Structural Change in Production Networks and Economic Growth*

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Abstract: We study structural change in production networks for intermediate inputs (input-output network) and new capital (investment network). For each network, we document that the share of output produced by goods sectors has declined since the 1950s, offset by a rising fraction of production by services sectors. We develop a multi-sector growth model to study these trends and show that our framework admits an aggregate balanced growth path with such structural change. Calibrating the model using disaggregated expenditure-side price data for the United States, we find that inputs to intermediates production are complements. However, in contrast to existing literature, we find that inputs to investment production are substitutes. Hence, structural change in production networks implies that resources endogenously reallocate to the slowest growing intermediates producers and the fastest growing investment producers. As a result, we show that investment-specific technical change accounts for an increasing share of U.S. aggregate growth, rising from 30-40% of growth prior to the 1980s to more than 70% since the year 2000. In addition, more than 20% of aggregate growth after 2000 stems from endogenous reallocation induced by structural change. At the same time, productivity growth within the input-output network has stagnated, accounting for the bulk of the recent slowdown in aggregate growth.

JEL: E23, O14, O40, O41

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1. Introduction

Production networks—the distribution of transactions between sectors that produce and purchase commodities used in production—play a central role in shaping economic fluctuations and growth.[1] However, with changes in technology and the organization of production, these networks change over time. This paper documents structural change in the production networks for investment and intermediates and assesses its implications for economic growth.

Our analysis proceeds in four steps. First, we show a decline in the share of both intermediate inputs and investment produced by goods sectors (i.e., manufacturing, construction), offset by an increase in the share produced by services sectors (e.g., financial services or professional/technical services). Second, we develop a multi-sector model with production networks for intermediate inputs and investment, and show that this environment admits an aggregate balanced growth path with structural change. Third, we calibrate the model to match the observed patterns of structural change in production networks. Finally, we use the model to quantify the growth contribution of each network and the importance of structural change for U.S. growth over the period 1947-2020.

Our analysis delivers several novel insights. We find that goods and services are substitutes in the production of investment, implying that investment expenditures endogenously reallocate toward the producers with the fastest total factor productivity (TFP) growth—the "frontiers" of growth, such as software production. For the production of intermediates, we find that goods and services are strong complements, implying that resources endogenously reallocate toward producers with the least TFP growth—the "bottlenecks" of growth, such as financial services. Because of these reallocation forces, investment-specific technical change—i.e., TFP growth in key production hubs of the investment production network—endogenously explains an increasing share of aggregate growth over time. We highlight these forces in a series of growth decompositions, showing that more than 20% of aggregate growth since the year 2000 can be accounted for by endogenous reallocation toward investment producers with the fastest TFP growth. In contrast, sluggish aggregate growth since 2000 is entirely accounted for by stagnating or even negative TFP growth among the producers of intermediate inputs.

The first step of our analysis uses national accounting data to document that, over time, goods sectors systematically produce a smaller share of output in each production network, offset by increased production

[1] Among many possible examples, see Acemoglu, Carvalho, Ozdaglar and Tahbaz-Salehi (2012), Baqaee and Farhi (2019), Foerster, Hornstein, Sarte and Watson (2019), Kopytov, Mishra, Nimark and Taschereau-Dumouchel (2021), vom Lehn and Winberry (2022).

in service sectors. The magnitude of these changes is large, similar to structural change observed in consumption expenditures, and these changes are widespread, occurring in nearly all sectors and in many countries. We also present several additional empirical facts regarding structural change in production works, including how a changing composition of purchasing sectors contributes to structural change and evidence that structural change in intermediate inputs does not merely reflect outsourcing of services tasks previously performed within establishments.

We then study these changes using an extension to the multi-sector neoclassical growth model, which is commonly applied to study structural transformation in consumption expenditures (see Herrendorf, Rogerson and Valentinyi, 2014, for a survey). In our model, each sector produces gross output using a combination of capital, labor, and intermediate inputs. Production networks are modeled as each sector's intermediates and investment being a constant elasticity of substitution (CES) bundle of inputs purchased from many different sectors. This setup implies that changes in relative prices across sectors, induced by changes in technology, can generate changes in the composition of intermediates and investment production—i.e., changes in production networks.

To develop intuition for the channels through which such changes contribute to aggregate growth, we characterize an aggregate balanced growth path (ABGP) consistent with structural change in production networks. Similar to Herrendorf, Rogerson and Valentinyi (2021), the existence of an ABGP only requires constant aggregate TFP growth, while the underlying sectoral TFP growth rates need not be constant.[2] Along the ABGP, aggregate growth is attributable to two channels: productivity growth in the production of investment and productivity growth in the production of intermediates used to produce investment. The composition of these two forces in aggregate growth depends on the elasticities of substitution between inputs used in the production of investment and intermediates.

While the empirical evidence on structural change in production networks can be summarized with just two sectors—goods and services—our model highlights that each sector's contribution to aggregate growth depends on the extent to which it produces consumption, investment, and intermediates. To allow for such heterogeneity, we calibrate our model using six sectors, where each sector is defined as either a goods or services sector that produces exclusively consumption, intermediates, or investment. This approach preserves the focus on the dominant patterns of structural change from goods toward services inputs, while simultane-

[2]This contrasts with Ngai and Pissarides (2007), who assume that all sectoral TFP terms grow at constant rates, precluding the coexistence of balanced growth and structural change in intermediates production. In fact, our framework requires that TFP growth in at least one sector is non-constant for an aggregate balanced growth path to exist.

ously allowing for sufficient flexibility to capture important heterogeneity in price- and productivity-growth trends specific to the production of consumption, investment, and intermediates.

To facilitate this six-sector calibration, we construct price series for goods and service sectors measured separately for sectors specializing in the production of consumption, investment, and intermediates in the U.S. from 1947-2020. Because these six sectors are not a simple partition of how sectoral activity is usually measured (i.e., using NAICS codes, as in the BEA's GDP by Industry database), we cannot use standard sectoral data sources to directly measure prices. Instead of using these income-side sectoral prices, we construct prices using expenditure-side national accounting data on 68 detailed consumption commodities and 30 detailed investment commodities, which we map to either goods or services sectors using BEA data identifying the sectors that make each commodity. Since there is no expenditure side national accounting data on intermediates expenditures—intermediates are not counted in GDP—we infer intermediates prices for each of our six sectors as a residual from the BEA's sectoral gross output prices, which are implicitly a weighted average of consumption, investment, and intermediates prices. Despite measuring these prices as residuals, we find that they almost perfectly match the U.S. Bureau of Labor Statistics' (BLS) producer price indexes (PPI) for intermediate inputs in the years they are available.

Based on these new price series, our calibration implies that goods and services inputs are strong complements (Leontief) in both consumption and intermediates production, but are substitutes in the production of investment. This means that productivity growth in the production of investment endogenously comprises a larger share of aggregate growth over time, as substitutability induces reallocation towards the inputs with the fastest TFP growth, and complementarity in intermediates production leads to reallocation toward the inputs with the slowest TFP growth. Thus, our results provide an optimistic view for future economic growth, in contrast to Baumol's (1972) prediction that structural change toward services implies aggregate economic growth will ultimately be governed by the growth rate of the least productive services sector.

Our result on the substitutability of investment inputs is novel, as previous studies find the best model fit when investment inputs are complements. We show that this difference is due to aggregation bias in price measurement. For example, recent contributions that allow for structural change in investment, such as Herrendorf et al. (2021), García-Santana, Pijoan-Mas and Villacorta (2021), and Sposi, Yi and Zhang (2021), all assume a single price for all commodities produced by the goods and services sectors, respectively. However, we show that the price trends of consumption, investment, and intermediate input commodities *within* goods and services sectors are heterogeneous. Consequently, since only about 5% of services output is used for investment on average, the price of investment produced by services sectors is not accurately represented in the aggregate price index for the services sector as a whole. This aggregation bias persists even when using more disaggregated services sectors. For example, in the professional/technical services sector, which is the leading producer of services investment, only 22% of output is used for investment, and the slow price growth for the investment commodities produced by this sector (e.g., software, intellectual property) is masked by the rapid price growth for other commodities (e.g., legal services, veterinary services). Thus, income-side accounting data on sectoral gross output prices is not sufficiently disaggregated to identify the price of output in sectors defined both by their product market (goods, services) and the use of their products (consumption, investment, and intermediates).

Finally, we use our calibrated model to analyze the forces driving U.S. economic growth over time. To do this, we derive a model expression for aggregate GDP measured as an index number, consistent with national accounting practices. This expression implies that aggregate growth is primarily determined by a combination of Domar-weighted productivity growth (i.e., Hulten's Theorem, Hulten, 1978; Baqaee and Farhi, 2019) and growth in the aggregate TFP series consistent with balanced growth. We analyze both growth in that aggregate TFP series and GDP growth overall, decomposing the contributions of TFP growth in the production of consumption, investment, and intermediates to aggregate growth.

We find that investment-specific technical change, i.e., TFP growth in goods and services sectors producing investment, explains 50% of U.S. aggregate GDP growth throughout 1947-2020 and has increased from explaining 30-40% of growth prior to the 1980s to more than 70% since the year 2000. Intermediates-specific technical change, in contrast, explains about 22% of aggregate growth over the entire sample but provides no (or even a negative) contribution to growth since 2000. In addition, we show that absent structural change in the production of investment, total aggregate growth since 2000 would have been about 20% lower.

Finally, our results imply a contrast between the aggregate growth slowdown of the 1970s and that of the 2000s: the former reflects slow TFP growth in the production of both investment and intermediates while the latter is entirely accounted for by stagnant or negative productivity growth in the production of intermediates. Thus, in contrast to Gordon (2016), who argues that recent slowdowns in aggregate growth represent a trend dating back to the 1970s, our analysis suggests that the growth slowdown of the 1970s is distinct from that of the 2000s. Moreover, although significant attention has been given to productivity disruptions along supply chains during the COVID-19 pandemic, our results suggest that weak productivity growth in the intermediates production network predates the pandemic by 20 years.

Related Literature. Our paper is at the intersection of two large literatures—the study of production networks as propagation mechanisms for economic fluctuations and growth and the study of long-run structural transformation from goods to services. Papers in the production networks literature have emphasized the role of static production networks in shaping fluctuations. The literature on structural change has typically focused on multi-sector models that either abstract from production networks (or treat them implicitly in a "value added" specification) or do not allow these networks to change over time. Our contribution lies in studying the intersection of these two phenomena.

While a significant literature has focused on how production networks propagate short-run fluctuations over the business cycle, their role in shaping long-run growth has also gained attention (e.g., Ngai and Samaniego, 2009; Moro, 2015; Foerster et al., 2019; Valentinyi, 2021). Our paper is closely related to Ngai and Samaniego (2009), who focus on how technological change in sectors producing intermediate goods can impact the composition of long-run growth via investment-specific technological change (as in Greenwood, Hercowitz and Krusell, 1997). Also closely related is recent work by Foerster et al. (2019), who focus on how production networks, especially the investment network, influence which sectors' TFP growth accounts for recent slowdowns in aggregate growth. Similar to Foerster et al. (2019), we find evidence that the proximate forces driving growth slowdowns in the 1970s and since 2000 are distinct. While some of our insights are similar, we highlight two important distinctions: first, our study allows for production networks to change endogenously over time, and second, our framework with structural change leads us to analyze patterns of growth across sectors defined not just by their product market (i.e., manufacturing, construction, services), but by the types of products they supply (i.e., consumption, investment, intermediates).

A second strand of literature focuses on structural change, but abstracts from explicitly modeling production networks. For example, Herrendorf, Rogerson and Valentinyi (2013) introduce value-added measures of structural change that implicitly embed indirect contributions to final production occurring via the input-output network. Absent using this approach, Herrendorf et al. (2013) and Herrendorf et al. (2014) argue that using expenditure-side prices to calibrate models of structural change is only appropriate when the model accounts for the input-output structure of the economy. We document that structural change in the network of intermediates production accounts for nearly half of the measured structural change in consumption and investment, motivating the importance of explicitly modeling the input-output network. Furthermore, since we explicitly model the input-output network, our use of expenditure-side prices is appropriate for calibration, being internally consistent with the model.

Our paper also relates to recent literature studying further disaggregation of the services sector. For example, recent work has split services into high-skill and low-skill services (Buera and Kaboski, 2012; Buera, Kaboski, Rogerson and Vizcaino, 2022), tradable and non-tradable services (Eckert et al., 2019), traditional and non-traditional services (Duarte and Restuccia, 2020), and stagnant and progressive services (Duernecker, Herrendorf and Valentinyi, 2021). Our six sector disaggregation considers a different division of services sectors, distinguishing subsectors by the type of product they produce and emphasizing the elasticity of substitution between goods and services in the production of each product. Since these sectors are not directly identifiable using standard sector definitions, our approach to measuring prices and productivity uses expenditure-side data, similar to how Greenwood et al. (1997) measure and quantify the importance of investment-specific technical change. Closest to our work, Duernecker et al. (2021) also provides some optimism about future growth, arguing that detailed services sectors are substitutes with each other within services consumption, implying reallocation towards high productivity growth services sectors.

Finally, our work relates to recent literature that has observed and begun to analyze structural change in production networks. Early work by Berlingieri (2013) and Galesi and Rachedi (2018) provide some evidence that services sectors have been rising in importance for intermediates production. Berlingieri (2013) argues that the rising importance of professional/technical services reflects increased outsourcing activity, which may contribute to rising services employment, and Galesi and Rachedi (2018) analyze how the change in the composition of intermediates production affects the effectiveness of monetary policy. More recently, work by Sposi (2019) and Sposi et al. (2021) extends models of structural change to endogenously account for changes in intermediates production to explain the hump-shaped rise and fall of manufacturing and the global distribution of manufacturing. In contrast, we focus on accounting for structural change in production networks and the implications of those changes for aggregate economic growth. Most closely related to our work, Herrendorf et al. (2021) and García-Santana et al. (2021) extend structural change models to incorporate structural change in investment. Our work builds on these analyses by further incorporating structural change in intermediates and allowing for heterogeneous prices of goods and services by use, allowing us to discover the substitutability of goods and services inputs in investment.

2. Data & Empirical Evidence

Our study of structural change focuses on two production networks, represented as matrices: the distribution over the production and purchases of intermediate inputs (the input-output network) and the distribution over the production and purchases of new capital (the investment network). The primary data sources for measuring these production networks in the U.S. are the Make and Use Tables from the BEA Input Output

Table 1: Average Share of Intermediates and Investment Production: Avg. 1947-2019

Goods Sectors (NAICS Codes)	% of Prod. Int.	Inv.	Services Sectors (NAICS Codes)	% of Prod. Int.	Inv.
Agriculture, forestry, fishing and hunting (11)	5.8	0.0	Wholesale trade (42)	5.8	3.8
Mining, except oil and gas (212)	1.1	0.2	Retail trade (44-45)	1.9	1.4
Support activities for mining (213)	0.0	0.3	Transport and warehousing (48-49, minus 491)	5.6	0.9
Construction (23)	1.9	37.0	Information (51)	4.6	6.2
Wood products (321)	1.5	0.5	Finance and insurance (52)	7.8	0.1
Non-metallic mineral products (327)	1.6	0.1	Real estate (531)	4.9	1.8
Primary metals (331)	4.8	0.1	Rental and leasing services (532-533)	1.1	0.0
Fabricated metal products (332)	4.2	1.1	Professional and technical services (54)	5.4	10.3
Machinery (333)	1.7	8.6	Management of companies and enterprises (55)	3.1	0.1
Computer and electronic products (334)	2.5	6.7	Administrative support and waste services (56)	3.3	0.1
Electrical equipment manufacturing (335)	1.2	1.0	Educational services (61)	0.3	0.2
Motor vehicles manufacturing (3361-3363)	3.3	9.1	Health services (62)	0.3	0.1
Other transportation equipment (3364-3369)	1.5	4.7	Arts, entertainment and recreation services (71)	0.4	0.1
Furniture and related manufacturing (337)	0.3	1.2	Accommodation services (721)	0.5	0.0
Misc. manufacturing (339)	0.9	1.3	Food services (722)	1.0	0.0
Food and beverage manufacturing (311-312)	4.9	0.1	Other private services (81)	1.7	0.1
Textile manufacturing (313-314)	2.0	0.3	Federal government (n/a, but incl. 491)	1.1	0.5
Apparel manufacturing (315-316)	0.7	0.0	State and local government (n/a)	1.4	0.9
Paper manufacturing (322)	2.6	0.0			
Printing products manufacturing (323)	1.2	0.6			
Chemical manufacturing (325)	4.6	0.4			
Plastics and rubber products (326)	1.8	0.1			

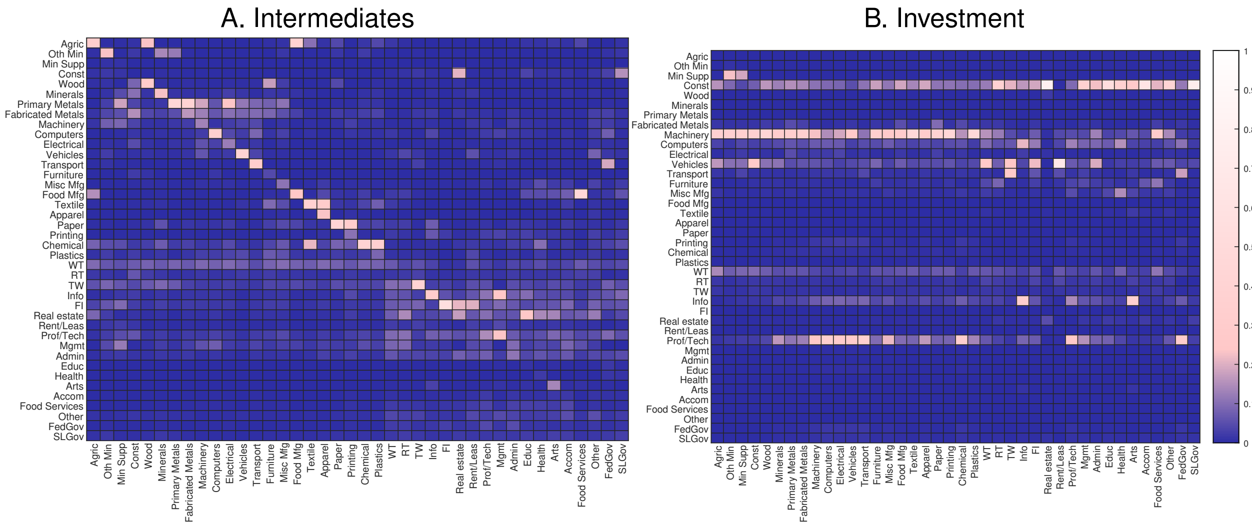
Notes: The table reports the average share of total intermediate and investment production by 40 consistent sectors. Individual components may not exactly sum to totals due to rounding. Sectors are classified according to the NAICS-based BEA codes, with 2007 NAICS codes listed in parentheses. Government sectors as defined by the BEA do not have naturally corresponding NAICS codes.

Database. These tables contain data for each sector on the value of gross output, value-added, intermediate input purchases (from each other sector), and final uses (consumption, investment, etc.) of each sector's production. The database begins in 1947, and we study patterns of change through 2020. For measuring the investment network, we additionally use data from vom Lehn and Winberry (2022), who combine BEA Input Output data with the BEA Fixed Assets tables to construct a time series of the investment network that covers a similar time frame and level of sector detail; we extend their data to run through 2020. Additional details regarding data sources and measurement of production networks are available in Appendix A.

Based on these sources, we compile a dataset that provides consistent coverage of U.S. production networks for 40 NAICS-defined sectors of the economy, including agriculture and government; Table 1 lists each of the 40 sectors and their corresponding NAICS codes.[3] We define "goods" sectors as all agriculture, mining, construction, and manufacturing sectors (22 in total) and "services" as all remaining sectors (18 in total). Our analysis will exclude three sectors whose outcomes are highly volatile due to fluctuations in the

[3]More recent vintages of the BEA Input Output database allow for greater sectoral detail, but given our interest in structural change over the long run, we focus on these 40 sectors, which can be observed throughout the period 1947-2020.

Figure 1: Heatmaps of Average Scaled Production Networks, 1947-2020



Notes: Panel A shows the scaled input-output network for intermediate goods; panel B shows the scaled investment network for new capital goods. Each (i, j) element in the matrix shows the fraction of total expenditure by sector j (columns) coming from producing sector i (rows).

price of oil—oil and gas extraction, utilities, and petroleum and coal manufacturing. These sectors' prices are very volatile over short-run and medium-run horizons, which can obfuscate trends in relative prices of intermediate inputs; we provide a further discussion in Appendix C. That said, we show in Appendix A that there are no long-run trends in the share of intermediates or investment produced by these sectors and, thus, the patterns of structural change we study are not significantly impacted by their exclusion.

Our data on production networks include imported intermediates, consumption, and investment, reflecting the full set of intermediate and investment purchases by each sector, regardless of geographic origin. Because the BEA only reports total spending on imports and doesn't differentiate how those imports are used, we can neither treat imports separately nor remove imports from the data within our measurement framework. That said, throughout the period 1947-2020, imports are never more than 8% of total consumption, intermediates, or investment in any given year, so changes in imports cannot account for the aggregated patterns of structural change we document below.

2.1. Average Production Networks

The production networks in our dataset are measured as a pair of matrices for each year t, where element (i, j) of each matrix reports expenditures by sector j on intermediates or investment produced by sector i in year t. Before presenting evidence on how these networks have changed over time, we first illustrate the average structure of production networks in the United States from 1947-2020. For visualization, it is convenient to consider scaled versions of the two networks, with elements s_{ijt}^M and s_{ijt}^X representing sector j's share of total expenditures on intermediates (M) or investment (X) made by sector i in year t.[4]

Figure 1 plots heatmaps of the scaled input-output network (panel A; matrix elements s_{ijt}^M) and the scaled investment network (panel B; matrix elements s_{ijt}^X), averaged over the entire sample period. These heatmaps show that both the input-output and investment networks are sparse; for any given sector, the majority of investment and intermediates are purchased from a small set of sectors. For the investment network, the distribution of investment producers is fairly similar across sectors. Most sectors purchase investment goods from a collection of prominent investment hubs—construction produces structures investment, machinery and vehicles manufacturing sectors produce a large amount of equipment investment, and information and professional/technical services produce software and other intellectual property investment.

However, for the input-output network, there is much more sector-specificity as to which sectors are important suppliers of intermediates. In particular, we observe significant homophily in the input-output network—goods sectors play a large role as intermediates suppliers for goods sectors and services sectors play a large role as intermediates suppliers for services sectors.

2.2. Structural Change in Production Networks

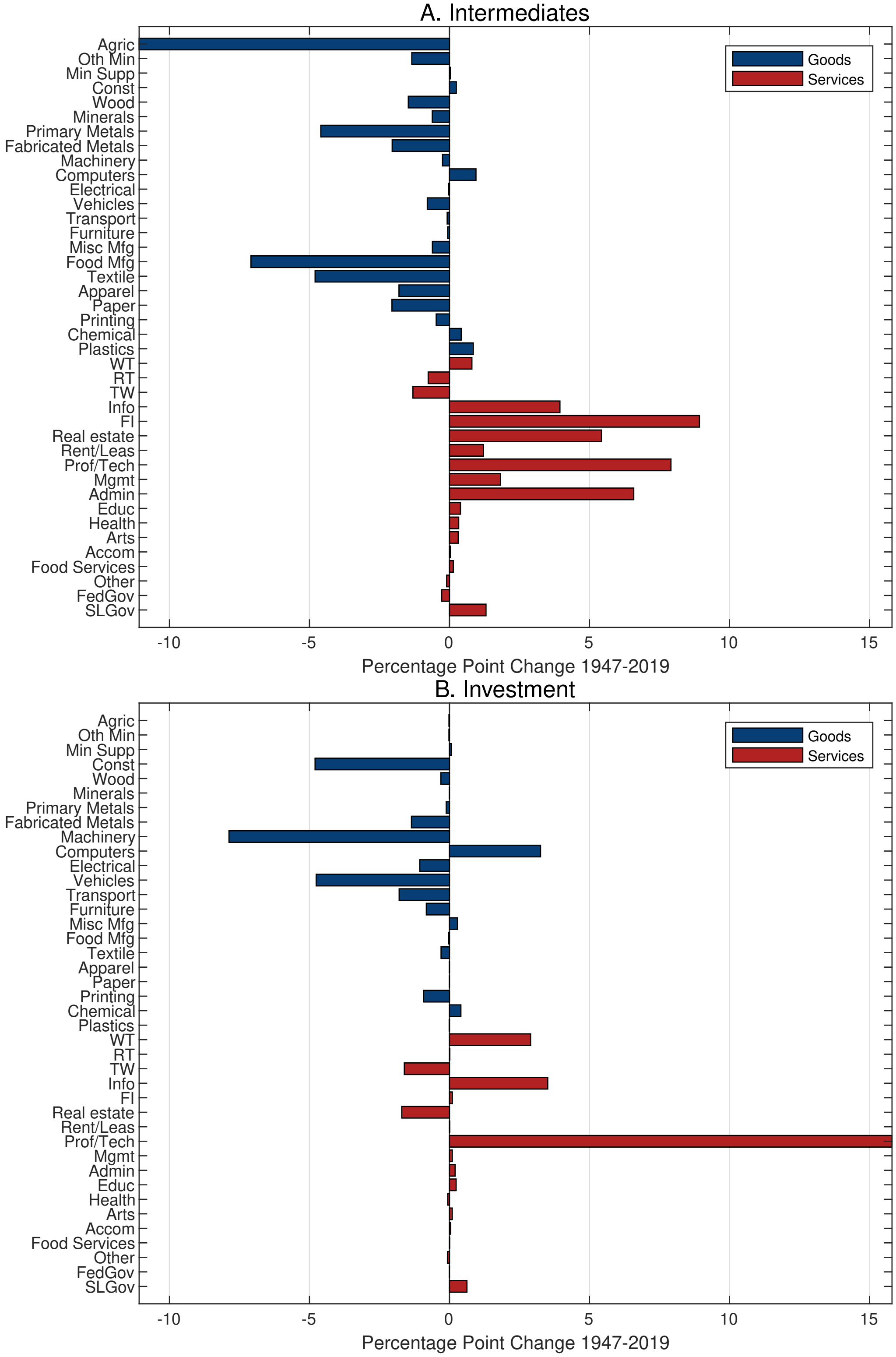
While Figure 1 shows the average structure of U.S. production networks, we now focus on how these networks change over time. We document patterns of change within these production networks both at the 40-sector level of disaggregation and at the two-sector level (goods and services).

Although normalizing by the total spending of sector j is convenient for visualizing a static network, this normalization ignores changes in the size distribution of purchasing sectors j when studying changes over time. Thus, to measure structural change in production networks, we analyze changes in the fraction of aggregate spending for intermediates or investment on products produced by sector i. Denoting these aggregate shares in each year t as s_{it}^M and s_{it}^X, we can express them as weighted averages of the elements of the scaled input-output and investment networks, s_{ijt}^M and s_{ijt}^X:

$$s_{it}^M = \sum_j \frac{P_{jt}^M M_{jt}}{\sum_k P_{kt}^M M_{kt}} s_{ijt}^M \quad \text{and} \quad s_{it}^X = \sum_j \frac{P_{jt}^X X_{jt}}{\sum_k P_{kt}^X X_{kt}} s_{ijt}^X, \tag{1}$$

[4] Formally, $s_{ijt}^M = \frac{P_{it} M_{ijt}}{\sum_l P_{lt} M_{ljt}}$, where P_{it} represents the price of sector i's intermediates at time t and M_{ijt} represents the quantity of intermediate inputs purchased from sector i by sector j, with an analogous expression for investment. We use information on total payments from sector j makes to sector i in year t, $P_{it} M_{ijt}$, as price and quantity are not separately observed in the network data; this notation allows for easy comparison to our model structure in the next section.

Figure 2: Changes in Production Share of Intermediates and Investment: 1947-2019



Notes: Each bar represents the change in the share of intermediates (panel A) or investment (panel B) produced by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

where $P^M_{jt} M_{jt}$ is defined as the total spending on intermediates by sector j, with $P^M_{jt} M_{jt} \equiv \sum_i P_{it} M_{ijt}$, and thus $\frac{P^M_{jt} M_{jt}}{\sum_k P^M_{kt} M_{kt}}$ is sector j's spending on intermediates as a fraction of aggregate intermediate spending (with expressions for investment defined analogously). Changes in s^M_{it} and s^X_{it} over time reflect both changes in production processes within sectors (changes in s^M_{ijt} and s^X_{ijt}) and changes in the composition of spending between sectors.

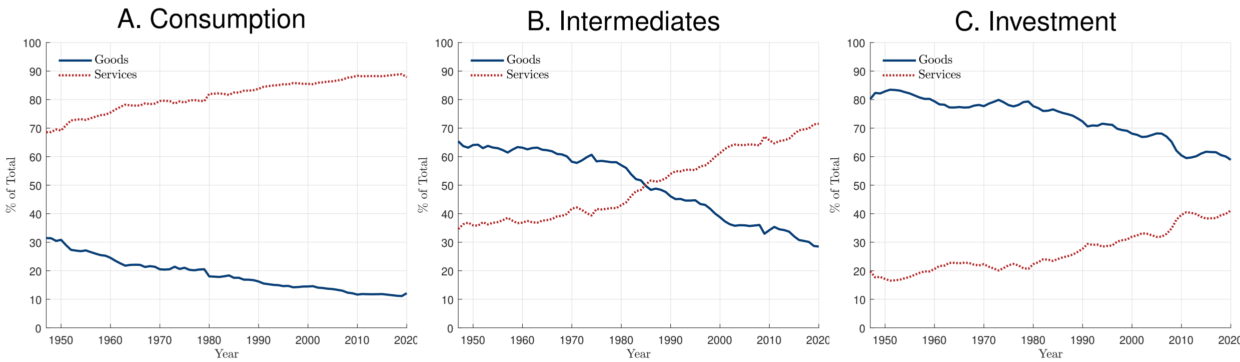
Figure 2 plots long differences (in percentage points) in s^M_{it} and s^X_{it} for each sector i from 1947 to 2019 (not 2020, to avoid any unusual endpoint effects with the onset of the COVID-19 pandemic). We observe that changes in sectoral production shares are primarily characterized by a decline in the share of production by goods sectors (blue bars) and an increase in the share of production by services sectors (red bars). However, the specific sectors whose production shares are changing the most are different for intermediates and investment. For intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative and waste services; the largest declines occurred in agriculture, primary metals, food and beverage manufacturing, and textile manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest declines were in machinery, construction, and motor vehicle manufacturing.[5]

Motivated by the general sectoral patterns in Figure 2, Figure 3 plots the time series of the share of total intermediates and investment spending on products made by aggregated goods or services sectors, s^M_{it} and s^X_{it}. For comparison with previous work on structural change in investment and consumption (e.g. Herrendorf et al., 2021), Figure 3 also plots the share of total consumption spending on products made by goods and services sectors. For each aggregate use, the fraction produced by the services sector is rising over time, with the services share of investment and consumption rising by roughly 20 percentage points and the services share of intermediates increasing by more than 35 percentage points.

The rising services share in each network may reflect increased spending on service inputs within sectors or increased total spending by sectors that use services intermediates more intensively. To understand these two dimensions of structural change, we conduct a shift-share decomposition of the increasing shares of investment and intermediates produced by services sectors (see Appendix A for details). Our decomposition finds that roughly 50% of the increased share of intermediates produced by services occurs due to

[5] While Figure 2 displays long differences, the time series of production shares for sectors whose production share of intermediates or investment has risen or fallen the most are shown in Appendix A.

Figure 3: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, 1947-2020



Notes: The figures plot the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue, solid line) and the services sector (red, dotted line). A full listing of sectors included in goods and services sectors can be seen in Table 1.

within-sector changes, whereas 75-100% of the increased share of investment produced by services occurs within sectors. These findings of a more significant role for between-sector changes in the services share of production accord with the production network patterns shown in Figure 1, which display more sector-specificity for intermediate input suppliers than for investment suppliers. These findings also account for why the magnitude of structural change in intermediates is larger than for consumption or investment.

Finally, we relate our stylized facts to the "value-added" measurement approach introduced by Herrendorf et al. (2013). Their approach implicitly includes structural change in the intermediates network by using input-output data to identify each sector's overall contribution to producing consumption or investment. Given the quantitatively large changes we document within the intermediates network (Figure 3), we ask what portion of value-added measured structural change can be attributed to the intermediates network. Our decomposition shows that structural change in the intermediates network alone accounts for roughly 50% of structural change in consumption and investment value-added (see Appendix A for details).

2.3. Outsourcing and International Evidence

A potential concern about the observed pattern of structural change in intermediates is that it may reflect outsourcing of services tasks, resulting in a change of *where* services tasks are performed and how they are recorded, rather than a change in the actual structure of production. That is, it may be that firms who originally produced services internally started outsourcing these tasks to other firms, with the newly outsourced tasks measured as services intermediates.

We provide three pieces of evidence to suggest that such outsourcing of services does not drive the observed structural change in intermediates, with details in Appendix A. First, the national accounting data we use measures economic activity at the establishment level. Thus, services provided to establishments within a firm by separate administrative offices and headquarters are already classified as intermediates produced by services. If an establishment now receives these services from a different firm, it would not impact the measured share of intermediates produced by services. Second, the average ratio of spending on intermediates relative to gross output (across sectors) has remained between 42-46% for nearly the entire sample window of 1947-2020, with little trend over time. Furthermore, within sectors, there is no correlation between increased intermediate spending (relative to gross output) and an increase in the share of intermediates purchased from services. Thus, structural change in the production of intermediates does not appear to have coincided with increased spending on intermediates.[6] Finally, we extend an argument made by Duernecker and Herrendorf (2022) to analyze structural transformation of the occupational distribution. Within all but one sector (agriculture), there is a systematic rise in workers employed in services occupations over the period 1950-2019, rather than a decline, which would result from systematic outsourcing of services tasks from goods to services sectors. Moreover, within sectors, there is no relationship between structural change in occupations and changes in the share of intermediates purchased from services sectors.

In addition, we document in Appendix A that the share of investment and intermediates spending purchased from services sectors is rising in nearly all sectors. Moreover, using data from the World Input Output Database (WIOD: Timmer, Dietzenbacher, Los, Stehrer and de Vries, 2015; Woltjer, Gouma and Timmer, 2021), we show that these changes in production networks are not unique to the United States but are observed more broadly throughout other high-income nations in Europe and Asia.

3. Model

To account for the patterns of structural change observed in the previous section, we now describe an N sector extension of the neoclassical growth model, which explicitly incorporates the production networks for intermediate inputs and investment. We present a general version of the model in this section and discuss additional assumptions necessary for a balanced growth path in Section 4.

[6] A limitation of this argument is that the ratio of spending on intermediates to total gross output is not exactly the cost share of intermediates in production due to the presence of markups. If markups rise over time, a constant ratio of intermediates spending could imply a rising cost share of intermediates, possibly reflecting increased outsourcing. However, the evidence on the markup trends is mixed (see Basu, 2019), making it difficult to know if a constant ratio of intermediates spending to gross output is masking a rise in cost shares.

3.1. Technology

For each sector j, gross output, Q_{jt}, is produced using capital, K_{jt}, labor L_{jt}, and a bundle of intermediate goods M_{jt} according to the following Cobb-Douglas production function:

$$Q_{jt} = A_{jt} \left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j} \right)^{\alpha_j} M_{jt}^{1-\alpha_j}, \tag{2}$$

where A_{jt} is exogenous TFP in sector j.

The intermediates bundle for each sector, M_{jt}, is produced by an "intermediates bundling" sector for sector j, which aggregates intermediate goods from all sectors using a CES technology:

$$M_{jt} = A_{jt}^{M} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}, \tag{3}$$

where ϵ_{Mj} is the elasticity of substitution between sectoral inputs in the production of intermediate goods for sector j, $\omega_{Mij} \in [0,1]$ (with $\sum_i \omega_{Mij} = 1$) determines the relative importance of inputs from each sector in producing intermediates, M_{ijt} represents intermediate inputs used in sector j purchased from sector i at time t, and A_{jt}^{M} represents exogenous intermediates-bundling TFP for sector j. We include this bundling TFP to allow for added flexibility when calibrating the model to fit the data; as we show in Section 6, movements in this TFP term play only a modest role in contributing to overall economic growth.

Capital is sector-specific and follows a standard law of motion, with depreciation rate δ_j in sector j:

$$K_{jt+1} = (1-\delta_j) K_{jt} + X_{jt}. \tag{4}$$

Investment, X_{jt}, is produced in an "investment bundling" sector for sector j's capital with the following aggregation technology:

$$X_{jt} = A_{jt}^{X} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}, \tag{5}$$

where ϵ_{Xj} is the elasticity of substitution between sectors in the production of investment in sector j, $\omega_{Xij} \in [0,1]$ (with $\sum_i \omega_{Xij} = 1$) determines the relative importance of inputs from each sector in producing investment, and X_{ijt} represents investment inputs used in sector j purchased from sector i at time t. A_{jt}^{X} represents exogenous investment-bundling TFP for sector j, which is included to allow for calibration flexibility.

3.2. Preferences

There is an infinitely lived representative household with preferences given by:

$$\sum_{t=0}^{\infty} \beta^t U(\{C_{it}\}_{i=1}^N), \tag{6}$$

where $0 < \beta < 1$ is the discount rate and C_{it} is consumption produced by sector i. We assume that the period utility function, $U(\{C_{it}\}_{i=1}^N)$ follows a log CES structure, with

$$U(\{C_{it}\}_{i=1}^N) = \ln\left(\left[\sum_i \omega_{Ci}^{1/\epsilon_C} C_{it}^{\frac{\epsilon_C - 1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C - 1}}\right), \tag{7}$$

where ϵ_C represents the elasticity of substitution between consumption goods and $\omega_{Ci} \in [0, 1]$ (with $\sum_i \omega_{Ci} = 1$) determines the relative importance of consumption goods from each sector in aggregate consumption. Our framework abstract from preferences over leisure, assuming the household inelastically supplies one unit of labor each period.

Our assumption of log CES preferences rules out income effects as a possible cause for structural transformation in consumption. However, given our focus on structural change in production networks, we make this assumption for ease of exposition and tractability.[7]

3.3. Equilibrium

We study the competitive equilibrium of this economy with representative profit-maximizing firms in all markets. The price of final output in each production sector j is denoted by P_{jt}; the price of the intermediates bundle is given by P_{jt}^M. The household owns the capital stock and accumulates capital in sector j by purchasing new investment goods from the investment bundling firm for sector j at price P_{jt}^X. The household rents sector-specific capital to each sector j at a rental price R_{jt}. Since labor is common to each sector and freely mobile, there is a single wage paid to the household, denoted by W_t. We provide a full listing of equilibrium conditions in Appendix B.

In equilibrium, all markets (final output, labor, capital, intermediate bundling, and investment bundling) clear. Final output produced by each sector can be used as consumption for the household or as an input to

[7] We could introduce non-homothetic PIGL preferences (as in Boppart, 2014), as the form of household preferences has no impact on either the growth rate of GDP or structural change in production networks along an aggregate balanced growth path (ABGP). In fact, for the case of two consumption sectors that we consider, the ABGP derived in the subsequent section is identical under PIGL preferences, similar to Herrendorf et al. (2021).

intermediate and investment bundles across sectors, implying a market clearing relationship of:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{8}$$

The extent to which each sector produces consumption, intermediates, and investment will depend on the calibration of preference and bundling parameters; in Section 5, we consider a calibration in which each sector j specializes in the production of only one final use: consumption, investment, or intermediates.

Given constant returns and competitive markets, it is straightforward to show that the price indices for the bundle of intermediate goods, P^M_{jt}, and the bundle of investment goods, P^X_{jt}, will be given by:

$$P^M_{jt} = \frac{1}{A^M_{jt}} \left(\sum_i \omega_{Mij} P_{it}^{1-\epsilon_{Mj}} \right)^{\frac{1}{1-\epsilon_{Mj}}} \tag{9}$$

$$P^X_{jt} = \frac{1}{A^X_{jt}} \left(\sum_i \omega_{Xij} P_{it}^{1-\epsilon_{Xj}} \right)^{\frac{1}{1-\epsilon_{Xj}}}. \tag{10}$$

Furthermore, straightforward manipulation of the first order conditions for each sector's production generates the following expression for the price of final output produced by each sector j:

$$P_{jt} = \frac{1}{A_{jt}} \left(\frac{R_{jt}}{\theta_j \alpha_j} \right)^{\theta_j \alpha_j} \left(\frac{W_t}{(1-\theta_j)\alpha_j} \right)^{(1-\theta_j)\alpha_j} \left(\frac{P^M_{jt}}{1-\alpha_j} \right)^{1-\alpha_j}. \tag{11}$$

Finally, we describe the equilibrium conditions that dictate structural change in production networks. Manipulating first order conditions for the intermediates and investment bundling sectors, we can derive the model equivalent of the shares of intermediates or investment spending by sector j on products made by sector i at time t, denoted s^M_{ijt} and s^X_{ijt} (as in Section 2):

$$s^M_{ijt} \equiv \frac{P_{it} M_{ijt}}{P^M_{jt} M_{jt}} = \omega_{Mij} \left(\frac{P_{it}}{A^M_{jt} P^M_{jt}} \right)^{1-\epsilon_{Mj}}, \tag{12}$$

$$s^X_{ijt} \equiv \frac{P_{it} X_{ijt}}{P^X_{jt} X_{jt}} = \omega_{Xij} \left(\frac{P_{it}}{A^X_{jt} P^X_{jt}} \right)^{1-\epsilon_{Xj}}. \tag{13}$$

These intermediates and investment expenditure shares depend on the relative prices of each sector's output, the CES scale (ω), and elasticity parameters (ϵ) in the bundling sectors. Thus, movements in relative prices across sectors can induce structural change in production networks. Furthermore, we can define the aggregate share of intermediates or investment spending on products made by sector i, s_{it}^{M} and s_{it}^{X}, using the expressions in equation (1). Thus, as described in Section 2, changes in production networks in the model reflect both changes in production processes within sectors (changes in s_{ijt}^{M} and s_{ijt}^{X}) and changes in the composition of spending between all sectors.

4. Balanced Growth Path

This section shows how the model can generate structural change along an aggregate balanced growth path (ABGP). We focus on an ABGP for two reasons: (1) balanced growth is generally consistent with empirical U.S. economic growth since the Industrial Revolution, and (2) the aggregate growth path representation facilitates a clearer understanding of the relationships between structural change and aggregate economic growth. In Section 6.3, we consider the model's implications for economic growth without some of the restrictions necessary to obtain an ABGP.

We study an ABGP in this economy in three steps. First, we describe the necessary assumptions for such a path to exist and the implications of these assumptions for prices and aggregate quantities. Second, we state and discuss a proposition establishing the existence and nature of an ABGP consistent with structural change in production networks. Finally, we analyze the connection between the aggregate growth rate and structural change in production networks along the ABGP.

4.1. Assumptions, Equilibrium Prices, and Aggregation

While the rich heterogeneity of our model in the previous section is appealing, it is also generally inconsistent with the existence of an ABGP—for example, Acemoglu and Guerrieri (2008) show that differential capital and labor intensities are only consistent with balanced growth in the limit.[8] The possibility of an ABGP requires assumptions that impose homogeneity in production functions and investment bundling across sectors. However, the existence of an ABGP does not require any assumptions regarding the structure of the input-output network (i.e., the intermediates bundling technologies).

We thus impose the following assumptions:

Assumption 1. *The parameters of the sectoral production functions are the same across all sectors, i.e.,* $\alpha_j = \alpha$ *and* $\theta_j = \theta$ *for all* j*.*

[8]That said, Herrendorf, Herrington and Valentinyi (2015) argue that capital share heterogeneity (as well as capital-labor substitution) is of second-order importance for explaining quantitative patterns of structural change.

Assumption 2. *The parameters governing the evolution of capital—both parameters of the investment bundling sectors and the depreciation rate—are the same for all sectors j, i.e., $\delta_j = \delta$, $\omega_{Xij} = \omega_{Xi}$ and $\epsilon_{Xj} = \epsilon_X$ for all j. Furthermore, each sector's investment bundling TFP is the same, i.e., $A^X_{jt} = A^X_t$.*

These assumptions allow us to simplify equilibrium price expressions. First, with common parameters in the investment bundling sectors, equation (10) implies that the price of new investment will be equated across sectors, i.e., $P^X_{jt} = P^X_t$. Furthermore, since the parameters governing the evolution of capital are also equated across sectors, there is now a single type of capital in the economy with a single rental rate, i.e., $R_{jt} = R_t$.[9] Finally, given common production function parameters (and a common wage by assumption), equation (11) implies that the relative price of final output in sectors i and j depends only on sectoral TFP differences and differences in the price of the intermediates bundles in those two sectors:

$$\frac{P_{jt}}{P_{it}} = \frac{A_{it}/\left(P^M_{it}\right)^{1-\alpha}}{A_{jt}/\left(P^M_{jt}\right)^{1-\alpha}} = \frac{\tilde{A}_i t}{\tilde{A}_{jt}}, \tag{14}$$

where $\tilde{A}_{jt}$ is an "adjusted" measure of productivity, with $\tilde{A}_{jt} \equiv \frac{A_{jt}}{\left(P^M_{jt}\right)^{1-\alpha}}$ that includes endogenous prices. Thus, price differences across sectors depend directly on productivity, A_{it}, and indirectly on differences in the marginal cost paid for intermediate inputs, P^M_{it}, in each sector. Without further assumptions, we cannot solve for the price of intermediates in closed form.[10] We consider a special case where a closed-form representation exists at the end of this section.

Lemma 1 relates the price of intermediates and investment bundles to the output prices and adjusted productivities of each sector:

Lemma 1. *Given assumptions 1 and 2, the price of sector j's product relative to the price of the bundle of*

[9] An implication of Assumption 2 is that, with a single capital input for all sectors of the economy, there is no scope for a compositional channel of structural change in the investment network. Our shift-share exercises reported in Section 2 imply that only a small fraction of structural change in the investment network comes from compositional movements. Hence, this assumption is consistent with empirical patterns of structural change.

[10] If we take aggregate investment as the numeraire, the price of the intermediate bundle in sector i is given by:

$$P^M_{it} = \frac{A^X_t}{A^M_{it}} \left(\sum_k \omega_{Xk} \left(\frac{A_{kt}}{(P^M_{kt})^{1-\alpha}} \right)^{\epsilon_X - 1} \right)^{\frac{1}{\epsilon_X - 1}} \left(\sum_k \omega_{Mki} \left(\frac{A_{kt}}{(P^M_{kt})^{1-\alpha}} \right)^{\epsilon_{Mi} - 1} \right)^{\frac{1}{1-\epsilon_{Mi}}}.$$

This produces a system of N non-linear equations that can be solved to obtain the price of each intermediates bundle.

intermediates in each sector, P_{it}^M, and to the price of the bundle of investment, P_t^X, can be written as:

$$\frac{P_{jt}}{P_t^X} = \frac{\tilde{B}_t^X}{\tilde{A}_{jt}} \tag{15}$$

$$\frac{P_{jt}}{P_{it}^M} = \frac{\tilde{B}_{it}^M}{\tilde{A}_{jt}} \tag{16}$$

where $\tilde{B}_{it}^M \equiv A_{it}^M \left(\sum_k \omega_{Mki} \tilde{A}_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ *and* $\tilde{B}_t^X \equiv A_t^X \left(\sum_k \omega_{Xk} \tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$.

Proof. See Appendix B. □

$\tilde{B}_t^X$ and $\tilde{B}_{it}^M$ represent aggregate investment and intermediates TFP, and are functions of the adjusted productivity in each sector and the exogenous bundling TFP. These aggregate TFPs capture all forces that expand the production frontier (and thus lower prices) for investment or intermediate inputs. The most important feature of these TFPs is the sum of adjusted sectoral productivities, $\tilde{A}_{it}$, weighted in proportion to that sector's weight (ω) in the CES bundling technology. Since these sums depend on adjusted sectoral productivities, aggregate investment and intermediates TFPs will grow both due to TFP growth in sectors producing investment or intermediates, A_{it}, or reductions in their costs for intermediate inputs, P_{it}^M.

Using these expressions for aggregate investment and intermediates TFP, we show that aggregate GDP can be represented by an aggregate production function, taking aggregate investment as the numeraire. Aggregate GDP in the model is the sum of value added in each sector, where nominal sectoral value added in sector j is defined as nominal sectoral gross output minus expenditures on intermediates:

$$P_{jt}^V V_{jt} = P_{jt} Q_{jt} - P_t^M M_{jt}. \tag{17}$$

V_{jt} represents real value added in sector j and P_{jt}^V is the price of value added. Thus, aggregate GDP, Y_t, denoted in units of the numeraire, is given by $Y_t = \sum_i P_{it}^V V_{it}$.

Given the above assumptions and price relationships, the following lemma shows that aggregate GDP can be expressed using an aggregate production function:

Lemma 2. *Given Assumptions 1 and 2, aggregate GDP, denoted in units of the numeraire (aggregate investment), is given by:*

$$Y_t = \sum_i P_{it}^V V_{it} = \mathcal{A}_t K_t^\theta \tag{18}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}}\left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$ *and* $K_t = \sum_j K_{jt}$*. Furthermore, the following aggregate equilibrium conditions hold:*

$$R_t = \theta \mathcal{A}_t K_t^{\theta-1} \tag{19}$$

$$W_t = (1-\theta)\mathcal{A}_t K_t^{\theta} \tag{20}$$

Proof. See Appendix B. □

Aggregate GDP in Lemma 2 has the same Cobb-Douglas production structure as sectoral real value added (with labor dropping out because it is unit supply in the aggregate). With aggregate investment as the numeraire, aggregate TFP, $\mathcal{A}_t$, only depends on investment-production TFP, $\tilde{B}_t^X$. Thus, only technological change in the production of investment expands the aggregate production frontier, either directly at final producers of investment or indirectly via their intermediate suppliers.

4.2. An Aggregate Balanced Growth Path

We adopt the same definition of an ABGP as Ngai and Pissarides (2007) and Herrendorf et al. (2021), where all aggregates denoted in units of the numeraire (aggregate investment) must grow at a constant rate. This means that K_t, Y_t, W_t, R_t, and X_t will grow at constant (though not necessarily equal) rates. Total consumption expenditures and total intermediates expenditures are defined in units of the numeraire, with $E_t^C = \sum_i P_{it}C_{it}$ and $E_t^M = \sum_i P_{it}^M M_{it}$. Thus, E_t^C and E_t^M also grow at a constant rate along the ABGP.

For any variable Z_t, the gross growth rate between time periods t and $t+1$ is defined as $\gamma_{t+1}^Z \equiv \frac{Z_{t+1}}{Z_t}$. Along the ABGP, we drop the time subscripts for variables growing at a constant rate. With these definitions, we state the following proposition:

Proposition 1. *Assume that Assumptions 1 and 2 hold and that* $\gamma_t^{\mathcal{A}} > \frac{1-\delta}{\beta}\ \forall t$*. An aggregate balanced growth path exists where*

$$\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma_t^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} \tag{21}$$

and $\gamma^R = 0$ *if and only if* $\gamma_t^{\mathcal{A}}$ *is constant.*

Proof. See Appendix B. □

Given the aggregate production function derived in Lemma 2, the result for the aggregate growth rate is unsurprising. As in the one-sector growth model, the economy's aggregate growth rate only depends on the growth rate of aggregate TFP. The requirement that $\gamma_t^{\mathcal{A}} > \frac{1-\delta}{\beta}$ is standard and holds for most reasonable parameter values.

One key feature of the aggregate balanced growth path in our model with an explicit input-output structure is similar to that of Ngai and Pissarides (2007): the aggregate growth rate of the economy with investment as the numeraire only depends on technological change that increases the production frontier for investment. However, our aggregate balanced growth path differs from that of Ngai and Pissarides (2007), who argue that endogenous structural change in production networks is inconsistent with an aggregate balanced growth path. The key distinction between the two frameworks lies in the assumptions made about productivity growth. While Ngai and Pissarides (2007) assume constant productivity growth in all sectoral TFP terms, A_{it}, the existence of our balanced growth path is based on the assumption that the combination of TFP growth in all sectors (including bundling sectors) generates constant growth in aggregate TFP, $\mathcal{A}_t$. Given the CES structure of investment and intermediates bundling, if all TFP terms grow at a constant rate, aggregate TFP cannot grow at a constant rate (see Herrendorf et al. (2021) for a further discussion). Whether or not this assumption is valid can be empirically assessed (we plot aggregate TFP in Section 6 and find it grows at a mostly constant rate), but most importantly, the aggregate balanced growth framework enables us to analyze more concretely the relationship between structural change and aggregate growth, which we take up next.

4.3. Implications of Balanced Growth for Structural Change and Growth Rates

Having established the existence of an aggregate balanced growth path, we now turn to understanding the relationship between economic growth and structural change along the ABGP. Because the ABGP is defined by constant aggregate growth, structural change will not impact the aggregate growth rate. However, we can analyze how the composition of aggregate growth is influenced by structural change. The intuition of these compositional effects is also informative for how structural change might affect the level of aggregate growth in environments where balanced growth restrictions are not imposed. We consider such off-balanced growth accounting exercises in Section 6.

While Proposition 1 requires no restrictions on the intermediates bundling processes across sectors, we are unable to write down a closed-form expression for aggregate TFP, $\mathcal{A}_t$. Thus, for just this subsection, we impose one additional assumption which allows for a closed-form expression to provide clearer intuition. This additional assumption and its implication for aggregate growth are described in Assumption 3 and

Lemma 3.

Assumption 3. *The parameters of the intermediates bundling sectors are the same for all sectors j, i.e., $\omega_{Mij} = \omega_{Mi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that each sector's intermediates bundling TFP is the same, i.e., $A^M_{jt} = A^M_t$*

Lemma 3. *Assume that assumptions 1-3 hold. Then, investment-production TFP, $\tilde{B}^X_t$, is given by $\tilde{B}^X_t = \left(B^X_t)\right)^{\alpha}\left(B^M_t\right)^{1-\alpha}$, where $B^X_t \equiv A^X_t\left(\sum_k \omega_{Xk} A^{\epsilon_X-1}_{kt}\right)^{\frac{1}{\epsilon_X-1}}$ and $B^M_t \equiv A^M_t\left(\sum_k \omega_{Mk} A^{\epsilon_M-1}_{kt}\right)^{\frac{1}{\epsilon_M-1}}$. Aggregate TFP, $\mathcal{A}_t$, is then given by: $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B^X_t \left(B^M_t\right)^{\frac{1-\alpha}{\alpha}}$.*

Proof. See Appendix B. □

Assumption 3 imposes homogeneity in intermediates bundling across sectors, implying that all sectors face the same price of intermediates bundles, P^M_t.[11] With this symmetry across sectors, we can separate investment-production TFP into two components—the direct effect of productivity growth at investment producers, B^X_t, and the indirect effect of productivity growth at intermediates producers (which reduces the price of the intermediates bundle), B^M_t. Thus, aggregate TFP, $\mathcal{A}_t$ depends on these two separable forces, and we can analyze how the relative importance of these two forces changes with structural change along the ABGP.

Given assumption 3, we can write the growth rate of aggregate TFP, and thus the growth rate of aggregate quantities on the ABGP, as $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} = \left(\gamma^{B^X}_t\left(\gamma^{B^M}_t\right)^{\frac{1-\alpha}{\alpha}}\right)^{\frac{1}{1-\theta}}$. The dependence of the aggregate growth rate on both $\gamma^{B^X}_t$ and $\gamma^{B^M}_t$ establishes a direct connection between the aggregate growth rate and structural change in production networks. These two growth rates can be expanded as follows:

$$\gamma^{B^M}_t = \gamma^{A^M}_t\left(\sum_i s^M_{it-1}(\gamma^A_{it})^{\epsilon_M-1}\right)^{\frac{1}{\epsilon_M-1}} \tag{22}$$

$$\gamma^{B^X}_t = \gamma^{A^X}_t\left(\sum_i s^X_{it-1}(\gamma^A_{it})^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}. \tag{23}$$

In addition to the production share parameters on capital (θ) and value-added (α), equations (22) and (23) illustrate that the growth rate of aggregates depends on four components: growth in intermediates-bundling TFP ($\gamma^{A^M}_t$), growth in investment-bundling TFP ($\gamma^{A^X}_t$), and two weighted sums of TFP growth in

[11] One consequence of assumption 3 is that because there is no longer heterogeneity in where sectors purchase intermediates from, there is no longer a composition channel for structural change. However, this channel remains present in our calibration and growth accounting exercises in subsequent sections.

each individual production sector i, where the weights are determined by the composition of intermediates and investment production (s_{it}^M and s_{it}^X).

As relative prices change over time due to different rates of TFP growth across sectors, there will be structural change in the production networks in the model, captured by changes in s_{it}^M and s_{it}^X, and how this structural change affects the composition of aggregate growth will depend on the elasticities of substitution in the CES aggregators for intermediates and investment, ϵ_M and ϵ_X. We establish how structural change and these elasticities of substitution impact aggregate growth with the following lemma:

Lemma 4. *Assume that assumptions 1-3 hold. Further, assume that there is weak positive dependence between the log of sectoral TFP and TFP growth across sectors (formally, $\mathbb{E}\left[ln(A_{it}) \mid \gamma_{it}^A = a\right]$ is weakly increasing in a with a probability measure across sectors of s_{it-1}^M).*

Holding all other parameters fixed, the growth rate of intermediates-production technical change, $\gamma_t^{B^M}$ is weakly increasing in ϵ_M. The same result holds for the growth rate of investment-production technical change, $\gamma_t^{B^X}$; all other parameters fixed, it is increasing in ϵ_X.

Proof. See Appendix B. □

Lemma 4 establishes that the higher the elasticity of substitution in either intermediates or investment, the faster intermediates- or investment-production technical change will grow. The intuition for this result can be seen from considering the limiting cases for these CES functions of growth rates when sector TFP growth rates are constant over time.[12] For example, if $\epsilon_M < 1$, implying gross complements, then as $t \to \infty$, the growth rate of intermediates-production technical change will converge to the *slowest* TFP growth rate among producers of intermediates. This is because movements in relative prices will ultimately cause s_{it}^M to converge to one for the slowest growing sector. In contrast, in the gross substitutes case (i.e., $\epsilon_M > 1$), this growth rate will converge to the *fastest* TFP growth rate, as all expenditures ultimately become concentrated on this sector.

Another implication of Lemma 4 and the above discussion occurs when one production network features complementarity in sectoral inputs and the other features substitutability. In this case, whichever network features gross substitutability will become endogenously more important for aggregate growth over time, as

[12]Lemma 4 does not require that sectoral TFP growth rates be constant over time; this is a convenient framing for intuition. The requirement is that there is weak positive dependence, which imposes that, on average, sectors with high TFP are also growing fast. Provided initial levels of TFP are normalized to 1, this implies that relative growth rates across sectors must be generally stable; if there are dramatic reversals in sectoral TFP growth rates, it is not possible to generally characterize the impact of ϵ_M and ϵ_X on $\gamma_t^{B^M}$ and $\gamma_t^{B^X}$.

resources are reallocated toward the fastest growing producers in that network ("frontiers"), while resources are allocated to the slowest growing producer in the other network ("bottlenecks").

Thus, structural change in production networks can affect the composition of aggregate growth along the balanced growth path, and potentially the level of aggregate growth in non-balanced growth environments. We discuss these relationships further when we analyze long-run growth patterns in Section 6 after we calibrate model parameters in Section 5.

5. Measurement, Calibration, and Structural Change

We now turn to the measurement of prices and calibration of model parameters, which will both feature prominently in shaping structural change and the composition of economic growth. This section focuses on the general procedures we use to calibrate the model for analyses consistent with the ABGP (i.e., when assumptions 1 and 2 hold, but not 3); additional measurement and calibration details are described in Appendix C.

The first question we must confront for measurement and calibration is the appropriate level of aggregation. While our model puts no restriction on the number of sectors we can analyze, we choose a level of aggregation that balances two considerations. On the one hand, as Section 2 highlights, the main patterns of structural change can conveniently be summarized within a parsimonious two-sector framework focusing on goods and services. This approach allows for straightforward interpretation of production elasticities and limits the number of parameters to be calibrated. On the other hand, our model emphasizes the potential importance of heterogeneity in productivity growth across sectors specializing in different types of products (consumption, investment, and intermediates). Specifically, evidence from the 40-sector data discussed in Section 2 suggests that, although services sectors increasingly produce all of these products, there is significant heterogeneity in which services subsectors are producing consumption, investment, and intermediates.

To balance these considerations, we focus on a six-sector aggregation, where each sector is defined by the interaction of the product market it operates in—goods or services—with the type of product it produces—consumption, investment, or intermediates. The resulting six sectors are goods-consumption (e.g., books, toys, food), goods-investment (e.g., buildings, machines, vehicles), goods-intermediates (e.g., primary metals, chemicals), services-consumption (e.g., education, health care), services-investment (e.g., software, R&D), and services-intermediates (e.g., financial services, wholesale trade). As a result, each sector in the model only produces consumption, investment, or intermediates, allowing for heterogeneity in productivity growth within goods and services sectors based on the final product they produce.

While this set of sectors has a natural representation in our model, it does not have a straightforward mapping into standard sectoral definitions. This poses a measurement challenge since standard national accounting data for NAICS sectors (e.g., the BEA's industry accounts, measured at the 40-sector level listed in Table 1) will not provide an accurate representation of the six sectors we study. As a result, a crucial step in our calibration is the construction of appropriate sectoral price series, which we will describe in Section 5.2.

5.1. Calibration Strategy

The non-CES aggregator parameters in the model are calibrated following a standard approach. We use expenditure data from 1947-2020 at the 40 sector level from the BEA Input-Output Database to calibrate production function parameters: α_j is calibrated using sectoral data on the ratio of nominal value added to nominal gross output; θ_j is calibrated using sectoral data on labor compensation (adjusted for taxes and self-employment) relative to nominal value added. Depreciation rates are calibrated using data on implied depreciation rates by sector as reported in the BEA Fixed Asset Accounts. With common production function parameters and depreciation rates across sectors (i.e. assumptions 1 and 2), we set α, θ, and δ to the average of these expenditure ratios across sectors. This yields values of $\alpha = 0.55$, $\theta = 0.34$, and $\delta = 0.08$. Finally, we set $\beta = 0.96$.

We calibrate the key CES aggregator parameters—the share parameters (ω_{Ci}, ω_{Xi}, ω_{Mij}) and the elasticity parameters (ϵ^C, ϵ^X, ϵ_j^M)— in two steps. First, for each sector, j, the share parameters are set to match the initial fraction of expenditures on consumption, intermediates, or investment purchased from sector i in the year 1947.[13] Then, we combine equations (12) and (13) with equations (9) and (10) (and analogous equations for consumption) to obtain the following expressions for the goods share of production of each

[13] In the case of intermediates, this requires observing the intermediate expenditure patterns for our six sector partition of the economy. We describe in Appendix C how we generate these by aggregating the intermediate expenditure patterns across our observed 40 sectors in proportion to each sector's role in producing consumption, investment, or intermediates within goods or services sectors.

product, which we use as estimating equations to identify the elasticities of substitution:

$$s^M_{gjt} = \frac{\omega_{Mgj} P^{1-\epsilon_{Mj}}_{g-m,t}}{\omega_{Mgj} P^{1-\epsilon_{Mj}}_{g-m,t} + (1-\omega_{Mgj}) P^{1-\epsilon^M_j}_{s-m,t}} \tag{24}$$

$$s^X_{gt} = \frac{\omega_{Xgj} P^{1-\epsilon^X}_{g-m,t}}{\omega_{Xg} P^{1-\epsilon_X}_{g-x,t} + (1-\omega_{Xg}) P^{1-\epsilon_X}_{s-x,t}} \tag{25}$$

$$s^C_{gt} = \frac{\omega_{Cg} P^{1-\epsilon^C}_{g-c,t}}{\omega_{Cg} P^{1-\epsilon_C}_{g-c,t} + (1-\omega_{Cg}) P^{1-\epsilon_C}_{s-c,t}} \tag{26}$$

where g–c, g–x, g–m, s–c, s–x, s–m identify our six sectors, g and s represent goods and services, and c, x, m represent consumption, investment, and intermediates. Given calibrated values for all ω parameters, we estimate equations (24), (25), and (26) via non-linear least squares using annual data moments on expenditure shares seen in Section 2 and sectoral prices, the measurement of which we describe next.

5.2. Measuring Prices

To estimate the elasticity parameters for our six-sector aggregation, we require separate price series for consumption, investment, and intermediates produced by goods and services sectors. These price series cannot be directly observed from standard sectoral data, as the BEA Industry accounts only report the price of gross output for each NAICS sector. These gross output prices are a weighted average of sectoral prices for consumption, investment, and intermediates commodities, and thus do not separately identify the price of different products made by a sector.[14] We solve this measurement challenge by constructing prices for our six sectors using prices of detailed commodities from the NIPA and highlight the insights from this approach relative to alternative measurement methods. In particular, we show how our prices derived from expenditure-side data compare with efforts to identify comparable prices using gross output price data from income-side national accounts.

We start with detailed expenditure-side data from the U.S. NIPA, which consistently covers expenditures and prices for 68 consumption commodities and 30 investment commodities over our entire sample 1947-2020. Examples of consumption commodities at this level of detail include household appliances, jewelry and watches, children's and infants' clothing, dental services, and purchased meals and beverages. Examples of investment commodities at this level of detail include non-mining structures, office and accounting

[14]Price heterogeneity across different products made by a sector largely reflects heterogeneity in the subsectors or industries that are aggregated together into the sectors we can observe in the data; we discuss this further below when discussing the limits to using gross output prices.

equipment, construction machinery, software, and R&D investment.

To map the prices of NIPA consumption and investment commodities into our six sectors, we use a combination of BEA Input Output data published in "bridge files" and "make tables." This combination "bridges" the income and expenditure sides of national accounts, mapping the NIPA consumption and investment commodities into the NAICS defined sectors that "make" them. This allows us to quantify the extent to which each detailed NIPA commodity is produced by goods or services sectors.[15]

Equipped with this mapping, we measure price growth for goods-consumption, services-consumption, goods-investment, and services-investment as the weighted average of price growth within each consumption and investment commodity, where the weights correspond to the sector's production share for each commodity. For example, we measure price growth for the goods consumption sector, g–c, using NIPA price and spending data on K consumption commodities (P_{kt}^C and $P_{kt}^C Q_{kt}^C$ for $k \in \{1, ..., K\}$) according to the formula:

$$\Delta \ln(P_{g-c,t}) = \sum_{k=1}^{K} \frac{\xi_{g-c,kt}^C P_{kt}^C Q_{kt}^C}{\sum_{\ell=1}^{L} \xi_{g-c,\ell t}^C P_{\ell t}^C Q_{\ell t}^C} \Delta \ln(P_{kt}^C) \tag{27}$$

where $\xi_{g-c,kt}^C$ is the fraction of commodity k's purchaser's value (averaged across years $t-1$ and t) that was produced by the goods consumption sector, g–c. As described above, this production share, $\xi_{g-c,kt}^C$, is identified from the combination of the BEA's consumption "bridge files" and "make tables." The calculations for the goods-investment, services-consumption, and services-investment sectors are analogous.

This measurement approach implies that relative price differences between goods and services sectors are driven by heterogeneity in the extent to which each sector produces different commodities (e.g., heterogeneity in $\xi_{g-c,kt}^C$). For example, even with heterogeneous price trends across NIPA consumption commodities k (e.g., the price of health care services rises much faster than the price of toys), the prices of services-consumption and goods-consumption would be equal if each commodity were produced 50% by goods sectors and 50% by services sectors. However, nearly two-thirds of all commodities we observe are

[15]These bridge files have been used in other work to map commodities reported in NIPA to production sectors. See, for example, Bils, Klenow and Malin (2013), vom Lehn and Winberry (2022), and Bergman, Jaimovich and Saporta-Eksten (2023). While vom Lehn and Winberry (2022) extend the BEA's bridge files for investment back to the year 1947, the bridge files for consumption are only available starting in 1997. We thus assume that the bridge file entries for consumption in years prior to 1997 are the same as the 1997 data. We note that variation in the bridge files over time, for the years we can observe them, is generally minimal. For example, vom Lehn and Winberry (2022) find that bridge file variation contributes very little to the overall measured changes in the investment network; changes in the amount of spending on each commodity generate the largest changes over time. Our results are robust to setting the bridge files to their average values over the entire sample; see Appendix C.

produced in large majority (more than 80%) by either goods or services, implying significant heterogeneity in the bridge/make files. Furthermore, the primary instances in which both goods and services sectors contribute significantly to the production of a consumption or investment commodity is when delivery of the commodity to the final user involves significant "margins" due to transportation, wholesale trade, or retail trade. These margins appear in expenditure-side price measures because they are purchaser prices, the final prices paid by consumers. The presence of such margins typically happens when the final product is itself a physical good.[16] Given this close link between goods-producing and margin sectors, if we were to reclassify these margin sectors as goods sectors, then over 90% of all commodities would be produced in large majority by either goods or services. The patterns of relative prices we measure are robust to this reclassification (see Appendix C).

The final two prices we need to measure are the prices of the goods- and services-intermediates sectors. Since intermediate inputs are not counted in GDP, we are unable to measure these sectors' prices using expenditure-side national accounting data.[17] However, gross output prices for each sector are implicitly an average of the price of consumption, investment, and intermediates produced by that sector. Thus, using the price of consumption and investment produced by goods or services, we identify the price of intermediates produced by goods or services as the residual in gross output prices obtained from the BEA's GDP by Industry database. This approach ensures that our measured prices are consistent with the aggregated prices of the goods and services sectors as a whole.

For example, we measure price growth in the goods-intermediates sector, g–m, as:

$$\Delta \ln(P_{g-m,t}) = \frac{1}{\zeta_{gt}^{M}} \left(\Delta \ln P_{gt}^{GO} - \zeta_{jt}^{C} \Delta \ln P_{g-c,jt} - \zeta_{jt}^{X} \Delta \ln P_{g-x,t} \right), \tag{28}$$

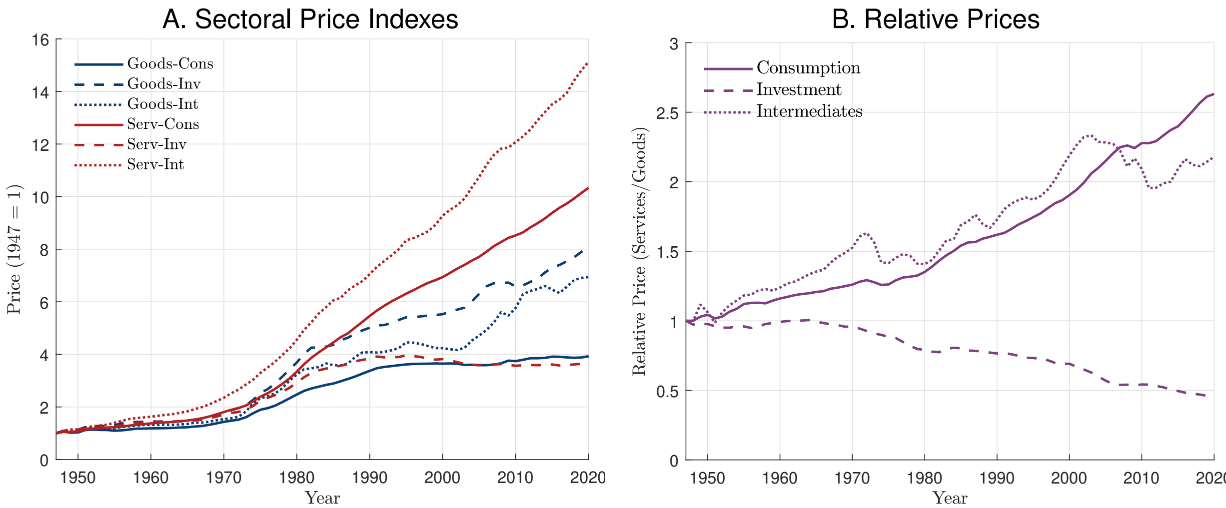
where ζ_{gt}^{i} represents the average share (between $t-1$ and t) of total gross output of the goods sector used for product i, $i \in (C, X, M)$ with $\zeta_{gt}^{M} + \zeta_{gt}^{C} + \zeta_{gt}^{X} = 1$, and $\Delta \ln P_{gt}^{GO}$ represents the log change of the gross output price for the total goods sector.[18] We use an analogous expression to infer the services-intermediates

[16] To give one such example, the bridge/make files indicate that of the 162 billion dollars spent on new motor vehicles in 1997, 71% of that value came from the motor vehicle manufacturing sector, 22% of that value came from the retail trade sector, 3% came from wholesale trade, 1% came from transportation and warehousing services, and the remaining 2% of production was spread across other sectors, such as fabricated metals and machinery manufacturing.

[17] As part of its GDP by Industry database, the BEA publishes the price of the intermediates bundle that each sector purchases. However, this bundle price does not allow us to cleanly identify the price of intermediate inputs produced by different sectors.

[18] As described in Appendix C, we adjust gross output prices (and to a lesser extent consumption and investment prices) for oil/energy price spillovers before performing this procedure. The qualitative patterns of relative price movements across goods and services sectors are robust to not making these corrections.

Figure 4: Prices of Goods and Services Consumption, Investment and Intermediates, 1947-2020



Notes: Panel A shows the time series of prices for each product (consumption, investment, or intermediates) produced by each sector (goods or services). Panel B shows the price of services divided by the price of goods for each product (consumption, investment, or intermediates).

price.

We note that the Producer Price Index (PPI) published by the U.S. Bureau of Labor Statistics (BLS) provides purchasers' prices for intermediate inputs produced by different sectors. These data are, in principle, exactly what we would want for our calibration. Unfortunately, the PPI data has incomplete coverage of services sectors (roughly 85% of the services sectors producing intermediates) and prices for intermediates produced by services sectors are only available starting in 2009. In Appendix C we document that our measurement procedure generates time series for intermediates prices that line up almost perfectly with the published PPI data for prices of both goods-produced and services-produced intermediates.

Figure 4 plots our price series for consumption, investment, and intermediates produced by goods and services sectors; panel A illustrates prices in levels (normalized to one in 1947) while panel B shows the price of services divided by the price of goods for each product. Unsurprisingly, the price of services consumption relative to goods rises significantly over time. This is consistent with existing literature using relative prices to explain structural transformation in consumption. We also observe that the relative price of services intermediates is rising significantly. Given the rising share of expenditures on both consumption and intermediates produced by services (see Figure 3), rising relative prices are consistent with complementarity between goods and services inputs to consumption and intermediates.

In stark contrast, Figure 4 shows that the relative price of services investment is falling. Given rising expenditures on services investment (see Figure 3), these falling prices suggest that goods and services inputs to investment are *substitutes*. The intuition for this finding is that investment inputs produced by services sectors are primarily information technology and intellectual property products (e.g., software and R&D), whose price has fallen significantly relative to the price of goods investment (e.g., equipment or structures).

This finding is robust to a wide variety of alternative specifications: we aggregate investment prices with user cost weights instead of investment expenditures as recommended by Holden, Gourio and Rognlie (2020); we focus exclusively on equipment and software, which feature the most overlap between goods and services production; we quality adjust investment prices as in Cummins and Violante (2002); and we use alternative implementations of bridge files for consumption and investment prices, including holding the values of bridge files constant across all time periods 1947-2020. In all cases, the price of investment by services sectors is declining relative to the price of investment produced by goods sectors (see Appendix C).

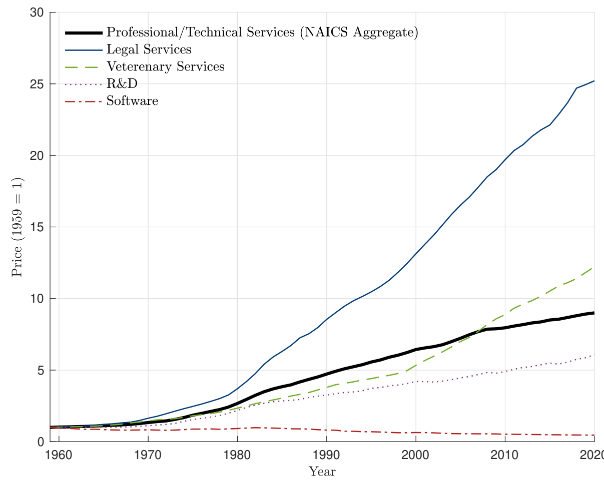
5.3. Aggregation Bias

Our finding of a declining relative price coincident with a rising expenditure share of services investment contrasts with the elasticities calibrated by Herrendorf et al. (2021), García-Santana et al. (2021) and Sposi et al. (2021), who use a single price for goods and services, respectively, to estimate that these inputs are complements in investment production. If we aggregate goods and services prices across consumption, investment, and intermediates, we similarly find that services are getting more expensive relative to goods as a whole. This is driven by the relative price of services to goods in consumption and intermediates. The reason for our different findings is aggregation bias; because investment is a small fraction of output—on average over the period 1947-2020, investment is 21% and 5% of gross output of goods and services, respectively—it is easy for aggregate price trends to be dominated by price movements in consumption and intermediates, masking the behavior of investment prices.

To highlight this argument, we explore an alternative approach to construct prices for our six sectors. We compute prices for each sector as a weighted average of gross output prices for the 40 consistent NAICS sectors in Table 1, where the weights are each sector's production share of consumption, investment, or intermediates within goods or services sectors (see Appendix C for details). Using this approach, we find similar results for most sectors, but sizable differences for the price of services-investment. Although the price of services-investment grows substantially less than the overall price of services when measured using gross output data, it still grows faster than goods-investment, implying a rising relative price of services.

We argue that measuring the price of services-investment using publicly available gross output prices

Figure 5: Heterogeneous Prices Within the Professional and Technical Services Sector



Notes: The thick line denotes the gross output price for the professional and technical services sector, obtained from the BEA GDP by Industry database. The thin lines are the prices of varied commodities produced by the professional and technical services sector. These detailed prices series are based on NIPA Tables 2.4.4U and 5.6.4.

is subject to significant aggregation bias, even when drawing on the disaggregated 40 sectors in Table 1. The gross output price data can properly identify the price of services-investment only if at least one of two conditions is met: 1) observed services sectors specialize in producing primarily one use (consumption, investment, or intermediates), implying no within-sector aggregation bias or 2) the price trends for consumption, investment, and intermediates within each sector are comparable. We present evidence that neither of these conditions is satisfied at the level of disaggregation available in the BEA GDP by Industry data over the entire sample. First, for the two services sectors that produce the most services investment (information and professional/technical services) only a small portion of gross output is used as investment (on average only 15% and 22%, respectively), suggesting the potential for significant aggregation bias.[19] Second, consider the set of commodities produced by the professional/technical services sector. While it is the largest and fastest growing producer of services-investment (primarily software and R&D), it also produces the consumption commodities of legal services and veterinary services (along with many other commodities).[20] Using underlying detail in the national accounts (only available since 1959), Figure 5 plots the time series of the prices of these commodities compared to the gross output price of the professional/technical services

[19]In more recent years, the GDP by Industry database has published price data for a more disaggregated set of industries. Although there is some added detail provided for investment-producing sectors, substantial evidence of aggregation bias persists. For example, using more detailed data beginning in 1963, the largest services-investment producer is miscellaneous professional/technical services, and only 24% of its gross output is used for investment. See Appendix C for more details.

[20]Based on more detailed Input-Output data available only in 2007 and 2012, legal services and veterinary services are the two biggest consumption commodities and software and R&D are the two biggest investment commodities produced by this sector.

Table 2: Calibrated Parameters of CES Bundling Aggregators

Bundling Technology	Elasticity of Subsitution (ϵ)	Goods Share in 1947 (ω)
Consumption	0	0.31
Investment	2.36	0.80
Intermediates		
Goods-Consumption	0.30	0.81
Goods-Investment	0.53	0.71
Goods-Intermediates	0.45	0.77
Services-Consumption	0	0.44
Services-Investment	0	0.34
Services-Intermediates	0	0.32

Notes: The table reports the calibrated parameter values for the CES aggregators corresponding to equations (3), (5), and (7). For each type of product (consumption, investment, intermediates), the parameter ϵ represents the elasticity of substitution between goods and services inputs, and the parameter ω represents the share parameter attached to goods inputs (with one minus that parameter being the share parameter attached to services). Given Assumption 2, there is only a single set of parameters for investment.

sector as a whole.[21]. Thus, while there has been little price growth in the investment commodities produced by this sector (software and R&D), their price trends are masked by the rising price of quantitatively large consumption commodities, such as legal services.

In summary, since our framework explicitly models the intermediate input network, both expenditure-side and income-side price data can be used to consistently measure prices. That said, for the six sectors we consider, the best measurement approach is the one that most precisely identifies price variation along both of the two dimensions that define them: whether the sector's output is a good or a service and whether the sector's output is used for consumption, investment, or as an intermediate input. Expenditure-side data directly distinguishes between final uses, and then relies on bridge/make files to distinguish between goods and services; income-side data directly distinguishes goods and services sectors, and then relies on more disaggregated sectors to distinguish between consumption, investment, and intermediates. Our evidence implies that the bridge/make files more precisely distinguish goods and services sectors on the expenditure-side than the underlying sectoral detail on the income-side distinguishes final use. Thus, expenditure-side data provides a more accurate measure of prices for our six sectors.

[21] In Appendix C, we also provide similar evidence within the information sector—the other primary producer of services-investment.

5.4. Elasticities of Substitution

Based on our data for sector-specific expenditures (Figure 3) and prices (Figure 4), we use the procedure described in Section 5.1 to calibrate parameter values for the CES bundling aggregators as displayed in Table 2. The table reports the parameters for the single CES aggregator in consumption and investment (given Assumption 2) and the parameters for each of the six sectors' CES bundling technologies. As anticipated in our above discussion of the time series patterns for relative prices and expenditures, the calibrated values for the elasticity parameters confirm that goods and services are complements in the production of consumption and intermediates (elasticities less than one), but substitutes in the production of investment (elasticities greater than one).

The elasticities for consumption and intermediates imply strong complementarity between goods and services, the best fit often given by Leontief aggregation, consistent with existing literature on structural change (e.g., Herrendorf et al., 2021; García-Santana et al., 2021; Sposi et al., 2021). However, the best fit for patterns of structural change in intermediates within the three goods sectors implies less complementarity than the Leontief specification.[22]

5.5. Structural Change

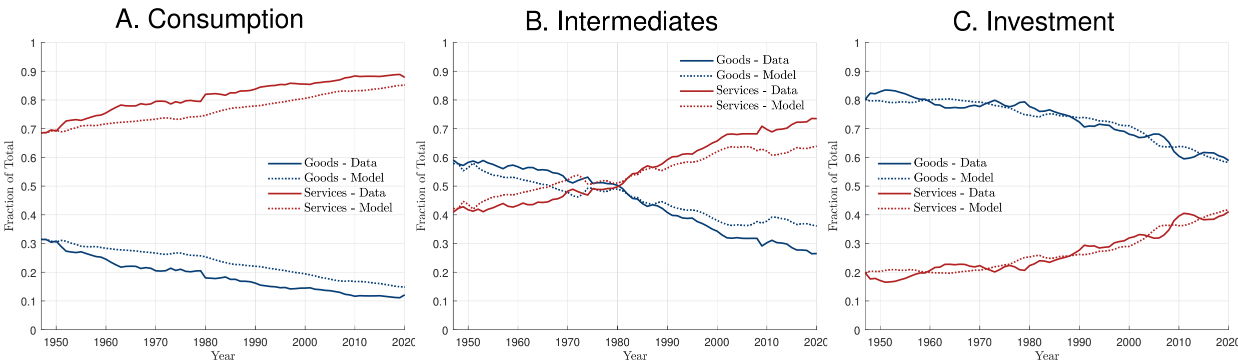
Given our calibration, we present the model's performance in accounting for patterns of structural change in Figure 6, taking as given measured price trends. Figure 6 presents the economy-wide goods and services production shares in consumption, intermediates, and investment.[23] We present the model fit for each of the six sectors' intermediates expenditure patterns in Appendix C.

Overall, the calibrated model provides a good fit to patterns of structural change, though a few comments are warranted. First, although the model accounts for much of the structural change in consumption, the model is unable to match the entire rise in the share of services. This is perhaps unsurprising, given that household preferences in our model do not feature any income effects, which are commonly argued to be important for explaining structural change in consumption (e.g., Boppart, 2014; Comin, Lashkari and

[22] We have also explored estimating investment aggregation elasticity parameters separately for each of our six sectors, and find that goods and services inputs to investment are substitutes in each sector, albeit with slightly lower elasticity values for services sectors. These results are available upon request.

[23] Intermediates, as discussed further in Appendix C, are constructed using the expenditure shares for each of our six sectors, aggregated using equation (1). Because we are considering the balanced growth equilibrium with common production parameters across sectors, each sector's share of total intermediate expenditures, $\frac{P_{jt}^{M} M_{jt}}{\sum_k P_{kt}^{M} M_{kt}}$, is equal to that sector's share of aggregate gross output. For consistency with the model, we aggregate sector-specific services expenditures shares using gross output weights; this generates slightly different empirical patterns of structural change than seen in Figure 3.

Figure 6: Model Calibration Fit to Structural Change Patterns in Consumption, Intermediates and Investment, 1947-2020



Notes: The figures plot the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue lines) and the services sector (red lines). Data series are solid lines; model series are dashed lines.

Mestieri, 2021).

Second, the model closely matches the long-run patterns in structural change in investment. Given that our model abstracts from adjustment costs and uncertainty about the future price of investment, it is not surprising that the model does not generate the short and medium-run dynamics observed in the data.

Finally, the model reproduces the majority (approx. 2/3) of the rising share of intermediates produced by services. The model fit is even stronger before 2009, explaining roughly 80% of the overall increase, but fails to capture a substantial portion of the increased share of services intermediates after 2009.[24] We show in Appendix C that the model results reproduce the contribution of within-sector and between-sector forces to structural change in intermediates. While the discrepancy in the aggregate may reflect the presence of income effects (i.e., scale effects) in intermediates structural change, the lack of perfect fit to the data may instead reflect the data limitations regarding intermediates prices, or additional heterogeneity in intermediates prices beyond what we can observe in the data. Thus, given the challenges in measuring intermediates prices, the model does a good job of capturing the overall pattern of structural change in intermediates.

6. Growth Accounting with Changing Production Networks

We now use our calibrated model to conduct two sets of growth accounting exercises to better understand the forces driving U.S. economic growth since 1947. First, we decompose the evolution of aggregate TFP along the ABGP, denoted $\mathcal{A}_t$ in Section 4, analyzing how the contributions of TFP growth in the production of intermediates and investment vary over time. Second, we characterize and decompose the growth rate of

[24] As shown in Appendix C, the change in model fit from 2009-2020 mostly occurs within goods sectors; the model consistently accounts for roughly 2/3 of the rising services share throughout the sample.

aggregate GDP measured in a way that is more consistent with national accounting conventions (as opposed to measuring GDP in units of investment). While this characterization requires assumptions 1 and 2, it does not impose that $\mathcal{A}_t$ grows at a constant rate, allowing us to study not only the composition of aggregate growth, but also what forces account for its slowdown in the 1970s and since the year 2000.

We emphasize two novelties of these growth accounting exercises relative to existing work. First, our six-sector calibration allows for new perspectives on the drivers of economic growth that are not possible in aggregations based on NAICS codes. Using measures of technology identified from the prices described in the previous section, we can more precisely identify the importance of productivity growth across both sectors and different final uses. Second, our model identifies how aggregate capital and TFP growth depend on structural change, allowing us to study growth counterfactuals where structural change did not occur to show how structural change in production networks impacts economic growth.

6.1. Measuring Technology

Before considering any growth accounting exercises, we must measure the TFP processes to be fed into the model. Specifically, we measure two types of TFP: TFP in each of the six sectors' production technologies (sectoral TFP) and the TFP in the bundling technologies (bundling TFP) for intermediate and investment inputs. We describe how we measure each of these in turn. Additional details are available in Appendix D.

First, we measure sectoral TFP by matching relative output prices in the model to relative output prices in the data, similar to García-Santana et al. (2021). That is, for five of our six sectors, we invert equation (14) and back out the sectoral TFP series that generate the relative price series observed in the data.[25] Since relative prices can only define technological change for all but one of our sectors, we infer TFP for the final sector by ensuring that the Domar-weighted sum of sectoral TFP growth in the model matches that same moment in the data (based on our 40 sector NAICS data, with Solow residuals constructed following the approach of vom Lehn and Winberry (2022)).

Second, we measure growth in intermediates bundling TFP using a log first-order approximation of the equilibrium intermediate input price (equation (9)), for years t and $t-1$.[26] When log-linearized around the average expenditure share in these two years, the resulting Tornqvist index can be rearranged to yield the

[25] To do this, we use the observed sectoral prices, P_{it}, and construct the price of the bundle of intermediate inputs implied by the model in equation (9), P_{it}^M, given these prices and the calibrated parameters reported in Table 2.

[26] Herrendorf et al. (2021) use a similar approach to measuring exogenous investment TFP.

following expression for growth in intermediates bundling TFP for sector i:

$$\Delta \ln(A_{it}^M) = -\left(\Delta \ln(P_{it}^M) - \sum_{j=g-m,s-m} \left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^M M_{it}}}\right) \Delta \ln(P_{jt})\right), \tag{29}$$

where g–m and s–m are the goods-intermediates and services-intermediates sectors, $\left(\overline{\frac{P_{jt}M_{ijt}}{P_{it}^M M_{it}}}\right)$ is the average between t and $t-1$ of the share of intermediate spending by sector i purchased from sector j, $\Delta \ln(P_{it}^M)$ is the price growth in the bundle of intermediates purchased by sector i, and $\Delta \ln(P_{jt})$ is the price growth in intermediates produced by sector j. We measure $\Delta \ln(P_{it}^M)$ using BEA GDP by Industry data on the price of intermediate bundles by sector, aggregated to the six-sector level.

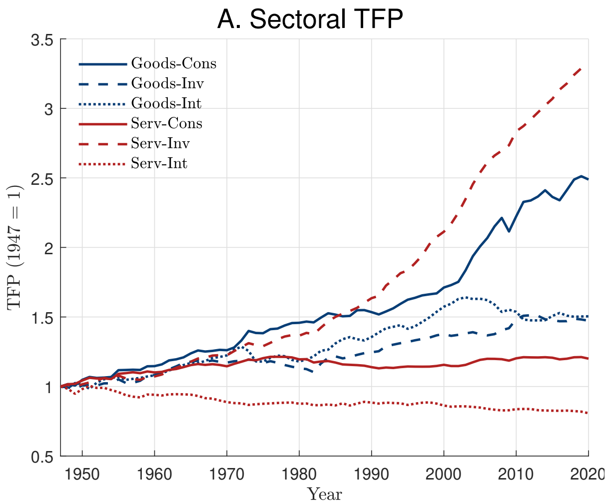
The appeal of using this log first-order approximation to measure intermediates bundling TFP is that it is measured independently of the model's calibrated parameters and fit. However, constructing intermediates bundling TFP using equation (9) directly (without approximation, which requires using calibrated parameters), generates nearly identical results. Furthermore, we expect growth in intermediates bundling TFP to be small, as measuring bundling TFP as a residual of observed intermediates bundle prices implies bundling TFP is largely aggregation/measurement error that is leftover in intermediate input bundle prices.

Finally, we could construct a series for investment bundling TFP using an analogous procedure as for intermediate bundling TFP, but because both the price of total investment and the price of goods and services investment are constructed from the same data source of expenditure side investment prices, there is approximately zero residual price growth. Thus, we set investment bundling TFP to be constant.[27]

Normalizing the level of all TFP terms to be 1 in 1947, Figure 7 displays sectoral TFP (panel A) and intermediates bundling TFP (panel B) for each sector over time. Given that sectoral TFP is calibrated using relative price data, it is unsurprising that growth in sectoral TFP illustrated in panel A of Figure 7 follows nearly the opposite ranking of growth in each sector's observed prices (panel A of Figure 4). That said, because the price of intermediates produced by services is growing faster than that produced by goods and because there is significant homophily in the input-output network (as seen in Section 2), the underlying technology growth in services sectors is faster than what is observed with relative prices alone. This explains, for example, why TFP in services-investment is significantly higher than in goods-consumption, despite the two sectors showing very similar price patterns in panel A of Figure 4. This highlights the importance of

[27]We have also explored measuring heterogeneity in investment-bundling TFP across sectors should assumption 2 be relaxed. Even then, there is little growth in investment-bundling TFP in each sector. Results are available upon request.

Figure 7: TFP by Sector, 1947-2020



Notes: Panel A shows the time series of sectoral TFP for each sector, A_{it}; panel B shows the time series of intermediates-bundling TFP, A_{it}^M.

accounting for underlying production networks in accurately identifying sectoral TFP growth.

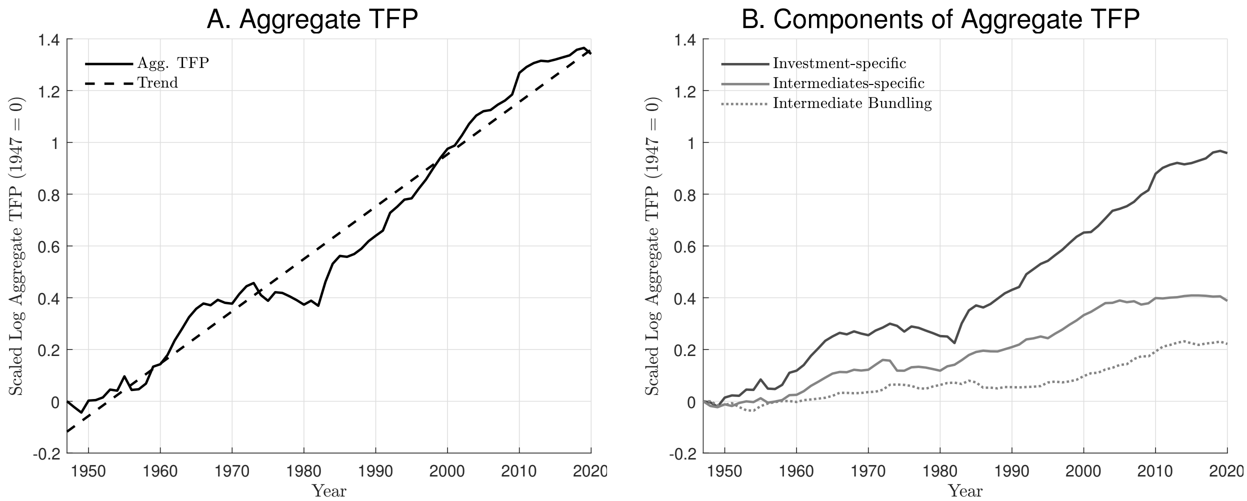
On average, intermediates bundling TFP growth is low (0.3% a year, averaged across sectors), and there is little heterogeneity in this growth across sectors (see panel B of Figure 4).

6.2. Growth Accounting along the ABGP

Using our series for sectoral TFP and technical change in intermediates bundling, we compute the time series for aggregate TFP, $\mathcal{A}_t$, as defined in Lemma 2. Given that GDP grows in proportion to $\mathcal{A}_t^{\frac{1}{1-\theta}}$, we plot $\frac{1}{1-\theta}\ln\mathcal{A}_t$ to facilitate easy comparison with our simulations for GDP growth in Section 6.3; panel A of Figure 8 plots $\frac{1}{1-\theta}\ln\mathcal{A}_t$. Similar to the aggregate technical change series constructed by Herrendorf et al. (2021), the long-run growth rate of $\mathcal{A}_t$ is approximately constant, although with significant medium-run fluctuations, most notably during the 1970s.

In panel B of Figure 8, we illustrate the growth patterns of three counterfactual series for $\mathcal{A}_t$, which are constructed analogously to those in panel A (logged and scaled), but under alternative paths for the underlying TFP processes. First, we construct $\mathcal{A}_t$ only using sectoral TFP growth in the goods-investment and services-investment sectors ("investment-specific technical change"), holding all other TFP series fixed at their initial values. Second, we construct $\mathcal{A}_t$ only using sectoral TFP growth in the goods-intermediates and services-intermediates sectors ("intermediates-specific technical change") and growth in intermediates bundling TFP, while holding fixed sectoral TFP for sectors producing investment. Finally, we construct $\mathcal{A}_t$ only using intermediates bundling TFP. Table 3 presents the corresponding numerical growth decomposition

Figure 8: Aggregate TFP and Its Composition, 1947-2020



Notes: Panel A shows the time series of aggregate TFP, $\mathcal{A}_t$, in logs (normalized to zero in 1947), with a linear trend line drawn through it; panel B shows the counterfactual evolution of aggregate TFP for three cases: (1) only TFP growth among investment producers ("investment-specific technical change"), (2) only TFP change in intermediates ("intermediates-specific technical change"), both at intermediates producers and from intermediates bundling, and (3) only intermediates-bundling TFP growth. For comparison to later results, all series are scaled by the coefficient $\frac{1}{1-\theta}$ (which is how aggregate TFP growth matters for aggregate GDP growth along the balanced growth path, as described in Proposition 1).

Table 3: TFP Growth Decomposition, 1947-2019

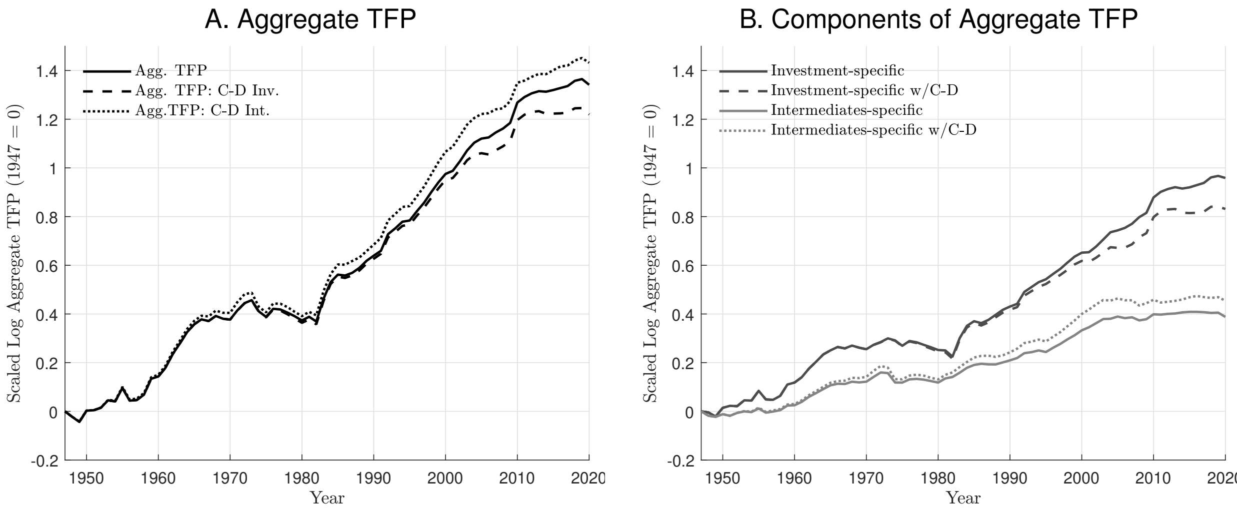
	Scaled Aggregate TFP Growth: $\Delta \ln(x) = \frac{1}{1-\theta}\Delta \ln \mathcal{A}_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%
All	1.36	100	0.23	100	0.60	100	0.39	100
Investment-Specific	0.97	71	0.13	58	0.40	66	0.32	81
Intermediates-Specific	0.41	30	0.09	41	0.21	36	0.07	19
Intermed. Bundling	0.23	17	0.07	29	0.03	6	0.13	34

Notes: The table shows long-run log changes in scaled aggregate TFP, $\frac{1}{1-\theta}\Delta \ln(\mathcal{A}_t)$, across different periods for four alternative simulations: (1) the full model simulation with all measured TFP series; three counterfactual simulations with only (2) sectoral TFP growth among investment producers ("investment-specific technical change"), (3) sectoral TFP growth among intermediates producers and intermediates-bundling technical change in each sector ("intermediates-specific technical change"), and (4) intermediates-bundling TFP growth. Counterfactual changes are also expressed as a percent of the change from the full model simulation; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

both for the entire sample and for three (approximately) twenty-year intervals beginning in 1960; we present decadal numbers beginning in 1950 in Appendix D.

We make three observations about the contributions of investment- and intermediates-specific technical

Figure 9: Aggregate TFP Growth, Cobb-Douglas Counterfactuals, 1947-2020



Notes: Panel A shows the time series of aggregate TFP, $\mathcal{A}_t$, in logs (normalized to zero in 1947), with two counterfactuals: one where the aggregation of investment inputs is Cobb-Douglas and one where the aggregate of intermediates inputs is Cobb-Douglas. Panel B shows the evolution of the components of aggregate TFP (either investment-specific technical change or intermediates-specific technical change) and how these change when aggregation is Cobb-Douglas in nature. When analyzing aggregate TFP growth from only investment, we only impose that investment aggregation is Cobb-Douglas and when analyzing aggregate TFP growth from only intermediates, we only assume that intermediates technical change is Cobb-Douglas. For comparison to later results, all series are scaled by the coefficient $\frac{1}{1-\theta}$, highlighting how aggregate TFP growth matters for aggregate GDP growth along the balanced growth path, as described in Proposition 1.

change to aggregate TFP growth. First, both sources of technical change significantly contribute to aggregate productivity growth. From Table 3, we see that over the entire sample of 1947-2019, investment-specific technical change contributes more than 70% of aggregate TFP growth while intermediates-specific technical change accounts for approximately 30%.

Second, for most of the sample, intermediates bundling technical change only contributes a modest amount to aggregate TFP growth, consistent with the small amount of growth observed in these series (panel B of Figure 7). Table 3 shows that this source of technical change contributes about 17% of total TFP growth in the postwar period. However, it has been rising in importance over time, accounting for 34% of TFP growth since the year 2000. Furthermore, since intermediates bundling technical change is larger than total intermediates-specific technical change since 2000, the contribution of the endogenous components of technical change in the production of intermediates is negative over this period.

Third, the importance of investment-specific technical change has been rising over time. Table 3 shows that this source of technical change accounts for up to 66% of aggregate TFP growth before 2000, but accounts for more than 80% since 2000. Furthermore, Figure 8 and Table 3 highlight that intermediates-specific technical change effectively stagnates after 2000. We revisit this observation when discussing the

Table 4: TFP Growth Decomposition, Cobb-Douglas Counterfactuals, 1947-2019

	Scaled Aggregate TFP Growth: $\Delta\ln(x) = \frac{1}{1-\theta}\Delta\ln\mathcal{A}_t$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%
A. All Sources of TFP Growth								
Baseline	1.36	100	0.23	100	0.60	100	0.39	100
Investment Cobb-Douglas	1.25	91	0.22	96	0.59	97	0.30	76
Intermediates Cobb-Douglas	1.45	106	0.24	105	0.68	112	0.38	99
B. Investment-Specific Technical change								
Baseline	0.97	100	0.13	100	0.40	100	0.32	100
Investment Cobb-Douglas	0.84	87	0.13	95	0.37	93	0.22	71
C. Intermediates-Specific Technical Change								
Baseline	0.41	100	0.09	100	0.21	100	0.07	100
Intermediates Cobb-Douglas	0.47	116	0.10	109	0.27	125	0.07	95

Notes: The table reports log changes in scaled aggregate TFP, $\frac{1}{1-\theta}\Delta\ln(\mathcal{A}_t)$, across different periods, and log changes in investment-specific and intermediates-specific technical change. The table also reports counterfactuals for the cases where either investment or intermediates aggregation is Cobb-Douglas, ruling out structural change in that network. For each period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

recent growth slowdown in Section 6.3.

As discussed in Section 4.3, the rising importance of investment-specific technical change could either reflect changing growth rates of underlying productivity series or the endogenous reallocation of resources across sectors. To explore the importance of endogenous reallocation, we consider a set of counterfactuals in which the investment or intermediates bundling technologies are Cobb-Douglas. This specification implies unitary elasticities of substitution and fixed expenditure shares, ruling out structural change.[28] Figure 9 presents the resulting counterfactual aggregate TFP series, illustrating how much of aggregate growth occurs when each production network is held fixed over time. The contribution of reallocation to aggregate productivity growth due to non-unitary elasticities of substitution is 100% minus the percent contributions reported in the second column of each panel in Table 4.

Figure 9 and Table 4 show that aggregate productivity growth is different under the Cobb-Douglas

[28] We take the limit as the elasticity of substitution goes to 1, using the calibrated values for the share parameters in the CES aggregator as the Cobb-Douglas exponents. When intermediates aggregation is Cobb-Douglas, we also recalibrate technology to match the relative prices observed in the data. Results without recalibrating are similar to those in Figure 9.

counterfactuals. When the aggregation of investment inputs is Cobb-Douglas, meaning that resources no longer reallocate to the fastest growing sector, over the entire postwar period, investment-specific technical change is 13% lower (panel B), and aggregate TFP growth is 9% lower (panel A). In contrast, when the aggregation of intermediates is Cobb-Douglas for each sector, intermediates-specific technical change is 16% higher (panel C) and aggregate TFP growth is 6% higher (panel A).

The importance of reallocation for productivity growth is more pronounced in recent years. Investment-specific technical change is 29% lower from 2000-2019 under the Cobb-Douglas counterfactual (panel B) and aggregate TFP growth is 24% lower (panel A), suggesting that the substitutability of investment inputs is important in accounting for recent aggregate growth. In contrast, the importance of reallocation forces in intermediates for aggregate TFP growth has been almost zero in the last 20 years (panel A).

6.3. GDP Growth Accounting off the ABGP

One potential shortcoming of the above analysis is that aggregate TFP, $\mathcal{A}_t$, corresponds to the aggregate growth rate of GDP only when GDP is measured in units of the numeraire—aggregate investment. As summarized in Lemma 2, measuring GDP this way eliminates any role for technical change in the production of consumption in aggregate growth. Aggregate growth only depends on direct technical change in the production of investment or on indirect technical change among the intermediate input suppliers to investment producers. Alternatively, we could choose aggregate consumption as the numeraire (as in Greenwood et al., 1997, or Foerster et al., 2019), but then any aggregate balanced growth path would be inconsistent with the Kaldor facts (see Duernecker et al., 2021). However, defining aggregate GDP in units of consumption or investment as the numeraire is inconsistent with how GDP is measured in national accounts, which utilize an index number based on growth in all subcomponents of GDP.

We develop an index-number representation of GDP that is more comparable to empirical measures of GDP in national accounting data. To do so, we define aggregate GDP in our model as a Tornqvist index, Y_t^{Index}, with aggregate GDP growth given by

$$\Delta\ln(Y_t^{Index}) = \sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta\ln(V_{it}), \tag{30}$$

where $\overline{\frac{P_{it}^Y V_{it}}{P_t^Y V_t}}$ is the average share of aggregate GDP from sector i in years t and $t-1$ and $\Delta\ln(V_{it})$ is real value added growth in sector i between t and $t-1$.[29]

Given that sectoral value added in our model is given by $V_{it} = A_{it}^{\frac{1}{\alpha_i}} K_{it}^{\theta_i} L_{it}^{1-\theta_i}$, assumptions 1 and 2 and the results of Lemma 2 (but no further balanced growth restrictions) imply that we can rewrite the above expression for GDP as:

$$\begin{aligned} \Delta \ln(Y_t^{Index}) &= \sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta \ln(V_{it}) \\ &= \sum_i \alpha \overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \left(\frac{1}{\alpha} \Delta \ln(A_{it}) + \theta \Delta \ln(K_{it}/L_{it}) + \Delta \ln(L_{it}) \right) \\ &\approx \sum_i \left(\overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \Delta \ln(A_{it}) \right) + \theta \Delta \ln(K_t) \\ &\approx \underbrace{\sum_i \left(\overline{\frac{P_{it} Q_{it}}{P_t^Y Y_t}} \Delta \ln(A_{it}) \right)}_{\text{(A) Hulten's Theorem term}} + \underbrace{\frac{\theta}{1-\theta} \left(\Delta \ln(\mathcal{A}_t) - \Delta \ln(R_t) \right)}_{\text{(B) Aggregate capital growth}} \end{aligned} \tag{31}$$

The approximation in the above expression follows from the approximation $\sum_i \overline{\frac{P_{it}^V V_{it}}{P_t^Y Y_t}} \Delta \ln(L_{it}) \approx 0$, consistent with our assumption of a fixed aggregate labor supply.

Equation (31) decomposes aggregate GDP growth into two terms: (A) Domar-weighted (sector gross output divided by GDP) productivity growth ("Hulten's Theorem", as described in (Baqaee and Farhi, 2019)), and (B) the growth rate of aggregate capital, which is given by the difference between the growth rate of aggregate TFP, $\mathcal{A}_t$, and the growth rate of the rental rate of capital, R_t.

Equation (31) holds for all equilibrium paths, not just balanced growth paths.[30] For example, an ABGP requires the additional assumption of constant aggregate TFP growth, which implies that the growth in the rental rate of capital is zero. Proposition 1 does not imply that growth in the aggregate GDP index,

[29] The U.S. national accounting system uses a Fisher ideal index number instead of a Tornqvist index. The two indices produce nearly identical results for U.S. data, and the Tornqvist index provides an expression for aggregate growth that is easier to analyze theoretically.

[30] An interesting question is how subsequent results change if we analyze GDP as an index number without imposing assumptions 1 or 2, requiring a common production structure and a single type of capital. Absent these assumptions, such an analysis would require numerical simulation along a transition path. We expect that results of such an exercise would not yield materially different results for two reasons. First, the evidence presented in Section 2 suggests that Assumption 2 is approximately satisfied in the U.S., given the high degree of similarity in the composition of investment purchases across all sectors. Second, as we show in Appendix C, if we construct production parameters for each of the six sectors in our study, capital share parameters are fairly similar across sectors, suggesting a limited role for heterogeneity. The one notable source of heterogeneity worth studying further would be that intermediate inputs receive a much higher Cobb-Douglas weight in goods sectors than in services sectors. For example, as argued in Moro (2015), this observation coupled with structural change would imply a lower aggregate growth rate with a reduction in the input-output multiplier as the economy transitions from goods to services.

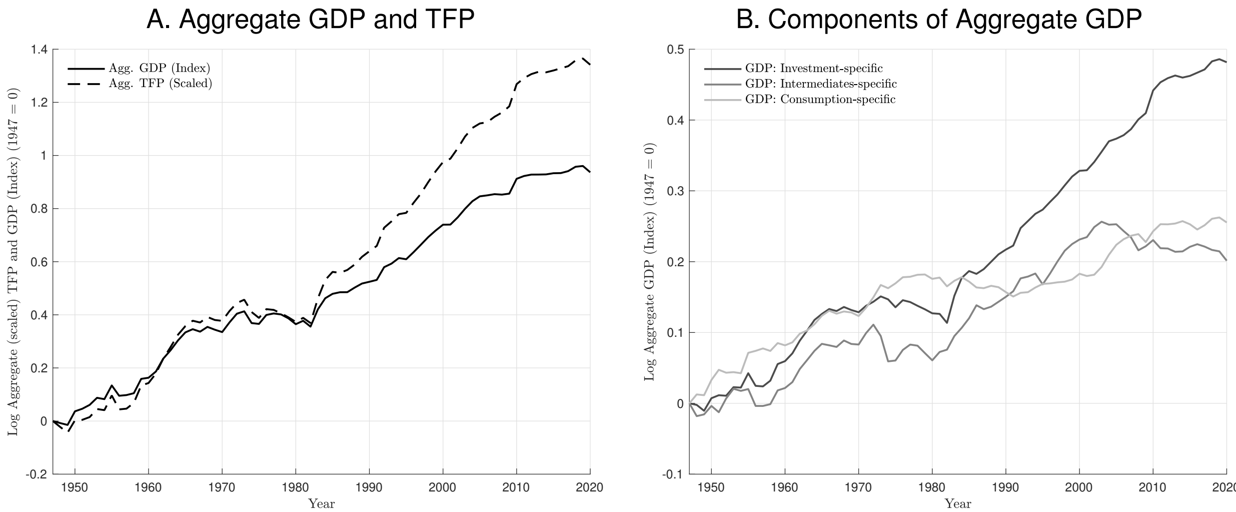
$\Delta \ln(Y_t^{Index})$, need be constant; constant aggregate growth in the GDP index would require the additional assumption that the Hulten's Theorem term also grows at a constant rate. As highlighted by expression (31), Hulten's Theorem does not hold in our model because aggregate capital and aggregate GDP can potentially grow at different rates. This is likely because the production of aggregate capital uses inputs from a different mix of sectors than aggregate GDP and aggregate capital growth is impacted by intermediates-bundling technical change, which is not present in the Hulten's Theorem term.

Given this more empirically consistent expression for aggregate GDP growth, we now revisit our decomposition of aggregate growth in Figure 8 and Table 3. We do so under the additional assumption that the change in the rental rate of capital is zero, which is empirically consistent with the U.S. experience over a long horizon (see for example Jones, 2016). Similarly to the previous section, we define the contribution of investment-specific technical change as the counterfactual GDP growth series where there is only TFP growth in goods-investment and services-investment; since TFP growth in these sectors also appears in the Hulten's Theorem term, the final contribution of investment-specific technical change to GDP growth will be different from its contribution to aggregate TFP growth. The contribution of intermediates-specific technical change is updated similarly. In addition to these two sources of aggregate growth, we also define "consumption-specific technical change" as the counterfactual GDP growth series where there is only sectoral TFP growth in goods-consumption and services-consumption sectors, which only impact GDP growth through the Hulten's Theorem term.

Figure 10 and Table 5 summarize our results. Panel A of Figure 10 shows that the two alternative GDP measures—log of the GDP index and scaled aggregate TFP—track each other closely through the mid-1980s, after which the index-number measure of GDP observes a substantial growth slowdown relative to our aggregate TFP series. We show in Appendix D that the index-number measure of GDP tracks empirical measures of GDP per worker very closely throughout the sample.

To understand the evolution of growth in the GDP index and its slowdown over time, panel B of Figure 10 and Table 5 document how technical change in consumption, investment, and intermediates contribute to growth in the GDP index over time. Given that consumption-specific technical change now contributes to aggregate growth and that intermediates-specific technical change receives extra weight in the GDP index (compared to aggregate TFP), the overall fraction of GDP growth accounted for by investment-specific technical change is smaller; 51% on average over the post-war period. However, the importance of investment-specific technical change is still rising over time, accounting for more than 70% of growth post-2000 compared to 33-54% from 1960-2000. In Appendix D, we show that a significant fraction (more than 20%) of

Figure 10: Aggregate GDP Growth (Index Number) and Its Composition, 1947-2020



Notes: Panel A shows the time series of aggregate GDP measured as an Index number, compared to aggregate GDP measured in units of aggregate investment, both are measured in logs (normalized to zero in 1947); panel B shows counterfactual evolutions of aggregate GDP for three cases: (1) only technical change among investment producers ("Investment-specific"), (2) only technical change in intermediates, both at intermediates producers and from intermediates bundling technical change ("Intermediates-specific"), and (3) only technical change among consumption producers ("Consumption-specific").

aggregate GDP growth post-2000 comes from reallocation forces in investment due to a non-unitary elasticity of substitution. Thus, absent endogenous reallocation of investment production to services sectors, the aggregate slowdown since 2000 would have been even worse. Interestingly, the more pronounced slowdown in aggregate GDP growth, especially post-2000, appears to be driven by stagnating intermediates-specific technical change.

This leads us to three observations that are potentially useful for the ongoing debate over the sources of slowing growth during the 1970s and since 2000, documented in a number of countries including the United States (see Syverson, 2017, for a recent review). First, growth in consumption-specific technical change is relatively stable throughout the entire period since 1947 and does not appear to materially contribute to either the growth slowdown of the 1970s or the 2000s. Second, slowing productivity growth in both production networks accounts for the slowdown in aggregate growth during the 1960s and 1970s. Third, the slowdown after 2000 is primarily attributable to stagnating or negative intermediates-specific technical change. In contrast to the arguments by Gordon (2016), this suggests that the recent slowdown in aggregate growth post-2000 may have different underlying causes than the slowdowns observed in earlier decades. Thus, future work aimed at understanding the origins of the slowdown in intermediates-technical change post-2000 is important for understanding the recent slowdown in aggregate growth.

Table 5: GDP Growth Decomposition, 1947-2019

	Aggregate GDP (Index) Growth: $\Delta\ln(x) = \Delta\ln(Y_t^{Index})$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%	$\Delta\ln(x)$	%
All	0.96	100	0.20	100	0.37	100	0.22	100
Investment-Specific	0.49	51	0.07	33	0.20	54	0.16	71
Intermediates-Specific	0.21	22	0.04	19	0.17	46	-0.02	-8
Consumption-Specific	0.26	27	0.09	47	0.01	2	0.08	36

Notes: The table shows long-run log changes in aggregate GDP measured as an index number, $\Delta\ln(Y_t^{Index})$ (following equation (31)), across different periods for four alternative simulations: (1) the full model simulation with all measured TFP series; three counterfactual simulations with TFP growth only from (2) investment producers ("investment-specific technical change"), (3) intermediates producers and exogenous intermediates-bundling TFP ("intermediates-specific technical change"), and (4) consumption producers ("consumption-specific technical change"). Counterfactual changes are also expressed as a percent of the change from the full model simulation; these may not exactly sum to 1, given the nonlinear relationships between individual technology series and the aggregates. For each time period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

7. Conclusion

This paper studies the intersection of structural change in production networks and economic growth. We document that the sectoral distribution of production in both intermediate and investment production networks has evolved, with services sectors producing a larger share of both intermediates and investment. To understand these patterns, we develop a framework that allows us to study structural change in both consumption and these production networks.

Explicitly modeling intermediates allows us to use expenditure-side prices on final commodities, rather than income-side sectoral gross output prices, to discipline the model in a way that is internally consistent (as discussed by Herrendorf et al., 2014). Specifically, we construct novel price series for goods and services, split by their use as final consumption, intermediates, or investment. Together with our stylized fact on the rising services share within investment production, these disaggregated price series imply that goods and services are substitutes in the production of investment, rather than complements, as found in previous studies. We show that aggregation bias accounts for differences in these findings, with expenditure-side prices able to capture price growth in the production of investment that is easily averaged out in gross output prices.

This finding has important implications for economic growth and provides useful insights regarding a concern initially brought up by Baumol (1972)—that structural change leads to a systematic reallocation from productive/innovative goods sectors to less innovative services sectors, eventually leading to an economy where the least productive sector dictates all economic progress. While the intermediates network in our framework does indeed appear to suffer from Baumol's "cost disease", we find the investment network to be the primary engine of growth in part because it is systematically shifting toward the most productive *services* (e.g., software development and R&D). Intuitively, complementarity in intermediates production leads to "bottleneck" growth, governed by the least productive sector, while substitutability in investment production generates "frontier growth", driven by the most productive sector. Our findings, similar to Duernecker, Herrendorf and Valentinyi (2017), thus provide some optimism for the impact of sectoral reallocation on aggregate economic growth.

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APPENDIX

Appendix A. Additional Empirical Results on Structural Change

Appendix A.1. Production Network Measurement Details

Our primary data source for measuring production networks and related data is the BEA Input-Output database, specifically, the time series of Make and Use tables from 1947-2020 and the time series of the investment network generated in vom Lehn and Winberry (2022). The Make and Use tables from the BEA Input Output database can be downloaded at the BEA's website here: https://www.bea.gov/industry/input-output-accounts-data; data from vom Lehn and Winberry (2022) containing the time series of the investment network can be found here: https://doi.org/10.7910/DVN/CALDHX. These data provide details for 40 NAICS-defined sectors of the economy, including agriculture and government (43 if energy/oil-intensive sectors are included); Table 1 lists each of the 40 sectors and their corresponding NAICS codes. More recent vintages of the BEA Input Output database allow for greater sectoral detail (and it is possible to construct more detailed investment networks for recent years), but given our interest in structural change over the long run, we focus on data on these 40 sectors which are available going back to 1947.

We use the Make and Use tables from the BEA to measure input-output relationships in the following way. The core of the Use table is a square matrix that reports intermediate input expenditures by different sectors (organized along columns) on specific commodities (organized along rows). These commodities are named and assigned NAICS codes based on which sectors are major producers of the given commodity, but more than one sector may be involved in the production of a given commodity. The mix of sectors that produce a given amount of each commodity is observed in the Make table, which is a square matrix reporting commodities along columns and the amounts of each commodity produced by each sector along rows. The final input-output matrix in each year is the matrix product of a scaled Make table, where each column is scaled by its sum (thus summing to 1) and the unscaled Use table.

The investment network data from vom Lehn and Winberry (2022) reports the matrix of sectoral spending and production of new investment; see that paper for construction details. We follow their procedures to extend the investment network through the year 2020. However, the raw investment matrix from vom Lehn and Winberry (2022) is still organized with commodities along each row, not sectors, and needs to also be adjusted using the Make matrix. Thus, the final investment network data we use is the product of the scaled Make matrix and the unscaled investment network data from vom Lehn and Winberry (2022).

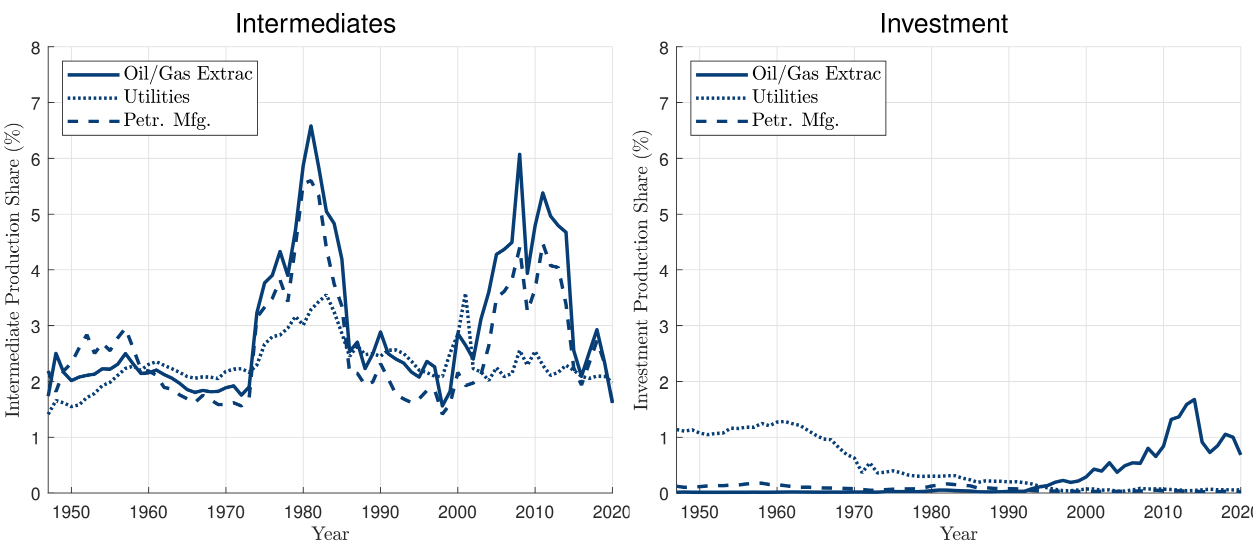
The Use tables from the BEA also contain information on the final uses of each commodity produced, including consumption. To measure structural change in consumption, we construct final consumption produced by each sector as the product of the scaled Make matrix and the sum of private final consumption and government consumption vectors in the Use table. The Use table also has information on the final use of each commodity as new investment, however, we use sums of the data from the investment network (which is closely tied to this figure) to compute the total production of investment by each sector.

Appendix A.2. Additional Empirical Results on Structural Change

In this subsection, we report four additional empirical results: data on energy/oil-intensive sectors' contributions to intermediates and investment production over time, time series detail for sectors whose production share of investment or intermediates has increased or decreased the most, detail on how the service shares of intermediates and investment is changing within all 40 sectors in our data, and the shift-share decomposition for the increased share of services in the production of investment or intermediates.

First, Figure A.1 reports the shares of intermediates and investment produced by each of the three sectors omitted from our analysis – oil/gas extraction, utilities, and petroleum manufacturing. Although there are large medium-run swings in these shares (particularly for intermediates), there is no long-run trend in how much these sectors produce of intermediates and only very slight (and off-setting) long-run trends in investment. Thus, as can be seen in Figure A.2, whether these sectors are included as part of the total goods sector or not, there are not large changes in the long-run structural change trends observed in consumption, intermediates, and investment.

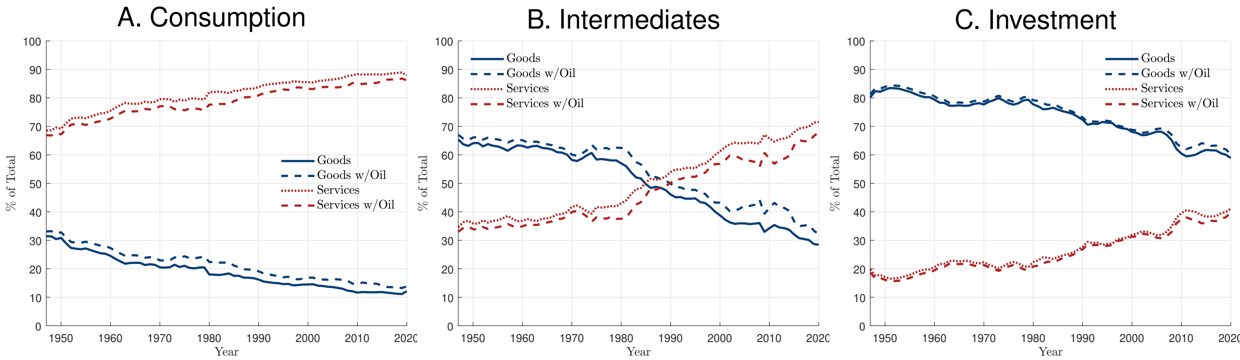
Figure A.1: Shares of Intermediates and Investment Produced by Oil-Intensive Sectors, 1947-2020



Notes: Panel A shows the share of total intermediates produced by oil-intensive sectors while panel B shows the share of investment produced by oil-intensive sectors.

Second, the four panels of Figure A.3 report the time series patterns of sectors whose share of production of intermediates or investment has increased the most (right panels) or decreased the most (left panels). As reported in Section 2, for intermediate goods, the largest increases in production share are in information services, finance/insurance, real estate, professional/technical services, and administrative support services; the largest decreases occurred in agriculture, primary metals, food and beverage manufacturing, textile manufacturing, and paper manufacturing. For investment, the largest increases occurred in professional/technical services, information services, and wholesale trade; the largest decreases are in machinery, construction, and motor vehicle manufacturing. While there is certainly heterogeneity in the changes over time in each of

Figure A.2: Trends in Production Share of Consumption, Intermediates and Investment, Goods vs. Services, With and Without Oil-Intensive Sectors, 1947-2020



Notes: The figures plot the fraction of total spending on consumption, intermediates and investment produced by the goods sector (blue lines) and the services sector (red lines). The dashed lines indicate these same fractions when oil-intensive sectors are included in the analysis (all part of the goods sector).

these production shares, each sector's changes in production shares appear to be part of a gradual long-run trend and not some single spike occurring in a particular year.

Third, in Figure A.4, we present bar charts showing the change in the services share of production of intermediates (top panel) and investment (bottom panel) within all 40 sectors in our data between 1947 and 2019. Although there is significant heterogeneity in how much the services share of production of intermediates or investment has changed in each sector, it is increasing in the vast majority of sectors.

Finally, we consider a shift-share decomposition of the increased share of intermediates and investment produced by services sectors. The rising share of services in the production of intermediates and investment reflects two changes. First, spending on intermediates and investment produced by services sectors now comprise a larger fraction of total intermediates and investment spending by each sector. Second, the growth of the services sector overall has led to increased aggregate spending on sectoral inputs purchased by services sectors (which is especially relevant for intermediates, given the evidence in Figure 1).

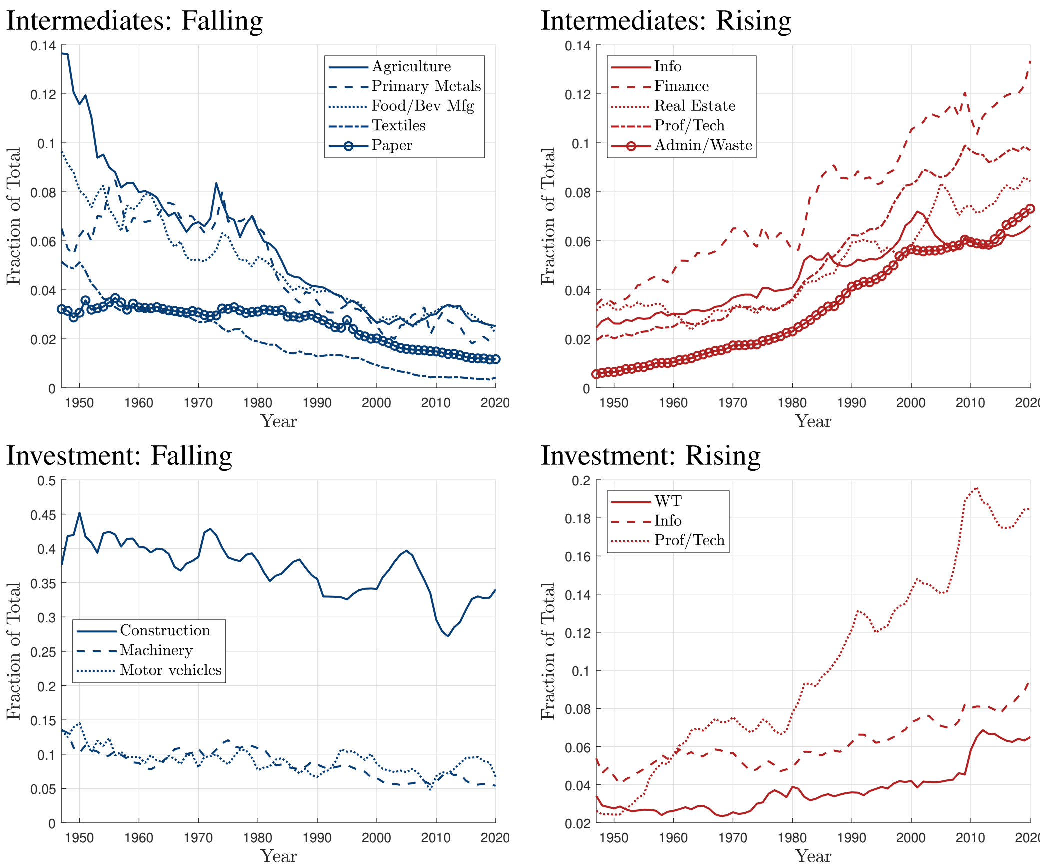
We quantify the importance of these two channels for structural changes in production networks by doing a shift-share decomposition of the services production share, isolating changes occurring within and between sectors. This decomposition can be expressed as:

$$\Delta Serv_t = \underbrace{\sum_j^N \left(\overline{\omega}_j \Delta Serv_{jt}\right)}_{\text{within}} + \underbrace{\sum_j^N \left(\overline{Serv}_j \Delta \omega_{jt}\right)}_{\text{between}},$$

where $\Delta x = x_{2019} - x_{1947}$ is the change in x and $\overline{x} = \frac{x_{2019}+x_{1947}}{2}$ is the average of x in the two periods 1947 and 2019. The results of this decomposition are presented in Tables A.1 (with changes in composition over just $N = 2$ sectors, goods and services) and A.2 (with changes in composition over all $N = 40$ sectors).

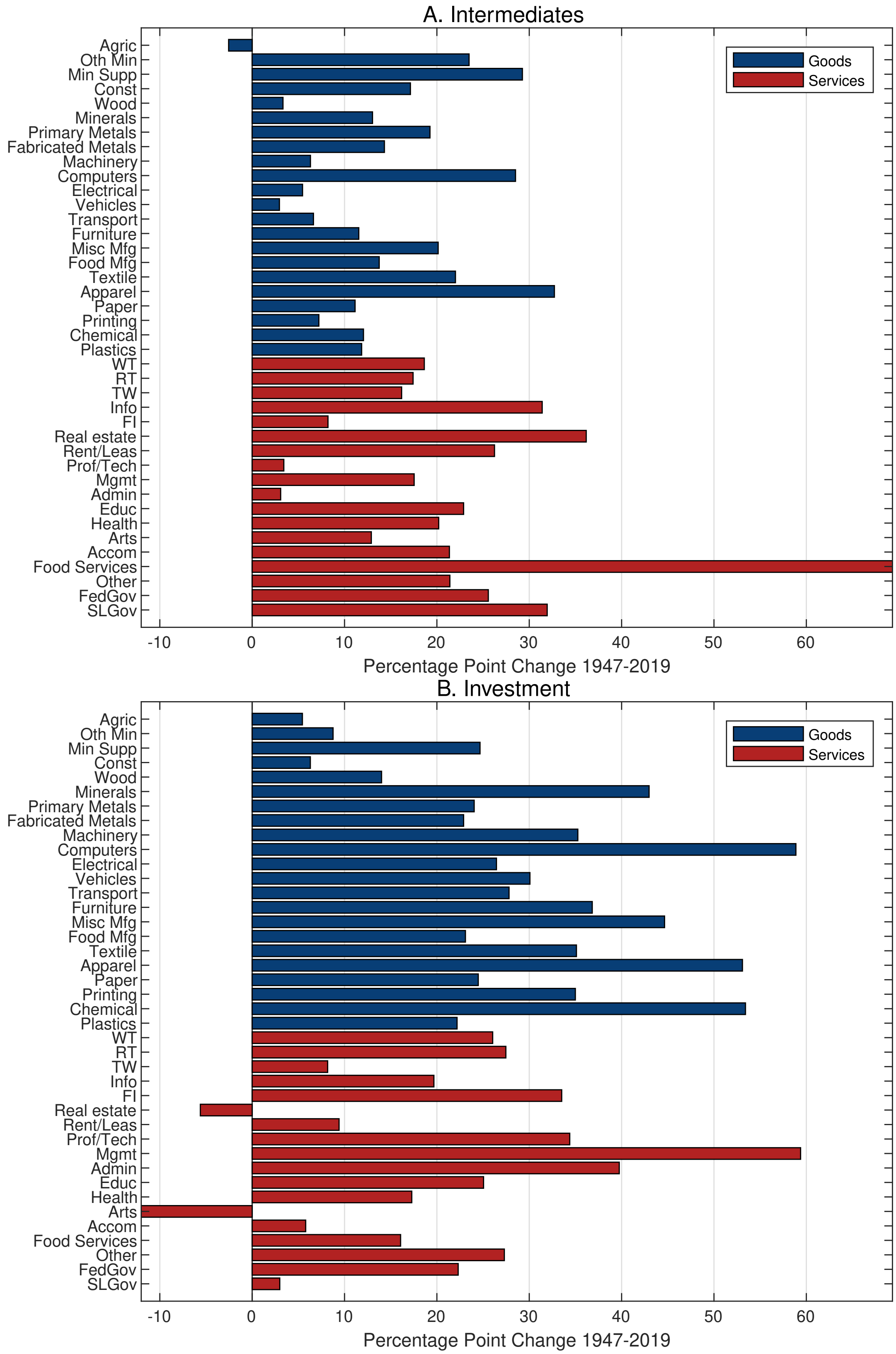
For investment, the large majority (75-100%) of all changes over time in the shares of investment produced by services are due to changes within sectors; for intermediates, the within-sector component con-

Figure A.3: Time Series Changes in Production Share of Intermediates and Investment, Additional Sector Detail



Notes: Each line represents a given sector's share of total production of intermediates (top rows) or investment (bottom rows). Right panels with red lines show sectors whose production share has increased the most; left panels with blue lines show sectors whose production share has decreased the most.

Figure A.4: Changes in Services Production Share of Intermediates and Investment Purchased by Each Sector: 1947-2019



Notes: Each bar represents the change in the services production share of intermediates (upper panel) or investment (lower panel) purchased by each sector between 1947 and 2019. Blue bars: goods sectors; red bars: services sectors.

Table A.1: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment

	1947	2019	Δ	Decomposition within	Decomposition between
Intermediates	0.35	0.71	0.37	0.19	0.17
				(53%)	(47%)
Investment	0.20	0.40	0.20	0.21	-0.01
				(104%)	(-4%)

Notes: The table reports the results of a shift-share decomposition of the share of services production over time. We decompose changes in the fraction of total intermediates (or investment) spending being produced by services, $Serv_t$, into changes in the share of spending on services within each purchasing industry j, $Serv_{jt}$, and the importance of industry j's spending on intermediates as a fraction of all intermediates expenditures, ω_{jt}. This decomposition can be expressed as:

$$\Delta Serv_t = \underbrace{\sum_j (\overline{\omega}_j \Delta Serv_{jt})}_{\text{within}} + \underbrace{\sum_j (\overline{Serv}_j \Delta \omega_{jt})}_{\text{between}},$$

where $\Delta x = x_{2019} - x_{1947}$ is the change in x and $\overline{x} = \frac{x_{2019}+x_{1947}}{2}$ is the average of x in the two periods 1947 and 2019. Individual components may not exactly sum to totals due to rounding.

Table A.2: Shift-Share Decomposition of Services Share of Production of Intermediates and Investment, 40 Sector Detail

	1947	2019	Δ	Decomposition within	Decomposition between
Intermediates	0.35	0.71	0.37	0.18	0.19
				(49%)	(51%)
Investment	0.20	0.40	0.20	0.15	0.05
				(73%)	(27%)

Notes: The table reports the shift-share decomposition described in Table A.1 for the production share of services, but with within-sector and between-sector changes across all 40 sectors in our data. Individual components may not exactly sum to totals due to rounding.

tributes roughly half of the change over time. This evidence accords with the production network patterns shown in Figure 1, which showed more sector-specificity in intermediate suppliers than investment suppliers across sectors, consistent with a more significant role for between-sector changes in the services share of production.

Appendix A.3. Within-Sector Structural Transformation of Occupations

This appendix provides labor market evidence that the patterns of structural change we observe in the intermediates network are not driven by the outsourcing of labor services from goods sector establishments. The concern is that the growth in services in intermediates is driven by goods producing establishments outsourcing services tasks such as janitorial, legal, or accounting services to services establishments. If this were the primary driver of structural change in intermediates, the following two patterns would be prevalent in data. First, we would expect a decline in services-task intensive occupations within goods producing industries. Second, we would expect that changes in services task intensity within sectors should be negatively correlated with changes in purchases of services intermediate inputs.

To investigate this concern, we utilize data from decennial U.S. Censuses for the years 1950-2010 and the 2019 American Community Survey (both provided by IPUMS 13.0) to document trends in the industry-specific concentration of services-task intensive occupations. We consider services-task intensive occupations as (1) managerial/professional/specialty, (2) technical/sales/admin, and (3) services occupations based on the IPUMS 13.0 `OCC1990` classification.

Using the IPUMS 13.0 `IND1990` industry aggregation, we construct 32 consistent sectors over the period 1950-2019 (following vom Lehn, 2018), as listed in Table A.3.[31] These sectors are a direct aggregation of the 40 NAICS 2007 sectors used in our main analysis and can therefore be directly compared over the entire sample. For each of these 32 sectors we construct two measures for the services-task intensity: the services employment share (total employment in services-task intensive occupations divided by total employment within the sector); and the services earnings share (total earnings of services-task intensive occupations divided by total earnings within the sector).

Table A.3 illustrates that employment and earnings shares of services intensive occupations within all but two (agriculture, and food services) of the 32 broad NAICS sectors have substantively increased over the period 1950-2019. This implies that the structural change patterns we document for the intermediates network do not coincide with a reduction in services workers in goods sectors.

An additional piece of suggestive evidence that outsourcing is not driving the patterns of structural change patterns in intermediates is that changes in occupational services-task intensity are not negatively correlated with changes in the purchases of intermediates produced by services sectors. To show this, we construct changes in our two measures of within-sector service intensity by decade and correlate these with changes in the each sector's share of intermediates expenditures produced by services. Specifically, we regress the change by decade in each measure of sectoral services task intensity on a constant, a full set of time effects (decade dummies), and the sectoral change by decade in the share of intermediates expenditures produced by services sectors.

Table A.4 summarizes the regression estimates, suggesting that within industry changes in the share of intermediates purchased from services appear to have no systematic correlation with concurrent changes in the service task intensity. If anything, while none of these correlations are statistically significant, the correlation for intermediates appears to be positive, rather than negative. This suggests that sectors experiencing more structural transformation in intermediates are not more likely to see a systematic decline of employment in services-task intensive occupations.

[31] IPUMS data does not allow us to distinguish the information services sector from the printing and publishing manufacturing sector throughout the postwar sample. We thus combine these two sectors into one and list it as a goods sector (though listing it as a services sector does not alter our results).

Table A.3: Change in Occupational Service Task Intensity Within Sectors, 1950-2019

	Perc. Pt. Ch.			Perc. Pt. Ch.	
Goods Producing Sectors (NAICS Codes)	Emp.	Earn.	Service Producing Sectors (NAICS Codes)	Emp.	Earn.
Ag./forestry/fishing/hunting (11)	-7.0	-8.6	Wholesale trade (42)	4.7	16.0
Mining, except oil and gas (212)	16.6	25.0	Retail trade (44-45)	5.7	9.2
Construction (23)	11.4	24.0	Transport and warehousing (48-49, minus 491)	9.7	20.2
Wood products (321)	9.3	21.3	Finance and insurance (52)	1.0	1.3
Non-metallic mineral products (327)	15.8	31.5	Real estate (531)	3.4	6.3
Primary and Fabricated Metals (331,332)	13.4	25.4	Prof./Tech./Rent./Mgmt/Admin. (54-56,532-533)	8.2	12.0
Machinery (333)	16.4	31.1	Educational services (61)	1.6	2.9
Computer and Electronic Mfg (335,334)	35.8	53.8	Health services (62)	2.3	4.2
Motor Vehicles Mfg (3361-3363)	12.9	30.3	Arts, ent. and rec. services (71)	5.3	4.4
Other transp. equipment (3364-3369)	28.9	43.8	Accommodation services (721)	1.5	3.3
Furniture and related MfG (337)	13.9	30.2	Food services (722)	-2.5	-2.4
Misc. manufacturing (339)	28.9	49.2	Other private services (81)	11.9	14.1
Food and Beverage Mfg (311-312)	8.0	23.3	Fed/State/Local Government (n/a, but incl. 491)	12.0	13.8
Textile manufacturing (313-314)	20.1	37.0			
Apparel manufacturing (315-316)	24.7	47.1			
Paper manufacturing (322)	11.5	25.4			
Printing and Information (51,323)	16.2	23.1			
Chemical manufacturing (325)	21.2	37.7			
Plastics and rubber products (326)	8.8	25.8			

Notes: The table reports the percentage point change in occupational employment and earnings shares bteween 1950 and 2019 within 32 consistent sectors. The sectors are a direct aggregation of the 40 NAICS 2007 sectors listed in Table 1 to map into the IPUMS IND1990 classification as suggested by vom Lehn (2018). Services intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, and (3) services occupations based on the IPUMS 13.0 OCC1990 classification. Data on employment are taken from the U.S. Census obtained from IPUMS USA.

Table A.4: Changes in Service Task Intensity and Structural Change

	ΔService Task Intensity	
	Employment Share (1)	Earnings Share (2)
Δ Share of Intermediates Expenditures from Services	2.036	2.806
	(3.207)	(4.680)
Obs.	224	224

Notes: The table reports results from linear regressions, where the left-hand side is the decadal change in the employment (column 1) or earnings share (column 2) of services-task intensive occupations within 32 consistent NAICS sectors (listed in Table A.3). The regressors are a constant, a full set of time effects (coefficients not reported), and the decadal change in the share of intermediates purchased from services sectors. Standard errors are reported in parentheses underneath the coefficients. The decadal changes are based on the years 1949, 1959, 1969, 1979, 1989, 1999, 2009, 2018. Data on employment and earnings are taken from the decadal U.S. Censuses for the years 1950-2010. For the final decade we prefer the 2019 ACS rather than the 2020 U.S. Census, to avoid the impact of the COVID-19 pandemic. All U.S. Census data are taken from IPUMS 13.0. Earnings and employment are reported for the previous year in each survey. Services intensive occupations are (1) managerial/professional/specialty, (2) technical/sales/admin, (3) service occupations based on the IPUMS 13.0 OCC1990 classification. Standard errors are reported in parentheses and confidence levels are indicated by $^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$.

Appendix A.4. Decomposing Value-Added Measures of Structural Change

We now analyze the importance of changes in the input-output network for "value-added" measures of structural change in consumption and investment. As explained in Herrendorf et al. (2013), a value-added approach to measuring sectoral production of consumption and investment focuses not only on the set of sectors producing the final product but also on the network of sectors contributing intermediate inputs needed to produce the product. Thus, value-added measures of structural change implicitly include structural change in intermediates and structural change in final producers of consumption and investment.

Value-added vectors of sectoral production of consumption and investment (denoted in current dollars), $\mathbf{c^{VA}}$ and $\mathbf{x^{VA}}$, are constructed using input-output data using the following equations:

$$\mathbf{c^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}\mathbf{c} \tag{A.1}$$

$$\mathbf{x^{VA}} = \mathbf{v}(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}\mathbf{x} \tag{A.2}$$

where $\mathbf{c}$ and $\mathbf{x}$ are vectors of final production of consumption and investment, respectively, by each sector, $\mathbf{I}$ is the identity matrix, $\mathbf{v}$ is a diagonal matrix of the share of value added in gross output in each sector, and $\mathbf{\Gamma}$ is a matrix of input-output relationships, where the $(i, j)th$ element of $\mathbf{\Gamma}$ is the ratio of intermediates purchased by sector j from sector i to the total gross output in sector j.[32]

We use data from the BEA Make and Use tables to construct value-added measures of consumption and investment (as described in equations (A.1) and (A.2). As noted above, the final vectors of consumption and investment are available from the Use tables and the investment network data. To compute the fraction of value added in gross output, we use data in the Use table on nominal value added and nominal gross output for each sector. To compute the total requirements matrix, or Leontief inverse, we scale our final input-output data (adjusted by the Make matrix) by the total gross output of each sector (as opposed to the total spending on intermediates), which gives us the matrix $\mathbf{\Gamma}$. Because we initially adjust both final consumption and the input-output data by the Make matrix, the formulas in equations (A.1) and (A.2) look slightly different from those reported in Herrendorf et al. (2013), but the methods are identical.[33]

Structural change in consumption value added or investment value added can occur because of changes in $\mathbf{v}$, the ratio of value added to gross output, changes in the total requirements matrix $(\mathbf{I} - \mathbf{\Gamma})^{\mathbf{-1}}$, the input-output network, or changes in $\mathbf{c}$ or $\mathbf{x}$, the final producers of consumption or investment goods. We consider a counterfactual decomposition where we allow only one of these three components to vary over time and hold the two other components fixed at their values in the initial year of our data, 1947. Since the construction of consumption and investment value-added is non-additive, the contributions of each of these three terms will not necessarily sum to one.

[32] We first compute all of these objects at the 40 sector level and then analyze structural change at the two sector level, aggregated up from consumption value added and investment value-added constructed at the 40 sector level.

[33] In principle, the total requirements matrix might change over time because the Make matrix has changed. We have explored counterfactuals holding the Make matrix fixed and found that changes in this matrix have virtually no impact on value-added measures of structural change.

Table A.5: Decomposing Structural Change in Services Share of Value Added Measures of Consumption and Investment

Services Share of:	1947	2019	Δ	% of Total
Consumption Value Added	0.68	0.87	0.20	
Final Prod. only	0.68	0.81	0.13	69%
Input-Output only	0.68	0.77	0.09	48%
VA share only	0.68	0.65	-0.02	-11%
Investment Value Added	0.36	0.54	0.18	
Final Prod. only	0.36	0.50	0.14	75%
Input-Output only	0.36	0.46	0.09	52%
VA share only	0.36	0.33	-0.03	-19%

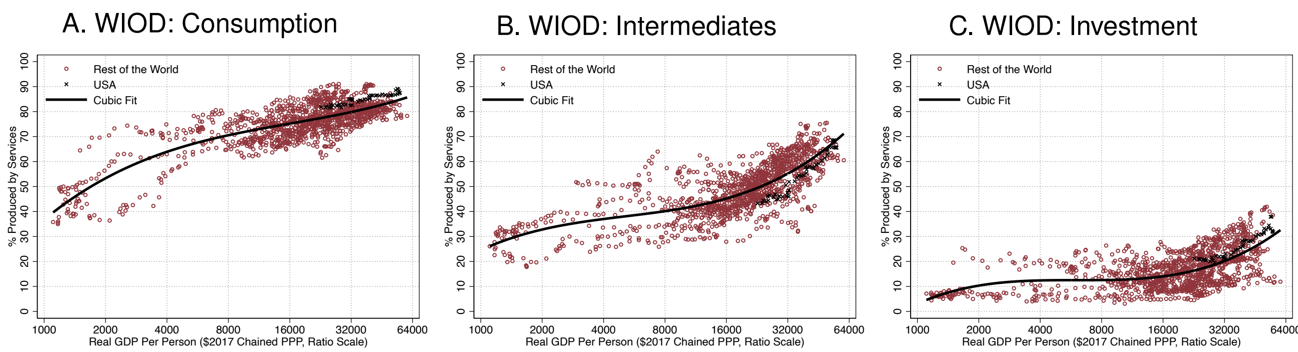
Notes: This table reports the share of value-added based consumption and investment produced by the services sector and how this changes over time due to changes in each component of the value-added measure (as seen in Equations (A.1) and (A.2)). Changes generated by each of the three components—final producers ("Final Prod. only"), the Total Requirements Matrix ("Input-Output only"), and value-added shares of gross output ("VA shares only")—are computed by holding fixed all other components at their values in 1947. The "% of total" column refers to the change in each component divided by the change in total consumption or investment value added. Because of the non-linear nature of the decomposition, the total of each component will not sum to the actual total.

Table A.5 presents the decomposition, highlighting the contribution of each of these three forces to the change in the share of consumption value-added and investment value-added produced by the services sector between 1947 and 2019. Changes in the input-output network account for 45-55% of the rising share of services production of consumption and investment value added. This total contribution is potentially slightly inflated because the contribution of each component sums to more than the total change in the services share of consumption and investment value added. However, if we compute the contribution of changes in the input-output network as a fraction of the sum of changes in each component, input-output changes still account for 40-50% of structural change in consumption and investment value added.

Appendix A.5. International Evidence

This appendix illustrates that the broad patterns of structural transformation in the production of consumption, intermediates, and investment that we find in BEA data for the United States are in fact a widespread phenomenon, occurring in many countries around the world. To do so, we obtain use tables analogous to those of the BEA for 41 countries (including the United States) from two waves of the World Input Output Database (WIOD: Timmer et al., 2015; Woltjer et al., 2021). The first wave reports data from 1965-2000 for 25 countries and 23 sectors, while the second covers data for 40 countries and 35 sectors over the period 1995-2011. The international use tables adhere to the 1993 version of the System of National Accounts (SNA), a comprehensive conceptual and accounting framework for compiling and reporting macroeconomic statistics developed by the United Nation's (UN) Intersecretariat Working Group on National Acccounts (ISWGNA). Industries are classified according to the International Standard Industrial Classification revision 3 (ISIC Rev. 3) and we aggregate these industries to 21 consistently defined sectors that can be grouped into goods and services. As in our main analysis, we exclude oil and utilities producing industries and split total expenditure on consumption, intermediates, and investment into the portion supplied by goods industries and the portion supplied by services industries.

Figure A.5: International Trends in Goods/Service Production Share of Consumption, Intermediates and Investment (1965-2011)



Notes: Panels A-C display the share of services in the production of consumption, investment, and intermediates using the use tables from the World Input Output Database (WIOD, see Timmer et al., 2015; Woltjer et al., 2021) plotted against real GDP per capita (taken from the Penn World Table 10.0: Feenstra, Inklaar and Timmer, 2015). Panels A-C additionally show a fitted cubic polynomial and also highlight the data points for the USA (black Xs).

To harmonize the two WIOD waves we measure the service/goods production shares starting with the level observed in the first year of the data and then construct shares in subsequent years by cumulating observed annual growth rates in these shares. For countries that span both WIOD waves, we use the growth rates from the 2013 WIOD starting with the year 2000. We drop two countries: Hong Kong, because its data ends in 1999 and Luxembourg because its services share of investment is an outlier (about twice as large as that of countries at similar levels of development, such as the United states).

Figure A.5 plots structural change in consumption, investment, and intermediates using the WIOD data. To facilitate comparison across countries, we plot the share of consumption, intermediates, and investment produced by the service sector against real GDP per capita (in $2017 chained PPP on a ratio scale), similar to illustrations provided by Galesi and Rachedi (2018). This figure reveals several notable insights. First, we highlight data for the United States (marked with black Xs) to illustrate that the range of values for the U.S. service shares constructed from WIOD data are very similar to those we observe from BEA data in Figure 3. Second, the fitted cubic trend lines suggest that the time series patterns observed in the United States are representative of the typical experience in other countries at similar levels of development (as measured by real GDP per capita). Third, the stylized fact of increasing services shares in the production of consumption, intermediates, and investment is present at all levels of development within the WIOD database, ranging from countries with initial GDP per person as low as 1,271 ($2017 chained PPP) to as high as 22,746 in 1965. It appears that structural transformation in the production of consumption is faster at lower levels of development, while that for intermediates and investment appears to accelerate at higher levels of development.

Appendix B. Equilibrium Conditions, Derivations, and Proofs

Appendix B.1. Household Problem

The household's problem is

$$\max_{C_{jt},K_{jt+1}} \sum_{t=0}^{\infty} \beta^t log\left(\left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C-1}}\right),$$

$$\text{s.t.} \sum_j P_{jt}C_{jt} + \sum_{j=1}^{N} P_{jt}^X \left(K_{jt+1} - (1-\delta_j)K_{jt}\right) \leq W_t + \sum_j R_{jt}K_{jt}.$$

The first order conditions for this problem, $\forall j$, are

$$\frac{P_{jt}^X}{E_t^C} = \frac{\beta}{E_{t+1}^C}\left(R_{jt+1} + P_{jt+1}^X(1-\delta_j)\right) \tag{B.1}$$

$$\frac{P_{jt}}{E_t^C} = \left(\omega_{Cj}\frac{C_t}{C_{jt}}\right)^{\frac{1}{\epsilon_C}} \frac{1}{C_t} \tag{B.2}$$

where total consumption, C_t, and total expenditures (denoted in units of the numeraire), E_t^C, are given by:

$$C_t \equiv \left[\sum_j \omega_{Cj}^{1/\epsilon_C} C_{jt}^{\frac{\epsilon_C-1}{\epsilon_C}}\right]^{\frac{\epsilon_C}{\epsilon_C-1}} \tag{B.3}$$

$$E_t^C = \sum_j P_{jt}C_{jt}. \tag{B.4}$$

Appendix B.2. Production Firm Problem

The profit maximization problem for the representative production firm in sector j is given by

$$\max_{L_{jt},K_{jt},M_{jt}} P_{jt}Q_{jt} - W_tL_{jt} - R_{jt}K_{jt} - P_{jt}^M M_{jt}.$$

where $Q_{jt} = A_{jt}\left(K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}\right)^{\alpha_j} M_{jt}^{1-\alpha_j}$.

The first order conditions for this problem are

$$W_t = \alpha_j(1-\theta_j)\frac{P_{jt}Q_{jt}}{L_{jt}} \tag{B.5}$$

$$R_{jt} = \alpha_j\theta_j\frac{P_{jt}Q_{jt}}{K_{jt}} \tag{B.6}$$

$$P_{jt}^M = (1-\alpha_j)\frac{P_{jt}Q_{jt}}{M_{jt}}. \tag{B.7}$$

Appendix B.3. Bundling Firm Problems

For each sector j, there are two bundling firms: one that produces the intermediate good used by sector j, M_{jt}, and one that produces the investment (purchased by the household) for capital specific to sector j, X_{jt}.

The profit maximization problem of the intermediates bundling firm for sector j is given by:

$$\max_{M_{ijt}} P^M_{jt} M_{jt} - \sum_i P_{it} M_{ijt},$$

where the bundle of intermediates used by sector j, M_{jt} is given by:

$$M_{jt} = A^M_{jt} \left(\sum_i \omega_{Mij}^{1/\epsilon_{Mj}} M_{ijt}^{\frac{\epsilon_{Mj}-1}{\epsilon_{Mj}}} \right)^{\frac{\epsilon_{Mj}}{\epsilon_{Mj}-1}}. \tag{B.8}$$

The first order conditions for this problem are, for each sector i:

$$P_{it} = P^M_{jt} \left(A^M_{jt}\right)^{1-\frac{1}{\epsilon_{Mj}}} \left(\omega_{Mij} \frac{M_{jt}}{M_{ijt}} \right)^{\frac{1}{\epsilon_{Mj}}}. \tag{B.9}$$

Obtaining the expression for the price of the intermediates bundle sold to sector j, P^M_{jt}, as reported in equation (9), follows from solving the first order conditions for M_{ijt}, plugging into the expression for M_{jt}, and solving for P^M_{jt}.

The profit maximization problem and first order conditions for the investment bundling are symmetric and are given by:

$$\max_{X_{ijt}} P^X_{jt} X_{jt} - \sum_i P_{it} X_{ijt}$$

$$P_{it} = P^X_{jt} \left(A^X_{jt}\right)^{1-\frac{1}{\epsilon_{Xj}}} \left(\omega_{Xij} \frac{X_{jt}}{X_{ijt}} \right)^{\frac{1}{\epsilon_{Xj}}} \tag{B.10}$$

where the bundle of investment for capital specific to sector j, X_{jt} is given by:

$$X_{jt} = A^X_{jt} \left(\sum_i \omega_{Xij}^{1/\epsilon_{Xj}} X_{ijt}^{\frac{\epsilon_{Xj}-1}{\epsilon_{Xj}}} \right)^{\frac{\epsilon_{Xj}}{\epsilon_{Xj}-1}}. \tag{B.11}$$

Similarly, the expression for the price of the investment bundle for sector j's capital shown in equation (10) can be obtained by solving the first order conditions for X_{ijt}, plugging into the expression for X_{jt}, and solving for P^X_{jt}.

Appendix B.4. Market Clearing Conditions

In equilibrium, each labor, capital, intermediate bundling and investment bundling market clears. To conserve on notation, market clearing is built into how the capital, intermediates and investment problems have been written down. With the household inelastically providing unitary labor supply each period, labor market clearing is simply given by $\sum_j L_{jt} = 1$. That leaves market clearing for final production in each sector j, which is given by:

$$C_{jt} + \sum_i M_{jit} + \sum_i X_{jit} = Q_{jt}. \tag{B.12}$$

We also note that the evolution of capital in each sector is given by the standard accumulation equation:

$$K_{jt+1} = (1-\delta_j)K_{jt} + X_{jt}. \tag{B.13}$$

Appendix B.5. Sectoral Value Added and Prices

For each production sector j, constant returns to scale implies

$$W_t L_{jt} + R_{jt}K_{jt} + P^M_{jt}M_{jt} = P_{jt}Q_{jt}. \tag{B.14}$$

Therefore, the accounting definition of nominal value added is simply

$$P^V_{jt}V_{jt} = P_{jt}Q_{jt} - P^M_{jt}M_{jt} = W_tL_t + R_{jt}K_{jt}. \tag{B.15}$$

To obtain real value added, we use a discrete-time application of the Divisia index definition, which differentiates the accounting definition of nominal value added holding prices fixed:

$$\begin{aligned} P^V_{jt}V_{jt}\Delta\ln V_{jt} &= P_{jt}Q_{jt}\Delta\ln Q_{jt} - P^M_{jt}M_{jt}\Delta\ln M_{jt} \\ \alpha_j\Delta\ln V_{jt} &= \Delta\ln Q_{jt} - (1-\alpha_j)\Delta\ln M_{jt} \\ \Delta\ln V_{jt} = \frac{1}{\alpha_j}&\Delta\ln A_{jt} + \theta_j\Delta\ln K_{jt} + (1-\theta_j)\Delta\ln L_{jt}. \end{aligned}$$

Cumulating this expression yields that real value added is given by $V_{jt} = A_{jt}^{\frac{1}{\alpha_j}}K_{jt}^{\theta_j}L_{jt}^{1-\theta_j}$.[34]

Finally, we can write the price index for value added in sector j, P^V_{jt}, as follows:

$$\begin{aligned} P^V_{jt} &= \frac{P_{jt}Q_{jt} - P^M_{jt}M_{jt}}{V_{jt}} = \frac{P_{jt}V_{jt}^{\alpha_j}\left(\left(\frac{(1-\alpha_j)P_{jt}}{P^M_{jt}}\right)^{\frac{1}{\alpha_j}}V_{jt}\right)^{1-\alpha_j} - P^M_{jt}\left(\frac{(1-\alpha_j)P_{jt}}{P^M_{jt}}\right)^{\frac{1}{\alpha_j}}V_{jt}}{V_{jt}} \\ &= P_{jt}^{\frac{1}{\alpha_j}}\left(P^M_{jt}\right)^{1-\frac{1}{\alpha_j}}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{1}{1-\alpha_j}-1\right) = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P^M_{jt}}\right)^{\frac{1-\alpha_j}{\alpha_j}} \end{aligned}$$

[34] We have normalized the implicit time-invariant constant in cumulating this expression to 1.

where we use the fact that $Q_{jt} = V_{jt}^{\alpha_j} M_{jt}^{1-\alpha_j}$ and the fact that $M_{jt} = \left(\frac{(1-\alpha_j)P_{jt}}{P_{jt}^M}\right)^{\frac{1}{\alpha_j}} V_{jt}$ (from the first order conditions for intermediates, shown in equation (B.7)).

Appendix B.6. Proof of Lemma 1

With Assumptions 1 and 2, we have $P_{jt}/P_{it} = \left[A_{it}/\left(P_{it}^M\right)^{1-\alpha}\right]/\left[A_{jt}/\left(P_{jt}^M\right)^{1-\alpha}\right] = \tilde{A}_{it}/\tilde{A}_{jt}$ where $\tilde{A}_{jt} \equiv A_{jt}/\left(P_{jt}^M\right)^{1-\alpha}$. With this relationship, the lemma is straightforward to prove by manipulation of the expression for the price of investment (equation (10), though now common to all sectors due to Assumptions 1 and 2):

$$\begin{aligned} P_t^X &= \frac{1}{A_t^X}\left(\sum_k \omega_{Xi} P_{kt}^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} = P_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\frac{P_{kt}}{P_{jt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} \\ &= P_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\frac{\tilde{A}_{jt}}{\tilde{A}_{kt}}\right)^{1-\epsilon_X}\right)^{\frac{1}{1-\epsilon_X}} = P_{jt}\tilde{A}_{jt}\frac{1}{A_t^X}\left(\sum_k \omega_{Xk}\left(\tilde{A}_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{1-\epsilon_X}}. \end{aligned}$$

Hence,

$$P_{jt}A_{jt} = P_t^X A_t^X \left(\sum_k \omega_{Xk}\left(\tilde{A}_{kt}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}.$$

Defining $\tilde{B}_X(t) \equiv A_t^X\left(\sum_k \omega_{Xk}\tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$, the result of the lemma obtains.

The proof for the price and technical change in intermediate goods follows identical steps as the above, with $\tilde{B}_{it}^M \equiv A_{it}^M\left(\sum_k \omega_{Mki}\tilde{A}_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$.

Appendix B.7. Proof of Lemma 2

Aggregate GDP, Y_t, denoted in units of the numeraire, is given by $Y_t = \sum_i P_{it}^V V_{it}$, where V_{it} is real value added in sector i and P_{it}^V is the price of value added in sector i.

As shown in Appendix B.4, sectoral real value added and its price, can be written as:

$$V_{jt} = A_{jt}^{\frac{1}{\alpha_j}} K_{jt}^{\theta_j} L_{jt}^{1-\theta_j}$$

$$P_{jt}^V = \frac{\alpha_j}{1-\alpha_j}(1-\alpha_j)^{\frac{1}{\alpha_j}}\left(\frac{P_{jt}^{\frac{1}{1-\alpha_j}}}{P_{jt}^M}\right)^{\frac{1-\alpha_j}{\alpha_j}}$$

Given Assumptions 1 and 2 (implying that $\alpha_j = \alpha$ for all sectors), and these expressions for real value added and its price, we can write aggregate GDP as:

$$Y_t = \sum_i P_{it}^V V_{it} = \sum_i \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\frac{A_{it} P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \left(\frac{K_{it}}{L_{it}} \right)^{\theta} L_{it}$$

Because of the common rental rate and wage, the capital to labor ratios will be equated across sectors, and with an aggregate labor supply of 1, will simply be equal to the aggregate stock of capital, $K_t = \sum_i K_{it}$. Further, from Lemma 1 and our choice of the price of investment as our numeraire, we have that $\frac{A_{it} P_{it}}{\left(P_{it}^M\right)^{1-\alpha}} = P_{it} \tilde{A}_{it} = \tilde{B}_t^X$. With this, we can rewrite the above expression for GDP as:

$$Y_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}} (K_t)^{\theta} \sum_i L_{it} = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}} K_t^{\theta} = \mathcal{A}_t K_t^{\theta}$$

where $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X \right)^{\frac{1}{\alpha}}$.

First order conditions for capital in each production sector give us $R_t = \theta\alpha \frac{P_{jt} Y_{jt}}{K_{jt}}$. Using our expressions for real sectoral value added and its price, as well as the result of Lemma 2, we can rewrite this first order condition as:

$$\begin{aligned} R_t &= \theta\alpha \frac{P_{jt} Y_{jt}}{K_{jt}} = \theta\alpha \frac{\frac{1}{\alpha} P_{jt}^V V_{jt}}{K_{jt}} = \theta \frac{\alpha}{1-\alpha} (1-\alpha)^{\frac{1}{\alpha}} \left(\frac{K_{jt}}{L_{jt}} \right)^{\theta-1} \left(\frac{P_{jt} A_{jt}}{(P_t^M)^{1-\alpha}} \right)^{\frac{1}{\alpha}} \\ &= \theta \frac{\alpha}{1-\alpha} (1-\alpha)^{\frac{1}{\alpha}} (\tilde{B}_t^X)^{\frac{1}{\alpha}} \left(\frac{K_t}{L_t} \right)^{\theta-1} = \theta \mathcal{A}_t K_t^{\theta-1} \end{aligned}$$

Applying the same algebraic steps to the first order condition for labor demand (equation (B.5)) generates the other equation in the lemma, $W_t = (1-\theta)\mathcal{A}_t K_t^{\theta}$.

Appendix B.8. Proof of Proposition 1

Given the if and only if statement in the proposition, we must prove both the necessary and sufficient directions. We start with the necessary direction, showing if an ABGP exists, this requires that $\gamma^{\mathcal{A}}$ is constant and that $\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$

The requirement that $\gamma^{\mathcal{A}}$ be constant follows immediately from the aggregate production function expression from Lemma 2, $Y_t = \mathcal{A}_t K_t^{\theta}$. If Y_t and K_t grow at constant rates, that means that $\mathcal{A}_t$ must as well. Thus, the remainder of this direction of the proof entails showing that the growth rates of K_t, Y_t, W_t, X_t, E_t^C and E_t^M are all equal to $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and that the growth rate of R_t is zero.

Taking the Euler equation from the household's problem (see Appendix B.1), we have that $\frac{E_{t+1}^C}{E_t^C} = \gamma_{t+1}^{E^C} = \beta \left(R_{t+1} + 1 - \delta \right)$. This implies that a constant growth rate in household expenditures implies a constant rental rate of capital, R_t, along the ABGP. Taking the ratio of first order conditions for capital in each sector, we have that $\frac{K_{jt}}{L_{jt}} = \frac{\theta}{1-\theta} \frac{W_t}{R_t}$.

With our assumptions of common parameters across sectors, capital to labor ratios are equated, and since $\sum_j L_{jt} = 1$, we can write the aggregate capital stock, K_t, as $K_t = \frac{\theta}{1-\theta}\frac{W_t}{R_t}$. Since R_t is constant along the ABGP, this implies that $\gamma^K = \gamma^W$.

Likewise, if we take the ratio of first order conditions for capital and intermediates at production firms, and rearrange, we have $K_{it} = \frac{\alpha\theta}{1-\alpha}\frac{1}{R_t}P^M_{it}M_{it}$. Summing this equation across sectors i, we have that:

$$K_t = \frac{\alpha\theta}{1-\alpha}\frac{1}{R_t}\sum_i P^M_{it}M_{it} \tag{B.16}$$

Thus, total expenditures on intermediates, $E^M_t \equiv \sum_i P^M_{it}M_{it}$, will grow at the same rate as the aggregate capital stock.

From Lemma 2, we have that $R_t = \theta\mathcal{A}_t K_t^{\theta-1}$. Taking the ratio of this simplified first order condition for capital across time periods yields $\frac{K_{t+1}}{K_t} = \left(\frac{R_{t+1}}{R_t}\frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\right)^{\frac{1}{1-\theta}}$ and $\gamma^K = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$, where the last step holds given the constant rental rate of capital along the ABGP. Taking the ratio of the aggregate production function from Lemma 2 across time periods yields $\frac{Y_{t+1}}{Y_t} = \frac{\mathcal{A}_{t+1}}{\mathcal{A}_t}\left(\frac{K_{t+1}}{K_t}\right)^{\theta}$ and $\gamma^Y = \gamma^{\mathcal{A}}\left(\gamma^{\mathcal{A}}\right)^{\frac{\theta}{1-\theta}} = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$, which thus implies $\gamma^Y = \gamma^K$.

Now, turning to the capital accumulation equation, if we divide by K_t, we have $\gamma^K = (1-\delta) + \frac{X_t}{K_t}$. Since γ^K is a constant, this requires that the RHS be constant, or in other words, $\gamma^X = \gamma^K$.

The only remaining condition to verify here is that aggregate consumption expenditures, E^C_t, grow at the same rate as aggregate capital. We can write GDP using expenditure side accounting as $Y_t = E^C_t + X_t$, since all these aggregates are denoted in units of the numeraire. Since we know that $\gamma^Y = \gamma^K = \gamma^X$ and γ^{E^C} is constant, then $\gamma^{E^C} = \gamma^Y = \gamma^K$. This finishes the necessity direction of the proof.

We now consider the sufficiency direction required for the proof. We now show that if $\gamma^{\mathcal{A}}$ is constant, then an ABGP exists. We do this by construction. We set $\gamma^K = \gamma^X = \gamma^Y = \gamma^{E^C} = \gamma^{E^M} = \gamma^W = \left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}}$ and we set R_t to be a constant such that the Euler Equation holds: $R_{t+1} = \frac{1}{\beta}\gamma^{E^C} - (1-\delta)$. Given our assumption that $\left(\gamma^{\mathcal{A}}\right)^{\frac{1}{1-\theta}} > \frac{1-\delta}{\beta}$, this will produce a non-negative rental rate for capital.

Then, given an initial value of $\mathcal{A}_t$, this value of R implies a unique value for K_0 from (the rewritten first order conditions). It is then straightforward to construct X_0 to satisfy capital accumulation, given K_0 and γ^K, and to construct E^M_0 using equation (B.16) above. Finally, we can determine the initial condition for expenditures, using the expenditure side accounting relationship, with $E^C_0 = Y_0 - X_0 = \mathcal{A}_0 K_0^{\theta} - K_0\left(\gamma^K - (1-\delta)\right)$. Lastly, to show that transversality holds, we need that $\lim_{t\to\infty}\beta^t\frac{K_t}{E^C_t} = 0$

Given that we have constructed the path such that $\gamma^K = \gamma^{E^C}$,$\frac{K_t}{E^C_t}$ will be a constant along this path and thus the limit will be satisfied. This completes the proof in the sufficiency direction.

Appendix B.9. Proof of Lemma 3

Given the assumption 3, that the parameters of the intermediates bundling sectors are the same for all sectors j, i.e. $\omega_{Mij} = \omega_{Xi}$ and $\epsilon_{Mj} = \epsilon_M$ for all j and that each sector's intermediates bundling TFP is the same, i.e. $A^M_{jt} = A^M_t$, we start by revisiting the result of Lemma 1. Assumption 3 implies that there is now a single intermediate good in the economy, with a single price, P^M_t. As a result, given our definition of $\tilde{A}_{it} \equiv \frac{A_{jt}}{\left(P_{jt}^M\right)^{1-\alpha}}$, we have that $P_{jt}/P_{it} = \tilde{A}_{it}/\tilde{A}_{jt} = A_{it}/A_{jt}$. Thus, we now have that $\tilde{B}_{it}^M = \tilde{B}_t^M$ and by the same logic as the above and the proof of Lemma 1, we have that $B_t^X/B_t^M = P_t^M/P_t^X$, where $B_{it}^M \equiv A_{it}^M \left(\sum_k \omega_{Mki} A_{kt}^{\epsilon_{Mi}-1}\right)^{\frac{1}{\epsilon_{Mi}-1}}$ and $B_t^X \equiv A_t^X \left(\sum_k \omega_{Xk} A_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}}$.

The final part left to show is that $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} B_t^X \left(B_t^M\right)^{\frac{1-\alpha}{\alpha}}$. Given that, in the more general case, $\mathcal{A}_t = \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{\alpha}} \left(\tilde{B}_t^X\right)^{\frac{1}{\alpha}}$, this amounts to showing that $\tilde{B}_t^X = \left(B_t^X\right)^{\alpha} \left(B_t^M\right)^{1-\alpha}$. Given the definition of $\tilde{B}_t^X$, we have that $\tilde{B}_X(t) = A_t^X \left(\sum_k \omega_{Xk} \tilde{A}_{kt}^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}} = A_t^X \left(\sum_k \omega_{Xk} \left(\frac{A_{kt}}{\left(P_t^M\right)^{1-\alpha}}\right)^{\epsilon_X-1}\right)^{\frac{1}{\epsilon_X-1}} = \frac{B_t^X}{\left(P_t^M\right)^{1-\alpha}} = \frac{B_t^X}{\left(\frac{B_t^X}{B_t^M}\right)^{1-\alpha}} = \left(B_t^X\right)^{\alpha} \left(B_t^M\right)^{1-\alpha}$. This completes the proof.

Appendix B.10. Proof of Lemma 4

We prove the result for intermediates and ϵ_M first; the result for investment with ϵ_X follows by the symmetry of the CES functions. For ease of exposition of the proof, we define $g^M(\epsilon_M)$ as follows:

$$g^M(\epsilon_M) = \left(\sum_i s_{it-1}^M (\gamma_{it}^A)^{\epsilon_M-1}\right)^{\frac{1}{\epsilon_M-1}}$$

Thus, the objective is to show that $g^M(\epsilon_M)$ is weakly increasing in ϵ_M. For ease of exposition, we also suppress the A superscript on γ_{it}^A and define $\gamma_i \equiv \gamma_{it}$.

We observe that $g^M(\epsilon_M)$ depends on ϵ_M in two ways—both directly, as an exponent on γ_{it}^A and in the exponent for the overall sum, but also indirectly, through its impact on s_{it-1}^M, which is itself a function of ϵ_M. With assumptions 1-3, s_{it-1}^M can be written as a function of exogenous values, including ϵ_M:

$$s_{it-1}^M(\epsilon_M) = \omega_{Mi} \frac{A_{it-1}^{\epsilon_M-1}}{\sum_j^N \omega_{Mj} A_{jt-1}^{\epsilon_M-1}}$$

Our goal is to show that for any $\epsilon_1 > \epsilon_2$, $g^M(\epsilon_1) \geq g^M(\epsilon_2)$. We show this in two steps. First, we define the function $\tilde{g}(\sigma, \epsilon_M) = \left(\sum_i s_{it-1}^M(\sigma) \gamma_{it}^{\epsilon_M-1}\right)^{\frac{1}{\epsilon_M-1}}$. We first show that for fixed σ and $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma, \epsilon_1) \geq \tilde{g}(\sigma, \epsilon_2)$. The second step defines the function $\hat{g}(\epsilon_M, \sigma) = \left(\sum_i s_{it-1}^M(\epsilon_M) \gamma_{it}^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ and shows that $\hat{g}(\epsilon_1, \sigma) \geq \hat{g}(\epsilon_2, \sigma)$. Then, given these two substeps, the final result follows from the following sequence of inequalities:

$$g(\epsilon_1) = \tilde{g}(\epsilon_1, \epsilon_1) \geq \tilde{g}(\epsilon_1, \epsilon_2) = \hat{g}(\epsilon_1, \epsilon_2) \geq \hat{g}(\epsilon_2, \epsilon_2) = g(\epsilon_2) \tag{B.17}$$

We note that the lemma makes the assumption of positive dependence in the form of $\mathbb{E}\left[ln(A_{it}) \mid \gamma_{it}^A = a\right]$ being weakly increasing in a. This assumption is not needed until Step 2, and so we demonstrate Step 1 for the more general case without this assumption.

Step 1: For $\epsilon_1 > \epsilon_2$, $\tilde{g}(\sigma,\epsilon_1) \geq \tilde{g}(\sigma,\epsilon_2)$. This first step of the proof follows from an application of Jensen's inequality. Jensen's inequality tells us that for any convex function, $\phi(x)$, any real valued function $h(x)$, and any set of non-negative weights a_i with $\sum_i a_i = 1$, $\sum_i a_i\phi(h(x_i)) \geq \phi(\sum_i a_i h(x_i))$. The inequality is reversed in the case where $\phi(x)$ is concave.

First, begin with the case where $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $\frac{\epsilon_1-1}{\epsilon_2-1} > 1$. Define $\phi(x) = x^{\frac{\epsilon_1-1}{\epsilon_2-1}}$. This function is convex because $\frac{\epsilon_1-1}{\epsilon_2-1} > 1$. Define $h(x) = x^{\epsilon_2-1}$ and $a_i = s^M_{it-1}(\sigma)$.

Jensen's inequality thus implies the following result:

$$\left(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_2-1}\right)^{\frac{\epsilon_1-1}{\epsilon_2-1}} = \tilde{g}(\sigma,\epsilon_2)^{\epsilon_1-1} \leq \left(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_1-1}\right) = \tilde{g}(\sigma,\epsilon_1)^{\epsilon_1-1}$$

Exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1-1}$, which is a positive exponent, completes the result for this case.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 < \frac{\epsilon_1-1}{\epsilon_2-1} < 1$, then it must be that $\epsilon_1 < 1$. In this case, $\phi(x)$ is now concave, which reverses the above inequality. However, because $\epsilon_1 < 1$, the step of exponentiating both sides of the inequality to the power $\frac{1}{\epsilon_1-1}$ again reverses the inequality and ensures the result holds.

If $\epsilon_1 \neq 1$ and $\epsilon_2 \neq 1$ and $0 > \frac{\epsilon_1-1}{\epsilon_2-1}$, then $\epsilon_2 < 1$ and $\epsilon_1 > 1$, and $\phi(x)$ is again convex and $\frac{1}{\epsilon_1-1}$ is a positive exponent, so the result still holds.

Finally, consider the case where either $\epsilon_1 = 1$ or $\epsilon_2 = 1$. Although $\tilde{g}(\epsilon_M)$ is undefined in this case, we consider instead the limiting result, defining $\tilde{g}(\sigma,0) = \prod_i \gamma_{it}^{s^M_{it-1}(\sigma)}$. Here we apply Jensen's inequality using $\phi(x) = \ln(x)$ and $h(x) = x^{\epsilon_1-1}$. If $\epsilon_2 = 1$ and $\epsilon_1 > 1$, then we have that:

$$(\epsilon_1-1)\ln(\tilde{g}(\sigma,\epsilon_1)) = \ln(\sum_i s^M_{it-1}(\sigma)\gamma_{it}^{\epsilon_1-1}) \geq (\epsilon_1-1)\sum_i s^M_{it-1}(\sigma)\ln(\gamma_{it}) = (\epsilon_1-1)\ln(g(\sigma,0))$$

Dividing both sides by $\epsilon_1 - 1$ and exponentiating both sides of the inequality yields the result.

In the case where $\epsilon_1 = 2$ and $\epsilon_2 < 1$, the same steps can be followed, replacing ϵ_1 with ϵ_2, but now since $\epsilon_2 < 1$, the final step of dividing both sides by $\epsilon_2 - 1$ will reverse the inequality, proving the result.

This completes step 1.

Step 2: For $\epsilon_1 > \epsilon_2$, $\hat{g}(\epsilon_1,\sigma) \geq \hat{g}(\epsilon_2,\sigma)$. To prove this inequality, we show that $\frac{\partial\hat{g}(\epsilon_M,\sigma)}{\partial\epsilon_M} \geq 0$.

Taking the partial derivative, we obtain the following result:

$$\begin{aligned}\frac{\partial\hat{g}(\epsilon_M,\sigma)}{\partial\epsilon_M} \quad = \quad & \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma}\sum_i\frac{\partial s^M_{it-1}(\epsilon_M)}{\partial\epsilon_M}\gamma_{it}^{\sigma-1} = \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma}\left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\ln(A_{it})\right. \\ & \left. -\left(\sum_i s^M_{it-1}\gamma_{it}^{\sigma-1}\right)\left(\sum_i s^M_{it-1}\ln(A_{it})\right)\right) = \frac{1}{\sigma-1}\hat{g}(\epsilon_M,\sigma)^{2-\sigma}\mathrm{Cov}\left(\ln(A_{it}),\gamma_{it}^{\sigma-1}\right)\end{aligned}$$

where $\mathrm{Cov}(\ln(A_{it}),\gamma_{it}^{\sigma-1})$ is the covariance between $\ln(A_{it})$ and $\gamma_{it}^{\sigma-1}$ where probability weights across sectors are defined by the shares s^M_{it-1}. Given the weak positive dependence assumption, that $\mathbb{E}\left[ln(A_{it}) \mid \gamma^A_{it} = a\right]$

is weakly increasing in a, the sign of this covariance term will be the the sign of $\sigma - 1$.[35] Thus, since this covariance has the same sign as $\sigma - 1$, the result will go through. Since we know that $\hat{g}(\epsilon_M, \sigma)^{2-\sigma} > 0$ and the entire expression is multiplied by $\frac{1}{\sigma-1}$, this ensures that $\frac{\partial \hat{g}(\epsilon_M,\sigma)}{\partial \epsilon_M} \geq 0$. This completes step 2 of the proof.

Given the successful completion of steps 1 and 2, the proof for intermediates follows from the inequalities in equation (B.17) and the proof for investment follows by symmetry.

Appendix C. Price Measurement Details and Additional Calibration Results

In this Appendix, we provide further detail regarding price measurement, robustness of our measurement procedures, and additional calibration details.

Appendix C.1. Measuring Consumption and Investment Prices by Sector

We measure the price of consumption and the price of investment produced by goods and services sectors using expenditure-side accounting data. We start with final expenditure data on the prices of and total expenditures on 68 consumption commodities (NIPA Tables 2.4.4 and 2.4.5) and 30 investment commodities (NIPA Tables 5.3.4, 5.3.5, 5.5.4, 5.5.5, 5.6.4). The 68 consumption commodities are the finest level of consumption commodity disaggregation possible over the postwar period (after omitting commodities produced primarily by the energy-intensive sectors omitted from our analysis: motor vehicle fuels, fuel oils and other fuels, water supply and sanitation services, electricity, and natural gas). The 30 investment commodities (two structures commodities, 24 equipment commodities, three intellectual property commodities, and residential commodities) are a subset of the 33 investment commodities used in vom Lehn and Winberry (2022). Due to data limitations, primarily on detailed investment prices (which were not studied in vom Lehn and Winberry (2022)), we combine light trucks and other trucks into a single commodity, all software (prepackaged and custom) into a single commodity, and residential structures and residential equipment into a single commodity.[36]

Equipped with this mapping, we measure price growth for goods-consumption, services-consumption, goods-investment, and services-investment as the weighted average of price growth within the 68 consumption and 30 investment commodities, where the weights correspond to the sector's production share for each commodity. For example, we measure price growth for the goods consumption sector, g–c, using NIPA price and spending data on K consumption commodities (P^C_{kt} and $P^C_{kt}Q^C_{kt}$ for $k \in \{1, ..., K\}$) according to

[35]This can be seen by an iterated expectations argument. Consider two random variables X and Y which have, without loss of generality, zero mean. The covariance of X and Y is $COV(X,Y) = \mathbb{E}[XY] = \mathbb{E}[X\mathbb{E}[Y \mid X]] = COV(X, \mathbb{E}[Y \mid X])$. If we assume that $\mathbb{E}[Y \mid X = x]$ is increasing in x, then, since the covariance of two increasing functions of X is positive, we have the result. Note that if the conditional expectation of Y is weakly increasing in X, then it will be weakly increasing in $X^{\sigma-1}$ if $\sigma - 1 > 0$ and weakly decreasing in X if $\sigma - 1 < 0$. When $\sigma - 1 < 0$, then we have the covariance of an increasing and a weakly decreasing function of X, which is weakly negative.

[36]The 33 investment commodities in vom Lehn and Winberry (2022) are approximately the finest level of disaggregation of investment commodities possible while accurately tracking each commodity's production by different sectors over time. It is possible to study patterns in more finely disaggregated non-mining structures, but the mix of sectors producing these structures is the same for each commodity. Thus, further disaggregation does not yield additional insight. For this case, we aggregate prices for detailed structures to a single investment price for non-mining structures using a Tornqvist index.

the formula:

$$\Delta \ln(P_{g-c,t}) = \sum_{k=1}^{K} \frac{\xi^C_{g-c,kt} P^C_{kt} Q^C_{kt}}{\sum_{\ell=1}^{L} \xi^C_{g-c,\ell t} P^C_{\ell t} Q^C_{\ell t}} \Delta \ln(P^C_{kt}) \tag{C.1}$$

where $\xi^C_{g-c,kt}$ is the fraction of commodity k's purchaser's value (averaged across years $t-1$ and t) that was produced by the goods consumption sector, g–c.

For consumption commodities, we use the BEA's published bridge file (available at `https://www.bea.gov/products/industry-economic-accounts/underlying-estimates`); for investment commodities, we use the bridge files constructed in vom Lehn and Winberry (2022), which we extend forward through the year 2020 using the most recent bridge file data from the BEA. Both of these bridge files are multiplied by scaled Make file, just as we do with other data (as described in Appendix A). Since we are interested primarily in the differences between goods and services production, we aggregate the final bridge files to two sectors, goods and services (as defined in Table 1).

One limitation of the consumption bridge file published by the BEA is that it begins in 1997. As a result, we apply the 1997 bridge file data to all years 1947-1996. However, there are minimal differences in our measured consumption prices if we use a fixed bridge file, averaged over the entire 1947-2020 sample, suggesting that movements in the bridge file are negligible in generating long-run trends.

We also modify the bridge files for investment from vom Lehn and Winberry (2022) by assuming that goods produce the entirety of structures (including residential investment). In the original bridge files from vom Lehn and Winberry (2022), a small fraction of structures (roughly 7% of non-mining structures and 11% of residential investment) is produced by services, primarily margin contributions from sectors like real estate and finance. However, because overall investment production by services is low compared to goods, especially early in the sample (as seen in Figure 3) and because investment spending on structures is large, absent any adjustment, a sizable portion of the services investment price is determined by structures prices, although the amount of total services investment production coming from structures is falling over time, from 1/3 in 1947 to less than 10% by 2020. This is unappealing because 1) it is unlikely that the price of structures investment reflects the price of these margins and 2) because the exact contribution of these services margins is determined in part by imputation (see Appendix A of vom Lehn and Winberry (2022) for further details). Thus, we set the services contribution to the production of structures equal to zero in our final bridge files. However, even without this adjustment, Appendix C.3 documents that services investment prices are still significantly decreasing relative to goods.

As discussed in Section 5, one concern with measuring the price of consumption and investment produced by goods and services using expenditure side accounting data is that there may be bias introduced when both goods and services contribute to the production of a commodity, but the final contributions of goods and services sectors have different prices. The primary instance where both goods and services sectors contribute significantly to a consumption or investment commodity is when delivery to the final user involves significant "margins" due to transportation, wholesale trade, or retail trade. This is more common with consumption than with investment commodities. If these margin sectors were reclassified as goods sectors, however, then over 90% of all commodities would be produced in large majority by either goods or services. Appendix C.3 documents that our observed patterns of relative prices are robust to such a reclassification.

For consistency with how we measure intermediates prices, we adjust final consumption and investment prices to remove the input-output network transmission of changes in oil/energy prices. We already omit consumption commodities where a large portion of the commodity is produced by one of the energy-intensive sectors. However, oil/energy price fluctuations may significantly impact the final prices of sectors that heavily rely on energy as an intermediate input. Thus, they may influence the final prices of goods or services consumption or investment.

We use the following approach to make this adjustment. In the following subsection, we describe a procedure for "purging" gross output prices (at the 40 sector level) of the impact of oil/energy prices transmitted through the input-output network. This procedure yields an adjusted gross output price for each sector j, $\tilde{P}_{jt}$, and a "adjustment term," given by the difference between the adjusted price and the original gross output price, $\tilde{P}_{jt} - P_{jt}$. With these sector-specific adjustment terms, we adjust the final price of each commodity using a weighted sum of sectoral adjustment terms, weighted by each sector's position in the bridge file for each commodity. Formally, the final commodity prices for each consumption (or investment) commodity k are given by $\tilde{P}_{kt} = P_{kt} + \sum_{j=1}^{N} \xi_{jkt}^{C} \left(\tilde{P}_{jt} - P_{jt}\right)$, where ξ_{jkt}^{C} is the final bridge file for commodity k (in the case of consumption). The impacts of this adjustment for consumption and investment prices are generally small, as can be seen in the subsequent subsection when we show price trends leaving in all oil/energy sectors.

Appendix C.2. Measuring Intermediates Prices

Appendix C.2.1. General Methodology

We measure intermediates prices using the procedure described in Section 5. Gross output prices by sector are implicitly an average of the price of consumption, investment, and intermediates produced by that sector. Thus, using the price of consumption and investment produced by goods or services, we identify the price of intermediates produced by goods or services as the residual in gross output prices. For example, we measure price growth in the goods-intermediates sector, g–m, as:

$$\Delta \ln(P_{g-m,t}) = \frac{1}{\zeta_{gt}^{M}} \left(\Delta \ln P_{gt}^{GO} - \zeta_{jt}^{C} \Delta \ln P_{g-c,jt} - \zeta_{jt}^{X} \Delta \ln P_{g-x,t}\right), \tag{C.2}$$

where ζ_{gt}^{i} represents the average share (between $t-1$ and t) of total gross output of the goods sector used for product i, $i \in (C, X, M)$ with $\zeta_{gt}^{M} + \zeta_{gt}^{C} + \zeta_{gt}^{X} = 1$, and $\Delta \ln P_{gt}^{GO}$ represents the log change of the gross output price for the total goods sector.

Appendix C.2.2. Additional Adjustments for Oil/Energy Price Fluctuations

Although we omit sectors closely tied to market fluctuations in oil/energy prices from our analysis, these fluctuations may have a nontrivial impact on gross output prices via intermediate inputs. That is, because oil/energy is an intermediate input for many sectors, the final price of those sectors' output may reflect fluctuations in these prices. To abstract from fluctuations in the price of oil/energy in analyzing long-run price trends, we adjust gross output prices for the impact of oil/energy prices operating through intermediate input prices.

Consider the following representation of the evolution of gross output and intermediates bundle prices for sector j:

$$\Delta \ln(P_{jt}) = \alpha_{jt}\Delta \ln(P_{jt}^{Y}) + (1-\alpha_{jt})\Delta \ln(P_{jt}^{M})$$
$$\Delta \ln(P_{jt}^{M}) = \sum_{i\notin E} s_{ijt}^{M}\Delta \ln(P_{it}) + \sum_{i\in E} s_{ijt}^{M}\Delta \ln(P_{it})$$

where P_{jt} is the gross output price of sector j, P_{jt}^{Y} is the (implicit) price of value added in sector j, P_{jt}^{M} is the price of the intermediates bundle purchased by sector j, and s_{ijt}^{M} represents elements of the input-output matrix for sector j at time t. The set E describes a set of sectors we wish to "exclude," yet still impact intermediate bundle prices and thus gross output prices in each sector.[37]

The following dynamics define the "adjusted" price series we seek to obtain:

$$\Delta \ln(\tilde{P}_{jt}) = \left(\alpha_{jt} + (1-\alpha_{jt})\sum_{i\notin E} s_{ijt}^{M}\right)\Delta \ln(P_{jt}^{Y}) + (1-\alpha_{jt})(1-\sum_{i\notin E} s_{ijt}^{M})\Delta \ln(\tilde{P}_{jt}^{M})$$
$$\Delta \ln(\tilde{P}_{jt}^{M}) = \sum_{i\notin E} \frac{s_{ijt}^{M}}{1-\sum_{k\in E} s_{kjt}^{M}}\Delta \ln(\tilde{P}_{it})$$

These two linear systems can be solved using matrix algebra, implying that the adjusted gross output price series can be written as a function of the original gross output prices, expenditure shares and the prices of the excluded sectors:

$$\Delta\overrightarrow{ln(\tilde{P}_t)} \quad = \Phi\left(\Delta\overrightarrow{ln(P_t)} - (I - diag(1-\alpha_t)\Gamma_t')^{-1}diag(1-\alpha_t)\left(\overrightarrow{s_t^{M,E}}\right)'\Delta\overrightarrow{P_t^{E}}\right)$$

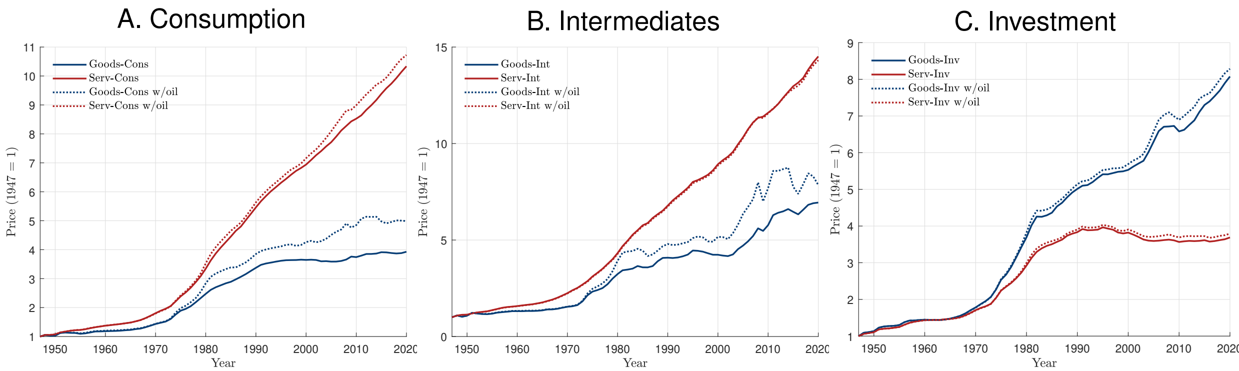
where

$$\Phi = (I - diag(1-\alpha_t)\Gamma_t')^{-1}diag\left(1 + \frac{(1-\alpha_t)\sum_{i\notin E} s_{it}^{M}}{\alpha_t}\right)(I - diag(1-\alpha_t)\Gamma_t'),$$

$\overrightarrow{ln(\tilde{P}_t)}$ is a vector of adjusted gross output prices, $\overrightarrow{ln(P_t)}$ is a vector of observed gross output prices, $diag(1-$

[37]Technically, the above representation only approximates how price measurement is done at the BEA for two reasons. First, the BEA uses Fisher indices instead of Tornqvist indices to compute price growth over time. However, the practical differences between these methodologies are negligible; the Tornqvist index is easier to analyze. Second, the above representation abstracts from imported intermediates, considering the gross output price of a sector j and an intermediate input made by that sector to be the same. This approximation is necessary to make the adjustments we propose. However, at least in the case of gasoline and oil, global movements and domestic movements in prices are very similar, suggesting minimal bias from ignoring the potential for differential oil prices from overseas.

Figure C.1: Sector Prices With and Without Oil Sectors Included



Notes: Solid lines denote baseline prices (as originally observed in Figure 4); dotted lines denote prices measured with oil/energy sectors included.

$\alpha_t)$ is a diagonal matrix with $1 - \alpha_{jt}$ along the diagonal, Γ_t is the input-output network with ij-th element s^M_{ijt}, $\overrightarrow{s_t^{M,E}}$ is a vector of the input-output network elements corresponding to excluded sectors, and $\overrightarrow{P_t^E}$ is a vector of prices of sectors to be excluded.

When constructing gross output prices at the sector level, from which intermediates prices will be inferred, we aggregate up these adjusted gross output prices across sectors. We then apply corrections to the consumption and investment prices (as described in the previous subsection) using the difference between adjusted and unadjusted gross output prices. The final intermediates prices for goods and services are then constructed as residuals of the difference in adjusted gross output prices and adjusted consumption and investment prices.

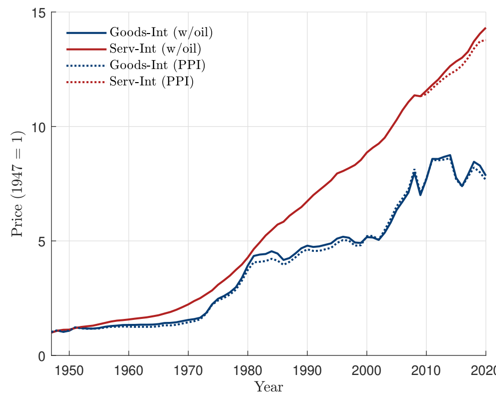
Figure C.1 plots the price of consumption, investment, and intermediates produced by goods and services sectors when oil/energy sectors are excluded and included. The impact of including oil sectors is greatest on goods sectors – the price of goods consumption rises more with the inclusion of energy-intensive commodities, and the price of goods intermediates is much more volatile. However, including oil sectors does not change the relative price patterns observed in the data and has a negligible impact on investment prices. The primary impact of including oil prices in our data would be to shrink the magnitude of the increase in the price of services-intermediates relative to goods-intermediates, which would worsen the calibration fit to structural change in intermediates.

Appendix C.2.3. Comparison to PPI Data

Although data from the PPI is ill-suited to our purposes, we can still compare our measured intermediates prices for the periods intermediate input prices are available from the PPI to validate our procedure.

The PPI publishes the prices of four broad types of intermediate inputs: processed goods, unprocessed goods, services, and construction. To construct a goods intermediate price from these data, we aggregate processed goods and unprocessed goods. We abstract from construction intermediates prices because they comprise only about 1% of all intermediates, and data on construction intermediates are only available

Figure C.2: Comparison of Measured Intermediates Prices to PPI Data



Notes: Solid lines denote measured prices including oil/energy sectors; dotted lines denote prices obtained from the Producer Price Index (PPI). PPI data on services intermediates prices are only available beginning in 2009.

beginning in 2009.[38] As PPI data is available monthly, we average monthly prices to the annual frequency to compare with our annual data.

Figure C.2 plots the goods and services intermediates price we obtain from our measurement procedure with the intermediates prices constructed from PPI data. We use our price series without adjustments for oil/energy prices (including all oil/energy sectors) because applying our adjustment procedures to the PPI data is not feasible. For services intermediates prices from the PPI, we normalize prices in the first year they are available (2009) to match our services intermediates price series (for comparison). For both series, our inferred intermediates prices align closely with those published in the PPI.

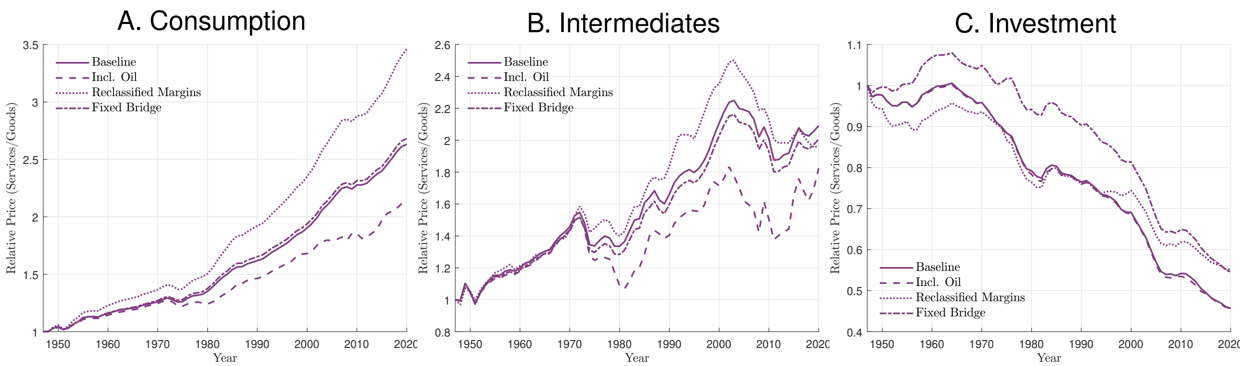
Appendix C.3. Robustness of Relative Price Measurement

This section considers the robustness of our relative price measures to various modifications to our measurement procedures. We first consider modifications that affect all three relative price series (consumption, investment, and intermediates). We then consider modifications specific to the measurement of investment prices.

First, we consider the following three robustness exercises: 1) not omitting any sectors producing oil/energy products or making any adjustment for oil/energy price fluctuations, 2) reclassifying margin contributions (i.e., wholesale trade, retail trade, and transportation/warehousing margins) to production as goods sectors, and 3) setting bridge files to their average value throughout the entire sample (i.e., no time variation in bridge files). The relative prices of services to goods in consumption, investment, and intermediates in these three cases are compared to our baseline relative prices in Figure C.3.

[38]The BLS chooses not to publish aggregate weights that would facilitate aggregation of these price series to a single price series for goods (or all intermediates in general). However, in correspondence with economists at the BLS, we found out that the approximate weight on processed goods is 80%, and the approximate weight on unprocessed goods is 20%. We use these approximate weights to aggregate the two price series.

Figure C.3: Relative Price (Services/Goods) Robustness for Consumption, Investment and Intermediates



Notes: Solid lines denote baseline relative prices (as originally observed in the right panel Figure 4); dashed lines are prices inclusive of all oil/energy sectors (excluded in the baseline), dotted lines denote prices where margin sectors have been reclassified as goods sectors, and dash-dotted lines indicate prices measured when the bridge file mapping production of each commodity into production sectors is held fixed at its average value across the years 1947-2020.

In each case, the alternative relative price series are qualitatively similar to our baseline measures.[39]

Second, we consider the following five robustness checks more focused on the relative price of investment: 1) using user cost weights to aggregate investment prices (i.e., rental services measures, constructed originally in vom Lehn and Winberry (2022) and extended through the year 2000), 2) applying price quality adjustments to investment prices based on an extension to Cummins and Violante (2002), 3) allowing for structures to be partially produced by services sectors (as discussed in Appendix C.1), and 4) focusing exclusively on investment prices for equipment and software (where there is the most overlap of investment being produced by both goods and services sectors).[40] We plot the relative price of services investment to goods investment under each of these four cases and our baseline results in Figure C.4. Under each modification, the relative price of services investment falls relative to the price of goods investment.[41]

Appendix C.4. Comparison to Gross Output Price Measurement

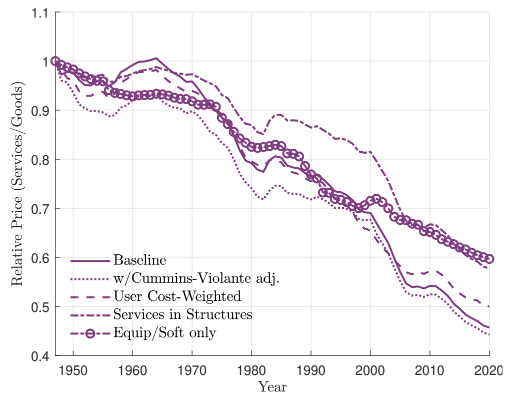
As discussed in Section 5, an alternative way to measure consumption, investment, and intermediates prices is to use gross output prices exclusively. The simplest approach to measuring the price of products produced by goods and services is to construct a single price for goods and a single price for services and assume this price applies equally to consumption, investment, and intermediates products. Such price series are constructed by separately aggregating up sectoral prices for goods and services sectors. This parsimonious approach is used by Herrendorf et al. (2021), García-Santana et al. (2021), and Sposi et al.

[39] The slightly different dynamics in the relative price of investment when holding bridge files fixed are largely attributable to changes in the importance of various margin sectors over time. If we both hold bridge files fixed and reclassify margins sectors, there is little discernible impact on the relative price of investment from holding the bridge file fixed over time.

[40] To extend the equipment investment quality adjustments presented in Cummins and Violante (2002), we take the percent difference between the quality-adjusted prices of each commodity and the actual prices from 1995-2000 and apply these to all years since 2000.

[41] Even if we focus only on equipment investment produced by goods and services, the relative price of investment produced by services still falls, including when reclassifying margin sectors as goods sectors.

Figure C.4: Robustness of Investment Price Measurement



Notes: The solid line denotes the baseline relative price of investment (services/goods); the dotted line is the relative price with quality adjustments to equipment prices as presented in Cummins and Violante (2002), the dashed line is the relative price where prices are aggregated with user cost of capital weights, the dash-dotted line is the relative price when services are allowed to contribute to the production of structures, and the line with circle markers is the relative price when only consider equipment and software investment prices.

(2021).[42]

With a single price for goods sectors and a single price for services sectors, a long literature has documented that the relative price of services is rising over time. This is unsurprising given our findings in Figure 4. Services prices are rising significantly faster than goods prices in intermediates and consumption, and these two uses make up the majority of gross output. Thus, aggregating the prices of consumption, investment, and intermediates into a single price masks heterogeneity in the price trends across different final uses.

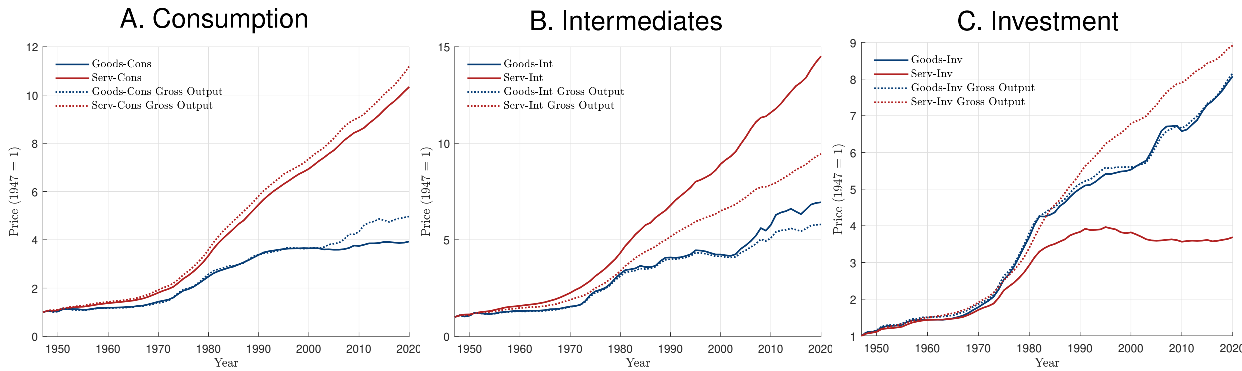
It is also possible to obtain consumption-, investment- and intermediates-specific prices for goods and services just using gross output data. Using input-output data, we can observe how each subsector within goods and services contributes to producing consumption, investment, and intermediates. We can then aggregate up sectoral gross output prices in proportion to how much each sector produces of consumption, investment, and intermediates. Formally, the price growth in use $u \in C, X, M$ produced by sector $j \in g, s$ can be measured from gross output data as:

$$\Delta \ln(P_{jt}^{u}) = \sum_{i \in j} s_{it}^{u,j} \Delta \ln(P_{it}) \tag{C.3}$$

where P_{jt}^{u} is price of use u produced by sector j at time t, i denotes the individual subsectors within j, $s_{it}^{u,j}$

[42]Herrendorf et al. (2021) measure the prices of goods and services in a "value-added" sense, embedding the input-output structure of the economy in how services and goods prices are aggregated. However, the overall trends in relative prices are qualitatively similar when not making this adjustment.

Figure C.5: Constructing Sector Prices Using Gross Output Data



Notes: Solid lines denote baseline prices; dotted lines denote prices constructed using gross output data according to equation (C.3).

is the share of all of sector j's production of use u that is produced by subsector i (averaged between time periods t and $t-1$), and P_{it} is the gross output price of sector i.

With this approach, variation in goods or services prices across consumption, investment, and intermediates is driven by heterogeneity in which subsectors produce each final product and heterogeneity in price growth in those subsectors. In principle, this gross output procedure could exactly replicate the prices we construct using expenditure side data if each detailed subsector only produces one of consumption, investment, and intermediates or if each of those products has the same final price. The equivalence of this gross output measurement approach and our expenditure-side measurement approach is ultimately a question of how disaggregated the source data is.

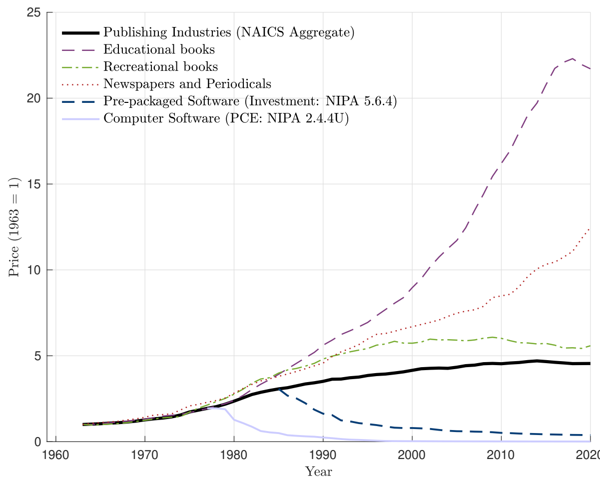
Figure C.5 plots the consumption, investment, and intermediates prices for goods and services sectors constructed using gross output data according to equation (C.3).[43] These gross output prices are noticeably different than those we construct using expenditure-side accounting data, particularly for investment. Although price growth in services-investment is clearly lower than price growth in other services sectors (and thus services as a whole), the services-investment price is rising over time relative to the goods-investment price, the opposite of our baseline findings.

However, measuring the price of services-investment using gross output prices is likely subject to significant aggregation bias, even when drawing on the disaggregated 40 sectors in Table 1. For the two services sectors that produce the most services investment (information and professional/technical services) only a small portion of gross output is used as investment (on average only 15% and 22%, respectively), suggesting the potential for significant aggregation bias. Moreover, as we show in Section 5, there is substantial price heterogeneity in the commodities produced by the professional/technical services sector, implying that using a single gross output price for this sector will overstate price growth in services-investment.

Since 1963, the BEA publishes gross output prices at a more disaggregated level, making it possible to analyze 62 non-oil/gas sectors, including greater detail within both the professional/technical services

[43] We use the oil/energy price adjusted gross output prices for consistency with how we measure our baseline prices.

Figure C.6: Heterogeneous Prices Within Publishing Industries



Notes: The thick line denotes the gross output price for the NAICS sector "publishing industries", obtained from the BEA GDP by Industry database. The thin lines are the prices of commodities produced by the publishing sector, based on NIPA Tables 2.4.4U (Personal Consumption Expenditures) and 5.6.4 (Investment).

and information sectors, which are the leading producers of services-investment.[44] However, this level of disaggregation does not resolve the core issues of aggregation bias. At this level of disaggregation, 7 of the 39 services sectors produce roughly 80% of all services investment.[45] Those seven sectors, listed in order of importance for producing services-investment (averaged over 1963-2020) are miscellaneous professional/technical services, wholesale trade, computer systems design and related services, broadcasting and telecommunications, publishing industries, retail trade, and motion picture and sound recording industries. However, despite this added detail in sectors, investment remains a small fraction of the uses of these sectors' gross output. For those 7 sectors, respectively, the share of output used for investment (averaged over 1963-2020) is 24%, 10%, 49%, 10%, 23%, 3%, and 29%. Thus, the added disaggregation in more recent data does not generate sectors that specialize in investment and reduce the possibility of aggregation bias.[46]

As an added example of price heterogeneity within this 62-sector level of disaggregation, consider the price of the publishing industries sector (within information). This sector produces investment in the form of pre-packaged software, but also produces many consumption products: consumer software, recreational books, educational books, and newspapers and periodicals. In Figure C.6, we plot the evolution of prices for

[44]The next leading producer of services-investment, wholesale trade, cannot be disaggregated further using these data.

[45]We compute this omitting the real estate margin contributions to investment, most of which are omitted in the construction of the investment network in vom Lehn and Winberry (2022). Much of the remaining 20% of investment represents R&D expenditures produced within sectors, and in all of these sectors, the share of gross output used for investment is less than 5% on average, suggesting that gross output prices are unlikely to accurately represent the price of investment made by these sectors.

[46]We do find, in results available upon request, that using more disaggregated sectors to measure the price of services-investment from gross output data generates a slower growing price series than what is plotted in Figure C.5, and is thus closer to our prices measured using expenditure side data. However, the evidence presented here explains why these more disaggregated data do not yet generate the exact same services-investment price as in our baseline using expenditure side data.

this subsector and the commodities it produces and see substantial price heterogeneity within this sector.[47] Although the gross output price of publishing industries is not growing very much over time, the price of pre-packaged software is substantially *falling* over time. However, because the price of newspapers and varied books are rising substantially over time, aggregation to a single price for this sector fails to capture the price of investment it produces.

The above exercises, coupled with those presented in Section 5 further illustrate the limitations of using gross output prices for generating prices by both production sector and product use. Put differently, both income-side accounting and expenditure-side accounting measure prices using the same underlying detailed data, but they aggregate these differently – income-side accounting aggregates prices by sectors and expenditure-side accounting aggregates prices by use. The nature of the added detail available from the bridge/make files enables us to cleanly disaggregate consumption and investment prices by producing sector. However, the added detail from more detailed production sectors in income side data is insufficient to properly identify prices by use. Thus, using expenditure-side data provides a superior approach to measuring prices by both production sector and product use.

Appendix C.5. Calibration Details

For our baseline calibration, we have the following parameters to calibrate: α, the Cobb-Douglas exponent on the intermediates bundle in each sector; θ, the Cobb-Douglas exponent on capital in each sector; δ, the depreciation rate; and the CES share and elasticity parameters in aggregating consumption, investment, and intermediates ($\omega_{Ci}, \omega_{Xi}, \omega_{Mij}, \epsilon^C, \epsilon^X$, and ϵ^M_j for $i \in g, s$ and $j \in g-c, g-x, g-m, s-c, s-x, s-m$). Even though our exercises primarily focus on balanced growth settings with limited sectoral heterogeneity, it is useful to understand the degree of underlying heterogeneity that we are abstracting from. As a result, we describe how we calibrate each of these parameters for each sector j in our data; for our baseline calibration, the parameter values we use are the unweighted averages of these calibrated parameters across sectors.

To calibrate model parameters for α_j, θ_j, and δ_j at the six-sector level, we first obtain implicit values for each model parameter at the 40-sector level (as described in Table 1) and then aggregate those values up to the six sectors we focus on in our calibration. To obtain implicit values for α_j, θ_j, and δ_j at the 40 sector level, we follow the procedures of vom Lehn and Winberry (2022). We calibrate α_j and θ_j using expenditure data from 1947-2020 at the 40 sector level from the BEA Input-Output Database and historical extensions to that database. We calibrate α_j using the ratio of nominal value added to nominal gross output in each sector; we calibrate θ_j using one minus the ratio of nominal labor compensation in each sector to nominal value added (minus taxes and adjusted for self-employment, as in vom Lehn and Winberry (2022)). We also follow the procedures of vom Lehn and Winberry (2022) in calibrating δ_j for each sector, using the implied depreciation rates for each NAICS sector published in the BEA Fixed Assets database.

Given implicit parameter values at the 40 sector level, we then aggregate up to the six sectors we consider in our calibration. We construct aggregate parameter values for each of our six sectors as a weighted average of the parameter values across the 40 observed NAICS sectors, where each sector is weighted according to

[47] The prices of consumer computer software and pre-packaged software are only available since 1977 and 1985, respectively.

the amount of each final product – consumption, investment, or intermediates – it produces among goods or services sectors. For example, to aggregate up parameter values for the goods-consumption sector, we weight parameter values for each goods subsector k with the weight $\frac{P_{kt}C_{kt}}{\sum_{l \in \text{Goods}} P_{lt}C_{lt}}$, where $P_{kt}C_{kt}$ is the total nominal production of consumption by goods subsector k. After we aggregate parameter values for each sector in each year, we take the average of these sectoral parameter values over each year from 1947-2020 and use that as the calibrated value of each parameter.

Table C.1 reports the calibrated parameter values for α_j, θ_j, and δ_j for each of the six sectors in our model; the calibrated values reported in Section 5 are simple averages of the value of each parameter across all six sectors.

Table C.1: Six-Sector Parameter Calibration

	θ	α	δ
Aggregate	0.34	0.55	0.08
A. Goods Sectors			
Consumption	0.36	0.36	0.09
Intermediates	0.21	0.46	0.12
Investment	0.35	0.42	0.09
B. Services Sectors			
Consumption	0.37	0.69	0.05
Intermediates	0.40	0.68	0.09
Investment	0.37	0.66	0.07

Notes: This table reports the calibrated parameter values for the Cobb-Douglas exponents in production (θ for capital within value-added, α for the value-added portion of gross output) and the depreciation rate of capital in each of the six sectors we study and the average of those used in the baseline calibration.

The remaining parameters to be calibrated involve the CES aggregators for consumption, investment, and intermediates, with share parameters (ω) and elasticity parameters (ϵ) for each aggregator. Since we assume these share parameters sum to one (i.e. for consumption, $\omega_{Cg} + \omega_{Cs} = 1$), we then only need to calibrate the share parameter corresponding to the goods sector. We calibrate them to match the initial fraction of expenditures on consumption, intermediates, or investment purchased from the goods sector in the year 1947.[48] In the case of intermediates, there are six share parameters to calibrate, ω_{Mgjt} for $j \in \{g-c, g-x, g-m, s-c, s-x, s-m\}$; the share of intermediates produced by the goods sector in each of our six sectors is computing by aggregating up expenditure shares in each of our 40 sectors proportionally to each sector's role in producing intermediates among either goods or services sectors (the same as how we aggregate up parameter values across 40 sectors for α_j, θ_j, and δ_j).

Finally, given values for the share parameters, we identify elasticity parameters by estimating equations the following three equations via non-linear least squares using annual data moments on expenditure shares and annual data on the price of consumption, investment, and intermediates produced by goods and services

[48] With prices normalized to one in the year 1947, the value of these share parameters is exactly the share of consumption, investment, or intermediates produced by the goods sector in 1947.

sectors:

$$
\begin{aligned}
s_{gjt}^{M} &= \frac{\omega_{Mgj} P_{g-m,t}^{1-\epsilon_{Mj}}}{\omega_{Mgj} P_{g-m,t}^{1-\epsilon_{Mj}} + (1-\omega_{Mgj}) P_{s-m,t}^{1-\epsilon_j^M}} \\
s_{gt}^{X} &= \frac{\omega_{Xgj} P_{g-m,t}^{1-\epsilon^X}}{\omega_{Xg} P_{g-x,t}^{1-\epsilon_X} + (1-\omega_{Xg}) P_{s-x,t}^{1-\epsilon_X}} \\
s_{gt}^{C} &= \frac{\omega_{Cg} P_{g-c,t}^{1-\epsilon^C}}{\omega_{Cg} P_{g-c,t}^{1-\epsilon_C} + (1-\omega_{Cg}) P_{s-c,t}^{1-\epsilon_C}}
\end{aligned}
$$

All final calibrated parameter values can be seen in Table 2. Table C.1 reports the calibrated parameter values for α_j, θ_j, and δ_j for each of the six sectors in our model.

To construct aggregate structural change in intermediates from the model, we aggregate up structural change in intermediates occurring in each of the six sectors (as in equation (12)). In our model, because the production technologies are identical across sectors, each sector's total share of intermediates expenditures, $\frac{P_{jt}^M M_{jt}}{\sum_i P_{it}^M M_{it}}$, is simply equal to that sector's share of total gross output, $\frac{P_{jt} Q_{jt}}{\sum_i P_{it} Q_{it}}$. Using the resource constraint, we can write the nominal gross output of any sector j as:

$$
P_{jt} Q_{jt} = \left(\xi_{jt} + \Lambda_{jt} \frac{X_t}{P_t^C C_t} \right) P_t^C C_t + \sum_{i=1}^{N} \Gamma_{jit} P_{it} Q_{it}
$$

where $\xi_{jt} = \frac{P_{jt} C_{jt}}{P_t^C C_t}$ equals the fraction of all consumption expenditures produced by sector j, $\Lambda_{jt} = \frac{\sum_{i=1}^{N} P_{jt} X_{jit}}{X_t}$ equals the fraction of all investment expenditures produced by sector j, and $\Gamma_{jit} = (1 - \alpha) \frac{P_{jt} M_{jit}}{P_{it}^M M_{it}}$ is the fraction of all investment expenditures by sector i produced by sector j (scaled by $1 - \alpha$).

In matrix notation, we can write this as:

$$
\overrightarrow{PQ} = (I - \Gamma)^{-1} \left(\overrightarrow{\xi} + \overrightarrow{\Lambda} \frac{X_t}{P_t^C C_t} \right) P_t^C C_t.
$$

Given that $P_t^C C_t$ is common to all sectors, all we need to obtain the gross output shares for each sector is Γ, $\overrightarrow{\xi}$, $\overrightarrow{\Lambda}$, and $\frac{X_t}{P_t^C C_t}$. With the exception of $\frac{X_t}{P_t^C C_t}$, the other objects are all outputs of the calibration procedure, determined by prices and CES parameters. The ratio of investment spending to consumption spending, $\frac{X_t}{P_t^C C_t}$, will be constant along the balanced growth path (given that investment and consumption expenditures grow at the same rate, as established in Proposition 1). Drawing from the details of the proof to Proposition 1 in Appendix B, we can write the equilibrium investment to consumption expenditures ratio as:

$$
\frac{X_t}{P_t^C C_t} = \frac{\gamma^K - (1 - \delta)}{\left(\frac{1}{\theta \beta} - 1 \right) \gamma^K - \left(\frac{1}{\theta} - 1 \right) (1 - \delta)}.
$$

Given values of δ, θ, β, and γ^K we can compute this ratio and thus compute the gross output shares for each sector along the balanced growth path. Note that γ^K is not a parameter we can set in advance, but a realized equilibrium object based on the technology processes fed into the model. Since these gross output shares are not needed to compute the balanced growth path growth rate, we compute these ex-post using the observed average growth rate of the capital stock along the balanced growth path implied by the technology series.

Because, in the model, gross output shares are identical to intermediate expenditure shares, we aggregate the comparison data presented in Figure 6 using gross output shares instead of intermediates expenditure shares. This generates a slight difference compared to the observed patterns of structural change in intermediates shown in Figure 3.

Appendix C.6. Additional Calibration Results

Our baseline calibration involves six sectors, each with a unique distribution of intermediates spending across goods and services intermediates. In Section 5, we reported the calibration fit to the model for aggregate intermediates expenditures. Here, we first report the model's fit to intermediate expenditure patterns in all six sectors and how the calibration compares to the shift-share decomposition exercises presented in Section 2.

In Figure C.7, we present the fit of the calibration to the share of intermediates purchased from goods and services sectors in each of our six sectors. In general, the model fit to structural change in intermediates is better in our three goods sectors before 2009; the model series account for 90% of the structural change in intermediates for goods sectors over through 2009, compared to 69% for services sectors. However, the decline in fit quality post-2009 appears to be primarily concentrated among goods sectors. The overall fit to structural change patterns through 2020 is similar across sectors, with the model explaining 2/3 of intermediates structural change in both goods and services over the entire sample.

In Table C.2, we compare a shift-share decomposition of our model's results for structural change in investment to a comparable shift-share decomposition on the data. Although the model does not replicate the entire rise in the share of intermediates produced by services, it does closely match the composition of changes over time due to within and between sector forces. As noted above in Appendix C.5, when comparing the model results to the data, we aggregate intermediate expenditures in each sector slightly differently than in the empirical results presented in Appendix C. Because there is no heterogeneity in the share of total intermediate spending in gross output across sectors, we use gross output shares to aggregate intermediate expenditure shares in the data; this accounts for the fact that the between sector contribution falls from 47% in Table A.1 to 41% in Table C.2.

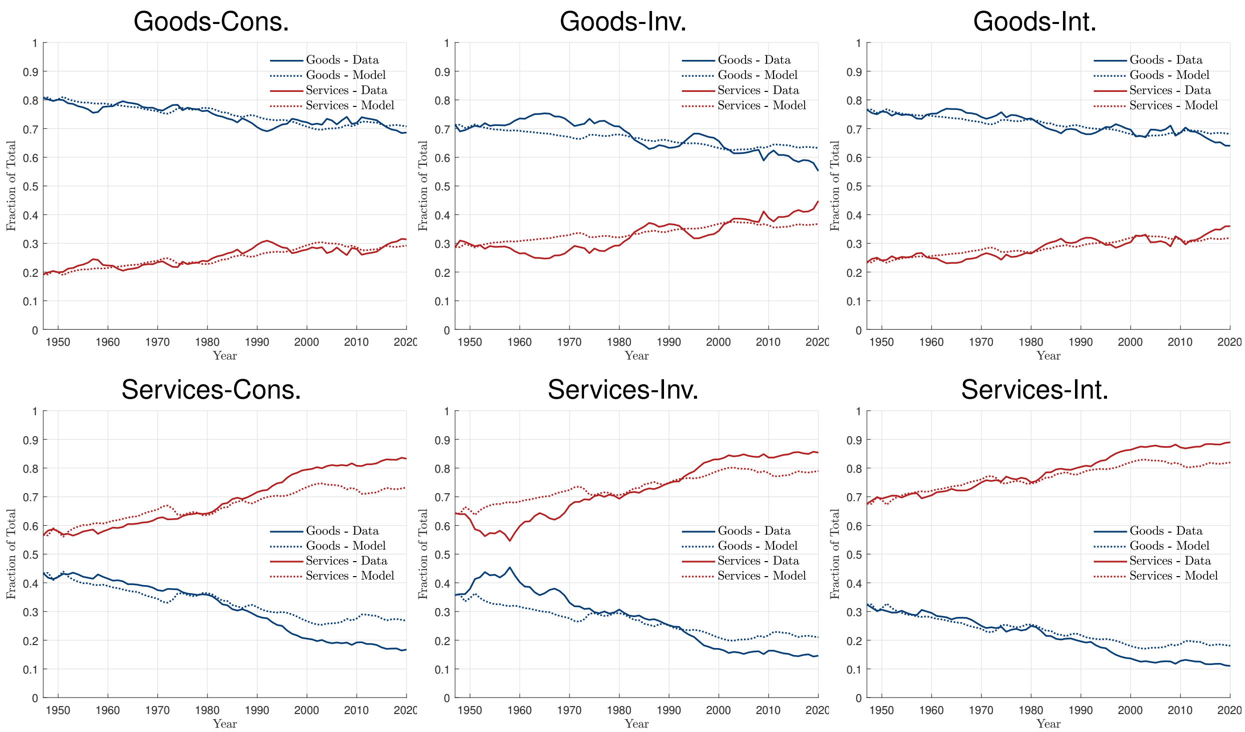
Appendix D. Details/Extensions to Accounting Exercises

Appendix D.1. Measuring Technological Change

Here we provide additional details for how we measure the exogenous TFP processes that we feed into our model in Section 6.

To measure growth in intermediates bundling TFP, we use BEA GDP by Industry data on the price of intermediate bundles by sector, aggregated up to the 6 sector level. To aggregate up intermediate prices bundles observed at the 40 sector level to the 6 sector level, we construct a weighted average of intermediate bundle price growth across goods or services sectors weighted by both that sector's total intermediate expenditures and role as a producer of consumption, investment or intermediates. For example, growth in

Figure C.7: Model Calibration Fit to Structural Change Patterns in Intermediates, 6 Sector Level, 1947-2020



Notes: Each panel plots the fraction of intermediates purchased from goods (blue lines) and services (red lines) Data series are solid lines; model series are dashed lines.

Table C.2: Shift-Share Decomposition of Services Share of Production of Intermediates, Model vs. Data

				Decomposition	
	1947	2019	Δ	within	between
Data	0.41	0.74	0.33	0.20	0.13
				(61%)	(39%)
Model	0.42	0.64	0.21	0.13	0.09
				(60%)	(40%)

Notes: The table reports the results of a shift-share decomposition of the share of services production of intermediates over time. See notes to Table A.1 for details. However, data figures are different from those in Table A.1 to be consistent with aggregation in the model; see the discussion in Appendix C.5.

the intermediate price bundle for services-consumption sector, $\Delta ln(P^M_{s-c,t})$ is aggregated up according to the equation:

$$\Delta ln(P^M_{s-c,t}) = \sum_{i \in \{Services\}} \frac{\frac{P_{it}C_{it}}{\sum_{k \in \{Services\}} P_{kt}C_{kt}} P^M_{it} M_{it}}{\sum_{l \in \{Services\}} \frac{P_{lt}C_{lt}}{\sum_{k \in \{Services\}} P_{kt}C_{kt}} P^M_{lt} M_{lt}} \Delta ln(P^M_{it})$$

where $P_{it}C_{it}$ is the value of consumption produced by sector i, $P^M_{lt}M_{lt}$ is the value of intermediates purchased by sector i, and P^M_{it} is the price of the intermediates bundle purchased by sector i. Similar to how we measure prices for consumption, intermediates, and investment, we adjust the intermediates price bundle for each to remove the impact of oil/energy price fluctuations. We do this by subtracting off the price of omitted sectors' output in proportion to each sector's use of omitted sectors as intermediates and by subtracting off a correction term for all other sectors' gross output prices (in proportion to each sector's use as an intermediate).

We measure sectoral TFP so that it is consistent with relative prices in the model, similar to García-Santana et al. (2021). However, since relative prices can only define technological change for all but one of our sectors, we infer TFP for the final sector by ensuring that the Domar-weighted sum of sectoral TFP growth in the model matches that same moment in the data, based on our 40 sector NAICS data. To construct the Solow residual for each of our 40 sectors we follow the approach of vom Lehn and Winberry (2022) and compute real gross output net of the primary inputs in log differences.

Formally, we construct sectoral TFP growth at the 40 sector level as:

$$\Delta\ln(A_{jt}) = \Delta\ln(Q_{jt}) - \alpha_j\theta_j\Delta\ln(K_{jt}) - \alpha_j(1-\theta_{jt})\ln L_{jt} - (1-\alpha_j)\ln M_{jt}. \tag{D.1}$$

Measures of real gross output, real intermediates, and employment are taken from the BEA GDP by Industry database and the real capital stock is constructed using the perpetual inventory method using sectoral real investment, implied depreciation rates, and initial values of the capital stock from the BEA Fixed Asset Accounts. Values of the production parameters are assigned using evidence on cost shares observed in BEA Input Output data—α_j using the ratio of nominal value added to nominal gross output in each sector and $1-\theta_j$ using the sectoral share of compensation in value added. To more precisely isolate changes in TFP from changes in the production technology or depreciation rates, we allow α_j, θ_j, and δ_j to vary over time in measurement. For the growth rate of TFP in any two years, we use the average of cost shares or implied depreciation rates for those two years. We normalize the level of all TFP terms to be 1 in 1947.

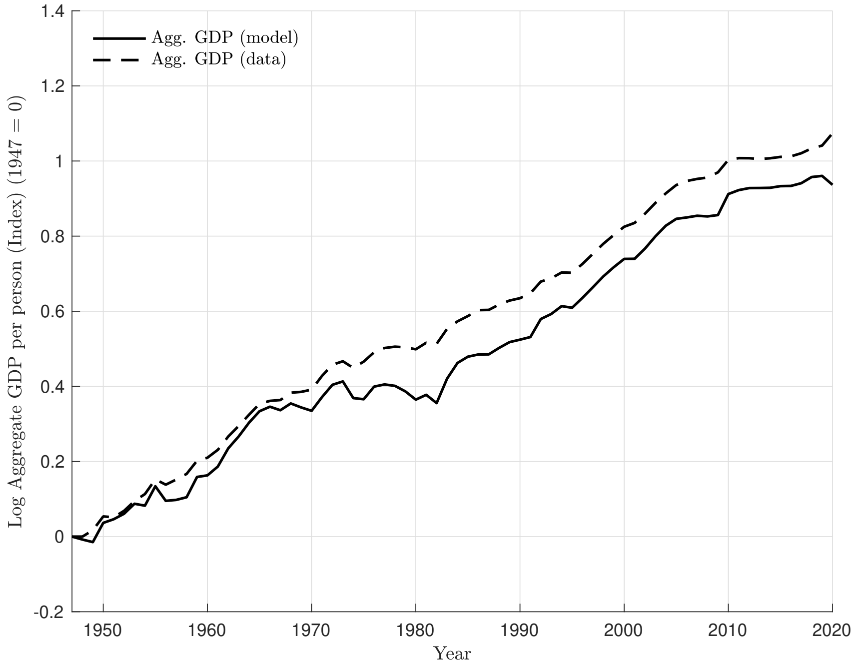
We then construct the Domar-weighted sum of sectoral TFP growth as $\sum_j \overline{\frac{P_{jt}Q_{jt}}{\sum_i P^V_{it}V_{it}}}\Delta ln\left(A_{jt}\right)$, where $\overline{\frac{P_{jt}Q_{jt}}{\sum_i P^V_{it}V_{it}}}$ is the Domar weight (nominal sectoral gross output divided by aggregate nominal GDP) averaged between time periods t and $t-1$.

Appendix D.2. Additional Growth Accounting Results

In this section, we present three additional results. First, we report our model constructed series for GDP measured as an index number (introduced in equation (31) in Section 6.3) opposite the data. Second, we report an extension of Table 3 and Table 5 where we report the composition of aggregate TFP (GDP) growth decade by decade, further highlighting the rising importance of investment-specific technical change over time. Third, we report an extension of Table 4 where we report counterfactuals with a unitary elasticities of substitution (i.e. Cobb-Douglas) in our GDP index number growth accounting framework.

Figure D.1 plots our model series for GDP measured as an index number opposite real GDP per worker in logs. Real GDP is constructed as the Tornqvist index of real value added from the 40 sectors we observe

Figure D.1: Aggregate GDP: Model Fit, 1947-2020



Notes: The figure compares model GDP with measured U.S. GDP per worker in logs (normalized to zero in 1947).

Table D.1: Aggregate GDP Growth, Cobb-Douglas Counterfactuals, 1947-2020

	Aggregate GDP Growth: $\Delta \ln(x) = \Delta \ln Y_t^{Index}$							
	1947-2019		1960-1980		1980-2000		2000-2019	
Sources of TFP growth	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%	$\Delta \ln(x)$	%
Baseline	0.96	100	0.20	100	0.37	100	0.22	100
Investment Cobb-Douglas	0.90	94	0.20	98	0.37	97	0.18	79
Intermediates Cobb-Douglas	1.03	107	0.21	106	0.43	114	0.22	101
Consumption Cobb-Douglas	1.02	106	0.21	103	0.39	104	0.26	116

Notes: The table shows long-run log changes in aggregate GDP measured as an index number, $\Delta \ln(Y_t^{Index})$, following equation (31), for different time periods. The table also reports counterfactuals for the cases where either investment, intermediates, or consumption aggregation is Cobb-Douglas, ruling out structural change in each network or the composition of consumption. For each time period, we show the long-run log change and the portion of aggregate growth accounted for by the counterfactual simulation in percent.

in the BEA data; total workers is defined as the total employment in those 40 sectors (as constructed in vom Lehn and Winberry (2022)). The model series for actual GDP closely matches the observed data.

Second, Panel A of Table D.2 shows decade by decade log changes in aggregate TFP, $\mathcal{A}_t$, and in the same sets of counterfactuals presented in Table 3—investment-specific technical change, intermediates-specific technical change, and only intermediates-bundling technical change. Panel B reports decade by decade log changes in the aggregate GDP Index, Y_t^{Index}, and in the same sets of counterfactuals as presented in Table 5—investment-specific, intermediates-specific, and consumption-specific technical change. From 1970-1980, log changes in GDP or its components are very low and/or negative, complicating interpretation. However, we observe a clear upward trend in the importance of investment-specific technical change over time, explaining as much as 90% of all growth between 2010-2019. In particular, notice that intermediates-specific technical change contributes negatively during the 2010s while investment-specific technical change overcompensates for the missing growth from the intermediates network.

Finally, Table D.1 shows the counterfactual log changes in GDP growth (measured as an index number) and its components where there is a unitary elasticity of substitution in different aggregation technologies – consumption, investment, and intermediates. In the counterfactuals featuring unitary elasticities of substitution, GDP growth over the entire sample 1947-2019 would be either 7% higher (when consumption or intermediates are Cobb-Douglas) or 6% lower (when investment is Cobb-Douglas). We also see that the importance of reallocation forces due to non-unitary elasticities is rising in recent decades—roughly 20% of aggregate GDP growth since 2000 is attributable to the reallocation of resources within investment, and growth would have been 16% higher over this period had there been no reallocation of resources within consumption.

Table D.2: Growth Decomposition by Decade, 1950-2020

	1950s		1960s		1970s		1980s		1990s		2000s		2010s	
Sources	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%	Δ	%
A. Growth Decomposition of Aggregate TFP along the ABGP: $\Delta = \frac{1}{1-\theta}\Delta \ln \mathcal{A}_t$														
All	0.14	100	0.23	100	-0.00	100	0.27	100	0.34	100	0.29	100	0.10	100
Investment-Specific	0.10	74	0.14	59	-0.00	85	0.18	67	0.22	66	0.23	77	0.09	92
Intermediates-Specific	0.04	26	0.10	41	-0.00	95	0.09	34	0.12	37	0.07	22	0.01	7
Intermed. Bundling	0.01	8	0.04	16	0.03	-702	-0.01	-3	0.04	13	0.09	32	0.04	39
B. Growth Decomposition of the GDP Index: $\Delta = \Delta \ln Y_t^{Index}$														
All	0.13	100	0.17	100	0.03	100	0.16	100	0.21	100	0.17	100	0.05	100
Investment-Specific	0.05	41	0.07	40	-0.00	-5	0.09	56	0.11	52	0.11	66	0.04	91
Intermediates-Specific	0.03	20	0.06	36	-0.02	-75	0.09	56	0.08	38	-0.00	-0	-0.02	-33
Consumption-Specific	0.05	39	0.04	24	0.05	176	-0.02	-12	0.03	12	0.06	35	0.02	40

Notes: The Table reports decade-by-decade versions of Tables 3 (panel A) and 5 (panel B).