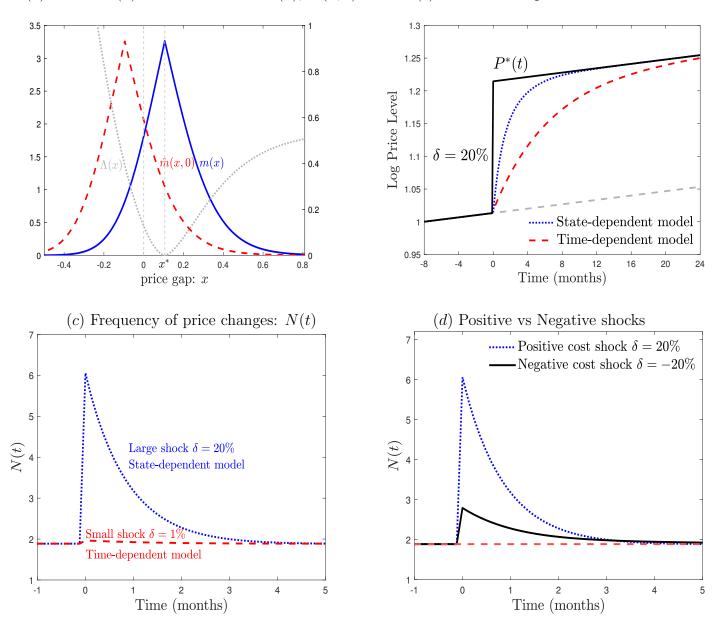


Figure 2: Calibrated hazard function $\Lambda(x)$, distribution m(x), and distribution $q(\Delta x)$

Note: Model calibration for the food and beverages sector of several countries. The first panel shows the calibrated hazard function (expressed as a monthly probability). The second panel shows the steady state distribution of price gaps using the average of the food and beverages sectors, see Table 1. The other panels show the distribution of the size of the price changes, $q(\Delta x)$, standardized at the COICOP level along with the calibrated model-implied distribution. See Table 1 for more information on the calibration.

Figure 3: Propagation of a Large Shock



(a) Hazard $\Lambda(x)$ and distributions m(x), $\hat{m}(x,0)$

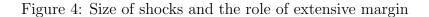
(b) Price level response to the shock

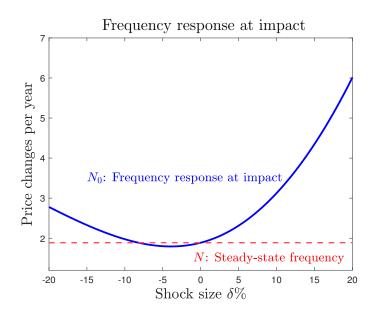
Note: The figure uses the average of the food and beverages sectors, see Table 1. The calibration matches a frequency of N = 1.9 price changes a year, the kurtosis and standard deviation of price changes of 2.3 and 27% respectively. Time is measured in months after the shock. Panel (a) has a dual scale: the left axis gives the density of price gaps; the right axis gives the hazard rate of a price gap (as the probability per month)

	Price-Setting Statistics						Model Calibration		
	Mean	St. Dev.	Kurtosis	Frequency	# obs.	σ	γ	κ	
	of Δx	of Δx	of Δx	Ν	(thousands)				
France	-0.002	0.15	2.4	2	503	0.21	2.54	0.09	
Germany	0.011	0.16	2.6	1.2	177	0.18	2.85	0.18	
Italy	0.007	0.26	2.2	1.5	93	0.32	2.09	0.13	
Netherlands	0.009	0.13	2.3	2.3	330	0.20	2.42	0.07	
Spain	0.011	0.16	2.9	2.1	350	0.23	3.40	0.12	
UK	0.007	0.31	2.1	0.8	95	0.28	1.89	0.24	
US	0.012	0.31	2.2	2.1	282	0.45	2.02	0.10	
Average	0.010	0.27	2.3	1.9	-	0.37	2.22	0.11	

Table 1: Price Setting Behavior for "Food and Beverages" before 2022

Source: The PriceStats data (period 2019-21) use a sample of daily changes in regular prices (excluding sales). The statistics are computed over the distribution of the size of price changes after dropping price changes larger than 1.50 log points in absolute value and products with less than 3 price spells for the period 2019-2021. Kurtosis is computed using equation (13). Frequency is annualized. The Average statistics are weighted by 2019 consumption at current USD. The calibration of parameters σ , γ , κ is discussed in the text. The drift of price gaps is $\mu = 2\%$. The additional parameters are set to canonical values: $\eta = 6$, $\rho = 0.05$.





Note: The figure uses the average of the food and beverages sectors, see Table 1.

Online Appendix: Large shocks travel fast

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A The model economy

Household problem. Households consume a composite good C made of varieties c_i with a constant elasticity of substitution $\eta > 1$

$$C_t = \left(\int_0^1 \left(A_{it}c_{it}\right)^{\frac{\eta-1}{\eta}} di\right)^{\frac{\eta}{\eta-1}}$$
(17)

where A_{it} denote shocks to preferences. The households maximize utility

$$\max_{C(t),c_i(t),H(t),M(t)} \quad \int_0^\infty e^{-\rho t} \left(\frac{C_t^{1-\epsilon}}{1-\epsilon} - \alpha H_t + \log\left(\frac{M_t}{P_t}\right)\right) dt$$

where $\alpha > 0$ is a labor disutility parameter and $\rho > 0$ is the discount factor, subject to the inter-temporal budget constraint

$$M_0 + \int_0^\infty Q_t \left(H_t W_t \left(1 + \tau_\ell \right) + \Pi_t + \tau_t - R_t M_t - \int_0^1 p_{it} c_{it} di \right) dt = 0$$
(18)

where R_t denotes the nominal interest rate, $Q_t = \exp(-\int_0^t R_s ds)$ is the discount factor, W_t is the nominal wage rate, τ_ℓ a labor income tax, Π_t the firms' profits, τ_t a lump-sum transfer, p_{it} the nominal price of variety *i* and *P* is the price index. The household first order conditions yield

$$C_t: e^{-\rho t} C_t^{-\epsilon} - \lambda Q_t P_t = 0$$
(19)

$$c_{it}: e^{-\rho t} C_t^{-\epsilon} C_t^{\frac{1}{\eta}} A_{it}^{\frac{\eta-1}{\eta}} c_{it}^{-\frac{1}{\eta}} - \lambda Q_t p_{it} = 0$$
(20)

$$H_t: -e^{-\rho t}\alpha + Q_t W_t (1+\tau_\ell) \lambda = 0$$
(21)

$$M_t: \quad e^{-\rho t} \frac{1}{M_t} - \lambda Q_t R_t = 0 \tag{22}$$

where λ is the lagrange multiplier of the intertemporal budget constraint.

Using equation (19) and equation (20)

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\eta} A_{it}^{\eta-1} C_t \tag{23}$$

Rearranging and differentiating equation (22) with respect to time we obtain

$$-\rho \frac{e^{-\rho t}}{M_t} - \frac{e^{-\rho t}}{M_t} \frac{\dot{M}_t}{M_t} = -\lambda Q_t R_t^2 + \lambda Q_t \dot{R}_t$$

where $\dot{Q}_t = -Q_t R_t$. Assume a monetary policy $M_t = M_0 \exp(\mu t)$. Then simplifying the above expression

$$\rho + \mu = R_t - \frac{\dot{R}_t}{R_t}$$

which is solved by $R_t = \rho + \mu$, all t. This implies, using equation (22)

$$\lambda = \frac{\exp(-\rho t)}{M_t Q_t R_t} = \frac{1}{M_0(\rho + \mu)}.$$

From equation (21), the nominal wage rate is

$$W_t = \exp(\mu t) \frac{\alpha}{1 + \tau_\ell} M_0(\rho + \mu),$$

with growth rate equal to μ . Using equation (20) and equation (21)

$$A_{it}c_{it} = \left(\frac{p_{it}}{W_t A_{it}}\right)^{-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta},$$

$$A_{it}c_{it} = \left(\frac{\eta}{\eta-1} K \left(\frac{E_t}{W_t}\right)^{\zeta} \frac{p_{it}}{p_{it}^*}\right)^{-\eta} \left(\frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta},$$

$$A_{it}c_{it} = e^{-\eta x_{it}} \left(K \left(\frac{E_t}{W_t}\right)^{\zeta} \frac{\eta}{\eta-1} \frac{\alpha}{1+\tau_\ell}\right)^{-\eta} C_t^{1-\epsilon\eta}.$$

where the second line uses the definition of profit-maximizing price and the fact $A_{it} = Z_{it}^{1-\zeta}$. Integrating over varieties,

$$C_{t}^{\frac{\eta-1}{\eta}} = \left(K \left(\frac{E_{t}}{W_{t}} \right)^{\zeta} \frac{\eta}{\eta-1} \frac{\alpha}{1+\tau_{\ell}} \right)^{1-\eta} C_{t}^{\frac{\eta-1}{\eta}-\epsilon(\eta-1)} \int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx,$$

$$C_{t}^{\epsilon(\eta-1)} = \left(K \left(\frac{E_{t}}{W_{t}} \right)^{\zeta} \frac{\eta}{\eta-1} \frac{\alpha}{1+\tau_{\ell}} \right)^{1-\eta} \int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx,$$

$$C_{t} = \left(K \left(\frac{E_{t}}{W_{t}} \right)^{\zeta} \alpha \right)^{-\frac{1}{\epsilon}} \left(\int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx \right)^{-\frac{1}{\epsilon(1-\eta)}}, \qquad (24)$$

which gives aggregate consumption as a function of the distribution of price gaps $\hat{m}(x,t)$ and prices. The labor subsidy is assumed to offset markups. Further, using the aggregate price

index implied by equation (23) we obtain

$$P_t^{1-\eta} = \int_0^1 \left(\frac{p_{it}}{A_{it}}\right)^{1-\eta} di,$$

$$P_t^{1-\eta} = \left(\frac{\eta}{\eta - 1} \operatorname{mc}_t\right)^{1-\eta} \int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x, t) dx,$$
(25)

where mc_t is short for $KE^{\zeta}W^{1-\zeta}$. Equation (25) gives an expression for real marginal costs as a function of the distribution of price gaps $\hat{m}(x,t)$.

The firm's profit function. The technology for firm *i* is Cobb-Douglas $y_i = (h_i/Z_i)^{1-\zeta} m_i^{\zeta}$ where h_i are units of labor and m_i are units of energy input. The marginal cost (and average cost) of producing y_i is $mc_i = K \cdot E^{\zeta} (W \cdot Z_i)^{1-\zeta}$ where $K = \zeta^{-\zeta} \cdot (1-\zeta)^{-(1-\zeta)}$. The profit function is

$$\Pi_{i} = (p_{i} - \mathrm{mc}_{i}) \left(\frac{p_{i}}{P}\right)^{-\eta} A_{i}^{\eta-1}C,$$

$$\Pi_{i} = \left(\frac{p_{i}}{\mathrm{mc}_{i}} - 1\right) \left(\frac{p_{i}}{PA_{i}}\right)^{-\eta} A_{i}^{-1}K \cdot E^{\zeta} \left(W \cdot Z_{i}\right)^{1-\zeta}C,$$

$$\Pi_{i} = \left(\frac{p_{i}}{\mathrm{mc}_{i}} - 1\right) \left(\frac{p_{i}}{PZ_{i}^{1-\zeta}}\right)^{-\eta} K \cdot E^{\zeta}W^{1-\zeta}C,$$

$$\Pi_{i} = \left(\frac{p_{i}}{\mathrm{mc}_{i}} - 1\right) \left(\frac{p_{i}}{\mathrm{mc}_{i}}\right)^{-\eta} P^{\eta} \left[K \cdot E^{\zeta}W^{(1-\zeta)}\right]^{1-\eta}C,$$

the first line uses equation (23), the second factorizes marginal cost out, the third uses the assumption that $A_i = Z_i^{1-\zeta}$ and the fourth rearranges. The flexible-price optimum is $p_i^* = \frac{\eta}{\eta-1} \text{mc}_i$. Let $x \equiv \log p_i/p_i^*$ denote the "price gap", namely the distance between the current price and the static profit maximizing price. Expressing profits as a function of the price gap gives

$$\Pi(x,t) = \left[e^x - \frac{\eta - 1}{\eta}\right] e^{-\eta x} \cdot P_t^{\eta} \left[\frac{\eta}{\eta - 1} K \cdot E_t^{\zeta} W_t^{(1-\zeta)}\right]^{1-\eta} C_t,$$

$$\frac{\Pi(x,t)}{P_t} = \left[e^x - \frac{\eta - 1}{\eta}\right] e^{-\eta x} \left[\frac{\eta}{\eta - 1} \frac{\mathrm{mc}_t}{P_t}\right]^{1-\eta} C_t,$$
(26)

where mc_t , again, is short for $KE_t^{\zeta}W_t^{1-\zeta}$. It is worth noting that all time dependent terms of real profits can be computed from the distribution of price gaps using equation (24) and equation (25). Notice how the assumption $A_i = Z_i^{1-\zeta}$ makes the profit function independent of the productivity shock, a feature that allows us to reduce the state space of the problem to a single scalar variable x. We define the flow cost function that represents forgone profits due to price gap x along a transition and in steady-state as

$$F(x,t) \equiv 1 - \frac{\Pi(x,t)}{\Pi_{ss}(0)}, \quad F(x) \equiv 1 - \eta \left[e^x - \frac{\eta - 1}{\eta}\right]e^{-\eta x},$$

where $\Pi_{ss}(0)$ are profits at x = 0 given steady state real marginal costs and consumption.

A second order expansion of the flow cost function around x = 0, yields the following quadratic approximation

$$F(x) \approx \frac{\eta(\eta - 1)}{2} x^2,$$

where the profits are expressed as a fraction of the maximized profits $\Pi_{ss}(0)$. Thus, profit maximization can be approximated by the minimization of the quadratic return function $\frac{\eta(\eta-1)}{2}x^2$.

The profit function $\Pi(x)$ also reveals that the aggregate consumption level C only has a second order effect on the firm's choice of the optimal price gap, since the cross partial derivative of $\frac{\partial^2 \Pi}{\partial x \partial C}$ is zero when evaluated around the optimal value x = 0. This fact, which is due to the constant elasticity assumption, implies that the steady state decision rules of the firm are not altered, up to a first order, by a small perturbation of the aggregate variable C. We note that the absence of strategic complementarities makes the analysis with steady state rule Λ very close to the one of a general equilibrium model with feedback to aggregates (see Appendix D).

The firm's price-setting problem. The firm's sequential problem consists in minimizing the flow costs from forgone profits and effort costs by choosing hazard rates $\ell(t)$ and the optimal reset point x^* according to

$$\upsilon(x) = \min_{\ell(t), x^*} \mathbb{E}\left\{\int_0^\infty e^{-\rho t} \left[F(x(t)) + (\kappa \ell(t))^\gamma\right] dt \,\middle| \, x(0) = x\right\}$$

s.t. $x(t) = x(0) - \mu t + \sigma z(t) + \sum_{\tau_i < t} \Delta x(\tau_i)$

where τ_i denotes the stopping times when a resetting opportunity arrives, $\Delta x(\tau_i) = x^* - x(\tau_i)$ is the price change, and $(\kappa \ell)^{\gamma}$ is the effort cost of choosing hazard rate ℓ with $\kappa > 0$ and $\gamma > 1$. This sequential formulation implies the HJB equation in equation (6).

B Proof of Proposition 1

Proof. Note that upon a stopping time τ (when adjustment occurs) we have $x(\tau) = x^* + \sigma z(\tau)$ where z(t) is a standard Brownian motion since $\mu = 0$. By Ito's lemma the stochastic process $u(t) \equiv (x(t) - x^*)^2$ follows the diffusion

$$du = \sigma^2 dt + 2\sigma \sqrt{u} \, dz$$

Notice that $u(t) - \sigma^2 t = 2\sigma \sqrt{u} z(t)$ is a martingale. Letting τ be a stopping time, we have

$$\mathbb{E}(u(\tau) - \sigma^2 \tau) = 0$$
 so that $\mathbb{E}(u(\tau)) - \sigma^2 \mathbb{E}(\tau) = 0$

Note that $\mathbb{E}(\Delta x) = \mathbb{E}(x^* - x(\tau)) = -\sigma \mathbb{E}z(\tau) = 0$, so that $\mathbb{E}(u(\tau)) \equiv Var(\Delta x)$ is the variance of the size of price changes. Then, using that the mean duration of price changes satisfies $\mathbb{E}(\tau) = 1/N$, we have

$$N \cdot \operatorname{Var}(\Delta x) = \sigma^2.$$

From the equation above and the stated relation between variance and frequency among the two economies, we immediately obtain $\tilde{\sigma}^2 = s \, \sigma^2$. Next we prove the scaling properties stated in the proposition. The steps are: we guess and verify a solution for the value function \tilde{v} using the HJB, then we obtain the hazard function $\tilde{\Lambda}$ and we guess and verify a solution to \tilde{m} using the hazard and the KFE. The HJB equation for the tilde economy satisfies

$$\tilde{\rho}\,\tilde{v}(x) = \tilde{B}x^2 + \frac{\tilde{\sigma}^2}{2}\tilde{v}''(x) + \tilde{C}(x) \quad \text{where} \quad \tilde{C}(x) \equiv \left(\frac{\tilde{v}(x) - \tilde{v}(x^*)}{\tilde{\kappa}\,\gamma}\right)^{\frac{\gamma}{\gamma-1}} \ (1-\gamma) < 0$$

Define $\beta \equiv \tilde{B}/B$ and consider $\tilde{\rho} = \frac{s}{a}\rho$, $\tilde{\sigma}^2 = s\sigma^2$ and $\tilde{\kappa} = \frac{\kappa}{s}a^{\frac{\gamma+1}{\gamma}}\beta^{\frac{1}{\gamma}}$. Guess that $\tilde{v}(x) = v\left(\frac{x}{\sqrt{a}}\right)\frac{a^2}{s}\beta$ and substitute in the equation above to obtain (after some algebra)

$$a\beta \ \rho \ v\left(\frac{x}{\sqrt{a}}\right) = a\beta \left(B\left(\frac{x}{\sqrt{a}}\right)^2 + \frac{\sigma^2}{2}v''\left(\frac{x}{\sqrt{a}}\right) + C\left(\frac{x}{\sqrt{a}}\right)\right)$$

This verifies our guess for \tilde{v} since the HJB for v holds. Note that given the value function and parameters we have

$$\tilde{\Lambda}(x) = \frac{1}{\tilde{\kappa}} \left(\frac{\tilde{v}(x) - \tilde{v}(x^*)}{\tilde{\kappa} \gamma} \right)^{\frac{1}{\gamma - 1}} = \frac{s}{a} \Lambda \left(\frac{x}{\sqrt{a}} \right).$$
(27)

Now guess a density function $\tilde{m}(x) = m\left(\frac{x}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$. Note that the density m solves $\int_{-\infty}^{\infty} m(x)dx = 1$ hence it follows that $\int_{-\infty}^{\infty} \tilde{m}(x)dx = \int_{-\infty}^{\infty} m\left(\frac{x}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}dx = 1$ which verifies the integration to one condition. Consider the Kolmogorov forward equation for the tilde economy

$$\tilde{\Lambda}(x)\tilde{m}(x) = \frac{\tilde{\sigma}^2}{2}\tilde{m}''(x), \qquad x \neq 0$$

and rewrite it using the guessed density, $\tilde{\sigma}^2 = s\sigma^2$, and equation (27). We obtain

$$\Lambda\left(\frac{x}{\sqrt{a}}\right)m\left(\frac{x}{\sqrt{a}}\right) = \frac{\sigma^2}{2}m''\left(\frac{x}{\sqrt{a}}\right), \qquad x \neq 0$$

which verifies our guess for \tilde{m} since the KFE for m holds. Our final result follows from

$$\tilde{q}(x) = \frac{\tilde{\Lambda}(x)\tilde{m}(x)}{\tilde{N}} = \frac{\Lambda\left(\frac{x}{\sqrt{a}}\right)m\left(\frac{x}{\sqrt{a}}\right)}{N}\frac{1}{\sqrt{a}} = q\left(\frac{x}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}.$$

C Detailed Data Description

In this Appendix we describe the data-cleaning process and provide information on the data coverage. We use granular price data collected from the websites of large multi-channel retailers that sell products both online and in brick-and-mortar stores. The data were collected on a daily basis by PriceStats, a private firm related to the Billion Prices Project at Harvard and MIT. Previous research has shown that price indices constructed with this data can closely track official CPI statistics in many countries (Cavallo and Rigobon (2016)).²⁰ We focus on the "Food and Beverages" sector and use data for all products sold by some of the largest food retailers in six European countries (France, Germany, Italy, Netherlands, Spain, and the United Kingdom) and the United States. The sample goes from January 1st, 2019, until July 22nd, 2023, and contains daily information on price spell duration and the size of price changes.

Data-cleaning process. To minimize the impact of scraping errors and compositional changes, we keep only the products that remain in the sample for at least 365 days. For each product, we fill all the missing prices with the last available price from the previous spell. We also drop all price changes larger than 1.5 log points and equal to 1 cent in local currency. We filter temporary price discounts (sales) by using a V-shaped algorithm that detects a price drop followed by an equal price increase, back to the original price, within 90 days. When we identify a sale, we replace the discounted price with the last observed pre-sale price to obtain a regular price spell for that product.

Data coverage. The final dataset contains information on 583,788 products from 58 retailers the "Food and Beverages" sector. For each country, we have products from eleven 3-digit COICOP sectors within the 1-digit "Food and Beverages" sector. As Table 2 shows, each product remains over 1000 days in the sample with an average of 8 distinct prices.

D General equilibrium

Equilibrium. The equilibrium is characterized by $\{\hat{v}, \hat{m}, C\}$ a value function for the firms' price-setting problem, a distribution of price gaps, and a path of aggregate consumption such that for each t

 $^{^{20}}$ Also see Cavallo, A. (2013). Online and official price indexes: Measuring Argentina's inflation. Journal of Monetary Economics 60(2), 152-165

 Table 2: Data Coverage

			Average Prices	Average
	Retailers	Unique Products	per Product	Product Life
France	12	147,115	9.41	998
Germany	7	$65,\!670$	7.26	963
Italy	5	36,490	7.04	988
Netherlands	8	80,614	9.98	1,001
Spain	9	$91,\!184$	8.65	1,051
United Kingdom	10	86,295	4.98	1,096
United States	7	76,420	9.05	1,034
All	58	583,788	8.05	1,019

$$\rho \hat{v}(x,t) - \hat{v}_t(x,t) = F(x,t;C(t)) - \mu \,\hat{v}_x(x,t) + \frac{\sigma^2}{2} \,\hat{v}_{xx}(x,t) + \min_{\ell,x^*} \{\ell \,(\hat{v}(x^*,t) - \hat{v}(x,t)) + (\kappa \,\ell)^\gamma\},\tag{28}$$

$$\hat{m}_t(x,t) = -\hat{\lambda}(x,t)\hat{m}(x,t) + \mu \,\hat{m}_x(x,t) + \frac{\sigma^2}{2} \,\hat{m}_{xx}(x,t), \quad x \neq \hat{x}^*(t), \tag{29}$$

$$C(t) = \left(K\left(\frac{E(t)}{W(t)}\right)^{\zeta} \alpha \right)^{-\frac{1}{\epsilon}} \left(\int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t) dx \right)^{-\frac{1}{\epsilon(1-\eta)}},$$
(30)

where F emphasizes how the firm problem depends on aggregate consumption C(t). Equation (28) takes a path of aggregate demand C and solves for \hat{v} given terminal condition $\lim_{t\to\infty} \hat{v}(x,t) = v(x)$, the steady-state value function. Policies $\hat{\lambda}, \hat{x}^*$ are obtained from equation (28) using the optimal return condition $\hat{v}_x(\hat{x}^*(t),t) = 0$ and the optimal hazard condition in equation (7). Equation (29) takes policies and solves for \hat{m} given initial condition $\hat{m}(x,0) = m(x+\delta)$, the displaced steady-state distribution.

Equilibrium is a fixed point problem. Policies λ, \hat{x}^* depend on the path C and the distribution depends on a path of policies. Equation (30) is the equilibrium condition coupling the HJB and KFE, requiring that the path C is consistent with both. This is the structure of a mean-field game where optimal decisions and aggregation are coupled only through a finite set of distributional moments, see Alvarez, Lippi, and Souganidis (2023). In this case, only through the aggregate consumption path.

Fixed point problem. Following Golosov and Lucas (2007), we construct an operator Γ over paths C such that a fixed point solves the coupled system of equations described by equations (28)-(30). Consider a path C, then the correspondence Γ maps C into a path ΓC

implied by the HJB and KFE. Specifically, for each t, $(\Gamma C)(t)$ is defined as

$$(\Gamma C)(t) = \left(K\left(\frac{E(t)}{W(t)}\right)^{\zeta}\alpha\right)^{-\frac{1}{\epsilon}} \left(\int_{\mathbb{R}} e^{(1-\eta)x} \hat{m}(x,t;C)dx\right)^{-\frac{1}{\epsilon(1-\eta)}}$$

where \hat{m} emphasizes the dependence on the input path C. Aggregate consumption path $C = \Gamma C$ and associated \hat{v} , \hat{m} solve equations (28) to (30).

Computation. To compute the equilibrium we use standard finite difference methods (Achdou et al., 2021). Iteration on the operator Γ converges in few steps. We set the two additional parameters of labor disutility and relative risk aversion to standard parameters $\alpha = 6$ and $\epsilon = 2$, following Golosov and Lucas (2007).

A shock to energy prices increases marginal cost by $\delta = 20\%$. We assume that energy prices and money grow at rate μ . Together these imply that (i) the shock to nominal marginal cost is permanent, drifting at rate μ for t > 0 and (ii) the relative price of energy to wages E(t)/W(t) increases permanently.

Results. Panel (a) and (b) of Figure 5 indicate that the general equilibrium response of frequency is very close to one using steady-state policy rules. Panel (a) depicts the hazards, as a monthly probability, at several points in time, in yearly units, and shows that they are close to the stationary hazard (dashed). Panel (b) shows that the response of frequency is slightly dampened due to a lower hazard at impact but reaches a comparable peak response of 5 price changes per year.

Panel (c) shows the response of output. At impact, the increase in relative costs E(t)/W(t) exactly offsets the increase in demand due to low price gaps. Afterwards, elevated relative costs and rising prices P(t) generate a permanent decrease in output. This is a canonical permanent supply shock.

The slightly dampened response of frequency is explained by the general equilibrium effects on the flow cost function (i.e. on the profit function). Equation (26) indicates that flow costs increase with output (are more negative) and decrease with real marginal costs, with an elasticity of 1 and $\eta - 1 = 5$, respectively. Real marginal costs (in equation (25)) increase by δ at impact and gradually revert to steady state, whereas output (in equation (24)) is unresponsive at impact and gradually declines by δ/ϵ relative to the pre-shock level. Quantitatively, the real marginal cost drives the dynamics of the flow cost, reducing the marginal value of repricing efforts, and thus the hazard (see panel (a)). This in turn, marginally dampens the response of frequency.

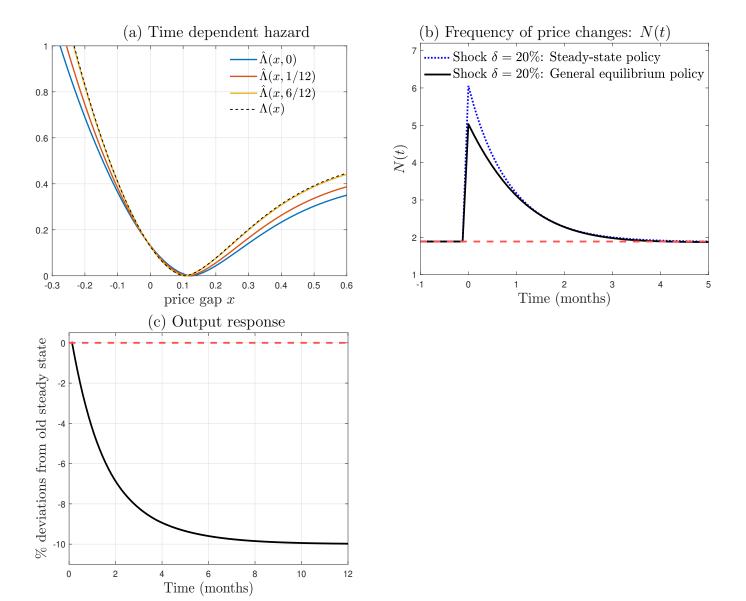


Figure 5: Propagation of a Large Shock in General Equilibrium

E More evidence on increased frequency of price changes

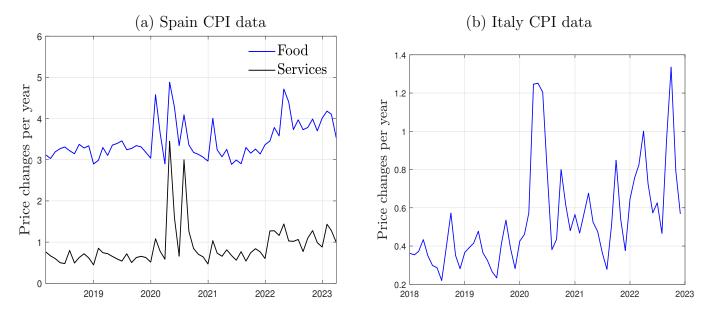


Figure 6: Increase in Frequency: Aggregate data

Sources: Left panel, Instituto Nacional de Estadistica and Bank of Spain staff calculations. Right panel, Istat and Bank of Italy staff calculations.