Markups, Taxes, and Rising Inequality*

Stéphane Auray[†] Aurélien Eyquem[‡]

Bertrand Garbinti[§] Jonathan Goupille-Lebret[¶]

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Abstract

We analyze income and wealth inequality dynamics through the lens of an heterogeneous-agent model with three key features: (i) an explicit link between firms' markups and top income shares, (ii) a granular representation of the tax and transfer system, and (iii) three assets with endogenous portfolio decisions. Using counterfactual analyzes, we look at how changes in markups, taxes, factor productivity, and asset prices affected inequalities between 1984 and 2018 in France. Rising markups account for the bulk of rising pretax income inequality. The drivers of rising wealth inequality are more complex. Rising markups and changes in taxes contribute to raise wealth inequality by increasing pretax income inequality and inequality in saving rates between wealth groups. Asset prices – the boom in housing prices – mechanically redistribute wealth from top and bottom wealth groups to the upper middle wealth group, whose wealth is mostly held in the form of housing, but are partly offset by a rise in saving rates of top wealth groups. Our results highlight the key role of endogenous saving decisions in response to exogenous variables as a key driver of wealth inequality.

Keywords: Heterogeneous Agents, Taxes, Market Power, Income Inequality, Wealth Inequality

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†CREST-ENSAI and Rennes School of Business, Campus de Kerlann, Bruz; e-mail: `stephane.auray@ensai.fr`.

‡Department of Economics, University of Lausanne, Internef, CH-1015 Lausanne, Switzerland. Email: `aurelien.eyquem@unil.ch`.

§CREST-ENSAE-Institut Polytechnique Paris, Palaiseau; e-mail: `bertrand.garbinti@ensae.fr`.

¶Univ Lyon, CNRS, GATE-LSE UMR 5824, F-69130 Ecully, France, and ENS de Lyon; e-mail: `jonathan.goupille-lebret@ens-lyon.fr`.

1 Introduction

How to explain income and wealth inequality dynamics? While the empirical literature has documented the rise in inequalities in many countries during the past decades (Alvaredo et al. (2017)), we still lack a clear understanding of the drivers behind this rise and the channels through which they operate. In this paper, we study how income inequality is formed and translates into wealth inequality dynamics.

To this end, we build an original heterogeneous-agent (HA) model with three key features: (*i*) an explicit link between firms' market power – markups – and top income shares, (*ii*) a granular representation of the tax and transfer system, and (*iii*) endogenous portfolio decisions leading to heterogeneity in wealth composition. We discipline the model based on French data and show that it fits the dynamics of income and wealth inequality between 1984 and 2018. Using counterfactual scenarios, we then look at how changes in markups, taxes, productivity, and asset prices affect inequality dynamics. Last, we propose a method that quantifies the contribution of various transmission channels to rising wealth inequality and highlights the key role of endogenous saving rates.

Regarding the model, we first introduce a key relation between markups and top income shares through entrepreneurial risk. On top of workers exposed to idiosyncratic risk *à la* Aiyagari (1994), we introduce a risky entrepreneurial process by which a small fraction of the population receives monopolistic profits. As markups have risen in France since the early 80s – although less than in the U.S. (see De Loecker, Eeckhout, and Unger (2020) and De Loecker and Eeckhout (2018)) – the rise in market power may explain part of the observed rise in top income shares.[1] Further, because entrepreneurs face large risks of losing the status, they secure large precautionary savings and become top wealth owners, which may contribute to explain rising top wealth shares.

Second, our model takes into account the granularity of the French tax and transfer system using a very rich and realistic set of time-varying flat and progressive taxes and transfers that applies to the relevant tax bases (payroll taxes on labor income, corporate taxes on profits, income taxes, taxes on consumption, taxes on wealth, and monetary transfers). Our tax system better replicates the mapping between pretax, disposable, and posttax income inequalities, and takes into account all the potential behavioral effects (direct, indirect, general equilibrium) of changes in taxes and transfers on the relevant margins (savings, consumption, and labor).

Third, we introduce two important features to match the top of the wealth distribution. Gaillard et al. (2023) show analytically that standard HA models can not account for large top wealth shares and have to incorporate non-homothetic wealth-dependent preferences (see also De Nardi (2004) or Francis (2009)) and scale-dependent returns to capital. Therefore, we introduce wealth

[1] See Boar and Midrigan (2019) and Deb (2024) for a similar approach based on U.S. data, and Eggertsson, Robbins, and Wold (2021) in a representative-agent model.

in the utility function, and generate increasing returns along the wealth distribution by considering three assets and endogenous portfolios. In our model, housing and financial services enter the utility function of households. Households then choose whether to rent or own their housing unit subject to minimum housing-size and constrained borrowing. Making the composition of individual portfolios endogenous, (*i*) provides a micro-founded explanation for increasing returns along the wealth distribution[2], (*ii*) helps match top wealth shares and the observed composition of portfolios along the wealth distribution, and (*iii*) allows to study the distributional effects of capital gains over time.

We use the model to study the rise in income and wealth inequality in France since 1984, the year just before inequalities started increasing. We calibrate the model's stationary equilibrium in 1984 using newly available wealth and income inequality series and fiscal data. These series are part of the "Distributional National Accounts" (DINA) project, and consist in long-term series of wealth, pretax and posttax national income that (*i*) are fully consistent with national accounts, (*ii*) cover the entire distribution, and (*iii*) provide detailed information on income, asset detention, portfolios as well as all taxes and transfers at the individual level (see Saez and Zucman (2016); Piketty, Saez, and Zucman (2018) for the U.S.; Garbinti, Goupille-Lebret, and Piketty (2018), Garbinti, Goupille-Lebret, and Piketty (2021) and Bozio et al. (2024) for France). We also exploit the richness of these time series to gauge the quality of our stationary equilibrium (in 1984) and dynamic simulations (from 1984 to 2018) on various dimensions (macroeconomic aggregates, income and wealth inequality, portfolios along the distribution, the aggregate and distributional structures of taxes).

The model fits the observed 1984 characteristics of income and wealth distributions from the bottom 50% to the top 1% shares, the composition of individual and aggregate wealth, the aggregate wealth-income ratio, the aggregate and distributional structure of taxes, as well as the usual macroeconomic ratios. Alternative specifications show that the introduction of entrepreneurs, wealth-in-utility, and of a granular and progressive tax system are all key to match the top and the bottom of the income and wealth distributions. In addition, considering three assets matters to match the aggregate wealth-income ratio, on top of being critical to generate increasing returns along the wealth distribution. It also matters in the dynamic simulations for the distributional effects of capital gains through heterogeneous wealth composition along the wealth distribution.

Starting from the 1984 stationary distribution, we then feed the model with a set of exogenous variables: changes in capital gains, taxation (taxes and transfers), markups, and others market forces such as the rate of capital depreciation or total factor productivity (TFP hereafter). Our dynamic simulations track the data very closely and reproduce two main facts regarding inequality dynamics: (*i*) the rise in income inequality is mostly driven by a significant increase in the top 1% income share and (*ii*) the rise in wealth inequality is mostly driven by a significant increase in the top 10% and top 1% wealth shares, at the expense of the bottom 50%.

[2]See Cao and Luo (2017) or Xavier (2020) for evidence based on U.S. data and Garbinti, Goupille-Lebret, and Piketty (2021) for similar evidence based on French data.

We then run counterfactual experiments to shed light on the respective contributions of the different exogenous variables to income and wealth inequality. We find that rising markups account for the bulk of rising income inequality. The drivers of rising wealth inequality are more complex and we propose a method to identify the channels through which exogenous variables affect wealth inequality. We find that rising markups and changes in taxes increase wealth inequality by increasing pretax income inequality and saving rate inequality among wealth groups. Capital gains – or changes in asset prices induced mainly by the housing boom – have ambiguous effects. While they carry large valuation effects that mechanically decrease wealth concentration, capital gains also increase saving rate inequality when interacting with rising income inequality, pushing up wealth inequality. More generally, our results highlight the key contribution of saving rate inequality to the dynamics of wealth inequality, and call for a thorough modeling of endogenous saving decisions.

Literature. Since the pioneering works of Bewley (1977), Huggett (1993), and Aiyagari (1994), a macroeconomic literature has developed and improved general equilibrium models with heterogeneous agents to reproduce the wealth and income inequality *at a given point in time* and explain their determinants (see De Nardi and Fella (2017); Benhabib and Bisin (2018) for a literature review). In particular, these contributions consider switching discount factors (Krusell and Smith (1998)), bequest motives (De Nardi (2004)), entrepreneurs (Cagetti and De Nardi (2006)), wealth in the utility function (Francis (2009)), original labor-income processes (Ferriere et al. (2023)), and stochastic jumps affecting the returns to assets (Benhabib, Cui, and Miao (2021)) as potential drivers of wealth inequality.[3] Gaillard et al. (2023) show that standard models are unable to match the concentration of wealth and require wealth-dependent preferences and scale-dependent returns to capital. Benhabib, Bisin, and Luo (2019) also point to the importance of inequality in saving rates to account for the observed dispersion of wealth in the U.S. Recently, a new line of papers investigates the *dynamics* of income inequality (Gabaix et al. (2016)) and wealth inequality (Kaymak and Poschke (2016) and Hubmer, Krusell, and Smith (2020)) for the U.S. We make progress on both fronts by developing a unified HA model with entrepreneurs, three assets, and wealth in utility that fits both the *level* and *dynamics* of income and wealth inequality in France. We also use our model to quantify the contribution of various mechanisms to the dynamics of wealth inequality depending on the exogenous variables.

Along the way, we make use of the recent technical advances in solving HA models based on continuous-time formulation of the heterogeneous-agent problem in solving the Hamilton-Jacobi-Bellman and Kolmogorov forward equations (see Achdou et al. (2022)), and rely on fast and fully non-linear dynamic simulations. An additional contribution to the literature is to further extend the approach of Berger et al. (2018) used by Fagereng et al. (2019), and solve a three-asset model including deposits, indivisible housing, and equity capital using a one-asset formulation. While multiple assets have been used to study the short-run effects of monetary or

[3]See Saez and Stantcheva (2018) for additional references on wealth in utility and potential microfoundations, including bequests and services from wealth.

fiscal policy within HA models (see Kaplan, Moll, and Violante (2018) and Kaplan and Violante (2021) among others), we show that having various classes of assets is key to account for wealth inequality dynamics through differential asset price dynamics.[4]

Our paper relates to the recent literature that empirically documents the rise of markups in the U.S. economy (De Loecker, Eeckhout, and Unger (2020)) and around the world (De Loecker and Eeckhout (2018)), and its implication for optimal regulation (Boar and Midrigan (2019) and Eeckhout et al. (2021)). We contribute to this literature by introducing a link between market power, firms' profits, and top income shares through entrepreneurs, and by quantifying the impact of rising market power on income and wealth inequality.[5] Eggertsson, Robbins, and Wold (2021) follow a similar path but focus on the aggregate macroeconomic implications of rising market power, while we consider both aggregate and distributional effects of rising markups. Deb (2024) establishes a tight relation between rising markups, declining business dynamism and top income shares. He builds on a static model that digs into the market-structure determinants of rising markups, but remains silent about dynamic aspects, such as wealth inequality.

Finally, our paper relates to the recent literature in applied economics dedicated to the construction of "Distributional National Accounts" for pretax and posttax income (see in particular Piketty, Saez, and Zucman (2018) for the U.S. and Garbinti, Goupille-Lebret, and Piketty (2018); Bozio et al. (2024) for France) and to the evolution and the determinants of wealth inequality (Saez and Zucman (2016); Martínez-Toledano (2020); Kuhn, Schularick, and Steins (2020); Garbinti, Goupille-Lebret, and Piketty (2021); Blanchet and Martínez-Toledano (2023)). Our paper provides an illustration of how such data can be used to discipline and validate HA models to analyze the dynamics of income and wealth inequality. In turn, it contributes to this literature by decomposing respective contributions of market and institutional factors to the dynamics of inequality, taking into account all their potential behavioral effects.

The paper is structured as follows. Section 2 presents the model and discusses the main assumptions. Section 3 details the calibration, presents the characteristics of the initial stationary equilibrium and presents some counterfactual stationary distributions. Section 4 presents the predictions of the dynamic model and its empirical fit when driven by a set of exogenous variables. Section 5 offers a decomposition of the main drivers of income and wealth inequality relying on counterfactual experiments, and proposes a method to identify the main transmission mechanisms behind rising wealth inequality that highlights the key role of behavioral (savings) responses to changes in exogenous variables.

[4] Kaplan, Moll, and Violante (2018) and subsequent papers rely on a liquid/illiquid divide among assets and adjustment costs because they seek to replicate the pattern of marginal propensities to consume in the event of short-lived shocks. In contrast, we consider different classes of assets (deposits, housing, and equity) that feature different rates of returns and capital gains over time, with potentially large effects on wealth inequality.

[5] The determinants of rising market power in the literature include the development of increasing returns from widening markets through trade or technologies (Autor et al. (2020)), the associated reallocation of market shares towards larger and more efficient firms (Baqaee and Farhi (2020)) or the falling demand elasticity driven by consumers becoming less price-sensitive (Döpper et al. (2021)).

2 Model

2.1 Overview

The model features heterogeneous households with uninsurable earnings and entrepreneurial risk along with a realistic taxation system that incorporates an extensive set of proportional and progressive taxes and transfers.

In the production sector, an intermediate good is produced using labor and capital, and sold to retailers. Retailers differentiate the intermediate good into varieties, and choose prices optimally under monopolistic competition, which resulting in markups and aggregate profits.

In the household sector, individuals can be workers or entrepreneurs. When they are workers, they face labor earning risk and supply labor endogenously. When they are entrepreneurs, they receive the monopolistic profits from firms and face a failure risk, *i.e.* the risk of becoming a regular worker. Entrepreneurial and labor earning risks push households to precautionary-save. An additional saving motive is also considered through the addition of wealth in the utility function.

Households face a positive net wealth constraint, and have access to three types of assets: deposits, gross housing subject to constrained borrowing, and physical capital. Deposits and housing services provide direct utility to all households, which justifies that returns on deposits and housing are lower than returns on capital. In equilibrium, these assumptions generate a realistic composition of portfolios along the wealth distribution, and imply that returns are increasing in wealth as in the data.

The model is presented and solved in continuous time. For the sake of clarity, time subscripts are omitted.

2.2 Firms

A representative firm produces an intermediate good y^m under perfect competition with the following technology:

$$y^m = \xi k^\alpha \ell^{1-\alpha} \tag{1}$$

where ξ is a measure of total factor productivity that grows at rate g_ξ, k the aggregate stock of capital and ℓ aggregate labor. The firm sells the intermediate good to the retailers at price φ. The associated after-tax profits are:

$$(1-\tau_\pi)\left(\varphi \xi k^\alpha \ell^{1-\alpha} - w\ell - \delta k\right) - r^k k \tag{2}$$

where r^k is the rental rate of physical capital and $\delta \in [0,1]$ is the depreciation rate of capital. Corporate profits are taxed at rate τ_π based on their total sales minus their wage bill with an allowance for depreciated capital. Maximization with respect to capital and labor gives:

$$\alpha \frac{\varphi y^m}{k} = \frac{r^k}{1-\tau_\pi} + \delta \text{ and } (1-\alpha)\frac{\varphi y^m}{\ell} = w \tag{3}$$

A unit-size continuum of retailers indexed in i then buy the intermediate good at price φ and differentiate it into varieties. Let $p(i)$ be the price set by retailer i for its variety and $y^d(i) = (p(i)/p)^{-\theta} y$ the demand for this variety, where $\theta > 1$ is the (potentially time-varying) elasticity of substitution between varieties, p is the aggregate price index and y the total demand for final goods. The optimal price $p(i)$ solves:

$$\max_{p(i)} \pi(i) = (1-\tau_\pi)\left(\frac{p(i)}{p} - \varphi\right)\left(\frac{p(i)}{p}\right)^{-\theta} y \tag{4}$$

Assuming symmetry across retailers ($p(i) = p$ and $y^d(i) = y = y^m$), the optimal pricing condition gives:

$$\frac{\theta}{\theta-1}\varphi = 1 \tag{5}$$

Total pretax profits are then given by $\pi = y/\theta$, and are fully redistributed to entrepreneurs. Those profits are treated as mixed-income and, following standard convention, classified as labor income (and taxed as such) for 70% and as capital income for the remaining 30%, and hence subject to the corporate tax rate.[6]

2.3 Households

The economy is populated by a unit-size continuum of heterogeneous households $j \in [0,1]$. There are two types of households, workers and entrepreneurs. Households switch types according to a two-state Markov process representing entrepreneurial dynamics. In addition, when households are workers, their individual productivity is subject to idiosyncratic shocks, as explained below. All variables involved in the household problem are expressed per-capita and relevant variables are deflated by a labor productivity index that grows at the exogenous rate g_ξ.

2.3.1 Income

As in Boar and Midrigan (2019) or Deb (2024), households can be either workers or entrepreneurs. When working, households supply ℓ^j units of labor and receive a real wage $w^j = we^{z^j}$, made of two components, a common component w resulting from the equilibrium of labor

[6]This is the standard convention used in the income inequality literature (Alvaredo et al. (2020), Bachas et al. (2022)) and it is closed to the estimates by Smith et al. (2019) that show that three-quarters of top pass-through profit in the U.S. can be classified as human capital income.

markets, and a log-normal Gaussian mixture process z^j as in Ferriere et al. (2023):[7]

$$\dot{z}^j = -\rho_z z^j + \epsilon_z^j \tag{6}$$

where $\exp(-\rho_z)$ measures the persistence of earning shocks ϵ_z^j and:

$$\epsilon_z^j \sim \begin{cases} N\left(\psi_1, \sigma_1^2\right) & \text{with probability } p_1, \\ N\left(\psi_2, \sigma_2^2\right) & \text{with probability } 1 - p_1. \end{cases} \tag{7}$$

with $\mathbb{E}\left(\epsilon_z^j\right) = p_1\psi_1 + (1 - p_1)\,\psi_2 = 0$ so that choosing ψ_1 and p_1 determines ψ_2.[8] This process can be thought of as a job-ladder process by which individual productivity most frequently increases (p_1 large and $\psi_1 > 0$) with little dispersion (σ_1^2 small) and decreases ($\psi_2 < 0$) in rare circumstances (p_2 small) such as unemployment spells, with large uncertainty (σ_2^2 large).

When households are entrepreneurs, they supply $\ell^j = 0$ units of labor and receive a fraction of the aggregate profits $\pi^j = \pi/e$, where π denotes aggregate profits and e is the equilibrium number of entrepreneurs. Running a business is thus a risky activity – remember being an entrepreneur or a worker follows a two-state Markov process – with a large payoff (profits per entrepreneur are large) but also with a positive probability of failure, as will be discussed further in the calibration section.

In addition to labor or entrepreneurial income, any household j can hold three assets: housing in quantity h^j, deposits in quantity m^j, and equity capital k^j. Households can also borrow an amount d^j from the deposit market at cost r^m, but only for the purpose of acquiring housing units. The total wealth of household j writes $a^j = k^j + p^h h^j - d^j + m^j$, where p^h is the relative price of housing – the price of non-durable goods being used as *numéraire*. The return on capital is r^k, the return on housing is r^h and the return on deposits is r^m. For reasons that we explain below, the model implies $r^m < r^h < r^k$.

As a result, labor and capital income of household j are given by:

$$\Phi_\ell^j = \left(1 - \tau_\ell^j\right)\left(w^j\left(1 - \mathbb{1}_{e^j}\right)\ell^j + \mathbb{1}_{e^j} 0.7\pi^j\right) \tag{8}$$

$$Y_k^j = r^k k^j + r^h p^h h^j + r^m\left(m^j - d^j\right) + \mathbb{1}_{e^j}(1 - \tau_\pi)0.3\pi^j \tag{9}$$

where $\mathbb{1}_{e^j}$ is an indicator function that equals 1 if household j is an entrepreneur and zero otherwise, and remember, τ_π denotes the tax rate on corporate profits. Variable τ_ℓ is the individual

[7]Variable z_t^j denotes the relative productivity of worker j and aggregate productivity is normalized in the stationary equilibrium so that the sum of labor income received by workers equals the aggregate labor income paid by firms.

[8]In the model all relevant variables, including individual productivity levels, are deflated by the average productivity growth rate so that $\mathbb{E}\left(\epsilon_z^j\right) = 0$ means that individual productivity levels are growing at the pace of aggregate productivity on average.

rate of non-contributive payroll taxes, *i.e.* the payroll taxes that do not finance unemployment and pension benefits.[9]

2.3.2 Preferences and optimization problem

Households derive utility from housing services s^j (see Iacoviello (2005), Kaplan and Violante (2014), Favilukis, Ludvigson, and Nieuwerburgh (2017), among many others) that can originate from owned housing or bought on goods market. We introduce a minimum size $h^{\min}$ for housing units. Households also derive utility from financial services provided by deposits (money).[10] Further, preferences also feature a warm-glow motive as in De Nardi (2004) or Francis (2009), to capture the fact that asset-rich households keep on saving beyond the precautionary motive they face. Finally, households can borrow up to a fraction ς of the housing value $d^j \leq \varsigma p^h h^j$. The optimization problem of household j thus writes:

$$\begin{aligned}
&\max_{k^j,h^j,s^j,d^j,m^j,c^j,\ell^j} \int_0^\infty e^{-\rho t} \left\{ \frac{\left(\Lambda^j\right)^{1-\gamma}}{1-\gamma} - \frac{\left(\ell^j\right)^{1+\zeta}}{1+\zeta} + \beta \log\left(\frac{a^j}{a} + \mu\right) \right\} dt \\
&\text{s.t.} \quad \text{budget} \qquad (1+\tau_c)\, c^j + r^h p^h s^j + \Delta^j = \left(1-\tau^j\right)\left(\Phi_\ell^j + \Upsilon_k^j\right) - \phi^j a^j + T^j \\
&\qquad\ \ \text{net wealth} \qquad a^j = k^j + p^h h^j - d^j + m^j, \\
&\qquad\ \ \text{net savings} \qquad \Delta^j = \dot{k}^j + p^h \dot{h}^j - \dot{d}^j + \dot{m}^j + g_\xi a^j \\
&\qquad\ \ \text{borrowing} \qquad m^j \geq 0,\, d^j \leq \varsigma p^h h^j, \\
&\qquad\ \ \text{bounds} \qquad a^j \geq 0,\, k^j \geq 0,\, h^j \in \left[h^{\min}, \infty\right),
\end{aligned} \tag{10}$$

with

$$\Lambda^j = \left(c^j\right)^{1-\kappa-\chi} \left(s^j\right)^{\kappa} \left(m^j\right)^{\chi} \tag{11}$$

where γ is the relative degree of risk-aversion. Given the expression for Λ^j, households derive utility from c^j, their consumption of non-durable goods, from s^j the size of the housing unit occupied which can be rented (if $h^j = 0$) or owned (if $h^j = s^j$) and m^j, the amount of liquid assets they hold. Further, households experience a disutility from supplying labor ℓ^j, and ζ denotes the inverse of the Frisch elasticity on labor supply. Finally, they derive utility from their net wealth a^j relative to the average net wealth in the economy a. Parameter μ makes wealth a luxury good as usual in the literature introducing bequest motives.

In the budget constraint, on the right-hand side, households receive labor and capital income Φ_ℓ^j and Υ_k^j respectively defined by Equation (8) and (9), and both sources of income are taxed at the progressive income tax rate τ^j. Net wealth a^j is taxed at the progressive rate ϕ^j, and T^j

[9] The model features households or dynasties with infinite lives and the model does not explicitly account for the situation of unemployed and retired households, they are implicitly considered as different types of households subject to income risk.

[10] There is a long tradition in macroeconomic of employing the money-in-the-utility approach (see Poterba and Rotemberg (1986) among many others. Goodfriend and McCallum (1988) develop a 'shopping-time' model to motivate the demand for money. For another exposition of the shopping-time model and its connection to the log-log money demand function used frequently in empirical work, see Lucas (2000).

denotes the progressive monetary transfer received from the government. On the left-hand side, household j allocates its income on non-durable goods c^j taxed at the rate τ_c and pays a rent r^h computed on the value of occupied gross housing $p^h s^j$. Household j also saves Δ^j in capital $\dot{k}^j$, gross housing $\dot{h}^j$, deposits $\dot{m}^j$, and contract housing debt $\dot{d}^j$. Last, remember that steady-state growth implies that level quantities, including individual wealth, increase at the rate g_ξ, so that $g_\xi a^j$ units of wealth have to be saved each period to keep the productivity-deflated level of wealth a^j at least constant in the stationary equilibrium.

2.3.3 Reformulation

The above problem can be reformulated as a one-asset problem while preserving the endogenous composition of household portfolios. More precisely, the problem can be split into a dynamic problem that consists in choosing the total level of expenditure and savings, and thus the total amount of net wealth, and a static expenditure minimization problem that allocates total expenditure into three main categories: non-durable goods, housing asset, and liquid asset. Recall that total wealth is:

$$a^j = k^j + p^h h^j - d^j + m^j, \tag{12}$$

The budget constraint can be rewritten as:

$$\dot{a}^j + g_\xi a^j + \overbrace{(1+\tau_c)\, c^j + R^{hj} s^j + R^{mj} m^j}^{P^{\Lambda j} \Lambda^j} = \left(1-\tau^j\right)\left(\Phi^j_\ell + \Phi^j_k\right) - \phi^j a^j + \Xi^j + T^j \tag{13}$$

where $P^{\Lambda j}$ is the true price index associated with Λ^j, to be defined later, and

$$\begin{aligned}
R^{hj} &= p^h \left((1-\mathbb{1}_{h^j})\, r_h + \mathbb{1}_{h^j} \left[\left(1-\tau^j\right)\left((1-\varsigma)\, r^k + \varsigma r^m\right) + \tau^j r^h \right] \right) && (14)\\
R^{mj} &= \left(1-\tau^j\right)\left(r^k - r^m\right) && (15)\\
\Phi^j_k &= r^k a^j + \mathbb{1}_{e^j}(1-\tau_\pi)0.3\pi^j && (16)
\end{aligned}$$

Variable R^{hj} denotes the cost of housing services, R^{mj} the cost of liquidity services, and Φ^j_k is an alternative measure of capital income. For the cost of housing services R^{hj}, it depends on whether household j rents ($\mathbb{1}_{h^j} = 0$) or owns ($\mathbb{1}_{h^j} = 1$) its housing unit. If household j rents, then $R^{hj} = r_h p^h$: occupying a housing unit of size s^j and worth $p^h s^j$ requires paying a rent $r_h p^h s^j$ each period. If household j is an occupying homeowner, then $\mathbb{1}_{h^j} = 1$, and the unit cost of housing is $R^{hj} = p^h \left[\left(1-\tau^j\right)\left((1-\varsigma)\, r^k + \varsigma r^m\right) + \tau^j r^h\right]$.[11] In this case, the rent payment is almost exactly offset by the housing return – housing returns are taxed so they do not fully cover the equivalent rent, hence the presence of $\tau^j r^h$ – and the housing cost becomes a weighted average of two components: (*a*) the interests paid on housing debt r^m for the fraction ς of debt-financed housing,

[11]Here, we consider that households who actually buy housing ($\mathbb{1}_{h^j} = 1$) borrow up to the limit, *i.e.* $d^j = \varsigma p^h h^j$, which is consistent with the fact that $r^m < r^h < r^k$.

and (b) the opportunity cost of not investing the equivalent amount in capital r^k for the fraction $1-\varsigma$ of savings-financed housing. The cost of liquidity services is $R^{mj}=\left(1-\tau^j\right)\left(r^k-r^m\right)$, an opportunity cost of not investing savings in capital. Both wedges R^{hj} and R^{mj} are positive in equilibrium because households derive utility from housing and liquidity services. Last, $\Xi^j=\dot{p}^h h^j$ captures the housing valuation gains.[12]

This reformulation has the major advantage of reducing the program of household j to the following one-asset problem:

$$\begin{aligned}&\max_{\Lambda^j,a^j,\ell^j}\int_0^\infty e^{-\rho_j t}\left\{\frac{\left(\Lambda^j\right)^{1-\gamma}}{1-\gamma}-\frac{\left(\ell^j\right)^{1+\zeta}}{1+\zeta}+\beta\log\left(\frac{a^j}{a}+\mu\right)\right\}dt\\ &\text{s.t.}\quad \dot{a}^j+P^{\Lambda j}\Lambda^j=\left(1-\tau^j\right)\left(\Phi^j_\ell+\Phi^j_k\right)-\left(\phi^j+g_\xi\right)a^j+\Xi^j+T^j\\ &\qquad\ a^j>0\end{aligned}\tag{17}$$

The impact of changes in the environment on portfolio compositions is entirely encapsulated in $P^{\Lambda j}$ while, as explained below, m^j, s^j and b^j are determined *after* Λ^j, ℓ^j, and $\dot{a}^j$ have been chosen by households. Note that the above dynamic problem also solves for the endogenous labor supply decisions of workers, implying:

$$\ell^j=\left[\frac{\left(1-\tau^j\right)\left(1-\tau^j_\ell\right)\left(1-\mathbb{1}^j{}_e\right)w^j}{P^{\Lambda j}\Lambda^j}\right]^{\frac{1}{\zeta}}\tag{18}$$

Workers' labor supply depends positively on the after-tax real wage and negatively on individual taxes through the substitution effect, where $1/\zeta$ is the elasticity of labor supply. Labor supply also varies negatively with aggregate expenditure $P^{\Lambda j}\Lambda^j$ through a standard wealth effect.

2.3.4 Endogenous portfolio choice

Once the above dynamic problem (17) is solved, the static problem involves choosing the composition of Λ^j subject to the relative costs of the three expenditure categories:

$$\begin{aligned}&\min_{c^j,s^j,m^j}\left(1+\tau_c\right)c^j+R^{hj}s^j+R^{mj}m^j\\ &\text{s.t.}\quad\left(c^j\right)^{1-\kappa-\chi}\left(s^j\right)^{\kappa}\left(m^j\right)^{\chi}=\overline{\Lambda}^j=\text{cst}\end{aligned}$$

and gives the following decision rule for housing services, financial services and the non-durable goods consumption:

$$m^{dj}=\chi P^j_\Lambda\Lambda^j/R^{mj}\ ,\ s^j=\kappa P^j_\Lambda\Lambda^j/R^{hj}\ \text{and}\ c^j=\left(1-\kappa-\chi\right)P^j_\Lambda\Lambda^j/\left(1+\tau_c\right),\tag{19}$$

[12] This term stems from the fact that $\dot{a}^j=\dot{k}^j+p^h\dot{h}^j+\dot{p}^h h^j-\dot{b}^j+\dot{m}^j$. The treatment of capital gains only matters in the dynamic setting and so is explained later on. In the stationary equilibrium, $\dot{p}^h=0$ and so $\Xi^j=0$.

with

$$P_\Lambda^j = \left(\frac{1+\tau_c}{(1-\kappa-\chi)}\right)^{1-\kappa-\chi}\left(\frac{R^{hj}}{\kappa}\right)^{\kappa}\left(\frac{R^{mj}}{\chi}\right)^{\chi}. \tag{20}$$

Overall, the above decision rules show that expenditure or demand in the three categories – non-durables, housing services or liquidity services – are increasing in aggregate expenditure Λ^j, decreasing in their weight in preferences and decreasing in their relative prices.

Let us now explain the home ownership condition. While housing demand is bounded from below given the minimum size of housing units, the upper bound is given by the borrowing constraint. Home ownership is then determined by the following conditions:[13]

$$h^j = \mathbb{1}_{h^j} s^j \text{ and } d^j = \zeta p^h h^j. \tag{21}$$

where $\mathbb{1}_{h^j} = 1$ if $h^{\min} \leq s^j \leq \frac{a^j - k^j - m^{dj}}{\zeta p^h}$ and $\mathbb{1}_{h^j} = 0$ otherwise. Last, assuming that only home-owners can buy capital implies:

$$k^j = \mathbb{1}_{h^j} \max(a^j - \left(p_h h^j - d^j\right) - m^{dj}, 0). \tag{22}$$

Any potentially remaining positive amount of wealth not allocated to housing or capital is then held in liquid form:

$$m^j = \max(a^j - \left(p_h h^j - d^j\right) - k^j, 0). \tag{23}$$

2.4 Tax and transfers system

The tradition in macroeconomic models is to pool the different taxes and monetary transfers together and consider a progressive tax schedule (see Heathcote, Storesletten, and Violante (2017) for a discussion). The latter usually features a level parameter and a progressivity parameter that both apply to the entire distribution of the tax base.[14]

Leveraging the richness of DINA series, we extend the standard approach as follows. First, we apply this approach for each type of tax and for monetary transfers *separately* based on their relevant tax bases (payroll taxes on labor income, income taxes on fiscal income, wealth taxes on wealth, and monetary transfers on fiscal income). Second, we consider varying level and progressivity parameters over the distribution of tax bases. More precisely, we split the distribution of each tax base in several segments and estimate parameters for each segment. Third, we estimate all the parameters of all the tax and transfer schedules for the reference year (1984) and for each subsequent year.

[13] Again, we consider that households who actually buy housing ($\mathbb{1}_{h^j} = 1$) borrow up to the limit, which is consistent with the cost of borrowing being systematically below the cost of renting.

[14] Heathcote, Storesletten, and Violante (2017) show that this simple functional form offers a good approximation of the tax and transfer system in the U.S. See also Cagetti and De Nardi (2009), Kaymak and Poschke (2016) or Hubmer, Krusell, and Smith (2020), among others, for applications focusing on inequality dynamics. Hubmer, Krusell, and Smith (2020) propose a more granular approach that fits a step-wise tax function on the distribution of personal income.

Each progressive tax rate $\mathcal{T}$ (income taxes, payroll taxes, wealth taxes, and monetary transfers) is household-specific, and we assume the following functional form for each segment s:

$$\mathcal{T}_s^j = 1 - \left(1 - \overline{\mathcal{T}}_s\right)\left(\frac{\mathcal{B}^j}{\overline{\mathcal{B}}_s}\right)^{-\eta_s} \tag{24}$$

For each type of tax or transfer $\mathcal{T} \in \{\tau, \tau_\ell, \phi, T\}$, the individual tax rate or transfer on segment s is described by a level parameter $\overline{\mathcal{T}}_s$ and a progressivity parameter η_s, where $\mathcal{B}$ is the relevant tax base and $\overline{\mathcal{B}}_s$ its average value on the segment s.[15]

Appendix B presents the complete methodology and the fit for each tax and transfer of the model. It shows that the French tax and transfer system features very different and non-linear progressivity patterns for different tax bases and time periods.[16]

While the traditional approach may be successful in matching the overall level of tax progressivity, it overlooks its granularity. As a consequence, it may fail to account for the effects of changes in taxes over time on the relevant margins (consumption, savings and labor supply). In Section 3.3, we show that using our approach over the traditional approach improves the mapping between pretax and posttax income distributions and is instrumental in matching the wealth distribution.

2.5 Government and market clearing

Next, we present the budget constraint of the government and the market clearing conditions.

First, the government uses the revenues from the different taxes and issues deposits $\dot{m}^s$ to finance monetary transfers as well as an exogenous amount of public good and services g. Its budget constraint yields:

$$\begin{aligned} g + \underbrace{\int_j \Omega^j T^j dj}_{\text{Transfers}} + r^m m^s \quad &= \dot{m}^s + \underbrace{\int_j \Omega^j \tau_\ell^j \left(w^j \left(1 - \mathbb{1}_{ej}\right)\ell^j + \mathbb{1}_{ej} 0.7\pi^j\right) dj}_{\text{Payroll tax}} \\ &+ \underbrace{\int_j \Omega^j \phi^j a^j dj}_{\text{Capital tax}} + \underbrace{\tau_\pi \left(\frac{r^k}{1-\tau_\pi} k + \int_j \Omega^j \mathbb{1}_{ej} 0.3\pi^j dj\right)}_{\text{Corporate tax}} \\ &+ \underbrace{\tau_c \int_j \Omega^j c^j dj}_{\text{Consumption tax}} + \underbrace{\int_j \Omega^j \tau^j \left(\Phi_\ell^j + \Upsilon_k^j\right) dj}_{\text{Income tax}} \end{aligned} \tag{25}$$

where Ω^j is the distribution of households with $\int_j \Omega^j = 1$.

[15] Payroll tax rates τ_ℓ^j are computed on labor income (see Equation (8)), income tax rates τ^j and monetary transfers T^j are computed on the total fiscal income $\Phi_\ell^j + \Upsilon_k^j$, and wealth tax rates ϕ^j are computed on a^j.

[16] See Figures 13 to 16 in Appendix B.

Second, the market clearing conditions of the capital, labor and deposit/housing debt markets are:

$$k = \int_j \Omega^j k^j dj \tag{26}$$

$$\ell = \int_j \Omega^j \left(1 - \mathbb{1}_{e^j}\right) \left(w^j / w\right) \ell^j dj \tag{27}$$

$$m^s = m - d = \int_j \Omega^j \left(m^j - d^j\right) dj \tag{28}$$

where deposit supply is controlled by the government and assumed to adjust to demand, given the interest rate r^m.

Third, given housing demand and exogenous housing prices, there exists an implicit housing supply h^s such that the housing market clears:[17]

$$h^s = h = \int_j \Omega^j h^j dj \tag{29}$$

These conditions ensure that the goods market clearing condition is met by Walras' law.[18] In equilibrium, the proportion of entrepreneurs in the economy is $e = \int_j \Omega^j \mathbb{1}_{e^j} dj$ so that profits per entrepreneur are:

$$\pi^j = \frac{y}{e\theta} \tag{30}$$

Finally, aggregate output, the real wage and the return on capital can be respectively expressed as:

$$y = \xi k^\alpha \ell^{1-\alpha} \text{ , } w = (1-\alpha) \frac{\theta - 1}{\theta} \frac{y}{\ell} \text{ and } r^k = (1 - \tau_\pi) \left(\alpha \frac{\theta - 1}{\theta} \frac{y}{k} - \delta\right) \tag{31}$$

2.6 Income concepts

Before we turn to the calibration and results, we define the different concepts of income we use, since their distribution are the basis of many objects we track in the paper. In line with the Distributional National Account literature, we use three basic income concepts in our analysis: pretax income, disposable income, and posttax income. By definition, aggregate pretax and posttax income are both equal to national income.[19] A full description of the income concepts is presented in Appendix A.

[17]As explained later on, we assume $p^h = 1$ in the stationary equilibrium, and that $\dot{p}^h$ is an exogenous variable in the dynamic simulations.

[18]The latter reads:

$$\underbrace{c + r^h p^h s}_{\text{Consumption}} + \underbrace{\dot{k} + \delta k + p^h \dot{h} + g_\xi a}_{\text{Investment}} + g = y + r^h p^h h$$

[19]National income is defined as GDP minus capital depreciation plus net foreign income, following standard national accounts guidelines (SNA 2008).

Pretax income is our benchmark concept to study the distribution of income before government intervention. It is defined as the sum of all income flows going to labor and capital, after taking into account the operation of the pension and unemployment insurance systems, but before taking into account other taxes and transfers. That is, we deduct pension and unemployment contributions and add pension and unemployment distributions. To recover the concept of pretax income in our model, we reassign corporate taxes and non-contributive payroll taxes to the labor and capital incomes of households.[20]

Disposable income is defined as pretax income minus all forms of taxes plus monetary transfers (T^j).

Posttax income is defined as the sum of all income flows going to labor and capital, after considering all forms of government interventions. It is equal to disposable income plus in-kind transfers and collective consumption expenditure net of the government balance budget (g in our model, rebated on a lump-sum basis).

3 Calibration and stationary equilibrium

We solve the model in two steps. The first step consists in finding a stationary equilibrium, including a stationary distribution of asset holdings, a composition of portfolios, and policy functions over an asset grid a^j where we have imposed that all variables of the model are constant.[21] We discretize the labor income risk process with 5 states – very low, low, medium, high and very high productivity. Along with the state of entrepreneurs, we have 6 states and use 501 grid points. We consider the economy to be in the stationary equilibrium in 1984 and use French data for this year to calibrate the model. The second step, used in the dynamic simulations, solves for the transition dynamics using a non-linear algorithm with a variety of exogenous variables to analyze their effects on aggregate and distributional dynamics. The details of both steps are given in Appendix C.

3.1 Calibration

The model is first solved along the first step and calibrated at an annual frequency using data or targeting data moments pertaining to the French economy in 1984, the year before inequalities start rising in the data. For some parameters, there is a direct mapping between the model's moment and the data. For other parameters, the mapping is too complex and we use a minimum distance method to set the parameters to the values that best fit the moments. In any case, the stationary equilibrium involves constant asset prices, *i.e.* $p^h = 1$.

[20] See Appendix A for more details.

[21] With labor-augmenting productivity growth, this implies that all quantities – except hours worked – grow at the exogenous rate of labor productivity g_{ξ}.

Earnings and transition probabilities. The growth rate of productivity is set to capture the average growth rate of national income per capita over the period, *i.e.* $g_\xi = 0.01$. The Gaussian mixture process for the productivity of workers is discretized using codes by Farmer and Toda (2017) and several parameters are set following Ferriere et al. (2023). We set the probability of 'good' (positive mean and small variance) shocks to $p_1 = 0.85$, the mean to $\psi_1 = 0.017$ and the variance to $\sigma_1^2 = 0.15$. The variance of 'bad' (negative mean and large variance) shocks is set to $\sigma_2^2 = 0.5$, and the process implies $\psi_2 = -0.0963$. The persistence of this process is driven by ρ_z and adjusted to match key moments of the distribution of pretax income using minimum distance methods (see dedicated paragraph below), which yields $\exp(-\rho_z) = 0.9343$. This large persistence aligns well with independent empirical evidence about labor inequality in France (see Kramarz, Nimier-David, and Delemotte (2022)). The discount factor ρ and the probability of becoming an entrepreneur p_{ew} do not map directly into observable moments, so they are also chosen to match distributional and aggregate moments. The probability of becoming a worker when an entrepreneur p_{we} is set to match the average business failure rate in France. Based on an extensive firm-level dataset for France, Aghion et al. (2018) report an overall stable exit rate of 12% between 1993 and 2015, so that $p_{we} = 0.12$.

Preferences. We set the relative risk-aversion parameter to its usual value $\gamma = 2$. In line with Gaillard et al. (2023), the utility weight of relative wealth is $\beta = 0.2$. The utility weights of financial services χ, housing services κ, and the scale parameter for wealth in utility μ are set to match observed moments using a minimum distance method. As shown by Francis (2009) and in our counterfactual analysis, the latter is critical in matching top percentiles of the wealth distribution. The minimum size of a housing unit h^{min} is set to 3 years of the average pretax income, *i.e.* $h^{min} = 3y$, and the borrowing constraint parameter implies a minimum 1/4 of housing to be financed by personal net wealth, the remaining $\varsigma = 3/4$ being borrowed. Finally, in line with evidence by Chetty et al. (2011) on intensive margin adjustments of labor supply, we impose a Frisch elasticity of $1/\zeta = 1/2.5 = 0.4$.

Firms, markups and asset returns. Recent micro evidence for the manufacturing sector in France by Bauer and Boussard (2020) suggests high markups, around 38.4% in 1984. However, aggregate markups also take into account (lower) markups in the service sector, and no markup at all in the public sector.[22] Hence, we assume a lower aggregate markup of 10%, implying $\theta = 11$, which is the value used by Kaplan, Moll, and Violante (2018). Data from national accounts point to an aggregate labor share of 0.755 in 1984. In our model, this implies adjusting the capital elasticity to $\alpha = 0.2624$, again based on our minimum distance procedure. The depreciation rate of capital is taken directly from national accounts data: $\delta = 0.1128$. The real interest rates on deposits and housing in 1984 are taken from the data and adjusted for risk as in the simplified procedure proposed by Kaplan and Violante (2014), which implies $r^m = 0.01$ and

[22] National accounting values public goods and services at their production costs.

$r^h = 0.0297$.[23] As already mentioned, the initial relative price of housing is normalized to $p^h = 1$. In equilibrium, the resulting net rental rate of capital is $r^k = 0.0595$.

Government. Our calibration focuses on effective – not statutory – tax rates. Using data from national accounts, the effective consumption tax (VAT) rate in 1984 was $\tau_c = 0.3161$. This number is higher than the statutory VAT rate because this tax captures several indirect taxes from the data. Similarly, the effective corporate tax rate was $\tau_\pi = 0.09$, and the amount of government expenditure on goods and services was $g/y = 0.2934$. To model the monetary transfers and each progressive taxes (payroll taxes, wealth taxes, income taxes), we rely on Equation (24) and estimate a level parameter and a progressivity parameter for each segment of the corresponding tax base distribution. This estimation relies on the French DINA series by Bozio et al. (2024), which provide detailed annual series of the joint distribution of pretax income, posttax income and wealth, and is broken down by income and tax categories. See Appendix B for a complete presentation of the methodology and the fit for each tax and transfer of the model.

Moments matching. The following parameters of the model are set to match key distributional and aggregate moments from the data. The discount factor ρ, the output elasticity of capital α, the persistence of the labor income risk $\exp(-\rho_z)$, the probability of becoming an entrepreneur when working (p_{ew}), and the following preference parameters χ (financial services), κ (housing services), and μ (scale parameter for wealth in utility) are all set together to match empirical moments. Our target moments are the following: the bottom 50%, middle 40%, top 10%, and top 1% shares of both pretax income and wealth, the aggregate shares of deposits (m/a) and housing (h/a) in total wealth, the wealth-income ratio a/y and the labor share computed as the aggregate labor pretax income divided by the aggregate pretax income.

Our moment matching procedure implies a discount factor $\rho = 0.0975$; an output elasticity of capital $\alpha = 0.262$ implying a labor share of 0.748 (against 0.755 in the data); a persistence of labor income risk $\exp(-\rho_z) = 0.9343$, and a probability of becoming an entrepreneur $p_{ew} = 0.002$, implying a stationary proportion $e = 1.63\%$ of entrepreneurs. Keeping in mind that the probability of losing the status is $p_{we} = 0.12$, the entrepreneurial process implies large precautionary savings for entrepreneurs: the status is quite risky and implies receiving more income than workers, since profits (roughly 10 percents of output) are shared among few individuals. The matching values of $\chi = 0.0342$ and $\kappa = 0.2681$ imply that deposits represent $m/a = 0.152$ of aggregate wealth (the exact value of the data) and housing $p^h h/a = 0.439$ (against 0.429 in the data). Finally, the wealth in utility scale parameter is $\mu = 11.2$. This calibration delivers a wealth-income ratio of 3.335, very close to the observed ratio (3.241). Parameter values are summarized in Table 1 and produce a very good fit with our targets.

Indeed, our calibration of the model delivers stationary distributions of income and wealth that reproduce several features of the data in 1984, as shown in Table 2. The model delivers a

[23] We consider the return on housing computed by Garbinti, Goupille-Lebret, and Piketty (2020). For each year from 1984 to 2018, we compute the risk-adjusted returns by dividing the returns by the standard deviation of the past 10 years. The average of these risk-adjusted returns between 1984 and 2018 are $r^m = 0.01$ and $r^h = 0.0297$.

Table 1: Parameter values and initial values of exogenous variables.

Parameters		
Steady-state growth rate	$g_\xi = 0.01$	(fixed)
Discount rate	$\rho = 0.0975$	(moments matching)
Persistence of labor income risk	$\exp(-\rho_z) = 0.9343$	(moments matching)
Probability of good labor income shock	$p_1 = 0.85$	(FGNV (2023))
Mean of good labor income shock	$\psi_1 = 0.017$	(FGNV (2023))
Variance of good labor income shock	$\sigma_1^2 = 0.15$	(fixed)
Mean of bad labor income shock	$\psi_2 = -0.0963$	(FGNV (2023))
Variance of bad labor income shock	$\sigma_2^2 = 0.5$	(fixed)
Prob. of becoming an entrepreneur	$p_{ew} = 0.002$	(moments matching)
Prob. of becoming a worker	$p_{we} = 0.12$	(ABBB (2022))
Relative risk-aversion	$\gamma = 2$	(fixed)
Weight of housing services in utility	$\kappa = 0.2681$	(moments matching)
Indivisible housing parameter	$h^{\min} = 3 \times y$	(fixed)
Borrowing constraint parameter	$\varsigma = 0.75$	(fixed)
Weight of financial services in utility	$\chi = 0.0342$	(moments matching)
Elasticity of labor supply	$1/\zeta = 1/2.5 = 0.4$	(CGDW (2011))
Weight of relative wealth in utility	$\beta = 0.2$	(GHWW (2023))
Wealth in utility scale parameter	$\mu = 11.2$	(moment matching)
Output elasticity of capital	$\alpha = 0.262$	(moment matching)
Initial values of exogenous variables		
Capital depreciation	$\delta = 0.1128$	(data)
Markups	$\theta/(\theta-1) = 1.1$	(KMV (2018))
Return on equity	$r^k = 0.0595$	(result)
Return on housing	$r^h = 0.0297$	(data)
Relative housing prices	$p^h = 1$	(normalization)
Return on deposits	$r^m = 0.01$	(data)
Government spending to output	$g/y = 0.2934$	(data)
Consumption tax rate	$\tau_c = 0.3161$	(data)
Corporate tax rate	$\tau_\pi = 0.09$	(data)
Progressive tax rates and transfers	See Appendix B	(data)

good match of the bottom 50%, middle 40%, top 10%, and top 1% shares of the pretax income and wealth. Further, although the model does not target any of the posttax shares of income, it matches them very well, showing that our assumptions regarding taxes and transfers capture key redistributive features of the French system.

Table 2: Moments from the data (1984) vs. model.

	Data (1984)			Model (1984)		
	Pretax	Posttax	Wealth	Pretax	Posttax	Wealth
Bottom 50%	0.230	0.337	0.081	0.240	**0.349**	0.081
Middle 40%	0.486	0.450	0.406	0.478	**0.443**	0.388
Top 10%	0.283	0.214	0.513	0.283	**0.208**	0.531
Top 1%	0.070	0.046	0.160	0.080	**0.051**	0.160
Share of deposits in agg. wealth		0.152			0.152	
Share of housing in agg. wealth		0.429			0.439	
Wealth to income ratio		3.241			3.335	

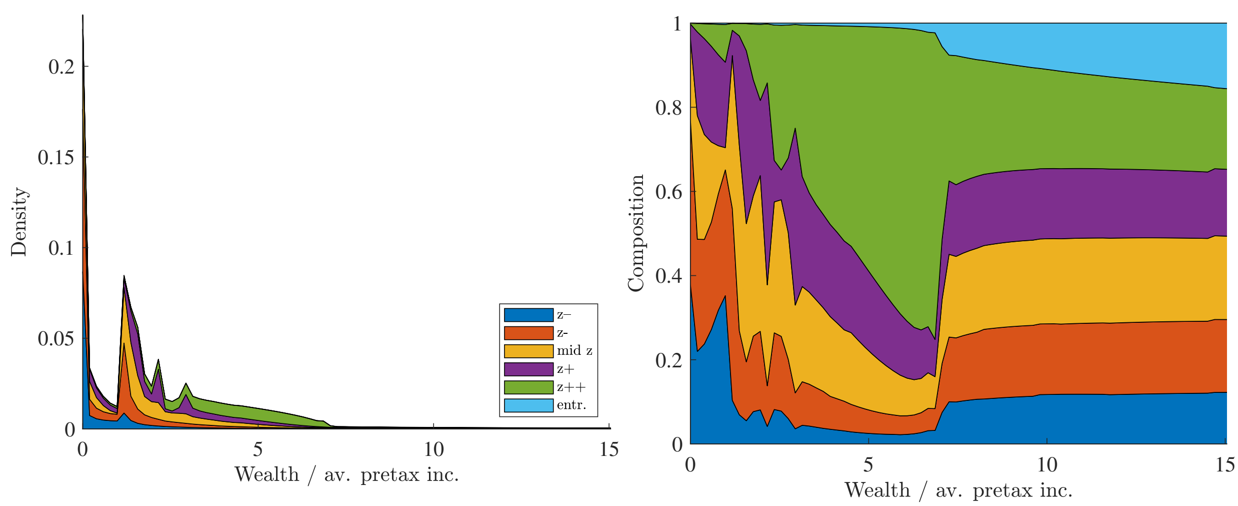
Note: Bold numbers are not targeted.

3.2 Initial stationary equilibrium

The calibration gives rise to the stationary distribution of households depicted in Figure 1.

Figure 1: Stationary distributions

(a) Stationary distributions (cumulated). (b) Composition of the population.



As expected, Panel (a) of Figure 1 shows that the stationary distribution is highly skewed to the left and fat-tailed at the right. The double peak of the stationary distribution – one at zero and one at 1.5 times the average pretax income – arises from the indivisible housing constraint and from the fact that owning its housing unit is cheaper than renting, as it involves a lower opportunity cost R^{hj}. Hence, households face incentives to hold just enough wealth to own their housing unit. Beyond that, the other features of our stationary distribution are relatively standard. The transition probabilities implied by our labor income process and the transition probabilities towards the status of entrepreneur determine the respective stationary proportions of household types: workers with very low or low productivity respectively represent 15.6% and 22.8% of the population, workers with an average productivity represent 24.2%, workers with high or very high productivity respectively make 17.7% and 18.1%, and entrepreneurs represent the remaining 1.6%.

Panel (b) of Figure 1 indicates that wealth increases with the level of productivity as the fraction of high-productivity workers increases with asset holdings. In addition, entrepreneurs are concentrated at the top of the wealth distribution. While they represent a small fraction of total population (1.6%), entrepreneurs are quite rich because they receive the total profits from firms – 10% of total output. In addition, they precautionary-save because the probability of becoming a worker is much larger than the probability for workers of becoming an entrepreneur. Hence, they represent a large share of top wealth owners, as shown by Panel (b) of Figure 1.

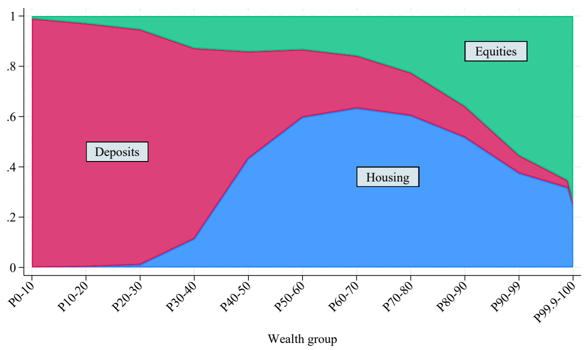
The model generates the policy functions depicted in Figure 17 in Appendix D. They help rationalize the shape and composition of the stationary distribution: entrepreneurs and highly productive workers at the top of the wealth distribution. They also show the usual determinants of labor supply, wealth and substitution effects. Last, Panels (d) and (e) of Figure 17 inform about the individual compositions of portfolios by types and wealth levels. They show that households at the bottom of the asset grid do not save enough to reach the threshold to become homeowners, and therefore keep their wealth in the form of liquid deposits. When households save enough to buy housing, they allocate almost all of their wealth to housing and then diversify their portfolios by holding capital. As a result of the varying composition of portfolios along the distribution of wealth, individual pretax returns are increasing in wealth (Panel (f) of Figure 17). How do these individual policy function interact with the distribution? Aggregating over households at the different levels of the asset grid, we obtain the aggregate portfolio compositions and portfolio returns along the wealth distribution depicted in Figure 2.

Clearly, households at the bottom of the wealth distribution hold only deposits. Above the third decile, households hold an increasingly large (up to 75%) share of their wealth in the form of housing, fewer deposits and the rest in capital. Above the seventh decile, the share of housing starts declining, the share of deposits continues to shrink, and the share of equity increases to reach 75% for the top 0.1% of the wealth distribution. These equilibrium portfolio compositions produce increasing returns along the wealth distribution, as in the data.[24] Since deposits carry the lowest returns (0.9%), households holding only deposits receive low returns. When portfolios start including housing, which carries a larger return (2.97%), portfolio returns start increasing. Returns further increase when households start holding equity, which carries the largest return

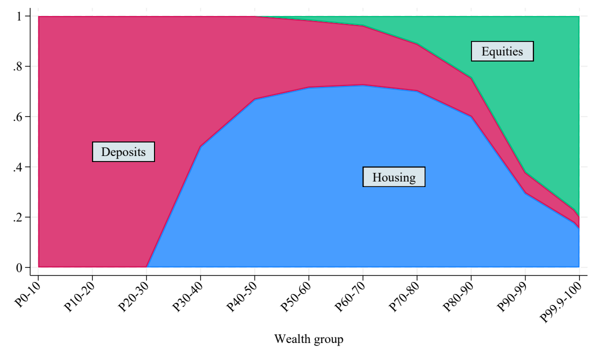
[24]See Garbinti, Goupille-Lebret, and Piketty (2021) for similar evidence from France, and Cao and Luo (2017) or Xavier (2020) for evidence based on U.S. data.

Figure 2: Portfolio composition and returns among wealth groups in 1984.

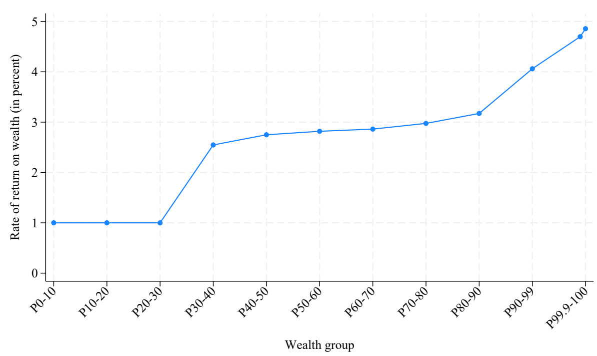
(a) Portfolio composition - data



(b) Portfolio composition - model



(c) Pretax rates of returns

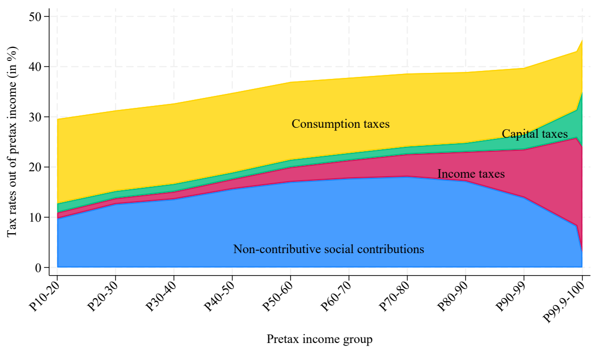


Notes: Series from Panel (a) come from Garbinti, Goupille-Lebret, and Piketty (2021). In Panel (c), rates of returns are computed by weighting each asset-specific rate of returns (housing, equities, and deposits) by the proportion of each asset in the wealth of the group.

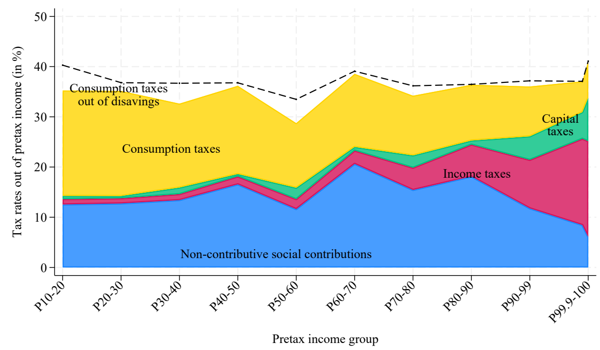
(5.95%). The differential composition of portfolios explains that top wealth owners receive returns that exceed those of bottom wealth owners by more than 3 percentage points.

Figure 3: Taxes paid (in % of pretax income) by wealth or pretax income groups, France 1984.

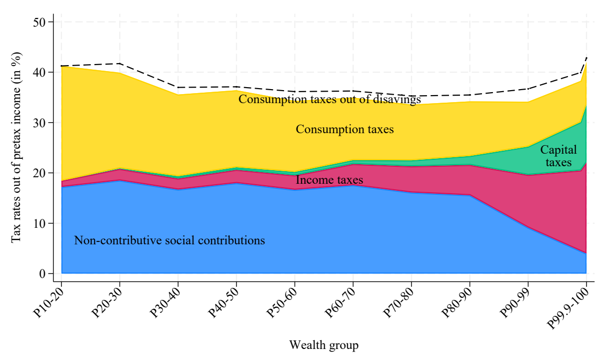
(a) Data (pretax income group)



(b) Model (pretax income group)



(c) Model (Wealth group)



Notes: Data from Panel (a) taken from Bozio et al. (2024).

Figure 3 reports the amount and composition of taxes paid by households ranked by income and wealth groups as a fraction of their pretax income. The consumption tax is regressive given that poorer households consume a larger fraction of their disposable income. Payroll taxes are also regressive at the top of the income distribution. Our results further show a strong progressivity of the income and wealth tax schedule. The latter is especially progressive at the very top because of large levels of wealth and because of large returns on wealth, driven by larger shares of equity in portfolios.

Overall, the model provides a close fit with the data on a large variety of dimensions. By design of our moment matching procedure, it fits the pretax income and wealth shares up to the top 1%, the aggregate composition of wealth, the aggregate wealth-income ratio and the labor share. It also fits non-targeted features of the data: the posttax income shares up to the top 1%, the composition of portfolios and increasing returns along the wealth distribution, and the composition of taxes paid along the pretax income distribution.

Table 3: Alternative specifications

	Base.	Counterfactual (Δ with baseline level)					
		$\theta \to \infty$	$\beta = 0$	$\zeta \to \infty$	Flat tax	One tax	One asset
Pretax inc.							
B50	0.24	$+0.02$	-0.01	-0.03	$+0.00$	$+0.00$	$+0.06$
M40	0.48	$+0.05$	$+0.02$	$+0.01$	$+0.00$	-0.00	-0.06
T10	0.28	-0.07	-0.00	$+0.02$	-0.01	$+0.00$	-0.00
T1	0.08	-0.06	-0.00	-0.00	$+0.00$	$+0.00$	$+0.00$
Posttax inc.							
B50	0.35	$+0.01$	-0.01	-0.01	-0.03	-0.02	$+0.11$
M40	0.44	$+0.03$	$+0.01$	$+0.00$	$+0.00$	-0.00	-0.10
T10	0.21	-0.04	-0.00	$+0.01$	$+0.02$	$+0.02$	-0.01
T1	0.05	-0.04	-0.00	-0.00	$+0.01$	$+0.01$	$+0.00$
Wealth							
B50	0.08	$+0.07$	-0.00	0.02	-0.01	-0.04	-0.07
M40	0.39	$+0.17$	$+0.06$	-0.02	-0.02	-0.02	-0.06
T10	0.53	-0.24	-0.06	0.01	$+0.04$	$+0.06$	$+0.13$
T1	0.16	-0.12	-0.03	-0.00	$+0.02$	$+0.03$	$+0.06$
Agg. ratios							
$m/\mathcal{W}$	0.15	-0.02	-0.01	0.00	$+0.09$	$+0.09$	-0.15
$h/\mathcal{W}$	0.44	$+0.04$	$+0.02$	0.00	-0.13	-0.15	-0.44
$k/\mathcal{W}$	0.41	-0.02	-0.01	-0.01	$+0.05$	$+0.06$	$+0.59$
$\mathcal{W}/\mathcal{Y}^{pretax}$	3.34	$+0.07$	-0.21	0.03	$+0.35$	$+0.16$	-1.89

Note: The table expresses the key moments targeted by our model. The first column refers to the baseline model and results, moments are expressed in levels. In the remaining columns, moments are expressed in difference from the baseline case. Column 2: no markups and thus no entrepreneurs. Column 3: wealth not in the utility function. Column 4: inelastic labor supply. Column 5: all progressive taxes and transfers replaced by flat rates that equal the average rates paid in the baseline model. Column 6: all progressive taxes and corporate taxes replaced by a single progressive income tax, as usually done in macro models (unique level and progressivity parameters estimated from the data). Monetary transfers are rebated lump-sum. Column 7: one asset (capital) is considered and thus $\chi = \kappa = 0$ in the utility function.

3.3 Alternative specifications

We now investigate the contribution of our assumptions to the results by reporting the distributions of income and wealth when we simplify or abstract from some of our assumptions. Table 3 reports the baseline moments in the first column and the difference between the alternative distribution and the baseline in the remaining columns. In particular, it allows us to study the role of saving motives and of a progressive tax system in explaining the level of income and wealth inequality in France in 1984. Positive numbers indicate that the alternative overshoots the baseline moments while negative numbers signal undershooting.

First, consider an alternative economy without markups assuming $\theta/(\theta-1) = 1$ by imposing $\theta \to \infty$ (Column (2) of Table 3). Doing so drives aggregate profits to zero, and thus extinguishes the primary source of income of entrepreneurs. It redistributes income and wealth from the top to the bottom of the distribution. The bottom 50% and middle 40% pretax income shares are larger (+2pp and +5pp respectively) than their baseline values, and the top 10% and 1%

shares shrink (-7pp and -6pp respectively). Because being an entrepreneur is risky and implies large precautionary savings, this movement is widely amplified for the wealth shares: removing markups results in undershooting the top 10% and top 1% wealth shares by 24pp and 12pp, respectively. We conclude that the presence of markups and entrepreneurs in the baseline model is central to match top income and wealth shares.

What if utility does not depend on wealth and therefore a saving motive is shut down? Column (3) of Table 3 reports the corresponding results. As expected, wealth in utility affects the distribution of wealth but not significantly the distribution of income. The main channel through which wealth in utility operates is that it prevents the saving rate from decreasing too fast with wealth. Its absence results in lower saving rates at the top of the wealth distribution and lowers the top 10% wealth share by 6pp and the top 1% wealth share by 3pp.

Assuming an inelastic supply of labor by setting $\zeta \to \infty$ (Column (4) of Table 3) slightly increases pretax income inequalities among workers: the bottom 50% share of pretax income is 3pp lower while the middle 40% and top 10% shares are respectively 1pp and 2pp higher. Because wealth effects dominate at the bottom of the wealth and income distributions, endogenous labor supply dampens the dispersion of labor income. Shutting down this channel thus raises labor income inequalities. The impact on posttax income inequalities stems from the impact on pretax income inequalities, and the effect on wealth inequalities is negligible.

The next alternative answers a longstanding question: What is the effect of tax and transfer progressivity on pretax income inequality? Our alternative shows not much. However, the effects on wealth inequality are more significant. Column (5) of Table 3 reports the results when we apply the (weighted) average tax or transfer rate to all households uniformly. Switching to a flat system of taxation leaves pretax income inequality almost unchanged but unsurprisingly raises posttax income inequality. A flat tax and transfer system has a more significant effect on wealth inequality: the bottom 50% and middle 40% wealth shares fall by 1pp and 2pp respectively, while the top 10% and top 1% wealth shares are 4pp and 2pp higher. These results are mostly driven by a flat taxation of capital, which favors wealth accumulation for top savers.

The two last alternative distributions highlight our contribution relative to standard macroeconomic models by respectively looking at a less granular tax schedule and at a model with capital as the only asset. In the former case, all progressive taxes – except transfers – are pooled with the corporate tax into a single income tax. A single average rate $\tau = 0.3075$ and a single progressivity parameter $\eta = -0.0589$ are estimated from French fiscal data in 1984. In the latter case, we make capital the only possible vehicle of savings by imposing $\chi = \kappa = 0$. With a single income tax including all the sources of taxation (Column (6) of Table 3), the model fails to replicate the distribution of posttax income to some extent and wealth to a larger extent. Just as when tax rates are flat, a system with only one tax understates the progressivity of the French tax and transfer system and results in increased posttax income inequalities, and substantially larger wealth inequalities: the bottom 50% and middle 40% wealth shares are 4pp and 2pp too low, and the top 10% and top 1% wealth shares are 6pp and 3pp too high. Last, by definition, a one-asset model – abstracting from deposits and housing – can not capture the composition of portfolios. It also widely fails to match the wealth-income ratio. Further, the returns on wealth become uniform along the wealth distribution. This makes households at the bottom of the income and wealth distributions richer in terms of income through higher returns on savings. It also makes them poorer in terms of wealth, as less savings are needed – given higher returns compared to the baseline model – to reach a given amount of self-insurance through savings. The bottom 50% and middle 40% pretax income shares are respectively 6pp above and 6pp below their baseline value. But the most striking effect is on wealth distribution: the one-asset model undershoots the bottom 50% and middle 40% wealth shares by 7pp and 6pp respectively, and overshoots the top 10% and top 1% wealth shares by 13pp and 6pp.

4 Dynamic simulations

Dynamic simulations of the model are now conducted (second step of the solution) feeding a sequence of changes to exogenous variables from 1984 to 2018. Because the Hamilton-Jacobi-Bellman equation is forward-looking, several current-period variables depend on their expected future path. We thus solve the model for an additional 30 years after 2018. An important contribution of our model is the endogenous composition of wealth portfolios along the distribution, which allows us to consider capital gains. We first discuss the (exogenous) dynamics of capital gains and how they are taken into account, and then discuss other exogenous variables.

4.1 Exogenous variables: capital gains

Our dynamic simulations consider exogenous capital gains, or asset prices, as a potential driver. Capital gains matter because, in the data, wealth levels and shares take account of real changes in asset prices – nominal changes deflated by inflation – whether these gains are realized – cashed by individuals – or unrealized. In France, between 1984 and 2018, gross housing prices and equity prices have respectively increased by more than 120% and 70%. Abstracting from capital gains would thus prevent our dynamic simulations from matching the spectacular rise in the observed wealth-income ratio. But capital gains also potentially matter for wealth shares.

Indeed, imagine that the composition of portfolios remains constant from 1984 onwards – an assumption Fagereng et al. (2019) coin as "saving by holding", because of large portfolio adjustment or transaction costs, behavioral biases or regulatory constraints. Imagine also that all capital gains are unrealized. Even in this extreme case, changes in asset prices have deep consequences, because housing wealth increases faster than capital wealth, and homeowners mechanically become wealthier than households holding other assets. But of course capital gains are changes in asset prices, and may also have effects on portfolio compositions. In our model, asset prices affect households decisions through two channels: (*i*) changes in the relative housing prices p^h alter the demand for housing services, the borrowing constraint and the conditions of home ownership more generally, and (*ii*) changes in asset prices – both housing and equity – change the value of individual wealth and thus the wealth tax rate ϕ^j, which affect savings decisions.

We thus proceed as follows. First, we incorporate pure valuation effects, captured by Ξ^j in our model, and treat them as in Fagereng et al. (2019). That is, we assume $\Xi^j = 0$ in the optimization problem of households and reflate housing and capital holdings by the observed housing gains $\dot{p}^h/p^h$ and equity capital gains $\dot{p}^k/p^k$ *after* optimization.[25] This increases both the left-hand side ($\dot{a}^j$) and right-hand side (Ξ^j) of the budget constraint of each household. Note that it also affects savings decisions because it changes the individual wealth applied to the wealth tax schedule, and thus the wealth tax rate ϕ^j paid by each household. Second, a fraction (20%) of the observed changes in housing prices is taken into account by households when optimizing, reflecting the (low) turnover of housing units.

4.2 Other exogenous variables

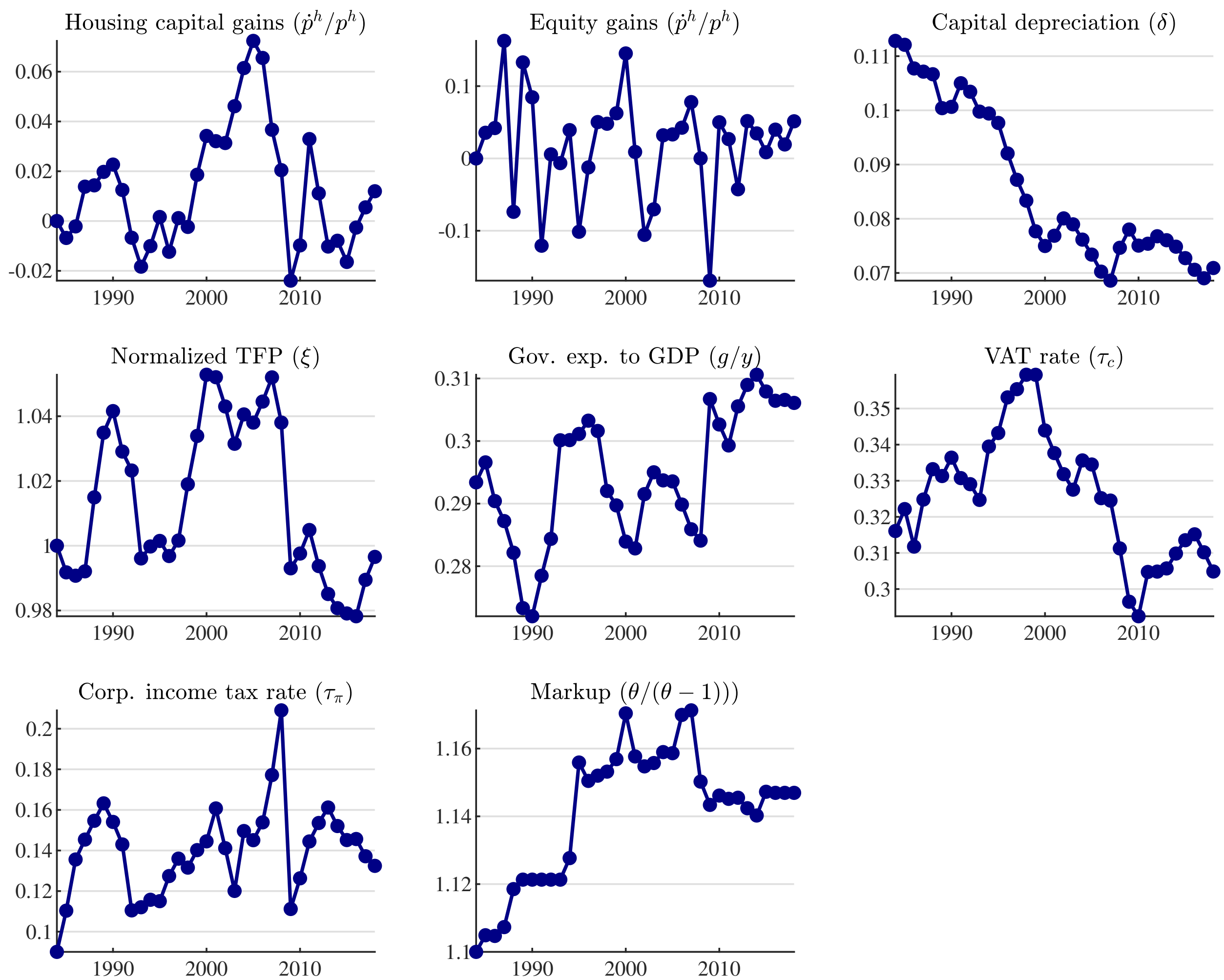
We also consider other exogenous variables. First, we introduce exogenous variations in aggregate markups between 1985 and 2016 based on the changes estimated by Bauer and Boussard (2020). We take their reported percentage variations and apply them to the 1984 level of markups. As in Boar and Midrigan (2019) and Deb (2024), rising market power may account for increase income and wealth inequality because markups are the only source of income for entrepreneurs, who are at the top of the income and wealth distributions. Second, we introduce time-varying parameters in all taxes, transfers, and government spending, for every year from 1984 to 2018, as changes in the level and progressivity of taxes and transfers may substantially affect income and wealth inequality dynamics. Last, we consider a time-varying depreciation rate of capital δ, derived using national account data, and a time-varying path for TFP. The level of TFP is taken from the long-term productivity database of Bergeaud, Cette, and Lecat (2016). Since the model already features labor productivity growth in the steady-state at rate g_ξ, we feed the model with log-deviations of TFP from an HP-filtered trend with $\lambda = 5000$.

Figure 4 reports the exogenous variables used to run dynamic simulations.[26] The model is solved for an additional 30 years after 2018. After 2018, when there are no data for exogenous variables anymore, the latter are assumed to remain equal at their 2018 values. Post-2018 capital gains are assumed to be zero, *i.e.* asset prices are stabilized at their 2018 levels for the later years.

[25] We take the housing capital gains and capital (equity & bonds) gains from Bozio et al. (2024).

[26] For level and progressivity parameters of progressive taxes and transfers, there are too many parameters to track but they are available upon request.

Figure 4: Exogenous variables

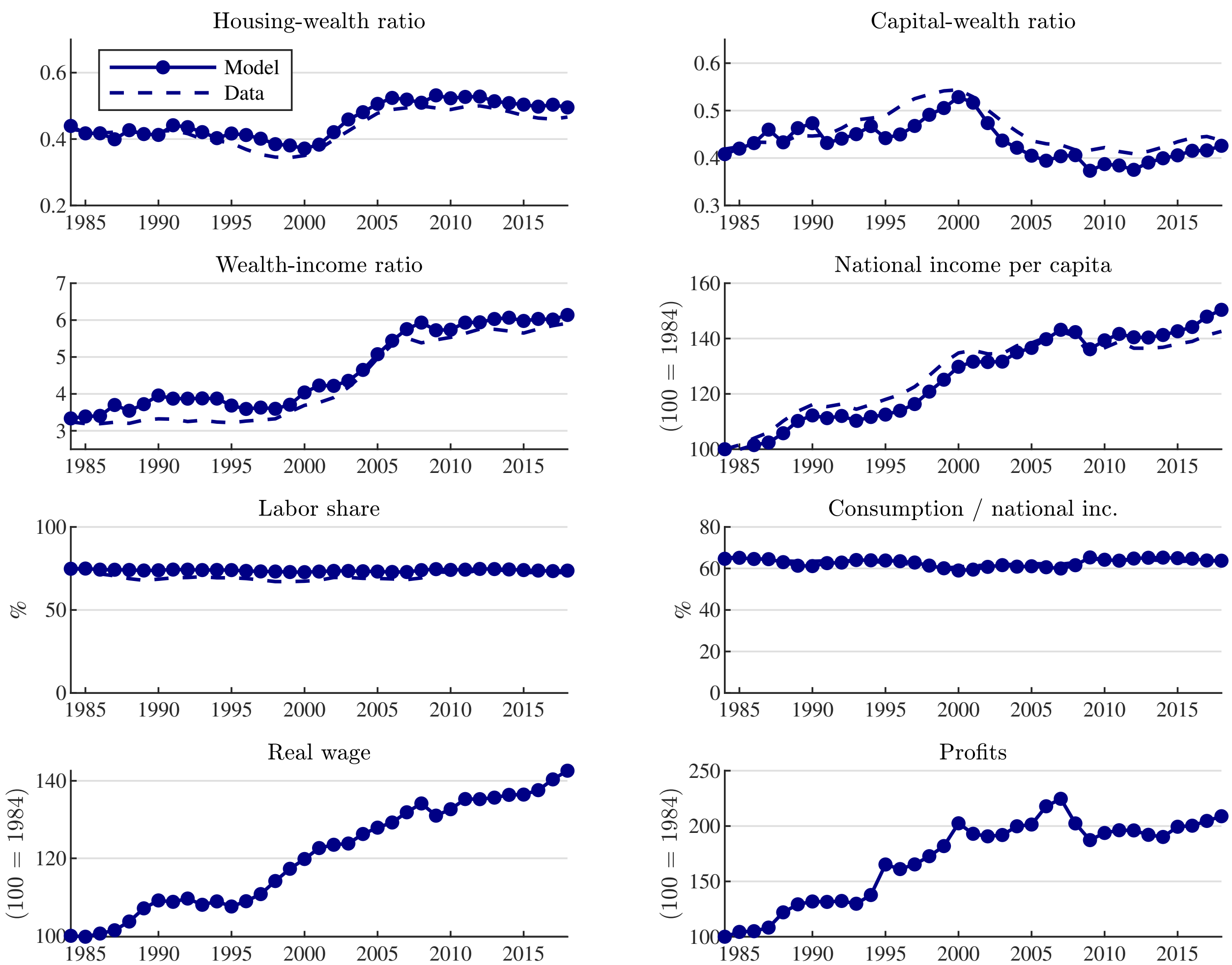


4.3 Results

Let us start by looking at the performance of our simulated model in replicating aggregate features of the data.

The top panels of Figure 5 report the evolutions of the aggregate housing-wealth, capital-wealth and wealth-income ratios, as well as the national income per capita. The model accounts very well for the fall of the housing-wealth ratio from 1984 to 2000 and for its rise after 2000. An opposite movement – a rise until 2000 and then a fall – of the capital-wealth ratio is observed and well reproduced by the model. Further, the observed wealth-income ratio rises from 3.2 in 1984 to almost 6 in 2018, an overall increase that our model matches closely. Finally, the dynamics of the national income are also matched very closely, an additional indication of the excellent performance of our model simulations.

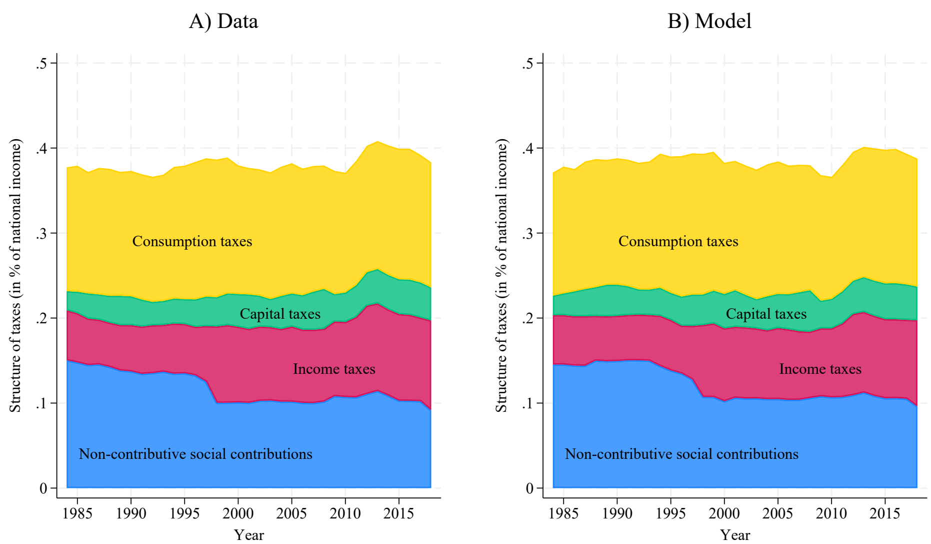
Figure 5: Macroeconomic variables



The bottom panels of Figure 5 report the dynamics of the labor share, the consumption-income ratio, and the normalized levels of the average real wage and aggregate profits. The model simulations track the observed dynamics of the labor share and of the consumption-income ratio very well. For the labor share, note that the rise in profits is relatively neutral because 70% of profits are recorded in the model (and taxed) as labor income while the remaining 30% are treated as capital income. Finally, since our simulations are driven by a significant increase in markups, profits increase more than the average real wage over the period.

Figure 6 reports the simulated and observed decomposition of taxes in percentage of national income over time. The model reproduces almost perfectly the evolution of the aggregate tax structure and of the aggregate tax level.

Figure 6: Structure of taxes



Finally, Figures 7 to 9 depict the performance of our simulated model in reproducing inequality dynamics. The model fits very closely the evolution of pretax income and posttax income shares for all income groups (bottom 50%, middle 40%, top 10% and top 1%). The observed dynamics of wealth shares are also very well captured by our model simulations for the bottom 50%, middle 40%, and top 10% wealth groups. For the top 1% wealth share, the overall increasing pattern is qualitatively well reproduced, but the model undershoots the large variations observed around 2000, resulting in a lower simulated top 1% wealth share in 2014 (19% vs. 24% in the data).

Figures 7 to 9 highlight two main facts regarding inequality dynamics: (*i*) a rise in income inequality driven mostly by a significant increase in the top 1% income share from 8% in 1984 to 11% in 2018 (+36% over the 1984-2018 period) and (*ii*) a rise in wealth inequality driven by a significant increase in the top 10% and top 1% at the expense of the bottom 50% and middle 40% wealth shares. We provide a deeper understanding of the key forces and transmission mechanisms in the next section.

Figure 7: Pretax income inequalities

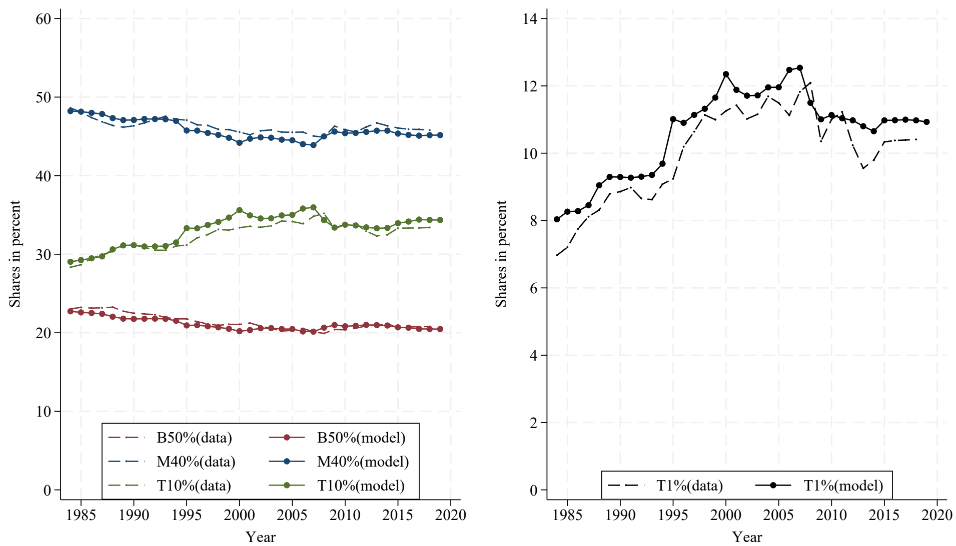


Figure 8: posttax income inequalities

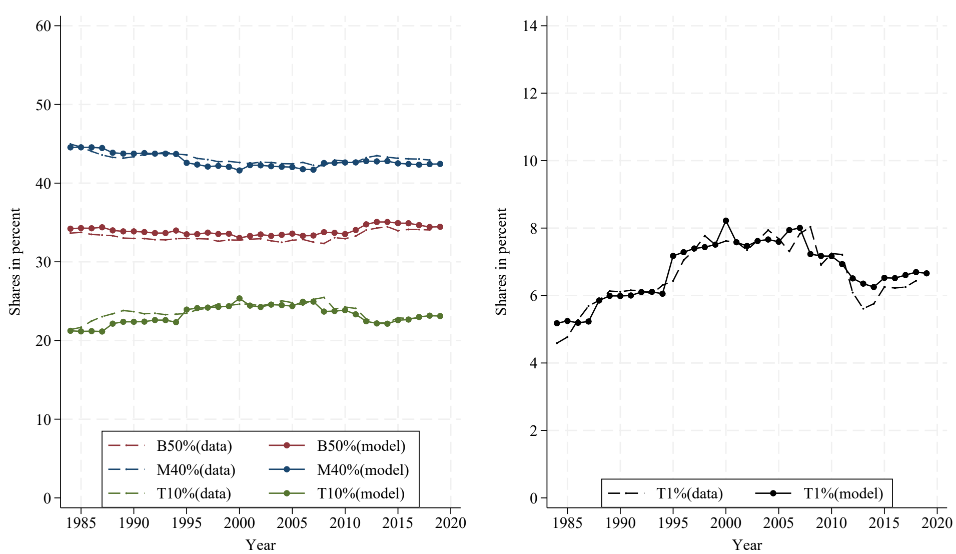
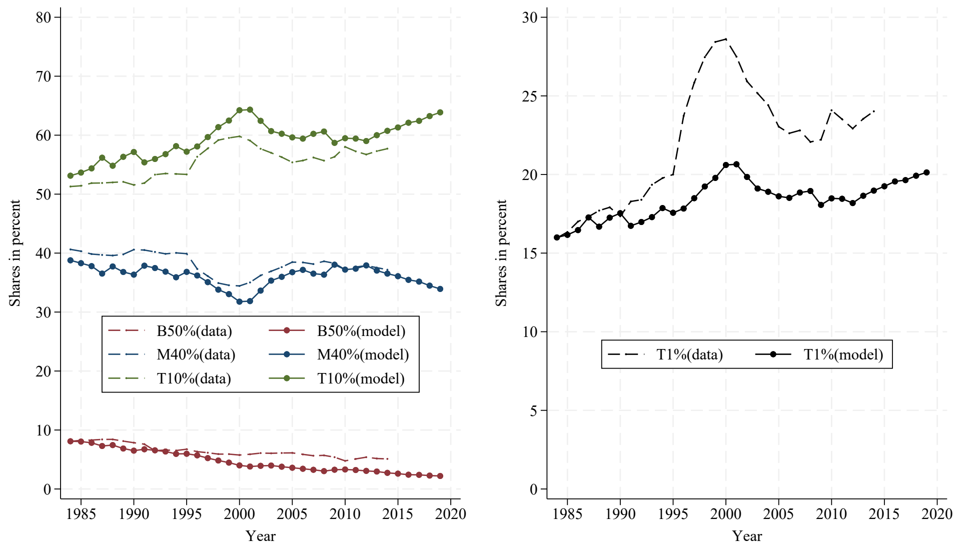


Figure 9: Wealth inequalities



5 Counterfactual dynamics

In this Section, we conduct counterfactual analyzes to shed light on how the different exogenous variables shape income and wealth inequality dynamics. We also propose a new method to identify the channels through which the different exogenous variables contribute to wealth inequality dynamics. In particular, the method allows to single out mechanical from behavioral and general equilibrium effect.

To do so, we group exogenous variables in four different categories: capital gains, taxes and transfers, markups, and others market forces (capital depreciation rate and TFP). We then run counterfactual simulations assuming that one or several groups of exogenous variables remain constant and equal to their 1984 level over the 1984-2018 period. Then we compare the resulting income and wealth shares with our benchmark series to quantify the respective contributions of each group of exogenous variables to the evolution of income and wealth inequalities.

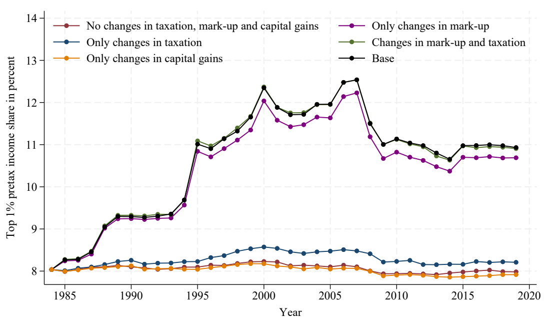
Furthermore, for each of the counterfactual scenario, we also propose a method that quantifies the contribution of transmission channels to wealth inequality dynamics. Our method is based on a simple wealth accumulation equation that identifies all the potential transmission channels: (i) aggregate pretax income dynamics, and the dynamics of inequality in (ii) pretax income, (iii) tax progressivity, (iv) saving rates, and (v) capital gains.

5.1 Income and wealth inequality dynamics by counterfactual scenario

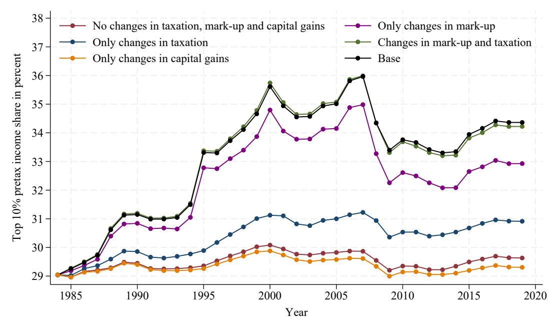
We first investigate the impact of our exogenous variables on inequality by focusing on the evolution of the top 1%, top 10%, middle 40%, and bottom 50% income shares (Figure 10) and wealth shares (Figure 11) by counterfactual scenario. The red curves represent the evolution of inequality in absence of changes in taxation, markups, and capital gains. In this case the model is only driven by changes in the depreciation rate of capital and TFP fluctuations. The blue, purple, and orange curves consider either changes in taxation or markups, or capital gains on top of changes in TFP and depreciation. The green curves combine both changes in markups and taxation but exclude changes in capital gains. Finally, the black curves depict our baseline scenario with all exogenous variables together.

Figure 10: Counterfactual pretax income shares

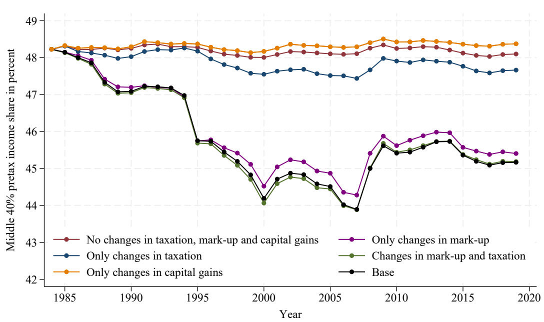
(a) Top 1% pretax income share



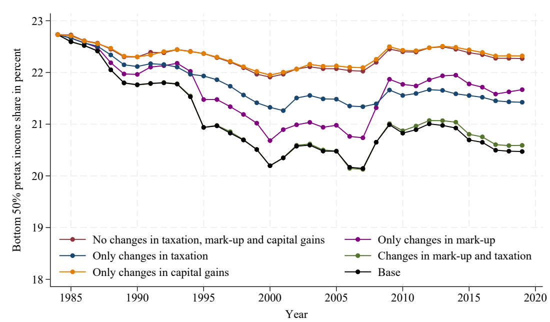
(b) Top 10% pretax income share



(c) Middle 40% pretax income share



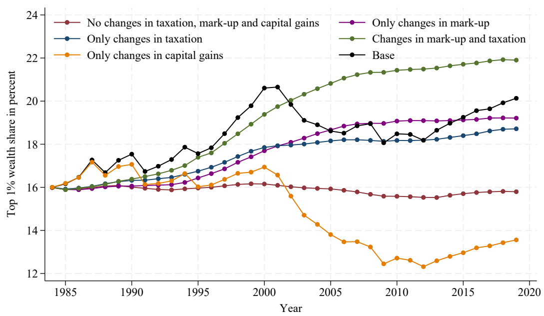
(d) Bottom 50% pretax income share



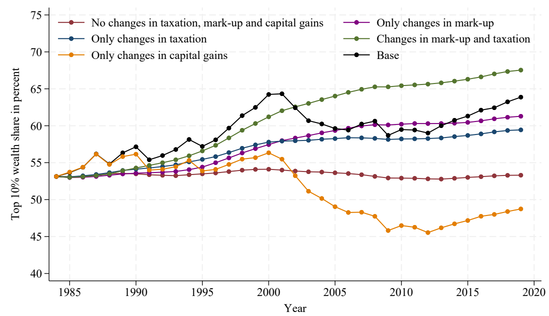
Considering only changes in TFP and capital depreciation (red curves), the Figures 10) and 11 show that income and wealth inequality would have remained almost stable between 1984 and 2018. It suggests that these exogenous variables matter for the dynamics of aggregate variables, but not for the dynamics of inequalities. For income inequality, (*i*) changes in markups are the main factor behind the increasing top 1% and top 10% pretax income shares, accounting for more than 90% of their rise. In contrast, (*ii*) changes in taxation play a limited role, and (*iii*) changes in capital gains have virtually no impact on pretax income inequality. For wealth inequality, changes in both taxation and markups play a significant role by increasing the top 1% and top 10% wealth shares at the expense of bottom 50% and middle 40% wealth shares. Adding changes in capital gains on top of other exogenous variables forces (differences between green and black curves) has a positive impact on the middle 40% wealth share at the expense of

Figure 11: Counterfactual wealth shares

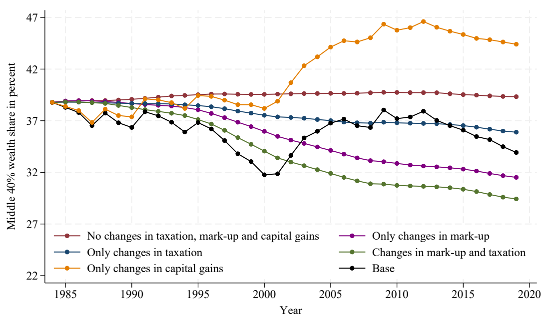
(a) Top 1% wealth share



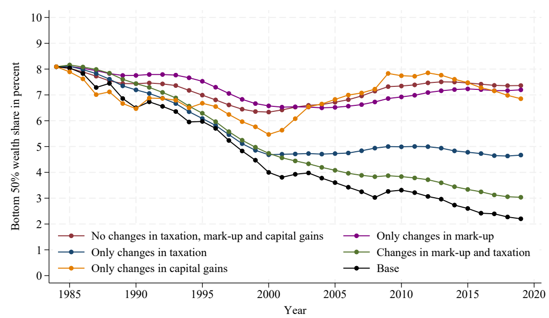
(b) Top 10% wealth share



(c) Middle 40% wealth share



(d) Bottom 50% wealth share



all other wealth groups. Overall, changes in taxation and markups have contributed to the rise in wealth inequality. Changes in capital gains have mitigated a part of this rise by reducing the inequality between the top 10% and middle 40% wealth group.

5.2 Quantifying transmission channels: method

A limitation of the above counterfactual analyzes is that they remain silent about the transmission mechanisms at play. For instance, rising markups redistribute income towards entrepreneurs at the top of the income distribution, but also lower wages and the return on capital. These movements may change pretax income inequality but also the tax rates faced by individuals, and induce behavioral responses, such as changes in their saving rates. The sign and strength of these behavioral effects are also affected by the tax and transfer system, which can change over time. Last, changes in capital gains can also induce strong behavioral responses beyond the mechanical effect that derives from the structure of portfolios along the wealth distribution.

To overcome this limitation, we leverage a simple equation for wealth accumulation that identifies the transmission channels behind wealth inequality dynamics. Wealth accumulation implies:

$$W_{it+1} = (1 + q_{it})(W_{it} + S_{it}) = (1 + q_{it})(W_{it} + s_{it}(1 - \tau_{it})sh^{Y}_{it}Y_t) \tag{32}$$

where W_{it} is the amount of wealth owned by wealth group i at time t, $(1 + q_{it})$ is the rate of capital gains of wealth group i, and S_{it} denotes savings. Savings S_{it} can further be split into four components: (*i*) the saving rate out of disposable income (s_{it}), (*ii*) the net-of-tax rate $(1 - \tau_{it})$, which accounts for both taxes and monetary transfers, (*iii*) the share of pretax income accruing to wealth group i (sh_{it}^{Y}), (*iv*) and aggregate pretax income (Y_t). As a consequence, $(1 - \tau_{it})sh_{it}^{Y}Y_t$ is the disposable income of wealth group i. Incidentally, we define S_{it} and s_{it} in the same way as Saez and Zucman (2016); Garbinti, Goupille-Lebret, and Piketty (2021). That is, we compute S_{it} and s_{it} as the *synthetic* savings and *synthetic* saving rates that account for the evolution of wealth of group i from t to $t+1$ given the observed values of the remaining variables (W_{it+1}, W_{it}, q_{it}, τ_{it}, sh_{it}^{Y} and Y_t). Using Equation (32), the evolution of the wealth share of group i is given by:

$$sh_{it+1}^{W} = \frac{W_{it+1}}{W_{t+1}} = \frac{(1+q_{it})}{(1+q_t)} \cdot \frac{(W_{it} + s_{it}(1-\tau_{it})sh_{it}^{Y}Y_t)}{(W_t + s_t(1-\tau_t)Y_t)} \tag{33}$$

This equation highlights that wealth inequality dynamics result from five complementary transmission channels: changes in (*i*) pretax income inequality (sh_{it}^{Y}), (*ii*) tax inequality (or progressivity, τ_{it} vs. τ_t) , (*iii*) saving rate inequality (s_{it} vs. s_t), (*iv*) capital gains inequality (q_{it} vs. q_t), and (*v*) aggregate pretax income (Y_t).

We use Equation (32) to decompose the evolution of wealth shares by transmission channel for each counterfactual scenario. Our five counterfactual scenarios are the same as those presented before: (1) without any changes in taxation, markups, and capital gains, (2) with only changes in capital gains, (3) with only changes in taxation, (4) with only changes in markups, and (5) with only changes in taxation and markups. These scenarios are then compared to the baseline scenario.

Our methodology consists in four steps for each counterfactual scenario. First, we fix all variables in Equation (32) to their 1984 values for each group *except* aggregate pretax income. The resulting wealth shares are computed and the results provide the relative contribution of aggregate pretax income to the evolution of wealth shares. Second, we allow each of the four following variables – saving rate, pretax income share, tax rate and rate of capital gains – to vary over time according to their simulated values. In this case, we keep the three other variables constant and equal to their 1984 value. We compute the resulting wealth shares, which then gives the first-order contribution of each of the variables/channels to the evolution of wealth shares. Third, we let two of the four variables/channels vary over time according to their simulated values, compute the wealth shares, and remove the first-order contributions to obtain the second-order contributions (interactions), which are then allocated to each of the three mechanisms proportionally to their respective first-order contributions. Last, we let three of the four variables/channels vary and proceed similarly to determine the third-order and the fourth-order contributions. The total contribution of each variable/channel is the sum of its first, second, third and fourth-order contributions.

5.3 Quantifying transmission channels: results

While the method can be used to decompose the wealth share evolution of any wealth group, we focus on the top 1% wealth share in Table 4, because it is the main driver of wealth inequality over the period.[27] This Table quantifies the contribution of each channel (pretax income inequality, tax progressivity, inequality in savings rates, and inequality in capital gains) to the changes in the top 1% wealth share for each counterfactual scenario. Figure 12 complements Table 4 by depicting the evolution of the transmission channels by counterfactual scenario.

Table 4: Transmission channels – Top 1% wealth share (1984-2018)

Counterfactual	Top 1%	Variations due to changes in			
Scenarios	Wealth Share Variation	Pretax Income Inequality	Tax progressivity	Saving rate inequality	Capital gains inequality
Base: Changes in markups, taxation and capital gains	26%	8%	0%	37%	-18%
Changes in markups and taxation	37%	8%	0%	29%	0%
Changes in markups	20%	8%	0%	12%	0%
Changes in taxation	17%	3%	0%	14%	0%
Changes in capital gains	-15%	0%	-1%	-4%	-11%
No changes in taxation, markups and capital gains	-1%	1%	0%	-2%	0%

Note: In the baseline scenario, the top 1% wealth share increases by 26% over the 1984-2018 period, of which 8 percentage points are due to changes in pretax income inequality.

In the counterfactual scenario with no changes in taxation, markups, and capital gains (last row of Table 4), the top 1% wealth share is almost constant between 1984 and 2018 (-1%).

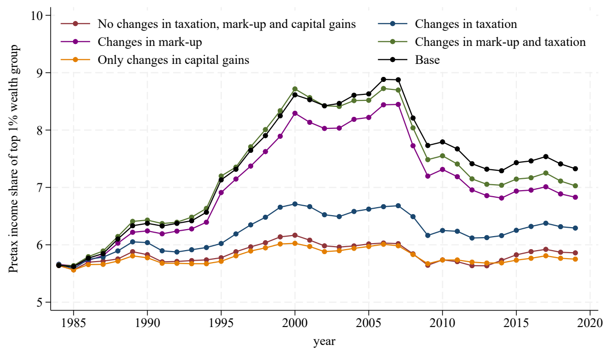
When considering only changes in capital gains (fifth row of Table 4), the top 1% wealth shares decreases by 15%. A large fraction of this decrease (11pp) is explained by the mechanical effect of asset prices on capital gains inequality. What is the mechanical effect of capital gains? Panel (e) of Figure 12 reports the cumulative rate of capital gains by wealth group over time. It shows that capital gains have been much more important for the middle 40% than other groups. This can be explained easily by variations in relative asset prices (housing prices rising more than equity prices) and stark differences in portfolio compositions among wealth groups (deposits for the bottom 50%, mostly housing for the middle 40% and equity capital for the top 10% and top 1%, see Panel (b) of Figure 2). Further, changes in capital gains also induce behavioral and general equilibrium effects on saving rate inequality, that contribute to further decrease the top 1% wealth share (-4pp).

While taxes and transfers were constant in the previous counterfactual scenarios, we now

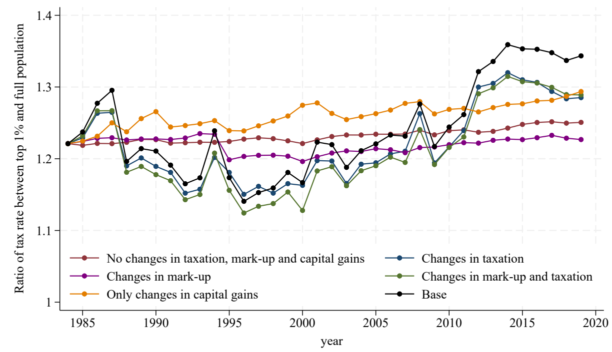
[27] The detailed table containing all the first, second and third-order contributions is available upon request.

Figure 12: Mechanisms of wealth inequality for the top 1% wealth group by scenario

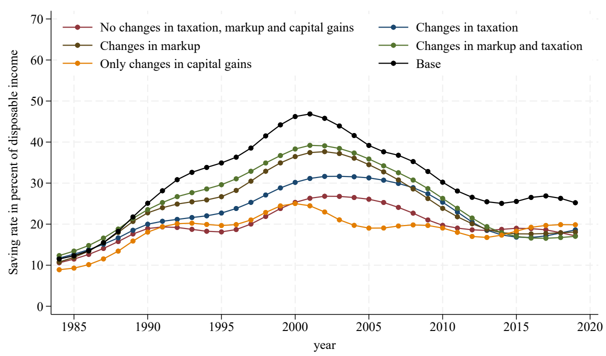
(a) Pretax income shares



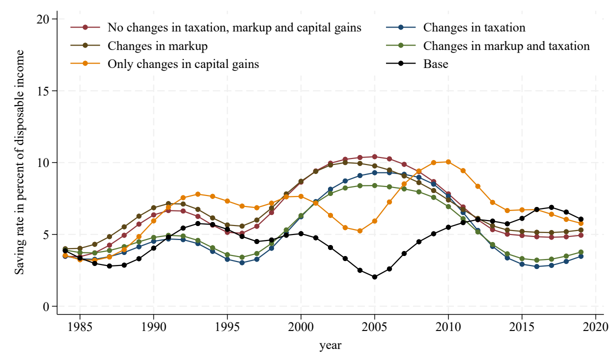
(b) Net tax rates: ratio top 1%/full pop.



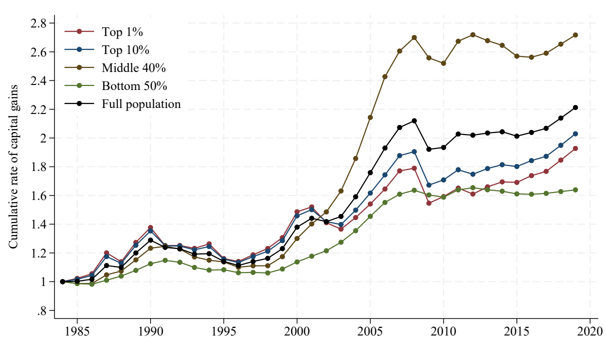
(c) Synthetic saving rate



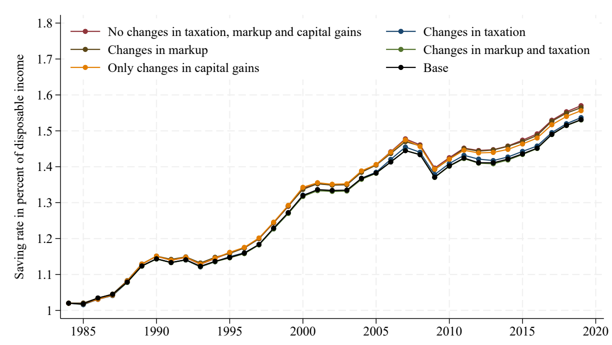
(d) Aggregate synthetic saving rate



(e) Rates of capital gains (Base. scenario)



(f) Aggregate pretax income



look at their specific impact over time (fourth row of Table 4). Doing so affects the dynamics of the economy directly through changes in tax progressivity, *i.e.* the mechanical effect of taxation, but also potentially through other transmission channels (pretax income and saving rate inequality) because of behavioral and general equilibrium effects.[28] When considering only changes in taxes, the top 1% wealth share increases by 17%. Notably, the impact of changes in taxes and transfers on wealth inequality is not predominantly driven by the mechanical effect of taxation (3rd column), but by raising saving rate inequality (+14pp, 4th column), and, to a lesser extent, by raising pretax income inequality (+3pp, 2nd column). Although tax rate progressivity may look broadly unchanged (see Panel (b) of Figure 12), changes in the composition of taxes paid can affect savings through changes in the after-tax return $(1-\tau^j)r^k - \phi$. For instance, if households from the bottom 50% of the wealth distribution pay less social security taxes and more income taxes, their after-tax return on savings drops, which lowers their saving rate. Similarly, if households from the top 1% of the wealth distribution pay less income taxes but more corporate taxes, their after-tax return rises, which boosts their saving rate. Both movements are at work and combine to account for the rise in saving rate inequality induced by changes in taxes, as seen from Panel (c) in Figure 12.

When considering changes in markups (third row of Table 4) instead of changes in taxation, the top 1% wealth share increases by 20%, 8pp of which are driven by the rise in pretax income inequality, and 12pp by the rise in saving rate inequality. In this case, the mechanical effect stems from the direct redistribution of several points of national income towards entrepreneurs at the top of the income distribution through profits, and from the mirror decline in factor payments to the bottom of the income distribution. Saving rates thus increase at the top of the wealth distribution and decline at the bottom, which magnifies the increase in the top 1% wealth share.

When considering both changes in taxation and markups (second row of Table 4), the top 1% wealth share increases by 37%, much above the baseline increase. The evolution of pretax income inequality accounts for 8pp while saving rate inequality accounts for 29pp of the total increase. Both figures roughly aggregate the effects of changes in markups and changes when taken in isolation from other factors.

Finally, when we introduce changes in capital gains along with changes in markup and taxation, the top 1% wealth share increases less (+26% instead of +37%). Table 4 shows that capital gains have a strong mechanical effect, and would, all else equal, decrease the top 1% wealth share by 18%. However, this strong negative mechanical effect of capital gains is partially offset by an increase in saving rate inequality (+37% instead of +29% in absence of capital gains). This additional 8pp contribution stands out because capital gains induce an increase in the inequality of saving rates by increasing the saving rate the top 1% wealth group and decreasing the aggregate saving rate (Panel (c) and (d) in Figure 12). In this scenario, the increase in markups and the

[28] The mechanical effect of taxation corresponds to the impact of taxation on the gap between pretax and disposable income inequality among wealth groups.

changes in taxes give enough room to the top 1% wealth group to increase its saving rate and therefore partially undo the negative mechanical effects of capital gains on their wealth share.

In summary, the increase in the top 1% wealth share between 1984 and 2018 results from two opposing forces. Changes in taxation and markups increase wealth inequality by increasing saving rate inequality and pretax income inequality. In contrast, changes in capital gains push down wealth inequality, but are partly offset by increased saving rate inequality. The first row of Table 4 is consistent with previous empirical work based on simple simulation exercises that stresses the key role of asset prices, saving rate inequality and pretax income inequality on the dynamics of wealth inequality (see Saez and Zucman (2016), Kuhn, Schularick, and Steins (2020), Garbinti, Goupille-Lebret, and Piketty (2021), Martínez-Toledano (2020)).

However, our framework carries deeper conclusions and three important contributions to this literature. First, although the mechanical effects of housing capital gains reduces top wealth shares, they are partially offset by behavioral and general equilibrium effects on saving rate inequality, especially when interacted with changing markups and taxes. Second, although the mechanical effects of taxation on top wealth shares is negligible, changes in taxes have strong effects on wealth inequality dynamics through the behavioral and general equilibrium effects they have on saving rate inequality. Third, in absence of changes in taxation, markups and capital gains, wealth inequality would have been stable. As a result, the main transmission channel of wealth inequality is the change in saving rate inequality, but only in response to additional changes in the environment (asset prices, taxation, and markup). More generally, these results point to the critical importance of endogenous saving decisions as a key driver of wealth inequality.

6 Conclusion

Unifying micro and macro remains an area of vast research potential. We build a rich, micro-founded macro model with three assets (deposits, gross housing and equity), labor-income and entrepreneurial risk, and a rich and realistic set of flat and progressive taxes and transfers. Thanks to newly available wealth and income inequality series and fiscal data, we calibrate our model and test its ability to fit the level and dynamics of wealth and income inequalities, the aggregate and distributional tax structure, the composition of wealth along the distribution, and key macroeconomic aggregates from 1984 to 2018. We then propose a method that quantifies the contribution of various transmission channels to wealth inequality dynamics in a counterfactual analysis. We show the importance of *(i)* markups in explaining the dynamics of income inequality and of *(ii)* endogenous saving decisions in response to exogenous variables as a key driver of wealth inequality.

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A Concept of pretax income

This work relies extensively on the long-term series of pretax income, posttax income and wealth developed for France within the "Distributional National Accounts" project. This project aims at combining national accounts, tax, and survey data in a comprehensive and consistent manner to build long-term series of inequalities that are unified over time and across countries, cover the entire distribution and are fully consistent with the National Accounts.

Complete methodological details about the construction of these series are provided in Garbinti, Goupille-Lebret, and Piketty (2021) for the wealth series, and Bozio et al. (2024) for pretax and posttax income series, along with a wide set of tabulated series, data files and computer codes. A complete presentation of the concepts and the general methodology to construct "Distributional National Accounts" series is provided by Alvaredo et al. (2020).

In this Appendix, we present the concept of pretax income and discuss its implications for the model.

Pretax vs. factor national income Pretax income is our benchmark concept to study the distribution of income before government intervention. It is defined as the sum of all income flows going to labor and capital, after taking into account the operation of the pension and unemployment insurance systems, but before taking into account other taxes and transfers. This concept should be benchmarked against the definition of factor income, which is equal to the sum of all income flows going to labor and capital, before considering the operation of the pension and unemployment system. The key difference between factor income and pretax income is the treatment of pensions, which are counted on a contribution basis for factor income and on a distribution basis for pretax income. In other words, pretax income includes pension and unemployment benefits, while it excludes contributive payroll taxes, *i.e.* the fraction of payroll taxes dedicated to the financing of the pension and unemployment system.

The main limitation of factor income is that retired individuals typically have very small factor income in countries using pay-as-you pension systems. As a result, inequality of factor income tends to look artificially large in countries and time periods with an older population. In contrast, pretax income inequality will not be affected by aging population nor by the design of the pension system.[29] However, the limitation of the concept of pretax income is that it does not incorporate the redistribution carried out by the pension and UI systems over the life-cycle.

Using the concept of pretax income yields three main implications for our model that we now discuss.

The incidence of taxes Computing pretax income requires to assign taxes that are not directly paid by households (corporate taxes and payroll taxes) into their income using tax incidence

[29] Note that pretax income is broader but conceptually similar to what most tax administrations attempt to tax, as pensions and unemployment benefits are largely taxable, while contributions are largely tax deductible.

assumptions. As pointed out by Saez and Zucman (2019), one need to distinguish current distributional analysis from tax reform distributional analysis. Current distributional analysis shows the current tax burden by income groups and should assign taxes on each economic factor without including behavioral responses. As such, taxes based on labor income (payroll taxes) should fall on the corresponding workers. Taxes based on wealth or capital income should fall on the owners of the corresponding assets.

Therefore, when we compute pretax income, we assign current payroll taxes and corporate taxes paid to the corresponding household income without incorporating any behavioral response (current distributional analysis). In contrast, we use our model and rely on a counterfactual analysis to study how changes in taxes affect the aggregates and the distributions of pretax income, posttax income and wealth relative to baseline through potential behavioral effects (direct, indirect, general equilibrium).

Pension and payroll taxes Because pretax income includes pension and unemployment benefits, the model should only include the fraction of payroll taxes that do not finance the pension and unemployment system to avoid double counting. Indeed, if we include all payroll taxes in the model and reassign them to pretax income, pretax income will include pension and unemployment contributions but also the corresponding benefits. Disposable and posttax income will also be inconsistent as pension and unemployment contributions will be subtracted but the corresponding benefits will not be added when going from pretax to disposable and posttax income. For simplicity we just assume equilibrium between contributions and distributions of the pension and unemployment systems, and neglect the incidence of pension and payroll taxes.

B Modeling the tax and transfer system

This Appendix provide a brief overview of the French tax and transfer system and presents in details the methodology used to estimate the different tax parameters.

B.1 Overview of the French tax and transfer system

The French tax system includes a large variety of taxes that we can regroup into five categories depending on the relevant tax basis: (*i*) taxes borne by pretax labor income ("non-contributive social contributions"), (*ii*) by total income (pretax labor and capital income) that we thus call "income tax", (*iii*) by capital ("wealth taxes"), (*iv*) by corporate profits ("tax on corporate profits"), and (*v*) by consumption ("consumption taxes").

Government spending can be decomposed into three distinct categories: monetary transfers, in-kind transfers, and collective consumption expenditure. Monetary transfers amount to about 4% of national income and include various types of housing benefits, family benefits, and social benefits. In-kind transfers are all transfers that are not monetary (or quasi-monetary) and can be individualized. They correspond to individual goods and services produced directly or reimbursed by government. In-kind transfers make up to 20% of national income (including 12.5% for health and 6.5% for education expenditure). Collective consumption expenditure regroups all consumption services that benefit to the community in general and cannot be individualized (spending on defense, police, the justice system, public infrastructure, etc.). It amounts to 10% of national income.

B.2 Fitting the progressivity of taxes and transfers

The DINA series contain a full decomposition of taxes and transfers that we exploit to compute the different tax rates that are used in our model.

We assume the functional forms given by Equation (24). Our approach consists in estimating two parameters for each tax: a level parameter ($\overline{T}$, τ_ℓ, τ, ϕ) and a progressivity parameter η. We refine this approach in two ways. First, to describe the effective tax schedule as accurately as possible, we compute aside the average tax rate for the top 0.1% of the distribution of the tax basis. As we will see, it is particularly relevant for the progressive income tax. Second, to allow for more flexibility and for a better fit to the actual effective tax rates, we fit the functional form on different segments of the distribution. That is, we estimate potentially different level and progressivity parameters on several subsets of the distribution of tax rates along the tax base. We proceed identically for transfers. We detail the tax concepts and results below.

B.3 Taxes & their progressivity

We gather taxes depending on the relevant tax basis. We start from the detailed categories of taxes (presented in detail in Bozio et al. (2024)) and we classify them in 5 broad categories: non-contributive payroll taxes, income taxes, wealth taxes, tax on corporate profits, and consumption taxes.[30] Hereafter, for taxes and transfers, we present our method for the year 1984. A similar approach is used for all other years for which we have data (1988 and each year from 1994 onwards).

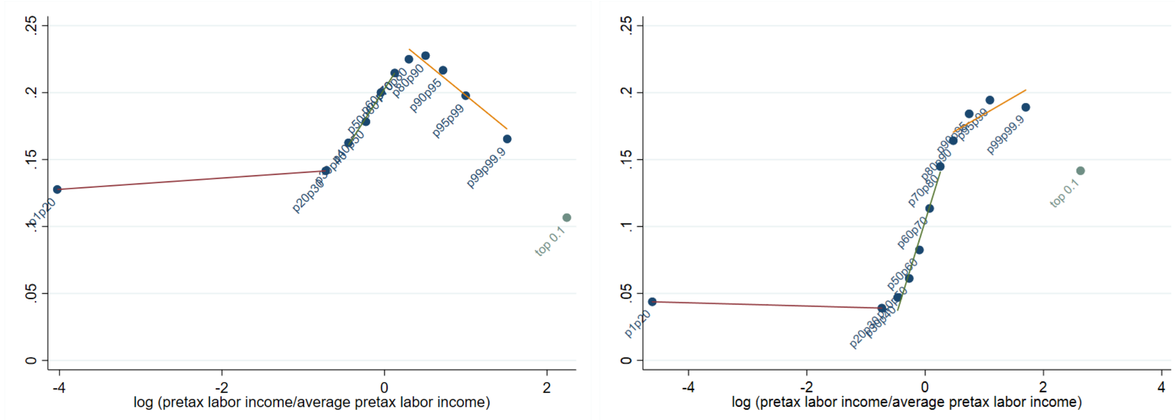
Payroll taxes τ_ℓ^j include all social security contributions that are not dedicated to the financing of the pension and unemployment systems as well as taxes on wages. Altogether, they make up to 11% of national income in 2018. They are applied to pretax labor income. For the different years for which we have data (1984, 1988 and each year from 1994), we estimate the different

[30] Note that we add pension and unemployment benefits to labor and capital incomes. Consequently, we do not add contributive payroll taxes to the analysis to avoid double-counting since these payroll taxes fund these benefits. In Garbinti, Goupille-Lebret, and Piketty (2018), we present another concept of income (factor income) where pension and unemployment benefits are not added to labor and capital incomes, and that allows to investigate the role of payroll taxes. One problem of that measure is that retired individuals typically have very small factor income in countries using pay-as-you pension systems such as France. As a result, inequality of factor income tends to rise mechanically with the fraction of old-age individuals in the population, which biases comparisons over time and across countries.

parameters on three segments of the distribution of pretax labor income (in addition to the average tax rate computed for the top 0.1%). Figure 13 shows how we fit the distribution of the non-contributive SSC for the year 1984 and 2018. It illustrates that our flexible non-linear specification allows for an excellent fit of the distribution of tax rates and improves significantly our ability to model tax rates as close as possible as those observed in the data. This goodness of the fit is similar for all subsequent years for which we have data (1984, 1988 and all years after 1994). Comparing panels (a) and (b) show the crucial importance of having time-varying parameters.

Figure 13: Individual SSC contributions τ_ℓ^j (% pretax labor income)

Panel (a): 1984 Panel (b): 2018

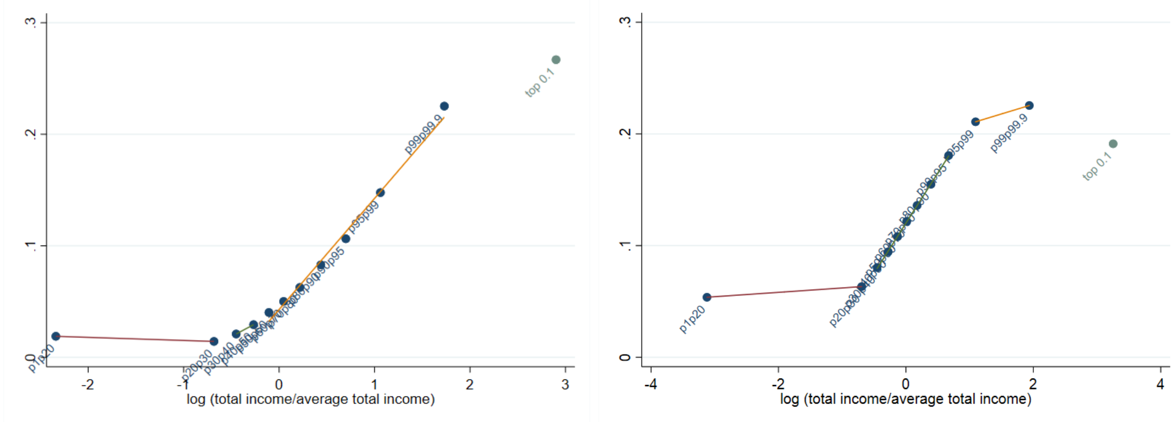


Income tax. We gather in the income tax τ^j taxes that are borne by total pretax income (labor and capital income, including profits). It thus includes both the income tax ("impôt sur le revenu des personnes physiques") and the "CSG" ("Contribution Sociale Généralisée", a flat tax) on capital and labor incomes. As for non-contributive SSC, we perform the estimation of the different level and progressivity parameters for three segments of the distribution of total pretax income. Figure 14 shows the fit of the distribution of the income tax SSC for the year 1984 and 2018. Here again, it clearly shows that our specification provides a great fit of the distribution of the tax rates, a feature that is similar for all other years of our sample.

Wealth tax. The wealth tax ϕ^j includes all taxes borne by assets. This corresponds to the wealth tax, the property tax and the estate tax. Note that it applies to the *level* of wealth. Further, note that in France, there exists a "tax shelter" (bouclier fiscal) to insure that the total tax burden cannot exceed 60 to 70% (depending on the period) of total income. This mechanism has been set to avoid that wealth taxes lead to a tax burden deemed too high. We deduct this tax shelter from the wealth tax. Figure 15 shows the fit of the wealth tax for 1984 and 2018. The tax rates appear aligned with a linear trend along the net wealth distribution, except for the bump observed from the 40th to the 60th percentiles. This is explained by the fact that property taxes apply to *gross* rather then *net* housing wealth. Here, we use the concept of *net* wealth (net of liabilities). It thus raises mechanically the tax rates for indebted individuals that are over-represented between the

Figure 14: Individual income tax rates τ^j (% total pretax income)

Panel (a): 1984 Panel (b): 2018



40th and 60th percentiles of the net wealth distribution. In an alternative specification, we take into account this bump by allowing for non-linearity but this does not change our results. This is likely due to the fact that the magnitude of the gap between the bump and the rest of the surrounding tax rates is small. We thus opt for the simplest specification and chose a linear one (up to the top 0.1% which is still considered aside).

Figure 15: Individual capital tax rates ϕ^j (% net wealth)

Panel (a): 1984 Panel (b): 2018

Tax on corporate profits. This tax directly applies to profits (Equation 2) and is a flat tax. We thus simply apply the same flat tax rate to firms profits. It is taken directly from national account every year.

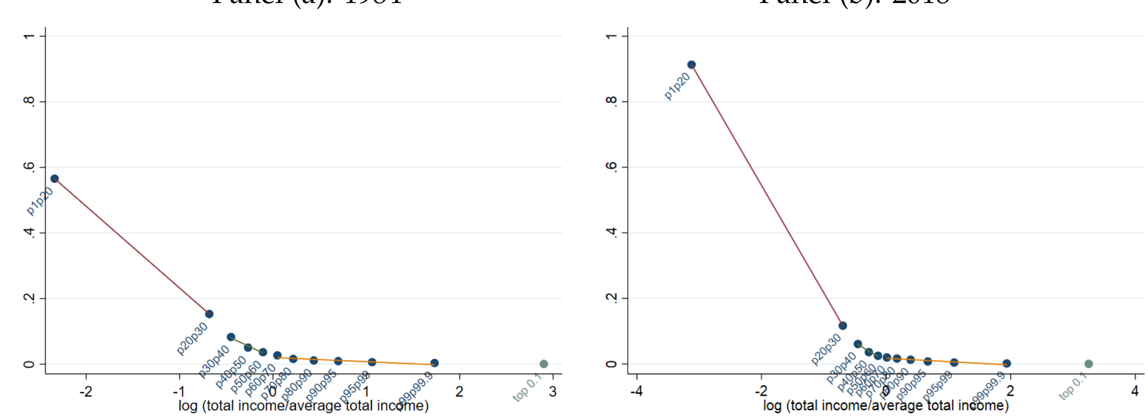
Consumption taxes. These taxes are borne by consumption (Equation 10). We consider here consumption taxes as a flat tax. This is not a significant departure from reality since the value added tax (VAT) represents the bulk of these taxes. Although the VAT has four different rates (ranging from 2.5 to 20%), the vast majority of goods are taxed at 20%. Consequently, as for the tax on corporate profits, the value of this flat tax is taken directly from national account every year.

B.4 Individual transfers & their progressivity

Individual transfers (or monetary transfers) T^j represent about 4% of national income and include the various types of housing benefits, family benefits, and social benefits.[31] Here again, we perform the estimation of the level and progressivity parameters for three segments of the distribution of total pretax income. Figure 16 shows how we fit the distribution of monetary transfers expressed as a percentage of pretax income for the year 1984 and 2018. Here, as for taxes, our non-linear specification allows for a very accurate representation of the transfers actually observed in the data over time.

Figure 16: Individual transfers T^j (% total pretax income, $Y^{pretax,j}$)

Panel (a): 1984 Panel (b): 2018



[31]The housing benefits regroup "Allocation de Logement Familiale" (ALF), "Allocation de Logement Personnalisée" (APL), and "Allocation de logement sociale" (ALS). The family benefits include "Allocation Familiale" (AF), "Complément Familial" (CF), "Allocation Pour Jeune Enfant" (APJE), "Prestation d'Accueil du Jeune Enfant" (PAJE), "Allocation de Rentrée Scolaire" (ARS), "Allocation d'Education de l'Enfant Handicapé" (AEEH), and "Allocation de Soutien Familial" (ASF). The social benefits regroup "Revenue de Solidarité Active"/"Prime d'Activité" (RSA/PPA), "Allocation Adulte Handicapé" (AAH), and "Allocation de Solidarité aux Personnes Agées" (ASPA).

C Solution method

Our solution method is fully non-linear and takes advantage of the continuous-time formulation of the heterogeneous-agent problem solving the Hamilton-Jacobi-Bellman and Kolmogorov forward equations. Our codes are adapted from those of Bence Bardoczy taken from the HACT project page maintained by Benjamin Moll: https://benjaminmoll.com/codes/.

C.1 Stationary equilibrium

The solution method uses an asset grid with 6 states (5 types of workers + entrepreneurs) and 501 grid points over an asset grid $a^j \in [0, 100]$. The algorithm solving for the stationary distribution is the following.

Starting from initial guesses for the steady-state level of capital k, total labor ℓ and taxes:

1. Compute output, y, capital rental rate r^k, aggregate real wage w and firms' aggregate profits π

2. Given the income tax schedule τ^j, the consumption tax rate τ_c, the rental rate of housing r^h and the rate on deposits r^m, compute the (household-specific) opportunity costs of housing R^j and deposits R^{mj} and the individual price indices P^j_Λ

3. Given the labor tax schedule τ^j_ℓ and the productivity levels z^j, compute individual labor income Φ^j_ℓ over the asset grid

4. Given the capital rental rate r^k and entrepreneurial profits $\pi^j = \pi/e$, compute capital income Φ^j_k over the asset grid

5. Given the income tax schedule τ^j, the capital tax schedule ϕ^j and the transfer schedule T^j, compute arbitrable income consistently with the budget constraint: $\left(1-\tau^j\right)\left(\Phi^j_\ell + \Phi^j_k\right) - \left(\phi^j + g_\xi\right) a^j + T^j$

6. Given the state transition matrix describing changes in workers' productivity or changes in status (worker to entrepreneur or vice-versa), solve the Hamilton-Jacobi-Bellman equation based on the utility function to determine the individual saving rules $\dot{a}^j$ and the individual expenditure rules Λ^j

7. Given Λ^j, P^j_Λ, R^{mj}, R^{hj} and τ_c compute the optimal rule for deposits m^{dj}, housing services s^j and the consumption of non-durables c^j

8. Given the minimum housing size and the borrowing constraint, determine the status of household j as renter ($\mathbb{1}_{h^j} = 0$) or homeowner ($\mathbb{1}_{h^j} = 1$=, and the amount of housing owned $h^j = \mathbb{1}_{h^j} s^j$ as well as housing debt d^j

9. Given the demand for deposit m^{dj} and housing owned h^j, compute holdings of capital $k^j = \mathbb{1}_{h^j}(a^j - (p^h h^j - d^j) - m^{dj})$
10. Adjust $m^j = a^j - (p^h h^j - d^j) - k^j$
11. Solve the Kolmogorov forward equation to get the distribution of households Ω^j over the asset grid
12. Update the distributions of labor income Φ^j_ℓ, and capital income Φ^j_k and Y^j_k and all the relevant measures of income
13. Update the progressive tax and transfer schedules τ^j_ℓ, τ^j, ϕ^j and T^j
14. Update aggregate labor $\ell = \int_j \Omega^j \left(1 - \mathbb{1}_{e^j}\right) \left(w^j/w\right) \ell^j dj$ and update the average level of labor productivity that guarantees $\int_j \Omega^j \left(1 - \mathbb{1}_{e^j}\right) w^j dj = w$
15. Compute the residual of capital-market clearing condition as the difference between the sum of individual capital detention $\int_j \Omega^j k^j dj$ and the aggregate stock of capital k
16. Adjust aggregate capital k using the above residual and iterate from 1. until the residual of the capital-market clearing condition is less than 0.1% of the aggregate capital stock.

Solving for the stationary equilibrium takes a few seconds.

C.2 Transition dynamics

The algorithm solving for the transitional dynamics is very similar to the one solving for the stationary distribution, and is based on updating the sequence of aggregate capital $\{k_t\}_{t=1}^{t=T}$, and all the relevant variables, given the path of exogenous variables

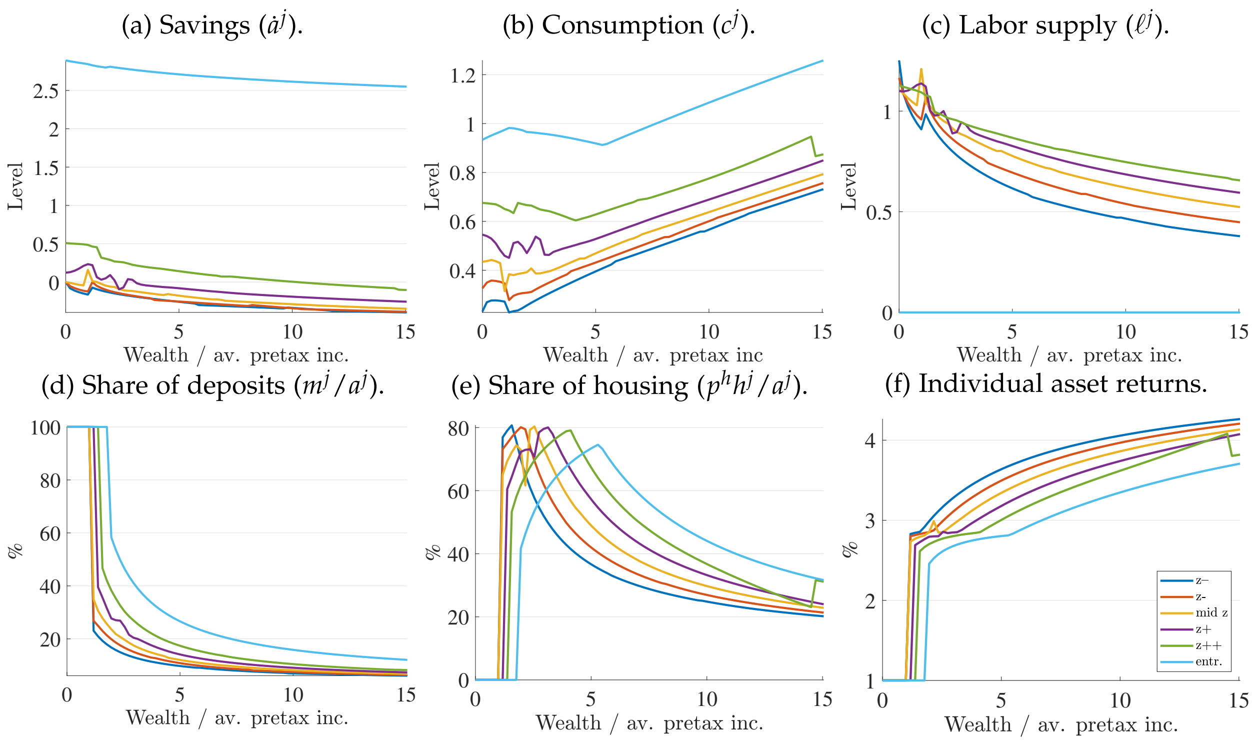
$$\left\{\theta_t, \delta_t, \dot{p}^h_t / p^h_t, p^h_t, g_t/y_t, \tau_{ct}, \tau_{\pi t}, \xi_t, \Gamma_t\right\}_{t=1}^{t=T}$$

where Γ_t captures all the parameters governing the progressive tax and transfer schedules. We start from the steady-state path of $\{k_t\}_{t=1}^{t=T} = k$ and update k_t using last iteration (time-varying) residuals until the maximum excess capital between $\{2 : T\}$ is strictly less than tolerance (0.1% of total capital stock). By definition, since capital is predetermined and the exogenous drivers are a surprise in the first period, errors at time $t = 1$ can not be brought to zero. Solving for the transition dynamics takes a few minutes depending on the exercise, nature of the exogenous drivers and length of the simulation.

D Policy functions

The model generates the policy functions depicted in Figure 17. Panel (a), (b) and (c) show the savings, consumption and labor supply schedules, Panel (d) and (e) the household-specific holdings of deposits and housing and Panel (f) the household-specific returns on wealth.

Figure 17: Policy functions



Panel (a) of Figure 17 shows that entrepreneurs are the largest savers in the model. First, these households receive large amounts of income. Second, given the low probability of becoming an entrepreneur and the relatively large probability of losing the status, a strong precautionary motive drives them to save up to 50% of their (large) disposable income. Workers also save to self-insure against earning risk, that is, the risk of transiting to lower levels of income. Hence, the second largest savers are the very high and high productivity workers. Saving rates are decreasing in wealth levels because once households have reached their target amount of precautionary saving, they stop saving. Workers with lower productivity (middle, low or very low) have overall negative savings, *i.e.* they use their existing wealth to sustain higher consumption taking into account the probability of upward income mobility.

Panel (c) shows that labor supply is driven both by a wealth effect – decreasing in the amount of wealth – and a substitution effect – increasing in the after-tax wage. At low levels of wealth, the wealth effect can be strong enough to overturn the substitution effect but the substitution effect dominates at larger levels of wealth. Panels (d) and (e) of Figure 17 inform about the individual compositions of portfolios by types and wealth levels. They show that households at the bottom of the asset grid do not save enough to reach the threshold to become homeowners, and therefore keep their wealth in the form of liquid deposits. When households save enough to buy housing, they allocate almost all of their wealth to housing and then diversify their portfolios by holding capital. As a result of the varying composition of portfolios along the distribution of wealth, individual pretax returns are increasing in wealth (panel (f)).

E Additional Tables and Figures

Figure 18: Counterfactual pretax income shares: constant taxes & markups & capital gains

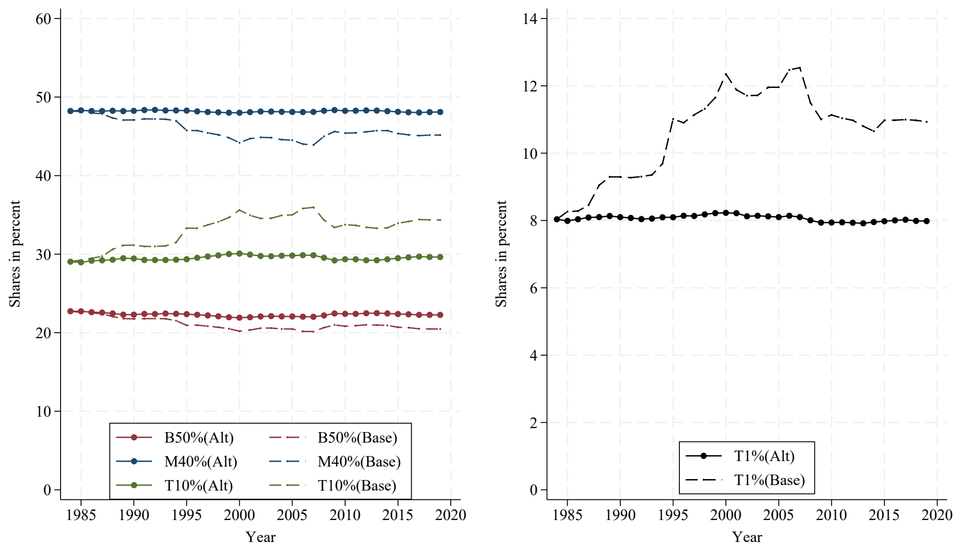


Figure 19: Counterfactual pretax income shares: only changes in taxes and transfers (constant markups & capital gains)

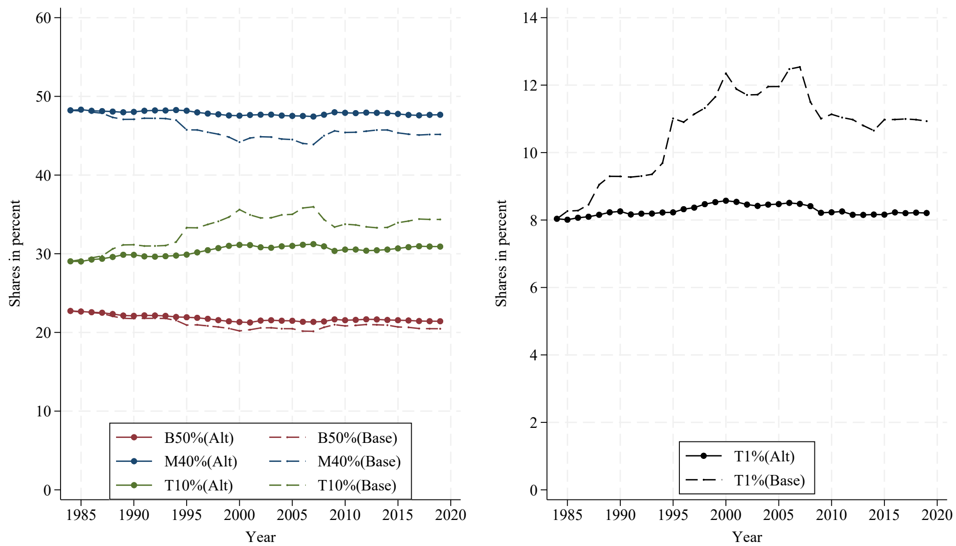


Figure 20: Counterfactual pretax income shares: only changes in markups (constant taxes and transfers & capital gains)

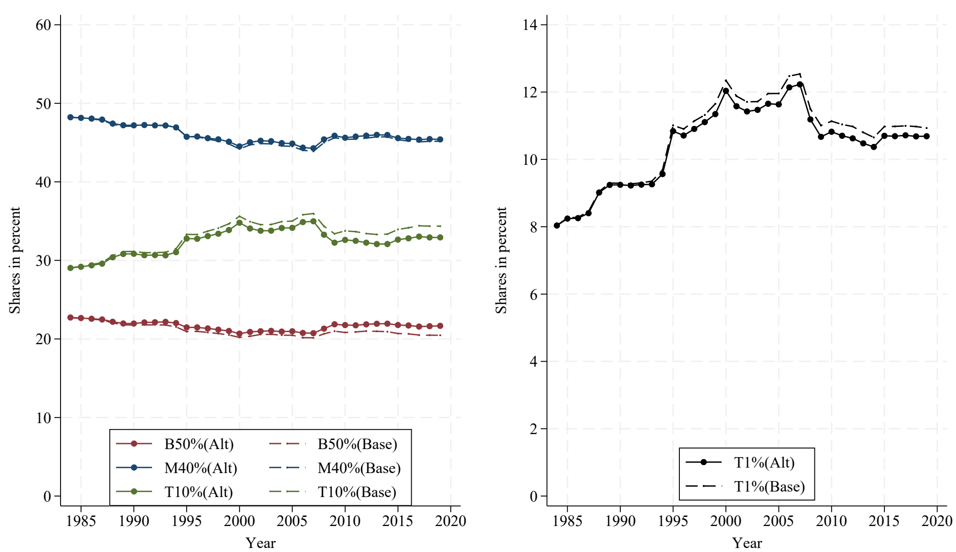


Figure 21: Counterfactual pretax income shares: Changes in both taxes and transfers & markups (constant capital gains)

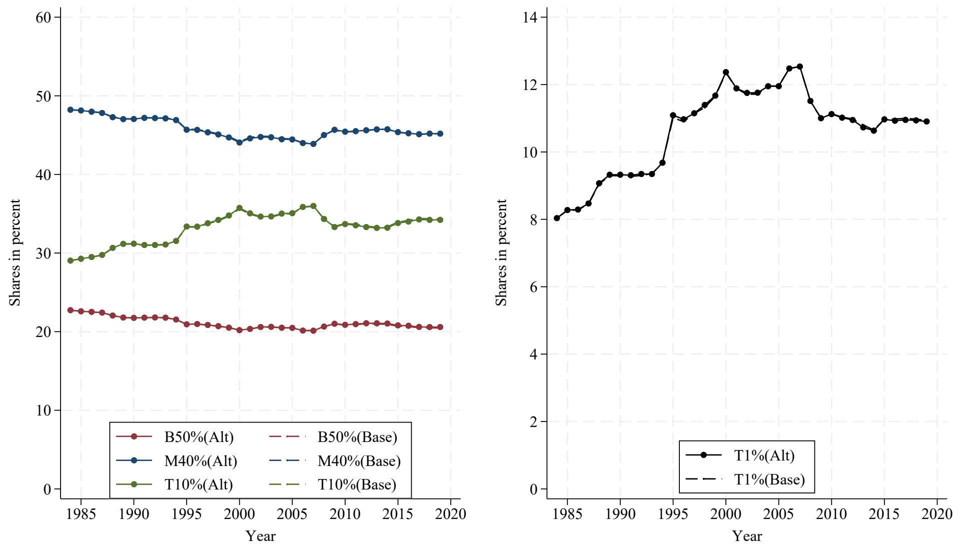


Figure 22: Counterfactual wealth shares: constant taxes & markups & capital gains

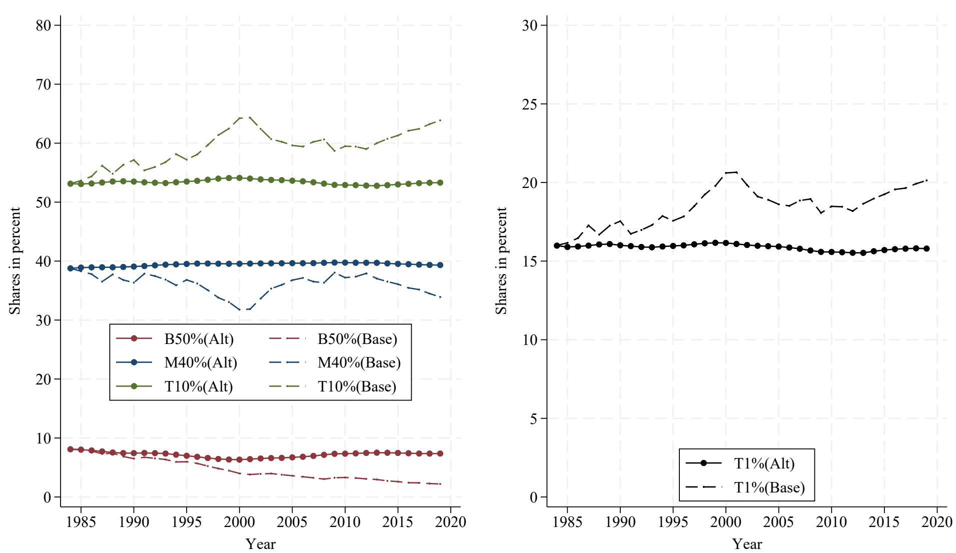


Figure 23: Counterfactual wealth shares: only changes in taxes and transfers (constant markups & capital gains)

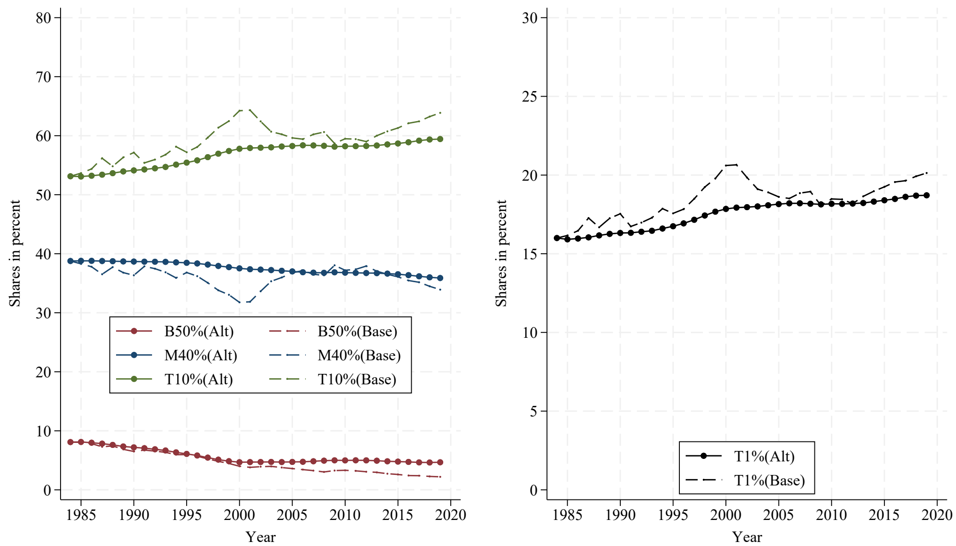


Figure 24: Counterfactual wealth shares: only changes in markups (constant taxes and transfers & capital gains)

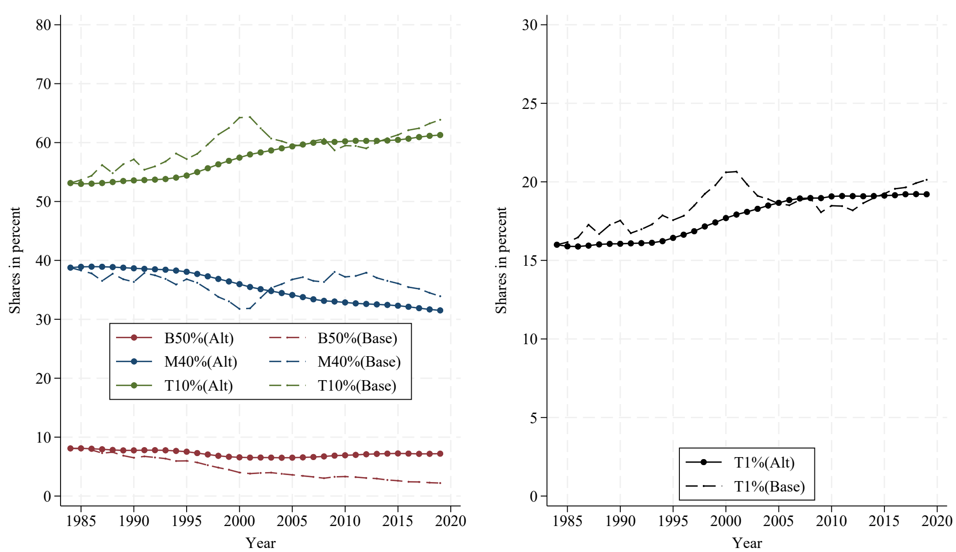


Figure 25: Counterfactual wealth shares: Changes in both taxes and transfers & markups (constant capital gains)